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Research Article

Adaptive Fuzzy Control for Stochastic Pure-Feedback Nonlinear Systems with Unknown Hysteresis and External Disturbance

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This paper solves the tracking control problem of a class of stochastic pure-feedback nonlinear systems with external disturbances and unknown hysteresis. By using the mean-value theorem, the problem of pure-feedback nonlinear function is solved. The direction-unknown hysteresis problem is solved with the aid of the Nussbaum function. The external disturbance problems can be solved by defining new Lyapunov functions. Using the backstepping technique, a new adaptive fuzzy control scheme is proposed. The results show that the proposed control scheme ensures that all signals of the closed-loop system are semiglobally uniformly bounded and the tracking error converges to the small neighborhood of origin in the sense of mean quartic value. Simulation results illustrate the effectiveness of the proposed control scheme.

1. Introduction

Hysteresis is widely found in mechanical equipment, which severely limits performance of the system and even leads to system instability. Therefore, the control problem of the system with hysteresis has been paid more and more attention. For the adaptive control system in [1–7], scholars study problems in different directions, such as hysteresis input in [1], dead zone input in [2], and time-delay input in [3]. There are several common hysteresis phenomena. The author solves the backlash-like hysteresis problem in [4]. The authors solve a class of traditional P-I hysteresis problems and propose an adaptive backstepping scheme in [5]. Scholars have studied a class of nonlinear systems with generalized P-I hysteresis inputs in [6, 7]. In addition, scholars have studied unmodeled dynamics deterministic systems in [8] and uncertain nonsmooth deterministic systems in [9]. The finite-time control problem of nonlinear deterministic systems is studied in [10]. However, the system studied above is a deterministic system and ignores the effects of stochastic disturbance.

Stochastic disturbance often occur in many systems, and the adaptive control problems of stochastic nonlinear systems are more difficult than those of deterministic nonlinear systems. Stochastic disturbance is added to the system, and differential operations on Lyapunov functions are more complicated. The research of stochastic nonlinear systems has been increasingly discussed in [11–21]. A finite time control method of switched stochastic systems is proposed in [11]. The control problem of nonlinear stochastic systems is discussed in [12–14]. Using the mean value theorem to solve the pure-feedback nonlinear function, the complexity of the system is increased. Further, more researchers have studied other types of stochastic nonlinear systems, such as from stochastic systems with unknown backlash-like hysteresis in [15] to pure-feedback stochastic nonlinear systems with unknown dead-zone input in [16], from SISO systems in [17] to pure-feedback MIMO systems in [18]. The stochastic systems with time-varying delays are proposed in [19, 20] and the stochastic systems with unknown direction hysteresis are proposed in [21]. The above studies have considered the effect of stochastic disturbance, without considering the external disturbances.

However, external disturbance often exists in practice. External disturbance cannot be ignored; it is also a source of system instability in practice. Scholars have extended the system without external disturbances in [22–25] to systems with external disturbances in [26–30], such as a determination system with external disturbances in [29] and a stochastic system with external disturbances in [30]. The system with external disturbances makes the design of the controller more difficult.

In this paper, the control problem of pure-feedback stochastic nonlinear system with external disturbance and unknown hysteresis is studied. For the determination system with external disturbance in [26], stochastic terms are not considered. The adaptive fuzzy control problem for stochastic nonlinear systems is studied in [14], without considering external disturbances. Therefore, a more general nonlinear system is processed in this paper. Furthermore, the difficulty is to deal with unknown direction hysteresis in [21]. The difficulty of this paper is how to solve the influence of external disturbance on the unknown direction hysteresis and ensure the stability of the nonlinear system. This problem can be solved by designing appropriate Lyapunov functions. The major contributions of this paper are described below:

(1) The tracking control problem of the stochastic pure-feedback nonlinear systems with stochastic disturbances, direction-unknown hysteresis, and external disturbances is solved in this paper.

(2) In the nth step of the backstepping design, the Lyapunov function with the external disturbance term $\Delta(t)$ is defined, and the external disturbance problem is solved. A new adaptive control scheme is proposed.

The remainder of this article is as follows. The second part puts forward the preparation work and the problem formulation. The third part is the design process of the adaptive control method. The fourth part gives the simulation example. The fifth part summarizes the full text.

2. Preparation and Problem Formulation

2.1. Preliminary Knowledge. The stochastic nonlinear system is expressed as follows:

$$dx = f(x,t)\,dt + h(x,t)\,dw, \tag{1}$$

where $x \in R^n$ is the state variable, $f : R^n \times R^+ \longrightarrow R^n$, $h : R^n \times R^+ \longrightarrow R^{n\times r}$ are continuous functions. w indicates that an independent r-dimension standard Brownian motion, which is defined on the complete probability space.

Definition 1 (see [31]). For a quadratic continuous differentiable function $V(x,t)$, define a derivative operator L expressed as follows:

$$LV = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f + \frac{1}{2} Tr\left\{h^T \frac{\partial^2 V}{\partial x^2} h\right\}, \tag{2}$$

where Tr is a trace of matrix.

Remark 2. The $\partial^2 V/\partial x^2$ in *Itô* correction term $(1/2)Tr\{h^T(\partial^2 V/\partial x^2)h\}$ makes the design of the control scheme in the stochastic system more complicated than the design of the control scheme in the determined system.

Lemma 3 (see [32]). *For the stochastic system (1), let $V(t)$ and $\zeta(t)$ are smooth functions defined on $[0,t_f)$, $V(t) \geq 0$, $\forall t \in [0,t_f)$; the function ξ satisfes $0 < |\xi(t)| \leq l < \infty$ (l is a constant); $N(.)$ is a Nussbaum-type function. The following inequality is satisfied:*

$$V(t) \leq e^{-ct} \int_0^t \left(\xi(t) N(\zeta)\dot{\zeta} + \dot{\zeta}\right) e^{c\tau} d\tau + \eta + M(t), \quad \forall t \in \left[0, t_f\right), \tag{3}$$

where η is a nonnegative variable, $M(t)$ is a real valued continuous local martingale. Then the functions $V(t)$, $\zeta(t)$ and $\int_0^t (\xi(t)N(\zeta)\dot{\zeta} + \dot{\zeta})d\tau$ are bounded on $[0,t_f)$.

Lemma 4 (see [33]). *For $\forall (x,y) \in R^2$, the following inequality is established:*

$$xy \leq \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q, \tag{4}$$

where $\varepsilon > 0$, $p > 1$, and $(p-1)(q-1) = 1$.

Lemma 5 (see [3]). *Consider the following dynamic system:*

$$\dot{\widehat{\theta}}(t) = -\gamma\widehat{\theta}(t) + k\rho(t), \tag{5}$$

where γ and k are positive constants and $\rho(t)$ is a positive function. Then for $\forall t \geq t_0$ and any bounded initial condition $\widehat{\theta}(t_0) \geq 0$, we have $\widehat{\theta}(t) \geq 0$.

2.2. Problem Formulation. This paper considers the following stochastic pure-feedback nonlinear system:

$$\begin{aligned} dx_i &= f_i(\overline{x}_i, x_{i+1})\,dt + \psi_i^T(\overline{x}_i)\,dw, \quad 1 \leq i \leq n-1, \\ dx_n &= \left(f_n(\overline{x}_n, u) + \Delta(t)\right)dt + \psi_n^T(\overline{x}_n)\,dw, \\ y &= x_1, \end{aligned} \tag{6}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ and $\overline{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$ are the state vectors, $y \in R$ is system output, and w is defined as (1). $f_i(.) : R^i \longrightarrow R$ and $\psi_i(.) : R^i \longrightarrow R^r (i = 1, 2, \ldots, n)$ are unknown nonlinear functions. $\Delta(t)$ is a bounded external disturbance. $u \in R$ is the system input and the output of an unknown Bouc-Wen hysteresis is defined as follows [1]:

$$u = H(v) = \mu_1 v + \mu_2 \zeta, \tag{7}$$

where μ_1 and μ_2 are unknown constants and have the same sign. $v(t) \in R$ is the input of the hysteresis. ζ is the auxiliary variable, $\zeta(t_0) = 0$. In [1] we know that ζ is bounded and can be expressed as:

$$|\zeta| \leq \sqrt[n]{\frac{1}{(\beta+\chi)}}, \tag{8}$$

where β, χ, n are the unknown hysteresis parameters, and $\beta > |\chi|$, $n > 1$.

Remark 6. This article has stochastic term $\psi_i^T(\bar{x}_i)dw$, and the hysteresis output u is different from [29]. If we ignore the external disturbance $\Delta(t)$, the results of this paper are the same as [21]. Therefore, this paper considers a more general nonlinear system.

