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Research Article

Exponential Stabilization of Coupled Hybrid Stochastic Delayed BAM Neural Networks: A Periodically Intermittent Control Method

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This paper considers exponential stabilization for a class of coupled hybrid stochastic delayed bidirectional associative memory neural networks (HSD-BAM-NN) with reaction-diffusion terms. A periodically intermittent controller is proposed to exponentially stabilize such an unstable HSD-BAM-NN, and sufficient conditions of the closed-loop BAM-NN system with exponential stabilization are derived by using Lyapunov-Krasovskii functional method, stochastic analysis techniques, and integral inequality property, which decide the basic parameters of the proposed controller. Furthermore, a framework to establish simulation algorithm with sampled states is presented to implement the stabilization controller. With a HSD-BAM-NN model of power synchronization in a photovoltaic (PV) array field, we illustrate numerical simulation results to verify the correctness and effectiveness of the proposed controller.

1. Introduction

Spurred by pioneering works on the neural networks models with BAM in Kosko [1–3], much attention has been paid to BAM-NN owning to its various applications in image processing, pattern recognition, automatic control [4, 5], associate memory, parallel computing [6], and optimization [7]. It is worth noting that the results of global stability for BAM models obtained in [1–3] require severe constraint conditions of symmetric connection weight matrix. In the neural networks with very large scale circuits, it is difficult for a practical NN system to satisfy the absolutely symmetric conditions in BAM models. Recently, the stability analysis of BAM neural networks has been paid considerable attention, and many stability conditions of such NN models have been reported in the published literature [8–22]. Since diffusion effects cannot be avoided in the neural networks, in a physical sense, when electrons are moving in asymmetric electromagnetic fields, it is more valuable to consider the alternative activation of neurons in an available space as well as in a given time interval. Therefore, the model of BAM neural networks could be formulated as partial differential equations (PDE) instead of only ordinary differential equations (ODE) [9, 11]. Based on PDE models, many contributions have been published focusing on the stability of BAM neural networks (BAM-NN) with reaction-diffusion terms by using Lyapunov functional method and LMI techniques [7, 9, 11–15, 17, 18, 23, 24].

Haykin [25] proposed that in real neural networks systems, synaptic transmission is a stochastic process with noise released by random electronic fluctuations of the neurotransmitters and other disturbance. Following the experimental conclusions, Hossain and Anagnostou [26] further stated that noise posed a basic problem for information processing which affected all aspects of nervous system function in a nervous system. Hence, it is also important to study the effects of noise perturbations existing in neutral networks dynamics. Literature [8] investigated the stability of stochastic NN for the first time, which guided followers exploring novel results of such problems, especially for stochastic NN with reaction-diffusion terms, and some striking published contributions can be found in [7, 11–15, 17, 18, 27].

It is worth mentioning that the hybrid BAM-NN driven by continuous-time Markov chains have been used to model many practical systems where they may experience abrupt changes in their structure and parameters. To consider the issues of system structures' abrupt changes, hybrid disturbance, and unreliable subsystem interconnections, the evolution dynamics of BAM-NN could be modeled as jump systems [28, 29]. As Markovian jump system was first introduced in [10], two fundamental components have formed original ideas of considering a jumping system as continuous state described by differential equations, and discrete-time state described by a continuous-time finite-state Markovian process [28–32]. For theory and techniques recently developed to analyze BAM-NN with Markovian jump factors, here we mention [11, 22, 24, 28, 29].

In recent literature, a variety of approaches have been published for the stabilization control of BAM neural networks with or without delays and reaction-diffusion terms which include feedback control [9, 21, 23, 32, 33], impulsive control [22, 24, 34], and intermittent control [35–38]. The type of control considered in this paper is intermittent control, which was first introduced to control nonlinear dynamical systems in [39] and has aroused much interest of researchers due to its merits in engineering applications. Different from continuous control approaches, intermittent control is more effective because the system output is measured intermittently rather than continuously [35]. Some novel contributions of intermittent control [35, 36, 38] based on the stability analysis of BAM-NN have been achieved in recent years. Nevertheless, the complex effects on BAM-NN with stimulation time-varying delays and stochastic reaction-diffusion terms have not been considered to use intermittent control in published results.

Motivated by the above discussion, the main purpose of this paper is to investigate BAM-NN with delays and reaction-diffusion terms and focus on its exponential stabilization by designing a periodically intermittent controller [35, 36, 38]. The main novelties in this paper can be highlighted as follows: Firstly, the model under consideration covers the frequently investigated models which often are characterized by special cases in structures or systematic functions. Secondly, most available results have been concerned with the stability problems, and in this technical note, stochastic stabilization is taken into account and a periodically intermittent controller with more flexible conditions is first proposed for such BAM-NN with stochastic reaction-diffusion terms. Finally, a numerical simulation method is designed to simulate the behavior of the time-varying delays stochastic hybrid partial differential equations, which enriches the theory of delayed stochastic partial differential equation.

Throughout the paper, we take $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, \mathbb{P})$ as a complete probability space with a filtration $\{\mathscr{F}\}_{t\geq 0}$ satisfying the usual conditions; i.e., $\{\mathscr{F}\}_{t\geq 0}$ is right-continuous and $\{\mathscr{F}\}_0$ contains all $\mathbb{P}$-null sets. Let $W(t)$, $t \geq t_0$, be one-dimensional Brownian motion defined on the probability space. Let $r(t)$, $t \geq 0$, be right-continuous Markov chain on the probability space taking values in a finite-state space $\mathbb{S} = \{1, 2, \ldots, \mathscr{N}\}$ with generator $\Gamma = (r_{ij})_{\mathscr{N}\times\mathscr{N}}$. And the transition probability from model i at time t to model j at time $t + \Delta$ is

$$\mathbb{P}\{r(t+\Delta) = j \mid r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta), & \text{if } i \neq j \\ 1 + \gamma_{ii}\Delta + o(\Delta), & \text{if } i = j \end{cases} \quad (1)$$

where $\Delta > 0$ and $\lim_{\Delta\to 0} o(\Delta)/\Delta = 0$. Here, $\gamma_{ij} \geq 0$, $i \neq j$, is the transition probability from model i to model j and $\gamma_{ii} = -\sum_{j=1,j\neq i}^{N} \gamma_{ij}$. We assume that the Markovian chain $r(\cdot)$ is independent of the Brownian motion $W(\cdot)$. It is well known that almost every sample path of $r(\cdot)$ is right-continuous step function with a finite number of simple jumps in any finite subinterval $\mathbb{R}_+$.

2. Preliminaries and Problem Formulation

In this paper, we consider the hybrid stochastic BAM neural network model with reaction-diffusion terms [11–13, 15], which are formulated as a couple of neurons' dynamics indexed by a pair superscripts of (n, m)

$$\begin{aligned} dx_i^{(n)}\left(t, \lambda^{(n)}\right) &= \Bigg(\sum_{k=1}^{l^{(n)}} \frac{\partial}{\partial \lambda_k^{(n)}} \cdot \left(D_{ik}^{(n)}(r(t)) \frac{\partial x_i^{(n)}\left(t, \lambda^{(n)}\right)}{\partial \lambda_k^{(n)}} \right) - a_i^{(n)}(r(t))\, x_i^{(n)}\left(t, \lambda^{(n)}\right) + \sum_{j=1}^{N^{(m)}} \left(b_{ji}^{(n)}(r(t))\, f_j^{(n)}\left(x_j^{(m)}\left(t, \lambda^{(m)}\right)\right)\right) \\ &\quad + \sum_{j=1}^{N^{(m)}} \left(c_{ji}^{(n)}(r(t))\, f_j^{(n)}\left(x_j^{(m)}\left(t - \tau^{(m)}(t), \lambda^{(m)}\right)\right)\right) \\ &\quad + u_i^{(n)}\left(t, \lambda^{(n)}, \lambda^{(m)}\right) \Bigg) dt + \sum_{j=1}^{N^{(m)}} h_{ji}^{(n)}\left(x_i^{(n)}\left(t, \lambda^{(n)}\right), x_j^{(m)}\left(t, \lambda^{(m)}\right), x_j^{(m)}\left(t - \tau^{(m)}(t), \lambda^{(m)}\right)\right) dW_j(t) \end{aligned} \quad (2)$$

where n, m denote the index of a couple of neuron networks (NN) with $N^{(n)}$ and $N^{(m)}$ neurons, respectively; i.e., $(n, m) = (1, 2)$ or $(2, 1)$. The notations $i = 1, 2, \ldots, N^{(n)}$ and $j = 1, 2, \ldots, N^{(m)}$ are, respectively, the indices of the neurons in the nth NN and the mth NN. $l^{(n)}$ is the dimension of space variable vector $\lambda^{(n)}$, and $\lambda^{(n)} = (\lambda_1^{(n)}, \lambda_2^{(n)}, \ldots, \lambda_{l^{(n)}}^{(n)}) \in \Omega_0 \subset \mathbb{R}^{l^{(n)}}$ where Ω_0 is a compact set and measurable with smooth boundary $\partial\Omega_0$ in space $\mathbb{R}^{l^{(n)}}$, $0 < mes\Omega_0 < +\infty$. The definition of $\lambda^{(m)}$ is the same as $\lambda^{(n)}$ and we let $\lambda = (\lambda^{(n)}, \lambda^{(m)})$. $x_i^{(n)} = x_i^{(n)}(t, \lambda^{(n)})$ and $x_j^{(m)} = x_j^{(m)}(t, \lambda^{(m)})$ are the states of the ith neuron in nth NN and the jth neuron in mth

NN at time $t(t \geq 0)$ and in space of $\lambda^{(n)}$, $\lambda^{(m)}$, respectively. For simplicity, we define $x^{(n)} = (x_1^{(n)}, \ldots, x_{N^{(n)}}^{(n)})^T \in \mathbb{R}^{N^{(n)}}$, $x^{(m)} = (x_1^{(m)}, \ldots, x_{N^{(m)}}^{(m)})^T \in \mathbb{R}^{N^{(m)}}$, and $x = ((x^{(n)})^T, (x^{(m)})^T)^T$ to denote the state vectors of the nth, mth, and whole coupled neural networks, respectively. $\tau_i^{(n)}(t)$ denotes the time delay satisfying $0 \leq \tau^{(n)}(t) \leq \tau^{(n)}, \dot{\tau}^{(n)}(t) \leq \tau_0^{(n)} < 1$ with constant $\tau^{(n)}, \tau_0^{(n)}$. $f_j^{(n)}(\cdot)$ denotes the activation function of the jth neuron in mth NN stimulating the ith neurons in the nth NN. $a_i^{(n)}(r(t)) > 0$ denotes changing rate of the ith neuron under the condition that neural network is disconnected and no external additional activation exists. $b_{ji}^{(n)}(r(t))$ is the connection weight of neurons in coupled NN and $c_{ji}^{(n)}(r(t))$ denotes the corresponding delayed connection weight. The smooth function $D_{ik}^{(n)}((r(t))) \geq 0$ is the transmission diffusion operator of neurons, $h_{ji}^{(n)}$ is the stochastic disturbance function of neurons, and $W_j(t)$ is the Brownian motions as noise acting on the transmission from the jth neuron to the ith neuron.

According to [9, 10], we present the following initial value condition for (2):

$$x_i^{(n)}\left(s, \lambda^{(n)}\right) = \phi_i\left(s, \lambda^{(n)}\right), \quad \left(s, \lambda^{(n)}\right) \in \left[-\tau^{(n)}, 0\right) \times \Omega_0 \tag{3}$$

which is with Dirichlet boundary value.

$$x_i^{(n)}\left(t_0, \lambda^{(n)}\right) = 0, \quad \left(t, \lambda^{(n)}\right) \in \left[-\tau^{(n)}, +\infty\right) \times \partial\Omega_0 \tag{4}$$

For convenience, we denote all the possible models of BAM-NN as $A^{(n)}(r(t)) = \operatorname{diag}(a_1^{(n)}(r(t)), \cdots, a_{N^{(n)}}^{(n)}(r(t)))$, $B^{(n)}(r(t)) = (b_{ji}^{(n)}(r(t)))_{N^{(n)} \times N^{(m)}}$, and $C^{(n))}(r(t)) = (c_{ji}^{(n)}(r(t)))_{N^{(n)} \times N^{(m)}}$.

