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Research Article

Stochastic Transportation Problem with Multichoice Random Parameter

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This paper deals with the situation of multiple random choices along with multiple objective functions of the transportation problem. Due to the uncertainty in the environment, the choices of the cost coefficients are considered multichoice random parameters. The other parameters (supply and demand) are replaced by random variables with Gaussian distributions, and each multichoice parameter alternative is treated as a random variable. In this paper, the Newton divided difference interpolation technique is used to convert the multichoice parameter into a single choice in the objective function. Then, the chance-constrained method is applied to transform the probabilistic constraints into deterministic constraints. Due to the consideration of multichoices in the objective function, the expectation minimization model is used to get the deterministic form. Moreover, the fuzzy programming approach with the membership function is utilized to convert the multiobjective function into a single-objective function. A case study is also illustrated for a better understanding of the methodology.

1. Introduction

An important application of the linear programming problem is the classical transportation problem, in which a homogeneous product is moved from one warehouse to another according to the product's availability and demand.

In real-life applications, there are numerous situations where decision-makers have to choose one among multiple choices. This kind of problem is recognized as a multichoice programming problem. To solve the multichoice programming problem, Chang [1] introduced the concept of binary variables to tackle the multichoice aspiration level for each objective set by the decision-maker. Subsequently, he also revised his model for multichoice programming problems in which binary variables are replaced with continuous variables [2].

Later, Mahapatra et al. [3] established a model for multichoice stochastic transportation problems with extreme value distributions utilizing binary variables. Quddoos et al. [4] considered a multichoice stochastic transportation problem where the cost coefficient of the objective function is of the multichoice type, and the random availability and demand of products are to follow a general form of distribution. Roy [5] presented the Lagrange interpolation polynomial (LIP) to convert the multichoice parameter into a single choice to deal with the circumstances of the multichoice parameter in the transportation problem. Pradhan and Biswal [6] proposed a linear programming model in which each multichoice parameter alternative was treated as a random variable, and they employed several forms of optimization models (V-model, fractile criterion model, probability maximization model, and E-model).

In today's highly competitive market, parameters are not always fixed; they fluctuate by nature. Stochastic programming deals with problems where the deterministic parameters of the transportation problem are replaced by random variables. Roy [7] proposed a problem in which the supply parameter is considered a random variable that follows the logistic distribution. Many researchers have proposed different models for multichoice stochastic transportation problems [8–10]. Li et al. [11] proposed a quasilinear stochastic programming model with reliability coefficients that is based on the expectation and the variance. This programming was used to analyze the performance of the stochastic transportation problem, indicating that the model is both operable and interpretable.

To deal with the imprecise situation, Zadeh [12] introduced the concept of fuzzy theory, which has been applied to various fields because of its dubious parameters. Kaufmann and Gupta [13] were the first to investigate the use of a fuzzy transportation problem in decision-making. By using the fuzzy membership function, Abd El-Wahed [14] determined the optimal compromise solution for the multiobjective transportation problem. Gani and Razak [15] presented a model for a two-stage cost-minimizing fuzzy transportation problem in which the parameters (supply and demand) are considered as fuzzy numbers. Rani and Gulati [16] considered a fully fuzzy multiobjective, multi-item, solid transportation problem in which the conveyance constraints are considered along with the other constraints of the classical transportation problem. Using the fuzzy programming technique, the fuzzy optimal compromise solution is presented. Ebrahimnejad [17] formulated a transportation problem in which the transportation cost, supplies, and demand are considered as interval-valued trapezoidal fuzzy numbers and also proposed a fuzzy linear programming approach for solving the same problem. Using signed distance ranking, the author made a comparison with the interval value fuzzy number and obtained the same results for both types of fuzzy numbers. Ojha et al. [18] considered transportation problems involving fuzzy stochastic costs within budget constraints. To obtain the deterministic problem of the defined problem, they used two different approaches (the α-cut of the fuzzy numbers and the credibility measure) as well as a genetic algorithm to solve the deterministic problem.

Acharya et al. [19] have identified a computational strategy for solving the fuzzy stochastic transportation problem. To convert stochastic transportation problems into deterministic problems, a strategy that includes conventional randomness is applied. Agrawal and Ganesh [20] dealt with the solution of the fuzzy fractional transportation problem in which the parameters of the transportation problem, supply and demand, are stochastic in nature and considered as fuzzy random variables that follow exponential distributions with fuzzy means and fuzzy variances. Mahapatra et al. [21] utilize fuzzy programming to turn a multiobjective function into a single-objective function. Because of its numerous applications, Maity and Kumar Roy [22] developed a model for solving multiobjective transportation problems with multichoice demand, and by using a fuzzy membership function, multiobjective problems are converted into a single objective. Kundu et al. [23] modeled a multiobjective multiitem solid transportation problem in which the coefficients for the objectives and the constraints are fuzzy numbers. To derive the crisp value, the expected values of the objective functions and the concept of the minimum fuzzy number are considered. For compromise solutions to objectives, fuzzy programming techniques and the global criterion method are used. Roy et al. [24] proposed a multiobjective transportation problem in an intuitionistic fuzzy environment in which all the parameters of the transportation problem are considered as intuitionistic fuzzy numbers. Two approaches to finding the optimal solution to the proposed problem are applied: intuitionistic fuzzy programming and goal programming. When telemedicine-based healthcare systems were required, Pal et al. [25] used the particle swarm optimization (PSO) algorithm and sparse PSO to store and transfer ECG recordings.