For the system (6), define

$$g_i(\bar{x}_i, x_{i+1}) = \frac{\partial f_i(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}, \quad i = 1, 2, \ldots, n, \tag{9}$$

with $x_{i+1} = u$.

Assumption 7. For $1 \le i \le n$, there is an unknown constant b_M such that

$$1 \le g_i(\bar{x}_i, x_{i+1}) \le b_M < \infty, \quad \forall (\bar{x}_i, x_{i+1}) \in R^i \times R. \tag{10}$$

By using the mean-value theorem, the pure-feedback nonlinear functions in (6) can be expressed as

$$f_i(\bar{x}_i, x_{i+1}) - f_i(\bar{x}_i, x_{i+1}^0) = g_i(\bar{x}_i, \eta_i)(x_{i+1} - x_{i+1}^0), \tag{11}$$

where $x_{n+1}^0 = u^0 = H(v^0)$, $x_{n+1} = u = H(v)$, and η_i is point between x_{i+1} and x_{i+1}^0.

Substituting (11) into (6), the control system can be rewritten as

$$\begin{aligned} dx_i &= \left(f_i(\bar{x}_i, x_{i+1}^0) + g_i(\bar{x}_i, \eta_i)(x_{i+1} - x_{i+1}^0)\right)dt \\ &\quad + \psi_i^T(\bar{x}_i)\,dw, \quad 1 \le i \le n-1, \\ dx_n &= \left(f_n(\bar{x}_n, u^0) + g_n(\bar{x}_n, \eta_n)(u - u^0) + \Delta(t)\right)dt \\ &\quad + \psi_n^T(\bar{x}_n)\,dw, \\ y &= x_1. \end{aligned} \tag{12}$$

2.3. Fuzzy Logic Systems. In order to approximate a continuous function $f(x)$ with a fuzzy logic system, consider the following fuzzy rules:

R^l: If x_1 is F_1^l and ... and x_n is F_n^l.

Then y is G^l, $l = 1, 2, \ldots, N$,

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the input of system, $y \in R$ is the output of the system, F_i^l and G^l are fuzzy sets in R, and N is the number of rules. The output form of the fuzzy logic system is as follows:

$$y(x) = \frac{\sum_{l=1}^N \Phi_l \prod_{i=1}^n \mu_{F_i^l}(x_i)}{\sum_{l=1}^N \left[\prod_{i=1}^n \mu_{F_i^l}(x_i)\right]}, \tag{13}$$

where

$$\begin{aligned} \Phi_l &= \max_{y \in R} \mu_{G^l}(y), \\ \Phi &= (\Phi_1, \Phi_2, \ldots, \Phi_N)^T. \end{aligned} \tag{14}$$

Letting

$$\begin{aligned} \xi_l(x) &= \frac{\prod_{i=1}^n \mu_{F_i^l}(x_i)}{\sum_{l=1}^N \left[\prod_{i=1}^n \mu_{F_i^l}(x_i)\right]}, \\ \xi(X) &= (\xi_1(x), \xi_2(x), \ldots, \xi_N(x))^T, \end{aligned} \tag{15}$$

the fuzzy system is written as

$$y(x) = \Phi^T \xi(x). \tag{16}$$

Lemma 8 (see [27]). *Let $f(x)$ be a continuous function defined on a compact set Ω. Then, for $\forall \varepsilon > 0$, there exists a fuzzy logic system $\Phi^T \xi(x)$ such that*

$$\sup_{x \in \Omega} \left| f(x) - \Phi^T \xi(x) \right| \le \varepsilon. \tag{17}$$

The goal of this paper is to design an adaptive controller, so that the system output y converges to the reference signal y_d and all signals of the closed-loop system are bounded.

Define a vector function as $\bar{y}_d^{(i)} = [y_d, y_d^{(1)}, \ldots, y_d^{(i)}]^T$, $i = 1, 2, \ldots, n$, where $y_d^{(i)}$ denotes the ith order derivative of y_d.

Assumption 9 (see [21]). The reference signal $y_d(t)$ and its order derivatives up to the nth time are continuous and bounded.

3. Adaptive Control Design

In this part, the adaptive fuzzy control is proposed by the backstepping technique, and the following coordinate transformation is defined to develop the backstepping technique:

$$\begin{aligned} z_1 &= y - y_d, \\ z_i &= x_i - \alpha_{i-1}, \quad i = 2, \ldots, n, \end{aligned} \tag{18}$$

where α_{i-1} is an intermediate function to be determined next.

In each step of the backstepping method, a fuzzy logic system $\Phi_i^T \xi_i(X_i)$ will be used to approximate an unknown function $\bar{f}_i$. We define a constant $\theta_i = \|\Phi_i\|^2$, the estimation error is $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, $\hat{\theta}_i$ as the estimation of θ_i, $i = 1, 2, \ldots, n$.

Step 1. For stochastic pure-feedback systems (12), according to $z_1 = x_1 - y_d$, we know that dynamic error is satisfied

$$\begin{aligned} dz_1 &= \left(f_1(\bar{x}_1, x_2^0) + g_1(\bar{x}_1, \eta_1)(x_2 - x_2^0) - \dot{y}_d\right)dt \\ &\quad + \psi_1^T dw. \end{aligned} \tag{19}$$

We choose Lyapunov function as follows:

$$V_1 = \frac{1}{4} z_1^4 + \frac{1}{2\lambda_1} \tilde{\theta}_1^2, \tag{20}$$

where λ_1 is a positive constant. By (2), (18), and (19), one has

$$\begin{aligned} &LV_1 \\ &= z_1^3\left(f_1(\bar{x}_1, x_2^0) + g_1(\bar{x}_1, \eta_1)(z_2 + \alpha_1 - x_2^0) - \dot{y}_d\right) \\ &\quad + \frac{3}{2} z_1^2 \psi_1^T \psi_1 - \frac{1}{\lambda_1} \tilde{\theta}_1 \dot{\hat{\theta}}_1. \end{aligned} \tag{21}$$

Applying Lemma 4 and Assumption 7, the following inequalities hold:

$$\frac{3}{2}z_1^2\psi_1^T\psi_1 \le \frac{3}{4}l_1^{-2}z_1^4\left\|\psi_1\right\|^4 + \frac{3}{4}l_1^2, \tag{22}$$

$$g_1\left(\overline{x}_1,\eta_1\right)z_1^3z_2 \le \frac{3}{4}b_M z_1^4 + \frac{b_M}{4}z_2^4, \tag{23}$$

where l_1 is a positive constant. Substituting (22) and (23) into (21), we can get

$$\begin{aligned} LV_1 \le{}& z_1^3\left(f_1\left(\overline{x}_1,x_2^0\right) + g_1\left(\overline{x}_1,\eta_1\right)\left(\alpha_1 - x_2^0\right) - \dot{y}_d \right.\\ &\left. + \frac{3}{4}b_M z_1 + \frac{3}{4}l_1^{-2}z_1\left\|\psi_1\right\|^4\right) + \frac{3}{4}l_1^2 + \frac{b_M}{4}z_2^4 - \frac{1}{\lambda_1} \\ &\cdot \widetilde{\theta}_1\dot{\widehat{\theta}}_1. \end{aligned} \tag{24}$$

Defining a new function $\overline{f}_1 = f_1 + (3/4)b_M z_1 - \dot{y}_d + (3/4)l_1^{-2}z_1\|\psi_1\|^4$, then the above inequality can be rewritten as

$$\begin{aligned} LV_1 \le{}& z_1^3\overline{f}_1 + z_1^3 g_1\left(\alpha_1 - x_2^0\right) + \frac{3}{4}l_1^2 + \frac{b_M}{4}z_2^4 \\ &- \frac{1}{\lambda_1}\widetilde{\theta}_1\dot{\widehat{\theta}}_1. \end{aligned} \tag{25}$$

