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Research Article

Parameter Identification and Adaptive Control of Uncertain Goodwin Oscillator Networks with Disturbances

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This paper investigates the dynamic properties of a differential equation model of mammals' circadian rhythms, including parameter identification, adaptive control, and outer synchronization. The circadian oscillator network is described by a Goodwin oscillator network, the couplings of which are from vasoactive intestinal polypeptides described by modified Van der Pol oscillators. We build up a drive-response system consisting of two networks with unknown parameters and disturbances. Then, we propose effective parameter updating laws to identify the unknown parameters and design adaptive control strategies to achieve outer synchronization in the drive-response system. As special cases, two succinct corollaries are presented for different instances. All the theoretical results are proved through strict mathematical deduction based on Lyapunov stability theory, and a numerical example is also carried out to illustrate the effectiveness.

1. Introduction

In the past decades, there has been tremendous interest in studying circadian rhythms of mammals at the cellular level [1–8]. Based on experimental findings of system biology and network biology, several gene regulatory network models have been established to describe the circadian rhythm system [3, 4]. Experimental evidence has shown that the circadian rhythms are controlled by a pacemaker located in the suprachiasmatic nucleus (SCN) of the hypothalamus [5] and the circadian oscillator is usually described by a Goodwin oscillator model, which describes a protein which represses the transcription of its own gene via an inhibitor [6]. Now, the Goodwin oscillator model and its variants have been widely adopted as one of the classic hypothetical genetic oscillators [7, 8]. The SCN consists of a dorsomedial shell and a ventrolateral core, and the ventrolateral core can be defined by cells containing vasoactive intestinal polypeptide (VIP) [9]. The circadian oscillators are coupled with each other via the rhythmic influence from VIP, and the VIP is required to maintain circadian synchrony of the SCN [10]. However, we know nothing about the dynamics of the VIP except its fundamental observational properties. Based on these observational properties, the Van der Pol oscillator was usually employed to describe the dynamics of the VIP [11]. In this paper, we will carry out a modified circadian rhythm network model with unknown parameters and investigate its several dynamical properties from the viewpoint of complex network dynamics with the help of nonlinear dynamics theory.

Recent years have seen significant advances in the study of complex network dynamics [12, 13], and its related studies will lead to more potential applications in the future. Synchronization is a kind of typical collective behaviors and basic motions in nature, which is one of the main research focuses in complex network science. From the viewpoint of mathematics, the core of synchronization is the stability of the zero solution of network error systems [14–16]. In previous studies, two effective methods are usually employed: the first one is to study synchronization induced by the mutual couplings between nodes [17, 18], and the second one is to design reasonable control laws [19–21]. A great number of researches on the first method have indicated that synchronization without external control needs certain requirement in both network structures and node dynamics. Therefore, a variety of external control approaches have been developed such as pinning control [22, 23], sliding mode control [24, 25], and feedback control [26, 27].

However, there are still lots of urgent and challenging problems in practical application. For instance, we often know very little about the exact values of system parameters, or there are some time-varying parameters. Under the effects of these uncertainties, the achieved synchronization might be destroyed and broken. Therefore, it is necessary to design an adaptive control law that adapts itself to these uncertainties, which is a popular control technique used for complex network models with unknown parameters [28–30]. The theoretical basis of adaptive control is parameter identification. As far as complex networks are concerned, three dynamic properties of uncertain networks need to be discussed, i.e., parameter identification, adaptive control, and outer synchronization [31]. In order to achieve each one of the three research goals above, Lyapunov stability is usually employed, which will show the convergence of the error systems and the parameter identification laws at the same time. Due to the convenience and effectiveness of adaptive control, it has been widely applied to many fields of science and technology, including secure communication, chaos generator design, biological systems, and information science.