Definition 1. A stochastic vector x is the solution of system (2)-(4) if it satisfies the following conditions:

(i) x is adapted to $\{\mathcal{F}_t\}_{\{t \geq 0\}}$,

(ii) for $T_0 \in \mathbb{R}_+, x \in C_{\mathcal{F}_0}^b([0, T_0] \times \Omega_0; \mathbb{R}^{(N^{(n)}+N^{(m)})})$,

$$\mathbb{E}\left(\max_{\lambda^{(n)} \in \Omega_0} \int_0^{T_0} \left(\sum_{n=1}^{2} \left(\left|x^{(n)}\right|^2 + \left|\nabla x^{(n)}\right|^2\right)\right) \mathrm{d}t\right) < +\infty, \tag{5}$$

(iii) for $t \in \mathbb{R}_+$,

$$\begin{aligned}
\int_{\Omega_0} x_i^{(n)} \mathrm{d}\lambda^{(n)} &= \int_{\Omega_0} \phi_i\left(0, \lambda^{(n)}\right) \mathrm{d}\lambda^{(n)} \\
&+ \int_{\Omega_0} \int_0^t \left(-a_i^{(n)}(r(s))\, x_i^{(n)}\left(s, \lambda^{(n)}\right)\right. \\
&+ \int_{\Omega_0} \int_0^t \sum_{k=1}^{l^{(n)}} \frac{\partial}{\partial \lambda_k^{(n)}} \left(D_{ik}^{(n)}(r(s)) \frac{\partial x_i^{(n)}\left(s, \lambda^{(n)}\right)}{\partial \lambda_k^{(n)}}\right) \mathrm{d}s\, \mathrm{d}\lambda^{(n)} \\
&+ \sum_{j=1}^{N^{(m)}} b_{ji}^{(n)}(r(s))\, f_j^{(n)}\left(x_j^{(m)}\left(s, \lambda^{(m)}\right)\right) + \sum_{j=1}^{N^{(m)}} c_{ji}^{(n)}(r(s)) \\
&\cdot \left. f_j^{(n)}\left(x_j^{(m)}\left(s - \tau^{(m)}(s), \lambda^{(m)}\right)\right)\right) \mathrm{d}s\, \mathrm{d}\lambda \\
&+ \int_{\Omega_0} \int_0^t \sum_{j=1}^{N^{(m)}} h_{ji}^{(n)}\left(x_i^{(n)}\left(s, \lambda^{(n)}\right), x_j^{(m)}\left(s, \lambda^{(m)}\right),\right. \\
&\left. x_j^{(m)}\left(s - \tau^{(m)}(s), \lambda^{(m)}\right)\right) \mathrm{d}W_j(s)\, \mathrm{d}\lambda.
\end{aligned} \tag{6}$$

Definition 2. The HSD-BAM-NN (2)-(4) is exponentially stable in pth moment if, for arbitrary model combinations of $(A(\tilde{r}), B(\tilde{r}), C(\tilde{r}))(\tilde{r} \in \mathbb{S})$, the states of the system satisfy

$$\limsup_{t \longrightarrow \infty} \frac{\log \mathbb{E}\left(\sum_{n=1}^{2}\left(\left\|x^{(n)}\right\|^p\right)\right)}{t} < 0 \tag{7}$$

where $\|x^{(n)}\|^p = (\int_{\Omega_0} |x^{(n)}|^p \mathrm{d}\lambda^{(n)})^{1/p}$, $p \geq 2$. To ensure the existence and uniqueness of the solution to system (2)-(4), we suppose the following assumptions.

Assumption 3. For $i = 1, \ldots, N^{(n)}$, arbitrary $s_1, s_2 \in \mathbb{R}$, the activation function $f_i^{(n)}(\cdot)$ is bounded with $f_i^{(n)}(0) = 0$ and

$$L_i^- \leq \frac{f_i^{(n)}(s_1) - f_i^{(n)}(s_2)}{s_1 - s_2} \leq L_i^+ \tag{8}$$

where $s_1 \neq s_2, L_i^-, L_i^+$ are constants.

Remark 4. In the above assumption, L_i^-, L_i^+ could be positive, negative, or zero. Such an assumption is weaker than the one in [14] where it demands $L_i^- \equiv -L_i^+$.

Assumption 5. For $s_1, s_2, s_3 \in \mathbb{R}$, there exits nonnegative constant $\sigma_{ji}^{(n)}$ satisfying the following.

$$\left(h_{ji}^{(n)}(s_1, s_2, s_3)\right)^2 \leq \sigma_{ji}^{(n)}\left(s_1^2 + s_2^2 + s_3^2\right) \tag{9}$$

As noted in [6, 10], even if the parameters or time-varying delay in neural networks are appropriately chosen, neural networks may lead to some phenomena such as instability, divergence, oscillation, and chaos [10, 11, 40]. In order to stabilize the BAM-NN system (2)-(4), we introduce the following periodically intermittent controller

$$u_i^{(n)}\left(t, \lambda^{(n)}\right) = \begin{cases} \sum_{\ell=1}^{N^{(n)}} K_{i\ell}^{(n)} x_\ell^{(n)}, & \mu T \leq t < \mu T + \delta \\ 0, & \mu T + \delta \leq t < (\mu + 1) T \end{cases} \tag{10}$$

where T denotes the control period, $\mu = 0, 1, 2, \cdots$ is the control periods number, $\delta (0 < \delta < T)$ is called the control time width, and $K_{i\ell}^{(n)}$ are the control gains.

Under the periodically intermittent controller (10), the closed-loop systems of (2)-(4) can be described as follows.

$$\begin{aligned}\mathrm{d}x_i^{(n)} &= \left(\sum_{k=1}^{l^{(n)}} \frac{\partial}{\partial\lambda_k^{(n)}}\left(D_{ik}^{(n)}(r(t))\frac{\partial x_i^{(n)}}{\partial\lambda_k^{(n)}}\right) - a_i^{(n)}(r(t))\right. \\ &\quad \cdot x_i^{(n)} + \sum_{j=1}^{N^{(m)}} b_{ji}^{(n)}(r(t))\, f_j^{(n)}\left(x_j^{(m)}\right) + \sum_{j=1}^{N^{(m)}} c_{ji}^{(n)}(r(t)) \\ &\quad \left.\cdot f_j^{(n)}\left(x_j^{(m)}\left(t-\tau^{(m)}(t),\lambda^{(m)}\right)\right) + \sum_{\ell=1}^{N^{(n)}} K_{i\ell}^{(n)} x_\ell^{(n)}\right)\mathrm{d}t \\ &\quad + \sum_{j=1}^{N^{(m)}} h_{ji}^{(n)}\left(x_i^{(n)}, x_j^{(m)},\right. \\ &\quad \left. x_j^{(m)}\left(t-\tau^{(m)}(t),\lambda^{(m)}\right)\right)\mathrm{d}W_j(t)\end{aligned} \tag{11}$$

For the pth moment of $x(t,\lambda)$, we cite the following lemma to derive the stability conditions of the system.

Lemma 6 (see [36]). *Let Ω_0 be a super cuboid set and $\lambda = (\lambda_k)_{1\times l}$, $|\lambda_k| \le \theta(\theta > 0)$, $k = 1,2,\ldots,l$. If $x(t,\lambda)$ is a real-valued continuous derivable function as $x \in \mathbb{C}((\mathbb{R}^+,\Omega_0);\mathbb{R})$, $x(t,\lambda)|_{\partial\Omega_0} = 0$, then*

$$\begin{aligned}&\int_{\Omega_0} |x(t,\lambda)|^p\,\mathrm{d}\lambda \\ &\quad \le \frac{p^2\theta}{4}\int_{\Omega_0} |x(t,\lambda)|^{p-2}\left|\frac{\partial x(t,\lambda)}{\partial\lambda_k}\right|^2\mathrm{d}\lambda.\end{aligned} \tag{12}$$

3. Main Results

In this section, we present the sufficient conditions for stability of the controlled HDS-BAM-NN (11). For simplicity, the following notations are used to state and prove the main results.

$$\begin{aligned}\varsigma_i^{(n)} &= \min_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(n)}\left\{\sum_{k=1}^{l^{(n)}}\frac{4(p-1)\underline{D}_{ik}^{(n)}}{p\theta_k^2} + p\tilde{a}_i^{(n)}\right. \\ &\quad - \sum_{j=1}^{N^{(m)}}\sum_{\overline{\ell}=1}^{p-1}\left(\left(\tilde{b}_{ji}^{(n)}\right)^{p\alpha_{\overline{\ell}ji}^{(n)}}\tilde{L}_j^{p\beta_{\overline{\ell}ji}^{(n)}} + \left(\tilde{c}_{ji}^{(n)}\right)^{p\xi_{\overline{\ell}ji}^{(n)}}\tilde{L}_j^{p\zeta_{\overline{\ell}ji}^{(n)}}\right) \\ &\quad - \frac{p-1}{2}\sum_{j=1}^{N^{(m)}}\left(\sum_{\overline{\ell}=1}^{p-2}\left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{\overline{\ell}ji}^{(n)}} + \left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{(p-1)ji}^{(n)}}\right. \\ &\quad \left.\left. + \left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{pji}^{(n)}}\right)\right) - \min_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(m)}\sum_{j=1}^{N^{(m)}}\left(\left(\tilde{b}_{ij}^{(m)}\right)^{p\overline{\alpha}_{pji}^{(n)}}\left(\tilde{N}_i\right)^{p\overline{\beta}_{pji}^{(n)}}\right. \\ &\quad \left. + \frac{p-1}{2}\left(\left|\sigma_{ij}^{(m)}\right|^{p\overline{\epsilon}_{(p-1)ij}^{(n)}} + \left|\sigma_{ij}^{(m)}\right|^{p\overline{\epsilon}_{pij}^{(n)}}\right)\right)\end{aligned} \tag{13}$$

$$\begin{aligned}k_i^{(n)} &= \min_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(n)}\left(\sum_{k=1}^{l^{(n)}}\frac{4(p-1)\underline{D}_{ik}^{(n)}}{p\theta_k^2} + p\tilde{a}_i^{(n)}\right. \\ &\quad - \sum_{j=1}^{N^{(m)}}\left(\sum_{\overline{\ell}=1}^{p-1}\left(\left(\tilde{b}_{ji}^{(n)}\right)^{p\alpha_{\overline{\ell}ji}^{(n)}}\tilde{L}_j^{p\beta_{\overline{\ell}ji}^{(n)}} + \left(\tilde{c}_{ji}^{(n)}\right)^{p\xi_{\overline{\ell}ji}^{(n)}}\tilde{L}_j^{p\zeta_{\overline{\ell}ji}^{(n)}}\right)\right. \\ &\quad \left.\left. + \frac{p-1}{2}\left(\sum_{\overline{\ell}=1}^{p-2}\left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{\overline{\ell}ji}^{(n)}} + \left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{(p-1)ji}^{(n)}} + \left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{pji}^{(n)}}\right)\right)\right) \\ &\quad - \min_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(m)}\sum_{j=1}^{N^{(m)}}\left(\left(\tilde{b}_{ij}^{(m)}\right)^{p\overline{\alpha}_{pij}^{(n)}}\left(\tilde{N}_i\right)^{p\overline{\beta}_{pij}^{(n)}}\right. \\ &\quad \left. + \frac{p-1}{2}\left(\left|\sigma_{ij}^{(n)}\right|^{p\overline{\epsilon}_{(p-1)ij}^{(n)}} + \left|\sigma_{ij}^{(n)}\right|^{p\overline{\epsilon}_{pij}^{(n)}}\right)\right)\end{aligned} \tag{14}$$

$$\begin{aligned}\nu_i^{(n)} &= \max_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(n)}\left(pK_{ii}^{(n)} + \sum_{\substack{\ell=1\\ \ell\ne i}}^{N^{(n)}}\sum_{\overline{\ell}=1}^{p-1}\left|K_{i\ell}^{(n)}\right|^{p\overline{\eta}_{\overline{\ell}i\ell}}\right. \\ &\quad \left. + \sum_{\substack{\ell=1\\ \ell\ne i}}^{N^{(n)}}\left|K_{\ell i}^{(n)}\right|^{p\overline{\eta}_{p\ell i}}\right)\end{aligned} \tag{15}$$

$$\begin{aligned}\eta_i^{(n)} &= \max_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(m)}\sum_{j=1}^{N^{(m)}}\left(\frac{p-1}{2}\left(\left|\sigma_{ij}^{(n)}\right|^{p\overline{\epsilon}_{(p-1)ij}^{(n)}} + \left|\sigma_{ij}^{(n)}\right|^{p\overline{\epsilon}_{pij}^{(n)}}\right)\right. \\ &\quad \left. + \left(\tilde{c}_{ij}^{(n)}\right)^{p\overline{\xi}_{pij}^{(n)}}\left(\tilde{N}_i\right)^{p\overline{\zeta}_{pij}^{(n)}}\right)\end{aligned} \tag{16}$$

In the above equations, $\tilde{a}_i^{(n)} = \min_{\tilde{r}\in\mathbb{S}} a_i^{(n)}(\tilde{r})$, $\tilde{b}_{ji}^{(n)} = \max_{\tilde{r}\in\mathbb{S}}|b_{ji}^{(n)}(\tilde{r})|$, $\tilde{c}_{ij}^{(n)} = \max_{\tilde{r}\in\mathbb{S}}|c_{ji}^{(n)}(\tilde{r})|$, $\underline{D}_{ik}^{(n)} = \min_{\tilde{r}\in\mathbb{S}} D_{ik}^{(n)}(\tilde{r})$, $\tilde{L}_j = \max\{|L_j^-|,|L_j^+|\}$, $\tilde{N}_i = \max\{|N_i^-|,|N_i^+|\}$, $\mu_{\tilde{r}}^{(n)} > 0$, $\mu_{\tilde{r}}^{(m)} > 0$, and other variable parameters are given as $\alpha_{\overline{\ell}ji}^{(n)}, \beta_{\overline{\ell}ji}^{(n)}, \xi_{\overline{\ell}ji}^{(n)}, \zeta_{\overline{\ell}ji}^{(n)}, \epsilon_{\overline{\ell}ji}^{(n)}, \overline{\alpha}_{\overline{\ell}ji}^{(n)}, \overline{\beta}_{\overline{\ell}ji}^{(n)}, \overline{\xi}_{\overline{\ell}ji}^{(n)}, \overline{\zeta}_{\overline{\ell}ji}^{(n)}, \overline{\epsilon}_{\overline{\ell}ji}^{(n)} \in (0,1)$ and satisfy that $\sum_{\overline{\ell}=1}^p \alpha_{\overline{\ell}ji}^{(n)} = \sum_{\overline{\ell}=1}^p \beta_{\overline{\ell}ji}^{(n)} = \sum_{\overline{\ell}=1}^p \xi_{\overline{\ell}ji}^{(n)} = \sum_{\overline{\ell}=1}^p \zeta_{\overline{\ell}ji}^{(n)} = \sum_{\overline{\ell}=1}^p \epsilon_{\overline{\ell}ji}^{(n)} = \sum_{\overline{\ell}=1}^p \overline{\epsilon}_{\overline{\ell}ij}^{(n)} = \sum_{\overline{\ell}=1}^p \overline{\alpha}_{\overline{\ell}ji}^{(n)} = \sum_{\overline{\ell}=1}^p \overline{\beta}_{\overline{\ell}ji}^{(n)} = \sum_{\overline{\ell}=1}^p \overline{\xi}_{\overline{\ell}ji}^{(n)} = \sum_{\overline{\ell}=1}^p \overline{\zeta}_{\overline{\ell}ij}^{(n)} = 1$.

