Research Article

Cosine Similarity Measure between Hybrid Intuitionistic Fuzzy Sets and Its Application in Medical Diagnosis

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In this paper, a cosine similarity measure between hybrid intuitionistic fuzzy sets is proposed. The aim of the paper is to investigate the cosine similarity measure with hybrid intuitionistic fuzzy information and apply it to medical diagnosis. Firstly, we construct the cosine similarity measure between hybrid intuitionistic fuzzy sets, and the relevant properties are also discussed. In order to obtain a reasonable evaluation in group decision, the weight of experts under different attributes is determined by the projection of individual decision information on the ideal decision information, where the ideal decision information is the average values of each expert's evaluation. Furthermore, we propose a decision method for medical diagnosis based on the cosine similarity measure between hybrid intuitionistic fuzzy sets, and the patient can be diagnosed with the disease according to the values of proposed cosine similarity measure. Finally, an example is given to illustrate feasibility and effectiveness of the proposed cosine similarity measure, which is also compared with the existing similarity measures.

1. Introduction

A similarity measure is an important tool for determining the degree of similarity between two objects in many fields, such as pattern recognition, medical diagnosis, and so on. Many similarity measures have been introduced [1–8]. Among them, some similarity measures of intuitionistic fuzzy sets (IFSs) have been proposed. For example, Li and Cheng [3] proposed a similarity measure between IFSs and applied it to pattern recognition. Huang and Yang [2] defined the similarity measure between IFSs based on the Hausdorff distance and used it to calculate the degree of similarity between IFSs. Nguen [9] proposed a new knowledge-based similarity measure between IFSs and applied it to pattern recognition. However, due to the complexity and uncertainty of the decision-making environment, the membership degree and nonmembership degree of IFS need to be expressed by interval rather than the numerical value. Motivated by this, Atanassov and Gargov [10] introduced the concept of interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of IFS. Xu [11] proposed some distance and similarity measures between IVIFSs and applied them to pattern recognition.

On the other hand, the cosine similarity measure based on Bhattacharyya distance was first proposed in Bhattacharyya [12]. Ye [7] proposed a cosine similarity measure for IFSs (C_{IFS}) and applied it to pattern recognition. Furthermore, Ye [13] proposed the cosine similarity measure for IVIFSs (C_{IVIFS}) and applied it to group decision-making problems. However, in the complex group decision-making problem, it is difficult to use a single value to express the alternative under all attributes. Because some attributes might be represented by IFSs, but other attributes are suitable to be represented by IVIFSs. At this time, the people should use hybrid intuitionistic fuzzy set to make a decision. However, the existing methods can not deal with the hybrid fuzzy information. As far as we know, no people studied the cosine similarity measure between hybrid IFSs. Motivated by this, we will introduce the cosine similarity measure with hybrid intuitionistic fuzzy information (C_{HIFS}) in this paper. This generalization makes the C_{HIFS} measure includes C_{IFS} measure and C_{IVIFS} measure as particular case.

In addition, applying the C_{HIFS} measure to group decision-making problems is very interesting. For example, Zhou and Wahab [14] use transmissibility incorporated

with cosine similarity measure to investigate the structural damage detection. Furthermore, Zhou et al. [15] apply transmissibility function with distance measure to separate the intact patterns apart from the damaged pattern. In group decision-making problems, the weight of the experts under different attributes can be obtained by using the projection of individual decision information on the ideal decision information. Then, we aggregate all individual decisions into a collective one and apply the proposed cosine similarity measure between hybrid intuitionistic fuzzy sets to medical diagnosis.

The rest of the paper is organized as follows. In Section 2, we review the cosine similarity measure for IFSs and IVIFSs. In Section 3, we propose the C_{HIFS} measure, some properties are also analyzed. In Section 4, we propose a decision method for medical diagnosis based on the cosine similarity measure between hybrid intuitionistic fuzzy sets. In Section 5, an example is given to illustrate the feasibility and effectiveness of the proposed C_{HIFS} measure. Finally, the conclusion and further research are discussed in Section 6.

2. Preliminaries

Throughout this paper, let $X = \{x_1, x_2, \dots, x_n\}$ be a finite universal set. In this section, we briefly review the IFSs and IVIFSs, the cosine similarity measure between IFSs, and the cosine similarity measure between IVIFSs.

2.1. Intuitionistic Fuzzy Set

Definition 1. Let X be a fixed set, an intuitionistic fuzzy set (IFS) A in X is defined as:

$$A = \left\{ \left(x_j, \mu_A(x_j), \nu_A(x_j) \right) \mid x_j \in X \right\}, \quad (1)$$

where the functions $\mu_A(x_j)$ and $\nu_A(x_j)$ represent the membership degree and nonmembership degree of the element x_j to the set A , respectively, such that $0 \leq \mu_A(x_j) + \nu_A(x_j) \leq 1 \forall x_j \in X$.