As far as the circadian rhythm model is concerned, it is more difficult to get the exact values of the system parameters, and this becomes one of the interesting and significant questions remaining open for discussion. In order to estimate or evaluate the unknown parameters existing in the circadian rhythm model, lots of researches have been carried out. Based on the time series data of a certain group of individuals in a circadian rhythm model, Tong [32] obtained the estimations of the group level, group amplitude, and group phase. Later, the estimations of the true values of unknown parameters were investigated for different circadian rhythm models [33–35]. Now, it has become an interesting and significant research direction in the fields of system biology. To the best of our knowledge, most of the previous results were based on the statistical method or experimental data, and few theoretical researches have been carried out. Motivated by the discussions mentioned above, this paper aims at providing theoretical estimations of unknown parameters existing in such a network. It may help us build more accurate mathematical models and better understand the circadian rhythms of mammals. Therefore, the subject of this paper has a certain degree of innovation, and it may also have some latent applications. By proposing appropriate parameter updating laws and adaptive control strategies, we identify the unknown parameters successfully and the network realizes outer synchronization. Based on Lyapunov stability theory and matrix theory, we give theoretical proof for adaptive outer synchronization of the Goodwin oscillator network with unknown parameters. As special cases, we present two succinct corollaries for different instances.

The organization of the remaining sections is as follows. In Section 2, some preliminaries are introduced, including the model descriptions of the Goodwin oscillator network with unknown parameters. In Section 3, adaptive controllers and parameter updating laws are designed, and their effectiveness is also proved theoretically. In Section 4, a simple example is provided to verify the validity of the theoretical results. In the last section, conclusions are provided to summarize the contributions of this paper and to highlight some interesting issues as a further work.

1.1. Model Descriptions. The Goodwin model describes a circadian oscillator consisting of three variables, which is illustrated in Figure 1. A clock gene mRNA (a) produces a clock protein (b), which activates a transcriptional inhibitor (c), and in turn inhibits the transcription of the clock gene. By the repression exerted by the inhibitor to the mRNA synthesis, the three variables build up a closed negative feedback loop.

The mathematical model for the circadian oscillator is given as follows, in which each variable is governed by a simple ordinary differential equation:

$$\begin{cases} \dot{a}(t) = v_1 K^n [K^n + c^n(t)]^{-1} - \delta_1 a(t), \\ \dot{b}(t) = v_2 a(t) - \delta_2 b(t), \\ \dot{c}(t) = v_3 b(t) - \delta_3 c(t), \end{cases} \tag{1}$$

where $a(t)$, $b(t)$, and $c(t)$ can be interpreted as the concentrations of clock genes, clock proteins, and transcriptional inhibitors, respectively; the constants v_1, v_2, and v_3 are the dimensionless transcription rates or translation rates; the constants δ_1, δ_2, and δ_3 are the dimensionless degradation rates of the chemical molecules; and K and n are the parameters of the Hill function. The equations above describe what is probably the simplest conceivable control process consistent with certain essential features of the genetic control of enzyme synthesis [6]. For instance, by choosing $K = 1$, $n = 10$, $v_1 = v_2 = v_3 = 1$, and $\delta_1 = \delta_2 = \delta_3 = 0.1$, the system (1) produces a damped oscillator.

In order to produce physiological rhythms, many researches considered the rhythmic influence of vasoactive intestinal polypeptides (VIP). Then, a Goodwin oscillator network model with a coupling term from VIP reads

$$\begin{cases} \dot{a}_i(t) = v_1 K^n [K^n + c_i^n(t)]^{-1} - \delta_1 a_i(t) + \sum_{j=1}^{N} c_{ij} m_j(t), \\ \dot{b}_i(t) = v_2 a_i(t) - \delta_2 b_i(t), \\ \dot{c}_i(t) = v_3 b_i(t) - \delta_3 c_i(t), \end{cases} \tag{2}$$

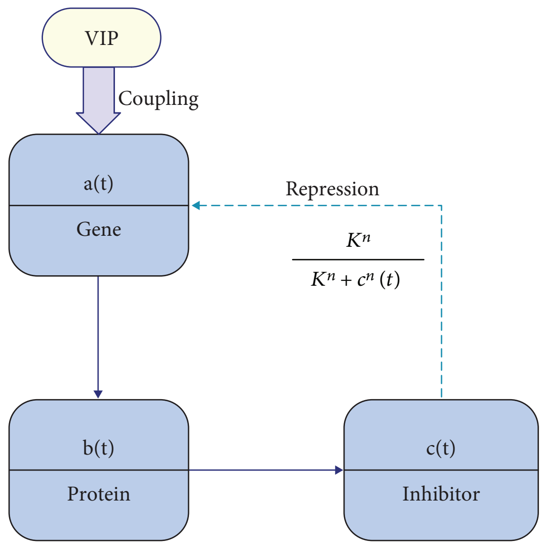

Figure 1: Scheme of a circadian oscillator modeled by a Goodwin oscillator. By the repression exerted by the inhibitor (c (t)) to the clock gene mRNA synthesis (a (t)), the three variables build up a closed negative feedback loop. The coupling from vasoactive intestinal polypeptide (VIP) is required to maintain circadian synchrony of the circadian oscillator.

