Hindawi
Complexity
Volume 2018, Article ID 5790681, 8 pages
https://doi.org/10.1155/2018/5790681

Research Article

Guaranteed Cost Finite-Time Control of Discrete-Time Positive Impulsive Switched Systems

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Received 6 July 2017; Revised 22 January 2018; Accepted 12 March 2018; Published 16 April 2018

Academic Editor: Peter Giesl

This paper considers the guaranteed cost finite-time boundedness of discrete-time positive impulsive switched systems. Firstly, the definition of guaranteed cost finite-time boundedness is introduced. By using the multiple linear copositive Lyapunov function (MLCLF) and average dwell time (ADT) approach, a state feedback controller is designed and sufficient conditions are obtained to guarantee that the corresponding closed-loop system is guaranteed cost finite-time boundedness (GCFTB). Such conditions can be solved by linear programming. Finally, a numerical example is provided to show the effectiveness of the proposed method.

1. Introduction

As a special kind of positive systems [1–3], the positive switched systems whose output and state are nonnegative whenever the initial condition and input are nonnegative have been found in many applications such as communication networks [4], viral mutation [5], and formation flying [6]. There have been many available results about continuous-time positive switched systems [7–11] and discrete-time positive switched systems [12–14].

However, most results mentioned above focus on the classical Lyapunov stability, which guarantees the stability in an infinite-time interval. Different from the Lyapunov stability concept, the finite-time stability requires that the states do not exceed a certain bound during a fixed finite-time interval. The paper [15] firstly defined the definition of finite-time stability (FTS) for linear deterministic systems. Recently, [16] firstly extended the concept of FTS to positive switched systems and gave some FTS conditions of positive switched systems. So far, there have been a few meaningful results about FTS of positive switched systems; see [17–20]. In these results, to make the best of the nature of positivity, the MLCLF approach has been widely used and became a powerful tool for the analysis and synthesis of positive switched systems. Due to the wide application of digital controllers, some researches have been done on the FTS of discrete-time positive switched systems. The paper [21] investigated the problem of robust finite-time stability and stabilization of a class of discrete-time positive switched systems. The paper [22] studied the problem of finite-time control of a class of discrete impulsive switched positive time-delay systems under asynchronous switching, but the effect of disturbance was ignored.

Moreover, in most of practical applications, the researchers are more interested in designing the control system which is not only finite-time stable but also guarantees an adequate level of performance. One method to this problem is the so-called guaranteed cost finite-time control. Some remarkable results have been presented; see [23–27]. These results mainly focus on nonpositive systems. Very recently, in [28], guaranteed cost finite-time control was extended to fractional-order positive switched systems and a cost function for fractional-order positive systems (or fractional-order positive switched systems) was proposed. In [29], the problem of guaranteed cost finite-time control for positive switched linear systems with time-varying delays was considered and a cost function of positive systems (or positive switched systems) was also presented. Based on [29], [30] extended guaranteed cost finite-time control to positive switched nonlinear systems with D-perturbation. It is worth noting that [28–30] are involved in continuous-time positive switched systems. However, the problem of guaranteed cost finite-time control for discrete-time positive impulsive switched systems is still open, which inspires us for this study.

In this paper, we consider the problem of GCFTB of discrete-time positive impulsive switched systems by constructing the MLCLF with average dwell time (ADT) technique. Firstly, the concept of guaranteed cost finite-time boundedness is extended to discrete-time positive impulsive switched systems. Secondly, a state feedback controller is designed and sufficient conditions are obtained to guarantee that the closed-loop system is GCFTB. Some sufficient conditions are obtained by linear programming.

The rest of the paper is organized as follows. Section 2 gives some necessary preliminaries and problem statements. In Section 3, the main results are given. In Section 4, a numerical example is provided. Section 5 concludes the paper.

Notations. The representation $A \succ 0$ ($\succcurlyeq 0$, $\prec 0$, $\preccurlyeq 0$) means that $a_{ij} > 0$ (≥ 0, <0, ≤ 0), which is also applying to a vector. $A \succ B$ ($A \succeq B$) means that $A - B \succ 0$ ($A - B \succeq 0$). R_+^n is the n-dimensional nonnegative (positive) vector space. $R^{n\times n}$ denotes the space of $n \times n$ matrices with real entries. I_l represents the l-dimensional vector $[1, \ldots, 1]^T$. A^T denotes the transpose of matrix A. 1-norm $\|x\|$ is defined by $\|x\| = \sum_{k=1}^{n} |x_k|$. N and N^+ are the sets of nonnegative and positive integers. Z^+ denotes the set of positive integers. Matrices are assumed to have compatible dimensions for calculating if their dimensions are not explicitly stated.