Based on the above literature, many studies have been carried out on the multichoice transportation problems with supply and demand parameters considered as random variables with different types of distributions. Roy [5] proposed the LIP to convert multiple choices into single choices. The existing approaches have their own limitations in converting multichoice into single-choice, probabilistic into deterministic, and multiobjective into a single objective. In this paper, a multiobjective transportation problem with multichoice random parameters is proposed in which the parameters, supply and demand, are treated as random variables that follow Gaussian distributions with known means and variances. The objective function's decision variable coefficients are multichoice parameters, and each alternative of the multichoice parameters is assumed to be a random variable with a Gaussian distribution. To find the solution to the proposed problem, the Newton divided difference interpolation polynomial (NDDIP) is used to obtain the optimal choice of the multichoice parameters. This technique was not used earlier in converting the multichoice parameters into single-choice ones. The main advantage of the NDDIP is that (i) the formula can be modified without affecting its previous formulation by adding another choice for the multichoice parameter at any step of the process; and (ii) the computation is simple, easy to understand, and takes less time to compute.

This paper is organized as follows: Section 2 contains the mathematical model of the problem, Section 3 describes the methodology, which includes the transformation technique and the fuzzy programming approach; a case study is presented in Section 4, and the concluding remarks are in Section 5. Table 1 represents the nomenclature which is used in this paper.

2. Problem Formulation

A multiobjective stochastic transportation problem is considered with a multichoice random parameter in which the product is shipped from various source locations to several destinations. In real-life applications, the decision depends on uncertainty; therefore, it makes sense that each alternative of the multichoice parameter is assumed as a random variable. The supply and demand are also assumed to be random variables due to certain factors, for example, the condition of the market, fluctuation in the availability and demand for the product, and variation in the price of the product. Hence, the mathematical model of a multiobjective transportation problem, in which the coefficients of the decision variables in the objective function are of the multichoice type and each alternative of the multichoice parameter is taken as a random variable, can be formulated as follows:

$$\min Z = \sum_{r=1}^{m} \sum_{s=1}^{n} \left\{ C_{rs}^{1}, C_{rs}^{2}, \cdots, C_{rs}^{p} \right\} y_{rs}, \quad (1)$$

Table 1: Nomenclature.

Z	Objective function.
r	Index of source locations.
s	Index of destinations.
C_{rs}	Transportation cost per unit from r^{th} source locations to s^{th} destination.
C_{rs}^{i}	Multichoice transportation cost per unit from r^{th} source locations to s^{th} destination.
$f_{C_{rs}}$	The function of interpolating polynomial choices C_{rs}.
p	Number of choices for the multichoice parameter.
w_r	Random availability of the product at r^{th} source location.
λ_{w_r}	Aspiration level for source constraints.
d_s	Random requirement of the product at s^{th} destination.
δ_{d_s}	Aspiration level for destination constraints.
y_{rs}	Number of units of the product that should be shipped from r^{th} source locations to s^{th} destination.
P	Probability.

subject to (s.t.)

$$P\left(\sum_{s=1}^{n} y_{rs} \le w_r\right) \ge 1-\lambda_{w_r} r = 1, 2, \cdots, m, \tag{2}$$

$$P\left(\sum_{r=1}^{m} y_{rs} \ge d_s\right) \ge 1-\delta_{d_s} s = 1, 2, \cdots, n, \tag{3}$$

$$y_{rs} \ge 0, r = 1, 2, \cdots, m, s = 1, 2, \cdots, n, \tag{4}$$

where y_{rs} (decision variable) denotes the number of units that should be transferred from m number of warehouses to n number of destination locations. These numbers are assumed to be deterministic.

The coefficients C_{rs}^{k}, $(k = 1, 2, \cdots, p)$ of the decision variables y_{rs} are treated as independent random variables, and the other parameters supply $w_r(r = 1, 2, \cdots, m)$ and demand $d_s(s = 1, 2, \cdots, n)$ are also treated as random variables that follow Gaussian distributions with known means and variances. Because the parameters are random, the constraints (2) and (3) are probabilistic in nature, with their aspiration level λ_{w_r} and δ_{d_s} $(0 < \lambda_{w_r}, \delta_{d_s} < 1)$, respectively.

3. Solution Methodology

3.1. NDDIP for Multichoice Parameters. The objective functions contain multichoice random parameters that are assumed to be independent random variables. To transform the multichoice parameters into a single choice, the NDDIP is used.

Introducing an integer variable $(0, 1, \cdots, p-1)$, for each choice $(C_{rs}^{1}, C_{rs}^{2}, \cdots, C_{rs}^{p})$, the interpolation polynomial is formed. If there are p number of choices for a parameter, each integer variable takes p number of nodes. Table 2 shows the considered multichoice parameter and, in Table 3, the divided difference (DD) table.

