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Research Article

Using Modified Intelligent Experimental Design in Parameter Estimation of Chaotic Systems

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Computational modeling plays an important role in prediction and optimization of real systems and processes. Models usually have some parameters which should be set up to the proper value. Therefore, parameter estimation is known as an important part of the modeling and system identification. It usually refers to the process of using sampled data to estimate the optimum values of parameters. The accuracy of model can be increased by adjusting its parameters to the optimum value which need a richer dataset. One simple solution for having a richer dataset is increasing the amount of data, but that can be costly and time consuming. When using data from animals or people, it is especially important to have a proper plan. There are several available methods for parameter estimation in dynamical systems; however there are some basic differences in chaotic systems due to their sensitivity to initial condition (butterfly effect). Accordingly, in this paper, a new cost function which is proper for chaotic systems is applied to the chaotic one-dimensional map. Then the efficiency of a newly introduced intelligent method experimental design in extracting proper data is investigated. The results show the success of the proposed method.

1. Introduction

Computational modeling plays an important role in the advancement of science by helping us to predict, optimize, simulate, and study the behavior of complex systems. There are many different ways to generate computational models but they can be divided into categories like white box and black box models. White box models are those in which every connection between input(s) and output(s) of a system can be tracked and analyzed correctly [1–3]. Unfortunately, understanding the interactions among components of a complex system in the real world bears some difficulties. Therefore, the choice is limited by two options: finding a more suitable way for modeling and not using any model. Since models have many advantages, an alternative kind of models called black box has been presented [1]. In fact, these models mimic the behavior of the original systems so they need to be trained, although their structures and parameters may have no relation to the actual structures of the systems. Thus, it is better to select the most appropriate model for the intended purpose. For example, two widely used structures which have shown good performance in real world applications are neural networks [4] and neurofuzzy models [5]. While the former is more efficient in extrapolation and more robust against high dimensional problems, the latter is very good for interpolation and providing better interpretation [1]. However, proper performance of the selected structure is highly dependent on the real experimental data used to train it [6–8]. The richness of the training set increases the accuracy of a model. If the dataset is not sufficiently rich, the model may be inaccurate, at least in the area where there was a paucity of training data [8]. A solution for obtaining a richer dataset is increasing the number of data points. However, in real problems, generation of additional data may require considerable effort, time, and money [7, 8]. Thus it is important to find a way to generate datasets that are as rich as possible using experimental design (ED) methods [6–8]. Although experimental design methods are not directly involved in the modeling procedure, they are equally important.

Control and synchronization of chaotic systems have attracted lots of interest in a variety of areas of science in recent years [9, 10]. They usually require tuning the parameters of the model. However, direct measurement of parameters in a real system is often difficult. Therefore, estimation of parameters from an observed chaotic scalar time series has become an active area of research [11–14]. A basic method for achieving this goal involves optimization in which the model parameters are chosen to minimize some cost functions. In this work, we investigate the efficiency of the recently proposed ED method called the Twilight Method Experimental Design (TMED) [15] for extracting rich data from a one-dimensional map. The main characteristic of the TMED is the use of prior knowledge obtained from the gathered data in extracting new data in a biologically inspired way. We show that this method not only has that advantage but also can be tuned for special purposes such as optimization, which usually is the main goal in modeling [16]. The remainder of the paper is organized as follows: in the next section we introduce the problem which involves the details about the chaotic map we deal with and the proper cost function which we want to optimize. In the third section, we describe the TMED in detail. Section 4 gives the numerical results, and Section 5 is the conclusion.

2. The Optimization Problem

Periodic flashes of light have been applied to examine properties of the visual system. The authors in [17] proposed a simple map (1) which can mimic the flicker vision of a salamander [17]. The model uses nonlinear feedback to account for period doubling in the ERG response to periodic flashes. For more detail about the model, see [17].

$$y(n+1) = \frac{\alpha}{A + By(n)^4} + C. \tag{1}$$

This system has fixed-point in every y^* which is correct in

$$\begin{gathered} y^* = \frac{\alpha}{A + By^{*4}} + C \longrightarrow \\ By^{*5} - CBy^{*4} + Ay^* - \alpha - CA = 0. \end{gathered} \tag{2}$$

This equation has 5 roots which can be real or complex. For each fixed-point, if $|y^*| < 1$ that fixed-point is stable and if it is $|y^*| > 1$, it is unstable. The values of parameters are chosen as $\alpha = 4.1$, $A = 1$, $B = 1$, and $C = -2.65$ according to [17].