2. Preliminaries and Problem Statements

Consider the following discrete-time positive impulsive switched systems:

$$
\begin{aligned}
x(k+1) &= A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + C_{\sigma(k)}w(k), \\
&\qquad k \neq k_m - 1,\ m \in Z^+ \\
x(k+1) &= E_{\sigma(k)}x(k), \quad k = k_m - 1,\ m \in Z^+,
\end{aligned} \tag{1}
$$

where $k \in N$, $x(t) \in R^n$ is the system state, and $u(t) \in R^l$ represents the control input. $\sigma(k)$ represents switching signal of system and takes values in a finite set $I = 1, 2, \ldots, S$, $S \in N^+$. In general, A_i, B_i, C_i, and E_i are the ith subsystem if $\sigma(k) = i \in I$. $k_0 = 0$ is the initial time. k_m ($m \in Z^+$) denotes the mth impulsive switching instant. Moreover, $\sigma(k) = i \in I$ means that the ith subsystem is active. $\sigma(k-1) = j$ and $\sigma(k) = i$ ($i \neq j$) indicate that k is a switching instant at which the system is switched from the jth subsystem to the ith subsystem. At switching instants, there exist impulsive jumps described by (1). A_p, B_p, C_p, and E_p are constant matrices with suitable dimensions, $w(k) \in R^l$ is the exogenous disturbance and defined as

$$
\sum_{k=0}^{T_f} \|w(k)\| \leq d, \tag{2}
$$

with a known scalar $d > 0$ and a given finite-time threshold value T_f.

Next, we will give some definitions and lemmas for system (1).

Definition 1. System (1) is said to be positive if for any switching signals $\sigma(k)$, any disturbance input $w(k) \succeq 0$, and control input $u(k) \succeq 0$, the corresponding trajectory satisfies $x(k) \succeq 0$ for all $k \geq 0$.

Lemma 2 (see [25]). *System (1) is positive if and only if $A_i \succeq 0$, $B_i \succeq 0$, $C_i \succeq 0$, and $E_i \succeq 0$, where $i \in I$.*

Definition 3. For any switching signal $\sigma(k)$ and any $t_2 \geq t_1 \geq 0$, let $N_\sigma(t_1, t_2)$ denote the switching numbers over the interval $[t_1, t_2)$. For given $t_\iota > 0$ and $n_0 > 0$, if the inequality

$$
N_\sigma(t_1, t_2) \leq n_0 + \frac{t_2 - t_1}{t_\iota}, \tag{3}
$$

holds, then t_ι is called an average dwell time, and n_0 is called a chattering bound. Generally, we choose $n_0 = 0$.

Definition 4 (finite-time stability (FTS)). For a given time T_f and two vectors $\alpha \succ \beta \succ 0$, discrete-time positive impulsive switched system (1) with $\omega(k) \equiv 0$ is said to be FTS with respect to $(\alpha, \beta, T_f, \sigma(k))$, if

$$
\begin{aligned}
&x^T(0)\beta \leq 1 \Longrightarrow \\
&x^T(k)\alpha < 1, \quad \forall k \in [0, T_f],
\end{aligned} \tag{4}
$$

where k is an any time point on the time interval $[0, T_f]$.

Definition 5 (finite-time boundedness (FTB)). For a given constant T_f, and two vectors $\alpha \succ \beta \succ 0$, discrete-time positive impulsive switched system (1) is said to be FTB with respect to $(\alpha, \beta, T_f, d, \sigma(k))$, where $w(t)$ satisfies (2), if

$$
\begin{aligned}
&x^T(0)\beta \leq 1 \Longrightarrow \\
&x^T(k)\alpha < 1, \quad \forall k \in [0, T_f],
\end{aligned} \tag{5}
$$

where k is an any time point on the time interval $[0, T_f]$.

Now we give some new definitions for our further study.

Definition 6. Define the cost function of discrete-time positive impulsive switched system (1) as follows:

$$
J = \sum_{s=0}^{T_f - 1} \left(x^T(s) R_1 + u^T(s) R_2\right), \tag{6}
$$

where $R_1 \succ 0$ and $R_2 \succ 0$ are two given vectors.

Remark 7. It should be noted that the proposed cost function is different from the general one, such as [26–28]; this definition provides a more useful description, because it takes full advantage of the characteristics of nonnegative states of discrete-time positive impulsive switched systems.

