Hindawi
Computational and Mathematical Methods
Volume 2022, Article ID 5089021, 14 pages
https://doi.org/10.1155/2022/5089021

Research Article

A Multidimensional Game Theory–Based Group Decision Model for Predictive Analytics

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Received 17 March 2022; Revised 13 July 2022; Accepted 18 August 2022; Published 16 September 2022

Academic Editor: Naeem Jan

An N-dimensional game theory–based model for multi-actor predictive analytics is presented in this article. The proposed model expands our previous work on two-dimensional group decision model for predictive analytics. The one-dimensional models are used for the problems where actors are interacting in a single issue space only. This is less than an ideal assumption since; in most cases, players' strategies may depend on the dynamics of multiple issues when dealing with other players. In this work, the one-dimensional model is expanded to N-dimensional model by considering different positions, and separate salience values, across different axes for the players. The model predicts an outcome for a given problem by taking into account stakeholder's positions in different dimensions and their conflicting perspectives. To illustrate the capability of the proposed model, three case studies have been presented.

1. Introduction

Originally developed to investigate complicated behavior in economics, game theory has found widespread use in politics, philosophy, military, sociology, and telecommunications due to its ability to explain complex dynamics among actors [1]. Game theory is used to model the interaction between agents in the process of a negotiation where each party considers its own interests. Policy experts typically use intuition to predict the future outcome of such problems; however, a mathematical framework is required to better forecast the outcome of a group decision problem of which the result is reproducible, explainable, and free of bias. Bruce Bueno de Mesquita (BDM) proposed one of the best models that used game theory to forecast a single event with several participants in 1980 [2]. He outlined the entire concept and provided illustrative case studies with outcomes in [3].

Authors in [4] studied the BDM model in depth and illustrated the reliability of its interpretations by reproducing the results using data provided in [3]. Another game theory–based approach called Preana incorporated the idea of reinforcement learning into its risk formulation [5]. It replicated the results reported by earlier studies and enabled agents to behave more logically at each round of the decision-making process. In addition, authors in [6] utilized the notion of game theory in bargaining problems to forecast the likely outcome of conflicts between Iran and the US over a variety of issues such as Iran's nuclear program.

The European Union Dataset is a collection of data based on expert interviews that covers a wide range of policy issues, as well as the position, salience, and capability of players. The first dataset (DEU I) was presented in [7] in which European Union legislative initiatives were considered issues to be investigated and provide explainability. The authors of the second dataset (DEU II) made a few minor changes to specific problems and added additional observations regarding the results of decisions that were still pending at the time the first dataset was presented [8]. The work reported in [9] supplemented the expert interviews with additional information, such as text analysis and media coverage, to assess factors such as salience during analysis of the EU legislative policies. Furthermore, [10] employed the EMU Positions dataset including positions and importance of the issues that were debated between 28 EU member states and institutions on economic and monetary reforms between 2010 and 2015, to extend the research in single case studies.

In general, these types of models, including the one presented here, indicate that using rational agents as actors, a negotiation or a group decision-making problem can be modeled. Each round, these actors change their positions in order to achieve the Nash equilibrium. Hence, having a reasonable and explainable forecast of the problem, stakeholders can dedicate more resources on their initial position, block other actors, make an alliance with critical actors, or take a more extreme position to achieve their goals.

All the models discussed above have only been able to analyze single event issues, which imply that the actors' attributes such as position and salience are only considered in relation to one particular issue. In this work, we propose expanding the one-dimensional game theory–based group decision model to an N-dimensional version. The mathematical formulation for the one-dimensional model is described in detail. Separate salience for each dimension is considered to provide the model more flexibility in distributing actor's capability across multiple axes. If bargaining problems were limited to a single issue or, more specifically, along one axis, the models discussed thus far could address the problem and forecast the outcome; however, negotiation problems are frequently multifaceted, necessitating that the parties involved to go through a wide range of topics while they bargain. In a recent work [11], we have proposed a two-dimensional model to address more complex problems involving multiple issues. In this paper, we have expanded this work to an N-dimensional space and modified some of our key formulations to include actors' salience as a weighting factor for better accuracy.

The rest of the paper will describe the proposed N-dimensional model in detail. Section 2 explains the structure of the one-dimensional model. The extended model's formulation is presented in Section 3. Section 4 provides the case studies and their analysis, and finally the last section concludes the paper and offers some suggestions for future research.

2. Background and Structure of the Model

This section aims to explain the structure of the BDM model. At first, the definitions of a few frequently used terms are provided. Next, the expected utilities and their formulation are presented. Following that, BDM model's voting procedure, offer categories, and offer selection will be addressed. For more details on the one-dimensional problem formulation, please see [4, 5].

2.1. Definition of Terms

(i) Problem: A problem can be defined by determining some actors and assigning their positions, capability, and salience, which are defined below. A problem may contain multiple issues

(ii) Issue: When multiple actors with mutual or conflicting interests are involved in a problem, one or more issues may arise. Each of these issues can be considered on its own axis in an N-dimensional problem

(iii) Actor: Actor, player, or stakeholder is an entity that utilizes all or part of its power to achieve a goal in regards to an issue of a given problem. Depending on the problem, players can be countries, organizations, individuals or even specific events

(iv) Position: An actor's stance or attitude toward an issue is referred to as position. In an N-dimensional problem, each actor takes a position on each dimension of the problem and promotes it as the problem's desired outcome from its own perspective

(v) Capability: Capability refers to the amount of power, wealth, or influence that an actor has and may use part or all of it in the direction of its goal

(vi) Salience: The importance of the problem to the actor is indicated by salience. Each actor is involved in multiple issues at the same time, and depending on the significance of each issue, it can exert some of its capability or power to address them

(vii) Utility: Utility is a function which measures the desirability of each position regarding the actor's supported position.

(viii) Risk: Risk is a mathematical parameter used to assess an actor's willingness to take risks. Players who hold positions with less support from other players are compromising security in order to achieve their objectives and could be considered ideological, ambitious, or simply risk-seeking. On the other hand, risk-averse players whose positions are closer to the current likely outcome, are more at ease, and do not have to be in conflict with other players to reach an agreement [6]

2.2. Parameter Normalization. Each actor in the model is assumed to have the following attributes: capability, salience, and position. Capability is a number between 0 and 1, indicating the degree of influence each actor can apply. The importance of the issue to each stakeholder is measured by salience, which ranges from 0 to 1, as well. The third attribute is the desired position which is taken by each actor along a single dimension. The collection of stakeholders' various points of view on each issue in the problem results in a spectrum of possible solutions; therefore, positions should be normalized in this space so that all the actors' attributes be in the range of $[0, 1]$. These parameters will serve as metric scales for each stakeholder when it comes to each dimension of the problem.