The intuitionistic fuzzy index $\pi_A(x_j) = 1 - \mu_A(x_j) - \nu_A(x_j)$, and we have $0 \leq \pi_A(x_j) \leq 1$. For example, $A = (0.4, 0.3)$ is an intuitionistic fuzzy number, and $\pi_A = 0.3$. The space of membership degree of IFS is shown in Figure 1.

In particular, when X has only one element, the IFS $A = \{(x_j, \mu_A(x_j), \nu_A(x_j)) \mid x_j \in X\}$ is reduced to $A = (\mu_A(x_j), \nu_A(x_j))$, which we call it an intuitionistic fuzzy number (IFN).

For any two IFSs $A = (x_j, \mu_A(x_j), \nu_A(x_j))$ and $B = (x_j, \mu_B(x_j), \nu_B(x_j))$, the following operations are true [16]:

- (1) $A + B = (\mu_A(x_j) + \mu_B(x_j) - \mu_A(x_j)\mu_B(x_j), \nu_A(x_j) + \nu_B(x_j) - \nu_A(x_j)\nu_B(x_j))$
- (2) $\lambda A = (1 - (1 - \mu_A(x_j))^\lambda, (1 - (1 - \nu_A(x_j))^\lambda), \lambda > 0$
- (3) $A = B$ if $\mu_A(x_j) = \mu_B(x_j), \nu_A(x_j) = \nu_B(x_j)$

The results of the operations $A + B$ and λA are still IFSs.

FIGURE 1: The membership degree of IFS.

2.2. Interval-Valued Intuitionistic Fuzzy Set

Definition 2. Let X be a fixed set, an interval-valued intuitionistic fuzzy set $\tilde{A}$ is defined as follows:

$$\tilde{A} = \left\{ \left(x_j, [\mu_{\tilde{A}L}(x_j), \mu_{\tilde{A}U}(x_j)], [\nu_{\tilde{A}L}(x_j), \nu_{\tilde{A}U}(x_j)] \right) \mid x_j \in X \right\}, \quad (2)$$

where

$$\mu_{\tilde{A}L}(x_j) \geq 0, \nu_{\tilde{A}L}(x_j) \geq 0; 0 \leq \mu_{\tilde{A}U}(x_j) + \nu_{\tilde{A}U}(x_j) \leq 1, \quad \forall x_j \in X. \quad (3)$$

The interval-valued intuitionistic fuzzy index is defined as $\pi_{\tilde{A}}(x_j) = [\pi_{\tilde{A}L}(x_j), \pi_{\tilde{A}U}(x_j)]$, where

$$\begin{aligned} \pi_{\tilde{A}L}(x_j) &= 1 - \mu_{\tilde{A}U}(x_j) - \nu_{\tilde{A}U}(x_j), \pi_{\tilde{A}U}(x_j) \\ &= 1 - \mu_{\tilde{A}L}(x_j) - \nu_{\tilde{A}L}(x_j). \end{aligned} \quad (4)$$

For example, $\tilde{A} = ([0.3, 0.4], [0.1, 0.4])$ is an interval-valued intuitionistic fuzzy number, the fuzzy index $\pi_{\tilde{A}}(x_j) = [0.2, 0.6]$.

Remark 1. If $\mu_{\tilde{A}L}(x_j) = \mu_{\tilde{A}U}(x_j), \nu_{\tilde{A}L}(x_j) = \nu_{\tilde{A}U}(x_j)$, then the interval-valued intuitionistic fuzzy set is reduced to intuitionistic fuzzy set.

When the set X has only one element, the IVIFS $\tilde{A} = \{(x_j, [\mu_{\tilde{A}L}(x_j), \mu_{\tilde{A}U}(x_j)], [\nu_{\tilde{A}L}(x_j), \nu_{\tilde{A}U}(x_j)]) \mid x_j \in X\}$ is reduced to $\tilde{A} = ([\mu_{\tilde{A}L}(x_j), \mu_{\tilde{A}U}(x_j)], [\nu_{\tilde{A}L}(x_j), \nu_{\tilde{A}U}(x_j)])$, which we call it an interval-valued intuitionistic fuzzy number (IVIFN).