In order to establish the sufficient conditions, the following two assumptions are further introduced.

Assumption 7. The following inequalities holds:

$$\begin{aligned}&\varsigma_i^{(n)} - \nu_i^{(n)} - \max_{\tilde{r}\in\mathbb{S}}\sum_{q=1}^{\mathcal{N}}\gamma_{\tilde{r}q}^{(n)}\mu_q^{(n)} - \frac{\eta_i^{(n)}}{1-\tau_0^{(n)}} > 0 \\ &k_i^{(n)} + \varrho_i^{(n)} - \max_{\tilde{r}\in\mathbb{S}}\sum_{q=1}^{\mathcal{N}}\gamma_{\tilde{r}q}^{(n)}\mu_q^{(n)} - \frac{\eta_i^{(n)}}{1-\tau_0^{(n)}} > 0\end{aligned} \tag{17}$$

where $\varrho_i^{(n)} > 0$.

Here we define a function

$$H_i(\widehat{\varepsilon}_i) = \varsigma_i^{(n)} - \nu_i^{(n)} - \max_{\widetilde{r}\in\mathbb{S}} \sum_{q=1}^{\mathcal{N}} \gamma_{\widetilde{r}q}^{(n)} \mu_q^{(n)} - \widehat{\varepsilon}_i \max_{\widetilde{r}\in\mathbb{S}} \mu_{\widetilde{r}}^{(n)} - \frac{\eta_i^{(n)} e^{\widehat{\varepsilon}_i \tau^{(n)}}}{1-\tau_0^{(n)}} \quad (18)$$

where $\widehat{\varepsilon}_i^{(n)} \in \mathbb{R}^+$. For $H_i(\widehat{\varepsilon}_i)$ being continuous derivable, by intermediate value theorem, we could conclude that there exists $\widehat{\varepsilon}_{i0} > 0$ satisfying $H_i(\widehat{\varepsilon}_i) > 0$ with $\widehat{\varepsilon}_i \in (0, \widehat{\varepsilon}_{i0})$. Similarly, choose $\widehat{\varepsilon}_{i1} > 0$ and $\widehat{\varepsilon}_i \in (0, \widehat{\varepsilon}_{i1})$ to have the following function.

$$F_i(\widehat{\varepsilon}_i) = k_i^{(n)} + \varrho_i^{(n)} - \max_{\widetilde{r}\in\mathbb{S}} \sum_{q=1}^{\mathcal{N}} \gamma_{\widetilde{r}q}^{(n)} \mu_q^{(n)} - \widehat{\varepsilon}_i \max_{\widetilde{r}\in\mathbb{S}} \mu_{\widetilde{r}}^{(n)} - \frac{\eta_i e^{\widehat{\varepsilon}_i \tau^{(n)}}}{1-\tau_0^{(n)}} \quad (19)$$

Let $\varepsilon = \min\{\widehat{\varepsilon}_{i0}, \widehat{\varepsilon}_{i1}\}$; we have the uniform inequality as $H_i(\varepsilon) > 0$, $F_i(\varepsilon) > 0$.

Assumption 8. The following inequality holds: $\varepsilon - \varrho(T-\delta)/\overline{\mu}T > 0$, where $\varrho = \max(\max_{1\le i\le N^{(n)}} \varrho_i^{(n)}, \max_{1\le j\le N^{(m)}} \varrho_j^{(m)})$, $\overline{\mu} = \min(\min_{\widetilde{r}\in\mathbb{S}} \mu_{\widetilde{r}}^{(n)}, \min_{\widetilde{r}\in\mathbb{S}} \mu_{\widetilde{r}}^{(m)})$.

Theorem 9. *Under Assumptions 3–5 and 7-8, the closed-loop system of (2)-(4) with the periodically intermittent controller (10) is exponentially stable in pth moment.*

Proof. Choose a candidate average Lyapunov-Krasovskii function [41]

$$\begin{aligned} &\widehat{V} : C\left([0,\infty)\times\Omega_0; \mathbb{R}^{N^{(n)}+N^{(m)}}\right) \times \mathbb{S} \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+ \\ &\widehat{V}(x, r(t), t) = \int_{\Omega_0} V(x, r(t), t)\, d\lambda, \end{aligned} \quad (20)$$

with

$$V(x, r(t), t) = \sum_{n=1}^{2} \left(\mu_{r(t)}^{(n)} \sum_{i=1}^{N^{(n)}} e^{\varepsilon t} \left|x_i^{(n)}\right|^p + \frac{e^{\varepsilon\tau^{(n)}(t)}}{1-\tau_0^{(n)}} \sum_{i=1}^{N^{(n)}} \eta_i^{(n)} \int_{t-\tau^{(n)}(t)}^{t} e^{\varepsilon s} \left|x_i^{(n)}\left(s, \lambda^{(n)}\right)\right|^p ds \right) \quad (21)$$

where $\mu_{r(t)}^{(n)} > 0$.

By the generalized Itô formula [42], we have the following.

$$\mathbb{E}\widehat{V}(x, r(t), t) = \mathbb{E}\widehat{V}(\phi, \psi, r(0), 0) + \mathbb{E}\int_0^t \int_{\Omega_0} \mathbb{L}V(x, r(s), s)\, d\lambda\, ds \quad (22)$$

For short expression, denote $r(t) = \widetilde{r}$, $x_{j,\tau}^{(m)} = x_j^{(m)}(t-\tau^{(m)}(t), \lambda^{(m)})$, $x_{i,\tau}^{(n)} = x_i^{(n)}(t-\tau^{(n)}(t), \lambda^{(n)})$. By Lemma 2.7 in [10], we can get the following.

$$\begin{aligned} \mathbb{L}V(x, \widetilde{r}, t) = \sum_{(n,m)=(1,2)}^{(2,1)} \Bigg(&\varepsilon\mu_{\widetilde{r}}^{(n)} \sum_{i=1}^{N^{(n)}} e^{\varepsilon t} \left|x_i^{(n)}\right|^p \\ &+ p\mu_{\widetilde{r}}^{(n)} e^{\varepsilon t} \sum_{i=1}^{N^{(n)}} \left|x_i^{(n)}\right|^{p-1} \left(\sum_{k=1}^{l^{(n)}} \frac{\partial}{\partial\lambda_k^{(n)}} \left(D_{ik}^{(n)}(\widetilde{r}) \frac{\partial x_i^{(n)}}{\partial\lambda_k^{(n)}} \right) \right. \\ &- a_i^{(n)}(\widetilde{r})\, x_i^{(n)} + \sum_{j=1}^{N^{(m)}} b_{ji}^{(n)}(\widetilde{r})\, f_j^{(n)}\left(x_j^{(m)}\right) \\ &\left. + \sum_{\ell=1}^{N^{(n)}} K_{i\ell}^{(n)} x_\ell^{(n)} + \sum_{j=1}^{N^{(m)}} c_{ji}(\widetilde{r})\, f_j^{(n)}\left(x_{j,\tau}^{(m)}\right) \right) + \mu_{\widetilde{r}}^{(n)} e^{\varepsilon t} \\ &\cdot \frac{p(p-1)}{2} \sum_{i=1}^{N^{(n)}} \left|x_i^{(n)}\right|^{p-2} \\ &\cdot \sum_{j=1}^{N^{(m)}} \left(h_{ji}^{(n)}\left(x_i^{(n)}, x_j^{(m)}, x_{j,\tau}^{(m)}\right) \right)^2 + \sum_{q=1}^{\mathcal{N}} \gamma_{\widetilde{r}q}^{(n)} \mu_q^{(n)} e^{\varepsilon t} \\ &\cdot \sum_{i=1}^{n} \left|x_i^{(n)}\right|^p + \frac{e^{\varepsilon\tau^{(n)}}}{1-\tau_0^{(n)}} \sum_{i=1}^{N^{(n)}} \eta_i^{(n)} e^{\varepsilon t} \left|x_i^{(n)}\right|^p - \frac{e^{\varepsilon\tau^{(n)}}}{1-\tau_0^{(n)}} \\ &\cdot \sum_{i=1}^{N^{(n)}} \eta_i e^{\varepsilon(t-\tau^{(n)}(t))} \left|x_{i,\tau}^{(n)}\right|^p \left(1-\dot{\tau}^{(n)}(t)\right) \\ &+ \sum_{q=1}^{\mathcal{N}} \gamma_{\widetilde{r}q}^{(n)} \frac{e^{\varepsilon\tau^{(n)}}}{1-\tau_0^{(n)}} \sum_{i=1}^{N^{(n)}} \eta_i^{(n)} \\ &\cdot \int_{t-\tau^{(n)}(t)}^{t} e^{\varepsilon s} \left|x_i^{(n)}\left(s, \lambda^{(n)}\right)\right|^p ds \Bigg) \end{aligned} \quad (23)$$

By employing the absolute value inequality and noticing that $\sum_{q=1}^{\mathcal{N}} \gamma_{\widetilde{r}q}^{(n)} = 0$, we can obtain the following.

$$\begin{aligned} \mathbb{L}V(x, \widetilde{r}, t) \le \sum_{(n,m)=(1,2)}^{(2,1)} \Bigg(\sum_{i=1}^{N^{(n)}} e^{\varepsilon t} \Bigg(&\varepsilon\mu_{\widetilde{r}}^{(n)} \left|x_i^{(n)}\right|^p \\ &+ p\mu_{\widetilde{r}}^{(n)} \left|x_i^{(n)}\right|^{p-1} \sum_{k=1}^{l^{(n)}} \frac{\partial}{\partial\lambda_k^{(n)}} \left(D_{ik}^{(n)}(\widetilde{r}) \frac{\partial x_i^{(n)}}{\partial\lambda_k^{(n)}} \right) \end{aligned}$$

$$+\left(K_{ii}-a_i^{(n)}(\widetilde{r})\right)x_i^{(n)}+p\mu_{\widetilde{r}}^{(n)}\left|x_i^{(n)}\right|^{p-1}\cdot\sum_{j=1}^{N^{(m)}}\left(\left|b_{ji}^{(n)}(\widetilde{r})\right|\left|f_j^{(n)}\left(x_j^{(m)}\right)\right|\right)+p\mu_{\widetilde{r}}^{(n)}\left|x_i^{(n)}\right|^{p-1}\cdot\sum_{j=1}^{N^{(m)}}\left(\left|c_{ji}^{(n)}(\widetilde{r})\right| f_j^{(n)}\left(x_{j,\tau}^{(m)}\right)\right|\right)+\mu_{\widetilde{r}}^{(n)}\cdot\frac{p(p-1)}{2}\left|x_i^{(n)}\right|^{p-2}\cdot\sum_{j=1}^{N^{(m)}}\sigma_{ji}^{(n)}\left(\left(x_i^{(n)}\right)^2+\left(x_i^{(m)}\right)^2+\left(x_{j,\tau}^{(m)}\right)^2\right)+\sum_{\substack{\ell=1\\ \ell\neq i}}^{N^{(n)}}p\mu_{\widetilde{r}}^{(n)}\left|x_i^{(n)}\right|^{p-1}\left|K_{i\ell}\right|\left|x_\ell^{(n)}\right|+\sum_{q=1}^{\mathcal{N}}\gamma_{\widetilde{r}q}^{(n)}\mu_q^{(n)}\left|x_i^{(n)}\right|^p+\frac{e^{\varepsilon\tau^{(n)}}}{1-\tau_0^{(n)}}\eta_i^{(n)}\left|x_i^{(n)}\right|^p-\eta_i^{(n)}\left|x_{j,\tau}^{(m)}\right|^p\Bigg)\Bigg) \tag{24}$$

