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Research Article

Modelling and Qualitative Analysis of Water Hyacinth Ecological System with Two State-Dependent Impulse Controls

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Water hyacinth and its ecological invasion have negative impacts on diversity of indigenous species and ecosystems, which becomes one of the hotspots in current ecological research. In this paper, a water hyacinth ecological system with two state-dependent impulse controls is studied. Firstly, we define the successor functions of semicontinuous dynamic system and give existence theorems of order-1 periodic solution and order-2 periodic solution of such system. Secondly, we analyze singular points of the system without impulsive state feedback control qualitatively and get the condition for focus point. Thirdly, we obtain the sufficient condition under which the system has an order-1 or order-2 periodic solution through the method of successor function and prove the stability of the order-1 or order-2 periodic solution by the analogue of Poincaré's criterion. Furthermore, some examples and numerical simulations are given to illustrate our results.

1. Introduction

The water hyacinth, a water plant with a showy purple flower, is a native of the Amazon Basin and is now treated as the most important nuisance aquatic plant worldwide. Rapid spread of water hyacinth causes serious environmental problems, which is recognized throughout the world. Water hyacinth, known as one of the top ten creatures into the enroach on grass, has spread by waters in the south of China in recent years, becoming a great ecological and social harm. The expansive water hyacinth sheets have blocked shipping lanes, prevented commercial ships from entering ports and local fishing vessels from leaving the shore, and interfered with the intake of water by industries. Meanwhile, the invasive weed has created anoxic conditions in the lake, thus raising toxicity and disease levels. The resulting stagnant water is an ideal breeding ground for malarial mosquitoes and schistosomiasis and a detriment for the functioning of fishing industries. As for the ecological harm, biological control, development, and utilization of water hyacinth, see [1–5] and the reference cited therein.

Nowadays, biological control measures are widely used to control water hyacinth. Biological control is to introduce natural enemies from the origin, establish populations, and control water hyacinth in the long term. The biological control of water hyacinth has the advantages of long-lasting prevention, low cost, and safe environment.

Recently, impulsive state feedback control differential equations have aroused scholars' enormous interest (see [6–20]). Chen et al. [6] and Chen [7] starting from a practical problem of the pest control established a class of pest prevention model with impulsive feedback control. The geometric theory of semicontinuous dynamical systems is presented, and by applying this theory, it is proved for this model to have at least one order-one periodic solution. Nie et al. [16–18] and Tian et al. [19] put forward a class of predator-prey models with state-dependent impulsive effects. By using Poincaré map and Lambert W function, the criteria for the existence and stability of a semitrivial solution and positive periodic solution of system are obtained. Zhao et al. [20] using successor functions and Poincaré-Bendixson theorem of impulsive differential equations studied the existence of periodical solutions to a predator-prey model with two state impulses. By stability theorem of periodic solution to impulsive differential equations, the stability conditions of periodic solutions to the system are obtained. As a continuation, a water hyacinth ecological system with Kuznets curve effect and two state-dependent impulse controls is put forward and numerical simulations are carried out based on the qualitative analysis on the proposed model.

This paper is organized as follows. In Section 2, a water hyacinth ecological system with Kuznets curve effect and two state-dependent impulses control is put forward, and some basic definitions are given. In Section 3, we first discuss the qualitative analysis of system (3) and then discuss the existence and stability of periodic solutions. In Section 4, we analyze our theoretical results by numerical simulations and give a brief discussion. Finally, conclusions are presented in Section 5.

2. Model Formulation and Preliminaries

2.1. Model Formulation. In ecosystems, water hyacinth can be farmed with fish. Water hyacinth can not only serve as the diet of fish but can also purify the water quality, which controls the pond water quality well. But there are also disadvantages. Too much water hyacinth, covering the water surface, will reduce the activity space of the fish. Most importantly, it consumes the oxygen in the water and prevents the entry of oxygen in the air; the fish are suffocated, causing the extinction of fish populations in water ecological system. Considering the water hyacinth as feed in fish farms and its characteristic of the ecological damage caused by its rapid spread, inspired by [21–24], we establish the water hyacinth ecological system with Kuznets curve effect.

$$\begin{cases} \dfrac{dx}{dt} = rx\left(1 - \dfrac{x}{K}\right) - \beta xy, \\ \dfrac{dy}{dt} = -dy + \delta(m - x)xy, \end{cases} \tag{1}$$

where r, K, β, d, δ, and m are all constants; $x(t)$ and $y(t)$ denote the water hyacinth (prey) and fish (predator) population, respectively, at any time t; r is the intrinsic growth rate of water hyacinth; K is the carrying capacity of water hyacinth; β is the coefficient of interspecific competition; d denotes the death rate of fish; and $\delta x(m - x)$ is the Kuznets curve effect function (see Figures 1–3). When the population of water hyacinth is less than m, it is conducive to the growth of fish; when the population of water hyacinth is greater than m, it will hinder the growth of fish.

The rapidly propagating water hyacinth will cover the water surface, causing intense intraspecific competition, leading to death of rot, pollution of water body, and aggravate eutrophication of water body, thus hindering the growth of other aquatic creatures and causing the imbalance of the ecological chain, irreversible damage to the ecosystem, loss of biodiversity, and frequent ecological disasters. The key to solving water hyacinth problem is to develop and make proper use of water hyacinth to avoid harm. At present, many places are trying to develop and utilize water hyacinth, using it as feed for livestock, poultry, grass carp, and so on, so as to turn harm into profit.

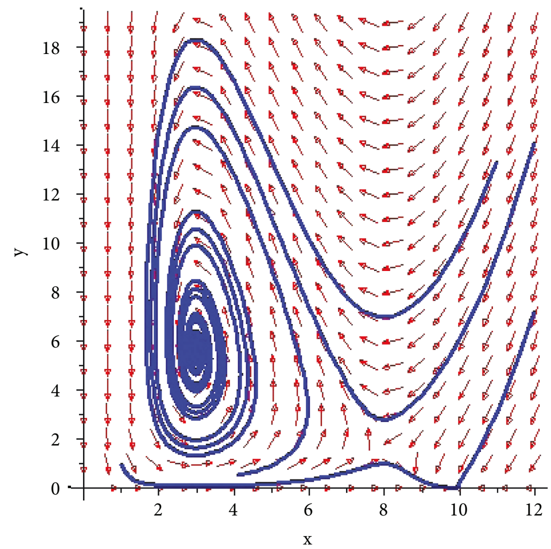

Figure 1: The vector diagram of system (8) where $r = 4$, $a = 0.4$, $b = 0.5$, $m_1 = 3$, and $m_2 = 8$.

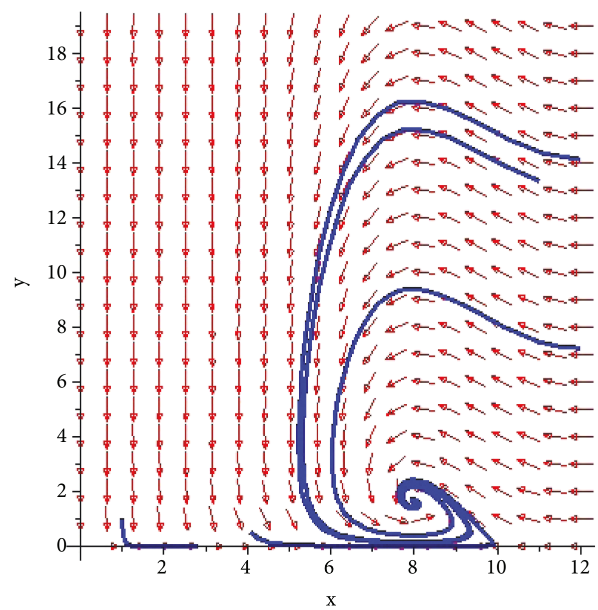

Figure 2: The vector diagram of system (8) where $r = 4$, $a = 0.4$, $b = 0.5$, $m_1 = 8$, and $m_2 = 12$.

According to the characteristics of water hyacinth ecosystem, the population size of water hyacinth is set at a controllable economic threshold (ET). When the population size of water hyacinth is less than ET_1, it will be detrimental to the growth of fish population. We consider using the methods of manually releasing water hyacinth and fishing fish population to control the ecological balance. When the population size of water hyacinth is greater than ET_2, it will lead to the extinction of fish population. We consider using the methods of manually fishing water hyacinth and releasing fish population to control the ecological balance.

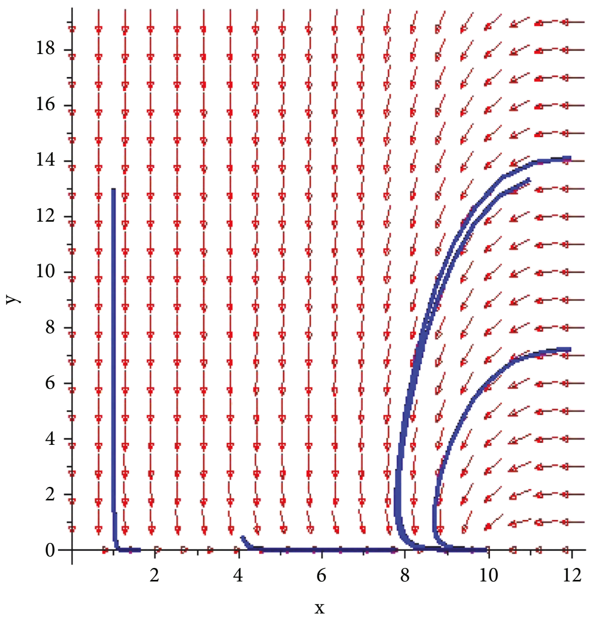

Figure 3: The vector diagram of system (8) where $r = 4$, $a = 0.4$, $b = 0.5$, $m_1 = 12$, and $m_2 = 14$.