where $(m_j(t), n_j(t))^{\mathrm{T}}$ is the concentration of VIP in the jth cell. The coupling matrix $C = (c_{ij})_{N\times N}$ represents the coupling configuration; if there is a coupling from cell i to cell j, then denote the weight of this coupling as $c_{ij} > 0$; otherwise, denote $c_{ij} = 0$. Different from most of the previous researches on complex networks, it is assumed that $c_{ii} \geq 0$ because the term $c_{ii}m_i(t)$ describes the coupling from VIP in the cell i to the Goodwin oscillator in the same cell. Throughout this paper, we further assume that the coupling matrix has equal row sums and equal column sums, i.e., there exists a nonnegative constant l such that $\sum_{i=1}^{N} c_{ij} = \sum_{j=1}^{N} c_{ij} = 2l$.

In many previous studies [11], the concentration dynamics of VIP was described by the following modified Van der Pol oscillators:

$$\begin{cases} \dot{m}_i(t) = \beta_1(n_i(t) - 2.5) + \beta_2(m_i(t) - 2.5) - \beta_3(m_i(t) - 2.5)^3 + ka_i(t), \\ \dot{n}_i(t) = \beta_1(m_i(t) - 2.5), \end{cases} \tag{3}$$

where β_p, $p = 1,2,3$, and k are constants.

For ease of notations, we denote $x_i(t) = (a_i(t), b_i(t), c_i(t))^{\mathrm{T}}$ and $y_i(t) = (m_i(t), n_i(t))^{\mathrm{T}}$ and rewrite the network (2)–(3) as follows:

$$\begin{cases} \dot{x}_i(t) = f_1(x_i(t))\alpha_1 + f_2(x_i(t))\alpha_2 + \sum_{j=1}^{N} c_{ij}\Gamma y_j(t), \\ \dot{y}_i(t) = g(y_i(t))\beta + k\Gamma^{\mathrm{T}} x_i(t), \end{cases} \tag{4}$$

where $\alpha_1 = (\nu_1, \nu_2, \nu_3)^{\mathrm{T}}$, $\alpha_2 = (\delta_1, \delta_2, \delta_3)^{\mathrm{T}}$, and $\beta = (\beta_1, \beta_2, \beta_3)^{\mathrm{T}}$ are all real-valued vectors; the coupling component matrix

$$\Gamma = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{5}$$

The matrix functions

$$\begin{aligned} f_1(x_i(t)) &= \mathrm{diag}\left\{K^n[K^n + c_i^n(t)]^{-1}, a_i(t), b_i(t)\right\} \in R^{3\times 3}, \\ f_2(x_i(t)) &= -\mathrm{diag}\left\{a_i(t), b_i(t), c_i(t)\right\} \in R^{3\times 3}, \\ g(y_i(t)) &= \begin{pmatrix} n_i(t) - 2.5 & m_i(t) - 2.5 & (m_i(t) - 2.5)^3 \\ m_i(t) - 2.5 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{6}$$

where $i = 1, 2, \ldots, N$. Assuming that the parameters α_1, α_2, and β, and the network topology matrices C and $\bar{C}$ are all unknown, we build the following response network:

$$\begin{cases} \dot{\bar{x}}_i(t) = f_1(\bar{x}_i(t))\bar{\alpha}_1(t) + f_2(\bar{x}_i(t))\bar{\alpha}_2(t) + \sum_{j=1}^{N} \bar{c}_{ij}\Gamma\bar{y}_j(t) + \Delta_x(t) + u_{xi}(t), \\ \dot{\bar{y}}_i(t) = g(\bar{y}_i(t))\bar{\beta}(t) + k\Gamma^{\mathrm{T}}\bar{x}_i(t) + \Delta_y(t) + u_{yi}(t), \end{cases} \tag{7}$$

