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*Research Article*

# New Results on Fuzzy Synchronization for a Kind of Disturbed Memristive Chaotic System

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This paper concerns the problem on the fuzzy synchronization for a kind of disturbed memristive chaotic system. First, based on fuzzy theory, the fuzzy model for a memristive chaotic system is presented; next, based on H-infinity technique, a multidimensional fuzzy controller and a single-dimensional fuzzy controller are designed to realize the synchronization of master-slave chaotic systems with disturbances. Finally, some typical examples are included to illuminate the correctness of the given control method.

## 1. Introduction

Since May 2008, by using nanotechnology physical techniques, HP laboratory research team successfully obtain the resistance with memory characteristic [1], which confirmed the concept of memristor proposed by Chua [2, 3]. As the fourth basic passive device, memristor establishes the relationship between the magnetic flux and the charge. It has been reported that memristor can be applied in the field of computer science [4], biological engineering [5], and electronic engineering [6]. Especially, memristor can be used to construct the chaotic circuits.

For chaotic circuits, the nonlinear device is the key component. In 2008, Itoh and Chua built the first memristor-based chaotic system by replacing the diode with a piecewise linear magnetron Chua's memristor [7]. Compared with the conventional nonlinear-device-based chaotic circuits, the memristor-based circuit has two main characteristics: first, the memristor-based circuit can produce the complicated dynamical behavior, which is different from the general chaotic dynamical behavior; secondly, the memristor-based circuit is more suitable to generate the high-frequency chaotic signal and have potential applications in chaotic secure communication, signal generator, and image process [8–12]. Hence, up to now, a number of memristor-based chaotic circuit with different structures are proposed. For example, the chaotic circuit with one memristor is studied in [13, 14], the chaotic circuit with two memristor is concerned in [15, 16], the integer-order chaotic memristor circuit is investigated in [17, 18], and the fractional chaotic memristor circuit are researched in [19, 20].

Chaos synchronization is a common phenomenon and can be found in biological systems, chemical reactions, power converters, secure communication system, and so on. Fuzzy technique is a powerful tool [21–27] and especially suitable for the chaos synchronization in the case that disturbances exist. For general fuzzy control, the control input is multidimensional and requires all system state information. However, in practical engineering, it is not easy to get all system state information. The multidimensional control can not only increase the control cost but also result in disturbance input problem. Hence, it is meaningful to design a single-dimensional fuzzy controller which is just based on one system state variable. In addition, disturbance inputs exist in actual system widely, which should be considered in synchronization control. All these motivate our research.

The paper is schemed as follows: the fuzzy model for a memristor-based chaotic circuit is constructed and the preliminary knowledge will be given in Section 2; a multidimensional fuzzy controller and a single-dimensional

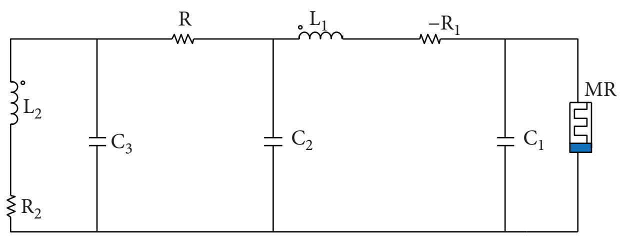


Figure 1: The memristor-based chaotic circuit.

fuzzy controller will be designed to achieve the chaos synchronization of the master-slave systems in Section 3; the typical simulation example will be included to validate the correctness of the scheme in Section 4; and finally, the paper will be concluded in Section 5.

Notations used in this paper are fairly standard. diag $\{\dots\}$ represents a block diagonal matrix, $R^n$ is the $n$-dimensional Euclidean space, $R^{n\times m}$ denotes the set of $n\times m$ real matrix, the superscript $T$ stands for matrix transposition, $\|\cdot\|_2$ refers to the Euclidean vector norm or the induced matrix 2-norm, and $\lambda_{\min}\{\cdot\}$ represents the maximum eigenvalue.

## 2. System Description and Preliminaries

First, consider a memristor-based circuit as Figure 1.

One can get the equivalent dynamic system as

$$\begin{aligned} C_1\frac{dV_{C_1}(t)}{dt} &= i_{L_1}(t) - W(\varphi(t))V_{C_1}(t),\\ L_1\frac{di_{L_1}(t)}{dt} &= V_{C_2}(t) - V_{C_1}(t) + R_1 i_{L_1}(t),\\ C_2\frac{dV_{C_2}(t)}{dt} &= \frac{1}{R}\left(V_{C_3}(t) - V_{C_2}(t)\right) - i_{L_1}(t),\\ C_3\frac{dV_{C_3}(t)}{dt} &= \frac{1}{R}\left(V_{C_2}(t) - V_{C_3}(t)\right) - i_{L_2}(t),\\ L_2\frac{di_{L_2}(t)}{dt} &= V_{C_3}(t) - R_2 i_{L_2},\\ \frac{d\varphi(t)}{dt} &= V_{C_1}(t). \end{aligned} \tag{1}$$