Although chaotic systems have random-like behavior in the time domain, they are ordered in state space and have a specific topology. Here, we propose using the geometrical similarity between these attractors as the objective function for parameter estimation [18, 19].

The first step to achieve our purpose is constructing a return map based on one real observed time series and the data from the model. Then, we perform the following steps:

(a) For each point of the real return map, we find its nearest neighbor in the model return map and calculate its Euclidean distance separation.

(b) For each point of the model return map, we find its nearest neighbor in the real return map and calculate its Euclidean distance separation.

We take the cost function as the average of those distances over the whole dataset.

3. Twilight Method Experimental Design

The TMED [15] is a kind of sequential experimental design inspired by evolutionary algorithms. In this algorithm, we first determine the number of data points we want to extract based on limitations such as cost and time (call this number M). Then we obtain an initial shadow of the system behavior by extracting some of those M data points in a way that they cover the input space area (here we have used factorial design). The number of these initial data points is N (obviously $N < M$). In other words, these N data are initial conditions for the TMED. We consider each data point as an individual in a population of honey bees. Now we want to produce new individuals (data) using the current population. To do this, an individual is selected as queen to reproduce a new offspring. Like almost all of evolutionary algorithms, we need to define a fitness function (FF) so that we can select the best individual as the queen. We are free to define a FF based on what we want the ED to accomplish. But they can be divided into two categories which are a FF for extracting data in nonsmooth areas of the system and another FF for extracting data in optimum areas of system (see [15] for more detail). Here our goal is to achieve the optimum value for model's parameters so we use second group of FFs which is for extracting data in optimum areas of system. The flowchart of the mentioned procedure can be seen in Figure 1.

3.1. FFs for Extracting Data in Optimum Areas of the System. In this stage, a new term is designed so that the local optimum (without losing generality, here minimum) points become prominent.

Heuristically, the higher the number of neighbor points that have a value greater than a point under consideration is, the greater the prominence of that point should be. Thus the fitness function is multiplied by a correction term as follows:

$$\left| \sum_{j=1}^{H} \left[0.5 \times \operatorname{sign}\left(f\left(x_i\right) - f\left(x_j\right) \right) \right] \right|, \tag{3}$$

where $[\cdot]$ represents the nearest integer function, x_i is the considered data, x_j is its jth neighbor, and f is the cost function.

If the value of the function f in x_i is less than the value of f at the neighbors of that point, $\operatorname{sign}(f(x_i) - f(x_j))$ is equal to -1 for all j. The nearest integer function changes numbers between -1 and 0 into -1. Also, values between 0 and 1 (which represent data points for which the value of f is greater than that at the point under consideration) are mapped into 0. Therefore, (3) describes the number of neighbors of a point