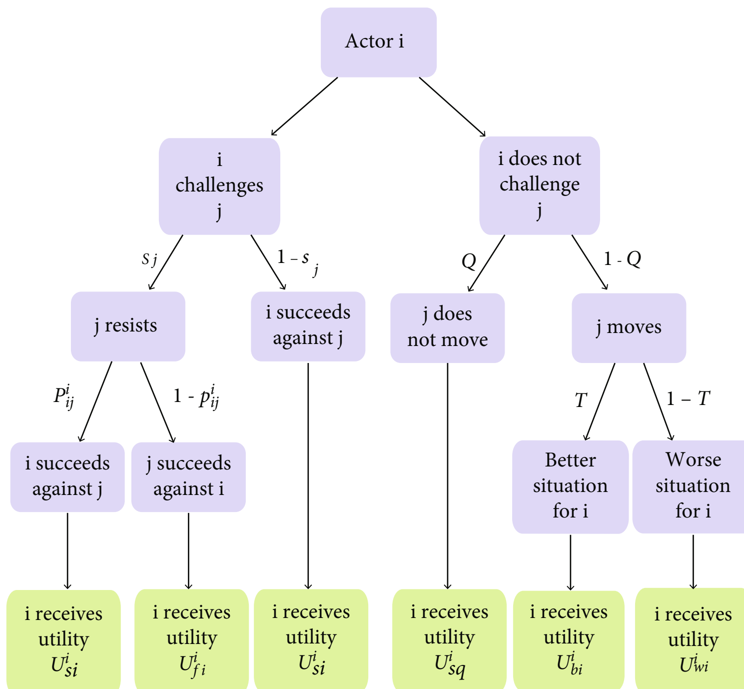

Figure 1: Game tree in expected utility model.

2.3. Expected Utility. The expected utility of actor i against actor j in this model, as outlined in [5], is the sum of the expected utilities when i is challenging j and when it is not challenging j. For better understanding, the structure of the model is represented as a tree in Figure 1. Two scenarios are included in the challenging case's expected utility. Actor i is challenged back by j in the first scenario, but j does not challenge actor i in the second scenario because the issue is not important enough to j, or it does not see a resealable chance to succeed on the issue. In the formulation used in (1) to (3), the probabilities of these scenarios are reflected using s_j and $(1-s_j)$. In the scenario of challenging, the utility gained in case of success and failure is notated by U^i_{si} and U^i_{fi}, respectively. On the other hand, the expected utility in the case of not challenging includes two different scenarios. In the first one, i will go through the status quo with probability of Q and gain its utility. In the second one, actor i will not go through the status quo with probability of $1-Q$; however, its situation would become better with T probability or worse with $(1-T)$ probability.

$$EU^i_{ij} = \left(EU^i_{ij}\right)_c - \left(EU^i_{ij}\right)_{nc}, \tag{1}$$

$$\left(EU^i_{ij}\right)_c = s_j\left(p^i_{ij}U^i_{si} + \left(1-p^i_{ij}\right)U^i_{fi}\right) + \left(1-s_j\right)U^i_{si}, \tag{2}$$

$$\left(EU^i_{ij}\right)_{nc} = QU^i_{sq} + (1-Q)\left(TU^i_{bi} + (1-T)U^i_{wi}\right). \tag{3}$$

The basic utility functions for one-dimensional problems, U^i_{si}, U^i_{fi}, U^i_{sq}, U^i_{bi}, and U^i_{wi}, are defined later in this section, and their multidimensional equivalents will be presented in the next section.

When actor i is considering an alternative position different from its current position, which is its desired outcome, the actor's utility function can be determined by a decreasing function of the distance between these two positions. Thus, the utility function from actor i's point of view is maximized when the alternative position equals to his current position and is minimized when they are towards the opposite end of the position spectrum.

$$u^i_{ij} = f\left(-\left|x_i - x_j\right|\right), \tag{4}$$

where x_i is actor i's position and f is any arbitrary descending function. The utility function u^i_{ij} demonstrates how much actor i attaches to his own policy portfolio. The specific function f that is used in our model is

$$f(\sigma) = 1 - 2(\sigma)^{r_i}, \tag{5}$$

where r_i is the risk parameter and will be defined later in this section. Substituting (5) into (4), the utility function would become

$$u^i_{ij} = 1 - 2\left|x_i - x_j\right|^{r_i}, \tag{6}$$

where $u^i_{ij} \in [-1, 1]$ and x_i and x_j are normalized so that x_i, $x_j \in [0, 1]$. According to [12], utilities for i's success and failure can be achieved by

$$U^i_{si} = 2 - 4\left[\frac{2 - \left(u^i_{ii} - u^i_{ij}\right)}{4}\right]^{r_i}, \tag{7}$$

$$U^i_{fi} = 2 - 4\left[\frac{2 - \left(u^i_{ij} - u^i_{ii}\right)}{4}\right]^{r_i}, \tag{8}$$

where $u^i_{ii} = 1$ since each actor is expected to cast the highest vote to his own policy. Substituting (6) into (7) and (8), we will have

$$U^i_{si} = 2 - 4\left[0.5 - 0.5\left|x_i - x_j\right|\right]^{r_i}, \tag{9}$$

$$U^i_{fi} = 2 - 4\left[0.5 + 0.5\left|x_i - x_j\right|\right]^{r_i}, \tag{10}$$

where $2 - 4(0.5)^{r_i} \le U^i_{si} \le 2$ and $-2 \le U^i_{fi} \le 2 - 4(0.5)^{r_i}$.

According to Figure 1, if actor i does not challenge j, j could either maintain its current position as status quo or move to make the situation better or worse for i. To meet the condition $U^i_{wi} \le U^i_{qi} \le U^i_{bi}$, the utilities U^i_{bi}, U^i_{wi}, and U^i_{sq} can be defined as follows:

$$U^i_{bi} = 2 - 4\left[\frac{4 - \left(u^i_{ii} - u^i_{ij}\right)_{t_n} - \left(u^i_{ii} - u^i_{ij}\right)_{t_0}}{8}\right]^{r_i}, \tag{11}$$

$$U^i_{wi} = 2 - 4\left[\frac{4 - \left(u^i_{ij} - u^i_{ii}\right)_{t_n} - \left(u^i_{ij} - u^i_{ii}\right)_{t_0}}{8}\right]^{r_i}, \tag{12}$$

where $2 - 4(0.5)^{r_i} \le U^i_{bi} \le 2$ and $-2 \le U^i_{wi} \le 2 - 4(0.5)^{r_i}$. The t_0 and t_n subscriptions refer to actor j's before and after position adjustment, respectively. If actor j is not challenged by actor i, j is expected to move to the median voter position. Therefore,

$$\left(u^i_{ij}\right)_{t_0} = 1 - 2\left|x_i - x_j\right|^{r_i}, \tag{13}$$

$$\left(u^i_{ij}\right)_{t_n} = 1 - 2\left|x_i - \mu\right|^{r_i}, \tag{14}$$

where μ is the median voter position which is the position with the most support and will be defined in Section 2.4. Substituting (13) and (14) into (11) and (12), we will have

$$U^i_{bi} = 2 - 4\left[0.5 - 0.25\left(\left|x_i - \mu\right| + \left|x_i - x_j\right|\right)\right]^{r_i}. \tag{15}$$

$$U^i_{wi} = 2 - 4\left[0.5 + 0.25\left(\left|x_i - \mu\right| + \left|x_i - x_j\right|\right)\right]^{r_i}. \tag{16}$$

where μ is the current median voter position. In the case that i does not challenge j and j does not move, the status quo utility is realized and is defined as

$$U^i_{sq} = 2 - 4\left[\frac{(4-0)}{8}\right]^{r_i} = 2 - 4(0.5)^{r_i}. \tag{17}$$

The parameter Q in (3) is reported to be set to 0.5 or 1 in different settings in various articles. For example, [12] considers Q to be 1, while [13] considers Q to be 0.5. $Q = 0.5$ denotes the most uncertain outcome of whether actor j will move or remain in place. In this work, we assumed $Q = 1$. The probability T indicated whether or not the situation gets better for i. If $dist(i, \mu) < dist(i, j)$, then it should be 1; Otherwise, T is supposed to be 0.