Define the state variable as

$$\begin{aligned} x_1(t) &= V_{C_1}(t),\\ x_2(t) &= Ri_{L_1}(t),\\ x_3(t) &= V_{C_2}(t),\\ x_4(t) &= V_{C_3}(t),\\ x_5(t) &= Ri_{L_2}(t),\\ x_6(t) &= \varphi(t). \end{aligned} \tag{2}$$

One can obtain the equivalent dynamical equation as

$$\begin{aligned} \dot{x}_1(t) &= r_1\left[x_2(t) - \bar{W}(x_6(t))x_1(t)\right],\\ \dot{x}_2(t) &= r_2[x_3(t) - x_1(t) + r_7x_2(t)],\\ \dot{x}_3(t) &= r_3[x_4(t) - x_3(t) - x_2(t)],\\ \dot{x}_4(t) &= r_4[x_3(t) - x_4(t) - x_5(t)],\\ \dot{x}_5(t) &= r_5[x_4(t) - r_6x_5(t)],\\ \dot{x}_6(t) &= x_1(t), \end{aligned} \tag{3}$$

with

$$\bar{W}(x_6(t)) = RW(\varphi(t)) = \begin{cases} a, & |x_6(t)| \le 1,\\ b, & |x_6(t)| > 1 \end{cases}, \tag{4}$$

where $x_i$, $i = 1, 2, \dots, 7$ is the state variable of the system and $r_i > 0$, $i = 1, 2, \dots, 7$ is the system parameter. The memristive system will possess the chaotic dynamical behavior when the system parameters are $r_1 = 5$, $r_2 = 2$, $r_3 = 2$, $r_4 = 4$, $r_5 = 3$, $r_6 = 0.1$, $r_7 = 0.8$, $a = 0.1$, and $b = 6$.

Next, consider the fuzzy modeling of the memristive chaotic system.

For $\dot{x}_1(t) = r_1(x_2(t) - \bar{W}(x_6)x_1(t))$.

*Rule 1.* If $x_1(t)$ is $H_{11}$, then

$$\dot{x}_1(t) = r_1(x_2(t) - ax_1(t)), \tag{5}$$

where $H_{11}$ means $|x_6(t)| \le 1$, and define

$$M_{11} = \begin{cases} 1, & |x_6(t)| \le 1,\\ 0, & |x_6(t)| > 1. \end{cases} \tag{6}$$

*Rule 2.* If $x_1(t)$ is $H_{12}$, then

$$\dot{x}_1(t) = r_1(x_2(t) - bx_1(t)), \tag{7}$$

where $H_{12}$ means $|x_6(t)| > 1$, and define

$$M_{12} = \begin{cases} 0, & |x_6(t)| \le 1,\\ 1, & |x_6(t)| > 1. \end{cases} \tag{8}$$

Hence, the fuzzy model of the memristive chaotic system is defined as

$$\begin{bmatrix} \dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\\ \dot{x}_4(t)\\ \dot{x}_5(t)\\ \dot{x}_6(t) \end{bmatrix} = \begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 & 0\\ 0 & M_{11} & 0 & 0 & 0 & 0\\ 0 & 0 & M_{11} & 0 & 0 & 0\\ 0 & 0 & 0 & M_{11} & 0 & 0\\ 0 & 0 & 0 & 0 & M_{11} & 0\\ 0 & 0 & 0 & 0 & 0 & M_{11} \end{bmatrix}$$

$$\cdot \begin{bmatrix} r_1[x_2(t) - ax_1(t)] \\ r_2[x_3(t) - x_1(t) + r_7 x_2(t)] \\ r_3[x_4(t) - x_3(t) - x_2(t)] \\ r_4[x_3(t) - x_4(t) - x_5(t)] \\ r_5[x_4(t) - r_6 x_5(t)] \\ x_1(t) \end{bmatrix} + \begin{bmatrix} M_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{12} \end{bmatrix} \cdot \begin{bmatrix} r_1[x_2(t) - bx_1(t)] \\ r_2[x_3(t) - x_1(t) + r_7 x_2(t)] \\ r_3[x_4(t) - x_3(t) - x_2(t)] \\ r_4[x_3(t) - x_4(t) - x_5(t)] \\ r_5[x_4(t) - r_6 x_5(t)] \\ x_1(t) \end{bmatrix}. \tag{9}$$

Above model can be rewritten as

$$\dot{x}(t) = \sum_{i=1}^{2} \Theta_i A_i x(t), \tag{10}$$

where

$$\Theta_i = \operatorname{diag}\{M_{1i}, M_{1i}, M_{1i}, M_{1i}, M_{1i}, M_{1i}\},$$
$$A_1 = \begin{bmatrix} -ar_1 & r_1 & 0 & 0 & 0 & 0 \\ -r_2 & r_2 r_7 & r_2 & 0 & 0 & 0 \\ 0 & -r_3 & -r_3 & r_3 & 0 & 0 \\ 0 & 0 & r_4 & -r_4 & -r_4 & 0 \\ 0 & 0 & 0 & r_5 & -r_5 r_6 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} -br_1 & r_1 & 0 & 0 & 0 & 0 \\ -r_2 & r_2 r_7 & r_2 & 0 & 0 & 0 \\ 0 & -r_3 & -r_3 & r_3 & 0 & 0 \\ 0 & 0 & r_4 & -r_4 & -r_4 & 0 \\ 0 & 0 & 0 & r_5 & -r_5 r_6 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