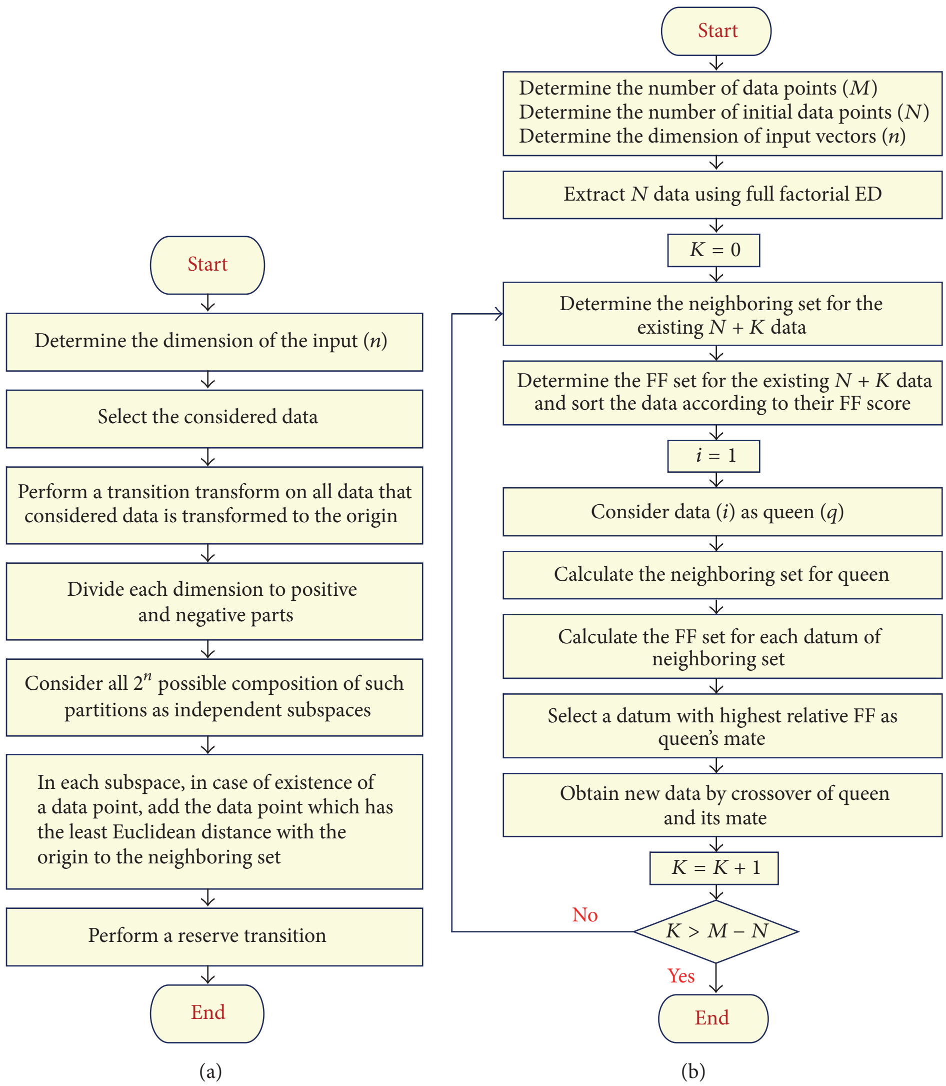

Figure 1: (a) Flowchart of the neighbor selection algorithm. (b) Flowchart of the proposed algorithm.

in which the value of f is less than that point. Finally, the FF will be

$$\mathrm{FF}(x_i) = \left|\sum_{j=1}^{H}\left[0.5 \times \operatorname{sign}\left(f(x_i) - f(x_j)\right)\right]\right| \times \left|\sum_{j=1}^{H}\left(f(x_i) - f(x_j)\right)\right| \times \prod_{j=1}^{H} d_j, \quad (4)$$

where $[\cdot]$ represents the nearest integer function, x_i is the considered data, x_j is its jth neighbor, f is the cost function, and d_j is the Euclidean distance between x_i and x_j.

This FF gives higher scores to points which (a) are local minimum, (b) have sharper slope with their neighbors, and (c) are located in areas with lower densities of data.

3.2. Modified TMED (MTMED): New Approach for Finding Optimum Value. An alternative method is provided in this section. In this approach we choose the best answer in each iteration as a queen. Then a new term (a noise) is added in order to construct new data. This new term is like a perturbation to seek the neighbor's point for better answer. Since the experimental design method is very sensitive to local minimum, this approach solves this problem by creating turbulence in the search space. So the only difference is that when the queen is selected (here queen is the point which has optimum value) and the best neighbor of queen is calculated, a little perturbation (white Gaussian noise) is added to the queen in the direction of the best neighbor to achieve new data.