2.4. Median Voter Position. The amount of support of each actor's position has to be evaluated in order to determine the median voter position. Using a voting process, actor i votes between positions j and k. This vote measures actor i's preference for j over k and can be obtained as follows:

$$v^i_{jk} = c_i s_i\left(u^i_{ij} - u^i_{ik}\right), \tag{18}$$

where u^i_{ij} is the utility function of actor i for challenging actor j from i's point of view and c_i and s_i are actor i's capability and salience, respectively. Substituting (6) into (18), we will get

$$v^i_{jk} = 2c_i s_i\left(\left|x_i - x_k\right| - \left|x_i - x_j\right|\right). \tag{19}$$

Using the preference level achieved by position j compared to position k, the Condorcet method of voting result would be the position which gains the highest number of votes in a pair-wise voting:

$$v_{jk} = \sum_{i=1, i\ne j,k}^{n} v^i_{jk}, \tag{20}$$

where v_{jk} is the votes cast for j versus k from all other players points of view. Using (20), median voter position μ can be obtained by finding the position with the most support.

2.5. Probability of Success. When two actors i and j challenge each other, the probability of success p_{ij} can be expressed as

$$p_{ij} = \frac{\sum_{k|u_{ki}>u_{kj}} v^k_{ij}}{\sum_{k=1}^{n}\left|v^k_{ij}\right|}. \tag{21}$$

The probability of success, p_{ij}, can be interpreted as the amount of support received by actor i in comparison to actor j [14]. By substituting (19) into (21), the i's probability of success over j can be achieved by

$$p_{ij} = \frac{\sum_{k|\arg>0} c_k s_k\left(\left|x_k - x_j\right| - \left|x_k - x_i\right|\right)}{\sum_{k=1}^{n} c_k s_k\left|\left(\left|x_k - x_j\right| - \left|x_k - x_i\right|\right)\right|}, \tag{22}$$

where c_k and s_k are the capability and salience of player k in the issue. The numerator calculates the expected level of support for i. The denominator calculates the sum of the support for both i and j, resulting in a fraction that obviously falls between 0 and 1.

2.6. Risk. The last parameter is the risk term, r_i, which is calculated using a risk value R_i. The risk value determines how much an actor is willing to risk to be distanced from the median voter position. If an actor's desired position is closer to the central position, its stance is more supported by others, has stronger alliance, and the actor is less likely required to have intense bargaining with others. This actor is considered as a risk averse player in the issue when it receives risk value R_i closer to -1. On the other hand, a risk seeking actor's R_i will be closer to 1 indicating that the actor has weaker support and could be attacked by others. Equation (23) expresses the formula for the risk value R_i:

$$R_i = \frac{2\sum_{j=1,j\neq i}^{n} EU_{ji}^{i} - \max_i \sum_{j=1,j\neq i}^{n} EU_{ji}^{i} - \min_i \sum_{j=1,j\neq i}^{n} EU_{ji}^{i}}{\max_i \sum_{j=1,j\neq i}^{n} EU_{ji}^{i} - \min_i \sum_{j=1,j\neq i}^{n} EU_{ji}^{i}}, \tag{23}$$

where $R_i \in [-1, 1]$. After achieving R_i, the risk term r_i can be calculated by a transformation in (24) which ensures that $0.5 < r_i < 2$. The risk term is considered to be 1 for all actors in the beginning stage as a risk neutral stance and will be calculated in each round.

$$r_i = \frac{1 - R_i/3}{1 + R_i/3}. \tag{24}$$

2.7. Offer Categories. The expected utilities are used to determine how the negotiation between the actors will proceed. Four different categories can occur when actors face each other: conflict, capitulation, compromise, or stalemate. These categories as well as the corresponding expected utilities are illustrated in Figure 2. Below, each of these scenarios is defined in further detail:

(i) Conflict: If each of the actors i and j believes that it has the upper hand in the confrontation, they are likely to pursue a conflict. This scenario happens, if $EU_{ij}^{i} > 0$ and $EU_{ji}^{j} > 0$. The outcome of the conflict can be considered as follows:

Actor j moves to i's position when $EU_{ij}^{i} > EU_{ji}^{j}$.
Actor i moves to j's position when $EU_{ij}^{i} < EU_{ji}^{j}$.

(ii) Capitulate: In the case where $EU_{ij}^{i} > 0$, $EU_{ji}^{j} < 0$, and $|EU_{ij}^{i}| < |EU_{ji}^{j}|$, actor j is expected to capitulate (acquiesce) when facing actor i. In this scenario, i tries (proposes) to force j to accept its current position

$$proposal_{ij}^{i} = x_i, \tag{25}$$

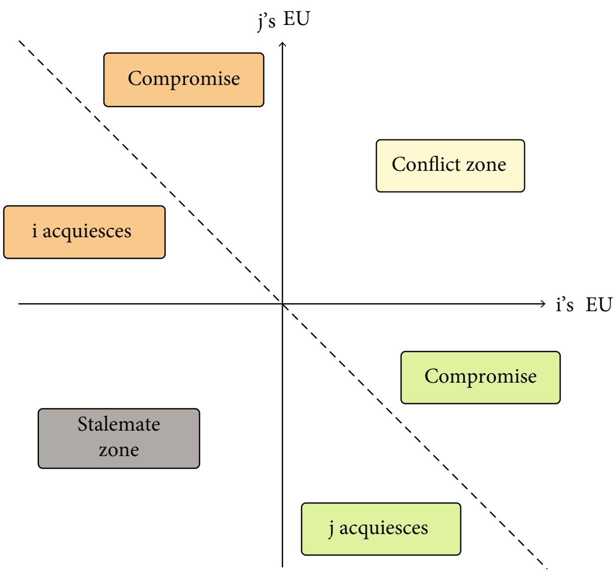

Figure 2: Different scenarios in expected utility model from i's viewpoint.

where x_i is the actor i's position and $proposal_{ij}^{i}$ is the position proposed to j by i, from i's point of view.

(iii) Compromise: This scenario happens if actor i has the upper hand and actor j is willing to agree with an acceptable offer from actor i. The stakeholders compromise in favor of actor i, if $EU_{ij}^{i} > 0$, $EU_{ji}^{j} < 0$, and $|EU_{ij}^{i}| > |EU_{ji}^{j}|$. Based on the compromise mentioned above, the offer is somewhere between i and j's positions, but closer to i's position

$$\Delta x = \left(x_i - x_j\right)\left|\frac{EU_{ij}^{j}}{EU_{ji}^{i}}\right|, \tag{26}$$

$$proposal_{ij}^{i} = x_j + \Delta x, \tag{27}$$

where Δx is the amount of position change when actor j accepts the compromise and x_i and x_j are actors i and j's position, respectively.

