

### Research Article

# Sliding-Mode Control of the Active Suspension System with the Dynamics of a Hydraulic Actuator

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An improved model of the active suspension system is proposed. Compared with the existing model of active suspension system, the dynamics of a hydraulic actuator in the active suspension system is fully considered in the proposed model. Based on the proposed model, a sliding-mode control method is designed to control the active suspension system. Stability proof and analysis of the closed-loop system of the active suspension is given by using Lyapunov stability theory. At last, the reliability and feasibility of the proposed sliding-mode control method are evaluated by computer simulation. Simulation research shows that the proposed sliding-mode control method can obtain good control performance for the active suspension system.

#### 1. Introduction

The main function of suspension system in the vehicle is to improve the riding comfort and the road-holding ability. By using the suspension system, acceleration amplitude of the sprung mass in a vehicle can be suppressed and tire deflection can be reduced. Therefore, the riding comfort and the roadholding ability mentioned above can be enhanced effectively. The suspension system includes the passive, semiactive, and active suspension. Compared with the passive and semiactive suspension, active suspension can regulate the suspension force according to the operation state of the vehicle. Therefore, in the research community and automotive industry, the active suspension system has attracted more and more attention of many researchers and engineers dedicated to improving vehicle performance.

In the past few years, some control methods of the active suspension system were proposed to improve the riding comfort and the road-holding ability of the vehicle. In [1], a quadratic-finite-horizon-optimal control method with the robust stability of the uncertain active suspension system is

presented. In [2], a new exponential stabilization criterion of the suspension system via the dynamic state feedback control is derived, and the optimization problem of exponential stabilization is solved by using the PSO method. In [3], a robust sampled-data  $H_\infty$  control method of the active suspension system is proposed. A Lyapunov functional approach is employed to establish the  $H_\infty$  performance. In [4, 5], a finite frequency approach of the active suspension is designed. The finite frequency approach can suppress the vibration more effectively for the concerned frequency range. In [6], the multiobjective  $H_{\infty}$  optimal control method with the actuator delay of the active suspension system is proposed, and the effectiveness of the proposed control method is demonstrated by using an actual example. In [7], an adaptive control method of the active suspension with unknown nonlinearities is designed. By using the proposed control method, the transient and steady-state suspension responses are guaranteed. In [8], an adaptive control method with the new robust adaptive law of the active suspension is proposed. The mitigation of the vertical and pitch displacements can be achieved with the proposed control to improve the ride

comfort. In [9], a robust adaptive control method of an antilock braking system with an active suspension is proposed, where a Takagi-Sugeno (T-S) fuzzy-neural network is introduced to estimate the unknown model. In [10], an indirect adaptive interval type-2 fuzzy-neural network (FNN) control method of the active suspension is designed to control the quarter-car suspension system. Both riding comfort and good handling can be achieved by using the proposed control method. In [11], a sampled-data  $H_{\infty}$  control method of the active suspension system is developed, and the T-S fuzzy model is introduced to estimate the uncertain unknown function in the active suspension system.

Although these existing works have obtained acceptable control performance for the active suspension in the vehicle, there are some important issues that should be discussed further. For example, the input control signal in the existing works for the active suspension control system is always considered as the suspension force. However, in the actual practice, suspension force cannot directly act as the suspension system. The suspension force is performed by using the hydraulic actuator. Therefore, the dynamics of the hydraulic actuator is an important factor that affects the operation performance of the active suspension system. However, in the existing works mentioned above for the active suspension control, the dynamics of the hydraulic actuator is rarely considered. To improve the control performance of the active suspension, an improved nonlinear model of the active suspension system is proposed. Compared with the existing model of the active suspension system, the dynamics of a hydraulic actuator in the active suspension system is fully considered in the proposed model. Based on the proposed model, a sliding-mode control method is presented to control the active suspension system.

