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Research Article

Establishment and Analysis of the Supernetwork Model for Nanjing Metro Transportation System

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In recent years, many researchers have applied complex network theory to urban public transport network to construct complex network and analyze its network performance. The original analysis method generally uses the Space L and Space R model to establish a simple link between public sites but ignores the organic link between the overall network system and the line subsystem. As an important part of urban public transport system, subway plays an important role in alleviating traffic pressure. In this paper, a supernetwork model of Nanjing metro network is established by using the supernetwork method. Three parameters, node-hyperedge degree, hyperedge-node degree, and hyperedge degree, are proposed to describe the model. The model is compared with the traditional Space L and Space P models. The study on the supernetwork model of Nanjing metro complex network shows that the network density, network centrality, and network clustering coefficient are large, and the average network distance is small, which meets the requirements of traffic planning and design. In this study, the subway line is considered as a subsystem and further simplified as a node, so that the complex network analysis method can be applied to the new supernetwork model, expanding the thinking of complex network research.

1. Introduction

With the rapid development of urban construction, more and more cities in China have opened the subway. As an important part of urban public transport system, subway plays an important role in relieving traffic pressure. Urban public transport system can be abstracted as complex network composed of stations and routes. The study of subway network is helpful to understand the evolution mechanism of public transport system and solve the problem of urban congestion.

Complex networks are characterized by complex structure and huge number of nodes. Watts and Strogatz were first proposed to have small world characteristics for complex networks [1]. Barabasi and Albert proposed scale-free power-law distribution properties [2]. Complex networks are applied to the construction and analysis of public transport network. An X L takes the bus line as the network node and uses Space R method to establish a multiweight bus road network model. By changing the different weights, the balance of the whole public transport network system is discussed [3]. From the static point of view of network topology, Bona A A D uses complex network theory to analyze the structure of public transport system in Curitiba, and compares it with the structure of public transport system in three big cities of China, including Shanghai, Beijing, and Guangzhou [4]. Manitz J proposes two methods for the cause estimation of delay in public transport networks. The application of the two methods in simulation research and in German railway system is examined [5].

Ouyang M takes China Railway System as an example, chooses three typical models based on complex network, and analyzes railway accessibility and virtual users based on traffic [6]. Zhang L has established a complex network of Jinan public transport lines by using the Space R method. It is found that the network has small world characteristics and large average clustering coefficient [7]. Mohmand Y T studied the structural characteristics of the Pakistani railway network, whose complex network shows the properties of the small world [8]. These studies include constructing complex networks of public transport networks and analyzing their structure and performance.

Complex network theory is also used to analyze the urban subway network. Based on the complex network theory, Ding

R explores the evolution process of Kuala Lumpur public rail transit network and evaluates the network performance changes in the face of different attack strategies [9]. Based on the trip data and operation schedule of Beijing subway system, Yang Y proposed a multilayer model to analyze the traffic flow pattern of subway network [10]. Feng J establishes a multilayer model of the workday and weekend flow distribution of the subway network based on the Beijing subway trip data and operation schedule [11].

Wu X established six metro complex networks in Beijing, London, Paris, Hong Kong, Tokyo, and New York and evaluated their topological efficiency and robustness [12]. Cats O establishes an evaluation model of public transport robustness and applies the model to Amsterdam urban rail transit network and evaluates the robustness of the network [13].Zhang J analyzed the complex network characteristics of the subway network in Beijing, Guangzhou, and Shanghai and studied the vulnerability of the subway network [14]. These studies include the construction of complex urban subway network, the analysis of its structural performance and robustness, and the establishment of subway network traffic flow model.

Supernetwork theory has been applied to various industries. Wang J P presents an improved hypernetwork model of knowledge diffusion algorithm and analyzes the performance of knowledge diffusion [15]. Zhao L constructs the knowledge supernetwork model of business incubators and studies the performance differences of knowledge services of different incubators by simulation [16]. Suo Q applies hypernetwork method to analyze user ratings in social networks and puts forward suggestions for collaboration in hypernetworks [17]. Cheng Q proposes a new method to reveal the community of supernetworks, which transforms the problem of community detection into the problem of DOT partitioning [18].

Wang F, taking WeChat as a sample, proposes an attractive and node-age inhomogeneous hypernetwork model [19]. Cheng Y puts forward the concept of supply-demand matching hypernetwork for manufacturing services in SOM system and reveals the matching relationship between each service and each task [20]. Lv T proposes a three-tier petroleum emergency dispatching network based on supernetwork model, which enhances regional emergency correlation by adding transfer management process [21]. Yamada T proposes a discrete network design problem based on supernetwork optimization of freight network [22]. The application of supernetwork analysis is focused on knowledge propagation model, community network analysis, and supply chain management.

Supernetwork analysis can also be seen in the subway network. Du W J puts forward a supernetwork model of urban public transport composed of conventional public transport network and urban rail transit network. Based on the external synchronization theory of coupled complex networks, the synchronization problem of urban public transport supernetwork model is studied [23]. Suo Q takes station representation as node and line representation as superedge. This paper presents a supernetwork model to describe the evolution mechanism of high-speed railway system [24]. At present, the analysis of supernetwork in metro network is limited to abstract network and simulation research, and the important parameters of metro supernetwork are not put forward and have not been applied to actual cases.

The complex network theory is used to model and analyze the urban subway network, generally using the Space L and Spacer methods. Both methods use subway stations as nodes, the Space L method establishes the links between adjacent stations on different lines, and the Space R method establishes the links between stations on the same line. The complex network model of urban subway is composed of the links between stations, and then the performance of the complex network can be analyzed. Once these two models are established, the relationship between the station and the line is neglected, and the change of the relationship between the stations is simply analyzed.