Let $\tilde{A} = (x_j, [\mu_{\tilde{A}L}(x_j), \mu_{\tilde{A}U}(x_j)], [\nu_{\tilde{A}L}(x_j), \nu_{\tilde{A}U}(x_j)])$ and $\tilde{B} = (x_j, [\mu_{\tilde{B}L}(x_j), \mu_{\tilde{B}U}(x_j)], [\nu_{\tilde{B}L}(x_j), \nu_{\tilde{B}U}(x_j)])$ be two IVIFSs, the following operations are true [17]:

- (1) $\tilde{A} + \tilde{B} = ([\mu_{\tilde{A}L}(x_j) + \mu_{\tilde{B}L}(x_j) - \mu_{\tilde{A}L}(x_j)\mu_{\tilde{B}L}(x_j), \mu_{\tilde{A}U}(x_j) + \mu_{\tilde{B}U}(x_j) - \mu_{\tilde{A}U}(x_j)\mu_{\tilde{B}U}(x_j)], [\nu_{\tilde{A}L}(x_j) + \nu_{\tilde{B}L}(x_j) - \nu_{\tilde{A}L}(x_j)\nu_{\tilde{B}L}(x_j), \nu_{\tilde{A}U}(x_j) + \nu_{\tilde{B}U}(x_j) - \nu_{\tilde{A}U}(x_j)\nu_{\tilde{B}U}(x_j)])$
- (2) $\lambda \tilde{A} = ([1 - (1 - \mu_{\tilde{A}L}(x_j))^\lambda, 1 - (1 - \mu_{\tilde{A}U}(x_j))^\lambda], [(1 - \nu_{\tilde{A}L}(x_j))^\lambda, (1 - \nu_{\tilde{A}U}(x_j))^\lambda]), \lambda > 0$

- (3) $\tilde{A} = \tilde{B}$ if $\mu_{\tilde{A}L}(x_j) = \mu_{\tilde{B}L}(x_j), \mu_{\tilde{A}U}(x_j) = \mu_{\tilde{B}U}(x_j),$
 $\nu_{\tilde{A}L}(x_j) = \nu_{\tilde{B}L}(x_j)$ and $\nu_{\tilde{A}U}(x_j) = \nu_{\tilde{B}U}(x_j)$

2.3. Cosine Similarity Measures for IFSs or IVIFSs.

Definition 3 (Ye [7]). Let $A = (\mu_A(x_j), \nu_A(x_j))$ and $B = (\mu_B(x_j), \nu_B(x_j))$ be two IFSs in X , the cosine similarity measure between A and B is defined as follows:

$$C_{IFS}(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \nu_B^2(x_i)}}. \quad (5)$$

The cosine similarity measure between two IFSs A and B satisfies the following properties:

- (1) $0 \leq C_{IFS}(A, B) \leq 1$
- (2) $C_{IFS}(A, B) = C_{IFS}(B, A)$
- (3) $C_{IFS}(A, B) = 1$ if $A = B$

Definition 4 (Ye [13]). Let $\tilde{A} = ([\mu_{\tilde{A}L}(x_j), \mu_{\tilde{A}U}(x_j)], [\nu_{\tilde{A}L}(x_j), \nu_{\tilde{A}U}(x_j)])$ and $\tilde{B} = ([\mu_{\tilde{B}L}(x_j), \mu_{\tilde{B}U}(x_j)], [\nu_{\tilde{B}L}(x_j), \nu_{\tilde{B}U}(x_j)])$ be two IVIFSs in X , the cosine similarity measure between two IVIFSs $\tilde{A}$ and $\tilde{B}$ is defined as follows:

$$C_{IVIFS}(\tilde{A}, \tilde{B}) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_{\tilde{A}L}(x_i)\mu_{\tilde{B}L}(x_i) + \mu_{\tilde{A}U}(x_i)\mu_{\tilde{B}U}(x_i) + \nu_{\tilde{A}L}(x_i)\nu_{\tilde{B}L}(x_i) + \nu_{\tilde{A}U}(x_i)\nu_{\tilde{B}U}(x_i) + \pi_{\tilde{A}L}(x_i)\pi_{\tilde{B}L}(x_i) + \pi_{\tilde{A}U}(x_i)\pi_{\tilde{B}U}(x_i)}{\sqrt{\mu_{\tilde{A}L}^2(x_i) + \mu_{\tilde{A}U}^2(x_i) + \nu_{\tilde{A}L}^2(x_i) + \nu_{\tilde{A}U}^2(x_i) + \pi_{\tilde{A}L}^2(x_i) + \pi_{\tilde{A}U}^2(x_i)} \cdot |H|}, \quad (6)$$

where

$$|H| = \sqrt{\mu_{\tilde{B}L}^2(x_i) + \mu_{\tilde{B}U}^2(x_i) + \nu_{\tilde{B}L}^2(x_i) + \nu_{\tilde{B}U}^2(x_i) + \pi_{\tilde{B}L}^2(x_i) + \pi_{\tilde{B}U}^2(x_i)}. \quad (7)$$

The cosine similarity measure between two IVIFSs $\tilde{A}$ and $\tilde{B}$ satisfies the following properties:

- (1) $0 \leq C_{IVIFS}(\tilde{A}, \tilde{B}) \leq 1$
- (2) $C_{IVIFS}(\tilde{A}, \tilde{B}) = C_{IVIFS}(\tilde{B}, \tilde{A})$
- (3) $C_{IVIFS}(\tilde{A}, \tilde{B}) = 1$ if $\tilde{A} = \tilde{B}$

3. Cosine Similarity Measure with Hybrid Intuitionistic Fuzzy Information

In this section, we will propose the cosine similarity measure with hybrid intuitionistic fuzzy information (C_{HIFS}) and some properties are also discussed.