System (3) is supposed as the master system, and the slave system is constructed as

$$\begin{aligned} \dot{y}_1(t) &= r_1\left(y_2(t) - \bar{W}(y_6(t))y_1(t)\right) + u_1(t) + w_1(t), \\ \dot{y}_2(t) &= r_2[y_3(t) - y_1(t) + r_7 y_2(t)] + u_2(t) + w_2(t), \\ \dot{y}_3(t) &= r_3[y_3(t) - y_4(t) - y_2(t)] + u_3(t) + w_3(t), \\ \dot{y}_3(t) &= r_4[y_4(t) - y_3(t) - y_5(t)] + u_4(t) + w_4(t), \\ \dot{y}_5(t) &= r_5[y_4(t) - r_6 y_5(t)] + u_5(t) + w_5(t), \\ \dot{y}_6(t) &= y_1(t) + u_6(t) + w_6(t), \end{aligned} \tag{12}$$

where $y(t) = (y_1(t), y_2(t), y_3(t), y_4(t), y_5(t), y_6(t))^T$ is the state variable vector of the slave system and $w(t) = (w_1(t), w_2(t), w_3(t), w_4(t), w_5(t), w_6(t))^T$ is the disturbance input of the slave system.

Hence, the fuzzy model of the slave system can be represented as

$$\dot{y}(t) = \sum_{i=1}^{2} \Theta_i A_i y(t) + w(t) + u(t), \tag{13}$$

where $u(t) = (u_1(t), u_2(t), u_3(t), u_4(t), u_5(t), u_6(t))^T$ is the synchronization fuzzy controller.

Define the synchronization error vector of the master-slave systems as

$$E(t) = y(t) - x(t), \tag{14}$$

where $E(t) = (e_1(t), e_2(t), e_3(t), e_4(t), e_5(t), e_6(t))^T$.

One can get the error dynamic system as

$$\begin{aligned} \dot{E}(t) = \dot{y}(t) - \dot{x}(t) &= \sum_{i=1}^{2} \Theta_i A_i y(t) - \sum_{i=1}^{2} \Theta_i A_i x(t) \\ &+ w(t) + u(t) = \sum_{i=1}^{2} \Theta_i A_i E(t) + w(t) + u(t). \end{aligned} \tag{15}$$

In this paper, the following lemmas are concerned:

**Lemma 1** *(see [28]). If $f(t) \in L_\infty \cap L_2$ and $\dot{f}(t) \in L_\infty$, one can get*

$$\lim_{t \to +\infty} f(t) = 0. \tag{16}$$

*Definition 1.* For nonzero $w(t) \in L_2[t_0, \infty]$ and under the assumption of zero initial condition, if there exists a positive scalar $\gamma$ such that

$$\|E(t)\|_2 \le \gamma \|w(t)\|_2. \tag{17}$$

Then, the slave system will synchronize to the master system with $H_\infty$ norm bound $\gamma$.

## 3. Main Results

Based on fuzzy theory and Lyapunov theory, a controller is presented as follows.

**Theorem 1.** *If there exist scalar $K_j^{\,i} > 0, i = 1, 2, j = 1, \ldots, 6$, design the multidimensional fuzzy controller with following control regulation*

$$u(t) = -\sum_{i=1}^{2} \Theta_i k_i E(t), \tag{18}$$

*with*

$$\begin{cases} k_1^{\,1} = K_1^{\,1} - r_1 a + 2, \\ k_2^{\,1} = K_2^{\,1} + \dfrac{(r_1 - r_2)^2}{4} + r_2 r_7 + 1, \\ k_3^{\,1} = K_3^{\,1} + \dfrac{(r_2 - r_3)^2}{4} - r_3 + 1, \\ k_4^{\,1} = K_4^{\,1} + \dfrac{(-r_3 - r_4)^2}{4} - r_4 + 1, \\ k_5^{\,1} = K_5^{\,1} + \dfrac{(r_5 - r_4)^2}{4} - r_5 r_6, \\ k_6^{\,1} = K_6^{\,1} + \dfrac{1}{4}, \end{cases}$$
$$\begin{cases} k_1^{\,2} = K_1^{\,2} - r_1 b + 2, \\ k_2^{\,2} = K_2^{\,2} + \dfrac{(r_1 - r_2)^2}{4} + r_2 r_7 + 1, \\ k_3^{\,2} = K_3^{\,2} + \dfrac{(r_2 - r_3)^2}{4} - r_3 + 1, \\ k_4^{\,2} = K_4^{\,2} + \dfrac{(-r_3 - r_4)^2}{4} - r_4 + 1, \\ k_5^{\,2} = K_5^{\,2} + \dfrac{(r_5 - r_4)^2}{4} - r_5 r_6, \\ k_6^{\,2} = K_6^{\,2} + \dfrac{1}{4}, \end{cases} \tag{19}$$

$$\begin{bmatrix} I - \sum_{i=1}^{2} \Theta_i K_i & \dfrac{I}{2} \\ * & -\gamma^2 I \end{bmatrix} < 0, \tag{20}$$

$$k_i = \operatorname{diag}\left\{k_1^{\,i}, k_2^{\,i}, k_3^{\,i}, k_4^{\,i}, k_5^{\,i}, k_6^{\,i}\right\}, \tag{21}$$

$$K_i = \operatorname{diag}\left\{K_1^{\,i}, K_2^{\,i}, K_3^{\,i}, K_4^{\,i}, K_5^{\,i}, K_6^{\,i}\right\}. \tag{22}$$