The supernetwork model can make up for this deficiency. Urban subway supernetwork is composed of main system and subsystem. The stations between the main systems are connected by some rules, which reflects the overall structure and performance of urban subway network. At the same time, the nodes in each subsystem are connected according to some rules, reflecting the connection between the lines and stations. When analyzing the performance of the main network of the metro supernetwork, the control rules of the line subsystem must be taken into account. In some cases, the line subsystem can be further simplified as a supernode, thus reflecting the overall relationship between lines. As an upgraded version of complex network, supernetwork can more effectively reflect the real structure and performance of urban subway network.

In recent years, Nanjing's public transport system has developed rapidly, opening a number of subway lines, more and more subway lines are also under construction. As an important part of public transport network system, subway can not only save traffic resources, but also provide a strong guarantee for the convenience of passengers. In this paper, the complex network of Nanjing metro is selected as the object. On the basis of the traditional complex network model Space L and Space P, the topology model of the supernetwork is constructed. In this paper, the subway supernetwork is described with the parameters of node degree, node-hyperedge degree, hyperedge-node degree, and hyperedge degree, and the network density, center degree, average distance, and clustering coefficient in complex network are extended to the theory of supernetwork, and the comparison with the traditional subway complex network model is made.

The subway can also be called the metro. In the general description of this article, it is called the subway. And according to the official name of Nanjing, it is called the metro. The supernetwork can also be called a hypernetwork. In the general description of this article, it is called the supernetwork. In the specific model, it is called hypernetwork, and its corresponding edge is also called hyperedge.

2. Nanjing Metro Network

The data of the Nanjing metro network mainly comes from the latest Nanjing bus line map issued by the Nanjing passenger transport management office and the latest tourism traffic

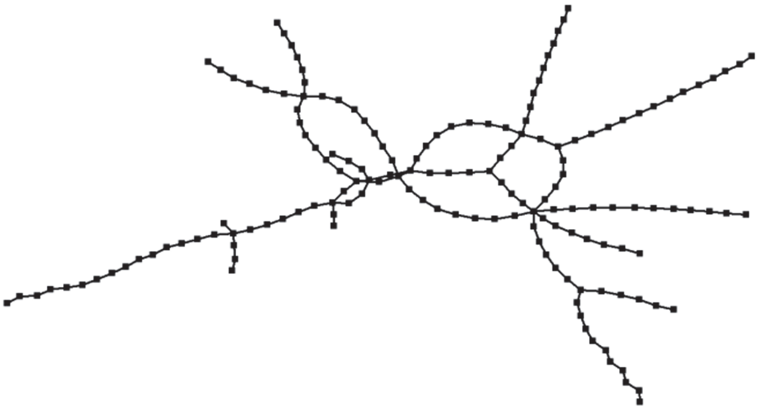

Figure 1: Space L spatial model of Nanjing metro complex network.

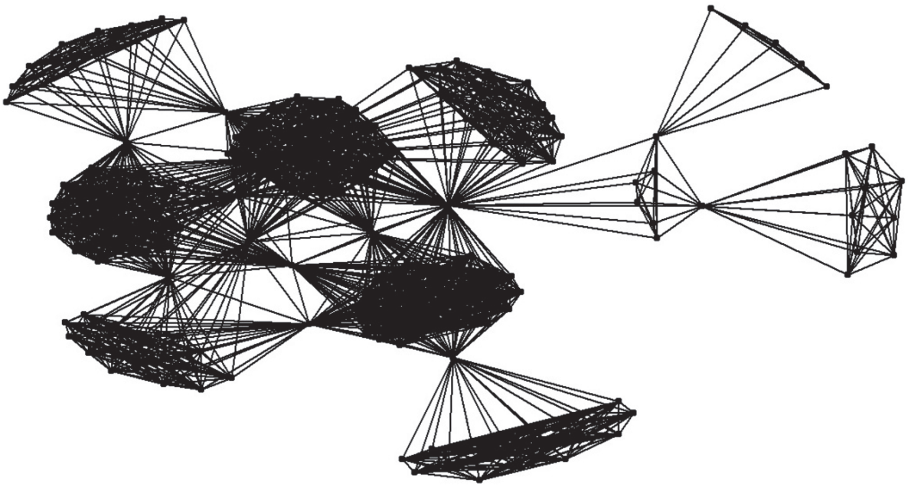

Figure 2: Space P spatial model of Nanjing metro complex network.

map of 2018 and the city map of Nanjing. The basic assumptions of Nanjing metro network topology are as follows.

The subway network is abstracted as an undirected network. There are differences between the upstream and downstream stations due to traffic control and other routes. Without considering the frequency of departure, the network is abstracted as a nonweighted network. The same name site is regarded as a docking site, ignoring the differences caused by the same location of individual sites but different locations. The temporary bus route diversion caused by road construction or other reasons, the cancellation or increase of subway stations, etc. shall not be considered.

There are two ways to describe the traditional traffic network topology: one is the Space L method, that is, the traffic site is regarded as a node, and if the two sites on a traffic line are adjacent, there is a link between them. Another is the Space P method, that is, the traffic network site as a node, if there is a direct traffic line between the two stations, they have a connection. From the definition, we can see that the network constructed by Space L method is the subnetwork constructed by Space P method [25].

Figure 1 shows the Space L spatial model of Nanjing metro complex network. It can be seen from the model that the number of nodes in the network is not much, and the topology map is not complex. This is because the subway network in Nanjing is still in the process of construction, and there are still more lines to be opened in the future to meet the needs of the residents. The network presents an obvious star structure extending from the center to the periphery. At the core of the network is the core residential area of the city, surrounded by suburbs and further county towns.