The rest of this paper is organized as follows. The dynamic model of quarter-car active suspension with the dynamics of hydraulic actuator is described in Section 2. The sliding-mode control method of the active suspension is designed in Section 3. Section 4 is the presentation and analysis of the simulation results. At last, some conclusions are given in Section 5.

## 2. System Description and Dynamic Model of the Active Suspension

A simplified quarter-car model of the active suspension system is shown in Figure 1.

In this section, sprung mass  $m_s$  is the mass of the vehicle body. The unsprung mass  $m_u$  is the assembled mass of the axle and wheel. When the vehicle is traveling, the tire is assured contact with the road surface. The tire can be modeled as a linear spring, whose stiffness coefficient is  $k_t$ .

According to Newton's second law, the dynamic equation of active suspension system is derived as

$$m_{\rm s}\ddot{z}_{\rm s} + b_{\rm s}(\dot{z}_{\rm s} - \dot{z}_{\rm u}) + k_{\rm s}(z_{\rm s} - z_{\rm u}) = F_{\rm a},$$
 (1)

$$m_{\rm u} \ddot{z}_{\rm u} + b_{\rm s} (\dot{z}_{\rm u} - \dot{z}_{\rm s}) + k_{\rm s} (z_{\rm u} - z_{\rm s}) + k_{\rm t} (z_{\rm u} - z_{\rm r}) = -F_{\rm a}.$$
 (2)



FIGURE 1: Simplified model of a quarter-car active suspension system.

In many existing works for the active control strategy of the suspension system, the hydraulic actuation force  $F_a$ is often considered as the control command in the closed-loop control system. However, hydraulic force is not a direct control command. In fact, the hydraulic cylinder is an actuator. The hydraulic force  $F_a$  is regulated by the servo-valve control current  $i_f$  in the hydraulic servo system. That is, in the actual control process of the suspension system, the current  $i_f$  is controlled to regulate the hydraulic actuation force, and the hydraulic force  $F_a$  is acted on the suspension system. Therefore, the dynamic characteristics between the  $i_f$  and  $F_a$  are an important factor to affect the operation performance of the active suspension system.

In this paper, the dynamic characteristics between the  $i_{\rm f}$  and  $F_{\rm a}$  are considered during the design process of the control method. The dynamic characteristics between  $i_{\rm f}$  and  $F_{\rm a}$  can be modeled as

$$\dot{F}_{a} = -k_{1}F_{a} + k_{2}(\dot{z}_{s} - \dot{z}_{u}) + k_{3}i_{f},$$
 (3)

where  $k_1$ ,  $k_2$ , and  $k_3$  are positive constants.  $k_1$ ,  $k_2$ , and  $k_3$  are determined by the servo-valve flow gain, the cross-section area of the cylinder, and some other parameters in the hydraulic cylinder.

In this paper, the control signal is selected as

$$u = i_{\rm f}.\tag{4}$$

State variables are defined as

$$x_{1} = z_{s},$$

$$x_{2} = \dot{z}_{s},$$

$$x_{3} = z_{u},$$

$$x_{4} = \dot{z}_{u},$$

$$x_{5} = F_{a}.$$
(5)

According to (1), (2), (3), (4), and (5), the system dynamics are rewritten as the following state-space form model:

$$\begin{aligned} \dot{x}_{1} &= x_{2}, \\ \dot{x}_{2} &= -\frac{b_{s}}{m_{s}}x_{2} + \frac{b_{s}}{m_{s}}x_{4} - \frac{k_{s}}{m_{s}}x_{1} + \frac{k_{s}}{m_{s}}x_{3} + \frac{1}{m_{s}}x_{5}, \\ \dot{x}_{3} &= x_{4}, \\ \dot{x}_{4} &= -\frac{b_{s}}{m_{u}}x_{4} + \frac{b_{s}}{m_{u}}x_{2} - \frac{k_{s}}{m_{u}}x_{3} + \frac{k_{s}}{m_{u}}x_{1} - \frac{k_{t}}{m_{u}}x_{3} + \frac{k_{t}}{m_{u}}z_{r} - \frac{1}{m_{u}}x_{5}, \\ \dot{x}_{5} &= -k_{1}x_{5} + k_{2}(x_{2} - x_{4}) + k_{3}u, \end{aligned}$$

$$(6)$$

$$y = z_{\rm s} - z_{\rm u} = x_1 - x_3, \tag{7}$$

where y is the suspension displacement.