Definition 8 (GCFTB). For a given time constant T_f and two vectors $\varsigma \succ \rho \succ 0$, consider discrete-time positive impulsive switched system (1) and cost function (6); if there exist a control law $u(t)$ and a positive scalar $J^\star$ such that the closed-loop system is FTB with respect to $(\alpha, \beta, T_f, d, \sigma(k))$ and the cost function satisfies $J \le J^\star$, then the closed-loop system is called GCFTB, where $J^\star$ is a guaranteed cost value and $u(t)$ is a guaranteed cost finite-time controller.

3. Main Results

3.1. Guaranteed Cost Finite-Time Boundedness Analysis. In this subsection, we will focus on the problem of GCFTB for discrete-time positive impulsive switched system (1) with $u(k) = 0$. The following theorem gives sufficient conditions of GCFTB for system (1) with $u(k) = 0$.

Theorem 9. *Consider the discrete-time positive impulsive switched system (1) with $u(k) = 0$, for a given time constant T_f, vectors $\alpha \succ \beta \succ 0$ and $R_1 \succ 0$; if there exist a set of positive vectors ν_i, ν_j, $i \ne j$, $i, j \in I$ and positive constants ϕ_1, ϕ_2, ς, $\xi > 1$, $\mu > 1$ such that the following inequalities hold:*

$$A_i^T \nu_i + R_1 - \xi \nu_i \prec 0, \tag{7}$$

$$C_i^T \nu_i - \varsigma I_l \prec 0, \tag{8}$$

$$E_i^T \nu_j - \mu \nu_i \prec 0, \tag{9}$$

$$\phi_1 \alpha \prec \nu_i \prec \phi_2 \beta, \tag{10}$$

$$\phi_1 > (\phi_2 + \varsigma d)\, \xi^{T_f}, \tag{11}$$

where $\nu_i = [\nu_{i1}, \nu_{i2}, \ldots, \nu_{in}]^T$ and ν_{ir} represents the ith elements of the vectors ν_i, respectively, then under the following ADT scheme:

$$T_\alpha > T_\alpha^* = \frac{T_f \ln \mu}{\ln(\phi_1) - \ln(\phi_2 + \varsigma d) - T_f \ln(\xi)}, \tag{12}$$

system (1) with $u(k) = 0$ is GCFTB with respect to $(\alpha, \beta, T_f, d, \sigma(k))$ and the guaranteed cost value of system (1) with $u(k) = 0$ is given by

$$J = \sum_{k=0}^{T_f - 1} x^T(k) R_1 \le J^* = \xi^{T_f} \mu^{T_f / T_\alpha^*} (\phi_2 + \varsigma d). \tag{13}$$

Proof. Construct the multiple linear copositive Lyapunov function (MLCLF) for system (1) with $u(k) = 0$ as follows:

$$V_i(x(k)) = x^T(k)\, \nu_{(i)}, \tag{14}$$

where $i \in I$.

Suppose a switching sequence $0 = k_0 \le k_1 \le \cdots \le k_m \le k_{m+1} \le \cdots \le T_f$. Without loss of generality, we assume that subsystem i is activated at the switching instant k_{m-1} and the subsystem j is activated at the switching instant k_m.

When $k \in [k_{m-1}, k_m - 1), m \in N$, $\sigma(k) = \sigma(k+1) = i$, along the trajectory of system (1) with $u(k) = 0$, the difference of the MLCLF is

$$\begin{aligned} &\triangle V_i(x(k)) + x^T(k) R_1 \\ &\quad = V_i(x(k+1)) - V_i(x(k)) + x^T(k) R_1 \\ &\quad = x^T(k) \left(A_i^T \nu_i + R_1 - \nu_i\right) + \omega^T(k) C_i^T \nu_i. \end{aligned} \tag{15}$$

From (7), (8) and $x(k) \succeq 0, k \in N$, we have

$$\begin{aligned} &\triangle V_i(x(k)) + x^T(k) R_1 \\ &\quad \le (\xi - 1) x^T(k) \nu_i + \varsigma \omega^T(k) I_l. \end{aligned} \tag{16}$$

It implies that

$$V_i(x(k+1)) \le \xi x^T(k) \nu_i + \varsigma \|\omega(k)\|_1. \tag{17}$$

When $k = k_m - 1, \sigma(k+1) = \sigma(k_m) = j, \sigma(k) = \sigma(k_m - 1) = i$, $i \ne j$. Along the trajectory of system (1) with $u(k) = 0$, we have

$$\begin{aligned} &V_j(x(k+1)) - \mu V_i(x(k)) \\ &\quad = x^T(k+1) \nu_j - \mu x^T(k) \nu_i \\ &\quad \le x^T(k) \left(E_i^T \nu_j - \mu \nu_i\right). \end{aligned} \tag{18}$$