Table 2: For multichoice parameter.

Nodes v_{rs}^{k}	0	1	$\cdots$	$p-1$
$f_{C_{rs}}(v_{rs})$	C_{rs}^{1}	C_{rs}^{2}	$\cdots$	C_{rs}^{p}

Table 3: Divided difference table.

v_{rs}^{k}	$f_{C_{rs}}(v_{rs}^{k})$	First DD	Second DD	Third DD
0	C_{rs}^{1}			
		$f[v_{rs}^{0}, v_{rs}^{1}]$		
1	C_{rs}^{2}		$f[v_{rs}^{0}, v_{rs}^{1}, v_{rs}^{2}]$	
		$f[v_{rs}^{1}, v_{rs}^{2}]$		$f[v_{rs}^{0}, v_{rs}^{1}, v_{rs}^{2}, v_{rs}^{3}]$
2	C_{rs}^{3}		$f[v_{rs}^{1}, v_{rs}^{2}, v_{rs}^{3}]$	
		$f[v_{rs}^{2}, v_{rs}^{3}]$		
3	C_{rs}^{4}			

Using Tables 2 and 3, the interpolation polynomial is written as

$$\begin{aligned} F_{C_{rs}}\left(v_{rs}; C_{rs}^{1}, C_{rs}^{2}, \cdots, C_{rs}^{p}\right) = {} & f\left[v_{rs}^{0}\right] + \left(v_{rs} - v_{rs}^{0}\right) \\ & \cdot f\left[v_{rs}^{0}, v_{rs}^{1}\right] + \left(v_{rs} - v_{rs}^{0}\right)\left(v_{rs} - v_{rs}^{1}\right) \\ & \cdot f\left[v_{rs}^{0}, v_{rs}^{1}, v_{rs}^{2}\right] + \cdots + \left(v_{rs} - v_{rs}^{0}\right) \\ & \cdot \left(v_{rs} - v_{rs}^{1}\right) \cdots \left(v_{rs} - v_{rs}^{p-1}\right) \\ & \cdot f\left[v_{rs}^{0}, v_{rs}^{1}, \cdots, v_{rs}^{p-1}\right]. \end{aligned} \tag{5}$$

After replacing the coefficients of the decision variables by their interpolation polynomial, the objective function is

$$\begin{aligned} \min Z = \sum_{r=1}^{m}\sum_{s=1}^{n} & \left\{ f\left[v_{rs}^{0}\right] + \left(v_{rs} - v_{rs}^{0}\right) f\left[v_{rs}^{0}, v_{rs}^{1}\right] \right. \\ & + \left(v_{rs} - v_{rs}^{0}\right)\left(v_{rs} - v_{rs}^{1}\right) f\left[v_{rs}^{0}, v_{rs}^{1}, v_{rs}^{2}\right] + \cdots + \left(v_{rs} - v_{rs}^{0}\right) \\ & \left. \cdot \left(v_{rs} - v_{rs}^{1}\right) \cdots \left(v_{rs} - v_{rs}^{p-1}\right) f\left[v_{rs}^{0}, v_{rs}^{1}, \cdots, v_{rs}^{p-1}\right] \right\} y_{rs}. \end{aligned} \tag{6}$$

Applying the NDDIP, the objective function is as follows:

$$\begin{aligned} \min Z = \sum_{r=1}^{m}\sum_{s=1}^{n} & \left\{ C_{rs}^{1} + \left(v_{rs} - v_{rs}^{0}\right)\left(C_{rs}^{2} - C_{rs}^{1}\right) + \cdots + \left(v_{rs} - v_{rs}^{0}\right) \right. \\ & \left. \cdot \left(v_{rs} - v_{rs}^{1}\right) \cdots \left(v_{rs} - v_{rs}^{p-1}\right) \left(\sum_{i=1}^{p} \frac{C_{rs}^{i}}{\prod_{i \ne j+1, j=0}^{p-1} \left(v_{rs}^{i-1} - v_{rs}^{j}\right)} \right) \right\} y_{rs}. \end{aligned} \tag{7}$$

3.2. Expectation Minimization Model (E-Model). The E-model can be used when the decision-maker wishes to minimize the expected value of the objective function. The expected value of the random variable is considered in place of the random variable. The coefficients C_{rs}^{i}, $(i = 1, 2, \cdots, p)$ of the decision variables y_{rs} are treated as independent random variables with mean $\mu^i_{C_{rs}}$ and variance $\sigma^2_{C_{rs}}$. The deterministic objective function with the expected value of the random variables is as follows:

$$\min Z = \sum_{r=1}^{m}\sum_{s=1}^{n} E\left\{ C^1_{rs} + \left(v_{rs} - v^0_{rs}\right)\left(C^2_{rs} - C^1_{rs}\right) + \cdots + \left(v_{rs} - v^0_{rs}\right) \cdot \left(v_{rs} - v^1_{rs}\right) \cdots \left(v_{rs} - v^{p-1}_{rs}\right) \left(\sum_{i=1}^{p} \frac{C^i_{rs}}{\prod_{i\neq j+1,\, j=0}^{p-1}\left(v^{i-1}_{rs} - v^j_{rs}\right)} \right) \right\} y_{rs}$$
$$= \sum_{r=1}^{m}\sum_{s=1}^{n} \left\{ E\left(C^1_{rs}\right) + \left(v_{rs} - v^0_{rs}\right)\left(E\left(C^2_{rs}\right) - E\left(C^1_{rs}\right)\right) + \cdots + \left(v_{rs} - v^0_{rs}\right) \cdot \left(v_{rs} - v^1_{rs}\right) \cdots \left(v_{rs} - v^{p-1}_{rs}\right) \left(\sum_{i=1}^{p} \frac{E\left(C^i_{rs}\right)}{\prod_{i\neq j+1,\, j=0}^{p-1}\left(v^{i-1}_{rs} - v^j_{rs}\right)} \right) \right\} y_{rs}. \tag{8}$$

Therefore,

$$\min Z = \left\{ \mu^1_{C_{rs}} + \left(v_{rs} - v^0_{rs}\right)\left(\mu^2_{C_{rs}} - \mu^1_{C_{rs}}\right) + \cdots + \left(v_{rs} - v^0_{rs}\right) \cdot \left(v_{rs} - v^1_{rs}\right) \cdots \left(v_{rs} - v^{p-1}_{rs}\right) \left(\sum_{i=1}^{p} \frac{\mu^i_{C_{rs}}}{\prod_{i\neq j+1,\, j=0}^{p-1}\left(v^{i-1}_{rs} - v^j_{rs}\right)} \right) \right\} y_{rs}. \tag{9}$$

3.3. Chance Constraints with Gaussian Distributions. As the constraints (2) and (3) of the defined problem are probabilistic in nature, therefore, we can not apply the usual solution procedure for solving the mathematical problem. The parameters supply $w_r (r = 1, 2, \cdots, m)$ and demand d_s $(s = 1, 2, \cdots, n)$ are random variables that follow Gaussian distributions with known means and variances. In order to fix the notations, we adopt the following notations: $w_r \sim N(\mu_{w_r}, \sigma^2_{w_r})$ and $d_s \sim N(\mu_{d_s}, \sigma^2_{d_s})$.

For every $r = 1, 2, \cdots, m$, consider the constraint (2):

$$P\left(\sum_{s=1}^{n} y_{rs} \leq w_r\right) \geq 1 - \lambda_{w_r} = P\left(w_r \leq \sum_{s=1}^{n} y_{rs}\right) \leq \lambda_{w_r} = P\left(\frac{w_r - \mu_{w_r}}{\sigma_{w_r}} \leq \frac{\sum_{s=1}^{n} y_{rs} - \mu_{w_r}}{\sigma_{w_r}}\right) \leq \lambda_{w_r} = P\left(\zeta_r \leq \frac{\sum_{s=1}^{n} y_{rs} - \mu_{w_r}}{\sigma_{w_r}}\right) \leq \lambda_{w_r}, \tag{10}$$

where $w_r - \mu_{w_r}/\sigma_{w_r} = \zeta_r$. Here, ζ_r is a standard Gaussian-distributed random variable with mean zero and unit variance. Therefore, the equation holds if and only if

$$\phi\left(\frac{\sum_{s=1}^{n} y_{rs} - \mu_{w_r}}{\sigma_{w_r}}\right) \leq \phi\left(-k_{\lambda_{w_r}}\right), \tag{11}$$

where ϕ is the cumulative density function of the standard Gaussian random variable, and $k_{\lambda_{w_r}}$ is such that $\phi(-k_{\lambda_{w_r}}) = \lambda_{w_r}$. By the increasing and bijectivity properties of ϕ, the constraint can be rewritten as

$$\frac{\sum_{s=1}^{n} y_{rs} - \mu_{w_r}}{\sigma_{w_r}} \leq -k_{\lambda_{w_r}}. \tag{12}$$

On simplifying, we get

$$\sum_{s=1}^{n} y_{rs} \leq \mu_{w_r} - k_{\lambda_{w_r}} \sigma_{w_r}, r = 1, 2, \cdots, m. \tag{13}$$

Thus, (13) represents the deterministic constraint of probabilistic constraint (2).

Consider the probabilistic constraint (3).