#### 3. Sliding-Mode Control of the Suspension System

3.1. Sliding-Mode Control Design. The control objective of the active suspension system is to make the actual suspension displacement y fast and accurately track its set point  $y_d$ .

The tracking error of a suspension system is defined as

$$e = y_{\rm d} - y. \tag{8}$$

s is the sliding-mode surface, which is described as

$$s = c_1 e + c_2 \dot{e} + \ddot{e},\tag{9}$$

where  $c_1$  and  $c_2$  are positive constants.

The sliding-mode control signal consists of two parts, including the equivalent control signal and the switching control signal [12]. The physical meaning of the equivalent control signal is to keep the trajectories of the dynamical system on the sliding-mode surface. It can be solved from  $\dot{s} = 0$ . The physical meaning of switching control is to make the trajectories of the dynamical system move towards the sliding-mode surface.

The sliding-mode control signal is

$$u = u_{\rm eq} + u_{\rm sw},\tag{10}$$

where  $u_{eq}$  is the equivalent control signal and  $u_{sw}$  is the switching control signal.

The derivative of *s* is given by

$$\dot{s} = c_1 \dot{e} + c_2 \ddot{e} + \ddot{e}. \tag{11}$$

From (7) and (8), the derivative of e is

$$\dot{e} = \dot{y}_{\rm d} - \dot{y} = \dot{y}_{\rm d} - (\dot{x}_1 - \dot{x}_3) = \dot{y}_{\rm d} - x_2 + x_4. \tag{12}$$

Neglecting the external disturbance  $(k_t/m_u)z_r$ , the second derivative of *e* is

$$\begin{split} \ddot{e} &= \ddot{y}_{d} - \dot{x}_{2} + \dot{x}_{4} = \ddot{y}_{d} + \left(\frac{k_{s}}{m_{s}} + \frac{k_{s}}{m_{u}}\right) x_{1} + \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) x_{2} \\ &+ \left(-\frac{k_{s}}{m_{s}} - \frac{k_{s}}{m_{u}} - \frac{k_{t}}{m_{u}}\right) x_{3} + \left(-\frac{b_{s}}{m_{s}} - \frac{b_{s}}{m_{u}}\right) x_{4} + \left(-\frac{1}{m_{s}} - \frac{1}{m_{u}}\right) x_{5}. \end{split}$$
(13)

The third derivative of e is

$$\begin{split} \ddot{e} &= \ddot{y}_{d} + \left(\frac{k_{s}}{m_{s}} + \frac{k_{s}}{m_{u}}\right) \dot{x}_{1} + \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \dot{x}_{2} + \left(-\frac{k_{s}}{m_{s}} - \frac{k_{s}}{m_{u}} - \frac{k_{t}}{m_{u}}\right) \dot{x}_{3} \\ &+ \left(-\frac{b_{s}}{m_{s}} - \frac{b_{s}}{m_{u}}\right) \dot{x}_{4} + \left(-\frac{1}{m_{s}} - \frac{1}{m_{u}}\right) \dot{x}_{5}. \end{split}$$
(14)