Applying the fundamental inequality, i.e., $a_1^p+a_2^p+\cdots+a_p^p \geq pa_1a_2\cdots a_p$, $(a_h \geq 0, h=1,2,\ldots,p)$, yields the following.

$$p\mu_{\widetilde{r}}^{(n)}\left|x_i^{(n)}\right|^{p-1}\sum_{j=1}^{N^{(m)}}\left(\left|b_{ji}^{(n)}(\widetilde{r})\right|\left|f_j^{(n)}\left(x_j^{(m)}\right)\right|\right)\leq\mu_{\widetilde{r}}^{(n)}\sum_{j=1}^{N^{(m)}}p\left|x_i^{(n)}\right|^{p-1}\widetilde{b}_{ji}^{(n)}\widetilde{L}_j\left|x_j^{(m)}\right|=\mu_{\widetilde{r}}^{(n)}\sum_{j=1}^{N^{(m)}}p\cdot\prod_{\overline{\ell}=1}^{p-1}\left(\left(\widetilde{b}_{ji}^{(n)}\right)^{\alpha_{\overline{\ell}ji}^{(n)}}\widetilde{L}_j^{\beta_{\overline{\ell}ji}^{(n)}}\left|x_i^{(n)}\right|\right)\left(\left(\widetilde{b}_{ji}^{(n)}\right)^{\alpha_{pji}^{(n)}}\widetilde{L}_j^{\beta_{pji}^{(n)}}\left|x_j^{(m)}\right|\right)\leq\mu_{\widetilde{r}}^{(n)}\sum_{j=1}^{N^{(m)}}\sum_{\overline{\ell}=1}^{p-1}\left(\widetilde{b}_{ji}^{(n)}\right)^{p\alpha_{\overline{\ell}ji}^{(n)}}\widetilde{L}_j^{p\beta_{\overline{\ell}ji}^{(n)}}\left|x_i^{(n)}\right|^p+\mu_{\widetilde{r}}^{(n)}\sum_{j=1}^{N^{(m)}}\left(\widetilde{b}_{ji}^{(n)}\right)^{p\alpha_{pji}^{(n)}}\widetilde{L}_j^{p\beta_{pji}^{(n)}}\left|x_j^{(m)}\right|^p \tag{25}$$

$$\mu_{\widetilde{r}}^{(n)}\sum_{\substack{\ell=1\\ \ell\neq i}}^{N^{(n)}}p\left|x_i^{(n)}\right|^{p-1}\left|K_{i\ell}\right|\left|x_\ell^{(n)}\right|=\mu_{\widetilde{r}}^{(n)}\sum_{\substack{\ell=1\\ \ell\neq i}}^{N^{(n)}}p\cdot\prod_{\overline{\ell}=1}^{p-1}\left(\left|K_{il}\right|^{\overline{\eta}_{\overline{\ell}il}}\left|x_i^{(n)}\right|\right)\left(\left|K_{i\ell}\right|^{\overline{\eta}_{pil}}\left|x_\ell^{(n)}\right|\right)=\mu_{\widetilde{r}}^{(n)}\sum_{\substack{\ell=1\\ \ell\neq i}}^{N^{(n)}}\left(\sum_{\overline{\ell}=1}^{p-1}\left|K_{il}\right|^{p\overline{\eta}_{\overline{\ell}il}}\left|x_i^{(n)}\right|^p+\sum_{\substack{\ell=1\\ \ell\neq i}}^{N^{(n)}}\left(\left|K_{il}\right|^{p\overline{\eta}_{\overline{\ell}il}}\left|x_\ell^{(n)}\right|^p\right)\right) \tag{26}$$

$$p\mu_{\widetilde{r}}^{(n)}\left|x_i^{(n)}\right|^{p-1}\sum_{j=1}^{N^{(m)}}\left(\left|c_{ji}^{(n)}(\widetilde{r})\right|\left|f_j^{(n)}\left(x_{j,\tau}^{(m)}\right)\right|\right)\leq\mu_{\widetilde{r}}^{(n)}\sum_{j=1}^{N^{(m)}}p\left(\left|x_i^{(n)}\right|^{p-1}\widetilde{c}_{ji}^{(n)}\widetilde{L}_j\left|x_{j,\tau}^{(m)}\right|\right)=\mu_{\widetilde{r}}^{(n)}\sum_{j=1}^{N^{(m)}}p\cdot\prod_{\overline{\ell}=1}^{p-1}\left(\left(\widetilde{c}_{ji}^{(n)}\right)^{\xi_{\overline{\ell}ji}^{(n)}}\widetilde{L}_j^{\zeta_{\overline{\ell}ji}^{(n)}}\left|x_i^{(n)}\right|\right)\left(\left(\widetilde{c}_{ji}^{(n)}\right)^{\xi_{pji}^{(n)}}\widetilde{L}_j^{\zeta_{pji}^{(n)}}\left|x_{j,\tau}^{(m)}\right|\right)\leq\mu_{\widetilde{r}}^{(n)}\sum_{j=1}^{N^{(m)}}\sum_{\overline{\ell}=1}^{p-1}\left(\widetilde{c}_{ji}^{(n)}\right)^{p\xi_{\overline{\ell}ji}^{(n)}}\widetilde{L}_j^{p\zeta_{\overline{\ell}ji}^{(n)}}\left|x_i^{(n)}\right|^p+\mu_{\widetilde{r}}^{(n)}\sum_{j=1}^{N^{(m)}}\left(\left(\widetilde{c}_{ji}^{(n)}\right)^{p\xi_{pji}^{(n)}}\widetilde{L}_j^{p\zeta_{pji}^{(n)}}\left|x_{j,\tau}^{(m)}\right|^p\right) \tag{27}$$

$$\mu_{\widetilde{r}}^{(n)}\frac{p(p-1)}{2}\left|x_i^{(n)}\right|^{p-2}\sum_{j=1}^{N^{(m)}}\sigma_{1ij}\left(\left(x_i^{(n)}\right)^2+\left(x_i^{(m)}\right)^2+\left(x_{j,\tau}^{(m)}\right)^2\right)=\mu_{\widetilde{r}}^{(n)}\frac{p(p-1)}{2}\sum_{j=1}^{N^{(m)}}p\left(\prod_{\overline{\ell}=1}^{p-2}\left|\sigma_{ji}^{(n)}\right|^{\epsilon_{\overline{\ell}ji}^{(n)}}\cdot\left|x_i^{(n)}\right|\right)\left(\left(\left|\sigma_{ji}^{(n)}\right|^{\epsilon_{(p-1)ji}^{(n)}}\left|x_i^{(n)}\right|\right)\left(\left|\sigma_{ji}^{(n)}\right|^{\epsilon_{pji}^{(n)}}\left|x_i^{(n)}\right|\right)+\left(\left|\sigma_{ji}^{(n)}\right|^{\epsilon_{(p-1)ji}^{(n)}}\left|x_j^{(m)}\right|\right)\left(\left|\sigma_{ji}^{(n)}\right|^{\epsilon_{pji}^{(n)}}\left|x_j^{(m)}\right|\right)+\left(\left|\sigma_{ji}^{(n)}\right|^{\epsilon_{(p-1)ji}^{(n)}}\left|x_{j,\tau}^{(m)}\right|\right)\left(\left|\sigma_{ji}^{(n)}\right|^{\epsilon_{pji}^{(n)}}\left|x_{j,\tau}^{(m)}\right|\right)\right)=\mu_{\widetilde{r}}^{(n)}\cdot\frac{p-1}{2}\sum_{j=1}^{N^{(m)}}\sum_{\overline{\ell}=1}^{p-2}\left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{\overline{\ell}ji}^{(n)}}\left|x_i^{(n)}\right|^p+\mu_{\widetilde{r}}^{(n)}\frac{p-1}{2}\cdot\sum_{j=1}^{N^{(m)}}\left(\left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{(p-1)ji}^{(n)}}+\left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{pji}^{(n)}}\right)\left(\left|x_i^{(n)}\right|^p+\left|x_j^{(m)}\right|^p+\left|x_{j,\tau}^{(m)}\right|^p\right) \tag{28}$$

Substituting (25)-(28) into (24) leads to the following.

$$\begin{aligned}\mathbb{L}V(x,\tilde{r},t) \le \sum_{(n,m)=(1,2)}^{(2,1)} \Bigg(\sum_{i=1}^{N^{(n)}} e^{\varepsilon t} \Bigg(\Bigg(\Bigg(\varepsilon\mu_{\tilde{r}}^{(n)} + p\mu_{\tilde{r}}^{(n)} k_{ii} - p\mu_{\tilde{r}}^{(n)} \widetilde{a_i^{(n)}} + \sum_{q=1}^{\mathcal{N}} \gamma_{\tilde{r}q}^{(n)} \mu_q^{(n)} + \mu_{\tilde{r}}^{(n)} \frac{p-1}{2} \sum_{j=1}^{N^{(m)}} \sum_{\tilde{\ell}=1}^{p-2} \left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{\tilde{\ell}ji}^{(n)}} \\
+ \mu_{\tilde{r}}^{(n)} \sum_{j=1}^{N^{(m)}} \sum_{\tilde{\ell}=1}^{p-1} \left(\left(\tilde{b}_{ji}^{(n)}\right)^{p\alpha_{\tilde{\ell}ji}^{(n)}} \tilde{L}_j^{p\beta_{\tilde{\ell}ji}^{(n)}} + \left(\tilde{c}_{ji}^{(n)}\right)^{p\xi_{\tilde{\ell}ji}^{(n)}} \tilde{L}_j^{p\zeta_{\tilde{\ell}ji}^{(n)}} \right) + \mu_{\tilde{r}}^{(n)} \sum_{\substack{\ell=1\\ \ell\neq i}}^{N^{(n)}} \sum_{\tilde{\ell}=1}^{p-1} \left|K_{i\ell}\right|^{p\overline{\eta}_{\tilde{\ell}il}} \\
+ \sum_{j=1}^{N^{(m)}} \Bigg(\mu_{\tilde{r}}^{(m)} \left(\tilde{b}_{ij}^{(m)}\right)^{p\overline{\alpha}_{pij}^{(n)}} \left(\tilde{N}_i\right)^{p\overline{\beta}_{pij}^{(n)}} + \Bigg(\mu_{\tilde{r}}^{(n)} \left(\left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{(p-1)ji}^{(n)}} + \left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{pji}^{(n)}} \right) + \mu_{\tilde{r}}^{(m)} \left(\left|\sigma_{ij}^{(n)}\right|^{p\overline{\epsilon}_{(p-1)ij}^{(n)}} + \left|\sigma_{ij}^{(n)}\right|^{p\overline{\epsilon}_{pij}^{(n)}} \right) \Bigg) \frac{p-1}{2} \Bigg) \Bigg) \\
\cdot \left|x_i^{(n)}\right|^p + \mu_{\tilde{r}}^{(n)} \sum_{\substack{\ell=1\\ \ell\neq i}}^{N^{(n)}} \left|K_{i\ell}\right|^{p\overline{\eta}_{pil}} \left|x_\ell^{(n)}\right|^p + \frac{e^{\varepsilon t}\eta_i^{(n)}}{1-\tau_0^{(n)}} + \mu_{\tilde{r}}^{(m)} \sum_{j=1}^{N^{(m)}} \left(\tilde{c}_{ij}^{(m)}\right)^{p\overline{\xi}_{pij}^{(n)}} \tilde{N}_i^{p\overline{\zeta}_{pij}^{(n)}} \left|x_{j,\tau}^{(m)}\right|^p + \mu_{\tilde{r}}^{(m)} \frac{p-1}{2} \\
\cdot \sum_{j=1}^{N^{(m)}} \left(\left(\left|\sigma_{ij}^{(n)}\right|^{p\overline{\epsilon}_{(p-1)ij}^{(n)}} + \left|\sigma_{ij}^{(n)}\right|^{p\overline{\epsilon}_{pij}^{(n)}} \right) \left|x_{j,\tau}^{(m)}\right|^p - \eta_i^{(n)} \left|x_{j,\tau}^{(m)}\right|^p \right) \Bigg) + \sum_{i=1}^{N^{(n)}} e^{\varepsilon t} p\mu_{\tilde{r}}^{(n)} \left|x_i^{(n)}\right|^{p-1} \left(\sum_{k=1}^{l^{(n)}} \frac{\partial}{\partial\lambda_k^{(n)}} \left(D_{ik}^{(n)}(\tilde{r}) \frac{\partial x_i^{(n)}}{\partial\lambda_k^{(n)}} \right) \right) \Bigg)\end{aligned} \tag{29}$$