Because $\overline{f}_1$ contains the unknown function f_1 and ψ_1, it cannot be directly controlled in practice. Therefore, according to Lemma 8, for any given $\varepsilon_1 > 0$, there exists a fuzzy logic system $\Phi_1^T\xi_1(X_1)$ such that

$$\overline{f}_1 = \Phi_1^T\xi_1\left(X_1\right) + \delta_1\left(X_1\right), \quad \left|\delta_1\left(X_1\right)\right| \le \varepsilon_1, \tag{26}$$

where $X_1 = (x_1, y_d, \dot{y}_d)$. According to Lemma 4, it follows that

$$\begin{aligned} z_1^3\overline{f}_1 &= z_1^3\Phi_1^T\xi_1\left(X_1\right) + z_1^3\delta_1\left(X_1\right) \\ &\le \frac{z_1^6}{2a_1^2}\left\|\Phi_1\right\|^2\xi_1^T\xi_1 + \frac{1}{2}a_1^2 + \frac{3}{4}z_1^4 + \frac{1}{4}\varepsilon_1^4 \\ &= \frac{z_1^6}{2a_1^2}\theta_1\xi_1^T\xi_1 + \frac{1}{2}a_1^2 + \frac{3}{4}z_1^4 + \frac{1}{4}\varepsilon_1^4, \end{aligned} \tag{27}$$

where a_1 is a positive parameter. We choose the following virtual control signal and adaptive law:

$$\alpha_1 = -\left(k_1 + \frac{3}{4}\right)z_1 - \frac{1}{2a_1^2}\widehat{\theta}_1 z_1^3\xi_1^T\xi_1 + x_2^0, \tag{28}$$

$$\dot{\widehat{\theta}}_1 = \frac{\lambda_1}{2a_1^2}z_1^6\xi_1^T\xi_1 - \gamma_1\widehat{\theta}_1, \quad \widehat{\theta}_1(0) \ge 0, \tag{29}$$

where k_1 and γ_1 are positive constants. Based on (28), Assumption 7, one has

$$z_1^3 g_1\left(\alpha_1 - x_2^0\right) \le -\left(k_1 + \frac{3}{4}\right)z_1^4 - \frac{1}{2a_1^2}z_1^6\widehat{\theta}_1\xi_1^T\xi_1. \tag{30}$$

Substituting (27), (29), and (30) into (25), we have

$$LV_1 \le -k_1 z_1^4 + \frac{b_M}{4}z_2^4 + \frac{3}{4}l_1^2 + \frac{1}{4}\varepsilon_1^4 + \frac{\gamma_1}{\lambda_1}\widetilde{\theta}_1\widehat{\theta}_1 + \frac{1}{2}a_1^2. \tag{31}$$

It is noted that

$$\frac{\gamma_1}{\lambda_1}\widetilde{\theta}_1\widehat{\theta}_1 \le -\frac{\gamma_1}{2\lambda_1}\widetilde{\theta}_1^2 + \frac{\gamma_1}{2\lambda_1}\theta_1^2. \tag{32}$$

Substituting (32) into (31), we have

$$LV_1 \le -k_1 z_1^4 - \frac{\gamma_1}{2\lambda_1}\widetilde{\theta}_1^2 + \varrho_1 + \frac{b_M}{4}z_2^4, \tag{33}$$

where $\varrho_1 = (3/4)l_1^2 + (1/2)a_1^2 + (1/4)\varepsilon_1^4 + (\gamma_1/2\lambda_1)\theta_1^2$.

Step 2. Since $z_2 = x_2 - \alpha_1$ and $It\hat{o}$ formula, one has

$$\begin{aligned} dz_2 ={}& \left(f_2\left(\overline{x}_2,x_3^0\right) + g_2\left(\overline{x}_2,\eta_2\right)\left(x_3 - x_3^0\right) - L\alpha_1\right)dt \\ &+ \left(\psi_2 - \frac{\partial\alpha_1}{\partial x_1}\psi_1\right)^T dw, \end{aligned} \tag{34}$$

with

$$\begin{aligned} L\alpha_1 ={}& \frac{\partial\alpha_1}{\partial x_1}\left[f_1\left(\overline{x}_1,x_2^0\right) + g_1\left(\overline{x}_1,\eta_1\right)\left(x_2 - x_2^0\right)\right] \\ &+ \frac{\partial\alpha_1}{\partial\widehat{\theta}_1}\dot{\widehat{\theta}}_1 + \sum_{i=0}^{1}\frac{\partial\alpha_1}{\partial y_d^{(i)}}y_d^{(i+1)} + \frac{1}{2}\frac{\partial^2\alpha_1}{\partial x_1^2}\psi_1^T\psi_1. \end{aligned} \tag{35}$$

Choose stochastic Lyapunov function as

$$V_2 = V_1 + \frac{1}{4}z_2^4 + \frac{1}{2\lambda_2}\widetilde{\theta}_2^2, \tag{36}$$

where λ_2 is a positive design constant. Using the similar procedure as (21), it follows that

$$\begin{aligned} LV_2 ={}& LV_1 + z_2^3\left(f_2\left(\overline{x}_2,x_3^0\right)\right. \\ &\left. + g_2\left(\overline{x}_2,\eta_2\right)\left(z_3 + \alpha_2 - x_3^0\right) - L\alpha_1\right) + \frac{3}{2}z_2^2\left(\psi_2\right. \\ &\left. - \frac{\partial\alpha_1}{\partial x_1}\psi_1\right)^T\left(\psi_2 - \frac{\partial\alpha_1}{\partial x_1}\psi_1\right) - \frac{1}{\lambda_2}\widetilde{\theta}_2\dot{\widehat{\theta}}_2. \end{aligned} \tag{37}$$

It is noticed that

$$\frac{3}{2}z_2^2\left\|\psi_2 - \frac{\partial\alpha_1}{\partial x_1}\psi_1\right\|^2 \le \frac{3}{4}l_2^{-2}z_2^4\left\|\psi_2 - \frac{\partial\alpha_1}{\partial x_1}\psi_1\right\|^4 + \frac{3}{4}l_2^2, \tag{38}$$

$$g_2\left(\overline{x}_2,\eta_2\right)z_2^3z_3 \le \frac{3}{4}b_M z_2^4 + \frac{b_M}{4}z_3^4, \tag{39}$$

where l_2 is a positive parameter. Substituting (33), (38), and (39) into (37), we have

$$\begin{aligned} LV_2 \le{}& -k_1 z_1^4 - \frac{\gamma_1}{2\lambda_1}\widetilde{\theta}_1^2 + \varrho_1 + z_2^3\left(f_2\left(\overline{x}_2,x_3^0\right)\right. \\ &+ g_2\left(\overline{x}_2,\eta_2\right)\left(\alpha_2 - x_3^0\right) - L\alpha_1 + b_M z_2 \\ &\left. + \frac{3}{4}l_2^{-2}z_2\left\|\psi_2 - \frac{\partial\alpha_1}{\partial x_1}\psi_1\right\|^4\right) + \frac{3}{4}l_2^2 + \frac{b_M}{4}z_3^4 - \frac{1}{\lambda_2} \\ &\cdot \widetilde{\theta}_2\dot{\widehat{\theta}}_2. \end{aligned} \tag{40}$$

Define a function as $\overline{f}_2 = f_2 - L\alpha_1 + b_M z_2 + (3/4)l_2^{-2} z_2 \|\psi_2 - (\partial\alpha_1/\partial x_1)\psi_1\|^4$. Furthermore, (40) can be rewritten as

$$LV_2 \le -k_1 z_1^4 - \frac{\gamma_1}{2\lambda_1}\tilde{\theta}_1^2 + \varrho_1 + z_2^3 \overline{f}_2 + z_2^3 g_2\left(\alpha_2 - x_3^0\right) + \frac{3}{4}l_2^2 + \frac{b_M}{4}z_3^4 - \frac{1}{\lambda_2}\tilde{\theta}_2\dot{\hat{\theta}}_2. \tag{41}$$

Since $\overline{f}_2$ contains the unknown function f_1, ψ_1, and ψ_2, it is not possible in practice. Thus, the fuzzy logic system $\Phi_2^T \xi_2(X_2)$ is used to approximate $\overline{f}_2$, where $X_2 = [\overline{x}_2^T, \hat{\theta}_1, \overline{y}_d^{(2)T}]^T \in \Omega_{Z_2}$. Due to Lemma 8, $\overline{f}_2$ can be written as

$$\overline{f}_2 = \Phi_2^T \xi_2\left(X_2\right) + \delta_2\left(X_2\right), \quad \left|\delta_2\left(X_2\right)\right| \le \varepsilon_2, \tag{42}$$

where ε_2 is any given positive constant. Repeating the method of (27), we have

$$z_2^3 \overline{f}_2 \le \frac{z_2^6}{2a_2^2}\theta_2 \xi_2^T \xi_2 + \frac{1}{2}a_2^2 + \frac{3}{4}z_2^4 + \frac{1}{4}\varepsilon_2^4, \tag{43}$$

where a_2 is a positive parameter. We choose the following virtual control signal and adaptive law:

$$\alpha_2 = -\left(k_2 + \frac{3}{4}\right)z_2 - \frac{1}{2a_2^2}\hat{\theta}_2 z_2^3 \xi_2^T \xi_2 + x_3^0, \tag{44}$$

$$\dot{\hat{\theta}}_2 = \frac{\lambda_2}{2a_2^2} z_2^6 \xi_2^T \xi_2 - \gamma_2 \hat{\theta}_2, \quad \hat{\theta}_2(0) \ge 0, \tag{45}$$

where k_2, γ_2 are design constants. Similar to (30), the following inequality is obtained:

$$z_2^3 g_2\left(\alpha_2 - x_3^0\right) \le -\left(k_2 + \frac{3}{4}\right)z_2^4 - \frac{1}{2a_2^2}z_2^6 \hat{\theta}_2 \xi_2^T \xi_2. \tag{46}$$