*Then, the slave system (12) can synchronize to the master system (3) with $H_\infty$ norm bound $\gamma$.*

*Proof 1.* With (18), the error dynamic system can be transformed as

$$\begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \\ \dot{e}_3(t) \\ \dot{e}_4(t) \\ \dot{e}_5(t) \\ \dot{e}_6(t) \end{bmatrix} = \begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{11} \end{bmatrix} \cdot \begin{bmatrix} r_1[e_2(t) - a e_1(t)] - k_1^{\,1} e_1(t) + w_1(t) \\ r_2[e_3(t) - e_1(t) + r_7 e_2(t)] - k_2^{\,1} e_2(t) + w_2(t) \\ r_3[e_4(t) - e_3(t) - e_2(t)] - k_3^{\,1} e_3(t) + w_3(t) \\ r_4[e_3(t) - e_4(t) - e_5(t)] - k_4^{\,1} e_4(t) + w_4(t) \\ r_5[e_4(t) - r_6 e_5(t)] - k_5^{\,1} e_5(t) + w_5(t) \\ e_1(t) - k_6^{\,1} e_6(t) + w_6(t) \end{bmatrix}$$
$$+ \begin{bmatrix} M_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{12} \end{bmatrix} \cdot \begin{bmatrix} r_1[e_2(t) - b e_1(t)] - k_1^{\,2} e_1(t) + w_1(t) \\ r_2[e_3(t) - e_1(t) + r_7 e_2(t)] - k_2^{\,2} e_2(t) + w_2(t) \\ r_3[e_4(t) - e_3(t) - e_2(t)] - k_3^{\,2} e_3(t) + w_3(t) \\ r_4[e_3(t) - e_4(t) - e_5(t)] - k_4^{\,2} e_4(t) + w_4(t) \\ r_5[e_4(t) - r_6 e_5(t)] - k_5^{\,2} e_5(t) + w_5(t) \\ e_1(t) - k_6^{\,2} e_6(t) + w_6(t) \end{bmatrix} \tag{23}$$

Choose the Lyapunov function candidate as

$$V(t) = \frac{1}{2}\left(e_1^{\,2}(t) + e_2^{\,2}(t) + e_3^{\,2}(t) + e_4^{\,2}(t) + e_5^{\,2}(t) + e_6^{\,2}(t)\right) \tag{24}$$

One can get the time derivative of $V(t)$ as

$$\dot{V}(t) = e_1(t)\left\{M_{11}\left[r_1[e_2(t) - a e_1(t)] - k_1^{\,1} e_1(t) + w_1(t)\right] + M_{12}\left[r_1[e_2(t) - b e_1(t)] - k_1^{\,2} e_1(t) + w_1(t)\right]\right\}$$