(iv) Stalemate: In a situation where both stakeholders believe they do not have the upper hand and cannot beat the other, neither is inclined to move from its current position. This scenario arises when $EU_{ij}^{i} < 0$, $EU_{ji}^{j} < 0$

2.8. Offer Selection. An entire set of actor interactions is represented by one round. Each actor will receive some offers from other actors at the conclusion of each round. To establish each actor's position in the following round, it is necessary for each actor to decide which offer should be accepted. The best option for each actor would be to select the offer that requires the minimum movement, according to Mesquita [15] and Baranick [16]. We utilized this idea in our implementations assuming that the minimum move must be greater than zero unless all the offers the actor receives are equal to its current position. Future research could result in a more efficient offer selection method. If no player can make an offer to the other actors that is greater than a certain threshold, we consider that the equilibrium has been attained [11].

3. Model Extension

In this section, our earlier two-dimensional model [11] is explained followed by an extension to the formulation to provide a model for N-dimensional problems. Attempts have been made to avoid repeating similar formulation, and to discuss only formulation that are modified for the N-dimensional model.

3.1. Two-Dimensional Model. The capability attribute in our 2D model is similar to the 1D model; however, the desired position and salience are assigned along x and y-axes for each actor [11].

In this model, each actor should take a stand in each of the two dimension, i.e., x-axis and y-axis. The position and salience vectors of actor i along the x and y-axes are represented as

$$p_i = (x_i, y_i), \tag{28}$$

$$s_i = \left(s_{i,x}, s_{i,y}\right), \tag{29}$$

So, the distance of i and j is calculated as

$$dist(i, j) = dist\left(p_i, p_j\right) = \frac{1}{\sqrt{2}}\sqrt{\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2}, \tag{30}$$

where $dist(i, j)$ is the distance function between the two positions, p_i and p_j.

Now that a two-dimensional position is described, the corresponding utility function can be defined. As mentioned earlier, the utility should be a decreasing function of the distance between the positions taken by actors i and j:

$$u_{ij} = 1 - 2(dist(i, j))^{r_i}, \tag{31}$$

where $i = 1, 2, .., n$ and n is the number of actors in the model and x_i and y_i are the actor i's positions along x and y-axes. The parameter r_i is the risk term for actor i. It should be noted that the positions along x and y-axes are normalized to the range of $[0, 1]$ in order to be used in the equation. The utility's superscript is intentionally omitted and will be added to the notation after adding salience to the equation later in (34).

If we had one salience for each actor, using (31) instead of (6) in (18), we could get

$$v^i_{jk} = 2c_i s_i (dist(i, k) - dist(i, j)). \tag{32}$$

In a two-dimensional problem, however, each actor has two separate salience along the x and y-axes, according to (29). Therefore, the weighted distance function $dist^k_w(i, j, s_k)$ from k's perspective will be defined based on actor k's salience towards each axis. Subsequently, the utility u^i_{ij} from actor i's perspective can be modified as follows:

$$dist^k_w(i, j, s_k) = \frac{\sqrt{s^2_{k,x}\left(x_i - x_j\right)^2 + s^2_{k,y}\left(y_i - y_j\right)^2}}{\sqrt{s^2_{k,x} + s^2_{k,y}}}, \tag{33}$$

$$u^i_{ij} = 1 - 2\left(dist^i_w(i, j, s_i)\right)^{r_i}, \tag{34}$$

Using (34) and adjusting (18), we have

$$v^i_{jk} = c_i\left(u^i_{ij} - u^i_{ik}\right), \tag{35}$$

Substituting (34) into (35), we can get

$$v^i_{jk} = 2c_i\left(\left(dist^i_w(i, k, s_i)\right)^{r_i} - \left(dist^i_w(i, j, s_i)\right)^{r_i}\right), \tag{36}$$

Equations (20) and (21) remain the same and are brought back here for convenience and readability purpose.

Thus, the probability of success p_{ij} can be achieved by

$$p_{ij} = \frac{\sum_{k|\arg>0} c_k\left(dist^k_w(k, j, s_k) - dist^k_w(k, i, s_k)\right)}{\sum^n_{k=1} c_k\left|dist^k_w(k, j, s_k) - dist^k_w(k, i, s_k)\right|}, \tag{37}$$

where c_k is the capability of actor k. It is worth noting that the salience s_k is contained within the weighted distance function $dist^k_w$.

The expected utilities EU^i_{ij} and $\left(EU^i_{ij}\right)_c$ used in (1) and (2) have to be split into two parts, one for each axis. However, $\left(EU^i_{ij}\right)_{nc}$ is not required to change:

$$EU^i_{ij,x} = \left(EU^i_{ij,x}\right)_c - \left(EU^i_{ij}\right)_{nc}, \tag{38}$$

$$EU^i_{ij,y} = \left(EU^i_{ij,y}\right)_c - \left(EU^i_{ij}\right)_{nc}, \tag{39}$$

$$\left(EU^i_{ij,x}\right)_c = s_{j,x}\left(p^i_{ij}U^i_{s_i} + \left(1 - p^i_{ij}\right)U^i_{f_i}\right) + \left(1 - s_{j,x}\right)U^i_{s_i}, \tag{40}$$

$$\left(EU^i_{ij,y}\right)_c = s_{j,y}\left(p^i_{ij}U^i_{si} + \left(1 - p^i_{ij}\right)U^i_{fi}\right) + \left(1 - s_{j,y}\right)U^i_{si}, \tag{41}$$

$$\left(EU^i_{ij}\right)_{nc} = QU^i_{sq} + (1 - Q)\left(TU^i_{bi} + (1 - T)U^i_{wi}\right), \tag{42}$$

Input: $[s_{i,d}]_{n\times N}$, $[c_{i,d}]_{n\times N}$, $[p_{i,d}]_{n\times N}$
$r_i \longleftarrow 1$ for all actors
while *threshold* $< 2.5\%$ **do**
  for $i, j, k \longleftarrow 1$ to n **do**
    Calculate v^i_{jk}
  end for
  for $i \longleftarrow 1$ to n **do**
    Calculate v_{jk}
  end for
  $votes = zeros(n, n)$
  for $k \longleftarrow 1$ to n **do**
    $votes = votes + v_{jk}$
  end for
  $index = \max(vote)$
  $\mu = p(:, index)$
  for $i, j \longleftarrow 1$ to n **do**
    Calculate $U_s(i, j)$, $U_f(i, j)$, $U_b(i, j)$, $U_w(i, j)$
  end for
  for $i \longleftarrow 1$ to n **do**
    Calculate $U_{sq}(i)$
  end for
  for $i, j, k \longleftarrow 1$ to n **do**
    Calculate p_{ij}
  end for
  for $i, j \longleftarrow 1$ to n **do**
    for $d \longleftarrow 1$ to N **do**
      Calculate $EU^i_{i,j,d}$
    end for
  end for
  Calculate $EU^i_{i,j}$
  Calculate R_i and r_i
  Calculate offer categories and proposals
  Choose proposal
end while

Algorithm 1: N-dimensional Model

Table 1: Complexity of different parts of the model.