From (9) and $x(k) \succeq 0, k \in N$, we have

$$V_j(x(k+1)) \le \mu V_i(x(k)), \quad i \ne j. \tag{19}$$

So, when $k \in [k_m, k_{m+1})$, from (19), we get

$$\begin{aligned} V_{\sigma(k)}(x(k)) &< \xi^{k-k_m} V_{\sigma(k_m)}(x(k_m)) \\ &\quad + \varsigma \sum_{s=k_m}^{k-1} \xi^{k-1-s} \|\omega(s)\|_1 \\ &< \mu \xi^{k-k_m} V_{\sigma(k_m - 1)}(x(k_m - 1)) \\ &\quad + \varsigma \sum_{s=k_m}^{k-1} \xi^{k-1-s} \|\omega(s)\|_1. \end{aligned} \tag{20}$$

Repeating the procedure of (20) and noting $\xi > 1$, we obtain

$$\begin{aligned} V_{\sigma(k)}(x(k)) &< \xi^{k-k_m} V_{\sigma(k_m)}(x(k_m)) \\ &\quad + \varsigma \sum_{s=k_m}^{k-1} \xi^{k-1-s} \|\omega(s)\|_1 \\ &< \mu \xi^{k-k_m} V_{\sigma(k_m-1)}(x(k_m-1)) \\ &\quad + \varsigma \sum_{s=k_m}^{k-1} \xi^{k-1-s} \|\omega(s)\|_1 \\ &< \mu \xi^{k-k_m} \xi^{k_m-1-k_{m-1}} V_{\sigma(k_{m-1})}(x(k_{m-1})) \\ &\quad + \varsigma \sum_{s=k_m}^{k-1} \xi^{k-1-s} \|\omega(s)\|_1 \\ &\quad + \mu\varsigma \sum_{s=k_{m-1}}^{k_m-1} \xi^{k_m-1-s} \|\omega(s)\|_1 \\ &< \mu \xi^{k-k_{m-1}} V_{\sigma(k_{m-1})}(x(k_{m-1})) \\ &\quad + \varsigma \sum_{s=k_m}^{k-1} \xi^{k-1-s} \|\omega(s)\|_1 \\ &\quad + \mu\varsigma \sum_{s=k_{m-1}}^{k_m-1} \xi^{k_m-1-s} \|\omega(s)\|_1 . \end{aligned} \tag{21}$$

By iterative operation, we get

$$\begin{aligned} V_{\sigma(k)}(x(k)) &< \mu^2 \xi^{k-k_{m-2}} V_{\sigma(k_{m-2})}(x(k_{m-2})) \\ &\quad + \varsigma \sum_{s=k_m}^{k-1} \xi^{k-1-s} \|\omega(s)\|_1 \\ &\quad + \mu\varsigma \sum_{s=k_{m-1}}^{k_m-1} \xi^{k_m-1-s} \|\omega(s)\|_1 \\ &\quad + \mu^2\varsigma \sum_{s=k_{m-2}}^{k_{m-1}-1} \xi^{k_m-1-s} \|\omega(s)\|_1 \le \cdots \\ &\le \mu^{N_\sigma(k,k_0)} \xi^{k-k_0} V_{\sigma(k_0)}(x(k_0)) \\ &\quad + \varsigma \sum_{s=k_m}^{k-1} \xi^{k-1-s} \|\omega(k)\|_1 \\ &\quad + \mu\varsigma \sum_{s=k_{m-1}}^{k_m-1} \xi^{k_m-1-s} \|\omega(s)\|_1 \\ &\quad + \mu^2\varsigma \sum_{s=k_{m-2}}^{k_{m-1}-1} \xi^{k_m-1-s} \|\omega(s)\|_1 + \cdots \\ &\quad + \mu^{N_\sigma(k-1,k_0)}\varsigma \sum_{s=k_0}^{k_1-1} \xi^{k_1-1-s} \|\omega(s)\|_1 \\ &= \mu^{N_\sigma(k,k_0)} \xi^{k-k_0} V_{\sigma(k_0)}(x(k_0)) \\ &\quad + \mu^{N_\sigma(k-1,s)}\varsigma \sum_{s=k_0}^{k-1} \xi^{k-1-s} \|\omega(s)\|_1 . \end{aligned} \tag{22}$$

According to (2), $\xi > 1$ and $\mu > 1$, we also have

$$V_{\sigma(k)}(x(k)) < \mu^{N_\sigma(k,k_0)} \xi^{k-k_0} \left(V_{\sigma(k_0)}(x(k_0)) + \varsigma d\right). \tag{23}$$