Let

$$\begin{aligned}\overline{\varsigma}_i^{(n)} = \min_{\tilde{r}\in\mathbb{S}} \mu_{\tilde{r}}^{(n)} \Bigg(p\tilde{a}_i^{(n)} \\
- \sum_{j=1}^{N^{(m)}} \sum_{\tilde{\ell}=1}^{p-1} \left(\left(\tilde{b}_{ji}^{(n)}\right)^{p\alpha_{\tilde{\ell}ji}^{(n)}} \tilde{L}_j^{p\beta_{\tilde{\ell}ji}^{(n)}} + \left(\tilde{c}_{ji}^{(n)}\right)^{p\xi_{\tilde{\ell}ji}^{(n)}} \tilde{L}_j^{p\xi_{\tilde{\ell}ji}^{(n)}} \right) \\
- \frac{p-1}{2} \sum_{j=1}^{N^{(m)}} \sum_{\tilde{\ell}=1}^{p-2} \left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{\tilde{\ell}ji}^{(n)}} - \frac{p-1}{2} \\
\cdot \sum_{j=1}^{N^{(m)}} \left(\left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{(p-1)ji}^{(n)}} + \left|\sigma_{ji}^{(n)}\right|^{p\epsilon_{pji}^{(n)}} \right) \Bigg) \\
- \min_{\tilde{r}\in\mathbb{S}} \mu_{\tilde{r}}^{(m)} \sum_{j=1}^{N^{(m)}} \Bigg(\left(\tilde{b}_{ij}^{(m)}\right)^{p\overline{\alpha}_{pij}^{(n)}} \tilde{N}_i^{p\overline{\beta}_{pij}^{(n)}} \\
+ \frac{p-1}{2} \left(\left|\sigma_{ij}^{(n)}\right|^{p\overline{\epsilon}_{(p-1)ij}^{(n)}} + \left|\sigma_{ij}^{(n)}\right|^{p\overline{\epsilon}_{pij}^{(n)}} \right) \Bigg)\end{aligned} \tag{30}$$

and by (22), we get the following inequality.

$$\begin{aligned}\mathbb{E}\widehat{V}(x,r(t),t) \le \mathbb{E}\widehat{V}(\phi,\psi,r(0),0) \\
- \sum_{(n,m)=(1,2)}^{(2,1)} \Bigg(\mathbb{E}\int_0^t \int_{\Omega_0} \sum_{i=1}^{N^{(n)}} e^{\varepsilon s} \left(\overline{\varsigma}_i^{(n)} - \varepsilon \max_{\tilde{r}\in\mathbb{S}} \mu_{\tilde{r}}^{(n)} - \nu_i^{(n)} - \max_{\tilde{r}\in\mathbb{S}} \sum_{q=1}^{\mathcal{N}} \gamma_{\tilde{r}q}^{(n)} \mu_q^{(n)} - \frac{e^{\varepsilon\tau^{(n)}}\eta_i^{(n)}}{1-\tau_0^{(n)}} \right) \left|x_i^{(n)}\left(s,\lambda^{(n)}\right)\right|^p d\lambda^{(n)} ds \\
- \mathbb{E}\int_0^t \sum_{i=1}^{n} e^{\varepsilon s} \int_{\Omega_0} p\mu_{r(s)} \left|x_i^{(n)}\left(s,\lambda^{(n)}\right)\right|^{p-1} \left(\sum_{k=1}^{l^{(n)}} \frac{\partial}{\partial\lambda_k^{(n)}} \left(D_{ik}^{(n)}(\tilde{r}) \frac{\partial x_i^{(n)}}{\partial\lambda_k^{(n)}} \right) \right) d\lambda^{(n)} ds \Bigg)\end{aligned} \tag{31}$$

According to Dirichlet boundary conditions, by Lemma 6, we can derive the integral part of evolution dynamics as follows.

$$\begin{aligned}\int_{\Omega_0} p\left|x_i^{(n)}\left(s,\lambda^{(n)}\right)\right|^{p-1} \left(\sum_{k=1}^{l^{(n)}} \frac{\partial}{\partial\lambda_k^{(n)}} \\
\cdot \left(D_{ik}^{(n)}(\tilde{r}) \frac{\partial x_i^{(n)}\left(s,\lambda^{(n)}\right)}{\partial\lambda_k^{(n)}} \right) \right) d\lambda^{(n)} \\
\le -\sum_{k=1}^{l^{(n)}} \frac{4(p-1) D_{ik}^{(n)}(\tilde{r})}{p\theta_k^2} \int_{\Omega_0} \left|x_i^{(n)}\left(s,\lambda^{(n)}\right)\right|^p ds \\
\le -\sum_{k=1}^{l^{(n)}} \frac{4(p-1) \underline{D}_{ik}^{(n)}}{p\theta_k^2} \int_{\Omega_0} \left|x_i^{(n)}\left(s,\lambda^{(n)}\right)\right|^p ds\end{aligned} \tag{32}$$

Using Itô's formula $(\mathbb{E}V(t))' = \mathbb{E}(\mathscr{L}V(t))$, we obtain

$$\begin{aligned}
&\mathbb{E}\widehat{V}(x,r(t),t)\le\mathbb{E}\widehat{V}(\phi,\psi,r(0),0)\\
&\quad-\sum_{(n,m)=(1,2)}^{(2,1)}\left(\mathbb{E}\int_0^t\int_{\Omega_0}\sum_{i=1}^{N^{(n)}}e^{\varepsilon s}\left(\overline{\varsigma}_i^{(n)}+\min_{\widetilde{r}\in\mathbb{S}}\mu_{\widetilde{r}}^{(n)}\sum_{k=1}^{l^{(n)}}\frac{4(p-1)\underline{D}_{ik}^{(n)}}{p\theta_k^2}-\varepsilon\max_{\widetilde{r}\in\mathbb{S}}\mu_{\widetilde{r}}^{(n)}-\nu_i^{(n)}-\max_{\widetilde{r}\in\mathbb{S}}\sum_{q=1}^{\mathcal{N}}\gamma_{\widetilde{r}q}^{(n)}\mu_q^{(n)}-\frac{e^{\varepsilon\tau^{(n)}}\eta_i^{(n)}}{1-\tau_0^{(n)}}\right)\left|x_i^{(n)}\left(s,\lambda^{(n)}\right)\right|^p d\lambda^{(n)}ds\right)\\
&\le\mathbb{E}\widehat{V}(\phi,\psi,r(0),0)-\sum_{(n,m)=(1,2)}^{(2,1)}\left(\mathbb{E}\int_0^t\int_{\Omega_0}\sum_{i=1}^{N^{(n)}}e^{\varepsilon s}\left(\overline{\varsigma}_i^{(n)}-\varepsilon\max_{\widetilde{r}\in\mathbb{S}}\mu_{\widetilde{r}}^{(n)}-\nu_i^{(n)}-\max_{\widetilde{r}\in\mathbb{S}}\sum_{q=1}^{\mathcal{N}}\gamma_{\widetilde{r}q}^{(n)}\mu_q^{(n)}-\frac{e^{\varepsilon\tau^{(n)}}\eta_i^{(n)}}{1-\tau_0^{(n)}}\right)\left|x_i^{(n)}\left(s,\lambda^{(n)}\right)\right|^p d\lambda^{(n)}ds\right)\\
&\le\mathbb{E}\widehat{V}(\phi,\psi,r(0),0)
\end{aligned}\tag{33}$$

where $(t,\lambda^{(n)})\in[\mu T,\mu T+\delta]\times\Omega_0$.

Similarly, when $(t,\lambda^{(n)})\in[\mu T+\delta,(\mu+1)T]\times\Omega_0$, we have

$$\begin{aligned}
&\mathbb{E}\widehat{V}(x,r(t),t)\le\mathbb{E}\widehat{V}(x(\mu T+\delta,\lambda),r(\mu T+\delta),\mu T\\
&\quad+\delta)-\mathbb{E}\int_0^t\int_{\Omega_0}\sum_{i=1}^{N^{(n)}}e^{\varepsilon s}\left[k_i^{(n)}+\varrho_i^{(n)}-\varepsilon\max_{\widetilde{r}\in\mathbb{S}}\mu_{\widetilde{r}}^{(n)}\right.\\
&\quad\left.-\nu_i^{(n)}-\max_{\widetilde{r}\in\mathbb{S}}\sum_{q=1}^{\mathcal{N}}\gamma_{\widetilde{r}q}^{(n)}\mu_q^{(n)}-\frac{e^{\varepsilon\tau^{(n)}}\eta_i^{(n)}}{1-\tau_0^{(n)}}\right]\\
&\quad\cdot\left|x_i^{(n)}\left(s,\lambda^{(n)}\right)\right|^p d\lambda\,ds\le\mathbb{E}\widehat{V}(x(\mu T+\delta,\\
&\quad\lambda),r(\mu T+\delta),\mu T+\delta)+\frac{\varrho}{\overline{\mu}}\mathbb{E}\int_0^t\widehat{V}(x,r(s),s)\,ds
\end{aligned}\tag{34}$$

where $\varrho=\max\{\max_{1\le i\le N^{(n)}}\varrho_i,\max_{1\le j\le N^{(m)}}\varrho_j^{(m)}\}$, $\overline{\mu}=\min\{\min_{l\in\mathbb{S}}\mu_{\widetilde{r}}^{(n)},\min_{l\in\mathbb{S}}\mu_{\widetilde{r}}^{(m)}\}$.

By using the Gronwall inequality to (34) it yields the following.

$$\begin{aligned}
&\mathbb{E}\widehat{V}(x,r(t),t)\\
&\quad\le\mathbb{E}\widehat{V}(x(\mu T+\delta,\lambda),r(\mu T+\delta),\mu T+\delta)\\
&\quad\cdot e^{(\varrho/\overline{\mu})(t-\mu T-\delta)}
\end{aligned}\tag{35}$$

From (33)-(35), we can conclude that

(I) for $(t,\lambda)\in([0,\delta),\Omega_0)$ and $(t,\lambda)\in([T,T+\delta),\Omega_0)$, by (33), we can, respectively, have

$$\mathbb{E}\widehat{V}(x,r(t),t)\le\mathbb{E}\widehat{V}(x(0,\lambda),r(0),0),\tag{36}$$

and

$$\begin{aligned}
\mathbb{E}\widehat{V}(x,r(t),t)&\le\mathbb{E}\widehat{V}(x(T,\lambda),r(T),T)\\
&\le\mathbb{E}\widehat{V}(x(0,\lambda),r(0),0)\,e^{(\varrho/\overline{\mu})(T-\delta)}
\end{aligned}\tag{37}$$

(II) for $(t,\lambda)\in([\delta,T),\Omega_0)$ and $(t,\lambda)\in([T+\delta,2T),\Omega_0)$, by (35), we can, respectively, have

$$\mathbb{E}\widehat{V}(x,r(t),t)\le\mathbb{E}\widehat{V}(x(0,\lambda),r(0),0)\,e^{(\varrho/\overline{\mu})(t-\delta)},\tag{38}$$

and

$$\begin{aligned}
&\mathbb{E}\widehat{V}(x,r(t),t)\\
&\quad\le\mathbb{E}\widehat{V}(x(T+\delta,\lambda),r(T+\delta),T+\delta)\,e^{(\varrho/\overline{\mu})(t-T-\delta)}\\
&\quad\le\mathbb{E}\widehat{V}(x(0,\lambda),r(0),0)\,e^{(\rho/\overline{\mu})(t-2\delta)}.
\end{aligned}\tag{39}$$

Repeating above procedure (I)-(II), for $(t,\lambda)\in[\mu T,\mu T+\delta)\times\Omega_0$, then $\mu\le t/T$ and

$$\begin{aligned}
\mathbb{E}\widehat{V}(x,r(t),t)&\le\mathbb{E}\widehat{V}(x(\mu T,\lambda),r(\mu T),\mu T)\\
&\le\mathbb{E}\widehat{V}(x(0,\lambda),r(0),0)\,e^{(\varrho/\overline{\mu}T)(T-\delta)t}
\end{aligned}\tag{40}$$

and for $(t,\lambda)\in[\mu T+\delta,(\mu+1)T)\times\Omega_0$, then $t/T<\mu+1$ and

$$\begin{aligned}
&\mathbb{E}\widehat{V}(x,r(t),t)\\
&\quad\le\mathbb{E}\widehat{V}(x(\mu T+\delta,\lambda),r(\mu T+\delta),\mu T+\delta)\\
&\quad\cdot e^{(\varrho/\overline{\mu})(t-\mu T-\delta)}\le\mathbb{E}\widehat{V}(x(0,\lambda),r(0),0)\,e^{(\varrho/\overline{\mu}T)(T-\delta)t}.
\end{aligned}\tag{41}$$

Hence, for arbitrary $(t,\lambda)\in[0,\infty)\times\Omega_0$,

$$\mathbb{E}\widehat{V}(x,r(t),t)\le\mathbb{E}\widehat{V}(x(0,\lambda),r(0),0)\,e^{(\varrho/\overline{\mu}T)(T-\delta)t}.\tag{42}$$