Component	Complexity
v^i_{jk}	$O(M^3N)$
v_{jk}	$O(M)$
μ	$O(M)$
$U_s(i, j)$, $U_f(i, j)$, $U_b(i, j)$, $U_w(i, j)$	$O(M^2N)$
$U_{sq}(i)$	$O(M)$
p_{ij}	$O(M^3N)$
$EU^i_{i,j,d}$	$O(M^2N)$
$EU^i_{i,j}$	$O(M^2N)$
R_i	$O(M^2)$
r_i	$O(M)$
Offer categories and proposals	$O(M^2N)$
Choosing proposals	$O(M^2)$

Table 2: Input data for case study I.

Players	Capability	Salience	Position X
Netherlands	0.08	0.8	4
Belgium	0.08	0.4	7
Luxembourg	0.03	0.2	4
Germany	0.16	0.8	4
France	0.16	0.6	10
Italy	0.16	0.6	10
UK	0.16	0.9	10
Ireland	0.05	0.1	7
Denmark	0.05	1.0	4
Greece	0.08	0.7	7

$$EU^i_{ij} = \frac{s_{i,x}EU^i_{ij,x} + s_{i,y}EU^i_{ij,y}}{\sqrt{s^2_{i,x} + s^2_{i,y}}}. \quad (43)$$

Next, the basic utilities including U^i_{si}, U^i_{fi}, U^i_{sq}, U^i_{bi}, and U^i_{wi} have to be defined to achieve the modified expected utilities in (40) to (42). If actor i challenges actor j, the basic utilities we had in (9) and (10) change as follows:

$$U^i_{si} = 2 - 4[0.5 - 0.5dist(i, j)]^{r_i}, \quad (44)$$

$$U^i_{fi} = 2 - 4[0.5 + 0.5dist(i, j)]^{r_i}, \quad (45)$$

where $dist(i, j)$ is the distance function between two positions p_i and p_j defined in (30). When actor j challenges i, U_{si} and U_{fi} are the utilities of i's success and failure, respectively.

When actor i does not challenge j, the utilities for the better and worse situations we had in (15) and (16) change as follows:

$$U^i_{bi} = 2 - 4[0.5 - 0.25(dist(i, \mu) + dist(i, j))]^{r_i}, \quad (46)$$

$$U^i_{wi} = 2 - 4[0.5 + 0.25(dist(i, \mu) + dist(i, j))]^{r_i}, \quad (47)$$

where μ is the median voter position. In the status quo case where i does not challenge j and j does not move, we get

$$U^i_{sq} = 2 - 4(0.5)^{r_i}, \quad (48)$$

The offer categories and the conditions remain almost the same as in one-dimensional model, except changing position's notation from x_i to p_i. Equation (49) is going to be used in the capitulate scenario where i forces j to accept its current position:

$$proposal^i_{ij} = p_i, \quad (49)$$

where $p_i = (x_i, y_i)$ is the actor i's position. Additionally, Equations (50) to (53) are now used in the compromise

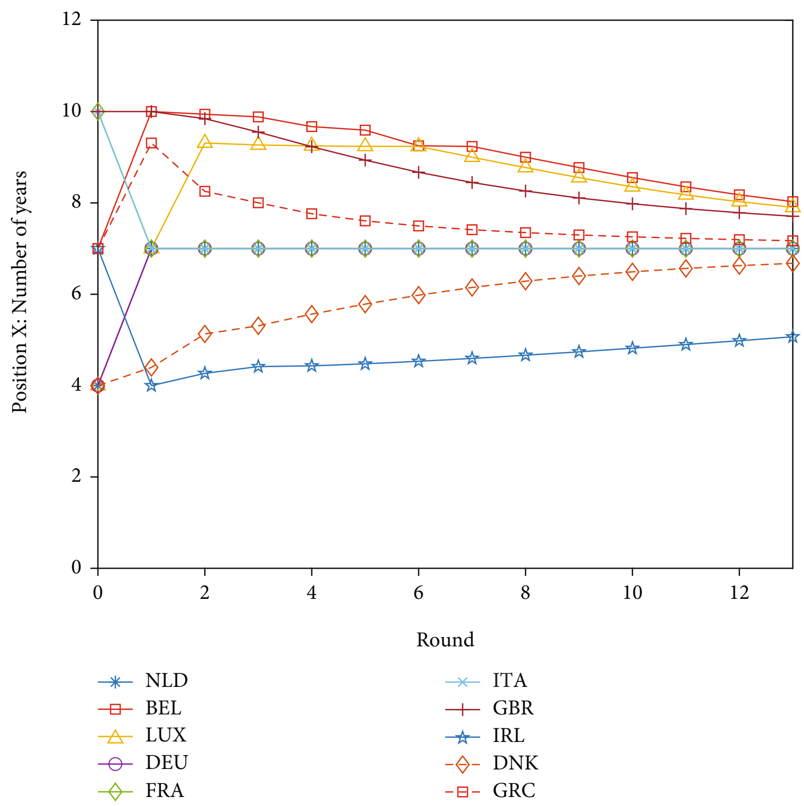

Figure 3: Players' positions after each round along the x-axis in case study I. Position X shows the number of years that would need to pass before the introduction of emission standards for medium sized automobiles.

Table 3: Input data for case study II.

Players	Capability	Sal X	Sal Y	Pos X	Pos Y
US	21	0.8	1.0	70	26
China	14	0.8	1.0	45	76
Japan	5	0.7	0.5	50	10
Germany	4	0.6	0.6	50	13
India	3	0.9	0.6	40	13
UK	3	0.8	0.5	70	10
France	3	0.7	0.3	70	5
Italy	2	0.7	0.3	60	6
Brazil	2	0.8	0.7	80	14
Canada	2	1.0	0.5	70	10
Saudi Arabia	1	1.0	0.5	80	10
Iraq	0.2	1.0	0.1	80	2
UAE	0.4	0.9	0.3	65	5
Russia	2	1.0	0.3	70	6

scenario where actor i needs to suggest an acceptable offer to actor j:

$$\Delta x = \left(x_i - x_j\right)\left|\frac{EU_{ij}^{j}}{EU_{ji}^{i}}\right|, \tag{50}$$

$$\Delta y = \left(y_i - y_j\right)\left|\frac{EU_{ij}^{j}}{EU_{ji}^{i}}\right|, \tag{51}$$

$$\Delta p = (\Delta x, \Delta y), \tag{52}$$

$$proposal_{ij}^{i} = p_j + \Delta p, \tag{53}$$

where Δp denotes the amount of position change that actor j accepts as a result of a compromise.

3.2. N-Dimensional Model. This section continues to extend the above two-dimensional model to an N-dimensional one. Here, we continue to only provide definitions and relations that needs to be updated, while the rest of the equations remain intact.