Substituting (43), (45) and (46) into (41), we have

$$LV_2 \le -k_1 z_1^4 - \frac{\gamma_1}{2\lambda_1}\tilde{\theta}_1^2 + \varrho_1 - k_2 z_2^4 + \frac{b_M}{4}z_3^4 + \frac{3}{4}l_2^2 + \frac{1}{4}\varepsilon_2^4 + \frac{\gamma_2}{\lambda_2}\tilde{\theta}_2\hat{\theta}_2 + \frac{1}{2}a_2^2. \tag{47}$$

It is noted that

$$\frac{\gamma_2}{\lambda_2}\tilde{\theta}_2\hat{\theta}_2 \le -\frac{\gamma_2}{2\lambda_2}\tilde{\theta}_2^2 + \frac{\gamma_2}{2\lambda_2}\theta_2^2, \tag{48}$$

(47), can be rewritten in the form

$$LV_2 \le -\sum_{j=1}^{2}\left(k_j z_j^4 + \frac{\gamma_j}{2\lambda_j}\tilde{\theta}_j^2\right) + \sum_{j=1}^{2}\varrho_j + \frac{b_M}{4}z_3^4, \tag{49}$$

where $\varrho_j = (\gamma_j/2\lambda_j)\theta_j^2 + (3/4)l_j^2 + (1/4)\varepsilon_j^4 + (1/2)a_j^2$, $j = 1, 2$.

Step i ($3 \le i \le n-1$). According to $z_i = x_i - \alpha_{i-1}$ and *Itô* formula, one has

$$\begin{aligned} dz_i &= \left(f_i\left(\overline{x}_i, x_{i+1}^0\right) + g_i\left(\overline{x}_i, \eta_i\right)\left(x_{i+1} - x_{i+1}^0\right) - L\alpha_{i-1}\right)dt \\ &\quad + \left(\psi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right)^T dw, \end{aligned} \tag{50}$$

with

$$\begin{aligned} L\alpha_{i-1} &= \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\left[f_j + g_j\left(x_{j+1} - x_{j+1}^0\right)\right] + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\theta}_j}\dot{\hat{\theta}}_j \\ &\quad + \sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(j)}}y_d^{(j+1)} + \frac{1}{2}\sum_{p,q=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_p \partial x_q}\psi_p^T\psi_q. \end{aligned} \tag{51}$$

We consider the following Lyapunov function:

$$V_i = V_{i-1} + \frac{1}{4}z_i^4 + \frac{1}{2\lambda_i}\tilde{\theta}_i^2, \tag{52}$$

where λ_i is a positive constant. Using the similar procedure as (21), it follows that

$$\begin{aligned} LV_i &= LV_{i-1} + z_i^3\left(f_i + g_i\left(z_{i+1} + \alpha_i - x_{i+1}^0\right) - L\alpha_{i-1}\right) \\ &\quad + \frac{3}{2}z_i^2\left(\psi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right)^T\left(\psi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right) \\ &\quad - \frac{1}{\lambda_i}\tilde{\theta}_i\dot{\hat{\theta}}_i. \end{aligned} \tag{53}$$

It is noticed that

$$\begin{aligned} &\frac{3}{2}z_i^2\left\|\psi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right\|^2 \\ &\quad \le \frac{3}{4}l_i^2 + \frac{3}{4}l_i^{-2}z_i^4\left\|\psi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right\|^4, \end{aligned} \tag{54}$$

$$g_i\left(\overline{x}_i, \eta_i\right)z_i^3 z_{i+1} \le \frac{3}{4}b_M z_i^4 + \frac{b_M}{4}z_{i+1}^4, \tag{55}$$

where l_i is a positive constant. Using the similar procedure as (24), we have

$$\begin{aligned} LV_i &\le -\sum_{j=1}^{i-1}\left(k_j z_j^4 + \frac{\gamma_j}{2\lambda_j}\tilde{\theta}_j^2\right) + \sum_{j=1}^{i-1}\varrho_j + z_i^3\Bigg(f_i \\ &\quad + g_i\left(\alpha_i - x_{i+1}^0\right) - L\alpha_{i-1} + b_M z_i \\ &\quad + \frac{3}{4}l_i^{-2}z_i\left\|\psi_i - \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}\psi_j\right\|^4\Bigg) + \frac{3}{4}l_i^2 + \frac{b_M}{4}z_{i+1}^4 \\ &\quad - \frac{1}{\lambda_i}\tilde{\theta}_i\dot{\hat{\theta}}_i. \end{aligned} \tag{56}$$

Define a function as $\overline{f}_i = f_i - L\alpha_{i-1} + b_M z_i + (3/4)l_i^{-2}z_i\|\psi_i - \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)\psi_j\|^4$. Furthermore, (56) can be rewritten as

$$\begin{aligned} LV_i &\le -\sum_{j=1}^{i-1}\left(k_j z_j^4 + \frac{\gamma_j}{2\lambda_j}\tilde{\theta}_j^2\right) + \sum_{j=1}^{i-1}\varrho_j + z_i^3\overline{f}_i \\ &\quad + z_i^3 g_i\left(\alpha_i - x_{i+1}^0\right) + \frac{3}{4}l_i^2 + \frac{b_M}{4}z_{i+1}^4 - \frac{1}{\lambda_i}\tilde{\theta}_i\dot{\hat{\theta}}_i. \end{aligned} \tag{57}$$

Similarly, The fuzzy logic system $\Phi_i^T \xi_i(X_i)$ is used to approximate $\overline{f}_i$, where $X_i = [\overline{x}_i^T, \overline{\widehat{\theta}}_{i-1}^T, \overline{y}_d^{(i)T}]^T \in \Omega_{Z_i}$, with $\overline{\widehat{\theta}}_{i-1} = [\widehat{\theta}_1, \widehat{\theta}_2, \ldots, \widehat{\theta}_{i-1}]^T$. According to Lemma 8, $\overline{f}_i$ can be expressed as

$$\overline{f}_i = \Phi_i^T \xi_i (X_i) + \delta_i (X_i), \quad |\delta_i (X_i)| \le \varepsilon_i, \tag{58}$$

where ε_i is any given positive constant. According to the method of (27), we can get

$$z_i^3 \overline{f}_i \le \frac{z_i^6}{2a_i^2} \theta_i \xi_i^T \xi_i + \frac{1}{2} a_i^2 + \frac{3}{4} z_i^4 + \frac{1}{4} \varepsilon_i^4, \tag{59}$$

where a_i is a design constant. We choose the following virtual control signal and adaptive law:

$$\alpha_i = -\left(k_i + \frac{3}{4}\right) z_i - \frac{1}{2a_i^2} \widehat{\theta}_i z_i^3 \xi_i^T \xi_i + x_{i+1}^0, \tag{60}$$

$$\dot{\widehat{\theta}}_i = \frac{\lambda_i}{2a_i^2} z_i^6 \xi_i^T \xi_i - \gamma_i \widehat{\theta}_i, \quad \widehat{\theta}_i (0) \ge 0, \tag{61}$$

where k_i and γ_i are positive parameters. Similar to (30), we have

$$z_i^3 g_i \left(\alpha_i - x_{i+1}^0\right) \le -\left(k_i + \frac{3}{4}\right) z_i^4 - \frac{1}{2a_i^2} z_i^6 \widehat{\theta}_i \xi_i^T \xi_i. \tag{62}$$

Similar to (31), we have

$$\begin{aligned} LV_i \le & -\sum_{j=1}^{i-1} \left(k_j z_j^4 + \frac{\gamma_j}{2\lambda_j} \widetilde{\theta}_j^2\right) + \sum_{j=1}^{i-1} \varrho_j - k_i z_i^4 + \frac{b_M}{4} z_{i+1}^4 \\ & + \frac{3}{4} l_i^2 + \frac{1}{4} \varepsilon_i^4 + \frac{\gamma_i}{\lambda_i} \widetilde{\theta}_i \widehat{\theta}_i + \frac{1}{2} a_i^2. \end{aligned} \tag{63}$$

It is noted that

$$\frac{\gamma_i}{\lambda_i} \widetilde{\theta}_i \widehat{\theta}_i \le -\frac{\gamma_i}{2\lambda_i} \widetilde{\theta}_i^2 + \frac{\gamma_i}{2\lambda_i} \theta_i^2. \tag{64}$$

The above inequality can be rewritten as

$$LV_i \le -\sum_{j=1}^{i} \left(k_j z_j^4 + \frac{\gamma_j}{2\lambda_j} \widetilde{\theta}_j^2\right) + \sum_{j=1}^{i} \varrho_j + \frac{b_M}{4} z_{i+1}^4, \tag{65}$$

where $\varrho_j = (\gamma_j/2\lambda_j)\theta_j^2 + (3/4)l_j^2 + (1/4)\varepsilon_j^4 + (1/2)a_j^2$, $j = 1, 2, \ldots, i$.