Now, we take the integrated control tactics into account for water hyacinth ecological system. Once the density of the water hyacinth population reaches the economic threshold (ET), we introduce impulsive state feedback control. Thus, we obtain the following system:

$$\begin{cases} \dfrac{dx}{dt} = rx\left(1 - \dfrac{x}{K}\right) - \beta xy, \\ \dfrac{dy}{dt} = -dy + \delta(m - x)xy, \\ \quad h_1 < x < h_2, \end{cases}$$
$$\begin{cases} \triangle x = (1 - q_1)h_2 - x, \\ \triangle y = -p_1 y, \\ \quad x = h_1, y > \dfrac{r}{\beta}\left(1 - \dfrac{h_1}{K}\right), \end{cases}$$
$$\begin{cases} \triangle x = -q_1 x, \\ \triangle y = \tau, \\ \quad x = h_2, y < \dfrac{r}{\beta}\left(1 - \dfrac{h_2}{K}\right), \end{cases} \tag{2}$$
$$\begin{aligned} &\Delta x(t) = x(t^+) - x(t), \\ &\Delta y(t) = y(t^+) - y(t), \\ &\quad h_1 < x(0) = x_0 < h_2, \\ &y(0) = y_0 > 0. \end{aligned}$$

This model describes that in productive practice for controlling water hyacinth, people always take such a strategy that when the water hyacinth arrives at a given ET h_1 or h_2, they began to use pulses to control the population of water hyacinths. ET h_1 denotes the economic value ET_1 with a small population of water hyacinth, and h_2 denotes the economic value ET_2 with a large population of water hyacinth ($h_1 < h_2$). When the population of water hyacinth is equal to h_1 and the population of fish is greater than $(r/\beta)(1 - (h_1/K))$, the method of impulse control is adopted to obtain economic benefits by harvesting fish, and meanwhile to increase the prey of fish, promoting the population of fish by releasing water hyacinth. $0 < p_1 < 1$ represents the harvest coefficient of fish, and $(1 - q_1)h_2 - x$ represents the quantity of water hyacinth. When the population of water hyacinth is equal to h_2 and the population of fish is less than $(r/\beta)(1 - (h_2/K))$, the mass reproduction of water hyacinth is likely to lead to the extinction of fish. At this time, the method of impulse control is adopted to reduce the population of water hyacinth by fishing it and at the same time to increase the population of fish by releasing fish. $0 < q_1 < 1$ represents the harvest coefficient of water hyacinth, and $\tau > 0$ is the release amount of fish.

Considering the biology background of system (2), we all assume that $(x(0), y(0)) \in \{h_1 < x(0) < h_2, y(0) > 0\}$ and we only discuss system (2) in the region $R_+^2 = \{(x, y) \mid x \geq 0, y \geq 0\}$.

For system (2), if $H_1 : m^2 > (4d/\delta)$, we change it to $\bar{x} = x$, $\bar{y} = \delta y$, $a = (r/K)$, $b = (\beta/\delta)$, $m_1 = ((m - \sqrt{m^2 - (4d/\delta)})/2)$, and $m_2 = ((m + \sqrt{m^2 - (4d/\delta)})/2)$. So we can derive equivalent system (3) from system (2) (let new variables be x, y, and t).

$$\begin{cases} \dfrac{dx}{dt} = x(r - ax - by), \\ \dfrac{dy}{dt} = y(x - m_1)(m_2 - x), \\ \quad h_1 < x < h_2, \end{cases}$$
$$\begin{cases} \triangle x = (1 - q_1)h_2 - x, \\ \triangle y = -p_1 \delta y, \\ \quad x = h_1, y > \dfrac{1}{b}(r - ah_1), \end{cases} \tag{3}$$
$$\begin{cases} \triangle x = -q_1 x, \\ \triangle y = \tau, \\ \quad x = h_2, y < \dfrac{1}{b}(r - ah_2), \end{cases}$$
$$\begin{aligned} &\quad h_1 < x(0) = x_0 < h_2, \\ &y(0) = y_0 > 0. \end{aligned}$$

On the basis of the ecological significance, system (3) should meet $r > am_2$, $m_1 < h_1 < m_2 < h_2$, and $(1 - q_1)h_2 > h_1$. We can get the relationship between the phase set and isocline as shown in the following figures (see Figures 4–6).

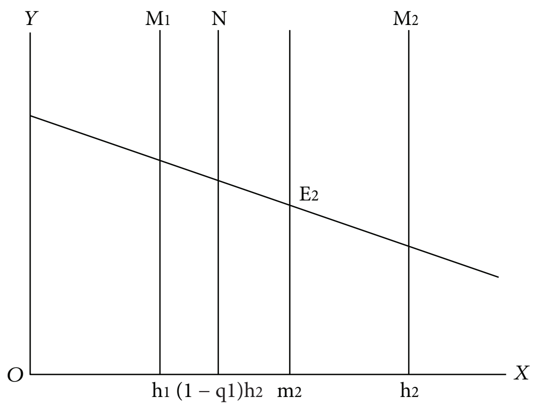

Figure 4: The case of $h_1 < (1 - q_1)h_2 < m_2 < h_2$ of system (3).

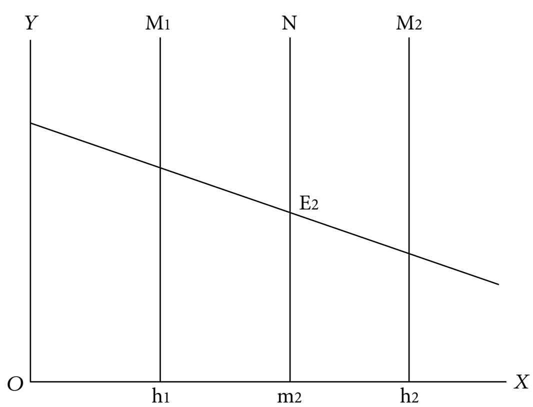

Figure 5: The case of $h_1 < (1 - q_1)h_2 = m_2 < h_2$ of system (3).

We first discuss the same position, i.e., $(1 - q_1)h_2 = m_2$ (see Figure 5); we change $p = p_1\delta$ and $q_1 = 1 - (m_2/h_2)$. So we can derive equivalent system (4) from system (3).

$$\begin{cases} \dfrac{dx}{dt} = x(r - ax - by), \\ \dfrac{dy}{dt} = y(x - m_1)(m_2 - x), \\ \quad h_1 < x < h_2, \end{cases}$$
$$\begin{cases} \triangle x = m_2 - h_1, \\ \triangle y = -py, \\ \quad x = h_1, y > \dfrac{1}{b}(r - ah_1), \end{cases} \tag{4}$$
$$\begin{cases} \triangle x = m_2 - h_2, \\ \triangle y = \tau, \\ \quad x = h_2, y < \dfrac{1}{b}(r - ah_2), \end{cases}$$
$$h_1 < x(0) = x_0 < h_2,$$
$$y(0) = y_0 > 0.$$

2.2. Preliminaries. In this section, we first introduce some definitions and lemmas of the geometric theory of the

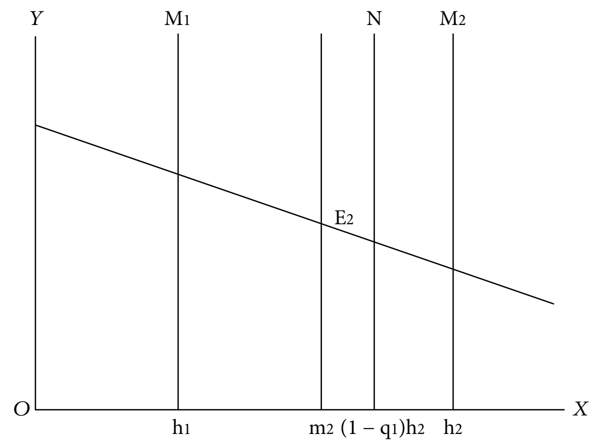

Figure 6: The case of $h_1 < m_2 < (1 - q_1)h_2 < h_2$ of system (3).

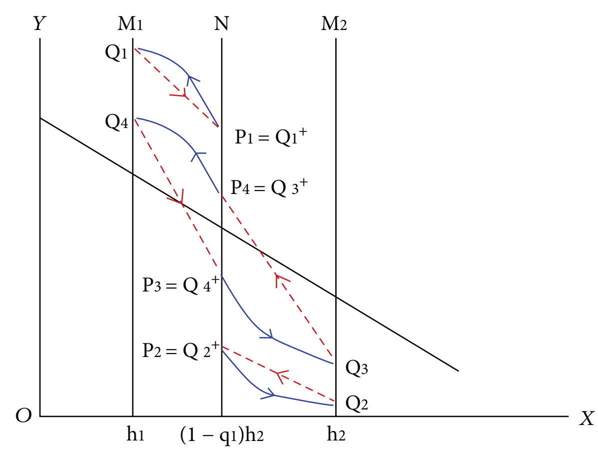

Figure 7: Define the periodic solution of system (3).

semicontinuous dynamic system, which will be useful for the latter discussion. And for more details, we can see [6–15] and the references cited therein.

Definition 1. A triple (X, Π, R^+) is said to be a semidynamical system if X is a metric space, R^+ is the set of all nonnegative real, and $\Pi(P, t)$: $X \times R^+ \to X$ is a continuous map such that (1) $\Pi(P, 0) = P$ for all $P \in X$; (2) $\Pi(P, t)$ is continuous for t and s; and (3) $\Pi(\Pi(P, s), t) = \Pi(P, s + t)$ for all $P \in X$ and s, $t \in R^+$. Sometimes, a semidynamical system (X, Π, R^+) is denoted by (X, Π).

Definition 2. Define the impulsive set $M_1 = \{(x, y) \in R_+^2 \mid x = h_1, y > ((r - ah_1)/b)\}$ and $M_2 = \{(x, y) \in R_+^2 \mid x = h_2, 0 < y < ((r - ah_2)/b)\}$; let $M = M_1 \cup M_2$; M is a closed subset. Define the continuous functions $I_1 : (h_1, y) \in M_1 \to (x^+, y^+) = ((1 - q_1)h_2, (1 - p)y) \in R_+^2$ and $I_2 : (h_2, y) \in M_1 \to (x^+, y^+) = ((1 - q_1)h_2, y + \tau) \in R_+^2$; thus, the phase set N can be defined $N = I_1(M_1) \cup I_2(M_2) = \{(x^+, y^+) \in R_+^2 \mid x^+ = (1 - q_1) h_2, y^+ > 0\}$. Then, (Ω, Π, M, I) is called an impulsive semidynamic system.

Definition 3. Define the periodic solution of the system (3) (see Figure 7):

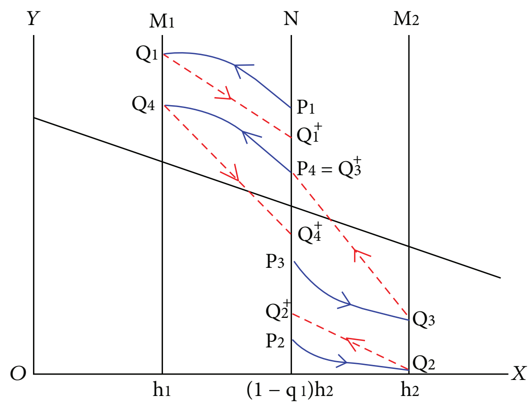

FIGURE 8: Successor function of system (3).