Substituting (7) and (8) into (14), the third derivative of e can be rewritten as

$$\begin{split} \ddot{e} &= \ddot{y}_{d} + \left[ -\left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{k_{s}}{m_{s}} - \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{k_{s}}{m_{u}} \right] x_{1} \\ &+ \left[ \left(\frac{k_{s}}{m_{s}} + \frac{k_{s}}{m_{u}}\right) - \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{b_{s}}{m_{s}} - \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{b_{s}}{m_{u}} - \left(\frac{1}{m_{s}} + \frac{1}{m_{u}}\right) k_{2} \right] x_{2} + \left[ \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{k_{s}}{m_{s}} + \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{k_{s}}{m_{u}} + \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{k_{t}}{m_{u}} \right] x_{3} + \left[ \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{b_{s}}{m_{s}} - \left(\frac{k_{s}}{m_{s}} + \frac{k_{s}}{m_{u}} + \frac{k_{t}}{m_{u}}\right) + \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{b_{s}}{m_{u}} + \left(\frac{1}{m_{s}} + \frac{1}{m_{u}}\right) k_{2} \right] x_{4} \\ &+ \left[ \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{1}{m_{s}} + \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) \frac{1}{m_{u}} + \left(\frac{1}{m_{s}} + \frac{1}{m_{u}}\right) k_{1} \right] x_{5} \\ &- \left(\frac{1}{m_{s}} + \frac{1}{m_{u}}\right) k_{3} u. \end{split}$$

$$\tag{15}$$

Substituting (12), (13), and (15) into (11), we have

$$\begin{split} \dot{s} &= c_{1}\dot{y}_{d} + c_{2}\ddot{y}_{d} + \ddot{y}_{d} \\ &+ \left[c_{2}\left(\frac{k_{s}}{m_{s}} + \frac{k_{s}}{m_{u}}\right) - \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{k_{s}}{m_{s}} - \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{k_{s}}{m_{u}}\right]x_{1} \\ &+ \left[-c_{1} + c_{2}\left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right) + \left(\frac{k_{s}}{m_{s}} + \frac{k_{s}}{m_{u}}\right) - \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{b_{s}}{m_{s}} \\ &- \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{b_{s}}{m_{u}} - \left(\frac{1}{m_{s}} + \frac{1}{m_{u}}\right)k_{2}\right]x_{2} + \left[c_{2}\left(-\frac{k_{s}}{m_{s}} - \frac{k_{s}}{m_{u}}\right)\right]x_{3} \\ &+ \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{k_{s}}{m_{s}} + \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{k_{s}}{m_{s}} + \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{k_{t}}{m_{u}}\right]x_{3} \\ &+ \left[c_{1} + c_{2}\left(-\frac{b_{s}}{m_{s}} - \frac{b_{s}}{m_{u}}\right) + \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{b_{s}}{m_{s}} - \left(\frac{k_{s}}{m_{s}} + \frac{k_{s}}{m_{u}} + \frac{k_{t}}{m_{u}}\right)\right) \\ &+ \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{b_{s}}{m_{u}} + \left(\frac{1}{m_{s}} + \frac{1}{m_{u}}\right)k_{2}\right]x_{4} + \left[c_{2}\left(-\frac{1}{m_{s}} - \frac{1}{m_{u}}\right)\right) \\ &+ \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{1}{m_{s}} + \left(\frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}}\right)\frac{1}{m_{u}} + \left(\frac{1}{m_{s}} + \frac{1}{m_{u}}\right)k_{1}\right]x_{5} \\ &- \left(\frac{1}{m_{s}} + \frac{1}{m_{u}}\right)k_{3}u. \end{split}$$