Step n. Based on the coordinate transformation $z_n = x_n - \alpha_{n-1}$ and $It\hat{o}$ formula, we can get

$$\begin{aligned} dz_n = & \left(f_n \left(\overline{x}_n, u^0\right) + g_n \left(\overline{x}_n, \eta_n\right) \left(u - u^0\right) + \Delta (t) \right. \\ & \left. - L\alpha_{n-1}\right) dt + \left(\psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \psi_j\right)^T dw, \end{aligned} \tag{66}$$

with

$$\begin{aligned} L\alpha_{n-1} = & \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \left[f_j + g_j \left(x_{j+1} - x_{j+1}^0\right)\right] \\ & + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \widehat{\theta}_j} \dot{\widehat{\theta}}_j + \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{(j+1)} \\ & + \frac{1}{2} \sum_{p,q=1}^{n-1} \frac{\partial^2 \alpha_{n-1}}{\partial x_p \partial x_q} \psi_p^T \psi_q. \end{aligned} \tag{67}$$

Consider stochastic Lyapunov function as follows:

$$V_n = V_{n-1} + \frac{1}{4} z_n^4 + \frac{1}{2\lambda_n} \widetilde{\theta}_n^2 + \frac{1}{2\sigma} \widetilde{\Delta}^2, \tag{68}$$

where λ_n and σ are positive constants. Denote $\widehat{\Delta}$ as the estimation of Δ, and the estimation error is $\widetilde{\Delta} = \Delta - \widehat{\Delta}$. Similar to procedure (21), it follows that

$$\begin{aligned} LV_n = & LV_{n-1} \\ & + z_n^3 \left(f_n + g_n \left(H (v) - u^0\right) + \Delta - L\alpha_{n-1}\right) + \frac{3}{2} \\ & \cdot z_n^2 \left(\psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \psi_j\right)^T \\ & \cdot \left(\psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \psi_j\right) - \frac{1}{\lambda_n} \widetilde{\theta}_n \dot{\widehat{\theta}}_n - \frac{1}{\sigma} \widetilde{\Delta} \dot{\widehat{\Delta}}. \end{aligned} \tag{69}$$

It is noticed that

$$\begin{aligned} & \frac{3}{2} z_n^2 \left\| \psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \psi_j \right\|^2 \\ & \quad \le \frac{3}{4} l_n^2 + \frac{3}{4} l_n^{-2} z_n^4 \left\| \psi_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \psi_j \right\|^4, \end{aligned} \tag{70}$$

where l_n is a positive constant. Based on (7), the the following inequality holds

$$\begin{aligned} g_n z_n^3 \left(H (v) - u^0\right) = & g_n z_n^3 \mu_1 \left(v - v^0\right) \\ & + g_n z_n^3 \mu_2 \left(\varsigma - \varsigma^0\right). \end{aligned} \tag{71}$$

As [21], an even Nussbaum-type function $N(\zeta) = \zeta^2 cos\zeta$ is defined, and the following equality holds:

$$v = -N (\zeta) \overline{v} + v^0, \tag{72}$$

$$\dot{\zeta} = -\gamma \overline{v} z_n^3, \tag{73}$$

where $\overline{v}$ is the auxiliary virtual controller and γ is a positive parameter. Then, the following equality can be obtained:

$$\begin{aligned} g_n z_n^3 \mu_1 \left(v - v^0\right) & = z_n^3 g_n \mu_1 \left(-N (\zeta) \overline{v} + v^0 - v^0\right) \\ & = -z_n^3 g_n \mu_1 N (\zeta) \overline{v} - z_n^3 \overline{v} + z_n^3 \overline{v} \\ & = -z_n^3 \left(g_n \mu_1 N (\zeta) + 1\right) \overline{v} + z_n^3 \overline{v}. \end{aligned} \tag{74}$$

According to Lemma 4 and (8), it follows that

$$g_n z_n^3 \mu_2 \left(\varsigma - \varsigma^0\right) \le \frac{3}{4} b_M \mu_2^{4/3} z_n^4 + \frac{1}{2} b_M (\beta + \chi)^{-4/n}. \tag{75}$$

Substituting (73)-(75) into (71), we have

$$g_n z_n^3 \left(H(v) - u^0\right) \le \frac{1}{\gamma}\left(g_n\mu_1 N(\zeta) + 1\right)\dot{\zeta} + z_n^3\overline{v} + \frac{3}{4} b_M \mu_2^{4/3} z_n^4 + \frac{1}{2} b_M (\beta + \chi)^{-4/n}. \tag{76}$$

According to (65) with $(i = n-1)$, (70), and (76), (69) can be rewritten as follows:

$$\begin{aligned} LV_n \le & -\sum_{j=1}^{n-1}\left(k_j z_j^4 + \frac{\gamma_j}{2\lambda_j}\widetilde{\theta}_j^2\right) + \sum_{j=1}^{n-1}\varrho_j + z_n^3\Bigg(f_n \\ & + \frac{1}{4} b_M z_n + \overline{v} + \frac{3}{4} b_M \mu_2^{4/3} z_n - L\alpha_{n-1} + \Delta \\ & + \frac{3}{4} l_n^{-2} z_n \left\|\psi_n - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\psi_j\right\|^4\Bigg) + \frac{1}{\gamma}\left(g_n\mu_1 N(\zeta)\right. \\ & \left. + 1\right)\dot{\zeta} + \frac{1}{2} b_M (\beta+\chi)^{-4/n} + \frac{3}{4} l_n^2 - \frac{1}{\lambda_n}\widetilde{\theta}_n\dot{\widehat{\theta}}_n - \frac{1}{\sigma} \\ & \cdot \widetilde{\Delta}_n \dot{\widehat{\Delta}}. \end{aligned} \tag{77}$$

Define a function as $\overline{f}_n = (1/4) b_M z_n + f_n + (3/4) b_M \mu_2^{4/3} z_n + \Delta - L\alpha_{n-1} + (3/4) l_n^{-2} z_n \|\psi_n - \sum_{j=1}^{n-1}(\partial\alpha_{n-1}/\partial x_j)\psi_j\|^4$. Furthermore, (77) can be rewritten as

$$\begin{aligned} LV_n \le & -\sum_{j=1}^{n-1}\left(k_j z_j^4 + \frac{\gamma_j}{2\lambda_j}\widetilde{\theta}_j^2\right) + \sum_{j=1}^{n-1}\varrho_j + z_n^3\overline{f}_n + z_n^3\overline{v} \\ & + \frac{1}{\gamma}\left(g_n\mu_1 N(\zeta) + 1\right)\dot{\zeta} + \frac{1}{2} b_M (\beta+\chi)^{-4/n} \\ & + \frac{3}{4} l_n^2 - \frac{1}{\lambda_n}\widetilde{\theta}_n\dot{\widehat{\theta}}_n - \frac{1}{\sigma}\widetilde{\Delta}_n\dot{\widehat{\Delta}}. \end{aligned} \tag{78}$$

For any positive constant $\varepsilon_n > 0$, the fuzzy logic system $\Phi_n^T \xi_n(X_n)$ existed, such that

$$\overline{f}_n = \Phi_n^T \xi_n(X_n) + \delta_n(X_n), \quad \left|\delta_n(X_n)\right| \le \varepsilon_n, \tag{79}$$