From (10) and (14), we have

$$\begin{aligned} V_{\sigma(k)}(x(k)) &= x^T(k)\, \nu_{\sigma(k)} \ge \phi_1 x^T(k)\, \alpha, \\ V_{\sigma(k_0)}(x(k_0)) &= x^T(0)\, \nu_{\sigma(k_0)} \le \phi_2 x^T(0)\, \beta. \end{aligned} \tag{24}$$

From (23) to (24) and $k \in [0, T_f]$, we obtain

$$\begin{aligned} x^T(k)\,\alpha &\le \frac{1}{\phi_1} \mu^{T_f/T_\alpha} \xi^{T_f} \left(\phi_2 x^T(0)\,\beta + \varsigma d\right) \\ &\le \frac{1}{\phi_1} \mu^{T_f/T_\alpha} \xi^{T_f} \left(\phi_2 + \varsigma d\right). \end{aligned} \tag{25}$$

Substituting (12) into (25), one has

$$x^T(k)\,\alpha < 1, \quad k \in \left[0, T_f\right]. \tag{26}$$

According to Definition 5, we conclude that system (1) with $u(k) = 0$ is FTB with respect to $(\alpha, \beta, T_f, d, \sigma(k))$.

Next, we will give the guaranteed cost value of system (1) with $u(k) = 0$.

When $k \in [k_{m-1}, k_m - 1)$, $m \in N$, according to (16), we know

$$V_i(x(k)) \le \xi x^T(k)\, \nu_i + \varsigma \|\omega(k)\|_1 - x^T(k)\, R_1. \tag{27}$$

Similar to the proof process of (17)–(22), for any $k \in [0, T_f]$ and $\mu > 1$, we can obtain

$$\begin{aligned} V_{\sigma(k)}(x(k)) &< \mu^{N_\sigma(k,k_0)} \xi^{k-k_0} V_{\sigma(k_0)}(x(k_0)) \\ &\quad + \mu^{N_\sigma(k,k_0)}\varsigma \sum_{s=0}^{k-1} \xi^{k-s} \|\omega(s)\|_1 \\ &\quad - \sum_{s=k_0}^{k-1} x^T(s)\, R_1. \end{aligned} \tag{28}$$

Noting that $V_{\sigma(k)}(x(k)) > 0$, (28) can be rewritten as

$$\begin{aligned} 0 &< \mu^{N_\sigma(k,k_0)} \xi^{k-k_0} V_{\sigma(k_0)}(x(k_0)) \\ &\quad + \mu^{N_\sigma(k,k_0)}\varsigma \sum_{s=0}^{k-1} \xi^{k-s} \|\omega(s)\|_1 - \sum_{s=0}^{k-1} x(s)^T R_1. \end{aligned} \tag{29}$$

Substituting (2) into (29) and letting $k = T_f$, we get

$$\sum_{s=0}^{T_f-1} x(s)^T R_1 < \mu^{N_\sigma(k,k_0)} \xi^{k-k_0} \left(V_{\sigma(k_0)}(x(k_0)) + \varsigma d\right) < \mu^{N_\sigma(T_f,k_0)} \xi^{T_f-k_0} \left(V_{\sigma(k_0)}(x(k_0)) + \varsigma d\right). \quad (30)$$

Then we can obtain

$$J = \sum_{s=0}^{T_f-1} x^T(s) R_1 \le J^* = \xi^{T_f} \mu^{T_f/T_\alpha^*} (\phi_2 + \varsigma d). \quad (31)$$

Therefore, according to Definition 8, we can conclude that system (1) with $u(k) = 0$ is GCGTB. Thus, the proof is completed. □

3.2. Guaranteed Cost Finite-Time Controller Design. In this subsection, we are concerned with the guaranteed cost finite-time controller design of discrete-time positive impulsive switched system (1). Under the controller $u(t) = K_{\sigma(k)}x(k)$, the corresponding closed-loop system is given by

$$\begin{aligned} x(k+1) &= \left(A_{\sigma(k)} + B_{\sigma(k)}K_{\sigma(k)}\right)x(k) + C_{\sigma(k)}w(k), \\ &\qquad k \ne k_m - 1,\ m \in Z^+, \\ x(k+1) &= E_{\sigma(k)}x(k), \quad k = k_m - 1,\ m \in Z^+. \end{aligned} \quad (32)$$

By Lemma 2, to guarantee the positivity of system (32), $A_i + B_iK_i \succeq 0$ should be satisfied, $\forall i \in I$. The following Theorem 10 gives some sufficient conditions to guarantee that the closed-loop system (32) is GCFTB.