4. Results

We have applied both TMED and MTMED on system (1), in order to find the optimum values of the parameters. In both,

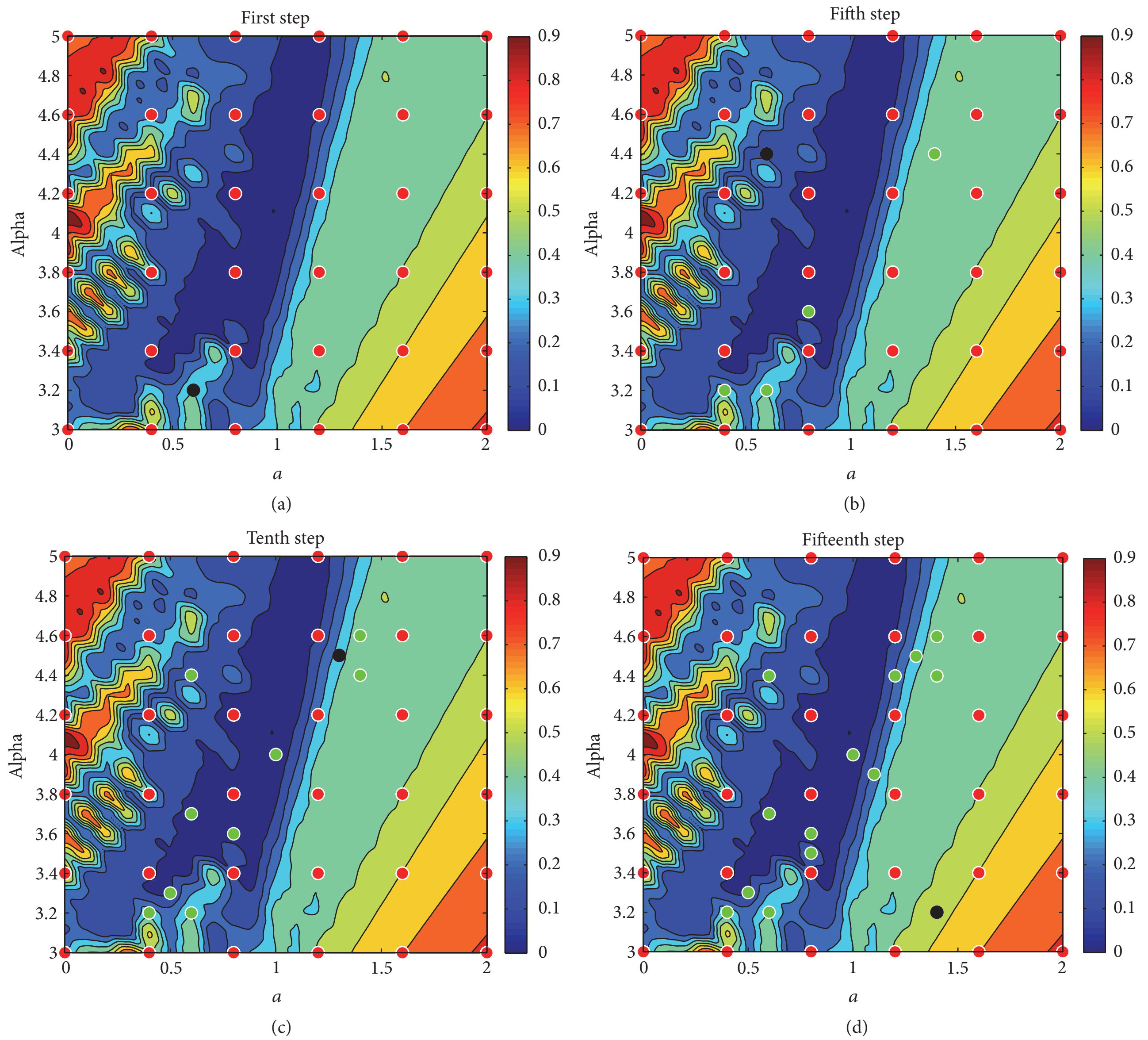

Figure 2: Process of finding the best parameters using TMED algorithm. (a), (b), (c), and (d) represent the first, 5th, 10th, and 15th iteration, respectively. It can be seen that the individuals (points) converge to the optimum area. Red, green, and black points show the first, newly generated, and queen data in every iteration.

the algorithm starts with 36 initial data items and then 15 more data items are created.