$$
\begin{aligned}
&+ e_2(t)\left\{M_{11} r_2[e_3(t) - e_1(t) + r_7 e_2(t)] - k_2{}^1 e_2(t)\right.\\
&+ w_2(t)\big] + M_{12}\big[r_2[e_3(t) - e_1(t) + r_7 e_2(t)]\\
&\left.- k_2{}^2 e_2(t) + w_2(t)\big]\right\} + e_3(t)\left\{M_{11}\big[r_3[e_4(t)\right.\\
&- e_3(t) - e_2(t)] - k_3{}^1 e_3(t) + w_3(t)\big]\\
&+ M_{12}\big[r_3[e_4(t) - e_3(t) - e_2(t)] - k_3{}^2 e_3(t)\\
&\left.+ w_3(t)\big]\right\} + e_4(t)\left\{M_{11}\big[r_4[e_3(t) - e_4(t)\right.\\
&- e_5(t)] - k_4{}^1 e_4(t) + w_4(t)\big] + M_{12}\big[r_4[e_3(t)\\
&\left.- e_4(t) - e_5(t)] - k_4{}^2 e_4(t) + w_4(t)\big]\right\}\\
&+ e_5(t)\left\{M_{11}\big[r_5[e_4(t) - r_6 e_5(t)] - k_5{}^1 e_5(t) + w_5(t)\big]\right.\\
&\left.+ M_{12}\big[r_5[e_4(t) - r_6 e_5(t)] - k_5{}^2 e_5(t) + w_5(t)\big]\right\}\\
&+ e_6(t)\left\{M_{11}\big[e_1(t) - k_6{}^1 e_6(t) + w_6(t)\big]\right.\\
&\left.+ M_{12}\big[e_1(t) - k_6{}^2 e_6(t) + w_6(t)\big]\right\}\\
={}& -r_1(M_{11}a + M_{12}b) e_1{}^2(t) + (r_1 - r_2) e_1(t) e_2(t)\\
&+ r_2 r_7 e_2{}^2(t) + (r_2 - r_3) e_2(t) e_3(t) - r_3 e_3{}^2(t)\\
&+ (r_3 + r_4) e_3(t) e_4(t) - r_4 e_4{}^2(t)\\
&+ (r_5 - r_4) e_4(t) e_5(t) - r_5 r_6 e_5{}^2(t) + e_1(t) e_6(t)\\
&- \sum_{j=1}^{6}\left(M_{11} k_j{}^1 e_j{}^2(t) + M_{12} k_j{}^2 e_j{}^2(t)\right) + w^T(t) E(t)\\
\leq{}& -\left[r_1(M_{11}a + M_{12}b) - 2 + M_{11} k_1{}^1 + M_{12} k_1{}^2\right] e_1{}^2(t)\\
&- \left[e_1(t) - (r_1 - r_2) e_2 \frac{t}{2}\right]^2 - \left[-\frac{(r_1 - r_2)^2}{4}\right.\\
&\left.- 1 - r_2 r_7 + M_{11} k_2{}^1 + M_{12} k_2{}^2\right] e_2{}^2(t)\\
&- \left[e_2(t) - (r_2 - r_3) e_3 \frac{t}{2}\right]^2 - \left[-\frac{(r_2 - r_3)^2}{4} - 1 + r_3\right.\\
&\left.+ M_{11} k_3{}^1 + M_{12} k_3{}^2\right] e_3{}^2(t) - \left[e_3(t) - (r_3 + r_4) e_4 \frac{t}{2}\right]^2\\
&- \left[-\frac{(r_3 + r_4)^2}{4} - 1 + r_4 + M_{11} k_4{}^1 + M_{12} k_4{}^2\right] e_4{}^2(t)\\
&- \left[e_4(t) - (r_5 - r_4) e_5 \frac{t}{2}\right]^2 - \left[-\frac{(r_5 - r_4)^2}{4} + r_5 r_6\right.\\
&\left.+ M_{11} k_5{}^1 + M_{12} k_5{}^2\right] e_5{}^2(t) - \left[e_1(t) - e_6 \frac{t}{2}\right]^2\\
&- \left[-\frac{1}{4} + M_{11} k_6{}^1 + M_{12} k_6{}^2\right] e_6{}^2(t) + w^T(t) E(t).
\end{aligned}
\tag{25}
$$

With (19), one can conclude that

$$
\dot{V}(t) \leq -E^T(t) \sum_{i=1}^{2} \Theta_i K_i E(t) + w^T(t) E(t). \tag{26}
$$

Consider the $H_\infty$ performance index as

$$
\begin{aligned}
J &= \int_{t_0}^{t_T} \left[E^T(t) E(t) - \gamma^2 w^T(t) w(t)\right] dt\\
&= \int_{t_0}^{t_T} \left[E^T(t) E(t) - \gamma^2 w^T(t) w(t) + \dot{V}(t)\right] dt\\
&\quad + V(t_0) - V(t_T).
\end{aligned}
\tag{27}
$$

For $V(t_0) = 0$ and $V(t_T) \geq 0$,

$$
\begin{aligned}
J &\leq \int_{t_0}^{t_T} \left[E^T(t) E(t) - \gamma^2 w^T(t) w(t) + \dot{V}(t)\right] dt\\
&= \int_{t_0}^{t_T} \eta^T(t) \Omega \eta(t) dt,
\end{aligned}
\tag{28}
$$

where

$$
\begin{aligned}
\eta(t) &= \left[E^T(t), w^T(t)\right]^T,\\
\Omega &= \begin{bmatrix} I - \sum_{i=1}^{2} \Theta_i K_i & \frac{I}{2}\\ * & -\gamma^2 I \end{bmatrix}.
\end{aligned}
\tag{29}
$$

Consider (20), it can be concluded that $J \leq 0$. Based on Definition 1, slave system (12) can synchronize to master system (3) with $H_\infty$ norm bound $\gamma$.

Next, consider the design for the single-dimensional fuzzy synchronization controller.

Construct the slave system as

$$
\begin{aligned}
\dot{y}_1(t) &= r_1\left(y_2(t) - \bar{W}(y_6(t)) y_1(t)\right),\\
\dot{y}_2(t) &= r_2[y_3(t) - y_1(t) + r_7 y_2(t)] + w(t) + u(t),\\
\dot{y}_3(t) &= r_3[y_4(t) - y_3(t) - y_2(t)],\\
\dot{y}_3(t) &= r_4[y_3(t) - y_4(t) - y_5(t)],\\
\dot{y}_5(t) &= r_5[y_4(t) - r_6 y_5(t)],\\
\dot{y}_6(t) &= y_1(t),
\end{aligned}
\tag{30}
$$

where $u(t)$ is the single-dimensional synchronization fuzzy controller.