Theorem 10. *Consider the discrete-time positive impulsive switched system (32), for a given time constant T_f, vectors $\alpha \succ \beta \succ 0$, $R_1 \succ 0$, and $R_2 \succ 0$; if there exist positive vectors ν_i, f_i, $i \in I$ and positive constants ϕ_1, ϕ_2, ς, $\xi > 1$, $\mu > 1$, such that ((8)–(11)) and the following inequalities hold:*

$$A_i + B_iK_i \succeq 0, \quad (33)$$

$$A_i^T\nu_i + f_i + R_1 + K_i^TR_2 - \xi\nu_i \prec 0, \quad (34)$$

where $f_i = K_i^TB_i^T\nu_i$, $\nu_i = [\nu_{i1}, \nu_{i2}, \ldots, \nu_{in}]^T$, and ν_{ir} represents the ith elements of the vectors ν_i, then under the following ADT scheme (12), the resulting closed-loop system (32) is GCFTB with respect to $(\alpha, \beta, T_f, d, \sigma(k))$ and the guaranteed cost value of system (32) is given by

$$J = \sum_{k=0}^{T_f-1} \left(x^T(k) R_1 + x^T(k) K_i^T R_2\right) \le J^* = \xi^{T_f} \mu^{T_f/T_\alpha^*} (\phi_2 + \varsigma d). \quad (35)$$

Proof. From (33), we know that $A_i + B_iK_i \succeq 0$. According to Lemma 2, system (32) is positive. Next, we prove the guaranteed cost finite-time stability of system (32).

Replacing R_1 in (27) with $R_1 + K_i^TR_2$, we have

$$V_i(x(k)) \le \xi x^T(k)\nu_i + \varsigma \|\omega(k)\|_1 - x^T(k)\left(R_1 + K_i^TR_2\right). \quad (36)$$

Similar to the proof process of (17)–(22), for any $k \in [0, T_f]$ and $\mu > 1$, we can obtain

$$V_{\sigma(k)}(x(k)) < \mu^{N_\sigma(k,k_0)} \xi^{k-k_0} V_{\sigma(k_0)}(x(k_0)) + \mu^{N_\sigma(k,k_0)} \varsigma \sum_{s=0}^{k-1} \xi^{k-s} \|\omega(s)\|_1 - \sum_{s=k_0}^{k-1} x^T(s)\left(R_1 + K_i^TR_2\right). \quad (37)$$

Noting that $V_{\sigma(k)}(x(k)) > 0$, (37) can be rewritten as

$$0 < \mu^{N_\sigma(k,k_0)} \xi^{k-k_0} V_{\sigma(k_0)}(x(k_0)) + \mu^{N_\sigma(k,k_0)} \varsigma \sum_{s=0}^{k-1} \xi^{k-s} \|\omega(k)\|_1 - \sum_{s=0}^{k-1} x(s)^T\left(R_1 + K_i^TR_2\right). \quad (38)$$

Substituting (2) into (38) and letting $k = T_f$, we get

$$\sum_{s=0}^{T_f-1} x(s)^T\left(R_1 + K_i^TR_2\right) < \mu^{N_\sigma(k,k_0)} \xi^{k-k_0} \left(V_{\sigma(k_0)}(x(k_0)) + \varsigma d\right) < \mu^{N_\sigma(T_f,k_0)} \xi^{T_f-k_0} \left(V_{\sigma(k_0)}(x(k_0)) + \varsigma d\right). \quad (39)$$

From (12), we can obtain

$$J = \sum_{s=0}^{T_f-1} x^T(s)\left(R_1 + K_i^TR_2\right) \le J^* = \xi^{T_f} \mu^{T_f/T_\alpha^*} (\phi_2 + \varsigma d). \quad (40)$$

The proof is completed. □

Next, an algorithm is presented to obtain the feedback gain matrices K_i, $i \in I$.

Algorithm 11.

Step 1. Inputting matrices $A_i, B_i, C_i, E_i, R_1, R_2, \alpha$, and β.

Step 2. By adjusting the parameters ξ, μ, then solving (8)–(11) and (34) via linear programming, positive vectors ν_p, f_p, and K_p can be obtained. If $K_p \succ 0$, turn to the next step. Otherwise, return to Step 1.

Step 3. Substituting ν_p and K_p into $f_p = K_p^TB_p^T\nu_p$, $\tilde{f}_p$ can be obtained. If $\tilde{f}_p - f_p \preceq 0$, then K_p are admissible. Otherwise, return to Step 1.