Figure 2 shows the Space P spatial model of Nanjing metro complex network. It can be seen from the model that because the model represents all sites on the same line, there are ten distinct clustering subgraphs, which actually represent 10 subway lines. These lines are linked by important nodes.

3. Hypergraph and Supernetwork Model

The concept of hypergraph is proposed by BERGE in 1970. This is the first time that the theory of undirected hypergraph is established systematically, and the application of hypergraph theory in operational research is studied by using matroids. Nodes in a supernetwork represent a given set of networks, while edges and arcs represent a combined movement and a combination of preferences in a given set, and

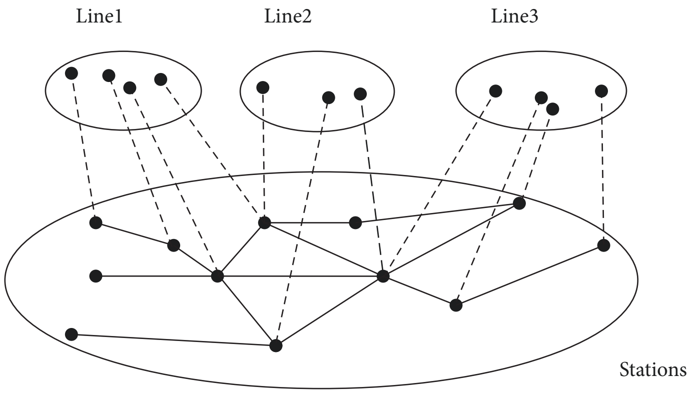

Figure 3: Supernetwork topology map of Nanjing metro.

the supernetwork uniquely represents all the combination of mobile and preference dominated by the rules [26].

The definition of hypergraph is as follows: suppose V is a finite set.

If $e_i \neq H(i = 1, 2, \ldots, m)$,

$$(1)\ \overset{m}{\underset{i=1}{Y}} e_i = V \qquad (1)$$

The two element relation $H = (V, E)$ is called a hypergraph.

The element of V, $\{v_1, v_2, \cdots, v_n\}$ is called the vertex of hypergraph, $E = \{E_1, E_2, \cdots, E_m\}$ is the edge set of hypergraph, and the set is called the edge of hypergraph.

Figure 3 shows the supernetwork topology map of Nanjing metro. The supernetwork model of the subway network consists of two parts, one is the subsystem network, which refers to the local railway lines, the other is the main system network, which refers to the overall network established between the railway stations. The main system and subsystems are independent and interrelated. Subsystem networks of the subway network include line 1, line 2, and line 3. The site on each route forms a line with certain rules. Lines, sites, and rules form the so-called hyperedge. Stations in the main system network of a metro network are associated with certain rules, such as the Space L and Space R methods for general complex network models. However, the constraints of subsystem networks are neglected once the complex network models using these two methods are established.

The supernetwork model of metro network is different, and the organic connection between the main system network and the subsystem network is always considered. Therefore, in the analysis of the supernetwork model of subway network, the relationship between nodes-hyperedge, hyperedge-node, and hyperedge-hyperedge is included. When each subway line is simplified into a supernode, a new superedge network model is formed. Unlike Space L and Space R, each node of the hyperedge network model represents a specific subway line. A general complex network analysis method is also applicable to the superedge network model.

In the hypergraph of the supernetwork of Nanjing metro, the neighborhood matrix A reflects the relationship between the subway station and the hyperedge of the subway line. The line of the A represents the subway station, and the column of the A is the subway line. If the site belongs to a certain line, there is a relationship between the two, and the assignment is 1 or 0. A is a symmetric matrix.

$$A_{m\times n} = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ \vdots \\ E_m \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & \cdots & v_n \end{array} \\ \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\ a_{21} & 0 & a_{23} & a_{24} & \cdots & a_{2n} \\ a_{31} & a_{32} & 0 & a_{34} & \cdots & a_{3n} \\ a_{41} & a_{42} & a_{43} & 0 & \cdots & a_{4n} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} & \cdots & 0 \end{bmatrix} \end{array} \qquad (2)$$

where N is the number of stations on the subway network and M is the number of subway lines. $v_i(i = 1, 2, 3, 4, \cdots, n)$ stands for the subway station,. $E_j(j = 1, 2, 3, 4, \cdots, m)$ represents the subway line. $a_{ij}(i = 1, 2, 3, 4, \cdots n; j = 1, 2, 3, 4, \cdots, m)$ represents the relationship between the site and the line.

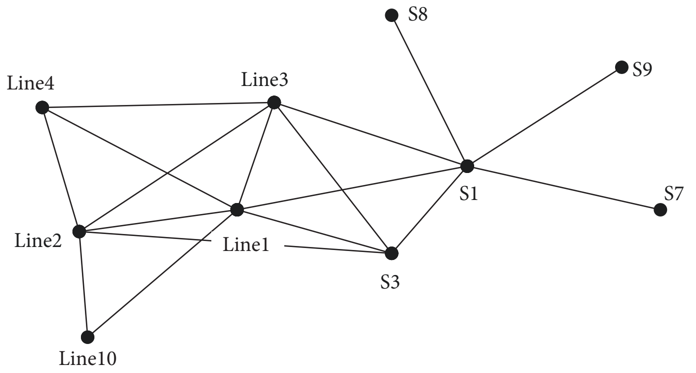

Figure 4: Topology of hyperedge-hyperedge relation in supernetwork of Nanjing metro.