(1) When there is a point P_1 at phase set N and a T_1 such that $\Pi(P_1, T_1) = Q_1 \in M_1$ and $Q_1^+ = I_1(Q_1) = P_1 \in N$, then $\Pi(P_1, T_1)$ is called order-1 periodic solution

(2) When there is a point P_2 at phase set N and a T_2 such that $\Pi(P_2, T_2) = Q_2 \in M_2$ and $Q_2^+ = I_2(Q_2) = P_2 \in N$, then $\Pi(P_2, T_2)$ is called order-1 periodic solution

(3) When there is a point P_3 at phase set N and a T_3 such that $\Pi(P_3, T_3) = Q_3 \in M_2$ and $Q_3^+ = I_2(Q_3) = P_4 \in N$, there exists a T_4 such that $\Pi(P_4, T_4) = Q_4 \in M_1$ and $Q_4^+ = I_1(Q_4) = P_3 \in N$, then $\Pi(P_3, T_3) + \Pi(P_4, T_4)$ is called order-2 periodic solution

Next, we will give the definition of the successor function of system (3). Let L be a coordinate axis defined at N; the origin point is the intersection point of line $L : x = (1 - q_1)h_2$ with axis $X : y = 0$; the positive direction is consistent with the positive direction of axis $Y : x = 0$, then we obtain a number axis d. For any $x \in d$, let $d(x) \in C^0$ be the coordinate of point x.

Definition 4 (see Figure 8).

(1) When there is a point P_1 at phase set N and a T_1 such that $\Pi(P_1, T_1) = Q_1 \in M_1$ and $Q_1^+ = I_1(Q_1) \in N$, then $f(P_1) = d(Q_1^+) - d(P_1)$ is called the successor function of the point P_1

(2) When there is a point P_2 at phase set N and a T_2 such that $\Pi(P_2, T_2) = Q_2 \in M_2$ and $Q_2^+ = I_2(Q_2) \in N$, then $f(P_2) = d(Q_2^+) - d(P_2)$ is called the successor function of the point P_2

(3) When there is a point P_3 at phase set N and a T_3 such that $\Pi(P_3, T_3) = Q_3 \in M_2$ and $Q_3^+ = I_2(Q_3) = P_4 \in N$, there exists a T_4 such that $\Pi(P_4, T_4) = Q_4 \in M_1$ and $Q_4^+ = I_1(Q_4) \in N$, then $g(P_3) = d(Q_4^+) - d(P_3)$ is called the successor function of the point P_3

Remark 1 (see Figure 7).

(1) If the successor function $f(P) = 0$, the trajectory $\Pi(P, T)$ with initial point P is an order-1 periodic solution of system (3)

(2) If the successor function $g(P) = 0$, the trajectory $\Pi(P, T)$ with initial point P is an order-2 periodic solution of system (3)

Lemma 1. *The successor function $f(P)$ and $g(P)$ is continuous.*

Lemma 2.

(1) Let the continuous dynamical system be (Π, X); if there are two points $P_1 \in N$ and $P_2 \in N$ at the phase set, they make $f(P_1)f(P_2) < 0$, then there must exist a point P between P_1 and P_2 so that $f(P) = 0$; thus, there must exist an order-1 periodic solution by point P

(2) Let the continuous dynamical system be (Π, X); if there are two points $P_1 \in N$ and $P_2 \in N$ at the phase set, they make $g(P_1)g(P_2) < 0$, then there must exist a point P between P_1 and P_2 so that $g(P) = 0$; thus, there must exist an order-2 periodic solution by point P

Lemma 3 (analogue of Poincaré's criterion) [25]. *The T-periodic solutions $x = \xi(t)$ and $y = \eta(t)$ of the system*

$$\begin{cases} \dfrac{dx(t)}{dt} = P(x, y), \\ \dfrac{dy(t)}{dt} = Q(x, y), \\ \quad \text{if } \varphi(x, y) \notin M, \end{cases} \qquad (5)$$
$$\begin{cases} \Delta x = \alpha(x, y), \\ \Delta y = \beta(x, y), \\ \quad \text{if } \varphi(x, y) \in M \end{cases}$$

are orbitally asymptotically stable if the Floquet multiplier μ_2 satisfies the condition $|\mu_2| < 1$, where

$$\mu_2 = \prod_{k=1}^{q} \Delta_k \exp\left(\int_0^T \left(\frac{\partial P}{\partial x}(\xi(t), \eta(t)) + \frac{\partial Q}{\partial x}(\xi(t), \eta(t))\right) dt\right), \qquad (6)$$

with

$$\Delta_k = \frac{P_+(\partial\beta/\partial y(\partial\varphi/\partial x) - \partial\beta/\partial x(\partial\varphi/\partial y) + (\partial\varphi/\partial x)) + Q_+(\partial\alpha/\partial x(\partial\varphi/\partial y) - \partial\alpha/\partial y(\partial\varphi/\partial x) + (\partial\varphi/\partial y))}{P(\partial\varphi/\partial x) + Q(\partial\varphi/\partial y)}, \qquad (7)$$

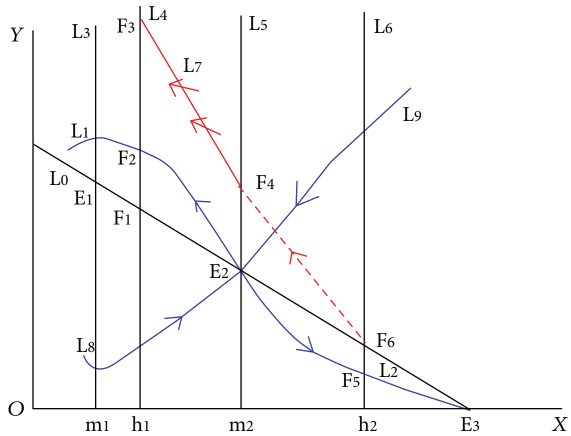

FIGURE 9: Structure diagram in R_2^+ of system (4).

and P, Q, $(\partial\alpha)/(\partial x)$, $(\partial\alpha)/(\partial y)$, $(\partial\beta)/(\partial x)$, $(\partial\beta)/(\partial y)$, $(\partial\varphi)/(\partial x)$, and $(\partial\phi)/(\partial y)$ are calculated at the point $(\xi(\tau_k),\eta(\tau_k))$, and $P_+ = P(\xi(\tau_k^+),\eta(\tau_k^+))$, $Q_+ = Q(\xi(\tau_k^+),\eta(\tau_k^+))$, and $\varphi(x,y)$ are a sufficiently smooth function with grad $\varphi(x,y) \neq 0$, and τ_k $(k \in N)$ are the moments for the jumps.

For the convenience of discussion, we need to define the following (see Figure 9).

$L_0 : r - ax - by = 0((dx/dt) = 0)$, $L_3 : x - m_1 = 0((dy/dt) = 0)$, $L_5 : x - m_2 = 0((dy/dt) = 0)$, $L : x - (1 - q_1)h_2 = 0$, $L_4 : x - h_1 = 0$, and $L_6 : x - h_2 = 0$.

The intersection point of L_0 and L_4 is denoted by $F_1(x_{F_1}, y_{F_1}) = F_1(h_1, (r - ah_1)/b)$.

The intersection point of L_0 and L_6 is denoted by $F_6(x_{F_6}, y_{F_6}) = F_6(h_2, (r - ah_2)/b)$. The phase point of $F_6(x_{F_6}, y_{F_6})$ at the phase set L_5 is denoted by $F_4(x_{F_4}, y_{F_4}) = F_4(m_2, ((r - ah_2)/b) + \tau)$. If $\tau > ((r - am_2)/b)$ of system (4), we define a function $W(x, y) = y + Kx - (Km_2 + ((r - ah_2)/b) + \tau)$, where $m_1 \le x \le m_2$. We have $(dW/dt)|_{W=0} = -K^2 x(m_2 - x) + K((x - m_1)(m_2 - x)^2 + x(a(h_2 - x) - b\tau)) + (((r - ah_2)/b) + \tau)(x - m_1)(m_2 - x)$. We can choose K enough large, such that $(dW/dt)|_{W=0} < 0$, so the straight line $L_7 : W = 0$ is nontangent. The intersection point of L_4 and L_7 is denoted by $F_3(x_{F_3}, y_{F_3}) = F_3(h_1, K(m_2 - h_1) + ((r - ah_2)/b) + \tau)$.

Assume that L_1, L_2, L_8, and L_9 are dividing lines of the saddle point E_2, where L_8 and L_9 are the stable flows of E_2 and L_1 and L_2 are the unstable flows of E_2. The intersection point of L_1 and L_4 is denoted by $F_2(x_{F_2}, y_{F_2})$. The intersection point of L_2 and L_6 is denoted by $F_5(x_{F_5}, y_{F_5})$.

3. Model Analysis

On the basis of the ecological significance, system (3) should meet $r > am_2$, $m_1 < h_1 < m_2 < h_2$, and $(1 - q_1)h_2 > h_1$. In this section, we will first study the qualitative characteristic of system (3) without the impulsive effect. Secondly, if $(1 - q_1)h_2 = m_2$, the sufficient conditions for the existence and stability of order-1 positive periodic solutions are analyzed. Thirdly, if $(1 - q_1)h_2 = m_2$, the sufficient conditions for the existence and stability of order-2 positive periodic solution are analyzed. Finally, if $(1 - q_1)h_2 \neq m_2$, the sufficient conditions for the existence of order-k $(k = 1, 2)$ positive periodic solution are analyzed.

3.1. Simple Qualitative Analysis of System (3) without Impulsive Effect. From system (3) without the impulsive effect, we obtain the following system:

$$\begin{cases} \dfrac{dx}{dt} = x(r - ax - by) = P(x, y), \\ \dfrac{dy}{dt} = y(x - m_1)(m_2 - x) = Q(x, y). \end{cases} \tag{8}$$

By $x(r - ax - by) = 0$ and $y(x - m_1)(m_2 - x) = 0$, we can obtain two boundary equilibriums $E_0(0, 0)$ and $E_3((r/a), 0)$ in system (8).

We can see that (i) there is no positive equilibrium if $r < am_1$; (ii) there is one positive equilibrium $E_1(m_1, (r - am_1)/b)$ if $am_1 < r < am_2$; and (iii) there are two positive equilibriums $E_1(m_1, (r - am_1)/b)$ and $E_2(m_2, (r - am_2)/b)$ if $r > am_2$.