4. Numerical Example

Consider the discrete-time positive impulsive switched system (1) with the parameters as follows:

$$A_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix},\quad B_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.1 \end{bmatrix},\quad C_1 = \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.3 \end{bmatrix},\quad E_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.4 & 0.2 \\ 0.3 & 0.2 \end{bmatrix},\quad B_2 = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},\quad C_2 = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.2 \end{bmatrix},\quad E_2 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$R_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix},\quad R_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix},\quad \alpha = \begin{bmatrix} 0.02 \\ 0.03 \end{bmatrix},\quad \beta = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}. \tag{41}$$

Choosing $T_f = 5$, $\xi = 1.1$, $\mu = 1.05$, $w(k) = [0.1(\cos(0.3k))^2, 0.1(\cos(0.3k))^2]^T$, $d = 1.2$. Solving the inequalities in Theorem 10 by linear programming, we have

$$\nu_1 = \begin{bmatrix} 0.1332 \\ 0.2021 \end{bmatrix},\quad \nu_2 = \begin{bmatrix} 0.1332 \\ 0.2023 \end{bmatrix},\quad f_1 = \begin{bmatrix} 0.1263 \\ 0.1432 \end{bmatrix},$$
$$f_2 = \begin{bmatrix} 0.1113 \\ 0.1328 \end{bmatrix},\quad K_1 = \begin{bmatrix} 0.0246 & 0.0252 \\ 0.0251 & 0.0254 \end{bmatrix},\quad K_2 = \begin{bmatrix} 0.0242 & 0.0251 \\ 0.0247 & 0.0253 \end{bmatrix}. \tag{42}$$

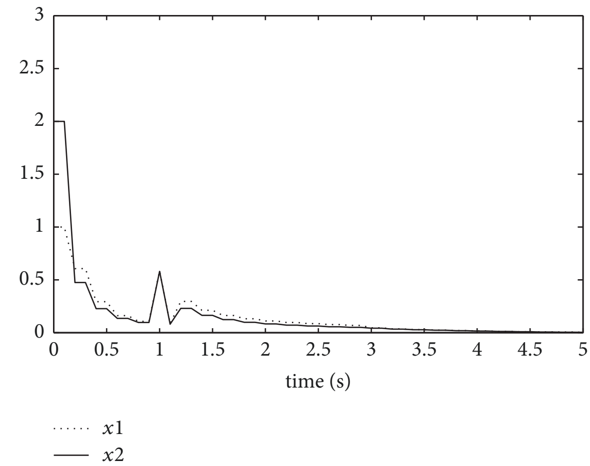

Figure 1: State trajectories of closed-loop system (1).

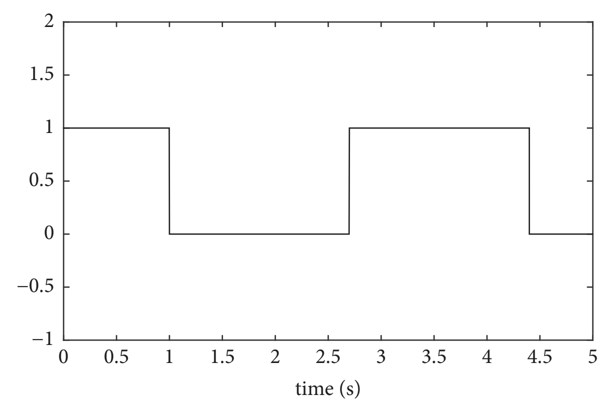

Figure 2: Switching signal of system (1) with ADT.

It is easy to confirm that $\tilde{f}_p - f_p \leq 0$ and (33) is satisfied; then K_p are admissible. According to (12), we get $T_\alpha^* = 1.7$.

The simulation results are shown in Figures 1–3, where the initial conditions of system (1) are $x(0) = [1, 2]^T$, which meet the condition $x^T(k)\alpha < 1$. The state trajectory of the closed-loop system is shown in Figure 1. The switching signal $\sigma(k)$ is depicted in Figure 2. Figure 3 plots the evolution of $x(t)\alpha$, which implies that the corresponding closed-loop system is GCFTB with respect to $(\alpha, \beta, T_f, d, \sigma(k))$, and the cost value $J^* = 3.72$, which can be obtained by (35).

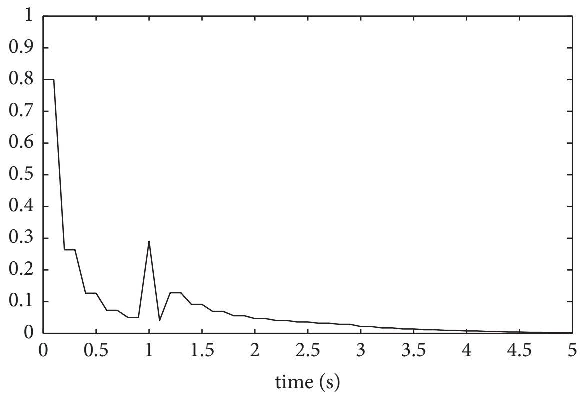

FIGURE 3: The evolution of $x^T(t)\alpha$ of system (1).