The study in this paper further simplifies the subway supernetwork and establishes the relationship between hyperedge and hyperedge. In hypergraph $S = (E, E)$, the neighborhood matrix B reflects the relationship between the subway hyperedges. The rows and columns of B represent the metro lines. If there is the same station between the two lines, there is a relationship between them. The assignment value is 1, otherwise it is 0. B is a symmetric matrix.

$$S_{m\times n} = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \\ E_4 \\ \vdots \\ E_m \end{array} \begin{array}{c} \begin{array}{cccccc} E_1 & E_2 & E_3 & E_4 & \cdots & E_m \end{array} \\ \begin{bmatrix} 0 & b_{12} & b_{13} & b_{14} & \cdots & b_{1m} \\ b_{21} & 0 & b_{23} & b_{24} & \cdots & b_{2m} \\ b_{31} & b_{32} & 0 & b_{34} & \cdots & b_{3m} \\ b_{41} & b_{42} & b_{43} & 0 & \cdots & b_{4m} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ b_{m1} & b_{m2} & b_{m3} & b_{m4} & \cdots & 0 \end{bmatrix} \end{array} \qquad (3)$$

where M is the number of subway lines. $E_i (i = 1, 2, 3, 4, \cdots, m)$ stands for subway lines, $E_j (j = 1, 2, 3, 4, \cdots, m)$ represents subway lines, and $b_{ij} (i = 1, 2, 3, 4, \cdots, m; j = 1, 2, 3, 4, \cdots, m)$ represents the relationship between the lines.

Figure 4 shows the topology of hyperedge-hyperedge relation in supernetwork of Nanjing metro. By comparing Figure 4 with Figure 2, we can see that the model is a simplified version of the Space P spatial model. In supernetwork model, the nodes are juxtaposed. After simplification, the hyperedge space model also forms a new complex network in which the nodes represent a line, the edges of which represent a common site between the lines. The method of analyzing general complex networks is applicable to the hyperedge space model.

4. The Degree of Complex Network

4.1. Node Degree. Those points adjacent to a point become a node's adjacent point; the number of adjacent points of a point is called the degree of the point, also known as the degree of association. The node degree is defined as the number of other nodes connected to the node. In fact, the degree of a point is also the number of lines connected to that point. If the degree of a point is 0, it is called an outlier. The node degree distribution can be described by the distribution function p (k), which indicates the probability that a randomly selected node is exactly k.

Figure 5 shows the probability distribution of node degree in Space L space of complex network in Nanjing metro. The formula is fitted to $y = 0.271x^{-1.64}$ through the data. It can be seen that the node degree distribution in the bus network Space L of Nanjing is close to the power-law distribution, which indicates that the subway network in Nanjing is a scale-free network in Space L.

According to the observation and analysis of the public transport network, the urban public transport network has the characteristics of growth and priority connectivity. Therefore, the public transport network will eventually form a scale-free network, and the distribution of the node degree in Figure 5 confirms this theory. In general, the greater the degree of a node means the more important the node is. As can be seen from Figure 5, the degree of most of the nodes is less than 6, and the node degree is basically 2. This is because the subway network structure is relatively simple and can not form a very complex network structure.

Figure 6 shows the probability distribution of node degree in Space P space of complex network in Nanjing metro. The formula is fitted to $y = 0.421x^{-0.89}$ through the data.

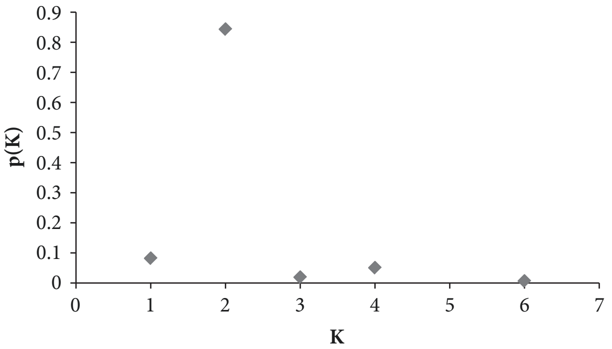

Figure 5: Probability distribution of node degree in Space L space of complex network in Nanjing metro.

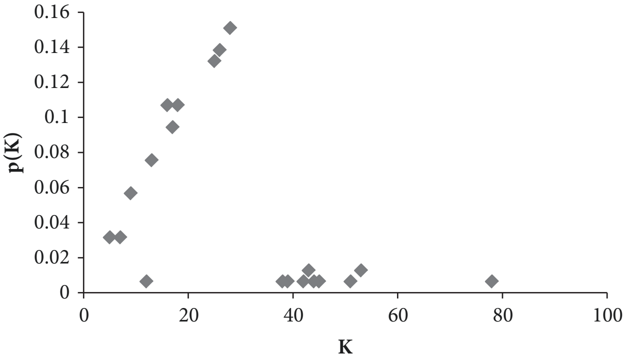

Figure 6: Probability distribution of node degree in Space P space of complex network in Nanjing metro.

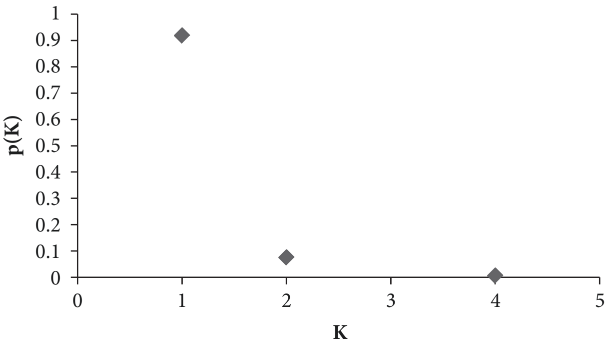

Figure 7: Probability distribution of hypernetwork node-hyperedge degree in Nanjing metro.