Letting

$$\begin{aligned} \alpha_{1} &= c_{2} \left( \frac{k_{s}}{m_{s}} + \frac{k_{s}}{m_{u}} \right) - \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{k_{s}}{m_{s}} - \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{k_{s}}{m_{u}}, \\ \alpha_{2} &= -c_{1} + c_{2} \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) + \left( \frac{k_{s}}{m_{s}} + \frac{k_{s}}{m_{u}} \right) - \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{b_{s}}{m_{s}} \\ &- \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{b_{s}}{m_{u}} - \left( \frac{1}{m_{s}} + \frac{1}{m_{u}} \right) k_{2}, \\ \alpha_{3} &= c_{2} \left( -\frac{k_{s}}{m_{s}} - \frac{k_{s}}{m_{u}} \right) + \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{k_{s}}{m_{s}} + \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{k_{s}}{m_{s}} \\ &+ \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{k_{t}}{m_{u}}, \\ \alpha_{4} &= c_{1} + c_{2} \left( -\frac{b_{s}}{m_{s}} - \frac{b_{s}}{m_{u}} \right) + \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{b_{s}}{m_{s}} \\ &- \left( \frac{k_{s}}{m_{s}} + \frac{k_{s}}{m_{u}} + \frac{k_{t}}{m_{u}} \right) + \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{b_{s}}{m_{u}} + \left( \frac{1}{m_{s}} + \frac{1}{m_{u}} \right) k_{2}, \\ \alpha_{5} &= c_{2} \left( -\frac{1}{m_{s}} - \frac{1}{m_{u}} \right) + \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{1}{m_{s}} + \left( \frac{b_{s}}{m_{s}} + \frac{b_{s}}{m_{u}} \right) \frac{1}{m_{u}} \\ &+ \left( \frac{1}{m_{s}} + \frac{1}{m_{u}} \right) k_{1}, \\ \beta &= \left( \frac{1}{m_{s}} + \frac{1}{m_{u}} \right) k_{3}, \end{aligned}$$

$$(17)$$

then (16) can be rewritten as

$$\dot{s} = c_1 \dot{y}_d + c_2 \ddot{y}_d + \ddot{y}_d + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \alpha_5 x_5 - \beta u.$$
(18)

Letting  $\dot{s} = 0$ , u in (18) is the equivalent of control signal  $u_{eq}$ . From (18),  $u_{eq}$  is

$$u_{\rm eq} = \frac{1}{\beta} [c_1 \dot{y}_{\rm d} + c_2 \ddot{y}_{\rm d} + \ddot{y}_{\rm d} + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \alpha_5 x_5].$$
(19)

Letting  $\overline{z_r} = (k_t/m_u)\dot{z_r}$  satisfies the following inequation:

$$\left|\overline{z_{\rm r}}\right| \le D. \tag{20}$$

According to the designing principle of the sliding-mode control, the switching control signal  $u_{sw}$  is designed as

$$u_{\rm sw} = \frac{1}{\beta} K sgn(s), \tag{21}$$

where K is a positive constant. K satisfies the following condition:

$$|\overline{z_{\rm r}}| \le D < K. \tag{22}$$

Therefore, sliding-mode control is designed as

$$u = \frac{1}{\beta} [c_1 \dot{y}_d + c_2 \ddot{y}_d + \ddot{y}_d + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \alpha_5 x_5] + \frac{1}{\beta} K sgn(s).$$
(23)

*Remark 1.* For stability, *K* is usually chosen to be conservatively large. This is not very desirable due to the chattering introduced. To improve the response time,  $c_1$  and  $c_2$  should be chosen to be conservatively large.

3.2. Stability Proof of the Closed-Loop System. The Lyapunov function candidate v is given as

$$\nu = \frac{1}{2}s^2. \tag{24}$$

The time derivative of v is

$$\dot{v} = s\dot{s}.$$
 (25)

Substituting (18) into (25), and considering the external disturbance  $(k_t/m_u)z_r$ , we have

$$\dot{\nu} = s\dot{s} = s \left[ c_{1}\dot{y}_{d} + c_{2}\ddot{y}_{d} + \ddot{y}_{d} + \alpha_{1}x_{1} + \alpha_{2}x_{2} + \alpha_{3}x_{3} + \alpha_{4}x_{4} + \alpha_{5}x_{5} - \beta u + \frac{k_{t}}{m_{u}}\dot{z}_{r} \right].$$
(26)