where $\bar{x}_i(t) = (\bar{a}_i(t), \bar{b}_i(t), \bar{c}_i(t))^{\mathrm{T}}$, $\bar{y}_i(t) = (\bar{m}_i(t), \bar{n}_i(t))^{\mathrm{T}}$, $\bar{\alpha}_1(t)$, $\bar{\alpha}_2(t)$, and $\bar{\beta}(t)$ are the updating laws of the unknown parameters in the network (4); $\Delta_x(t), \Delta_y(t)$ are the external disturbances such as wind and noise; and $u_{xi}(t), u_{yi}(t)$ are the adaptive controllers left to be designed in the next section, where $i = 1, 2, \ldots, N$. Network (4) and network (7) form a drive-response system; the next section will design adaptive controllers to make the drive-response system realize the outer synchronization and design parameter updating laws to identify the unknown parameters.

2. Adaptive Control Schemes for Outer Synchronization

Let us first carry out the definition of outer synchronization, two hypotheses, and a lemma to prove the effectiveness of our results.

Definition 1. The system (4)–(7) is said to achieve outer synchronization if

$$\begin{aligned} \lim_{t\to\infty} \|e_{xi}(t)\| &= 0, \\ \lim_{t\to\infty} \|e_{yi}(t)\| &= 0, \end{aligned} \tag{8}$$

where $e_{xi}(t) = \bar{x}_i(t) - x_i(t)$ and $e_{yi}(t) = \bar{y}_i(t) - y_i(t)$, where $i = 1, \dots, N$.

Hypothesis 1. For any $x \in R^3$ and $y \in R^2$, denote $X = (x^T, y^T)^T \in R^5$, and

$$F(X, \alpha_1, \alpha_2, \beta) = \left[(f_1(x)\alpha_1 + f_2(x)\alpha_2)^T, \left(g(y)\beta + k\Gamma^T x\right)^T \right]^T. \tag{9}$$

Suppose that there exists a positive constant L such that

$$(Y - X)^T [F(Y, \alpha_1, \alpha_2, \beta) - F(X, \alpha_1, \alpha_2, \beta)] \le L(Y - X)^T (Y - X) \tag{10}$$

holds for any $X, Y \in R^5$.

Since the function $F(X, \alpha_1, \alpha_2, \beta)$ of the network (4) is differentiable and the oscillator is a damped oscillator, there exists a positive constant L satisfying Hypothesis 1 in theory.

Hypothesis 2. For the disturbances $\Delta_x(t)$ and $\Delta_y(t)$, suppose that there exist two positive constants $0 \le \rho_x, \rho_y < \infty$ such that

$$\|\Delta_x(t)\| \le \rho_x, \left\|\Delta_y(t)\right\| \le \rho_y. \tag{11}$$

The second hypothesis guarantees the boundedness of the disturbances, and this paper yields adaptive controllers that are robust against all possible bounded disturbances.

Lemma 1. *For any vectors $x \in R^3$ and $y \in R^2$ and the matrix Γ defined by (5), the following inequality holds:*

$$2x^T \Gamma y \le x^T \Gamma \Gamma^T x + y^T \Gamma^T \Gamma y. \tag{12}$$

Proof. Denote $x = (a, b, c)^T$ and $y = (m, n)^T$; it follows from (5) that $\Gamma^T x = (a, 0)^T$, $\Gamma y = (m,0,0)^T$, and $2x^T \Gamma y = 2ma$. After a simple deduction, one gets that

$$x^T \Gamma \Gamma^T x + y^T \Gamma^T \Gamma y = a^2 + m^2. \tag{13}$$

Based on the inequality $a^2 + m^2 \ge 2am$, the lemma is proved.

Now, with the help of the preceding preliminaries and Lyapunov stability theory, we turn to prove the following theorem.

Theorem 1. *Under Hypothesis 1 and Hypothesis 2, let the parameter updating laws*

$$\begin{cases} \dot{\bar{\alpha}}_p(t) = -\sum_{j=1}^{N} f_p\left(\bar{x}_j(t)\right) e_{xj}(t), \quad p = 1, 2, \\ \dot{\bar{\beta}}(t) = -\sum_{j=1}^{N} g^T\left(\bar{y}_j(t)\right) e_{yj}(t), \end{cases} \tag{14}$$