The Jacobian matrix at equilibrium $E(x, y)$ is given by

$$J(E) = \begin{pmatrix} r - 2ax - by & -bx \\ y((m_1 + m_2) - 2x) & (x - m_1)(m_2 - x) \end{pmatrix}. \tag{9}$$

Theorem 1. *$E_0(0, 0)$ is the saddle point; if $am_1 < r < am_2$ holds, then $E_3(r/a, 0)$ is the saddle point; if $r < am_1$ or $r > am_2$ holds, then $E_3((r/a), 0)$ is the stable node point.*

Theorem 2. *$E_1(m_1, (r - am_1)/b)$ is the stable node point, if $r > am_1$; $E_2(m_2, (r - am_2)/b)$ is the saddle point, if $r > am_2$.*

Theorem 3. *There exists no limit cycle in system (8) in Ω.*

Proof 1. Let the straight line $l_1 : x - (r/a) - \varepsilon = 0$ $(\varepsilon > 0)$; we have $(dl_1/dt) = (dx/dt)|_{x=(r/a)+\varepsilon} = -((r/a) - \varepsilon)(a\varepsilon - by) < 0$, then the straight line segment is nontangent, and the path line of system (8) thereon is always to the left direction. Define a function $F(x, y) = (m_1 + m_2)x + by - M$, where $0 \le x \le (r/a) + \varepsilon$. We have $(dF/dt)|_{F=0} = (m_1 + m_2)(dx/dt) + b(dy/dt) \le -a(m_1 + m_2)(x - ((r + m_1 m_2)/2a))^2 + (((m_1 + m_2)(r + m_1 m_2)^2)/4a) - m_1 m_2 M$. We can choose M enough large, that is $M > \max\{((m_1 + m_2)(r + m_1 m_2)^2)/4am_1 m_2, (m_1 + m_2)((r/a) + \varepsilon)\}$ such that $(dF/dt) < 0$, so the straight line $l_2 : F = 0$ is nontangent, the trajectory of system (8) from the upper right of l_2 through l_2 into the lower left. The straight line l_2 intersects axis y and l_1 at points A and B, respectively.

For system (8), we construct a Bendixson ring $O\widehat{ABC}O$ including E_i $(i = 1,2,3,4)$. Define $\overline{OA}$, $\overline{AB}$, $\overline{BC}$, and $\overline{CO}$ as the length of line $l_3 : x = 0$, l_2, l_1, and $l_4 : y = 0$, respectively. Since $\overline{OA}$ and $\overline{CO}$ are the length of orbit of system (8), the orbits of system (8) go through into the interior of the Bendixson ring from the outer of $\overline{AB}$ and $\overline{BC}$. Hence, we know that system (8) is uniformly bounded.

Let the Dulac function $D(x, y) = x^{-1}y^{-1}$; we have $((\partial(PD))/\partial x) + ((\partial(QD))/\partial x) = -ay^{-1} < 0$; by the Bendixson-

Dulac theory, system (8) does not exist a limit cycle in $\Omega = \{(x, y) \mid 0 \le x \le (r/a) + \varepsilon, 0 < (m_1 + m_2)x + by < M\}$. The proof is completed.

The illustration of vector fields of system (8) can be seen in Figures 1–3.

3.2. Existence and Stability of Order-1 Periodic Solution of System (4)

Theorem 4. *If $0 < p < 1 - ((r - am_2)/by_{F_2})$ and $\tau > ((r - am_2)/b)$, there exists an order-1 periodic solution in system (4).*

Proof 2. If $(1-p)y_{F_2} > y_{E_2}$ (see Figure 10), there exists a point $B_1(m_2, y_{B_1}) \in N$, satisfying $0 < y_{B_1} - y_{E_2} < (1-p)y_{F_2} - y_{E_2}$. The trajectory of system (4) Γ_1 over B_1 with $x = h_1$ at point C_1, which is pulsed to $x = m_2$ and the phase point, is $B_2(m_2, y_{B_2})$, so $f(B_1) = y_{B_2} - y_{B_1} > 0$.

There exists another point $B_3(m_2, y_{B_3}) \in N$, satisfying $y_{B_3} \gg 0$. The trajectory of system (4) Γ_2 over B_3 with $x = h_1$ at point C_2, which is pulsed to $x = m_2$ and the phase point, is $B_4(m_2, y_{B_4})$, so $f(B_3) = y_{B_4} - y_{B_3} < 0$.

Therefore, there must exist a point $B(m_2, y_B) \in N$, which satisfies $y_{B_1} < y_B < y_{B_3}$, so that $f(B) = 0$. By Lemma 2, we know that system (4) has an order-1 periodic solution. Thus, the proof is completed.

Theorem 5. *For system (4), if $((a(m_2 - h_1))/by_{F_2}) < p < 1 - ((r - am_2)/by_{F_2})$, the order-1 periodic solution to it is stable.*

Proof 3. According to Lemma 3 (see Figure 10), let $x = \xi_1(t)$ and $y = \eta_1(t)$ be a periodic solution to system (4) and $\xi_1(0) = m_2$ and $\eta_1(0) = y_B$; $\xi_1(T) = x_C = h_1$ and $\eta_1(T) = y_C$; we have $\xi_1^+ = \xi_1(T+0) = m_2$ and $\eta_1^+ = \eta_1(T+0) = (1-p)\eta_1(T) = (1-p)y_C = \eta_1(0) = y_B$

$$\begin{aligned} P(x, y) &= x(r - ax - by), \\ Q(x, y) &= y(x - m_1)(m_2 - x), \\ \alpha_1(x, y) &= m_2 - h_1, \\ \beta_1(x, y) &= -py, \\ \varphi_1(x, y) &= x - h_1. \end{aligned} \tag{10}$$

Then

$$\begin{aligned} \frac{\partial P}{\partial x} &= r - 2ax - by, \\ \frac{\partial Q}{\partial x} &= (x - m_1)(m_2 - x), \\ \frac{\partial \alpha_1}{\partial x} &= \frac{\partial \alpha_1}{\partial y} = 0, \\ \frac{\partial \beta_1}{\partial x} &= 0, \\ \frac{\partial \beta_1}{\partial y} &= -p, \\ \frac{\partial \varphi_1}{\partial x} &= 1, \\ \frac{\partial \varphi_1}{\partial y} &= 0, \\ \Delta_1 &= \frac{P_+(\partial\beta_1/\partial y(\partial\varphi_1/\partial x) - (\partial\beta_1/\partial x)(\partial\varphi_1/\partial y) + (\partial\varphi_1/\partial x)) + Q_+((\partial\alpha_1/\partial x)(\partial\varphi_1/\partial y) - (\partial\alpha_1/\partial y)(\partial\varphi_1/\partial x) + (\partial\varphi_1/\partial y))}{P(\partial\varphi_1/\partial x) + Q(\partial\varphi_1/\partial y)} \\ &= \frac{P_+(\xi_1(T+0), \eta_1(T+0))(1-p)}{P(\xi_1(T), \eta_1(T))} = \frac{P(B)(1-p)}{P(C)} = \frac{m_2(r - am_2 - b(1-p)y_C)(1-p)}{h_1(r - ah_1 - by_C)}, \\ \int_0^T \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial x}\right) dt &= \int_0^T ((r - 2ax - by) + (x - m_1)(m_2 - x)) dt = \int_{m_2}^{h_1} \frac{1}{x} dx + \int_{(1-p)y_C}^{y_C} \frac{1}{y} dy - \int_0^T axdx \\ &= \ln \frac{h_1}{m_2} + \ln \frac{1}{1-p} - \int_0^T axdx, \\ \mu_2 = \Delta_1 \exp\left\{\int_0^T \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial x}\right) dt\right\} &= \frac{m_2(r - am_2 - b(1-p)y_C)(1-p)}{h_1(r - ah_1 - by_C)} \exp\left\{\ln \frac{h_1}{m_2} + \ln \frac{1}{1-p} - \int_0^T axdx\right\} \\ &= \frac{m_2(r - am_2 - b(1-p)y_C)(1-p)}{h_1(r - ah_1 - by_C)} \frac{h_1}{m_2} \frac{1}{1-p} \exp\left\{-\int_0^T axdx\right\} \le \frac{r - am_2 - b(1-p)y_C}{r - ah_1 - by_C}. \end{aligned} \tag{11}$$

Since $(x(t), y(t))$ is the periodic solution of system (4), if $p > ((a(m_2 - h_1))/by_{F_2}) > ((a(m_2 - h_1))/by_C)$ and conditions of Theorem 4 hold, then $|\mu_2| < 1$. Therefore, the order-1 periodic solution of system (4) is stable, if $((a(m_2 - h_1))/by_{F_2}) < p < 1 - ((r - am_2)/by_{F_2})$. Thus, the proof is completed.

Theorem 6. *If $0 < \tau < ((r - am_2)/b) - y_{F_5}$ and $p > 1 - ((r - am_2)/by_{F_3})$, there exists an order-1 periodic solution in system (4).*

Proof 4. If $0 < \tau < ((r - am_2)/b) - y_{F_5}$ (see Figure 11), there exists a point $A_1(m_2, y_{A_1}) \in N$, satisfying $0 < y_{A_1} < \tau$. The trajectory of system (4) Γ_3 over A_1 with $x = h_2$ at point D_1, which is pulsed to $x = m_2$ and the phase point, is $A_2(m_2, y_{A_2})$, so $f(A_1) = y_{A_2} - y_{A_1} > 0$.

There exists another point $A_3(m_2, y_{A_3}) \in N$, satisfying $0 < y_{E_2} - y_{A_3} \ll 1$. The trajectory of system (4) Γ_4 over A_3 with $x = h_2$ at point D_2, which is pulsed to $x = m_2$ and the phase point, is $A_4(m_2, y_{A_4})$, so $f(A_3) = y_{A_4} - y_{A_3} < 0$.

Therefore, there must exist a point $A(m_2, y_A) \in N$ and it satisfies $y_{A_1} < y_A < y_{A_3}$, so that $f(A) = 0$. By Lemma 2, we know that system (4) has an order-1 periodic solution. Thus, the proof is completed.