Substituting the controller (23) into (26), we have

$$\dot{v} = s\dot{s} = s \left\{ c_{1}\dot{y}_{d} + c_{2}\ddot{y}_{d} + \ddot{y}_{d} + \alpha_{1}x_{1} + \alpha_{2}x_{2} + \alpha_{3}x_{3} + \alpha_{4}x_{4} + \alpha_{5}x_{5} - [c_{1}\dot{y}_{d} + c_{2}\ddot{y}_{d} + \ddot{y}_{d} + \alpha_{1}x_{1} + \alpha_{2}x_{2} + \alpha_{3}x_{3} + \alpha_{4}x_{4} + \alpha_{5}x_{5}] - Ksgn(s) + \frac{k_{t}}{m_{u}}\dot{z}_{r} \right\},$$
  
$$\dot{v} = s\dot{s} = s \left[ -Ksgn(s) + \frac{k_{t}}{m_{u}}\dot{z}_{r} \right] = -kssgn(s) + s\frac{k_{t}}{m_{u}}\dot{z}_{r} = -k|s| + s\frac{k_{t}}{m_{u}}\dot{z}_{r}.$$
(27)

Considering the inequation (20), we have

$$\dot{\nu} \le 0.$$
 (28)

Therefore, according to the Lyapunov stability theory [13–15], the closed-loop control system is stable from (28).

#### 4. Simulation Results

In this section, a simulation analysis case is given to evaluate the effectiveness and applicability of the controller design method described above.



FIGURE 2: Vertical road profile, that is,  $z_r = 0.1 \sin(t)$ .

TABLE 1: Parameter values.

| m <sub>s</sub> = 275      | $m_{\rm u} = 50$   | $k_{\rm t} = 157,750$ |
|---------------------------|--------------------|-----------------------|
| $k_{\rm s} = 12,015$      | $b_{\rm s} = 1200$ | $c_1 = 10$            |
| <i>c</i> <sub>2</sub> = 5 |                    |                       |

A sinusoidal roadway is introduced as the input disturbance signal in the suspension system, which is assumed as  $z_r = 0.1 \sin(t)$ . The vertical road profile is shown in Figure 2.

Some main parameters of active suspension in the simulation are shown in Table 1.

With the active suspension control, the vibration of the suspension displacement can be effectively suppressed. This effective suppression can avoid damage to the vehicle structure safety. Meanwhile, the service life of the suspension system is also extended with the effective suppression of the vibration of the suspension displacement. In this section, the proposed sliding-mode control method is applied to control the suspension system, which is as the active control. In Figure 3, the control performances of the active control and the passive control are shown for comparison. Suspension passive control means that the suspension force is fixed.

In Figure 3, the solid line is the suspension displacement by using the proposed sliding-mode control method as the active control. The dashed line denotes the suspension displacement response with the passive control. It can be observed that the vibration of the suspension displacement is suppressed to within a small range with the active control.

The control input signal, that is, the servo-valve control current  $i_f$  in the hydraulic servo system, is shown in Figure 4.

#### 5. Conclusions

The dynamical model of the active suspension system is proposed in this paper. The dynamics of the hydraulic system is introduced in the proposed model. Based on the improved model, the sliding-mode control method is designed as the active control for the suspension system. Stability of the closed-loop control system is proven via the Lyapunov stability theory. From the simulation case results, we know that the proposed sliding-mode control method can obtain the satisfactory operation performance for the active suspension system.



FIGURE 3: Suspension displacement of the active control and passive control.



FIGURE 4: The servo-valve control current  $i_{\rm f}$  in the hydraulic servo system.

#### Nomenclature

- $m_{\rm s}$ : Sprung mass
- $m_{\rm u}$ : Unsprung mass
- $k_t$ : Equivalent stiffness coefficient of the tire
- $b_s$ : Equivalent damping coefficient of the damper
- *k*<sub>s</sub>: Equivalent stiffness coefficient of the spring: hydraulic force
- $z_s$ : Vertical displacement of the sprung mass
- $z_{u}$ : Vertical displacements of the unsprung mass
- $z_{\rm r}$ : Vertical road profile.

#### **Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

#### 6

#### **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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