Definition 5. Let A be fuzzy set (FS) in $X = \{x_1, x_2, \dots, x_n\}$, I and II be two subsets of the attribute set X , such that $I \cup II = X, I \cap II = \emptyset$. If $x_j \in I$, the value of fuzzy set A is characterized by IFSs, if $x_j \in II$, the values of fuzzy set A is

characterized by IVIFSs, then A is called hybrid intuitionistic fuzzy sets (HIFSs).

Definition 6. Let $A = \{(x_j, \mu_A(x_j), \nu_A(x_j)) | x_j \in X\}$ and $B = \{(x_j, \mu_B(x_j), \nu_B(x_j)) | x_j \in X\}$ be two hybrid intuitionistic fuzzy sets, such that if the same attributes $x_j \in I$, $(\mu_A(x_j), \nu_A(x_j))$, and $(\mu_B(x_j), \nu_B(x_j))$ are IFSs, if the same attribute $x_j \in II$, $([\mu_{AL}(x_j), \mu_{AU}(x_j)], [\nu_{AL}(x_j), \nu_{AU}(x_j)])$, and $([\mu_{BL}(x_j), \mu_{BU}(x_j)], [\nu_{BL}(x_j), \nu_{BU}(x_j)])$ are IVIFSs, which we call A and B the same type hybrid intuitionistic fuzzy sets.

Definition 7. Suppose A and B are the same type hybrid intuitionistic fuzzy sets, that is, if $x_j \in I$, $(\mu_A(x_j), \nu_A(x_j))$, and $(\mu_B(x_j), \nu_B(x_j))$ are IFSs, if the same attribute $x_j \in II$, $([\mu_{AL}(x_j), \mu_{AU}(x_j)], [\nu_{AL}(x_j), \nu_{AU}(x_j)])$, and $([\mu_{BL}(x_j), \mu_{BU}(x_j)], [\nu_{BL}(x_j), \nu_{BU}(x_j)])$ are IVIFSs, then the cosine similarity measure between hybrid intuitionistic fuzzy sets A and B is defined as follows:

$$C_{HIFS}(A, B) = \frac{1}{n} \left[\sum_{x_j \in I} \frac{\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i)}{\sqrt{\mu_A^2(x_i) + \nu_A^2(x_i) + \pi_A^2(x_i)}\sqrt{\mu_B^2(x_i) + \nu_B^2(x_i) + \pi_B^2(x_i)}} \right. \\ \left. + \sum_{x_j \in II} \frac{\mu_{AL}(x_i)\mu_{BL}(x_i) + \mu_{AU}(x_i)\mu_{BU}(x_i) + \nu_{AL}(x_i)\nu_{BL}(x_i) + \nu_{AU}(x_i)\nu_{BU}(x_i) + \pi_{AL}(x_i)\pi_{BL}(x_i) + \pi_{AU}(x_i)\pi_{BU}(x_i)}{\sqrt{\mu_{AL}^2(x_i) + \mu_{AU}^2(x_i) + \nu_{AL}^2(x_i) + \nu_{AU}^2(x_i) + \pi_{AL}^2(x_i) + \pi_{AU}^2(x_i)} \cdot \sqrt{\mu_{BL}^2(x_i) + \mu_{BU}^2(x_i) + \nu_{BL}^2(x_i) + \nu_{BU}^2(x_i) + \pi_{BL}^2(x_i) + \pi_{BU}^2(x_i)}} \right]. \quad (8)$$

Remark 2. If $I = \emptyset$, then C_{HIFS} measure is reduced to C_{IVIFS} measure.

Remark 3. If $II = \emptyset$, then C_{HIFS} measure is reduced to C_{IFS} measure.

Theorem 1. The cosine similarity measure between two hybrid intuitionistic fuzzy sets A and B satisfies the following properties:

- (1) $0 \leq C_{\text{HIFS}}(A, B) \leq 1$
- (2) $C_{\text{HIFS}}(A, B) = C_{\text{HIFS}}(B, A)$
- (3) $C_{\text{HIFS}}(A, B) = 1$ if $A = B$

Proof

- (1) It is obvious that the property (1) is true according to the cosine value in $[0, 1]$
- (2) Because the multiplication of numbers satisfies the commutative law, if the positions of A and B are exchanged in the computation of cosine measure, the result values will not change, so the property (3) is true.
- (3) If $A = B$, $x_i \in I$, we have $\mu_A(x_i) = \mu_B(x_i)$ and $\nu_A(x_i) = \nu_B(x_i)$.