By calculation, we obtain

$$\begin{aligned}
&\mathbb{E}\widehat{V}(x,r(t),t)\ge e^{\varepsilon t}\overline{\mu}\mathbb{E}\left(\int_{\Omega_0}\sum_{i=1}^{N^{(n)}}\left|x_i^{(n)}\right|^p d\lambda^{(n)}\right.\\
&\quad\left.+\int_{\Omega_0}\sum_{j=1}^{N^{(m)}}\left|x_j^{(m)}\right|^p d\lambda^{(n)}\right)
\end{aligned}\tag{43}$$

$$\begin{aligned}
&\mathbb{E}\widehat{V}(x(0,\lambda),r(0),0)\\
&\quad\le\max_{\widetilde{r}\in\mathbb{S}}\mu_{\widetilde{r}}^{(n)}\mathbb{E}\int_{\Omega_0}\sum_{i=1}^{N^{(n)}}\left|\phi_i\left(0,\lambda^{(n)}\right)\right|^p d\lambda^{(n)}\\
&\quad+\sup_{-\tau^{(n)}\le s\le 0}\left(\left(\max_{1\le i\le N^{(n)}}\eta_i^{(n)}\right)\frac{\tau^{(n)}e^{\varepsilon\tau^{(n)}}}{1-\tau_0^{(n)}}\right.\\
&\quad\left.\times\mathbb{E}\int_{\Omega_0}\sum_{i=1}^{N^{(n)}}\left|\phi_i\left(s,\lambda^{(n)}\right)\right|^p d\lambda^{(n)}\right)
\end{aligned}$$

$$+\max_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(m)}\mathbb{E}\int_{\Omega_0}\sum_{j=1}^{N^{(m)}}\left|\psi_j\left(0,\lambda^{(n)}\right)\right|^p \mathrm{d}\lambda^{(n)}$$
$$+\sup_{-\tau^{(n)}\le s\le 0}\left[\left(\max_{1\le j\le N^{(m)}}\eta(m)_j\right)\frac{\tau^{(n)}e^{\varepsilon\tau^{(n)}}}{1-\tau_0^{(n)}}\right.$$
$$\left.\cdot\mathbb{E}\int_{\Omega_0}\sum_{j=1}^{N^{(m)}}\left|\psi_j\left(s,\lambda^{(n)}\right)\right|^p \mathrm{d}\lambda^{(n)}\right]=\mu_0 \quad (44)$$

so

$$\mathbb{E}\left(\int_{\Omega_0}\sum_{i=1}^{N^{(n)}}\left|x_i^{(n)}\right|^p \mathrm{d}\lambda^{(n)}+\int_{\Omega_0}\sum_{j=1}^{N^{(m)}}\left|x_j^{(m)}\right|^p \mathrm{d}\lambda^{(n)}\right)$$
$$\le\frac{\mu_0}{\overline{\mu}}e^{-(\varepsilon-(\varrho/\overline{\mu}T)(T-\delta))t}. \quad (45)$$

Under Assumption 8, Theorem 9 holds.

In Theorem 9, if we let $\varsigma_i^{(n)}=k_i^{(n)}$, then $\alpha_{\overline{\ell}ji}^{(n)}=\alpha_{\ell ji}^{(n)}$, $\beta_{\overline{\ell}ji}^{(n)}=\beta_{\ell ji}^{(n)}$, $\xi_{\overline{\ell}ji}^{(n)}=\xi_{\ell ji}^{(n)}$, $\zeta_{\overline{\ell}ji}^{(n)}=\zeta_{\ell ji}^{(n)}$, $\epsilon_{\overline{\ell}ji}^{(n)}=\epsilon_{\ell ji}^{(n)}$, $\overline{\epsilon}_{\overline{\ell}ji}^{(n)}=\overline{\epsilon}_{\ell ji}^{(n)}$, $\overline{\alpha}_{\overline{\ell}ji}^{(n)}=\overline{\alpha}_{\ell ji}^{(n)}$, $\overline{\beta}_{\overline{\ell}ji}^{(n)}=\overline{\beta}_{\ell ji}^{(n)}$, $\overline{\xi}_{\overline{\ell}ji}^{(n)}=\overline{\xi}_{\ell ji}^{(n)}$, $\overline{\zeta}_{\overline{\ell}ji}^{(n)}=\overline{\zeta}_{\ell ji}^{(n)}$, $\varrho_i=-\nu_i^{(n)}$. Thus, we can obtain the following corollary. □

Corollary 10. *Under Assumptions 3–5, system (2)-(4) with periodically intermittent controllers (10) is exponentially stable in pth moment if the following conditions hold:*

(I) $\nu_i^{(n)}<0$, $\varsigma_i^{(n)}-\nu_i^{(n)}-\max_{\tilde{r}\in\mathbb{S}}\sum_{q=1}^{\mathcal{N}}\gamma_{\tilde{r}q}^{(n)}\mu_q^{(n)}-\eta_i^{(n)}/(1-\tau_0^{(n)})>0$.

(II) $\varepsilon-\overline{\nu}(T-\delta)/\overline{\mu}T>0$, *where* $\overline{\nu}=\max\{\max_{1\le i\le N^{(n)}}|\nu_i^{(n)}|, \max_{1\le j\le N^{(m)}}|\nu_j^{(m)}|\}$, $\overline{\mu}=\min\{\min_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(n)},\min_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(m)}\}$.

In Theorem 9, letting $\varsigma_i^{(n)}=1/p$, $\varsigma_j^{(m)}=1/p$ *which means that* $\alpha_{\overline{\ell}ji}^{(n)}=1/p$, $\beta_{\overline{\ell}ji}^{(n)}=1/p$, $\xi_{\overline{l}ji}^{(n)}=1/p$, $\zeta_{\overline{l}ji}^{(n)}=1/p$ *and* $\epsilon_{\overline{\ell}ji}^{(n)}=1/p$, *we can get the following.*

$$\tilde{\varsigma}_i^{(n)}=\tilde{k}_i^{(n)}=\min_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(n)}\left(\sum_{k=1}^{l^{(n)}}\frac{4(p-1)\underline{D}_{ik}^{(n)}}{p\theta_k^2}+p\tilde{a}_i^{(n)}\right.$$
$$-(p-1)\sum_{j=1}^{N^{(m)}}\left(\tilde{b}_{ji}^{(n)}\tilde{L}_j+\tilde{c}_{ji}^{(n)}\tilde{L}_j\right)$$
$$\left.-\frac{p(p-1)}{2}\sum_{j=1}^{N^{(m)}}\left|\sigma_{ji}^{(n)}\right|\right)-\min_{\tilde{r}\in\mathbb{S}}\left(\mu_{\tilde{r}}^{(m)}\right)_l \quad (46)$$
$$\cdot\sum_{j=1}^{N^{(m)}}\left(\tilde{b}_{ij}^{(m)}\tilde{N}_i+(p-1)\left|\sigma_{ij}^{(n)}\right|\right)$$

We can also have a further extended corollary.

Corollary 11. *Under Assumptions 3 to 5, the BAM-NN system (2)-(4) with periodically intermittent controllers (10) is exponentially stable in pth moment if the following conditions hold:*

(I) $\tilde{\varsigma}_i^{(n)}-\tilde{\nu}_i^{(n)}-\max_{\tilde{r}\in\mathbb{S}}\sum_{q=1}^{\mathcal{N}}\gamma_{\tilde{r}q}^{(n)}\mu_q^{(n)}-\tilde{\eta}_i^{(n)}/(1-\tau_0^{(n)})>0$.

(II) $\tilde{\varsigma}_i^{(n)}+\tilde{\varrho}_i^{(n)}-\max_{\tilde{r}\in\mathbb{S}}\sum_{q=1}^{\mathcal{N}}\gamma_{\tilde{r}q}^{(n)}\mu_q^{(n)}-\tilde{\eta}_i^{(n)}/(1-\tau_0^{(n)})>0$, $\tilde{\varrho}_i^{(n)}>0$.

(III) $\tilde{\varepsilon}-\tilde{\varrho}(T-\delta)/\overline{\mu}T>0$, *where* $\tilde{\varrho}=\max\{\max_{1\le i\le N^{(n)}}|\tilde{\varrho}_i^{(n)}|, \max_{1\le j\le N^{(m)}}|\tilde{\varrho}_j^{(m)}|\}$, $\overline{\mu}=\min\{\min_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(n)},\min_{\tilde{r}\in\mathbb{S}}\mu_{\tilde{r}}^{(m)}\}$.

Remark 12. Unlike the result in [22], which only considered the mean square exponential stability of stochastic BAM-NN with time-varying delays and reaction-diffusion terms, we design a periodically intermittent controller to exponentially stabilize the unstable neural network in pth moment. Moreover, the controller (10) is linear which can be easily implemented in practice.

Algorithm 13. For the periodically intermittent controller (10) of the BAM-NN system (2)-(4), we summarize the following algorithm to implement the controller.

(1) By instigating a practical system's structure and dynamics' characterization with its disturbance, we can develop a model of the BAM-NN system, i.e., $\mathbb{S}$, γ_{ij}. *Supposing and examining the transition probability* $\mathbb{P}(\cdot)$ *as well as the main parameters of each mode, we can have* $A^{(n)}(r(t))$, $B^{(n)}(r(t))$, $C^{(n))}(r(t))$ *and other relations of their states.*

(2) By using (13)-(16) and Corollary 11, we can calculate the values of stabilization indices of the system and the controller gains K_{ij} *with the parameters of BAM-NN.*

(3) Choose a numerical solution to the stochastic partial differential equation (SPDE) to simulate the sample states of BAM-NN. Here we use a so-called estimation-correction method, which is based on the following main steps [43].

Firstly*, with given steps* H, L *of simulation time interval* $t\in[0,t_f]$ *and space* $\lambda_k\in[\lambda_{mink},\lambda_{maxk}]\in\Omega_0$, *we can get the grids of time and space, i.e.,* $t_h=h\Delta t$, $h=0,1,2,\ldots,H$, $\lambda_l=l\Delta\lambda$, $l=0,1,2,\ldots,L$, *by which we define a numerical solution to BAM-NN as* $x_l^h=x(t_h,\lambda_l)=[x^{(1)}(t_h,\lambda_l),x^{(2)}(t_h,\lambda_l)]^T$. Then we denote $\Sigma F(x_l^h)=A(r(t_h))x_l^h+B(r(t_h))f(x_l^h)+C(r(t_h))f(x_l^{h-\tau_h})+Kx_l^h+h(x_l^h,x_l^{h-\tau_h})((W(t_h)-W(t_{h-1}))/\Delta t)$, where $\tau_h\approx\tau(t_h)/\Delta t$ is an integer times of $\tau(t_h)$ to Δt.

Secondly*, according to the basic formulation of SPDE (2) with its discretized time and space, we have an estimation formula for the numerical solution* x_l^h

$$x_l^h+\frac{\Delta t}{2}\Sigma F\left(x_l^h\right)=-\frac{\rho}{2}Dx_{l+1}^{h+1/2}+(I+\rho D)x_l^{h+1/2}-\frac{\rho}{2}Dx_{l-1}^{h+1/2} \quad (47)$$

where $x_l^{h+1/2}=x(t_h+(1/2)\Delta t,\lambda_l)$, $\rho=\Delta t/\Delta\lambda^2$, *I is an identity matrix with suitable dimensions. By the boundary function (4), we can calculate the values of* $x_0^{h+1/2}$ *and* $x_L^{h+1/2}$. *Then we can solve three diagonal linear equations from (47) to get an estimated value of* $x_l^{h+1/2}$.

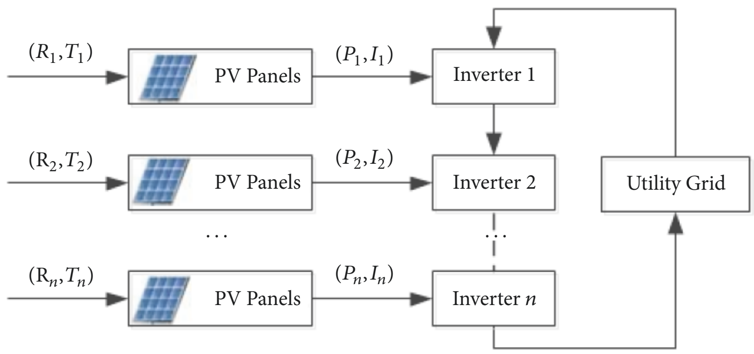

Figure 1: Equivalent topology structure of a grid-connected PV power system with series multiple inverters.

***Finally**, based on the estimated value of $x_l^{h+1/2}$, we have a further correction formula as follows.*

$$\begin{aligned} &-\frac{\rho}{2}Dx_{l+1}^{h+1} + (I+\rho D)\,x_l^{h+1} - \frac{\rho}{2}Dx_{l-1}^{h+1} \\ &\quad = \frac{\rho}{2}Dx_{l+1}^{h} + (I-\rho D)\,x_l^{h} + \frac{\rho}{2}Dx_{l-1}^{h} \\ &\qquad + \Delta t \Sigma F\left(x_l^{h+1/2}\right) \end{aligned} \tag{48}$$

Similarly, we also transfer formulation (48) to three diagonal linear equations and calculate the numerical solution x_l^h with permitted calculation errors.