**Theorem 2.** *If there exist scalar $k^i > 0, i = 1, 2,$ design the single-dimensional fuzzy controller with following control regulation*

$$
u(t) = -\sum_{i=1}^{2} M_{1i} k_i e_2(t), \tag{31}
$$

*where*

$$k_i = \left(k^i + r_7\right) r_2$$
$$\cdot \begin{bmatrix} -(M_{11}a + M_{12}b) & 0 & 0 & 0 \\ 0 & 1 - \left(M_{11}k^1 + M_{12}k^2\right) & 0 & \dfrac{1}{2r_2} \\ 0 & 0 & -r_6 & 0 \\ 0 & \dfrac{1}{2r_2} & 0 & -\gamma^2 \end{bmatrix} \leq 0. \tag{32}$$

*Then, slave system (30) with any initial conditions can synchronize to master system (3) with $H_\infty$ norm bound $\gamma$.*

*Proof 2.* With (31), the error dynamic system can be transformed as

$$\begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \\ \dot{e}_3(t) \\ \dot{e}_4(t) \\ \dot{e}_5(t) \\ \dot{e}_6(t) \end{bmatrix} = \begin{bmatrix} M_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{11} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{11} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{11} \end{bmatrix}$$
$$\cdot \begin{bmatrix} r_1[e_2(t) - ae_1(t)] \\ r_2[e_3(t) - e_1(t) + r_7 e_2(t)] + w(t) - k^1 e_2(t) \\ r_3[e_4(t) - e_3(t) - e_2(t)] \\ r_4[e_3(t) - e_4(t) - e_5(t)] \\ r_5[e_4(t) - r_6 e_5(t)] \\ e_1(t) \end{bmatrix}$$
$$+ \begin{bmatrix} M_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & M_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{12} \end{bmatrix}$$
$$\cdot \begin{bmatrix} r_1[e_2(t) - be_1(t)] \\ r_2[e_3(t) - e_1(t) + r_7 e_2(t)] + w(t) - k^2 e_2(t) \\ r_3[e_4(t) - e_3(t) - e_2(t)] \\ r_4[e_3(t) - e_4(t) - e_5(t)] \\ r_5[e_4(t) - r_6 e_5(t)] \\ e_1(t) \end{bmatrix}. \tag{33}$$

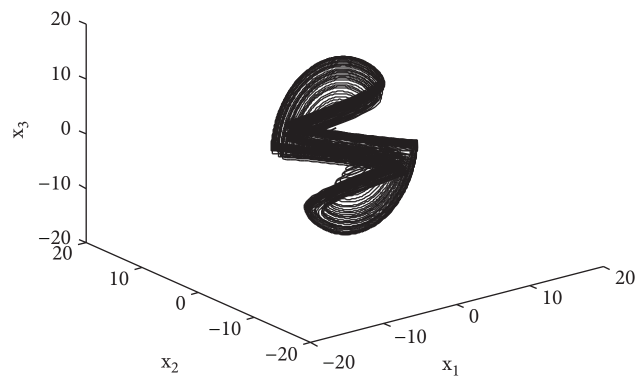


Figure 2: Attractor of the memristive chaotic system.

Choose the Lyapunov function candidate as

$$V_0(t) = \frac{1}{2r_1} e_1{}^2(t) + \frac{1}{2r_2} e_2{}^2(t) + \frac{1}{2r_3} e_3{}^2(t) + \frac{1}{2r_4} e_4{}^2(t) + \frac{1}{2r_5} e_5{}^2(t). \tag{34}$$

One can get the time derivative of $V_0(t)$ as

$$\begin{aligned} \dot{V}_0(t) &= \frac{1}{r_1} e_1(t)\dot{e}_1(t) + \frac{1}{r_2} e_2(t)\dot{e}_2(t) + \frac{1}{r_3} e_3(t)\dot{e}_3(t) \\ &\quad + \frac{1}{r_4} e_4(t)\dot{e}_4(t) + \frac{1}{r_5} e_5(t)\dot{e}_5(t) \\ &= e_1(t)\{M_{11}[e_2(t) - ae_1(t)] + M_{12}[e_2(t) - be_1(t)]\} \\ &\quad + e_2(t)\{M_{11}[e_3(t) - e_1(t) + r_7 e_2(t)] \\ &\quad + M_{12}[e_3(t) - e_1(t) + r_7 e_2(t)] + w(t)/r_2 + u(t)/r_2\} \\ &\quad + e_3(t)\{M_{11}[e_4(t) - e_3(t) - e_2(t)] \\ &\quad + M_{12}[e_4(t) - e_3(t) - e_2(t)]\} \\ &\quad + e_4(t)\{M_{11}[e_3(t) - e_4(t) - e_5(t)] \\ &\quad + M_{12}[e_3(t) - e_4(t) - e_5(t)]\} \\ &\quad + e_5(t)\{M_{11}[e_4(t) - r_6 e_5(t)] + M_{12}[e_4(t) - r_6 e_5(t)]\} \\ &= e_1(t)\{M_{11}[e_2(t) - ae_1(t)] + M_{12}[e_2(t) - be_1(t)]\} \\ &\quad + e_2(t)\{M_{11}\left[e_3(t) - e_1(t) - k^1 e_2(t)\right] \\ &\quad + M_{12}\left[e_3(t) - e_1(t) - k^2 e_2(t)\right]\} \\ &\quad + e_3(t)\{M_{11}[e_4(t) - e_3(t) - e_2(t)] \\ &\quad + M_{12}[e_4(t) - e_3(t) - e_2(t)]\} \\ &\quad + e_4(t)\{M_{11}[e_3(t) - e_4(t) - e_5(t)] \\ &\quad + M_{12}[e_3(t) - e_4(t) - e_5(t)]\} \\ &\quad + e_5(t)\{M_{11}[e_4(t) - r_6 e_5(t)] \\ &\quad + M_{12}[e_4(t) - r_6 e_5(t)]\} + e(t)_2 w(t)/r_2 \\ &= -(M_{11}a + M_{12}b)e_1{}^2(t) - \left(M_{11}k^1 + M_{12}k^2\right)e_2{}^2(t) \\ &\quad - (e_3(t) - e_4(t))^2 - r_6 e_5{}^2(t) + e_2(t)w(t)/r_2. \end{aligned} \tag{35}$$