and the adaptive controllers

$$\begin{cases} u_{xi}(t) = -\eta_{xi}(t) e_{xi}(t) - \gamma_{xi}(t) \operatorname{sign}\left[e_{xi}(t)\right] + \sum_{j=1}^{N} p_{ij}(t) \Gamma \bar{y}_j(t), \\ \dot{\eta}_{xi}(t) = k_{xi} e_{xi}^T(t) e_{xi}(t), \quad k_{xi} > 0, \\ \dot{\gamma}_{xi}(t) = \xi_{xi} \|e_{xi}(t)\|_1, \quad \xi_{xi} > 0, \\ u_{yi}(t) = -\eta_{yi}(t) e_{yi}(t) - \gamma_{yi}(t) \operatorname{sign}\left[e_{yi}(t)\right], \\ \dot{\eta}_{yi}(t) = k_{yi} e_{yi}^T(t) e_{yi}(t), \quad k_{yi} > 0, \\ \dot{\gamma}_{yi}(t) = \xi_{yi} \left\|e_{yi}(t)\right\|_1, \quad \xi_{yi} > 0, \\ \dot{p}_{ij}(t) = -e_{xi}^T(t) \Gamma \bar{y}_j(t). \end{cases} \tag{15}$$

Then, the following conclusions hold:

(i) The parameter updating laws (14) satisfy

$$\begin{aligned} &\lim_{t\to\infty} \bar{\alpha}_1(t) = \alpha_1, \\ &\lim_{t\to\infty} \bar{\alpha}_2(t) = \alpha_2, \\ &\lim_{t\to\infty} \bar{\beta}(t) = \beta \end{aligned} \tag{16}$$

(ii) There exist constants $\eta_{xi}^*, \eta_{yi}^*, \gamma_{xi}^*, \gamma_{yi}^*$ such that the adaptive controllers (15) satisfy

$$\begin{aligned} &\lim_{t\to\infty} \eta_{xi}(t) = \eta_{xi}^*, \\ &\lim_{t\to\infty} \eta_{yi}(t) = \eta_{yi}^*, \\ &\lim_{t\to\infty} \gamma_{xi}(t) = \gamma_{xi}^*, \\ &\lim_{t\to\infty} \gamma_{yi}(t) = \gamma_{yi}^*, \\ &\lim_{t\to\infty} p_{ij}(t) = c_{ij} - \bar{c}_{ij} \end{aligned} \tag{17}$$

(iii) The system (4)–(7) achieves outer synchronization, i.e.,

$$\lim_{t\to\infty} \|e_{xi}(t)\| = \lim_{t\to\infty} \left\|e_{yi}(t)\right\| = 0 \tag{18}$$

where $\|e_{xi}(t)\|_1 = e_{xi}(t) \operatorname{sign}\left[e_{xi}(t)\right]$ and $\|e_{yi}(t)\|_1 = e_{yi}(t) \operatorname{sign}\left[e_{yi}(t)\right]$, where $i, j = 1, 2, \dots, N$.

Proof. Denote $e_i(t) = (e_{xi}^{\mathrm{T}}(t), e_{yi}^{\mathrm{T}}(t))^{\mathrm{T}}$, $\tilde{\alpha}_p(t) = \overline{\alpha}_p(t) - \alpha_p$, $p = 1, 2$, and $\tilde{\beta}(t) = \overline{\beta}(t) - \beta$, and consider the following Lyapunov function:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{19}$$

where

$$\begin{aligned} V_1(t) &= \frac{1}{2}\sum_{i=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left[p_{ij}(t) + \bar{c}_{ij} - c_{ij}\right]^2, \\ V_2(t) &= \frac{1}{2}\sum_{p=1}^{2}\tilde{\alpha}_p^{\mathrm{T}}(t)\tilde{\alpha}_p(t) + \frac{1}{2}\tilde{\beta}^{\mathrm{T}}(t)\tilde{\beta}(t), \\ V_3(t) &= \frac{1}{2}\sum_{i=1}^{N}\Big[k_{xi}^{-1}(\eta_{xi}(t) - \eta_{xi}^*)^2 + k_{yi}^{-1}\left(\eta_{yi}(t) - \eta_{yi}^*\right)^2 \\ &\quad + \xi_{xi}^{-1}(\gamma_{xi}(t) - \gamma_{xi}^*)^2 + \xi_{yi}^{-1}\left(\gamma_{yi}(t) - \gamma_{yi}^*\right)^2\Big], \end{aligned} \tag{20}$$

where $\eta_{xi}^*, \eta_{yi}^*, \gamma_{xi}^*, \gamma_{yi}^*$ are positive constants left to be chosen, where $i = 1, 2, \ldots, N$. The aim of the proof is to select appropriate constants to ensure that $\dot{V}(t) < 0$.