Theorem 7. *For system (4), if $((a(h_2 - m_2))/b) < \tau < ((r - am_2)/b) - y_{F_5}$, the order-1 periodic solution to it is stable.*

Proof 5. According to Lemma 3 (see Figure 11), let $x = \xi_2(t)$ and $y = \eta_2(t)$ be a periodic solution to system (4) and $\xi_2(0) = x_A = m_2$ and $\eta_2(0) = y_A$; $\xi_2(T) = x_D = h_2$ and $\eta_2(T) = y_D$; we have $\xi_2^+ = \xi_2(T + 0) = m_2$ and $\eta_2^+ = \eta_2(T + 0) = \eta_2(T) + \tau = y_D + \tau = \eta_2(0) = y_A$

$$\begin{aligned}
P(x, y) &= x(r - ax - by),\\
Q(x, y) &= y(x - m_1)(m_2 - x),\\
\alpha_2(x, y) &= m_2 - h_2,\\
\beta_2(x, y) &= \tau,\\
\varphi_2(x, y) &= x - h_2.
\end{aligned} \tag{12}$$

Then

$$\begin{aligned}
\frac{\partial P}{\partial x} &= r - 2ax - by,\\
\frac{\partial Q}{\partial x} &= (x - m_1)(m_2 - x),\\
\frac{\partial \alpha_2}{\partial x} &= 0,\\
\frac{\partial \alpha_2}{\partial y} &= 0,\\
\frac{\partial \beta_2}{\partial x} &= 0,\\
\frac{\partial \beta_2}{\partial y} &= 0,\\
\frac{\partial \phi_2}{\partial x} &= 1,\\
\frac{\partial \phi_2}{\partial y} &= 0,\\
\Delta_1 &= \frac{P_+((\partial\beta_2/\partial y)(\partial\varphi_2/\partial x) - (\partial\beta_2/\partial x)(\partial\varphi_2/\partial y) + (\partial\varphi_2/\partial x)) + Q_+((\partial\alpha_2/\partial x)(\partial\varphi_2/\partial y) - (\partial\alpha_2/\partial y)(\partial\varphi_2/\partial x) + (\partial\varphi_2/\partial y))}{P(\partial\varphi_2/\partial x) + Q(\partial\varphi_2/\partial y)}\\
&= \frac{P_+(\xi_2(T+0), \eta_2(T+0))}{P(\xi_2(T), \eta_2(T))} = \frac{P(A)}{P(D)} = \frac{m_2(r - am_2 - b(y_D + \tau))}{h_2(r - ah_2 - by_D)},\\
\int_0^T \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial x}\right) dt &= \int_0^T ((r - 2ax - by) + (x - m_1)(m_2 - x)) dt = \int_{m_2}^{h_2} \frac{1}{x} dx + \int_{y_D+\tau}^{y_D} \frac{1}{y} dy - \int_0^T axdx\\
&= \ln \frac{h_2}{m_2} + \ln \frac{y_D}{y_D + \tau} - \int_0^T axdx,\\
\mu_2 &= \Delta_1 \exp\left\{\int_0^T \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial x}\right) dt\right\} = \frac{m_2(r - am_2 - b(y_D + \tau))}{h_2(r - ah_2 - by_D)} \exp\left\{\ln \frac{h_2}{m_2} + \ln \frac{y_D}{y_D + \tau} - \int_0^T axdx\right\}\\
&= \frac{m_2(r - am_2 - b(y_D + \tau))}{h_2(r - ah_2 - by_D)} \frac{h_2}{m_2} \frac{y_D}{y_D + \tau} \exp\left\{-\int_0^T axdx\right\} \le \frac{r - am_2 - b(y_D + \tau)}{r - ah_2 - b\eta_2} \frac{y_D}{y_D + \tau} < \frac{r - am_2 - b(y_D + \tau)}{r - ah_2 - by_D}.
\end{aligned} \tag{13}$$

Since $(x(t), y(t))$ is the periodic solution of system (4), if $\tau > ((a(h_2 - m_2))/b)$ and conditions of Theorem 6 hold, then $|\mu_2| < 1$. Therefore, the order-1 periodic solution of system (4) is stable, if $((a(h_2 - m_2))/b) < \tau < (r - am_2/b) - y_{F_5}$. Thus, the proof is completed.

3.3. Existence and Stability of Order-2 Periodic Solution of System (4)

Theorem 8. *If $\tau > (r - am_2/b)$ and $p > 1 - ((r - am_2)/by_{F_3})$, there exists an order-2 periodic solution in system (4).*

Proof 6. If $\tau > ((r - am_2)/b)$ and $p > 1 - ((r - am_2)/by_{F_3})$ (see Figure 12), there exists a point $A_3(m_2, y_{A_3}) \in N$, satisfying $y_{A_3} = (1-p)y_{F_3} < y_{E_2}$. The trajectory of system (4) Γ_2 over A_3 with $x = h_2$ at point D_2, which is pulsed to $x = m_2$ and the phase point, is $B_2(m_2, y_{B_2}) \in N$. We know that $y_{E_2} < y_{B_2} = y_{D_2} + \tau < y_{F_4}$; the trajectory of system (4) Γ_4 over B_2 with $x = h_1$ at point C_2, which is pulsed to $x = m_2$ and the phase point, is $A_4(m_2, y_{A_4}) \in N$, so $g(A_3) = y_{A_4} - y_{A_3} < 0$.

There exists another point $A_1(m_2, y_{A_1}) \in N$, satisfying $0 < y_{A_1} \ll 1$. The trajectory of system (4) Γ_1 over A_1 with $x = h_2$ at point D_1, which is pulsed to $x = m_2$ and the phase point, is $B_1(m_2, y_{B_1}) \in N$. We know that $y_{B_1} = y_{D_1} + \tau > y_{E_2}$; the trajectory of system (4) Γ_3 over B_1 with $x = h_1$ at point C_1, which is pulsed to $x = m_2$ and the phase point, is $A_2(m_2, y_{A_2}) \in N$, so $g(A_1) = y_{A_2} - y_{A_1} > 0$.

So there must exist a point $A(m_2, y_A) \in N$, satisfying $y_{A_1} < y_A < y_{A_3}$, so that $g(A) = 0$. By Lemma 2, we know that system (4) has an order-2 periodic solution. The proof is completed.

Theorem 9. *For system (4), if $p > 1 - ((r - am_2)/by_{F_3})$ and $\tau > ((r - am_2)/b) + ((r - ah_2)/b)$, the order-2 periodic solution to it is stable.*

Proof 7. According to Lemma 3 (see Figure 12), let $x = \xi(t)$ and $y = \eta(t)$ be a periodic solution to system (4) and $\xi(0) = x_A = m_2$ and $\eta(0) = y_A$; $\xi(T_1) = x_D = h_2$ and $\eta(T_1) = y_D$; we have $\xi(T_1 + 0) = m_2 = x_B$, $\eta(T_1 + 0) = y_D + \tau = y_B$, $\xi(T_1 + T_2) = h_1 = x_C$, and $\eta(T_1 + T_2) = y_C$. We also have $\xi(T_1 + T_2 + 0) = m_2 = x_A$ and $\eta(T_1 + T_2 + 0) = (1-p)y_C = y_A$

$$
\begin{aligned}
P(x, y) &= x(r - ax - by),\\
Q(x, y) &= y(x - m_1)(m_2 - x),\\
\alpha_1(x, y) &= m_2 - h_1,\\
\beta_1(x, y) &= -py,\\
\varphi_1(x, y) &= x - h_1,\\
\alpha_2(x, y) &= m_2 - h_2,\\
\beta_2(x, y) &= \tau,\\
\varphi_2(x, y) &= x - h_2.
\end{aligned}
\tag{14}
$$

Then

$$
\begin{aligned}
\frac{\partial P}{\partial x} &= r - 2ax - by,\\
\frac{\partial Q}{\partial x} &= (x - m_1)(m_2 - x),\\
\frac{\partial \alpha_1}{\partial x} &= \frac{\partial \alpha_1}{\partial y} = 0,\\
\frac{\partial \beta_1}{\partial x} &= 0,\\
\frac{\partial \beta_1}{\partial y} &= -p,\\
\frac{\partial \varphi_1}{\partial x} &= 1,\\
\frac{\partial \varphi_1}{\partial y} &= 0,\\
\frac{\partial \alpha_2}{\partial x} &= 0,\\
\frac{\partial \alpha_2}{\partial y} &= 0,\\
\frac{\partial \beta_2}{\partial x} &= 0,
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial \beta_2}{\partial y} = 0,\\
&\frac{\partial \varphi_2}{\partial x} = 1,\\
&\frac{\partial \varphi_2}{\partial y} = 0,\\
&\Delta_1 = \frac{P_+((\partial\beta_1/\partial y)(\partial\varphi_1/\partial x) - (\partial\beta_1/\partial x)(\partial\varphi_1/\partial y) + (\partial\varphi_1/\partial x)) + Q_+((\partial\alpha_1/\partial x)(\partial\varphi/\partial y) - (\partial\alpha_1/\partial y)(\partial\varphi_1/\partial x) + (\partial\varphi_1/\partial y))}{P(\partial\varphi_1/\partial x) + Q(\partial\varphi_1/\partial y)}\\
&\quad = \frac{P_+(\xi(T_1+T_2+0), \eta(T_1+T_2+0))(1-p)}{P(\xi(T_1+T_2), \eta(T_1+T_2))} = \frac{(1-p)P(A)}{P(C)} = \frac{m_2(r - am_2 - b(1-p)y_C)(1-p)}{h_1(r - ah_1 - by_C)},\\
&\Delta_2 = \frac{P_+((\partial\beta_2/\partial y)(\partial\varphi_2/\partial x) - (\partial\beta_2/\partial x)(\partial\varphi_2/\partial y) + (\partial\varphi_2/\partial x)) + Q_+((\partial\alpha_2/\partial x)(\partial\varphi_2/\partial y) - (\partial\alpha_2/\partial y)(\partial\varphi_2/\partial x) + (\partial\varphi_2/\partial y))}{P(\partial\varphi_2/\partial x) + Q(\partial\varphi_2/\partial y)}\\
&\quad = \frac{P_+(\xi(T_1+0), \eta(T_1+0))}{P(\xi(T_1), \eta(T_1))} = \frac{P(B)}{P(D)} = \frac{m_2(r - am_2 - b(y_D+\tau))}{h_2(r - ah_2 - by_D)},\\
&\int_0^{T_1+T_2}\left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial x}\right)dt = \int_0^{T_1+T_2}((r - 2ax - by) + (x - m_1)(m_2 - x))dt\\
&\quad = \int_0^{T_1}((r - 2ax - by) + (x - m_1)(m_2 - x))dt + \int_{T_1}^{T_1+T_2}((r - 2ax - by) + (x - m_1)(m_2 - x))dt\\
&\quad = \int_{m_2}^{h_2}\frac{1}{x}dx + \int_{(1-p)y_C}^{y_D}\frac{1}{y}dy + \int_{m_2}^{h_1}\frac{1}{x}dx + \int_{y_D+\tau}^{y_C}\frac{1}{y}dy - \int_0^{T_1+T_2} axdx,\\
&\mu_2 = \Delta_1\Delta_2 \exp\left\{\int_0^{T_1+T_2}\left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial x}\right)dt\right\}\\
&\quad = \frac{m_2(r - am_2 - b(1-p)y_C)(1-p)}{h_1(r - ah_1 - by_C)}\frac{m_2(r - am_2 - b(y_D+\tau))}{h_2(r - ah_2 - by_D)}\frac{h_2}{m_2}\frac{y_D}{(1-p)y_C}\frac{h_1}{m_2}\frac{y_C}{y_D+\tau}\\
&\quad = \frac{r - am_2 - b(1-p)y_C}{r - ah_1 - by_C}\frac{r - am_2 - b(y_D+\tau)}{r - ah_2 - by_D}\frac{y_D}{y_D+\tau} < \frac{r - am_2 - b(1-p)y_C}{r - ah_1 - by_C}\frac{r - am_2 - b(y_D+\tau)}{r - ah_2 - by_D}.
\end{aligned}
\tag{15}
$$

Obviously, $|\mu_2| < 1$, if $p > 1 - ((r - am_2)/by_{F_C})$ and $\tau > ((r - am_2)/b) + (r - ah_2/b)$ and conditions of Theorem 8 hold. Therefore, the order-2 periodic solution of system (4) is stable, if $p > 1 - ((r - am_2)/by_{F_3})$ and $\tau > ((r - am_2)/b) + ((r - ah_2)/b)$. Thus, the proof is completed.