Here, the actors' position and salience have to be specified for each dimension of the problem. Therefore, (28), (29), (30), and (33) will be modified as follows:

$$p_i = (x_{i,1}, x_{i,2}, \cdots, x_{i,N}), \tag{54}$$

$$s_i = (s_{i,1}, s_{i,2}, \cdots, s_{i,N}), \tag{55}$$

$$dist(i,j) = \sqrt{\frac{\left(x_{i,1} - x_{j,1}\right)^2 + \left(x_{i,1} - x_{j,2}\right)^2 + \cdots + \left(x_{i,N} - x_{j,N}\right)^2}{N}}, \tag{56}$$

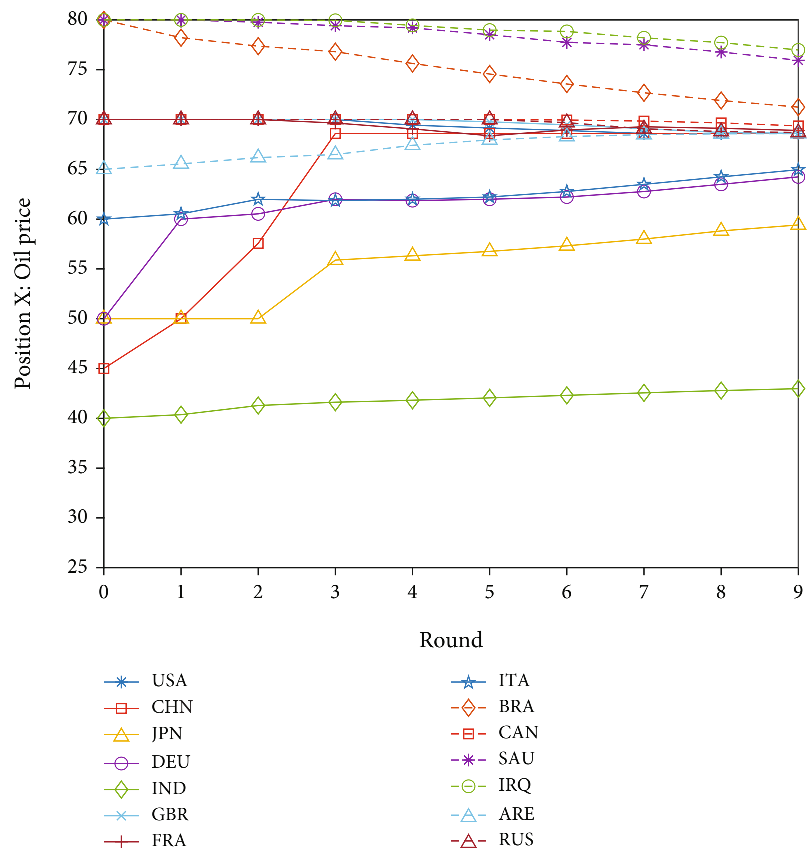

Figure 4: Player positions after each round along the x-axis in case study II. Position X shows the oil price in U.S. dollars desired by each player.

$$dist_w(i,j,s_k) = \sqrt{\frac{s_{k,1}^2 \left(x_{i,1} - x_{j,1}\right)^2 + s_{k,2}^2 \left(x_{i,1} - x_{j,2}\right)^2 + \cdots + s_{k,N}^2 \left(x_{i,N} - x_{j,N}\right)^2}{s_{k,1}^2 + s_{k,2}^2 + \cdots + s_{k,N}^2}}, \quad (57)$$

where N is the number of problem dimensions and p_i and s_i denote actor i's position and salience vector, respectively. Also, $x_{i,d}$ and $s_{i,d}$ are actor i's position and salience along dimension d, respectively. The function $dist(i,j)$ is the distance function which measures the distance between actors i and j's positions. The function $dist_w(i,j,s_k)$ represents the weighted distance between actors i and j regarding actor k's salience vector.

The expected utilities in (38) to (43) can be extended as following:

$$EU_{ij,d}^i = \left(EU_{ij,d}^i\right)_c - \left(EU_{ij}^i\right)_{nc}, \quad (58)$$

$$\left(EU_{ij,d}^i\right)_c = s_{j,d}\left(p_{ij}^i U_{s_i}^i + \left(1 - p_{ij}^i\right) U_{f_i}^i\right) + \left(1 - s_{j,d}\right) U_{s_i}^i, \quad (59)$$

$$\left(EU_{ij}^i\right)_{nc} = QU_{sq}^i + (1 - Q)\left(TU_{bi}^i + (1 - T)U_{wi}^i\right), \quad (60)$$

$$EU_{ij}^i = \frac{s_{i,1} EU_{ij,1}^i + s_{i,2} EU_{ij,2}^i + \cdots + s_{i,1} EU_{ij,N}^i}{\sqrt{s_{i,1}^2 + s_{i,2}^2 + \cdots + s_{i,N}^2}}, \quad (61)$$

where $d = 1, 2, \cdots, N$ and N is the problem dimension.

Supposing the position vector $p_i = (x_{i,1}, x_{i,2}, \cdots, x_{i,N})$, the proposal in the compromise scenario can be defined as

$$\Delta x_d = \left(x_{i,d} - x_{j,d}\right) \left|\frac{EU_{ij}^j}{EU_{ji}^i}\right|, \quad (62)$$

$$\Delta p = (\Delta x_1, \Delta x_2, \cdots, \Delta x_N), \quad (63)$$

$$proposal_{ij}^i = p_j + \Delta p, \quad (64)$$

where Δx_d is the amount of movement along dimension d, while Δp is the vector of movement that actor j accepts through the compromise.

3.3. Complexity Analysis. When dealing with problems with large number of issues (dimensions) and/or large number of actors per issue/dimension, the computational complexity of the problem becomes increasingly important. To demonstrate the analysis of the model's time complexity, a simple pseudo code is provided in Algorithm 1. Table 1 shows the complexity of different parts of the algorithm. It can be seen that the overall computational complexity of the algorithm is

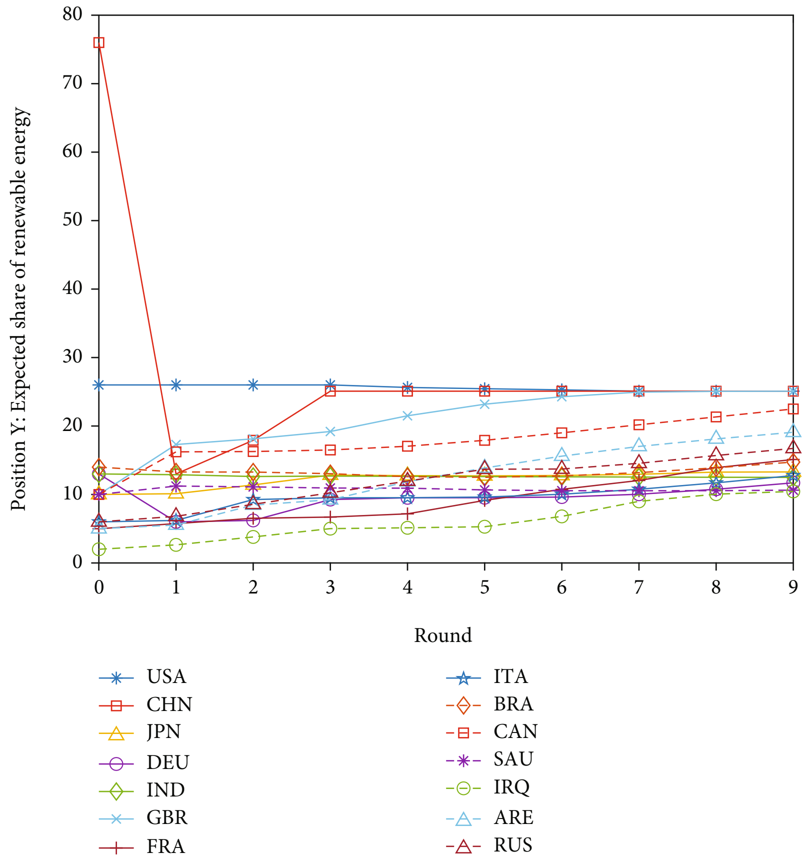

Figure 5: Player positions after each round along the y-axis in case study II. Position Y shows the expected share for renewable energy in relation to the whole energy market.