By the parameter updating laws (14) and the controllers (15), the derivative of $V_2(t)$ and $V_3(t)$ along the trajectories of (4)–(7) can be calculated as follows:

$$\dot{V}_2(t) = -\sum_{p=1}^{2}\sum_{j=1}^{N}\tilde{\alpha}_p^{\mathrm{T}}(t)f_p(\bar{x}_j(t))e_{xj}(t) - \sum_{j=1}^{N}\tilde{\beta}^{\mathrm{T}}(t)g^{\mathrm{T}}\left(\bar{y}_j(t)\right)e_{yj}(t), \tag{21}$$

$$\begin{aligned} \dot{V}_3(t) = \sum_{i=1}^{N}\Big[&(\eta_{xi}(t) - \eta_{xi}^*)e_{xi}^{\mathrm{T}}(t)e_{xi}(t) \\ &+ \left(\eta_{yi}(t) - \eta_{yi}^*\right)e_{yi}^{\mathrm{T}}(t)e_{yi}(t) + (\gamma_{xi}(t) - \gamma_{xi}^*)\|e_{xi}(t)\|_1 \\ &+ \left(\gamma_{yi}(t) - \gamma_{yi}^*\right)\|e_{yi}(t)\|_1\Big]. \end{aligned} \tag{22}$$

It follows from Hypothesis 1 that the derivative of $V_1(t)$ can be written as

$$\begin{aligned} \dot{V}_1(t) &= \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\Big[F\left(\bar{X}_i(t), \overline{\alpha}_1(t), \overline{\alpha}_2(t), \overline{\beta}(t)\right) \\ &\quad - F(X_i(t), \alpha_1, \alpha_2, \beta)\Big] \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} e_{xi}^{\mathrm{T}}(t)\left[\bar{c}_{ij}\Gamma\bar{y}_j(t) - c_{ij}\Gamma y_j(t)\right] \\ &\quad + \sum_{i=1}^{N}\Big[e_{xi}^{\mathrm{T}}(t)\Delta_x(t) + e_{yi}^{\mathrm{T}}(t)\Delta_y(t) + e_{xi}^{\mathrm{T}}(t)u_{xi}(t) \\ &\quad + e_{yi}^{\mathrm{T}}(t)u_{yi}(t)\Big] - \sum_{i=1}^{N}\sum_{j=1}^{N}\left[p_{ij}(t) + \bar{c}_{ij} - c_{ij}\right]e_{xi}^{\mathrm{T}}(t)\Gamma\bar{y}_j(t) \\ &\le \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\Big[F\left(\bar{X}_i(t), \overline{\alpha}_1(t), \overline{\alpha}_2(t), \overline{\beta}(t)\right) \\ &\quad - F\left(\bar{X}_i(t), \alpha_1, \alpha_2, \beta\right) + Le_i(t)\Big] \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N}\left[c_{ij}e_{xi}^{\mathrm{T}}(t)\Gamma e_{yj}(t) - p_{ij}(t)e_{xi}^{\mathrm{T}}(t)\Gamma\bar{y}_j(t)\right] \\ &\quad + \sum_{i=1}^{N}\Big[e_{xi}^{\mathrm{T}}(t)\Delta_x(t) + e_{yi}^{\mathrm{T}}(t)\Delta_y(t) + e_{xi}^{\mathrm{T}}(t)u_{xi}(t) \\ &\quad + e_{yi}^{\mathrm{T}}(t)u_{yi}(t)\Big], \end{aligned} \tag{23}$$