3.4. Existence of Order-k $(k = 1, 2)$ Periodic Solution of System (3)

Case I. $(1 - q_1)h_2 < m_2$ (see Figure 13).

For notation simplicity, let the intersection of the phase set N and L_1 be $F_8(x_{F_8}, y_{F_8}) = F_8((1 - q_1)h_2, y_{F_8})$. The intersection point of L_8 and L is denoted by $F_7(x_{F_7}, y_{F_7}) = F_7((1 - q_1)h_2, y_{F_7})$.

Theorem 10. *If $0 < p < 1 - (by_{F_7}/(r - ah_1))$ and $\tau > y_{F_7}$, there exists an order-1 periodic solution in system (3).*

Proof 8. The proof is similar to that of Theorem 4 and omitted thereby.

Theorem 11. *If $p > 1 - (y_{F_7}/y_{F_3})$ and $\tau < y_{F_7} - y_{F_5}$, there exists an order-1 periodic solution in system (3).*

Proof 9. The proof is similar to that of Theorem 6 and omitted thereby.

Theorem 12. *If $p > 1 - (y_{F_7}/y_{F_3})$ and $\tau > y_{F_7}$, there exists an order-2 periodic solution in system (3).*

Proof 10. The proof is similar to that of Theorem 8 and omitted thereby.

Case II. $(1 - q_1)h_2 > m_2$ (see Figure 14).

For notation simplicity, let the intersection of the phase set N and L_9 be $F_9(x_{F_9}, y_{F_9}) = F_9((1 - q_1)h_2, y_{F_9})$. The intersection point of L_2 and L is denoted by $F_{10}(x_{F_{10}}, y_{F_{10}}) = F_{10}((1 - q_1)h_2, y_{F_{10}})$.

Theorem 13. *If $0 < p < 1 - (y_{F_9}/y_{F_2})$ and $\tau > y_{F_9}$, there exists an order-1 periodic solution in system (3).*

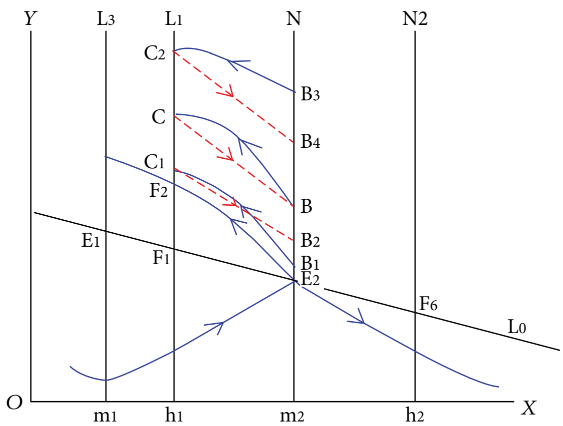

FIGURE 10: There exists an order-1 periodic solution of system (4) with $(1-p)y_{F_2} > y_{E_2}$.

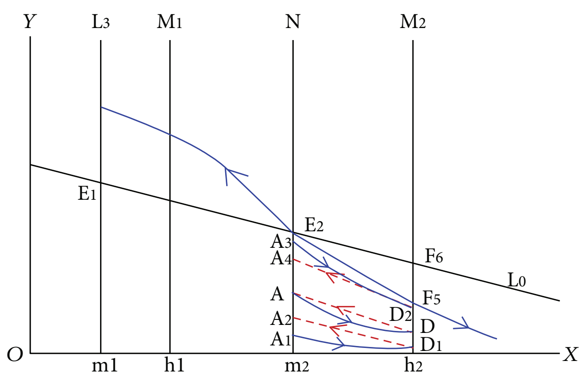

FIGURE 11: There exists an order-1 periodic solution of system (4) with $y_{F_5} + \tau < y_{E_2}$.

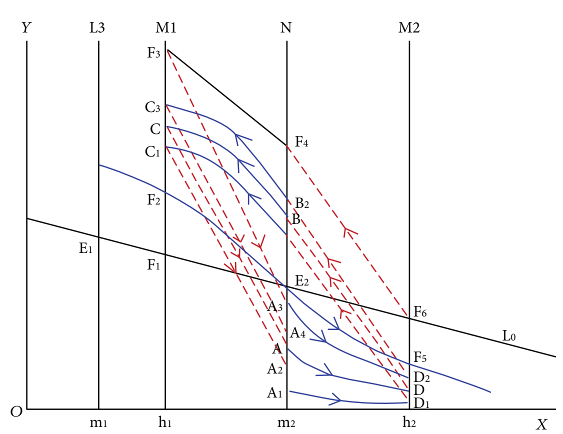

FIGURE 12: There exists an order-2 periodic solution of system (4) with $(1-p)y_{F_3} < y_{E_2}$ and $\tau > y_{E_2}$.

Proof 11. The proof is similar to that of Theorem 4 and omitted thereby.

Theorem 14. *If $p > 1 - (y_{F_9}/y_{F_3})$ and $\tau < y_{F_9} - y_{F_6}$, there exists an order-1 periodic solution in system (3).*

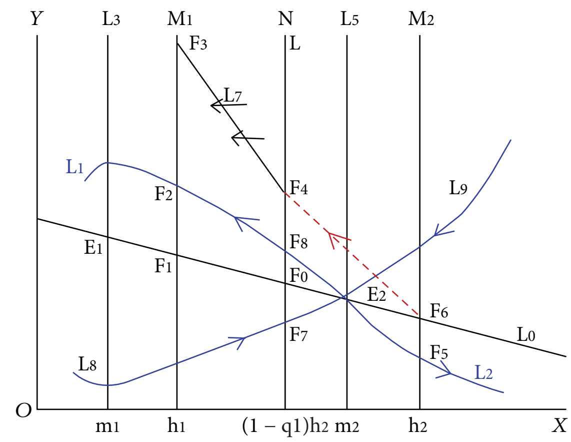

FIGURE 13: Structure diagram in R_2^+ of system (3) with $(1-q_1)h_2 < m_2$.

Proof 12. The proof is similar to that of Theorem 6 and omitted thereby.

Theorem 15. *If $p > 1 - (y_{F_9}/y_{F_3})$ and $\tau > y_{F_9}$, there exists an order-2 periodic solution in system (3).*

Proof 13. The proof is similar to that of Theorem 8 and omitted thereby.

4. Numerical Simulations and Discussion

In this section, a specific example is presented to verify the theoretical results obtained in the previous section by considering the change of the control parameters p, τ, and q_1. From Figures 15–25, we know that the numerical simulation results are consistent with the theoretical results.

In system (3), let $r = 4$, $a = 0.4$, $b = 0.5$, $m_1 = 3$, $m_2 = 8$, $h_1 = 6$, $h_2 = 9$, $0 < p = p_1\delta < 1$, $\tau > 0$, and $0 < q_1 < 1 - (h_1/h_2)$; we get the following system:

$$\begin{cases} \dfrac{dx}{dt} = x(4 - 0.4x - 0.5y), \\ \dfrac{dy}{dt} = y(x-3)(8-x), \\ \quad 6 < x < 9, \end{cases}$$
$$\begin{cases} \triangle x = 3 - 9q_1, \\ \triangle y = -py, \\ \quad x = 6, y > 3.2, \end{cases} \qquad (16)$$
$$\begin{cases} \triangle x = -9q_1, \\ \triangle y = \tau, \\ \quad x = 9, y < 0.8, \end{cases}$$
$$6 < x_0 < 9,$$
$$y_0 > 0.$$

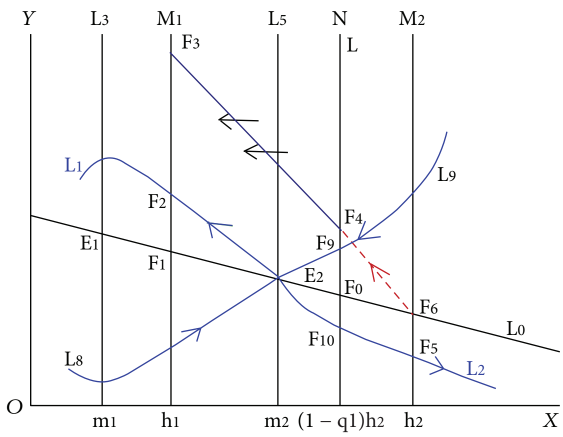

FIGURE 14: Structure diagram in R_2^+ of system (3) with $(1-q_1)h_2 > m_2$.

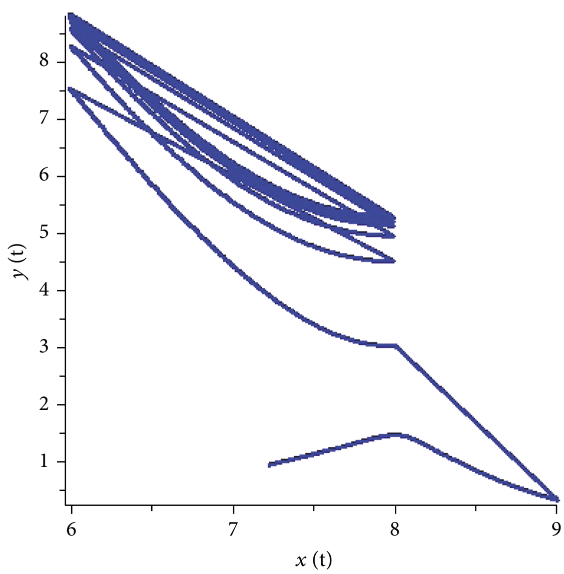

FIGURE 15: There exists an order-1 periodic solution in system (16) with $q_1 = (1/9)$, $p = 0.4$, and $\tau = 2.7$.