If $A = B$, $x_i \in II$, we have $\mu_{AL}(x_i) = \mu_{BL}(x_i)$, $\mu_{AU}(x_i) = \mu_{BU}(x_i)$, $\nu_{AL}(x_i) = \nu_{BL}(x_i)$, and $\nu_{AU}(x_i) = \nu_{BU}(x_i)$, then $C_{\text{HIFS}}(A, B) = 1$ is obvious obtained.

4. Multiple-Attribute Group Decision-Making with the Cosine Similarity Measure between Hybrid Intuitionistic Fuzzy Sets

In this section, we will apply the C_{HIFS} measure between hybrid intuitionistic fuzzy sets to medical diagnosis. The C_{HIFS} measure can be applied in many situations, such as pattern recognition, medical diagnosis, and so on. The main motivation for considering this model is that the representation of the decision information is very complex. We need several doctors correctly to evaluate the symptoms of the disease. The doctor usually provides his/her preferences for symptoms with IFSs or IVIFSs. Suppose that doctors are good at different diagnostic skills, we can obtain the weights of doctors based on the projection of individual decision on the ideal decision; then, all individual diagnosis decisions are aggregated into a collective one. At last, we apply the C_{HIFS} measure to medical diagnosis.

In a given pathology, suppose that a set of symptoms $S = (s_1, s_2, \dots, s_n)$, a set of diagnoses $A = (A_1, A_2, \dots, A_m)$ and a set of medical experts $E = (e_1, e_2, \dots, e_t)$. Assume that a patient has all the symptoms, which can be represented by the hybrid intuition fuzzy set $\tilde{B}$, our aim is to diagnose what kind of diagnoses the patient $\tilde{B}$ belong to.

In order to solve this problem, we first introduce some relevant concepts.

Definition 8. Let $A' = (a_{ij})_{m \times n} = (\mu_{A_i}(x_j), \nu_{A_i}(x_j))_{m \times n}$ be a decision matrix, I and II be two subsets of the attribute set $X = \{x_j | j = 1, 2, \dots, n\}$, such that $I \cup II = X$ and $I \cap II = \emptyset$. If the attribute $x_j \in I$, then the evaluation values a_{ij} are IFSs, if the attribute $x_j \in II$, then the evaluation values a_{ij} are IVIFSs. In this case, A' is called a hybrid intuitionistic fuzzy matrix.

Definition 9. Let $A' = (A_1, A_2, \dots, A_m)^T$ and $B' = (B_1, B_2, \dots, B_m)^T$ be two hybrid intuitionistic fuzzy matrices, where $A_i = (a_{i1}, a_{i2}, \dots, a_{in})$ and $B_i = (b_{i1}, b_{i2}, \dots, b_{in})$ ($i = 1, 2, \dots, m$), if they satisfy the following conditions:

- (1) $a_{ij} = (\mu_{A_i}(x_j), \nu_{A_i}(x_j))$ about x_j in A_i if and only if $b_{ij} = (\mu_{B_i}(x_j), \nu_{B_i}(x_j))$ about x_j in B_i
- (2) $a_i = ([\mu_{A_iL}(x_j), \mu_{A_iU}(x_j)], [\nu_{A_iL}(x_j), \nu_{A_iU}(x_j)])$ about x_j in A_i if and only if $b_{ij} = ([\mu_{B_iL}(x_j), \mu_{B_iU}(x_j)], [\nu_{B_iL}(x_j), \nu_{B_iU}(x_j)])$ about x_j in B_i , then A_i and B_i are the same type vector, A' and B' are the same type hybrid intuitionistic fuzzy matrices.

Now, suppose that a set of diagnoses $A' = (A_1, A_2, \dots, A_m)$, where A_i is represented by IFS $A_i = (x_j, \mu_{A_i}(x_j), \nu_{A_i}(x_j))$ or IVIFS $A_i = (x_j, [\mu_{A_iL}(x_j), \mu_{A_iU}(x_j)], [\nu_{A_iL}(x_j), \nu_{A_iU}(x_j)])$ ($i = 1, 2, \dots, m$). We should diagnose what kind of disease the patient $\tilde{B}$ belongs to. Furthermore, assume that the patient $\tilde{B}$ is represented by the same type intuitionistic fuzzy set as A_i . In the following, we will present the method for application of C_{HIFS} measure to medical diagnosis, which involves the following steps:

Step 1. Each medical expert provides his/her individual decision matrix about the relation between the diagnosis and the symptoms.