(4) Comparing the sampled data profiles of the simulation states from (1)-(3) in Algorithm 13 with the measured data profiles from the aimed practical system, we can identify the validness of the model and performance of the controlled BAM-NN system. Furthermore, we can improve the parameters and control gains to obtain a better model by repeating steps (1)-(3).

4. Numerical Simulation of an Illustrative Application

For a grid-connected photovoltaic (PV) power generation system with series-connected inverters, which is illustrated by it is equivalent topology structure in Figure 1, every branch-circuit with a group of PV panels penetrates power into utility grid via a inverter. Since the current/power (I_i, P_i) generated by the ith group of PV panels is strongly dependent on operating conditions and field factors, such as sun geometric locations, their irradiation levels (R_i) of the sun and the ambient temperature (T_i) stochastically fluctuate with the environmental factors of PV power fields. Thus it is technically necessary to maintain power synchronization of series-connected inverters to improve their output power and standard voltage and current [44]. For this purpose, we need to develop a model with power and current difference between every couple-connected PV inverter based on basic photovoltaic model of PV panels [44, 45], which can be modeled as a HSD-BAM-NN ((2)-(4)), and the power/current difference $(\Delta P_{ij}, \Delta I_{ij})$ is taken as states $x(t)$ with irradiation levels (R_i) as the space variable λ. It is assumed that the temperature difference can be ignored. By [44, 45], we formulate every part of BAM-NN ((2)-(4)) and calculate the parameters in the model; i.e., the modeled BAM-NN (2) has the same coefficient matrix, which is denoted as follows.

$$\begin{aligned} D_r &= \begin{pmatrix} \begin{bmatrix} 4.96 & 6.34 \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 8.05 & 6.76 \end{bmatrix} \end{pmatrix} \\ A_r &= \begin{pmatrix} \begin{bmatrix} 0.28 & 0 \\ 0 & 0.18 \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 0.42 & 0 \\ 0 & 0.35 \end{bmatrix} \end{pmatrix} \\ B_r &= \begin{pmatrix} \begin{bmatrix} 0.3 & 0.4 \\ 12 & 0.3 \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} 0.2 & 0.2 \\ 10 & -0.5 \end{bmatrix} \end{pmatrix} \\ C_r &= \begin{pmatrix} \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.3 \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} -0.2 & 0.2 \\ -0.4 & -0.1 \end{bmatrix} \end{pmatrix} \end{aligned} \tag{49}$$

Firstly, we get the initial value conditions

$$\begin{aligned} x^{(1)}\left(s, \lambda^{(1)}\right) &= e^{s}\left(\cos\left(2\pi\lambda^{(1)}\right) - 1\right) \\ x^{(2)}\left(s, \lambda^{(2)}\right) &= e^{2s}\sin\left(4\pi\lambda^{(2)}\right) \end{aligned} \tag{50}$$

where $(s, \lambda^{(2)}) \in [-2, 0] \times \Omega_0$, $\Omega_0 = [0.5, 8.6]$. And the boundary value functions are taken as follows.

$$\begin{aligned} x^{(1)}(t, 0.5) &= 2.3, \\ x^{(1)}(t, 8.6) &= 6.95 \\ x^{(2)}(t, 0.5) &= 1.8, \\ x^{(2)}(t, 8.6) &= 0.34, \\ & t \geq -2 \end{aligned} \tag{51}$$

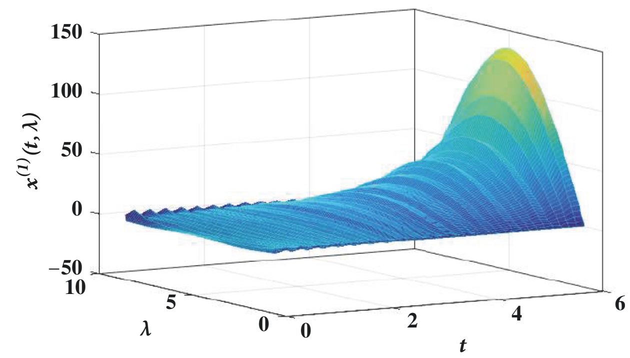

Figure 2: Unstable sample state surface of $x^{(1)}(t,\lambda)$ in HSD-BAM-NN's simulation with no control.

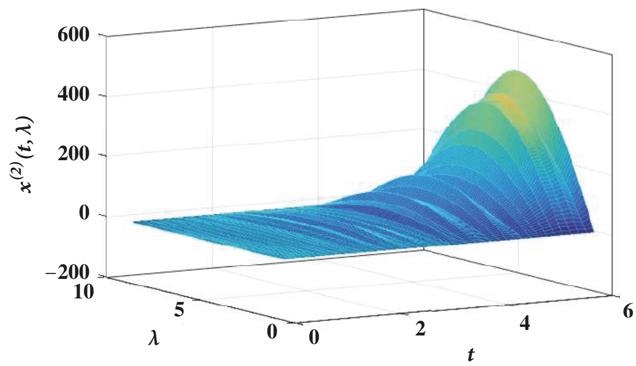

Figure 3: Unstable sample state surface of $x^{(2)}(t,\lambda)$ in HSD-BAM-NN's simulation with no control.

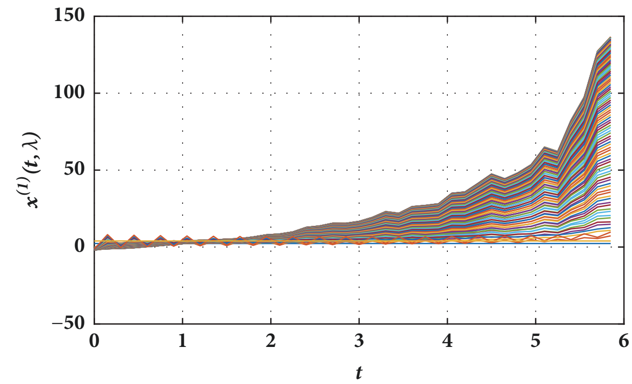

Figure 4: Unstable sample state profiles of $x^{(1)}(t,\lambda)$ in HSD-BAM-NN's simulation with no control.

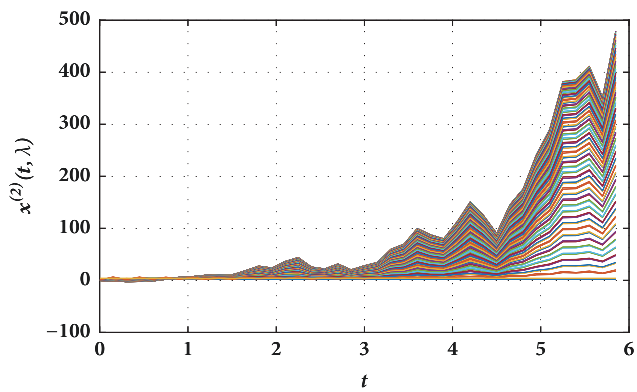

Figure 5: Unstable sample state profiles of $x^{(2)}(t,\lambda)$ in HSD-BAM-NN's simulation with no control.

Denote $\tau^{(1)}(t) = 0.8\sin((\pi/4)t + 0.2)$, $\tau^{(2)}(t) = 1/(1 + 0.3e^{t})$; the generator of the Markovian chain is as follows.

$$\Gamma = \begin{pmatrix} -\frac{4}{5} & \frac{4}{5} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \tag{52}$$

The activation functions are $f(x) = (3/4)\sin x + (1/4)x$, $g(x) = (1/2)(|x+1| - |x-1|)$, and the stochastic disturbed functions are $h_1^2(s_1,s_2,s_3) \le 0.01(s_1^2 + s_2^2 + s_3^2)$, $h_2^2(s_1,s_2,s_3) \le 0.02(s_1^2 + s_2^2 + s_3^2)$.

According to the given structure and parameters of the BAM-NN (2)-(4), we can perform the numerical simulations in instability and stabilization cases.

(1) Instability. Using Algorithm 13 in Section 3, the sample states $x(t,\lambda)$ of BAM-NN (2) are calculated, and the surfaces of (t,λ) versus $x(t,\lambda)$ are shown in Figures 2 and 3, while the t versus $x(t,\lambda)$ profile curves are shown in Figures 4 and 5. These figures show the instability behavior.

(2) Stabilization. Let $p = 2, \mu_1^{(1)} = \mu_1^{(2)} = 1, \mu_2^{(1)} = \mu_2^{(2)} = 2$. By (13)-(16), we can calculate that $L^- = -1/2, L^+ = 1, N^- = 0, N^+ = 1, \tau^{(n)} = 1, \tau_0^{(n)} = 1/4, \tilde{\varsigma}_i^{(n)} = \tilde{k}_i = -1.43, \tilde{\eta}_i = 2.84, \tilde{\varsigma}_j^{(2)} = \tilde{k}_j^{(2)} = -1,33, \tilde{\eta}_j^{(2)} = 2.82, j = 1,2$. The gain coefficients of periodically intermittent controllers (10) are obtained as follows by sufficient conditions

$$K = -\begin{pmatrix} 2.95 & 2.86 \\ 3.42 & 1.6 \end{pmatrix} \tag{53}$$

Furthermore, we can get the indices' values of the stable conditions of HSD-BAM-NN with $\tilde{\nu}_1^{(1)} = \tilde{\nu}_2^{(1)} = -18.56, \tilde{\nu}_1^{(2)} = \tilde{\nu}_2^{(2)} = -25.02, \tilde{\varsigma}_i^{(1)} - \tilde{\nu}_i^{(1)} - \max_{\tilde{r}\in\mathbb{S}} \sum_{q=1}^{\mathcal{N}} \gamma_{\tilde{r}q}^{(1)} \mu_q^{(1)} - \eta_i^{(1)}/(1-\tau_0^{(1)}) > 0, \tilde{\varsigma}_i^{(2)} - \tilde{\nu}_i^{(2)} - \max_{\tilde{r}\in\mathbb{S}} \sum_{q=1}^{\mathcal{N}} \gamma_{\tilde{r}q}^{(2)} \mu_q^{(2)} - \eta_i^{(2)}/(1-\tau_0^{(2)}) > 0, i = 1,2$. Also, we have $\tilde{\varepsilon}_1^{(1)} = \tilde{\varepsilon}_2^{(1)} = 3.15, \tilde{\varepsilon}_1^{(2)} = \tilde{\varepsilon}_2^{(2)} = 8.399$. From (III) in Corollary 11, we know that $\tilde{\varepsilon} - \tilde{\nu}(T-\delta)/\overline{\mu}T > 0$. Letting $\delta = 3.6, T = 5$, we can calculate the sample stabilized states of the controlled system. The trend surfaces of (t,λ) versus $x(t,\lambda)$ are shown in Figures 6 and 7 with the profile curves of t versus $x(t,\lambda)$ in Figures 8 and 9. From the states' trends, we can see that an unstable system (2)-(4) can be stabilized by the controller (10) with appropriate parameters, which theoretically shows that the output power of connected PV panels can be synchronized to a given level with its BAM-NN model, even though the power stochastically fluctuate.

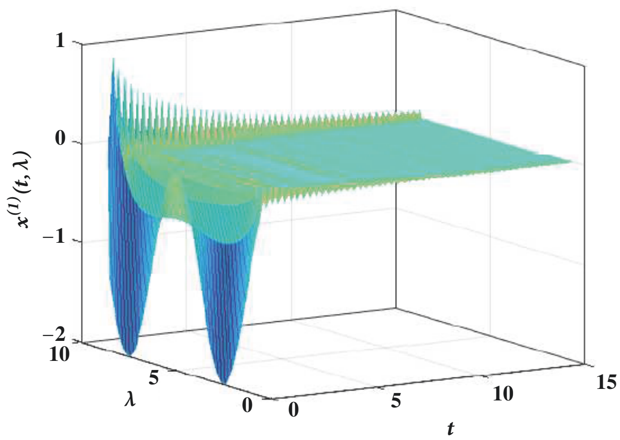

Figure 6: Sample state surface of $x^{(1)}(t,\lambda)$ in HSD-BAM-NN's simulation with controller (10).

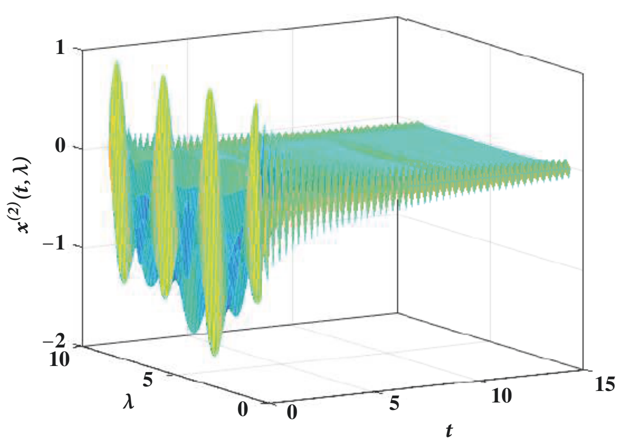

Figure 7: Sample state surface of $x^{(2)}(t,\lambda)$ in HSD-BAM-NN's simulation with controller (10).