Obviously (see Figure 9), we know that $E_0(0,0)$ and $E_2(8,1.6)$ are saddle points and $E_1(3,5.6)$ and $E_3(10,0)$ are stable node points. Let $L_7 : W(x,y) = y + Kx - (8K + 0.8 + \tau) = 0$; we have $F_1(6,3.2)$, $F_3(6, 2K + \tau + 0.8)$, $F_4(8,0.8 + \tau)$, $F_6(9,0.8)$, and $(x_0, y_0) = (7.21,0.9)$.

4.1. Verification for the Case $(1-q_1)h_2 = m_2$

Case I. Let $q_1 = (1/9)$, $p = 0.4$, and $\tau = 2.7$, then $h_1 < (1-q_1)h_2 = m_2 < h_2$, and the impulsive set and the phase set are $M = M_1 \cup M_2 = \{(x,y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x,y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x,y) \in R_+^2 \mid x = 8, y \geq 0\}$. According to Theorem 4, there exists an order-1 periodic solution in system (16), which can be seen in Figure 15. We can observe that there exists an order-1 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 6 and 8).

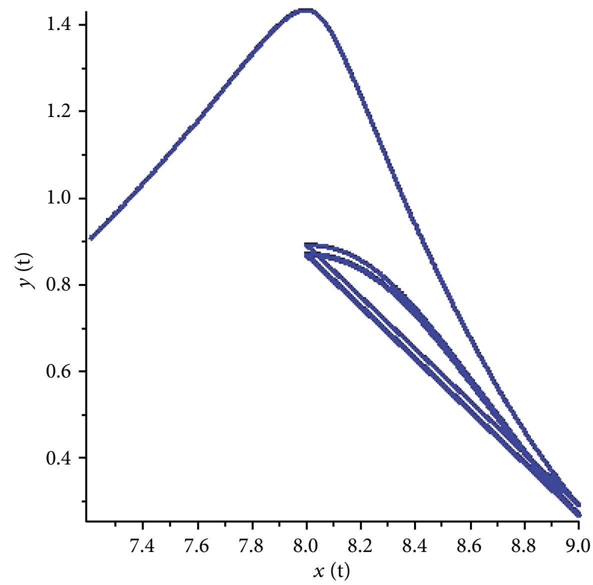

FIGURE 16: There exists an order-1 periodic solution in system (16) with $q_1 = (1/9)$, $p = 0.8$, and $\tau = 0.6$.

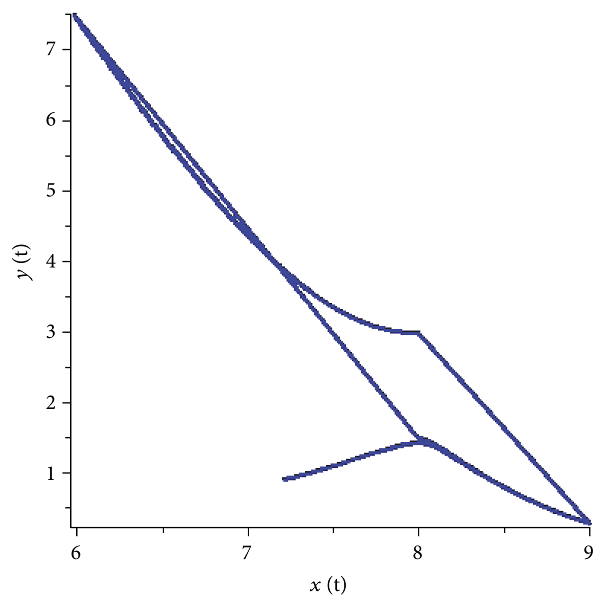

FIGURE 17: There exists an order-2 periodic solution in system (16) with $q_1 = (1/9)$, $p = 0.8$, and $\tau = 2.7$.

Case II. Let $q_1 = (1/9)$, $p = 0.8$, and $\tau = 0.6$, then $h_1 < (1-q_1)h_2 = m_2 < h_2$, and the impulsive set and the phase set are $M = M_1 \cup M_2 = \{(x,y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x,y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x,y) \in R_+^2 \mid x = 8, y \geq 0\}$. According to Theorem 6, there exists an order-1 periodic solution in system (16), which can be seen in Figure 16. We can observe that there exists an order-1 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 8 and 9).

Case III. Let $q_1 = (1/9)$, $p = 0.8$, and $\tau = 2.7$, then $h_1 < (1-q_1)h_2 = m_2 < h_2$, and the impulsive set and the phase set are

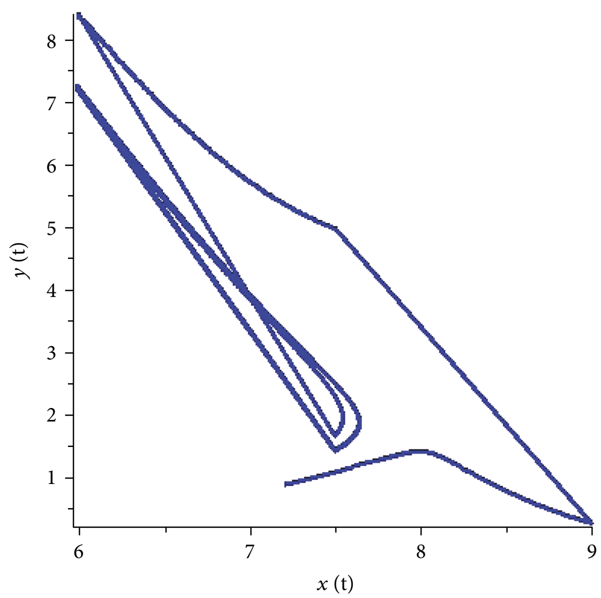

Figure 18: There exists an order-1 periodic solution in system (16) with $q_1 = (1/6)$, $p = 0.8$, and $\tau = 4.7$.

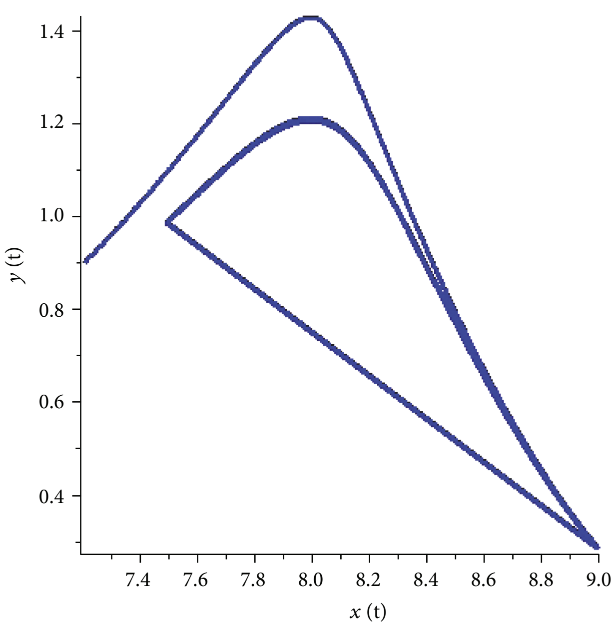

Figure 19: There exists an order-1 periodic solution in system (16) with $q_1 = (1/6)$, $p = 0.9$, and $\tau = 0.7$.

$M = M_1 \cup M_2 = \{(x, y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x, y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x, y) \in R_+^2 \mid x = 8, y \geq 0\}$. According to Theorem 8, there exists an order-2 periodic solution in system (16), which can be seen in Figure 17. We can observe that there exists an order-2 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 6 and 9).

4.2. Verification for the Case $(1 - q_1)h_2 < m_2$

Case I. Let $q_1 = (1/6)$, $p = 0.8$, and $\tau = 4.7$, then $h_1 < (1 - q_1) h_2 < m_2 < h_2$, and the impulsive set and the phase set are

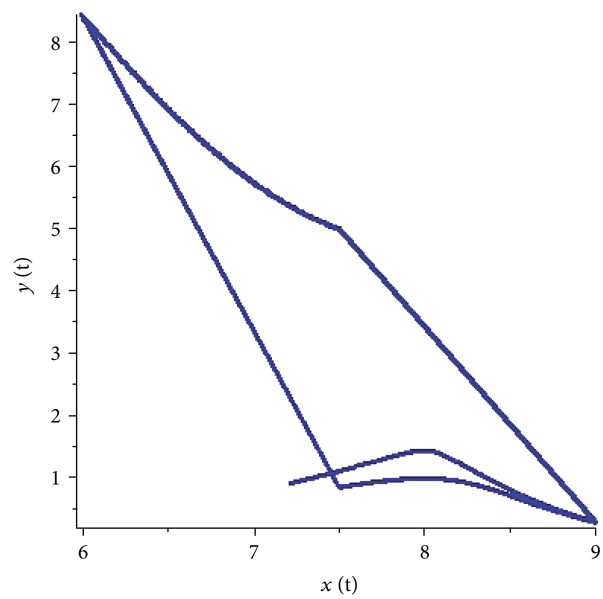

Figure 20: There exists an order-2 periodic solution in system (16) with $q_1 = (1/6)$, $p = 0.9$, and $\tau = 4.7$.

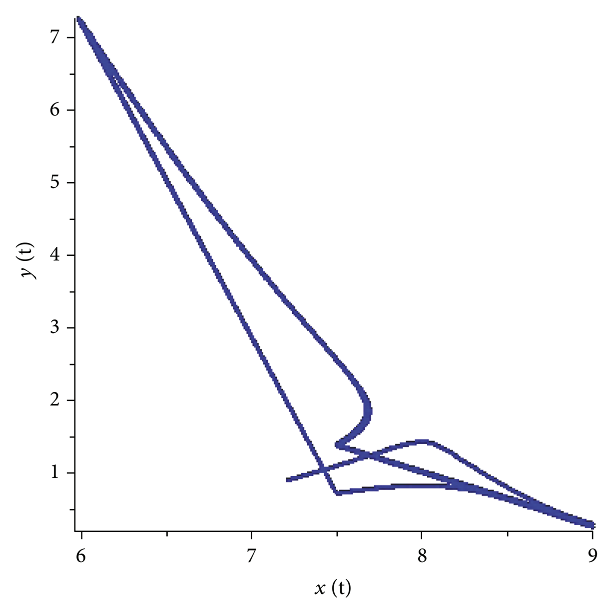

Figure 21: There exists an order-2 periodic solution in system (16) with $q_1 = (1/6)$, $p = 0.9$, and $\tau = 1.1$.