Figure 2 shows the process of finding the best parameters using TEDM algorithm. Parts (a), (b), (c), and (d) represent the first, 5th, 10th, and 15th iteration, respectively. It can be seen that the individuals (points) converge to the optimum area. Red, green, and black points show the first, newly generated, and queen data in every iteration.

The final population is shown in Figure 3.

Figure 4 shows the process of finding the best parameters using MTEDM algorithm. Parts (a), (b), (c), and (d) represent the first, 5th, 10th, and 15th iteration, respectively. It can be seen that the individuals (points) converge to the optimum area. Red, green, and black points show the first, newly generated, and queen data in every iteration.

The final population is shown in Figure 5.

Comparing Figures 3 and 5, it can be seen that although TMED tries to converge to better areas, it cannot do it in 51 steps (36 + 15), while MTMED does it very well.

5. Conclusion

Modeling of systems and processes plays an essential role in their optimization and prediction. Since many real systems are complex and we do not know the exact relations among their components, black box models have increasingly been

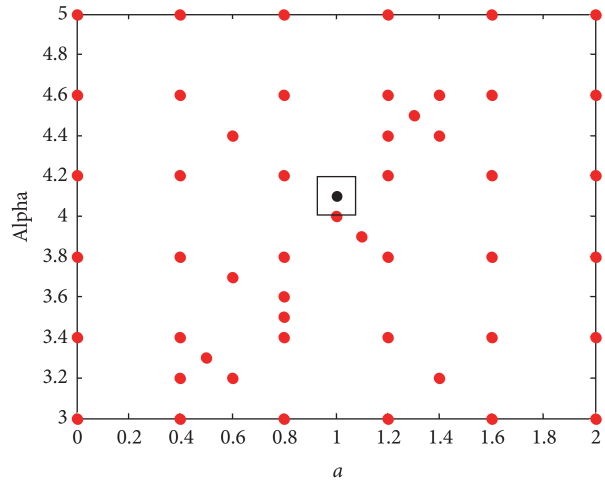

Figure 3: Final population (51 individuals) resulted from TMED algorithm. The black dot is the optimum location.