Step 2. According to the expert's diagnostic decision matrix $R_k = (r_{ij}^k)_{m \times n}$, the ideal decision information should be close to the opinions of most doctors; then, we define the ideal relation $R^* = (r_{ij}^*)_{m \times n}$ between the diagnosis A_i ($i = 1, 2, \dots, m$) and the symptom s_j ($j = 1, 2, \dots, n$) as follows:

$$R^* = (r_{ij}^*)_{m \times n} = \begin{matrix} & s_1 & s_2 & \cdots & s_n \\ \begin{matrix} A_1 \\ A_2 \\ \vdots \\ A_m \end{matrix} & \begin{pmatrix} r_{11}^* & r_{12}^* & \cdots & r_{1n}^* \\ r_{21}^* & r_{22}^* & \cdots & r_{2n}^* \\ \vdots & \vdots & \cdots & \vdots \\ r_{m1}^* & r_{m2}^* & \cdots & r_{mn}^* \end{pmatrix} \end{matrix} \quad (9)$$

$$\text{If } j \in I, r_{ij}^* = (\mu_{ij}^*, \nu_{ij}^*) = \left(\frac{1}{t} \sum_{k=1}^t \mu_{ij}^{(k)}, \frac{1}{t} \sum_{k=1}^t \nu_{ij}^{(k)} \right). \quad (10)$$

$$\begin{aligned} \text{If } j \in II, r_{ij}^* &= (\tilde{\mu}_{ij}^*, \tilde{\nu}_{ij}^*) \\ &= \left(\left[\frac{1}{t} \sum_{k=1}^t \mu_{ijL}^{(k)}, \frac{1}{t} \sum_{k=1}^t \mu_{ijU}^{(k)} \right], \left[\frac{1}{t} \sum_{k=1}^t \nu_{ijL}^{(k)}, \frac{1}{t} \sum_{k=1}^t \nu_{ijU}^{(k)} \right] \right). \end{aligned} \quad (11)$$

Step 3. Medical experts may give unreasonable assessments when they encounter unfamiliar symptoms. So, it is not very reasonable to assume that each expert has equal weights. In order to obtain a reasonable evaluation, the weights of medical experts under different attributes are obtained by the projection of the individual evaluation on the ideal evaluation r_{ij}^* . The greater the weight of the expert is, the closer the evaluation value is to the ideal evaluation.

The projection of each decision on the ideal decision is given by

$$\text{if } j \in \text{I}, \Pr j_{r_{ij}^*}^{r_{ij}^{(k)}} = \frac{\mu_{ij}^{(k)} \mu_{ij}^* + \nu_{ij}^{(k)} \nu_{ij}^* + \pi_{ij}^{(k)} \pi_{ij}^*}{\sqrt{\mu_{ij}^{*2} + \nu_{ij}^{*2} + \pi_{ij}^{*2}}}, \quad (12)$$

$$\text{if } j \in \text{II}, \Pr j_{r_{ij}^*}^{r_{ij}^{(k)}} = \frac{\mu_{ijL}^{(k)} \mu_{ijL}^* + \mu_{ijU}^{(k)} \mu_{ijU}^* + \nu_{ijL}^{(k)} \nu_{ijL}^* + \nu_{ijU}^{(k)} \nu_{ijU}^* + \pi_{ijL}^{(k)} \pi_{ijL}^* + \pi_{ijU}^{(k)} \pi_{ijU}^*}{\sqrt{\mu_{ijL}^{*2} + \mu_{ijU}^{*2} + \nu_{ijL}^{*2} + \nu_{ijU}^{*2} + \pi_{ijL}^{*2} + \pi_{ijU}^{*2}}}. \quad (13)$$

Then the weight of medical expert's evaluation on different symptoms can be defined as

$$w_{ij}^{(k)} = \frac{\Pr j_{r_{ij}^*}^{r_{ij}^{(k)}}}{\sum_{k=1}^t \Pr j_{r_{ij}^*}^{r_{ij}^{(k)}}}, \quad k = 1, 2, \dots, t; \quad (14)$$

$$i = 1, 2, \dots, m; j = 1, 2, \dots, n.$$

Step 4. According to the recognition principle of maximum degree of cosine similarity measure, the process of diagnosis $\tilde{B}$ to A_k is derived by $k = \text{Max}_{1 \leq i \leq m} (C_{\text{HIFS}}(A_i, \tilde{B}))$.

5. Numerical Example

In this section, the proposed cosine similarity measure between hybrid IFSs is applied in medical diagnosis to demonstrate its effectiveness.