Table 4: Input data for case study III.

Players	Capability	Sal X	Sal Y	Sal Z	Pos X	Pos Y	Pos Z
Israel	0.05	1.0	0.5	0.7	0.00	0.50	0.0
US	1.00	0.9	0.6	0.9	0.05	0.50	0.1
UK	0.20	0.7	0.5	0.6	0.10	1.00	0.2
France	0.30	0.7	0.6	0.5	0.20	1.75	0.3
Germany	0.30	0.6	0.6	0.4	0.30	1.75	0.3
China	0.50	0.7	1.0	0.9	0.60	2.25	0.7
Russia	0.40	0.7	0.5	0.9	0.80	2.25	0.9
Iran	0.01	1.0	1.0	1.0	1.00	2.50	1.0

$O(M^3N)$, where M and N are the number of actors and the dimension of the problem, respectively.

4. Evaluation Through Case Studies

We have presented various case studies in this section to demonstrate the effectiveness of the proposed model. First, we have evaluated our model by comparing its results for a one-dimensional problem against a well-known one-dimensional model presented by BDM to see if it produces comparable results. In order to assess the proposed model's capability for offering a solution in both two- and three-dimensional space, two additional case studies are explored in collaboration with subject matter expert (Ambassador Michael Gfoeller, former US ambassador in the Middle East). The median voter position of the last round, which indicates the position with the strongest support, is used to decide the problem's outcome in all of these cases. In these case studies, at 2.5% deviation of the final position, along each direction, the equilibrium is considered to be achieved.

4.1. Case Study I. The first case study, which is one-dimensional, attempts to estimate the number of years that must pass until medium-sized cars are subject to pollution regulations. Table 2 provides an overview of the players as well as their initial position, salience, and capability along the only dimension of the problem, i.e., along x-axis. The players' positions along x-axis are represented in Figure 3. Our model forecasts the result to be 7.0 years, which is in line with the reported value of [4, 11]. While the actual delay is 8.33, the obtained result demonstrates how well the proposed model works when used to solve one-dimensional issues.

It is worth noting that the one-dimensional model can show how much each actor can influence the final outcome of the issue by altering the amount of power used during bargaining. This method of explanation can aid actors in better understanding and forecasting the negotiation process.

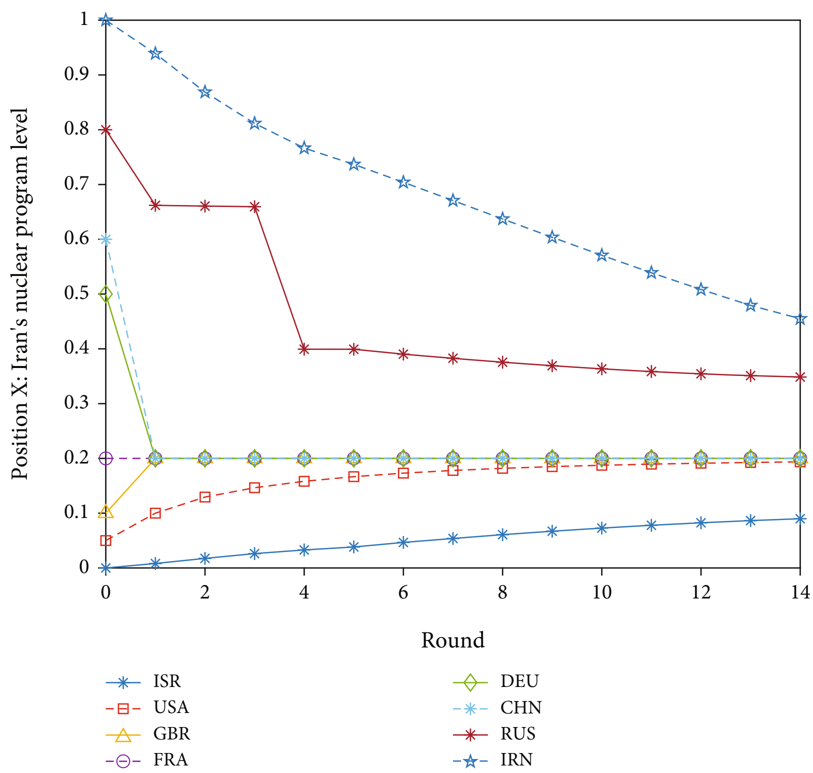

FIGURE 6: Player positions after each round along the x-axis in case study III. Position X shows Iran's nuclear program level expected by different players so that 0 indicates abandoning the program and 1 indicates having a nuclear weapon.

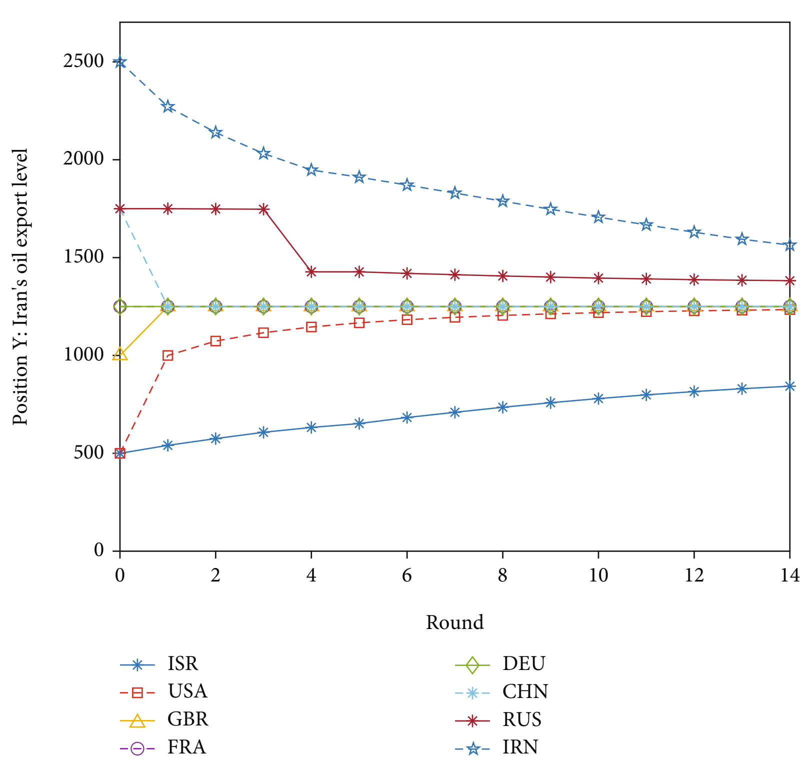

FIGURE 7: Player positions after each round along the y-axis in case study III. Position Y shows Iran's oil export level in thousand barrels per day desired by each player.