$M = M_1 \cup M_2 = \{(x, y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x, y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x, y) \in R_+^2 \mid x = 7.5, y \geq 0\}$. According to Theorem 10, there exists an order-1 periodic solution in system (16), which can be seen in Figure 18. We can observe that there exists an order-1 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 6 and 7.5).

Case II. Let $q_1 = (1/6)$, $p = 0.9$, and $\tau = 0.7$, then $h_1 < (1 - q_1)h_2 < m_2 < h_2$, and the impulsive set and the phase set are $M = M_1 \cup M_2 = \{(x, y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x, y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x, y) \in R_+^2 \mid x = 7.5, y \geq 0\}$. According

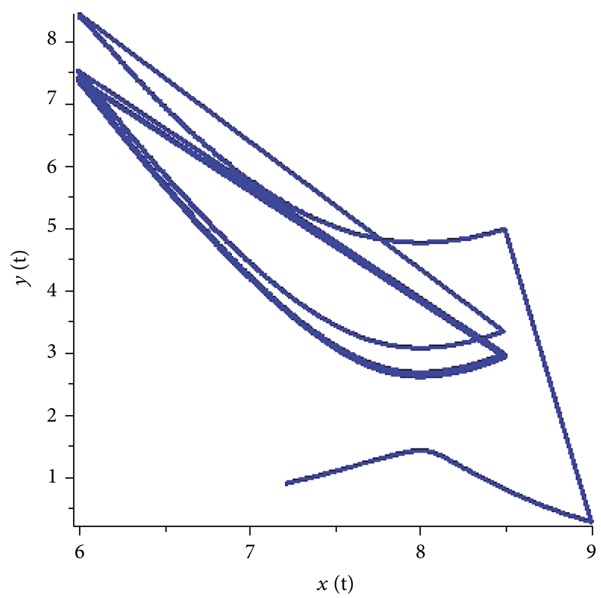

Figure 22: There exists an order-1 periodic solution in system (16) with $q_1 = (1/18)$, $p = 0.6$, and $\tau = 4.7$.

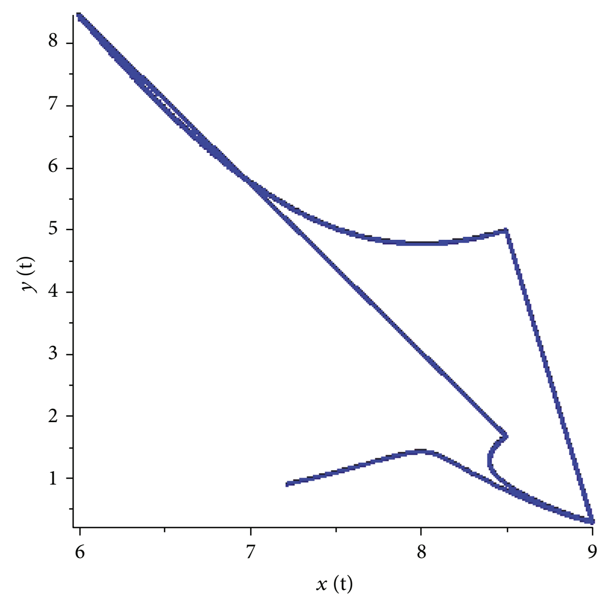

Figure 24: There exists an order-2 periodic solution in system (16) with $q_1 = (1/18)$, $p = 0.8$, and $\tau = 4.7$.

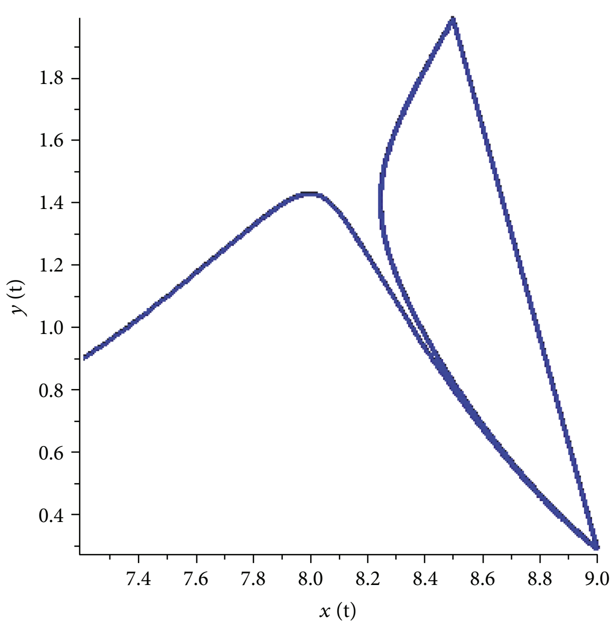

Figure 23: There exists an order-1 periodic solution in system (16) with $q_1 = (1/18)$, $p = 0.8$, and $\tau = 1.7$.

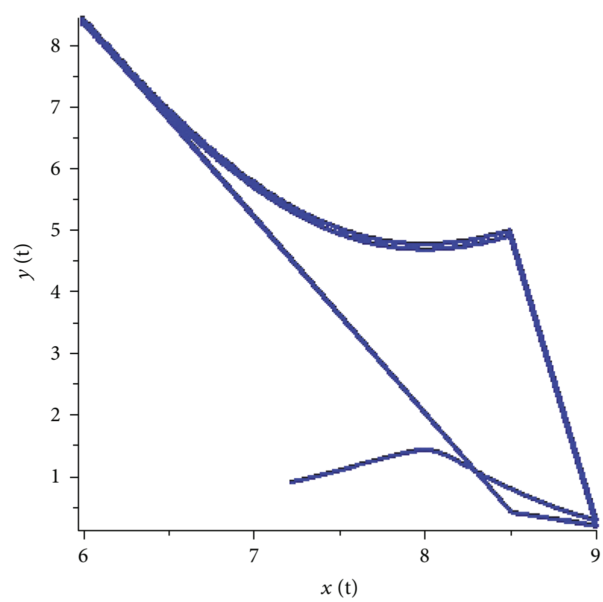

Figure 25: There exists an order-2 periodic solution in system (16) with $q_1 = (1/18)$, $p = 0.95$, and $\tau = 4.7$.

to Theorem 11, there exists an order-1 periodic solution in system (16), which can be seen in Figure 19. We can observe that there exists an order-1 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 7.5 and 8).

Case III. Let $q_1 = (1/6)$, $p = 0.9$, and $\tau = 4.7$, then $h_1 < (1 - q_1)h_2 < m_2 < h_2$, and the impulsive set and the phase set are $M = M_1 \cup M_2 = \{(x, y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x, y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x, y) \in R_+^2 \mid x = 7.5, y \geq 0\}$. According to Theorem 12, there exists an order-2 periodic solution in system (16), which can be seen in Figure 20. We can observe that there exists an order-2 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 6 and 8).

Case IV. Let $q_1 = (1/6)$, $p = 0.9$, and $\tau = 1.1$, then $h_1 < (1 - q_1)h_2 < m_2 < h_2$, and the impulsive set and the phase set are $M = M_1 \cup M_2 = \{(x, y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x, y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x, y) \in R_+^2 \mid x = 7.5, y \geq 0\}$. According to Theorem 12, there exists an order-2 periodic solution in system (16), which can be seen in Figure 21. We can observe that there exists an order-2 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 6 and 8).

4.3. Verification for the Case $(1-q_1)h_2 > m_2$

Case I. Let $q_1 = (1/18)$, $p = 0.6$, and $\tau = 4.7$, then $h_1 < m_2 < (1-q_1)h_2 < h_2$, and the impulsive set and the phase set are $M = M_1 \cup M_2 = \{(x, y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x, y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x, y) \in R_+^2 \mid x = 8.5, y \geq 0\}$. According to Theorem 13, there exists an order-1 periodic solution in system (16), which can be seen in Figure 22. We can observe that there exists an order-1 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 6 and 8.5).

Case II. Let $q_1 = (1/18)$, $p = 0.8$, and $\tau = 1.7$, then $h_1 < m_2 < (1-q_1)h_2 < h_2$, and the impulsive set and the phase set are $M = M_1 \cup M_2 = \{(x, y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x, y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x, y) \in R_+^2 \mid x = 8.5, y \geq 0\}$. According to Theorem 14, there exists an order-1 periodic solution in system (16), which can be seen in Figure 23. We can observe that there exists an order-1 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 8.5 and 9).

Case III. Let $q_1 = (1/18)$, $p = 0.8$, and $\tau = 4.7$, then $h_1 < m_2 < (1-q_1)h_2 < h_2$, and the impulsive set and the phase set are $M = M_1 \cup M_2 = \{(x, y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x, y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x, y) \in R_+^2 \mid x = 8.5, y \geq 0\}$. According to Theorem 15, there exists an order-2 periodic solution in system (16), which can be seen in Figure 24. We can observe that there exists an order-2 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 6 and 9).

Case IV. Let $q_1 = (1/18)$, $p = 0.95$, $\tau = 4.7$, then $h_1 < m_2 < (1-q_1)h_2 < h_2$, and the impulsive set and the phase set are $M = M_1 \cup M_2 = \{(x, y) \in R_+^2 \mid x = 6, y > 3.2\} \cup \{(x, y) \in R_+^2 \mid x = 9, y < 0.8\}$ and $N = \{(x, y) \in R_+^2 \mid x = 8.5, y \geq 0\}$. According to Theorem 15, there exists an order-2 periodic solution in system (16), which can be seen in Figure 25. We can observe that there exists an order-2 periodic solution of system (16) which lies between the phase set and the impulse set (i.e., between 6 and 9).

From Figures 1, 15–25, we know that at the initial point (7.21, 0.9) in the system without pulse control, fish species will be extinct, which is harmful to biodiversity. In order to maintain biodiversity, we introduce the state feedback control and put forward the economic threshold (ET) of ecological control to effectively control the amount of water hyacinth population which can prevent the extinction of fish and can fully employ water hyacinth as feedstuff for continuous development of fishery production, changing the harm into benefit.

5. Conclusions

In the system with pulse control, we gain order-1 periodic solutions and order-2 periodic solution of the system, respectively, verifying the accuracy of Theorems 4, 6, 8, and 10–15. At the same time, by using the comprehensive control strategies, such as periodically dropping and harvest, we achieve the diversity of species in water hyacinth ecological system.

In the future, from the viewpoint of optimization, we will further consider the costs and profits under domination of the market economy and further improve the model, thus realizing the effective protection of the diversity of species in water hyacinth ecology and obtaining the biggest economic profits.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

No potential conflict of interest was reported by the authors.

Acknowledgments

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