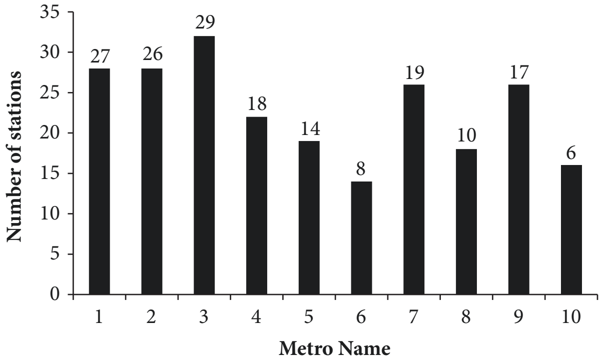

Figure 8: Distribution of hyperedge-node degree of hypernetwork in Nanjing metro.

It can be seen that the node degree distribution in the subway network Space P of Nanjing is close to the power-law distribution. As can be seen from Figure 6, the degree of most of the nodes is less than 60 and the degree of node concentration is between 10 and 30, which indicates the number of other sites connected by the node through the subway line.

4.2. Node-Hyperedge Degree. Node-hyperedge degree is defined as the number of hyperedges that contain the node. As shown in Figure 3, we can see that there is a node belonging to line 1 and line 2, and the node's node-hyperedge degree is 2. The node-hyperedge degree distribution can be described by the distribution function P (k), which represents the probability that the node-hyperedge degree of a randomly selected node is exactly K.

Figure 7 shows the probability distribution of hypernetwork node-hyperedge degree in Nanjing metro. The formula is fitted to $y = 0.916x^{-3.59}$ through the data. It can be seen that the probability distribution of node-hyperedge degree of Nanjing metro network is close to power-law distribution. From Figure 7, we can see that node-hyperedge degree is actually 1, 2, and 4. The number of stations containing stations is usually 1, meaning that most subway stations only have one route to go through. A few subway stations, as important transfer sites, have two routes to go through. This is determined by the nature of the subway network, the structure presents star type radiation, and the overlapping sites are few.

4.3. Hyperedge-Node Degree. The hyperedge-node degree is defined as the number of nodes contained in a superedge. In subway hypernetwork, this parameter represents the number of subway stations contained in a line.

Figure 8 shows the distribution of hyperedge-node degree of hypernetwork in Nanjing metro. The name of the subway is 1 to 10, representing Nanjing metro 1, 2, 3, 4, 10, and suburban railway lines s1, s3, s7, s8, and s9. The degree of hyperedge-node is between 6 and 29. The subway lines in the center of the city usually have more stations, and the distance between stations is shorter, which effectively meets the needs of the residents in the central area. The suburban subway lines have fewer stations, and the distance between stations is longer, connecting the suburbs, remote counties, and airports.

4.4. Hyperedge Degree. The hyperedge degree refers to the number of other hyperedges adjacent to the hyperedge, that is, the number of other hyperedges that have common nodes with the hyperedge. In subway hyperedge network, this parameter represents the number of other subway lines connected by a subway line.

Table 1: Degree distribution of main sites in Nanjing metro.

Space	Space L		Space P		Node-hyperedge	
Ranking	Node	Degree	Node	Degree	Node	Degree
1	Nanjing south railway station	6	Nanjing south railway station	78	Nanjing south railway station	4
2	Yuantong	4	NanJing Railway Station	53	Yuantong	2
3	Jimingsi	4	Daxinggong	53	Youfangqiao	2
4	NanJing Railway Station	4	Xinjiekou	51	Xinjiekou	2
5	Jinma Road	4	Jimingsi	45	Xiangyu Road South	2
6	Taifeng Road	4	Taifeng Road	44	Taifeng Road	2
7	Xinjiekou	4	Youfangqiao	43	NanJing Railway Station	2
8	Gulou	4	Gulou	43	Lukou airport	2
9	Daxinggong	4	Jinma Road	42	Jinma Road	2
10	Xiangyu Road South	3	Andemen	39	Jimingsi	2
11	Andemen	3	Yuatong	38	Gulou	2
12	Youfangqiao	3	Chengxin Road	28	Daxinggong	2

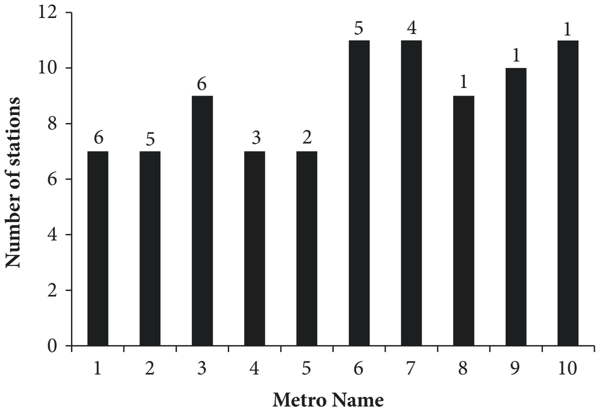

Figure 9: Hyperedge degree distribution of the hypernetwork in Nanjing metro.

Figure 9 shows the hyperedge degree distribution of the hypernetwork in Nanjing metro. The name of the subway is 1 to 10, representing Nanjing metro 1, 2, 3, 4, 10, and suburban railway lines s1, s3, s7, s8, and s9. The value of the superedge is between 1 and 6. Metro lines 1, 2, and 3 have a higher node-hyperedge degree and have a better switching function. The suburban subway s1 has a superedge of 5, because it connects the suburbs, airports, railway stations, and remote county towns. Metro line s3 has a superedge of 4, because it connects some suburban lines.