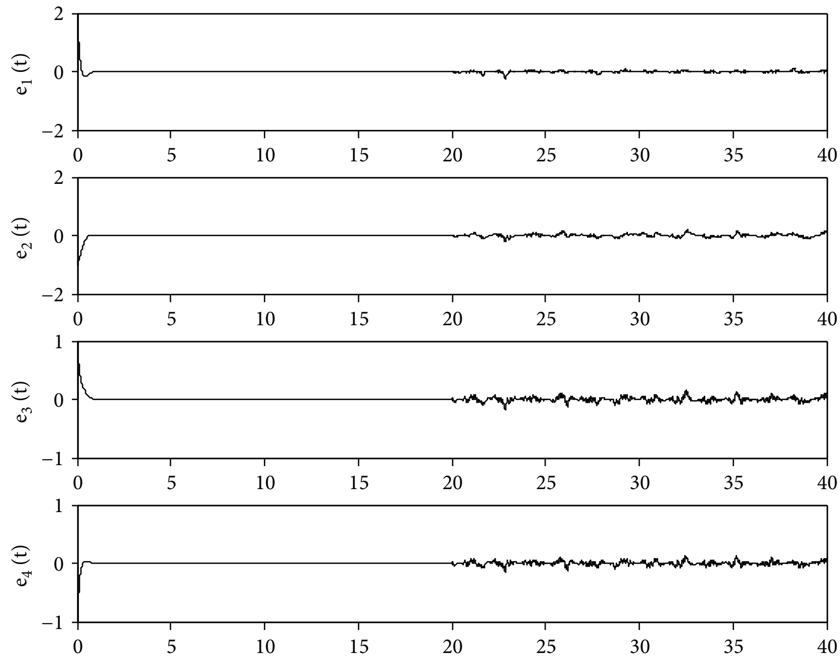


Figure 3: Time response of synchronization error variables with multidimensional fuzzy controller.

With (31), one can conclude that

$$\dot{V}_0(t) \leq -\bar{E}^T(t)\sum_{i=1}^{2}\overline{\Theta}_i\bar{K}_i\bar{E}(t) + \frac{1}{r_2}w(t)e_2(t), \tag{36}$$

where

$$\begin{aligned}\bar{K}_1 &= \operatorname{diag}\left\{a, k^1, r_6\right\} \geq 0,\\ \bar{K}_2 &= \operatorname{diag}\left\{b, k^2, r_6\right\} \geq 0,\\ \bar{E} &= \left[e_1(t), e_2(t), e_5(t)\right]^T,\\ \overline{\Theta}_i &= \operatorname{diag}\left\{M_{1i}, M_{1i}, M_{1i}\right\}.\end{aligned} \tag{37}$$

Consider the $H_\infty$ performance index as

$$\begin{aligned}J &= \int_{t_0}^{t_T}\left[e_2{}^2(t) - \gamma^2 w^T(t)w(t)\right]dt\\ &= \int_{t_0}^{t_T}\left[e_2{}^2(t) - \gamma^2 w^T(t)w(t) + \dot{V}_0(t)\right]dt\\ &\quad + V(t_0) - V(t_T).\end{aligned} \tag{38}$$

For $V(t_0) = 0$ and $V(t_T) \geq 0$,

$$J \leq \int_{t_0}^{t_T}\left[e_2{}^2(t) - \gamma^2 w^T(t)w(t) + \dot{V}_0(t)\right]dt = \int_{t_0}^{t_T}\eta^T(t)\Omega\eta(t)dt, \tag{39}$$

where

$$\eta(t) = \left[\bar{E}^T(t), w^T(t)\right]^T,$$
$$\Omega = \begin{bmatrix} -(M_{11}a + M_{12}b) & 0 & 0 & 0\\ 0 & 1-\left(M_{11}k^1 + M_{12}k^2\right) & 0 & \dfrac{1}{2r_2}\\ 0 & 0 & -r_6 & 0\\ 0 & \dfrac{1}{2r_2} & 0 & -\gamma^2\end{bmatrix}. \tag{40}$$

Consider (32), it can be concluded that $J \leq 0$. Based on Definition 1, slave system (30) can synchronize to master system (3) with $H_\infty$ norm bound $\gamma$.