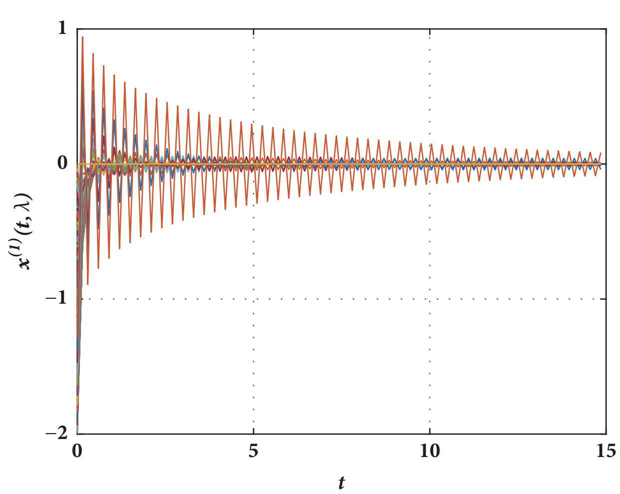

Figure 8: Sample state profiles of $x^{(1)}(t,\lambda)$ in HSD-BAM-NN's simulation with controller (10).

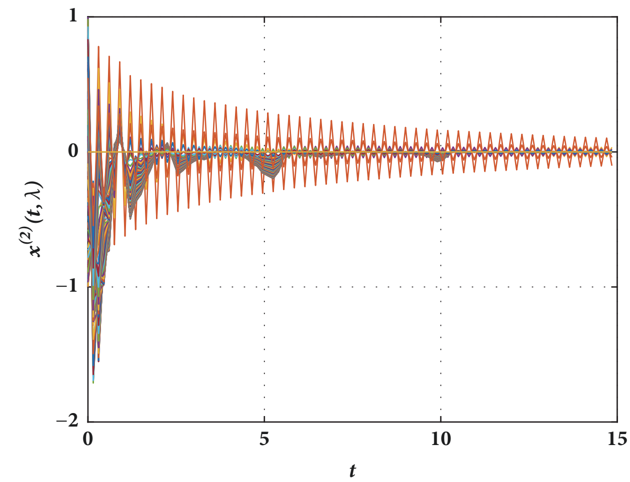

Figure 9: Sample state profiles of $x^{(2)}(t,\lambda)$ in HSD-BAM-NN's simulation with controller (10).

5. Conclusion

In this paper, a hybrid stochastic delayed BAM neural network is considered for its stabilization problem and a periodically intermittent controller is designed to stabilize an unstable HSD-BAM-NN with an exponential convergence property. The sufficient conditions of exponential stabilization of HSD-BAM-NN are derived by Lyapunov-Krasovskii functional method, stochastic analysis techniques, and integral inequality. And the framework is established to give a solution algorithm to the sufficient conditions. The simulation results of the grid-connected photovoltaic (PV) power generation system verify the effectiveness of the proposed controller.

Data Availability

The data underlying the research and Matlab program code used in numerical simulation are available by email to the first author (pengyj@scut.edu.cn).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] B. Kosko, "Adaptive bidirectional associative memories," *Applied Optics*, vol. 26, no. 23, pp. 4947–4960, 1987.

[2] B. Kosko, "Bidirectional associative memories," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 18, no. 1, pp. 49–60, 1988.

[3] B. Kosko, *Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence*, Prentice Hall, New Jersey, NJ, USA, 1992.

[4] C. Yang, Y. Jiang, Z. Li, W. He, and C.-Y. Su, "Neural control of bimanual robots with guaranteed global stability and motion precision," *IEEE Transactions on Industrial Informatics*, 2017.

[5] Y. Jiang, C. Yang, J. Na, G. Li, Y. Li, and J. Zhong, "A brief review of neural networks based learning and control and their applications for robots," *Complexity*, vol. 2017, Article ID 1895897, 14 pages, 2017.

[6] A. Browne, B. D. Hudson, D. C. Whitley, M. G. Ford, and P. Picton, "Biological data mining with neural networks: implementation and application of a flexible decision tree extraction algorithm to genomic problem domains," *Neurocomputing*, vol. 57, pp. 275–293, 2004.

[7] Q. Luo, F. Deng, X. Mao, J. Bao, and Y. Zhang, "Theory and application of stability for stochastic reaction diffusion systems," *Science China Information Sciences*, vol. 51, no. 2, pp. 158–170, 2008.

[8] X. Liao and X. Mao, "Stability of stochastic neural networks," *Neural,Parallel & Science Computer*, vol. 4, pp. 205–224, 1996.

[9] J. Cao and L. Wang, "Exponential stability and periodic oscillatory solution in BAM networks with delays," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 13, no. 2, pp. 457–463, 2002.

[10] X. Mao, "Exponential stability of stochastic delay interval systems with Markovian switching," *IEEE Transactions on Automatic Control*, vol. 47, no. 10, pp. 1604–1612, 2002.

[11] Q. Song and J. Cao, "Global exponential stability and existence of periodic solutions in BAM networks with delays and reaction-diffusion terms," *Chaos, Solitons and Fractals*, vol. 23, no. 2, pp. 421–430, 2005.

[12] B. T. Cui and X. Y. Lou, "Global asymptotic stability of BAM neural networks with distributed delays and reaction-diffusion terms," *Chaos, Solitons and Fractals*, vol. 27, no. 5, pp. 1347–1354, 2006.

[13] X. Lou, B. Cui, and W. Wu, "On global exponential stability and existence of periodic solutions for BAM neural networks with distributed delays and reaction-diffusion terms," *Chaos, Solitons and Fractals*, vol. 36, no. 4, pp. 1044–1054, 2008.

[14] P. Balasubramaniam and C. Vidhya, "Global asymptotic stability of stochastic BAM neural networks with distributed delays and reaction-diffusion terms," *Journal of Computational and Applied Mathematics*, vol. 234, no. 12, pp. 3458–3466, 2010.

[15] Q. Zhu, X. Li, and X. Yang, "Exponential stability for stochastic reaction-diffusion BAM neural networks with time-varying and distributed delays," *Applied Mathematics and Computation*, vol. 217, no. 13, pp. 6078–6091, 2011.

[16] X. Li and J. Jia, "Global robust stability analysis for BAM neural networks with time-varying delays," *Neurocomputing*, vol. 120, pp. 499–503, 2013.

[17] Z. Quan, L. Huang, S. Yu, and Z. Zhang, "Novel LMI-based condition on global asymptotic stability for BAM neural networks with reaction-diffusion terms and distributed delays," *Neurocomputing*, vol. 136, pp. 213–223, 2014.

[18] R. Rao, S. Zhong, and X. Wang, "Stochastic stability criteria with LMI conditions for Markovian jumping impulsive BAM neural networks with mode-dependent time-varying delays and nonlinear reaction-diffusion," *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 1, pp. 258–273, 2014.

[19] F. Wang, D. Sun, and H. Wu, "Global exponential stability and periodic solutions of high-order bidirectional associative memory (BAM) neural networks with time delays and impulses," *Neurocomputing*, vol. 155, pp. 261–276, 2015.

[20] S. Senthilraj, R. Raja, F. Jiang, Q. Zhu, and R. Samidurai, "New delay-interval-dependent stability analysis of neutral type BAM neural networks with successive time delay components," *Neurocomputing*, vol. 171, pp. 1265–1280, 2016.

[21] Z. Pu and R. Rao, "Exponential stability criterion of high-order BAM neural networks with delays and impulse via fixed point approach," *Neurocomputing*, vol. 292, pp. 63–71, 2018.

[22] J. Zhu and J. Sun, "Stability of quaternion-valued impulsive delay difference systems and its application to neural networks," *Neurocomputing*, vol. 284, pp. 63–69, 2018.

[23] Z. Zhang, K. Liu, and Y. Yang, "New LMI-based condition on global asymptotic stability concerning BAM neural networks of neutral type," *Neurocomputing*, vol. 81, pp. 24–32, 2012.

[24] Q. Zhu, R. Rakkiyappan, and A. Chandrasekar, "Stochastic stability of Markovian jump BAM neural networks with leakage delays and impulse control," *Neurocomputing*, vol. 136, pp. 136–151, 2014.

[25] Haykin, *Neural Networks*, Prentice-Hall, Englewood-Cliffs, NJ, USA, 1994.

[26] F. Hossain and E. N. Anagnostou, "Assessment of a stochastic interpolation based parameter sampling scheme for efficient uncertainty analyses of hydrologic models," *Computers & Geosciences*, vol. 31, no. 4, pp. 497–512, 2005.

[27] F. Jiang, H. Yang, and Y. Shen, "On the robustness of global exponential stability for hybrid neural networks with noise and delay perturbations," *Neural Computing & Applications*, vol. 24, no. 7-8, pp. 1497–1504, 2014.

[28] X. Feng, K. A. Loparo, Y. Ji, and H. J. Chizeck, "Stochastic stability properties of jump linear systems," *IEEE Transactions on Automatic Control*, vol. 37, no. 1, pp. 38–53, 1992.

[29] P. Vadivel, R. Sakthivel, K. Mathiyalagan, and P. Thangaraj, "New passivity criteria for fuzzy BAM neural networks with Markovian jumping parameters and time-varying delays," *Reports on Mathematical Physics*, vol. 71, no. 1, pp. 69–91, 2013.

[30] H. Geng, Z. Wang, Y. Liang, Y. Cheng, and F. E. Alsaadi, "State estimation for asynchronous sensor systems with Markov jumps and multiplicative noises," *Information Sciences*, vol. 417, pp. 1–19, 2017.

[31] H. Geng, Y. Liang, and Y. Cheng, "Target state and markovian jump ionospheric height bias estimation for OTHR tracking systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, pp. 1–13, 2018.

[32] L. Wang, L. Zhang, and X. Ding, "Global dissipativity of a class of BAM neural networks with both time-varying and continuously distributed delays," *Neurocomputing*, vol. 152, pp. 250–260, 2015.

[33] R. Sakthivel, A. Arunkumar, K. Mathiyalagan, and S. M. Anthoni, "Robust passivity analysis of fuzzy Cohen-Grossbert BAM neural networks with time-varying delays," *Applied Mathematics and Computation*, vol. 218, no. 7, pp. 3799–3809, 2011.

[34] Z. Huang and Q.-G. Yang, "Exponential stability of impulsive high-order cellular neural networks with time-varying delays," *Nonlinear Analysis: Real World Applications*, vol. 11, no. 1, pp. 592–600, 2010.

[35] X. Yang and J. Cao, "Stochastic synchronization of coupled neural networks with intermittent control," *Physics Letters A*, vol. 373, no. 36, pp. 3259–3272, 2009.

[36] C. Hu, J. Yu, H. Jiang, and Z. Teng, "Exponential lag synchronization for neural networks with mixed delays via periodically intermittent control," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 20, no. 2, Article ID 023108, pp. 1–8, 2010.

[37] C. Hu, J. Yu, H. Jiang, and Z. Teng, "Exponential synchronization for reaction-diffusion networks with mixed delays in terms of p-norm via intermittent driving," *Neural Networks*, vol. 31, pp. 1–11, 2012.

[38] S. Dharani, R. Rakkiyappan, and J. H. Park, "Pinning sampled-data synchronization of coupled inertial neural networks with reaction-diffusion terms and time-varying delays," *Neurocomputing*, vol. 227, pp. 101–107, 2017.

[39] M. Zochowski, "Intermittent dynamical control," *Physica D: Nonlinear Phenomena*, vol. 145, no. 3-4, pp. 181–190, 2000.

[40] S. Zhu and Y. Shen, "Robustness analysis of global exponential stability of neural networks with Markovian switching in the presence of time-varying delays or noises," *Neural Computing and Applications*, vol. 23, pp. 1563–1571, 2013.

[41] Q. Luo, F. Deng, J. Bao, B. Zhao, and Y. Fu, "Stabilization of stochastic Hopfield neural network with distributed parameters," *Science China Information Sciences*, vol. 47, no. 6, pp. 752–762, 2004.

[42] X. Mao and C. Yuan, *Stochastic Differential Equations with Markovian Switching*, Imperial College Press, London, UK, 2006.

[43] D. Sharp, S. Habib, and M. Mineev, "Numerical methods for stochastic partial differential equations," U.S. Department of Energy, Office of Scientific and Technical Information, United States, 1999.

[44] G. K. Singh, "Solar power generation by PV (photovoltaic) technology: a review," *Energy*, vol. 53, pp. 1–13, 2013.

[45] Y. Shan, J. Hu, Z. Li, and J. M. Guerrero, "A model predictive control for renewable energy based ac microgrids without any pid regulators," *IEEE Transactions on Power Electronics*, vol. 33, no. 11, pp. 9122–9126, 2018.


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