5.1. Illustration of the Cosine Similarity Measures for Hybrid IFSs. Assume that a set of diagnosis $A = \{A_1 \text{ (viral fever)}, A_2 \text{ (typhoid)}, A_3 \text{ (stomach problem)}, A_4 \text{ (chest problem)}\}$ and a set of symptoms $S = \{s_1 \text{ (temperature)}, s_2 \text{ (stomach pain)}, s_3 \text{ (cough)}, s_4 \text{ (chest pain)}\}$. Suppose a patient has all the symptoms, which can be represented by the following hybrid intuitionistic fuzzy information (data obtained through a survey of doctors):

$$\tilde{B} = \{(s_1, 0.5, 0.4), (s_2, 0.6, 0.2), (s_3, [0.5, 0.6], [0.2, 0.3]), (s_4, 0.4, 0.2)\}. \quad (15)$$

There are three medical experts evaluate each diagnosis with all the symptoms, which are represented by the hybrid IFSs, the results are shown in Tables 1–3.

By step 3 in Section 4, applying (12)–(14), we can calculate the weights of each medical expert for the diagnosis with respect to different symptoms, which are obtained in Tables 5–7.

From the previous formula $C_{\text{HIFS}}(A', B)$, we can calculate the cosine similarity measure between $\tilde{A}_i$ ($i = 1, 2, 3, 4$) and $\tilde{B}$ as follows:

$$\begin{aligned} C_{\text{HIFS}}(\tilde{A}_1, \tilde{B}) &= 0.9674, \\ C_{\text{HIFS}}(\tilde{A}_2, \tilde{B}) &= 0.9477, \\ C_{\text{HIFS}}(\tilde{A}_3, \tilde{B}) &= 0.9140, \\ C_{\text{HIFS}}(\tilde{A}_4, \tilde{B}) &= 0.9511. \end{aligned} \quad (16)$$

We can conclude that the diagnosis of the patient $\tilde{B}$ is viral fever (A_1).

5.2. Comparison Analysis. In this subsection, the existing cosine similarity measure is used to compare with the same numerical example. In the numerical example, the decision information is represented with hybrid IFS, we can transform it into a unified form. For example, the relation between the diagnosis and the symptoms under the attribute s_3 of experts is IVIFSs, and if we use the cosine similarity measure C_{IFS} proposed by Ye [7] to calculate the numerical example, we should convert the corresponding IVIFSs to IFS according to the midpoints of IVIFSs. For example, $([0.4, 0.6], [0.1, 0.3])$ can be converted to (0.5) . Then using the cosine similarity measure C_{IFS} proposed by Ye [7], we can obtain the corresponding cosine similarity measure values: $C_{\text{IFS}}(A_1, \tilde{B}) = 0.9691$, $C_{\text{IFS}}(A_2, \tilde{B}) = 0.9546$, $C_{\text{IFS}}(A_3, \tilde{B}) = 0.9377$, and $C_{\text{IFS}}(A_4, \tilde{B}) = 0.9586$. That is to say, the diagnosis of the patient $\tilde{B}$ is still the viral fever A_1 . The proposed cosine similarity between hybrid IFS in this paper produces the same results as the existing methods. This means that the proposed method is feasible and effective, and it has some advantages in solving multiple criteria decision-making problems. On one hand, the method is more convenient to make decision for decision makers, who can express their preferences over the decision information by IFS or IVIFS simultaneously. On the other hand, because the information conversion will be lost in decision-making process, there are no information conversions between IFSs and IVIFSs in this model, the alternatives will be ranked directly based on the original decision information.

6. Conclusion

The paper proposed the cosine similarity measure between hybrid intuitionistic fuzzy sets, and the proposed method

TABLE 1: The relation between the diagnosis and the symptoms—expert 1.

	s_1	s_2	s_3	s_4
A_1	(0.5, 0.4)	(0.5, 0.3)	([0.4, 0.6], [0.1, 0.3])	(0.4, 0.4)
A_2	(0.7, 0.3)	(0.7, 0.2)	([0.3, 0.5], [0.4, 0.5])	(0.6, 0.2)
A_3	(0.8, 0.1)	(0.6, 0.4)	([0.6, 0.7], [0.2, 0.3])	(0.6, 0.3)
A_4	(0.7, 0.2)	(0.5, 0.2)	([0.5, 0.7], [0.1, 0.2])	(0.5, 0.3)

TABLE 2: The relation between the diagnosis and the symptoms—expert 2.

	s_1	s_2	s_3	s_4
A_1	(0.4, 0.5)	(0.6, 0.2)	([0.5, 0.6], [0.2, 0.3])	(0.3, 0.4)
A_2	(0.5, 0.2)	(0.7, 0.2)	([0.4, 0.7], [0.1, 0.3])	(0.7, 0.1)
A_3	(0.6, 0.2)	(0.5, 0.1)	([0.5, 0.7], [0.1, 0.2])	(0.6, 0.2)
A_4	(0.7, 0.1)	(0.4, 0.3)	([0.3, 0.6], [0.2, 0.4])	(0.4, 0.3)