For every $s = 1, 2, \cdots, n$, we have

$$P\left(\sum_{r=1}^{m} y_{rs} \geq d_s\right) \geq 1 - \delta_{d_s}. \tag{14}$$

Applying the same procedure, we get

$$= P\left(d_s \leq \sum_{r=1}^{m} y_{rs}\right) \geq 1 - \delta_{d_s} = P\left(\eta_s \leq \frac{\sum_{r=1}^{m} y_{rs} - \mu_{d_s}}{\sigma_{d_s}}\right) \geq 1 - \delta_{d_s}, \tag{15}$$

where $d_s - \mu_{d_s}/\sigma_{d_s} = \eta_s$. Here, η_s is the standard Gaussian distributed random variable with mean zero and unit variance. Therefore, the considered equation holds if and only if

$$\phi\left(\frac{\sum_{r=1}^{m} y_{rs} - \mu_{d_s}}{\sigma_{d_s}}\right) \geq \phi\left(g_{\delta_{d_s}}\right), \tag{16}$$

where ϕ is the cumulative density function of the standard Gaussian random variable and $g_{\delta_{d_s}}$ is such that $\phi(g_{\delta_{d_s}}) = 1 - \delta_{d_s}$. The constraint can be rewritten as

$$\frac{\sum_{r=1}^{m} y_{rs} - \mu_{d_s}}{\sigma_{d_s}} \geq g_{\delta_{d_s}}. \tag{17}$$

On simplifying, we obtain

$$\sum_{r=1}^{m} y_{rs} \geq \mu_{d_s} + g_{\delta_{d_s}} \sigma_{d_s} s = 1, 2, \cdots, n. \tag{18}$$

Here, (18) represents the deterministic constraint of probabilistic constraint (3).

Using the methodology given in Subsections 3.1, 3.2, and 3.3 and Equations (9), (13), and (18), the deterministic mathematical model of the proposed problem is

$$\min Z = \left\{ \mu^1_{C_{rs}} + \left(v_{rs} - v^0_{rs}\right)\left(\mu^2_{C_{rs}} - \mu^1_{C_{rs}}\right) + \cdots + \left(v_{rs} - v^0_{rs}\right) \cdot \left(v_{rs} - v^1_{rs}\right) \cdots \left(v_{rs} - v^{p-1}_{rs}\right) \left(\sum_{i=1}^{p} \frac{\mu^i_{C_{rs}}}{\prod_{i\neq j+1,\, j=0}^{p-1} \left(v^{i-1}_{rs} - v^j_{rs}\right)} \right) \right\} y_{rs}, \tag{19}$$

s.t.

$$\begin{aligned} &\sum_{s=1}^{n} y_{rs} \le \mu_{w_r} - k_{\lambda_{w_r}} \sigma_{w_r}, r = 1, 2, \cdots, m, \\ &\sum_{r=1}^{m} y_{rs} \ge \mu_{d_s} + g_{\delta_{d_s}} \sigma_{d_s} s = 1, 2, \cdots, n, \\ &\sum_{r=1}^{m} \left(\mu_{w_r} - k_{\lambda_{w_r}} \sigma_{w_r}\right) \ge \sum_{s=1}^{n} \left(\mu_{d_s} + g_{\delta_{d_s}} \sigma_{d_s}\right), \\ &z_{rs} \ge 0, 0 \le v_{rs} \le p - 1 r = 1, 2, \cdots, m s = 1, 2, \cdots, n. \end{aligned} \tag{20}$$

3.4. Fuzzy Programming Approach. To convert the multiobjective function into the single objective function, the fuzzy programming approach [26] and then the single objective function are solved with the constraints to get the compromise solution to the said problem. The steps for applying the fuzzy programming approach are as follows:

Step 1: firstly, convert the multichoice random parameter to its deterministic form using the NDDIP, which is discussed in Subsection 3.1

Step 2: consider only one objective at a time and ignore the others. Then, determine the values for that objective function, and it will be repeated for all objectives

Step 3: construct a payoff matrix for every objective function at different solutions, which were obtained in Step 2, say

$$\begin{array}{c} \\ y_1 \\ y_2 \\ \vdots \\ y_t \end{array} \begin{array}{c} \begin{matrix} Z_1 & Z_2 & \cdots & Z_t \end{matrix} \\ \begin{pmatrix} Z_1(y_1) & Z_2(y_1) & \cdots & Z_t(y_1) \\ Z_1(y_2) & Z_2(y_2) & \cdots & Z_t(y_2) \\ \vdots & \vdots & \ddots & \vdots \\ Z_1(y_t) & Z_2(y_t) & \cdots & Z_t(y_t) \end{pmatrix} \end{array}, \tag{21}$$

where $y_t = (y^t_{11}, y^t_{12}, \cdots, y^t_{rs})$ is the solution vector for t^{th} objective function.

Step 4: construct a membership function for every objective function. Let l_t and u_t denote the lower and upper bounds, respectively, corresponding to the t^{th} objective function. Then, the membership function for t^{th} objective function is

$$\mu(Z_t(y)) = \begin{cases} 0, & \text{if } Z_t(y) \ge u_t, \\ \dfrac{u_t - Z_t(y)}{u_t - l_t}, & \text{if } l_t < Z_t(y) < u_t, \\ 1, & \text{if } Z_t(y) \le l_t. \end{cases} \tag{22}$$

Step 5: now, using the membership function $(\mu(Z_t(y)))$, the transformed single objective deterministic problem can be stated as $\max \eta$, s.t. $\eta \le \mu(Z_t(y))$, and along with deterministic constraints

Step 6: Solve the deterministic single objective model and find the compromise solution

4. A Case Study

A food product production company wants to transfer its products from its plants to vendors with the least possible transportation cost and time. A product is transferred from 3 different plants (w_1, w_2, and w_3) to 4 different vendors (d_1, d_2, d_3, and d_4) based on the requirement and availability of the product. Due to certain factors, such as weather conditions, the expectations of the market, and the sale of the product, supply and demand are never fixed. They are uncertain in nature. Therefore, supply and demand are treated as independent random variables that follow Gaussian distributions with known means and variances. The values of their means, variances, and aspiration levels are given in Tables 4 and 5, respectively.