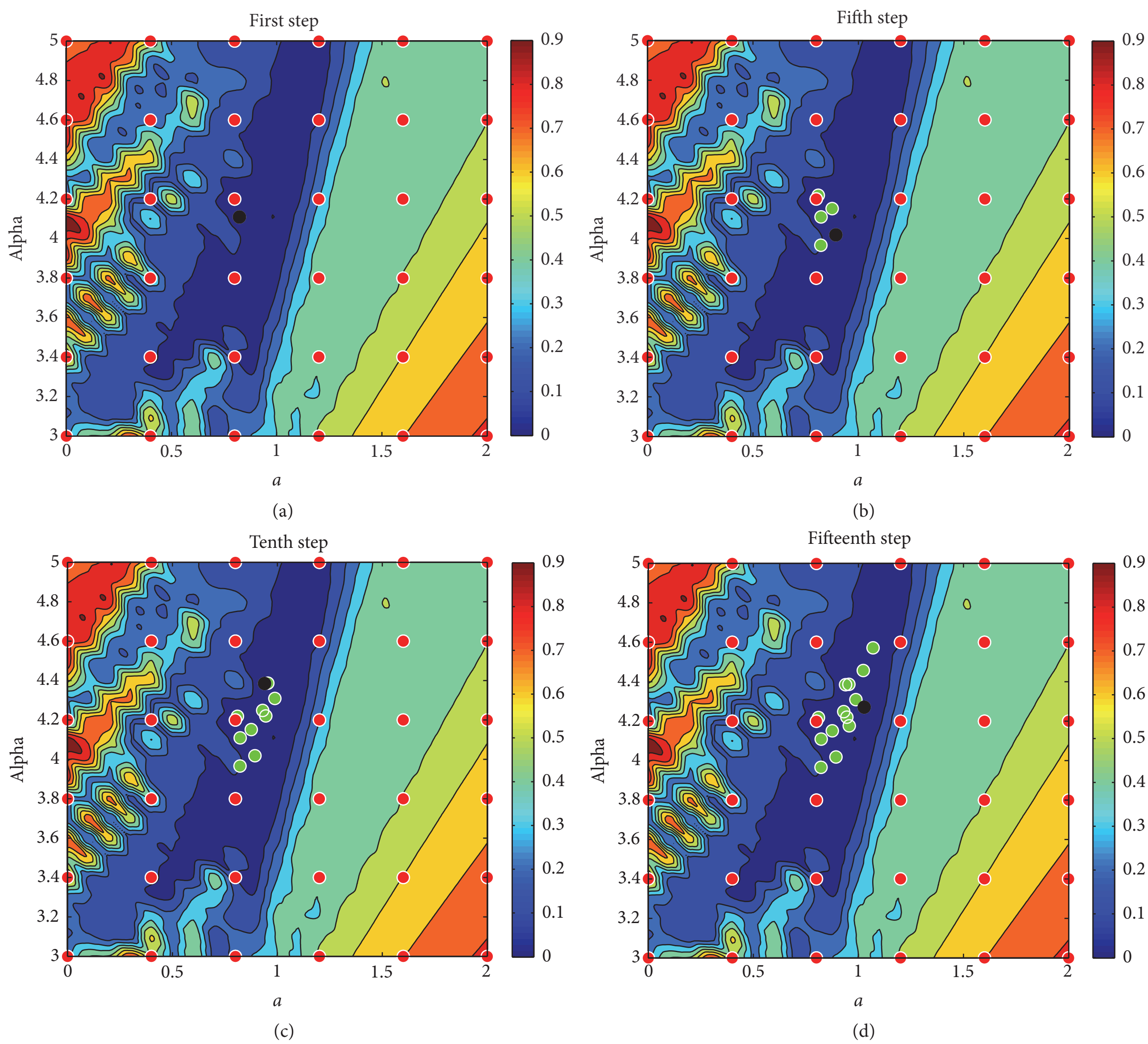

Figure 4: Process of finding the best parameters using MTMED algorithm. (a), (b), (c), and (d) represent the first, 5th, 10th, and 15th iteration, respectively. It can be seen that the individuals (points) converge to the optimum area. Red, green, and black points show the first, newly generated, and queen data in every iteration.

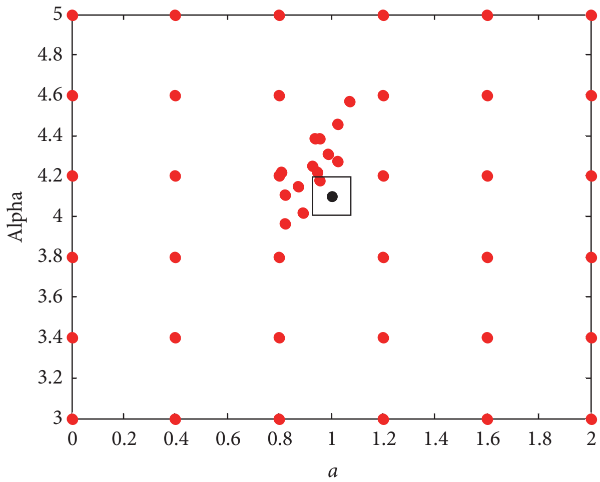

Figure 5: Final population (51 individuals) resulted from MTMED algorithm. The black dot is the optimum location.