TABLE 3: The relation between the diagnosis and the symptoms—expert 3.

	s_1	s_2	s_3	s_4
A_1	(0.5, 0.3)	(0.6, 0.2)	([0.4, 0.6], [0.2, 0.3])	(0.5, 0.4)
A_2	(0.7, 0.2)	(0.4, 0.4)	([0.5, 0.7], [0.1, 0.3])	(0.6, 0.3)
A_3	(0.6, 0.3)	(0.7, 0.3)	([0.6, 0.8], [0.1, 0.2])	(0.7, 0.2)
A_4	(0.5, 0.2)	(0.5, 0.3)	([0.3, 0.6], [0.1, 0.4])	(0.6, 0.1)

According to step 2 in Section 4, applying (10) and (11), respectively, the ideal relation between the diagnosis and the symptoms are shown in Table 4.

TABLE 4: The ideal relation between the diagnosis and the symptoms.

	s_1	s_2	s_3	s_4
A_1^*	(0.467, 0.4)	(0.567, 0.233)	([0.433, 0.6], [0.167, 0.3])	(0.4, 0.4)
A_2^*	(0.633, 0.233)	(0.6, 0.267)	([0.4, 0.633], [0.2, 0.367])	(0.633, 0.2)
A_3^*	(0.667, 0.2)	(0.6, 0.267)	([0.567, 0.733], [0.133, 0.233])	(0.633, 0.233)
A_4^*	(0.633, 0.167)	(0.467, 0.267)	([0.367, 0.633], [0.133, 0.333])	(0.5, 0.233)

TABLE 5: The weights of the medical expert 1 for A_i with respect to s_j .

	s_1	s_2	s_3	s_4
A_1	0.3427	0.3155	0.3391	0.3333
A_2	0.3614	0.3614	0.3052	0.3223
A_3	0.3761	0.3466	0.3293	0.3263
A_4	0.3458	0.3395	0.3314	0.3313

TABLE 6: The weights of the medical expert 2 for A_i with respect to s_j .

	s_1	s_2	s_3	s_4
A_1	0.3371	0.3423	0.3311	0.3148
A_2	0.2841	0.3614	0.3474	0.3531
A_3	0.3097	0.2821	0.3196	0.3193
A_4	0.3594	0.3210	0.3276	0.3106

TABLE 7: The weights of the medical expert 3 for A_i with respect to s_j .

	s_1	s_2	s_3	s_4
A_1	0.3202	0.3422	0.3298	0.3519
A_2	0.3545	0.2772	0.3474	0.3246
A_3	0.3142	0.3713	0.3511	0.3544
A_4	0.2948	0.3395	0.3410	0.3581

When the weight values of the experts are determined, the aggregated evaluating decision results provided by different experts are obtained in Table 8.

TABLE 8: The aggregated relation between the diagnosis and the symptom.

	s_1	s_2	s_3	s_4
$\bar{A}_1$	(0.4683, 0.3933)	(0.5708, 0.2273)	([0.4351, 0.6], [0.1581, 0.3])	(0.4093, 0.4)
$\bar{A}_2$	(0.5316, 0.2316)	(0.6364, 0.2424)	([0.4097, 0.6494], [0.1527, 0.3506])	(0.6386, 0.1786)
$\bar{A}_3$	(0.6918, 0.175)	(0.6172, 0.2431)	([0.5704, 0.7398], [0.1256, 0.2286])	(0.6388, 0.2283)
$\bar{A}_4$	(0.6513, 0.1559)	(0.4699, 0.2614)	([0.3739, 0.6364], [0.1255, 0.3179])	(0.5115, 0.2024)

would be quite good for some real-world applications, such as pattern recognition and medical diagnosis. Through the proposed cosine similarity measure, we can classify the patient $\bar{B}$ in one of the diagnosis $A_1, A_2, \dots, A_m$. Finally, a numerical example illustrated the application and efficiency of the developed approach, which is also compared to the existing methods. In future research, we expect to develop further extensions of the C_{HIFS} measure by adding the new characteristic, such as ordered weighted averaging operator, and we will also consider other applications of the proposed C_{HIFS} measure.

Abbreviations

IFS: Intuitionistic fuzzy set
 IVIFS: Interval-valued intuitionistic fuzzy set
 HIFS: Hybrid intuitionistic fuzzy set
 C_{IFS} : Cosine similarity measure for intuitionistic fuzzy set
 C_{IVIFS} : Cosine similarity measure for interval-valued intuitionistic fuzzy set
 C_{HIFS} : Cosine similarity measure for hybrid intuitionistic fuzzy set.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication for the paper.

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