Similarly, we can obtain

$$z_n^3 \overline{f}_n \le \frac{z_n^6}{2a_n^2}\theta_n \xi_n^T \xi_n + \frac{1}{2} a_n^2 + \frac{3}{4} z_n^4 + \frac{1}{4}\varepsilon_n^4, \tag{80}$$

where a_n is a positive parameter. We choose the following virtual control signal and adaptive law:

$$\overline{v} = -\left(k_n + \frac{3}{4}\right) z_n - \frac{1}{2a_n^2}\widehat{\theta}_n z_n^3 \xi_n^T \xi_n, \tag{81}$$

$$\dot{\widehat{\theta}}_n = \frac{\lambda_n}{2a_n^2} z_n^6 \xi_n^T \xi_n - \gamma_n \widehat{\theta}_n, \tag{82}$$

$$\dot{\widehat{\Delta}} = -\sigma_d \widehat{\Delta}, \tag{83}$$

where k_n, γ_n, and σ_d are positive constants. Substituting (80)-(83) into (78), we have

$$\begin{aligned} LV_n \le & -\sum_{j=1}^{n-1}\left(k_j z_j^4 + \frac{\gamma_j}{2\lambda_j}\widetilde{\theta}_j^2\right) + \sum_{j=1}^{n-1}\varrho_j \\ & + \frac{1}{\gamma}\left(g_n\mu_1 N(\zeta) + 1\right)\dot{\zeta} + \frac{1}{2} b_M (\beta+\chi)^{-4/n} \\ & + \frac{3}{4} l_n^2 + \frac{1}{2} a_n^2 + \frac{1}{4}\varepsilon_n^4 - k_n z_n^4 + \frac{\gamma_n}{\lambda_n}\widetilde{\theta}\widehat{\theta} + \frac{\sigma_d}{\sigma}\widetilde{\Delta}\widehat{\Delta}. \end{aligned} \tag{84}$$

Furthermore,

$$\begin{aligned} \frac{\gamma_n}{\lambda_n}\widetilde{\theta}_n\widehat{\theta}_n &\le -\frac{\gamma_n}{2\lambda_n}\widetilde{\theta}_n^2 + \frac{\gamma_n}{2\lambda_n}\theta_n^2, \\ \frac{\sigma_d}{\sigma}\widetilde{\Delta}\widehat{\Delta} &\le -\frac{\sigma_d}{2\sigma}\widetilde{\Delta}^2 + \frac{\sigma_d}{2\sigma}\Delta^2. \end{aligned} \tag{85}$$

The above inequality can be rewritten as

$$\begin{aligned} LV_n \le & -\sum_{j=1}^{n}\left(k_j z_j^4 + \frac{\gamma_j}{2\lambda_j}\widetilde{\theta}_j^2\right) + \sum_{j=1}^{n}\varrho_j - \frac{\sigma_d}{2\sigma}\widetilde{\Delta}^2 \\ & + \frac{1}{\gamma}\left(g_n\mu_1 N(\zeta) + 1\right)\dot{\zeta} + \frac{1}{2} b_M (\beta+\chi)^{-4/n}, \end{aligned} \tag{86}$$

where $\varrho_j = (\gamma_j/2\lambda_j)\theta_j^2 + (3/4)l_j^2 + (1/4)\varepsilon_j^4 + (1/2)a_j^2$, $j = 1, 2, \ldots, n-1$, and $\varrho_n = (\gamma_n/2\lambda_n)\theta_n^2 + (3/4)l_n^2 + (1/4)\varepsilon_n^4 + (1/2)a_n^2 + (\sigma_d/2\sigma)\Delta^2$.

The control design of the adaptive fuzzy logic system has been completed by using the backstepping technique. The main theorem is described below.

Theorem 10. *Consider the stochastic pure-feedback nonlinear system (6) with Assumptions 7–9. For bounded initial conditions, combine with the virtual control signal and the adaptation law (60)-(61) and (81)-(83) guarantee that*

(i) *all the signals of the closed-loop system are semi-globally uniformly bounded on* $[0, t_f)$, $\forall t_f > 0$;

(ii) *the steady-state tracking errors* z_j *converge to* Ω_Z *in the sense of mean quartic value, which is defined as*

$$\Omega_Z = \left\{ z_j \mid \sum_{j=1}^{n} E\left[\left|z_j\right|^4\right] \le 4\rho \right\}, \tag{87}$$

where ρ *is defined in (97).*

Proof. (i) Let Lyapunov function as $V = V_n$, defining $c = \min\{\sigma_d, 4k_j, \gamma_j, j = 1, 2, \ldots, n\}$ and $d = \sum_{j=1}^{n}\varrho_j + (1/2) b_M (\beta + \chi)^{-4/n}$. According to (86), it follows that

$$LV \le -cV + \frac{1}{\gamma}\left(g_n\mu_1 N(\zeta) + 1\right)\dot{\zeta} + d, \quad t \ge 0. \tag{88}$$

Multiplying V by e^{ct} and based on $It\hat{o}$ formula, one has

$$d\left(e^{ct}V\right) = e^{ct}(cV + LV)\,dt + e^{ct}M(t)\,dw, \tag{89}$$

where

$$M(t) = \frac{\partial V}{\partial z_1}\psi_1^T(x_1) + \sum_{i=2}^{n}\frac{\partial V}{\partial z_i}\left(\psi_i(\overline{x}_i) - \sum_{j=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_j}\psi_i(\overline{x}_i)\right)^T. \tag{90}$$

According to (88)-(89), the following inequality can be obtained:

$$d\left(e^{ct}V\right) \le e^{ct}\left(\frac{1}{\gamma}(g_n\mu_1 N(\zeta)+1)\dot{\zeta} + d\right)dt + e^{ct}M(t)\,dw. \tag{91}$$

Integrating (91) on $[0,t]$, one has

$$e^{ct}V(t) - V(0) \le \int_0^t e^{c\tau}\frac{1}{\gamma}(g_n\mu_1 N(\zeta)+1)\dot{\zeta}\,d\tau + \frac{d}{c}\left(e^{ct}-1\right) + \int_0^t e^{c\tau}M(\tau)\,dw. \tag{92}$$

Furthermore,

$$\begin{aligned} V(t) &\le e^{-ct}\frac{1}{\gamma}\int_0^t e^{c\tau}(g_n\mu_1 N(\zeta)+1)\dot{\zeta}\,d\tau + e^{-ct}V(0) \\ &+ \frac{d}{c}\left(1-e^{-ct}\right) + e^{-ct}\int_0^t e^{c\tau}M(\tau)\,dw, \\ &\forall t \in \left[0,t_f\right). \end{aligned} \tag{93}$$

Let $\xi(t) = g_n\mu_1$; according to Assumption 7 and the definition of μ_1, we have $0 < |\xi(t)| \le b_M|\mu_1| < +\infty$. Thus, according to the boundedness of $\xi(t)$ and Lemma 3, we can get that $V(t)$ and ζ and $\int_0^t(g_n\mu_1 N(\zeta)+1)\dot{\zeta}d\tau$ are bounded on $[0,t_f)$.

Next according to the definition of $V(t)$, we can get that z_j and $\widetilde{\theta}_j$ and $\widetilde{\Delta}$ are bounded on $[0,t_f)$. Thereby, $\widehat{\theta}_j$ and $\widehat{\Delta}$ are also bounded on $[0,t_f)$. Due to $z_1 = x_1 - y_d$ and y_d being bounded, we can get that x_1 is bounded. Based on α_1 is the function of z_1 and $\widehat{\theta}_1$, thus $x_2 = z_2 + \alpha_1$ is also bounded. Furthermore, we know that $x_j, j = 3,4,\ldots,n$ is bounded. Thus, all the signals of the closed-loop system are semiglobally uniformly bounded on $[0,t_f)$, $\forall t_f > 0$.

(ii) Define $\beta = \sup(1/\gamma)\int_0^t|(g_n\mu_1 N(\zeta)+1)\dot{\zeta}|d\tau$.

$$\begin{aligned} &e^{-ct}\frac{1}{\gamma}\int_0^t e^{c\tau}(g_n\mu_1 N(\zeta)+1)\dot{\zeta}\,d\tau \\ &= \frac{1}{\gamma}\int_0^t e^{c(\tau-t)}(g_n\mu_1 N(\zeta)+1)\dot{\zeta}\,d\tau \\ &\le \frac{1}{\gamma}\int_0^t (g_n\mu_1 N(\zeta)+1)\dot{\zeta}\,d\tau \le \beta, \quad t \ge 0. \end{aligned} \tag{94}$$

By taking expectation on (93) and applying (94), $E[w] = 0$, we can get

$$E[V(t)] \le \beta + e^{-ct}E[V(0)] + \frac{d}{c}\left(1-e^{-ct}\right), \quad t \ge 0, \tag{95}$$

and therefore,

$$E[V(t)] \le \beta + \frac{d}{c}, \quad t \longrightarrow +\infty. \tag{96}$$

Let

$$\rho = \beta + \frac{d}{c}, \tag{97}$$

and further, due to $V = V_n$ in the definition of (68), we have

$$E\left(\sum_{j=1}^{n} z_j^4\right) \le 4E[V(t)] \le 4\rho, \quad t \longrightarrow +\infty. \tag{98}$$

Thus, the steady-state tracking errors z_j converge to Ω_Z in the sense of mean quartic value. □

4. Simulation Example

In this section, a simulation example is given to prove the effectiveness of the proposed adaptive control scheme.