Figure 10 shows the probability distribution of hypernetwork hyperedge degree in Nanjing metro. If the hyperedge degree distribution is described by the distribution function p (k), the probability of a hyperedge of a randomly selected hyperedge is exactly k. From the probability distribution map, the hyperedge does not obey the power-law distribution, but it is similar to the two power function after fitting.

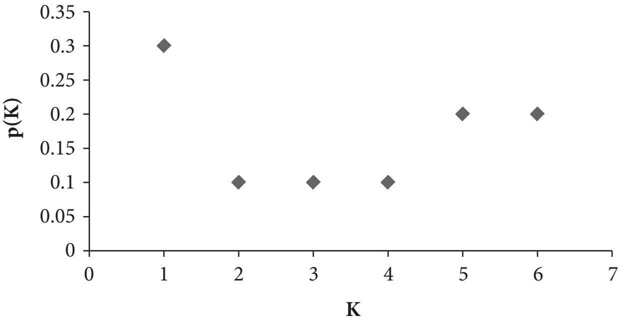

Figure 10: Probability distribution of hypernetwork hyperedge degree in Nanjing metro.

4.5. Analysis of Public Hub Sites. Table 1 shows the highest degree of 12 nodes in Space L, Space P, and node-hyperedge space. These sites, known as public pivot points, play a vital role in the urban public transport network, connected to not only a large number of subway stations, but also a number of bus stations and many of the lines through the site.

It can be seen from Table 1 that the node degree of Space P space is 6. The maximum node degree of Space P space is 78, and the range of numerical fluctuation is large. The node-hyperedge space is 2 except for one node with 4. In the three spaces, the most important is the Nanjing south railway station, which connects the suburban, railway station, and the airport's subway lines, which has played an important role in the transfer. In the three spaces, the top ranking sites are basically unchanged. These are important public hub sites.

5. Spatial Characteristics of Metro Network

In this paper, three spatial models of urban subway network are constructed by using the space L, space R and superedge

Table 2: Spatial characteristics of the complex network of Nanjing metro.

characteristics	Space L	Space P	Hyperedge Space
Network size	159	159	10
Network density (%)	1.31	13.69	37.78
Network centrality (%)	2.52	36.13	36.11
Network average distance	16.77	2.34	1.82
Network clustering coefficient (%)	0	95.8	67.6

space methods. The superedge network model simplifies each subway line into a supernode. If each line has the same station, the supernodes are connected. When analyzing the performance of Space L and Space R models in subway networks, the general spatial characteristic parameters include network size, network density, network center degree, network average distance, and network clustering coefficient [9, 10]. The analysis method is also applicable to the superedge network model.

Table 2 shows the spatial characteristics of Nanjing metro complex network and uses the network size, network density, network center degree, network average distance, and network clustering coefficient of five indicators. In this paper, three models of Space L, Space P, and hyperedge space are selected for comparison. The hyperedge space model is shown in Figure 4. The network size of Space L and Space P space is 159, which means that there are 159 subway stations. The size of the network in the hyperedge space is 10, which means that there are 10 subway lines.

5.1. Network Density. Network density refers to the degree of closeness between nodes in a network. Network G's network density d (G) is defined as

$$d(G) = \frac{2M}{[N(N-1)]} \tag{4}$$

where M is the number of connections actually owned in the network and N is the number of network nodes. The range of network density is [0, 1]. When the network is completely connected, the network density is 1, while the actual network density is usually much less than 1.

As can be seen from Table 2, the network density of Space L is relatively low, because the subway lines are still relatively small, and the structure presents a star-shaped loose structure. The network density of Space P is the result of the characteristics of the structure model, so the connection between the stations on each line has been established, and the density value of the network is improved. In fact, there are relatively few links between the lines. The density of network in hyperedge space is relatively high, because the model reflects the relationship between subway lines, and the distribution is more balanced in the whole region.

5.2. Network Centrality. Degree centrality is divided into node centrality and network centrality. The former refers to the degree of centrality among the nodes in which the nodes are directly connected to them, while the latter focuses on the central degree of the whole network, representing the degree of centralization of the entire network, that is, the extent to which the entire network organizes the operation around a node or a group of nodes. The degree centrality C_D (v_i) of node v_i is defined as

$$C_D(V_i) = \frac{k_i}{N-1} \tag{5}$$

In all networks containing N nodes, assume that network $\mathbf{G}_{\text{optimal}}$ maxims the following formula:

$$H = \sum_{i=1}^{N} \left[C_D(V_{\max}) - C_D(V_i)\right] \tag{6}$$

In the formula, v_i is the node of the network $\mathbf{G}_{\text{optimal}}$, and V $_{\max}$ represents the node with the largest degree of centrality in the network $\mathbf{G}_{\text{optimal}}$.

For a network G containing N nodes, $\mathrm{V}_{\max}$ means that it has the largest degree of centrality. Figure $\mathbf{G}_{\text{optimal}}$ for star network, the degree centrality C_D of network G is defined as

$$C_D = \frac{1}{N-2} \sum_{i=1}^{N} \left[C_D(V_{\max}) - C_D(V_i)\right] \tag{7}$$

As can be seen from Table 2, the network centrality of Space L is relatively small, which is due to loose structure; no node has a larger degree of node. The network center of Space P is relatively large, because the connection between lines is associated with all sites on different lines and thus presents better centrality. The network center of the hyperedge space is relatively large, because some important metro lines are effectively connected to other suburban metro lines, such as line 1, line 2, and line 3 of the Nanjing metro.

5.3. Network Average Distance. In mathematics, physics, and sociology, the small world network is a type of mathematical graph, in which most of the nodes are not adjacent to each other, but most of the nodes can arrive at a few steps from any other point. Small world networks are usually measured by means of two parameters: average distance and clustering coefficient. The small world standard has a small network average distance L and a high clustering coefficient C.