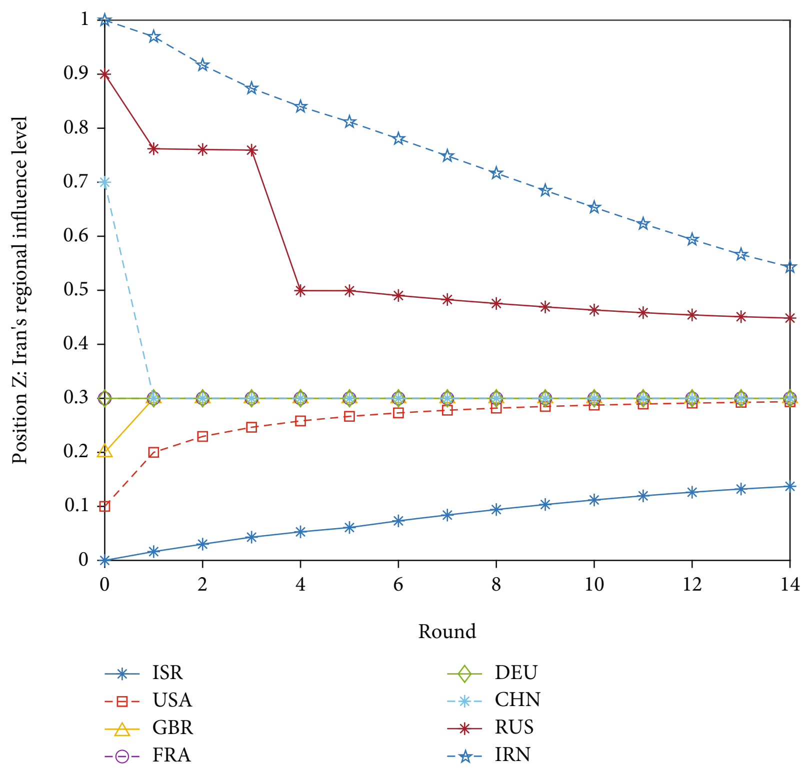

Figure 8: Player positions after each round along the z-axis in case study III. Position Z shows Iran's regional influence level.

4.2. Case Study II. We examined the effectiveness of the suggested approach for two-dimensional problems, a subset of N-dimensional problems, in our second case study. According to [11], each player in this problem has a desired position on the x-axis in regards to the oil price and is negotiating over its portion of the market for renewable energy on the y-axis. The model allows for simultaneous consideration of the oil prices that each player is targeting and their respective shares of the renewable energy market. It should be noted that renewable energy will ultimately overtake the energy market in the future, despite the fact that many players are reluctant to make the switch. Each oil pricing position is set by the level of production as well as the significance of oil to each actor's industry and budget. Actors are often selected from countries with the most developed economies or highest level of oil production [11].

In the previous version of the model [11], each actor could only set one value for the salience parameter. Therefore, if an actor has different salience values for its positions, there will not be much flexibility, and a compromised value will have to be chosen to represent both dimensions. The current proposed model, on the other hand, gives the problem designer more flexibility by allowing him or her to set the salience parameters for each axis separately.

Players' attributes based on the data obtained on October 2nd are listed in Table 3 [11]. At that time, the Brent crude oil price was \$39.27 per barrel [17]. After the proposed model is run, players settle on a price of \$68.6 per barrel, while the outcome for the y-axis representing the renewable energy market share is 25.1 percent of the total energy market. Figures 4 and 5 illustrate actors' positions after each round along x- and y-axes, respectively. These results were evaluated as reasonable by the collaborating subject matter expert, mentioned above.

The outcome of the model can be used in a variety of ways. It can be used to see how changing different parameters, such as position or capability, affects the end result. In two-dimensional problems, the results can also be utilized to determine how much actors are willing to give up on one dimension of the problem to get closer to their desired position on the other. To put it in another way, the presented multidimensional model allows for more in-depth analysis of bargaining across multiple dimensions, which is certainly a valuable expansion to the classical single dimension models.

4.3. Case Study III. Our third case study, on the x-axis, is the negotiation between Iran, the 5 + 1 group, and Israel over Iran's nuclear program, while on the y-axis and z-axis, the players are considering Iran's oil exports and regional influence levels, respectively. This case study exemplifies how several actors might have similar or divergent viewpoints on various aspects of a negotiating problem. Although most of the actors' positions on Iran's nuclear program are close to the US's position, they do not necessarily agree with the US on the level of Iranian oil exports or regional influence. Regarding the oil export level issue on the y-axis of this multidimensional problem, the US, which produces the most oil worldwide, is interested in expanding its market; therefore, it makes sense for it to make other countries to cut back on their oil outputs. On the other hand, significant oil importers such as China are interested to diversify their portfolio of oil imports. Iran's regional influence is another issue that countries have different interests in it. Countries such as the US view Iran's regional role as interference and want Iran to have the least influence in the region, while countries such as Russia benefit from Iran's activities.

Table 4 contains actors' position, capability, and salience in the problem. Position X represents the actors' stance on Iran's nuclear program, with 0 denoting complete abandonment and 1 denoting the achievement of a nuclear weapon. Position Y reflects players' position on Iran's oil exports in million barrels per day. The data corresponding to these two dimensions of the problem are well discussed in [11]. Position Z demonstrates actors' standing towards Iran's regional influence level, with 0 denoting no activity in the region and 1 denoting the highest level possible. The players' positions in each round are depicted in Figure 6–8. The median voter position results for the last round indicate that the players would agree to a level of 0.2 for Iran's nuclear program (x-axis), 1,250,000 barrels per day for oil exports (y-axis), and 0.3 for Iran's regional influence (z-axis). As can be seen, and according to the subject matter expert working with our team, the three-dimensional model provides a reasonable outcome when three different positions for each actor are considered simultaneously.

It should be noted that the current focus of this study is not on the details of the data, but rather on its capability to model complex problems and find the most plausible outcome. The outcome can assist decision-makers in modifying their initial positions and anticipating the most likely outcome of the problem.

5. Conclusions and Future Work

Real-world negotiations typically involve making concessions on several issues. Modeling these types of problems was not feasible under the assumptions of the existing one-dimensional model. To expand the one-dimensional game theory–based group decision model to an N-dimensional version, the mathematical formulations are modified to include actors' attributes along multiple issue. Additionally, separate salience values for each dimension are considered to make the model more flexible and distribute actor's capability (power) across multiple axes. We showed that in complex problems, the proposed model allows for more in-depth analysis of bargaining across multiple dimensions. To illustrate the effectiveness of the proposed model, three case studies were examined. The first one was a one-dimensional problem, and the findings demonstrate that the new model, like the earlier models, can predict the likely outcome of such problems. Furthermore, two case studies with more than one issue were explored to demonstrate the model's efficiency and explainability.

In the future, we intend to conduct more evaluation of this model, working closely with subject matter experts. We will also look into performing further research to modify and extend this model as needed, with a particular focus on changes to the offer selection component and the replacement of spike-like positions with more flexible options.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

Authors would like to thank Ambassador Michael Gfoeller (Retired) for his contribution to this work as a subject matter expert.

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