Example. Consider a second-order pure-feedback stochastic system with unknown hysteresis and external disturbance

$$\begin{aligned} dx_1 &= \left(x_1^2 + 5x_2 + 0.5\sin {x_1}^2\right)dt + \sin x_1 dw, \\ dx_n &= \left({x_2}^2 \sin {x_2}^2 + 3u + \sin t^2\right)dt + \cos(x_1x_2)\,dw, \\ y &= x_1, \end{aligned} \tag{99}$$

where x_1 and x_2 are the state variable, t is the time variable, y is the system output, and u is the unknown hysteresis output. These hysteresis parameters are chosen as $\mu_1 = 0.5$, $\mu_2 = 1.2$, $\beta = 1.5$, $\chi = 0.8$, and $n = 2$. It is obvious that (99) satisfies the assumptions of 7-9. The system control target is that the system output y is tracked to the reference signal $y_d = 0.6\sin(0.5x) + 0.5\sin(x)$.

For all state variables, the fuzzy sets are defined on the interval $[-1.5, 1.5]$, the fuzzy membership functions are as follows:

$$\begin{aligned} \mu_{F_i^1}(x) &= \exp\left(-0.5(x+1.5)^2\right), \\ \mu_{F_i^2}(x) &= \exp\left(-0.5(x+1)^2\right), \\ \mu_{F_i^3}(x) &= \exp\left(-0.5(x+0.5)^2\right), \\ \mu_{F_i^4}(x) &= \exp\left(-0.5x^2\right), \\ \mu_{F_i^5}(x) &= \exp\left(-0.5(x-0.5)^2\right), \\ \mu_{F_i^6}(x) &= \exp\left(-0.5(x-1)^2\right), \\ \mu_{F_i^7}(x) &= \exp\left(-0.5(x-1.5)^2\right). \end{aligned} \tag{100}$$

The virtual control signal and the adaptive law are selected as follows:

$$\alpha_1 = -\left(k_1 + \frac{3}{4}\right)z_1 - \frac{1}{2a_1^2}\widehat{\theta}_1 z_1^3\xi_1^T\xi_1 + x_2^0,$$

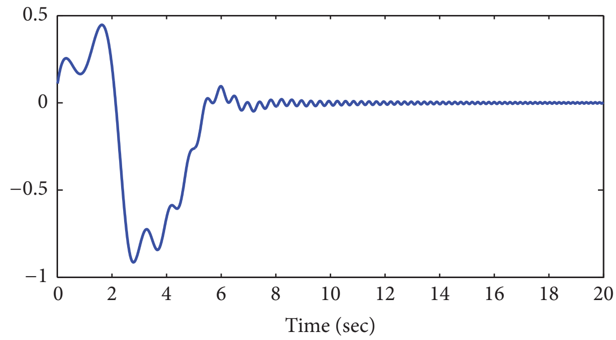

FIGURE 1: Tracking error $z_1 = y - y_d$.

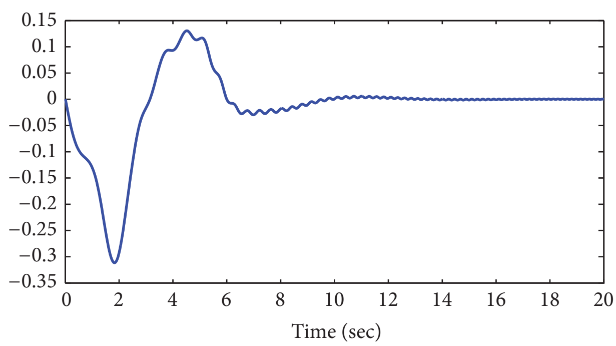

FIGURE 2: State variable x_2.

$$
\begin{aligned}
\dot{\widehat{\theta}}_1 &= \frac{\lambda_1}{2a_1^2} z_1^6 \xi_1^T \xi_1 - \gamma_1 \widehat{\theta}_1, \quad \widehat{\theta}_1(0) \geq 0, \\
\overline{v} &= -\left(k_2 + \frac{3}{4}\right) z_2 - \frac{1}{2a_2^2} \widehat{\theta}_2 z_2^3 \xi_2^T \xi_2, \\
\dot{\widehat{\theta}}_2 &= \frac{\lambda_2}{2a_2^2} z_2^6 \xi_2^T \xi_2 - \gamma_2 \widehat{\theta}_2, \\
\dot{\widehat{\Delta}} &= -\sigma_d \widehat{\Delta},
\end{aligned}
\tag{101}
$$

where $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$. The Nussbaum function is chosen as $N(\zeta) = \zeta^2 \cos \zeta$; its control law is $v = -N(\zeta)\overline{v} + v^0$, $\dot{\zeta} = -\gamma \overline{v} z_2^3$. The control parameters are designed as $k_1 = k_2 = 10$, $a_1 = a_2 = 2$, $\lambda_1 = \lambda_2 = 8$, $\gamma_1 = \gamma_2 = 0.5$, $\gamma = 8$, $\sigma_d = 0.25$, and $x_2^0 = v^0 = 0$. The initial condition is selected as $[x_1(0), x_2(0)]^T = [0.1, 0.2]^T$, $\widehat{\Delta}(0) = 0$, and $[\widehat{\theta}_1(0) = \widehat{\theta}_2(0)]^T = [0, 0]^T$. The simulation results are shown in Figures 1–5. Figure 1 shows the tracking error $z_1 = y - y_d$. Figure 2 shows the state variable x_2. Figure 3 shows the true control input $\overline{v}$. Figure 4 presents the adaptive parameter $\widehat{\theta}_2$. Figure 5 presents estimation of disturbance bound $\widehat{\Delta}$.

5. Conclusion

In this paper, we study a class of stochastic pure-feedback nonlinear systems with bounded external disturbance and unknown hysteresis. This paper holds that stochastic disturbance and external disturbance exist simultaneously. By using the characteristics of the Nussbaum function, the unknown hysteresis problem is solved. Based on the approximation ability of fuzzy logic system, a new adaptive fuzzy control scheme is proposed. The scheme ensures that all signals of the closed-loop system are bounded and the tracking error converges to small domain of the origin. Finally, the simulation results show the effectiveness of the proposed scheme.

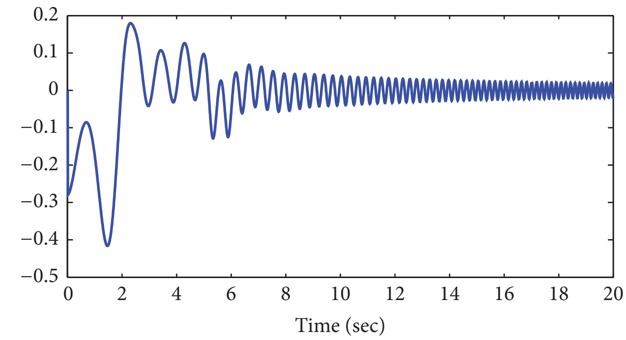

FIGURE 3: The true control input $\overline{v}$.

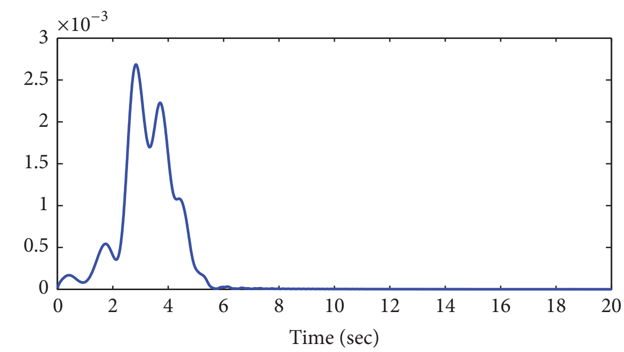

FIGURE 4: The adaptive parameter $\widehat{\theta}_2$.