and then,

$$\begin{aligned} \dot{V}_1(t) &\le \sum_{i=1}^{N} e_i^{\mathrm{T}}(t)\Big[F\left(\bar{X}_i(t), \overline{\alpha}_1(t), \overline{\alpha}_2(t), \overline{\beta}(t)\right) \\ &\quad - F\left(\bar{X}_i(t), \alpha_1, \alpha_2, \beta\right) + Le_i(t)\Big] \\ &\quad + \sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}e_{xi}^{\mathrm{T}}(t)\Gamma e_{yj}(t) \\ &\quad + \sum_{i=1}^{N}\left[e_{xi}^{\mathrm{T}}(t)\Delta_x(t) + e_{yi}^{\mathrm{T}}(t)\Delta_y(t)\right] \\ &\quad - \sum_{i=1}^{N}\left[\eta_{xi}(t)e_{xi}^{\mathrm{T}}(t)e_{xi}(t) + \gamma_{xi}(t)\|e_{xi}(t)\|_1\right] \\ &\quad - \sum_{i=1}^{N}\left[\eta_{yi}(t)e_{yi}^{\mathrm{T}}(t)e_{yi}(t) + \gamma_{yi}(t)\|e_{yi}(t)\|_1\right]. \end{aligned} \tag{24}$$

Combining the identities $\sum_{i=1}^{N} c_{ij} = \sum_{j=1}^{N} c_{ij} = 2l$,

$$\begin{aligned} &F\left(\bar{X}_i(t), \overline{\alpha}_1(t), \overline{\alpha}_2(t), \overline{\beta}(t)\right) - F\left(\bar{X}_i(t), \alpha_1, \alpha_2, \beta\right) \\ &\quad = \left(\tilde{\alpha}_1^{\mathrm{T}}(t)f_1(\bar{x}_i(t)) + \tilde{\alpha}_2^{\mathrm{T}}(t)f_2(\bar{x}_i(t)), \tilde{\beta}^{\mathrm{T}}(t)g^{\mathrm{T}}(\bar{y}_i(t))\right)^{\mathrm{T}}, \end{aligned} \tag{25}$$

and Lemma 1 together yields

$$\begin{aligned} \dot{V}_1(t) &\le \sum_{p=1}^{2}\sum_{i=1}^{N}\tilde{\alpha}_p^{\mathrm{T}}(t)f_p(\bar{x}_i(t))e_{xi}(t) + \sum_{i=1}^{N}\left[\tilde{\beta}^{\mathrm{T}}(t)g^{\mathrm{T}}(\bar{y}_i(t))e_{yi}(t)\right] \\ &\quad + L\sum_{j=1}^{N} e_i^{\mathrm{T}}(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} c_{ij}\Big[e_{xi}^{\mathrm{T}}(t)\Gamma\Gamma^{\mathrm{T}}e_{xi}(t) \\ &\quad + e_{yj}^{\mathrm{T}}(t)\Gamma^{\mathrm{T}}\Gamma e_{yj}(t)\Big] + \sum_{i=1}^{N}\Big[e_{xi}^{\mathrm{T}}(t)\Delta_x(t) - \gamma_{xi}(t)\|e_{xi}(t)\|_1 \\ &\quad + e_{yi}^{\mathrm{T}}(t)\Delta_y(t) - \gamma_{yi}(t)\|e_{yi}(t)\|_1\Big] - \sum_{i=1}^{N}\Big[\eta_{xi}(t)e_{xi}^{\mathrm{T}}(t)e_{xi}(t) \end{aligned}$$

$$+\eta_{yi}(t)e_{yi}^{\mathrm{T}}(t)e_{yi}(t)\Big] \le \sum_{p=1}^{2}\sum_{j=1}^{N}\tilde{\alpha}_p^{\mathrm{T}}(t)f_p(\bar{x}_j(t))e_{xj}(t) + \sum_{j=1}^{N}\left[\tilde{\beta}^{\mathrm{T}}(t)g^{\mathrm{T}}\left(\bar{y}_j(t)\right)e_{yj}(t)\right] + \sum_{i=1}^{N}\Big[e_{xi}^{\mathrm{T}}(t)\Delta_x(t) - \gamma_{xi}(t)\|e_{xi}(t)\|_1 + e_{yi}^{\mathrm{T}}(t)\Delta_y(t) - \gamma_{yi}(t)\left\|e_{yi}(t)\right\|_1\Big] + \sum_{i=1}^{N}\Big[(L+l-\eta_{xi}(t))e_{xi}^{\mathrm{T}}(t)e_{xi}(t) + \left(L+l-\eta_{yi}(t)\right)e_{yi}^{\mathrm{T}}(t)e_{yi}(t)\Big]. \tag{26}$$