## 4. Example and Simulation

First, consider the dynamics of the memristive chaotic system, and the simulation result is shown in Figure 2.

Next, we study the synchronization control of the master-slave systems. In the simulation, the system initial values are $y(0) = [1, -1, 0.5, -1, 2, -1]^T$ and $x(0) = 0.001 \times [1, 1, 1, 1, 1, 1]^T$. The disturbance input is $w(t) = w_i(t) = 2\sin(2t)\sin\left(e^t/(t+1)\right)$, $t \geq 20$s. Let $\gamma = 0.4$, and based on Theorem 1, the control parameters for the multidimensional fuzzy controller are $k_1 = \operatorname{diag}\{4.06, 7.41, 1.56, 8.56, 2.21, 2.81\}$ and $k_2 = \operatorname{diag}\{-35.43, 7.41, 1.56, 8.56, 2.21, 2.81\}$; the simulation result is shown in Figure 3. Then, based on Theorem 2, the control parameters for the single-dimensional fuzzy controller are $k_1 = 9.73$ and $k_2 = 7.38$; the simulation result is shown in Figure 4.

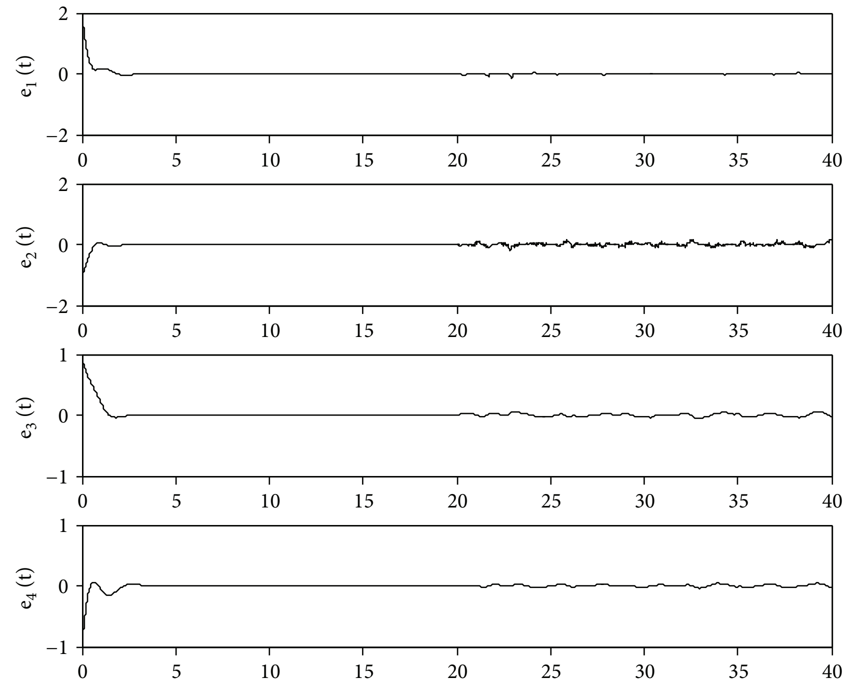


Figure 4: Time response of synchronization error variables with single-dimensional fuzzy controller.

*Remark 2.* Figure 3 depicts the time response of the synchronization error variables of the memristive master-slave systems with the multidimensional fuzzy controller. Figure 4 depicts the time response of the synchronization error variables of the master-slave systems with the single-dimensional fuzzy controller. It can be seen that although there exists just one disturbance $w(t)$ for single-dimensional fuzzy control, the disturbance has impact on all error synchronization variables. In addition, it can be seen that both controllers can be able to realize the synchronization of the master-slave systems; the multidimensional fuzzy controller has the better control performance and realizes the chaos synchronization during 2.0 seconds. However, the possession of the good control performance is at the cost of the acquirement of all system state information. In addition, multidimensional control may introduce more disturbance input. The single-dimensional synchronization controller has the general control performance but requires just one system state information, which can decrease the control cost and the disturbance input. Hence, two kinds of controllers are useful and recommended for the different applied cases.

*Remark 3.* For the nonlinear disturbed chaotic system, the fuzzy modeling technique is adopted to realize the exact linearization control, which can eliminate the constraint on the system nonlinear term, compared with the general nonlinear control method; in addition, H-infinity approach is introduced to deal with the case that disturbances exist.

## 5. Conclusion

This paper focuses on the fuzzy synchronization for a new memristive chaotic system with disturbances. Based on fuzzy theory and Lyapunov stability theory, we have built the fuzzy model for the memristive chaotic system. Then, by using H-infinity technique, we have presented two kinds of fuzzy controllers for the possible application in chaos synchronization of slave-master systems. Finally, we have included some example to demonstrate the effectiveness of the given fuzzy controllers. In addition, the proposed results can be extended to the memristive chaotic control system with daelay or event trigger, which is our future work.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this paper.

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