The cost and duration of transportation are affected by a variety of factors, such as mode of transportation, road collection tax, road condition, and fluctuations in fuel prices, among others. These are multichoice random parameters with Gaussian distributions in terms of mean and variance. The problem has two objective functions (transportation cost and transportation time), and they will be held simultaneously. The values for the cost and time random parameters are given in Tables 6 and 7, respectively.

Using the methodology and the data, the mathematical formulation of the said problem is as follows:

Cost objective function:

$$\begin{aligned} \min Z_1 = {} & \{9 - 2v_{11} + 3v_{11}(v_{11} - 1)\} y_{11} \\ & + \{12 - 2v_{12} + 2.5v_{12}(v_{12} - 1)\} y_{12} \\ & + \{10 - 2v_{13} + 1.5v_{13}(v_{13} - 1)\} y_{13} \\ & + \{15 - 3v_{14} + 2v_{14}(v_{14} - 1)\} y_{14} \\ & + \{14 - 4v_{21} + 0.5v_{21}(v_{21} - 1)\} y_{21} \\ & + \{6 + v_{22} + 1.5v_{22}(v_{22} - 1)\} y_{22} \\ & + \{15 - 4v_{23} + 2.5v_{23}(v_{23} - 1)\} y_{23} \\ & + \{16 - 2v_{24} - 2.5v_{24}(v_{24} - 1)\} y_{24} \\ & + \{20 - 2v_{31} + 0.5v_{31}(v_{31} - 1)\} y_{31} \\ & + \{13 - 3v_{32} + v_{32}(v_{32} - 1)\} y_{32} \\ & + \{7 + v_{33} + 0.5v_{33}(v_{33} - 1)\} y_{33} \\ & + \{6 + 5v_{34} - 4v_{34}(v_{34} - 1)\} y_{34}. \end{aligned} \tag{23}$$

Table 4: For supply: mean, variance, and their aspiration level.

Mean (μ_{w_r})	Variance $(\sigma^2_{w_r})$	Probabilities (λ_{w_r})
$\mu_{w_1} = 13$	$\sigma^2_{w_1} = 3$	$\lambda_{w_1} = 0.01$
$\mu_{w_2} = 13$	$\sigma^2_{w_2} = 2$	$\lambda_{w_2} = 0.02$
$\mu_{w_3} = 13$	$\sigma^2_{w_3} = 7$	$\lambda_{w_3} = 0.03$

Table 5: For demand: mean, variance, and their aspiration level.

Mean (μ_{d_s})	Variance $(\sigma^2_{d_s})$	Probabilities (δ_{d_s})
$\mu_{d_1} = 7$	$\sigma^2_{d_1} = 5$	$\delta_{d_1} = 0.04$
$\mu_{d_2} = 5$	$\sigma^2_{d_2} = 3$	$\delta_{d_2} = 0.05$
$\mu_{d_3} = 6$	$\sigma^2_{d_3} = 2$	$\delta_{d_3} = 0.06$
$\mu_{d_4} = 4$	$\sigma^2_{d_4} = 1$	$\delta_{d_4} = 0.07$

Table 6: For cost random parameters.

$c^1_{11} \sim N(9,3)$	$c^2_{11} \sim N(7,2)$	$c^3_{11} \sim N(11,4)$
$c^1_{12} \sim N(12,6)$	$c^2_{12} \sim N(10,3)$	$c^3_{12} \sim N(13,6)$
$c^1_{13} \sim N(10,5)$	$c^2_{13} \sim N(8,2)$	$c^3_{13} \sim N(9,3)$
$c^1_{14} \sim N(15,7)$	$c^2_{14} \sim N(12,5)$	$c^3_{14} \sim N(13,4)$
$c^1_{21} \sim N(14,2)$	$c^2_{21} \sim N(10,3)$	$c^3_{21} \sim N(7,2)$
$c^1_{22} \sim N(6,1)$	$c^2_{22} \sim N(7,4)$	$c^3_{22} \sim N(11,4)$
$c^1_{23} \sim N(15,8)$	$c^2_{23} \sim N(11,4)$	$c^3_{23} \sim N(12,5)$
$c^1_{24} \sim N(16,7)$	$c^2_{24} \sim N(18,10)$	$c^3_{24} \sim N(15,7)$
$c^1_{31} \sim N(20,11)$	$c^2_{31} \sim N(18,9)$	$c^3_{31} \sim N(17,4)$
$c^1_{32} \sim N(13,5)$	$c^2_{32} \sim N(10,4)$	$c^3_{32} \sim N(9,4)$
$c^1_{33} \sim N(7,3)$	$c^2_{33} \sim N(8,2)$	$c^3_{33} \sim N(10,3)$
$c^1_{34} \sim N(6,1)$	$c^2_{34} \sim N(11,4)$	$c^3_{34} \sim N(8,3)$