5. Conclusions

In this paper, we have considered the issue of guaranteed cost finite-time control for discrete-time positive impulsive switched systems. Based on the ADT approach, a guaranteed cost finite-time controller is constructed to guarantee that the closed-loop system is GCFTB. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The authors are thankful for the support of the National Natural Science Foundation of China (under Grants nos. U1404610, 61473115, and 61374077) and Young Key Teachers Plan of Henan Province (2016GGJS-056).

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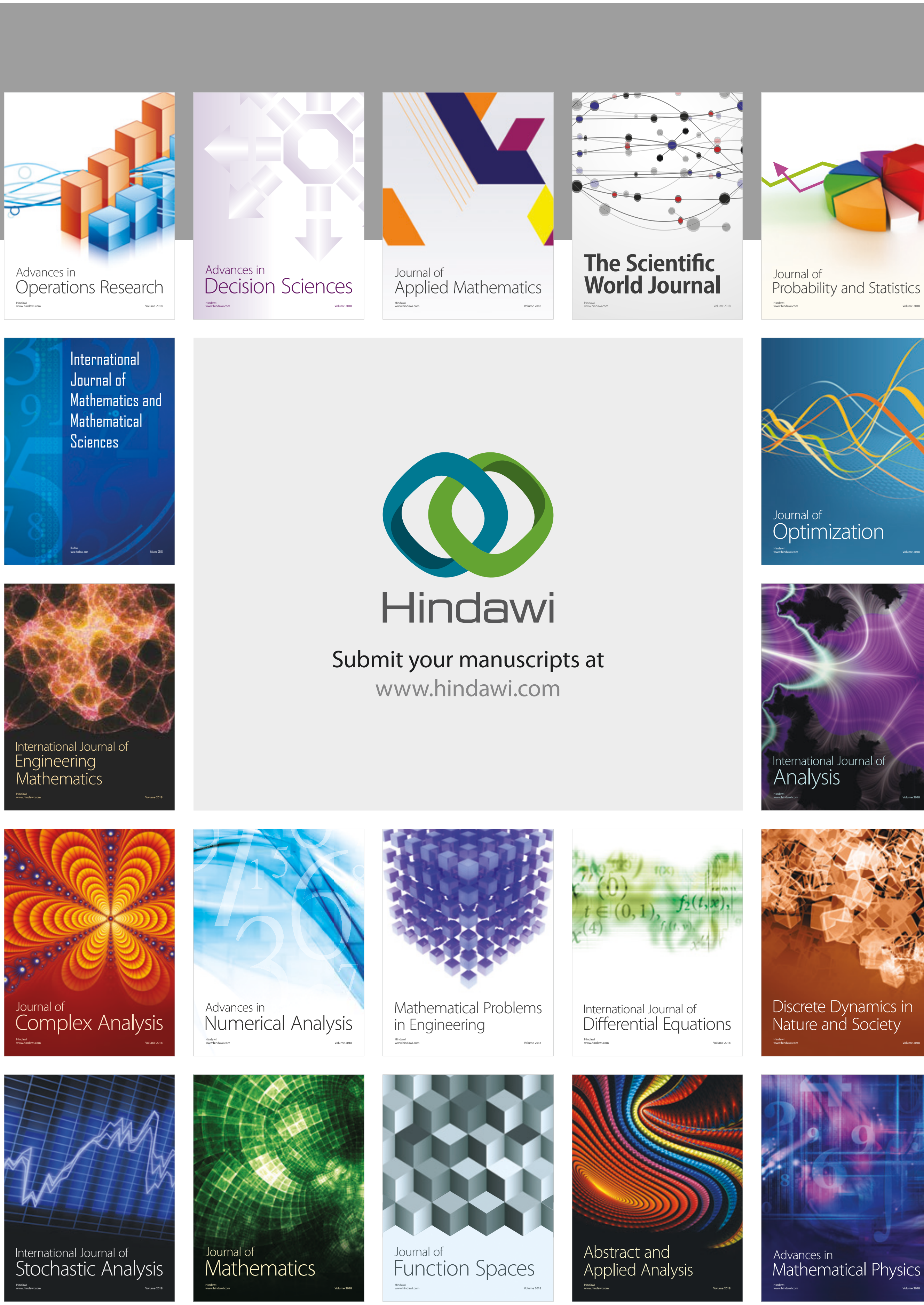
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