of interest in modeling. A proper black box model requires a rich dataset for training. Having a rich dataset may be difficult due to cost and time. In this paper, we have investigated the efficiency of a newly proposed experimental design method on gathering proper data for parameter estimation of a chaotic one-dimensional map. This method is a biologically inspired intelligent method which can be tuned to select data according to the purpose of the modeling. We are not aware of any such intelligent ED method. Considering the possible costs of gathering biological data which deal with human health, we believe that any improvement in ED techniques is of great value. Furthermore, there are some basic differences in parameter estimation of chaotic systems due to their sensitivity to initial condition or butterfly effect. Accordingly, a new cost function which is proper for chaotic systems is used in this work. We have tested the proposed methods on a chaotic one-dimensional map which is a simple model of nonlinear feedback to account for period doubling in the ERG response to periodic flashes, and the results clearly show its efficiency.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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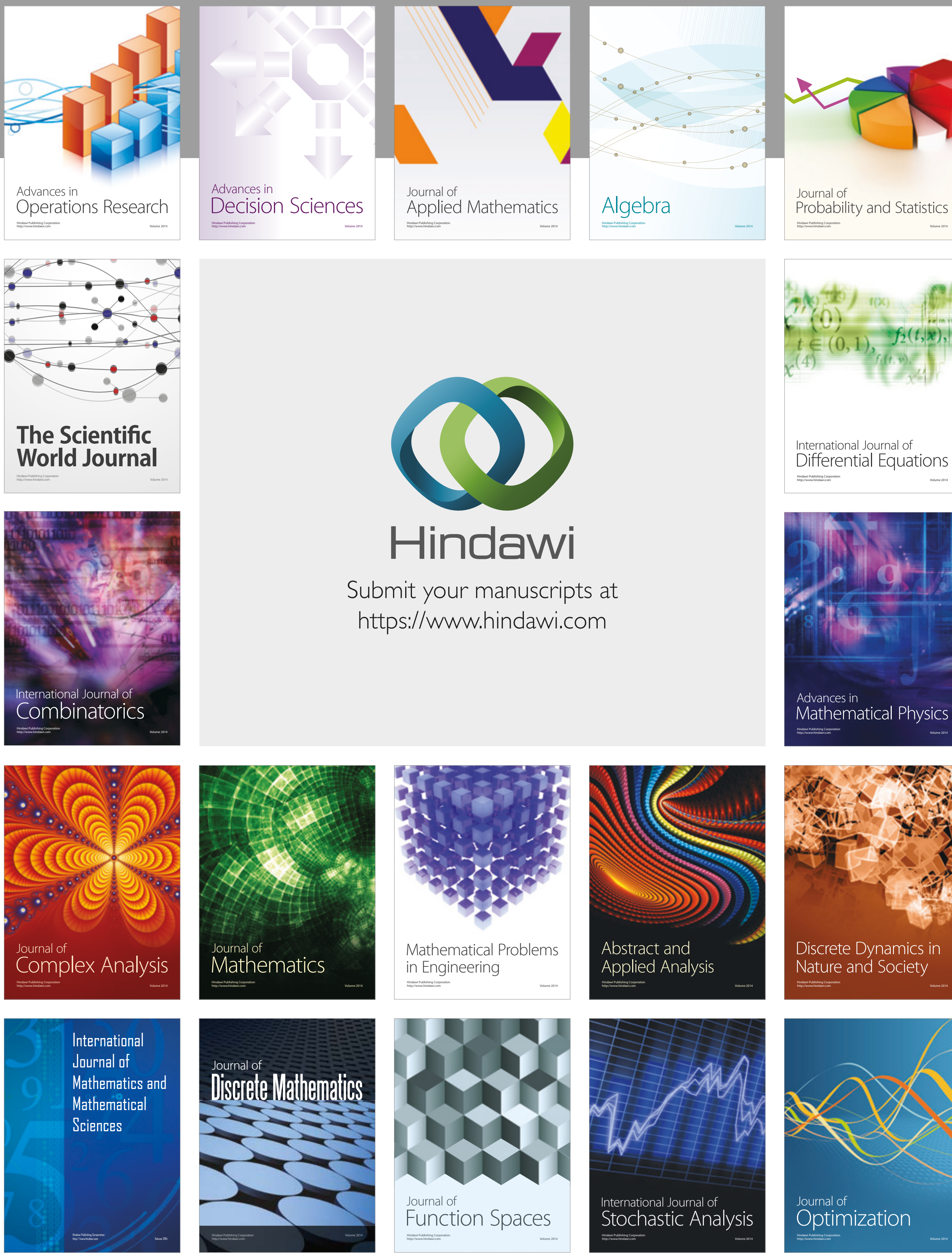
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