Time objective function:

$$\begin{aligned}\min Z_2 = {} & \{4 - 2u_{11}\}y_{11} + \{7 - 3u_{12} + 2u_{12}(u_{12} - 1)\}y_{12} \\ & + \{6 - u_{13} + 1.5u_{13}(u_{13} - 1)\}y_{13} \\ & + \{8 - u_{14} + u_{14}(u_{14} - 1)\}y_{14} \\ & + \{9 - 4u_{21} + 1.5u_{21}(u_{21} - 1)\}y_{21} \\ & + \{4 + 2u_{22} - 0.5u_{22}(u_{22} - 1)\}y_{22} \\ & + \{10 - 3u_{23} + 2.5u_{23}(u_{23} - 1)\}y_{23} \\ & + \{11 - 4u_{24} - 5u_{24}(u_{24} - 1)\}y_{24} \\ & + \{15 - 3u_{31} + 3.5u_{31}(u_{31} - 1)\}y_{31} \\ & + \{9 - 2u_{32} - 0.5u_{32}(u_{32} - 1)\}y_{32} \\ & + \{3 + 3u_{33} - 2u_{33}(u_{33} - 1)\}y_{33} \\ & + \{4 + 3u_{34} - 2.5u_{34}(u_{34} - 1)\}y_{34},\end{aligned} \tag{24}$$

Table 7: For time random parameters.

$t^1_{11} \sim N(4,1)$	$t^2_{11} \sim N(6,2)$	$t^3_{11} \sim N(8,3)$
$t^1_{12} \sim N(7,2)$	$t^2_{12} \sim N(4,1)$	$t^3_{12} \sim N(5,2)$
$t^1_{13} \sim N(6,3)$	$t^2_{13} \sim N(5,3)$	$t^3_{13} \sim N(7,4)$
$t^1_{14} \sim N(8,3)$	$t^2_{14} \sim N(7,3)$	$t^3_{14} \sim N(8,2)$
$t^1_{21} \sim N(9,3)$	$t^2_{21} \sim N(5,1)$	$t^3_{21} \sim N(4,1)$
$t^1_{22} \sim N(4,3)$	$t^2_{22} \sim N(6,2)$	$t^3_{22} \sim N(7,4)$
$t^1_{23} \sim N(10,5)$	$t^2_{23} \sim N(7,2)$	$t^3_{23} \sim N(9,3)$
$t^1_{24} \sim N(11,5)$	$t^2_{24} \sim N(15,8)$	$t^3_{24} \sim N(9,4)$
$t^1_{31} \sim N(15,7)$	$t^2_{31} \sim N(12,5)$	$t^3_{31} \sim N(16,9)$
$t^1_{32} \sim N(9,3)$	$t^2_{32} \sim N(7,2)$	$t^3_{32} \sim N(4,1)$
$t^1_{33} \sim N(3,1)$	$t^2_{33} \sim N(6,2)$	$t^3_{33} \sim N(5,2)$
$t^1_{34} \sim N(4,1)$	$t^2_{34} \sim N(7,4)$	$t^3_{34} \sim N(5,3)$

s.t.

$$y_{11} + y_{12} + y_{13} + y_{14} \leq 8.84, \tag{25}$$

$$y_{21} + y_{22} + y_{23} + y_{24} \leq 12.03, \tag{26}$$

$$y_{31} + y_{32} + y_{33} + y_{34} \leq 15.2, \tag{27}$$

$$y_{11} + y_{21} + y_{31} \geq 11.02, \tag{28}$$

$$y_{12} + y_{22} + y_{32} \geq 7.94, \tag{29}$$

$$y_{13} + y_{23} + y_{33} \geq 8.26, \tag{30}$$

$$y_{14} + y_{24} + y_{34} \geq 5.5, \tag{31}$$

$$y_{rs} \geq 0 \quad r = 1,2,3, s = 1,2,3,4, \tag{32}$$

$$0 \leq v_{rs} \leq 2, v_{rs} \in \mathbb{Z} \quad r = 1,2,3, s = 1,2,3,4, \tag{33}$$

$$0 \leq u_{rs} \leq 2, u_{rs} \in \mathbb{Z} \quad r = 1,2,3, s = 1,2,3,4. \tag{34}$$

Solving the objective 1, {(23)} along with the constraints (25)–(33), the ideal solution of the model is

$$y_1 = (7,0,1,0,4,0,8,0,0,0,0,8,6), \tag{35}$$

and for the objective 2, {(24)} with the constraints (25)–(32) and (34), then the ideal solution is

$$y_2 = (7,0,1,0,4,8,0,0,0,0,8,6). \tag{36}$$

Using the obtained solution, we formulated a payoff matrix that is shown below:

$$\begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{cc} Z_1 & Z_2 \end{array} \\ \begin{pmatrix} 225 & 115 \\ 253 & 101. \end{pmatrix} \end{array} . \tag{37}$$