Distance refers to the total number of lines that a node must pass through in its path to another node, i.e., the length of the shortcut between two points. Mean distance represents the average distance between all pairs of points in a graph. The overall reachability of the network is better than the average distance, but the connectivity of the whole network cannot be truly reflected by the connected distance in the case that the whole network is not in the connected state, but in the case of multiple subgraphs. Although many real networks have large number of nodes, the average distance is surprisingly small. This is the so-called small world effect.

For the undirected simple graph, the formula is as follows:

$$L = \frac{2}{N(N-1)} \sum_{i=1}^{N} \sum_{j=i+1}^{N} d_{ij} \tag{8}$$

where L is the average distance of the network, N is the total number of nodes, and the distance from node i to node j.

As you can see from Table 2, the network average distance of Space L is larger because it represents the length between one site and another, and the space model's star structure determines the distance to the suburb. The average distance of Space P is 2.34, which means the average transfer is 2.34 times from one subway station to another. Considering the shortcut of the subway, the transfer efficiency is still high. The average distance in the hyperspace is 1.82, which means that it is more efficient to transfer from one subway line to another through 1.82.

5.4. Network Clustering Coefficient. According to the graph theory, the clustering coefficient is the coefficient that represents the degree of node clustering in a graph. Evidence shows that in the real network, especially in a specific network, the nodes tend to establish a set of close organizational systems because of the relative high density connection points. In real-world networks, this probability is often greater than the average probability of randomly setting up a connection between two nodes.

First of all, we look at the definition of the clustering coefficient of the nodes. If the node v_i is connected directly with the k_i node, the maximum number of possible edges between the k_i nodes for the undirected network is $k_i\ (k_i - 1)/2$, while the actual number of edges is M_i.

$$C = \sum_{i=1}^{n} C_i = \sum_{i=1}^{n} \frac{2M_i}{k_i (k_i - 1)} \tag{9}$$

The network clustering coefficient C is the average clustering coefficient C_i of all nodes i. It is obvious $0 \leq C \leq 1$, where k_i represents the number of all adjacent nodes of node i and N represents the number of all nodes.

It can be seen from Table 2 that the clustering coefficient of Space L is zero because of the loose topology of space. Space P's network clustering coefficient is large because the sites on each line are set up to connect when building the model. The network clustering coefficient of the hyperedge space is relatively large, because the subway lines in some urban areas have played an important connection with the lines of the suburbs, airports, and railway stations.

6. Conclusions

When using the general complex network method to analyze the network, the nodes are often regarded as independent, ignoring the small group effect of the network. Hypergraph and hypernetwork method make up for this deficiency to a certain extent. This paper chooses Nanjing metro complex network as the research object, establishes the space L, space P, and hypernetwork model, and compares the three network structures. The hypernetwork model reflects the relationship between the subway station and the subway lines and the relationship between the subway lines. The analysis shows that the network density, network centrality, and network centrality of the metro hypernetwork in Nanjing are large, and the average distance of the network is small, which is in line with the ideal traffic planning and design.

The public transport hub sites extracted from the hypernetwork model are similar to the other two models. This paper only analyzes the complex network of Nanjing metro and can further expand to the bus system, shared bicycle system, and uses the hypernetwork model to establish a higher level, more complex system, and analyze the connection. The application of the hypernetwork model is only an undirected simple network, and the relationship established is only a subordinate relationship between the line and the site. The future hypernetwork model can be extended to a directed weighted network, to establish a more complex model to consider travel costs and travel preferences and to apply to solving other traffic problems.

Appendix

Nanjing Metro Data

The data of the Nanjing metro network mainly comes from the latest Nanjing bus line map issued by the Nanjing passenger transport management office and the latest tourism traffic map of 2018 and the city map of Nanjing. The basic assumptions of Nanjing metro network topology are as follows.

By May 2018, Nanjing metro network has opened 10 lines, 1, 2, 3, 4, and 10 and the suburban railway lines S1, S3, S7, S8, S9, which are composed of 159 subway stations. The opening order is 1, 2, 10, S1, S8, 3, 4, S3, S9, and S7.

The metro data are as follows:

Line 1: maigaoqiao, hongshandongwuyuan, nanjingzhan, xinmofanmalu, xuanwumen, gulou, zhujianglu, xinjiekou, zhangfuyuan, sanshanjie, zhonghuamen, andemen, tianlongsi, ruanjiandadao, huashenmiao, nanjingnanzhan, shuanglongdadao, hedingqiao, shengtailu, baijiahu, xiaolongwan, zhushanlu, tianyindadao, longmiandadao, nanyidajiangsujingmaoxueyuanzhan, nanjingjiaoyuan, zhongguoyaokedaxue

Line 2: youfangqiao, yurundajie, yuantong, aotidong, xinglongdajie, jiqingmendajie, yunjinlu, mochouhu, hanzhongmen, shanghailu, xinjiekou, daxinggong, xianmen, minggugong, muxuyuan, xiamafang, xiaolingwei, zhonglingjie, maqun, jinmalu, xianhemen, xuezelu, xianlinzhongxin, yangshangongyuan, nandaxianlinxiaoqu, jingtianlu

Line 3: linchang, xinghuolu, dongdachengxianxueyuan, taifenglu, tianruncheng, liuzhoudonglu, shangyuanmen, wutangguangchang, xiaoshi, nanjingzhan, nanjinglinyedaxuexinzhuang, jimingsi, fuqiao, daxinggong, changfujie, fuzimiao, wudingmen, yuhuamen, qiazimen, daminglu, mingfaguangchang, nanjingnanzhan, hongyundadao, shengtaixilu, tianyuanxilu, jiulonghu, chengxindadao, dongdajiulonghuxiaoqu, mozhoudonglu