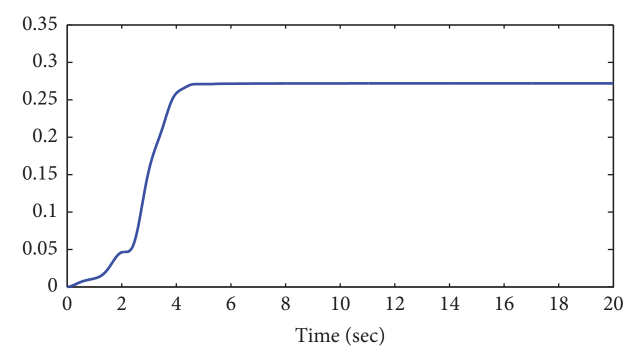

FIGURE 5: Estimation of disturbance bound $\widehat{\Delta}$.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

[1] J. Zhou, C. Wen, and T. Li, "Adaptive output feedback control of uncertain nonlinear systems with hysteresis nonlinearity," *IEEE Transactions on Automatic Control*, vol. 57, no. 10, pp. 2627–2633, 2012.

[2] M.-C. Pai, "Chaos control of uncertain time-delay chaotic systems with input dead-zone nonlinearity," *Complexity*, vol. 21, no. 3, pp. 13–20, 2015.

[3] M. Wang, S. Zhang, B. Chen, and F. Luo, "Direct adaptive neural control for stabilization of nonlinear time-delay systems," *Science China Information Sciences*, vol. 53, no. 4, pp. 800–812, 2010.

[4] H. Wang, P. X. Liu, and S. Liu, "Adaptive neural synchronization control for bilateral teleoperation systems with time delay and backlash-like hysteresis," *IEEE Transactions on Cybernetics*, 2017.

[5] C.-Y. Su, Q. Wang, X. Chen, and S. Rakheja, "Adaptive variable structure control of a class of nonlinear systems with unknown Prandtl-Ishlinskii hysteresis," *IEEE Transactions on Automatic Control*, vol. 50, no. 12, pp. 2069–2074, 2005.

[6] G. Gu, L. Zhu, and C. Su, "Modeling and Compensation of Asymmetric Hysteresis Nonlinearity for Piezoceramic Actuators With a Modified Prandtl–Ishlinskii Model," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 3, pp. 1583–1595, 2014.

[7] C. Hua and Y. Li, "Output feedback prescribed performance control for interconnected time-delay systems with unknown Prandtl-Ishlinskii hysteresis," *Journal of The Franklin Institute*, vol. 352, no. 7, pp. 2750–2764, 2015.

[8] H. Wang, P. X. Liu, S. Li, and D. Wang, "Adaptive Neural Output-Feedback Control for a Class of Nonlower Triangular Nonlinear Systems With Unmodeled Dynamics," *IEEE Transactions on Neural Networks and Learning Systems*, 2017.

[9] X. Zhao, X. Wang, G. Zong, and H. Li, "Fuzzy-approximation-based adaptive output-feedback control for uncertain nonsmooth nonlinear systems," *IEEE Transactions on Fuzzy Systems*, pp. 1–12, 2018.

[10] F. Wang, B. Chen, C. Lin, J. Zhang, and X. Meng, "Adaptive Neural Network Finite-Time Output Feedback Control of Quantized Nonlinear Systems," *IEEE Transactions on Cybernetics*, 2017.

[11] F. Wang, B. Chen, Y. Sun, and C. Lin, "Finite time control of switched stochastic nonlinear systems," *Fuzzy Sets and Systems*, 2018.

[12] H. Wang, K. Liu, X. Liu, B. Chen, and C. Lin, "Neural-based adaptive output-feedback control for a class of nonstrict-feedback stochastic nonlinear systems," *IEEE Transactions on Cybernetics*, vol. 45, no. 9, pp. 1977–1987, 2015.

[13] X. Liu, Y. Li, and W. Zhang, "Stochastic linear quadratic optimal control with constraint for discrete-time systems," *Applied Mathematics and Computation*, vol. 228, pp. 264–270, 2014.

[14] H. Li, L. Bai, Q. Zhou, R. Lu, and L. Wang, "Adaptive fuzzy control of stochastic nonstrict-feedback nonlinear systems with input saturation," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 2185–2197, 2017.

[15] H. Q. Wang, B. Chen, K. F. Liu, X. P. Liu, and C. Lin, "Adaptive neural tracking control for a class of nonstrict-feedback stochastic nonlinear systems with unknown backlashlike hysteresis," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 25, no. 5, pp. 947–958, 2014.

[16] H. Wang, B. Chen, and C. Lin, "Adaptive fuzzy control for pure-feedback stochastic nonlinear systems with unknown dead-zone input," *International Journal of Systems Science*, vol. 2013, Article ID 773470, 2013.

[17] Y. Gao, S. Tong, and Y. Li, "Fuzzy adaptive output feedback DSC design for SISO nonlinear stochastic systems with unknown control directions and dead-zones," *Neurocomputing*, vol. 167, pp. 187–194, 2015.

[18] Y.-J. Liu and S. C. Tong, "Adaptive fuzzy identification and control for a class of nonlinear pure-feedback MIMO systems with unknown dead zones," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 5, pp. 1387–1398, 2015.

[19] Q. Zhou, P. Shi, S. Xu, and H. Li, "Observer-based adaptive neural network control for nonlinear stochastic systems with time delay," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 24, no. 1, pp. 71–80, 2013.

[20] Z. X. Yu, S. G. Li, and H. B. Du, "Razumikhin-Nussbaum-lemma-based adaptive neural control for uncertain stochastic pure-feedback nonlinear systems with time-varying delays," *International Journal of Robust and Nonlinear Control*, vol. 23, no. 11, pp. 1214–1239, 2013.

[21] F. Wang, Z. Liu, Y. Zhang, and C. L. P. Chen, "Adaptive fuzzy control for a class of stochastic pure-feedback nonlinear systems with unknown hysteresis," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 1, pp. 140–152, 2016.

[22] X. Zhang, X. Liu, and Y. Li, "Adaptive fuzzy tracking control for nonlinear strict-feedback systems with unmodeled dynamics via backstepping technique," *Neurocomputing*, vol. 235, pp. 182–191, 2017.

[23] N. Wang, J.-C. Sun, and Y.-C. Liu, "Direct adaptive self-structuring fuzzy control with interpretable fuzzy rules for a class of nonlinear uncertain systems," *Neurocomputing*, vol. 173, pp. 1640–1645, 2016.

[24] Y. Hou and S. Tong, "Adaptive fuzzy output-feedback control for a class of nonlinear switched systems with unmodeled dynamics," *Neurocomputing*, vol. 168, pp. 200–209, 2015.

[25] G. Nagamani, S. Ramasamy, and P. Balasubramaniam, "Robust dissipativity and passivity analysis for discrete-time stochastic neural networks with time-varying delay," *Complexity*, vol. 21, no. 3, pp. 47–58, 2016.

[26] N. Wang, J.-C. Sun, and M. J. Er, "Tracking-error-based universal adaptive fuzzy control for output tracking of nonlinear systems with completely unknown dynamics," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 2, pp. 869–883, 2018.

[27] B. Chen, X. Liu, K. Liu, and C. Lin, "Direct adaptive fuzzy control of nonlinear strict-feedback systems," *Automatica*, vol. 45, no. 6, pp. 1530–1535, 2009.

[28] Z. Liu, F. Wang, Y. Zhang, X. Chen, and C. L. P. Chen, "Adaptive tracking control for a class of nonlinear systems with a fuzzy dead-zone input," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 1, pp. 193–204, 2015.

[29] B. Ren, S. S. Ge, C. Su, and T. H. Lee, "Adaptive neural control for a class of uncertain nonlinear systems in pure-feedback form with hysteresis input," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 39, no. 2, pp. 431–443, 2009.

[30] S. Tong, Y. Li, Y. Li, and Y. Liu, "Observer-based adaptive fuzzy backstepping control for a class of stochastic nonlinear strict-feedback systems," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 41, no. 6, pp. 1693–1704, 2011.

[31] S.-J. Liu, J.-F. Zhang, and Z.-P. Jiang, "Decentralized adaptive output-feedback stabilization for large-scale stochastic nonlinear systems," *Automatica*, vol. 43, no. 2, pp. 238–251, 2007.

[32] S. S. Ge, F. Hong, and T. H. Lee, "Adaptive neural control of nonlinear time-delay systems with unknown virtual control coefficients," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. 34, no. 1, pp. 499–516, 2004.

[33] H. Wang, B. Chen, X. Liu, K. Liu, and C. Lin, "Robust adaptive fuzzy tracking control for pure-feedback stochastic nonlinear systems with input constraints," *IEEE Transactions on Cybernetics*, vol. 43, no. 6, pp. 2093–2104, 2013.