The membership function for each objective has been formulated using a payoff matrix:

$$\mu(Z_1(y)) = \begin{cases} 0, & \text{if } Z_1(y) \geq 253, \\ \dfrac{253 - Z_1(y)}{253 - 225}, & \text{if } 225 < Z_1(y) < 253, \\ 1, & \text{if } Z_1(y) \leq 225, \end{cases}$$
$$\mu(Z_2(y)) = \begin{cases} 0, & \text{if } Z_2(y) \geq 115, \\ \dfrac{115 - Z_2(y)}{115 - 101}, & \text{if } 101 < Z_2(y) < 115, \\ 1, & \text{if } Z_2(y) \leq 101. \end{cases} \quad (38)$$

Using the fuzzy programming approach (described in Subsection 3.4), multiobjective functions are converted into single objectives. The nonlinear problem can be stated as

max η

s.t.

$$\begin{aligned} 28\eta \leq{} & (9 - 2v_{11} + 3v_{11}(v_{11} - 1))y_{11} + (12 - 2v_{12} + 2.5v_{12}(v_{12} - 1))y_{12} \\ & + (10 - 2v_{13} + 1.5v_{13}(v_{13} - 1))y_{13} + (15 - 3v_{14} + 2v_{14}(v_{14} - 1))y_{14} \\ & + (14 - 4v_{21} + 0.5v_{21}(v_{21} - 1))y_{21} + (6 + v_{22} + 1.5v_{22}(v_{22} - 1))y_{22} \\ & + (15 - 4v_{23} + 2.5v_{23}(v_{23} - 1))y_{23} + (16 - 2v_{24} - 2.5v_{24}(v_{24} - 1))y_{24} \\ & + (20 - 2v_{31} + 0.5v_{31}(v_{31} - 1))y_{31} + (13 - 3v_{32} + v_{32}(v_{32} - 1))y_{32} \\ & + (7 + v_{33} + 0.5v_{33}(v_{33} - 1))y_{33} + (6 + 5v_{34} - 4v_{34}(v_{34} - 1))y_{34}, \\ 14\eta \leq{} & (4 - 2u_{11})y_{11} + (7 - 3u_{12} + 2u_{12}(u_{12} - 1))y_{12} \\ & + (6 - u_{13} + 1.5u_{13}(u_{13} - 1))y_{13} + (8 - u_{14} + u_{14}(u_{14} - 1))y_{14} \\ & + (9 - 4u_{21} + 1.5u_{21}(u_{21} - 1))y_{21} + (4 + 2u_{22} - 0.5u_{22}(u_{22} - 1))y_{22} \\ & + (10 - 3u_{23} + 2.5u_{23}(u_{23} - 1))y_{23} + (11 - 4u_{24} - 5u_{24}(u_{24} - 1))y_{24} \\ & + (15 - 3u_{31} + 3.5u_{31}(u_{31} - 1))y_{31} + (9 - 2u_{32} - 0.5u_{32}(u_{32} - 1))y_{32} \\ & + (3 + 3u_{33} - 2u_{33}(u_{33} - 1))y_{33} + (4 + 3u_{34} - 2.5u_{34}(u_{34} - 1))y_{34}, \end{aligned} \quad (39)$$

and (25)–(34).

The aforementioned nonlinear deterministic model is solved using LINGO 11.0 software. We obtain the following solution for each objective with aspiration level $\eta = 1$: $y_{11} = 8$, $y_{21} = 3$, $y_{22} = 8$, $v_{24} = 1$, $y_{33} = 5$, and $y_{34} = 5$, and the rest of the decision variables are zero. It shows that the number of units transported from 1st plant to 1st vendor is 8 units; from 2nd plant to 1st, 2nd and 4th vendors are 3, 8, and 1 units, respectively; similarly, from 3rd plant to 3rd, 4th vendors are 5 and 5 units, respectively, for both the objectives (minimum transportation cost and least transporting time). The obtained total transportation cost is $(Z_1 =)197$ units; the total transportation time is $(Z_2 =)87$ units.

5. Conclusion

In this paper, a probabilistic mathematical model for multi-objective, multichoice random transportation problems was developed. The chance-constrained technique was used to convert probabilistic constraints to deterministic constraints. The E-model was applied to the deterministic form of the objective function because the coefficients of the objective function are considered multichoice random parameters, and if the expected value of the objective function is optimized, the entire objective can be optimized. Also, in the objective functions, multichoice parameters were replaced with the interpolation polynomial obtained by the Newton divided difference interpolation. The defined problem contains multiple objectives, so a fuzzy programming approach was used to transform the multiobjective function into a single objective. Finally, a nonlinear deterministic programming problem has been solved, and the solution obtained for multiple objectives simultaneously satisfies all the objectives.

Real-world problems involving transportation, such as lowering maintenance costs in a business environment, logistic management, production planning, and supply chain management, are the applications of the proposed methodology.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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