Line 4: longjiang, nanyiershicaochang, yunnanlu, gulou, jimingsi, jiuhuashan, gangzicun, jiangwangmiao, wangjiawan, jubaoshan, suningzongbuxuzhuang, jinmalu, huitonglu, lingshan, dongliu, mengbei, xiganghuashu, xianlinhu

Line 10: andemen, xiaoxing, zhongsheng, yuantong, aotizhongxin, mengdudajie, lüboyuan, jiangxinzhou, linjianglu, pukouwanhuicheng, nanjinggongyedaxue, longhualu, wendelu, yushanlu

S1: nanjingnanzhan, cuipingshan, fochengxilu, jiyindadao, zhengfangzhonglu, xiangyulubei, xiangyulunan, lukoujichang

S3: gaojiachong, linshan, qiaolinxincheng, shiqihe, shuanglong, lanhuatang, maluowei, liucun, tianbao, gaomiaolu, wuhoujie, pingliangdajie, yongchulu, youfangqiao, jiaxi, chunjianglu, tiexinqiao, jingmingjiayuan, nanjingnanzhan

S7: lukoujichang, jichangdong, zhetang, zhetangxinqu, jinshan, tuanshan, lishui, zhongshandonglu, jinlonglu, wuxiangshan

S8: taishanxincun, taifenglu, gaoxinkaifaqu, xinxigongchengdaxue, xiejiadian, dachang, getang, changlu, huagongyuan, liuhekaifaqu, longchi, xiongzhou, fenghuangshangongyuan, fangzhouguangchang, shenqiao, babaiqiao, jinniuhu

S9: xiangyulunan, tongshan, shiqiu, mingjue, tuanjiewei, gaochun

Data Availability

The Nanjing metro data used to support the findings of this study are included within Appendix.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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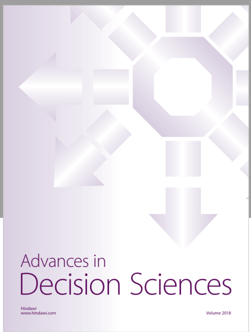
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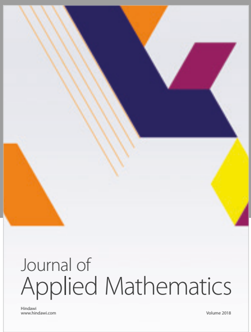
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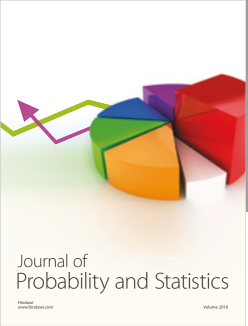
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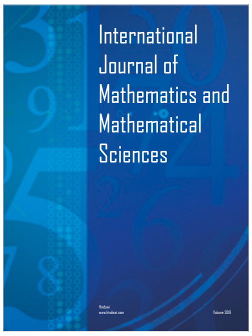
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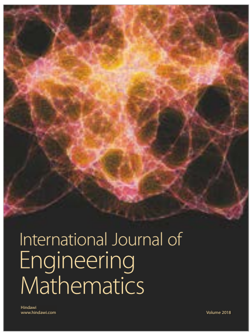
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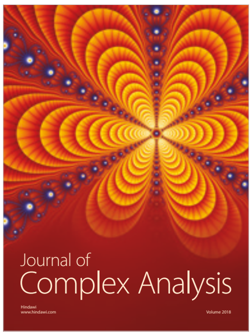
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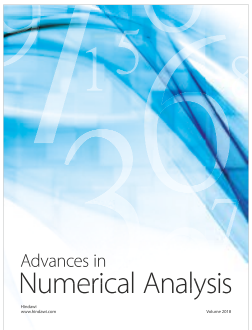
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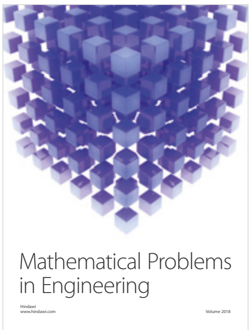
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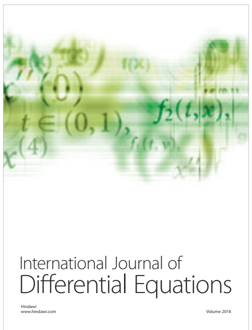
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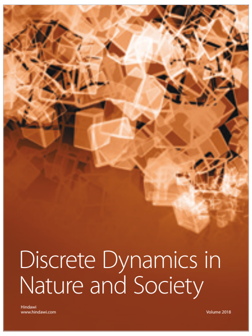
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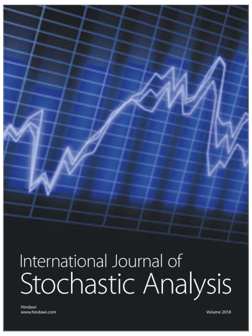
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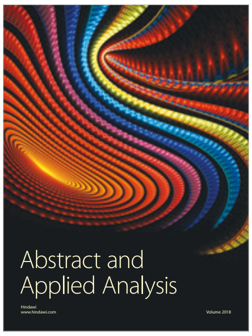
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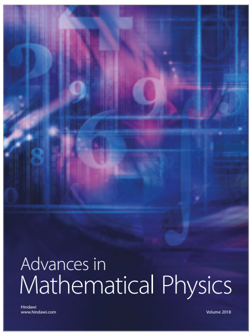
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