Then, one concludes from the inequalities (21), (22), and (26) that

$$\dot{V}(t) \le \sum_{i=1}^{N}\Big[e_{xi}^{\mathrm{T}}(t)\Delta_x(t) - \gamma_{xi}^*\|e_{xi}(t)\|_1 + e_{yi}^{\mathrm{T}}(t)\Delta_y(t) - \gamma_{yi}^*\left\|e_{yi}(t)\right\|_1\Big] + \sum_{i=1}^{N}\Big[(L+l-\eta_{xi}^*)e_{xi}^{\mathrm{T}}(t)e_{xi}(t) + \left(L+l-\eta_{yi}^*\right)e_{yi}^{\mathrm{T}}(t)e_{yi}(t)\Big]. \tag{27}$$

Choosing the constants $\gamma_{xi}^*, \gamma_{yi}^*, \eta_{xi}^*, \eta_{yi}^*$ large enough to ensure that

$$\rho_x - \gamma_{xi}^* < 0, \rho_y - \gamma_{yi}^* < 0, L+l-\eta_{yi}^* < 0, L+l-\eta_{yi}^* < 0, \tag{28}$$

one finally proves that $\dot{V}(t) < 0$. Based on Lyapunov stability theory, $V_2(t)$ indicates the validity of the first item, $V_3(t)$ indicates the validity of the first two equalities of the second item, the second term of $V_1(t)$ indicates that $\lim_{t\to\infty}p_{ij}(t) = c_{ij} - \bar{c}_{ij}$, and the first term of $V_1(t)$ indicates the validity of the third item. Hence, the three items are all proved.

As special cases, when some of the three unknown parameters are given and fixed, Theorem 1 remains valid. For example, assuming that the parameters α_1 and α_2 are both determined, one gets the following corollary.

Corollary 1. *Consider the system (4)–(7) with $\overline{\alpha}_1(t) = \alpha_1$ and $\overline{\alpha}_2(t) = \alpha_2$ under Hypothesis 1 and Hypothesis 2 hold. Choose the adaptive controllers (15) and the parameter updating the law*

$$\overline{\beta}(t) = -\sum_{j=1}^{N}g^{\mathrm{T}}\left(\bar{y}_j(t)\right)e_{yj}(t). \tag{29}$$

The three statements in Theorem 1 still hold.

Proof. Choose the following Lyapunov function:

$$W(t) = V_1(t) + W_2(t) + V_3(t), \tag{30}$$

where $V_1(t)$ and $V_3(t)$ are defined in the proof of Theorem 1, and

$$W_2(t) = \frac{1}{2}\tilde{\beta}^{\mathrm{T}}(t)\tilde{\beta}(t). \tag{31}$$

Analogous to the proof of Theorem 1, it follows from Lemma 1 that

$$\dot{V}_1(t) \le \sum_{i=1}^{N}\Big[e_{xi}^{\mathrm{T}}(t)\Delta_x(t) - \gamma_{xi}(t)\|e_{xi}(t)\|_1 + e_{yi}^{\mathrm{T}}(t)\Delta_y(t) - \gamma_{yi}(t)\left\|e_{yi}(t)\right\|_1\Big] + \sum_{i=1}^{N}\Big[(L+l-\eta_{xi}(t))\|e_{xi}(t)\|^2 + \left(L+l-\eta_{yi}(t)\right)\|e_{yi}(t)\|^2\Big] + \sum_{j=1}^{N}\tilde{\beta}^{\mathrm{T}}(t)g^{\mathrm{T}}\left(\bar{y}_j(t)\right)e_{yj}(t), \tag{32}$$

and

$$\dot{W}_2(t) = -\sum_{j=1}^{N}\tilde{\beta}^{\mathrm{T}}(t)g^{\mathrm{T}}\left(\bar{y}_j(t)\right)e_{yj}(t). \tag{33}$$

Thus, one gets

$$\dot{W}(t) \le \sum_{i=1}^{N}\Big[e_{xi}^{\mathrm{T}}(t)\Delta_x(t) - \gamma_{xi}^*\|e_{xi}(t)\|_1 + e_{yi}^{\mathrm{T}}(t)\Delta_y(t) - \gamma_{yi}^*\left\|e_{yi}(t)\right\|_1\Big] + \sum_{i=1}^{N}\Big[(L+l-\eta_{xi}^*)e_{xi}^{\mathrm{T}}(t)e_{xi}(t) + \left(L+l-\eta_{yi}^*\right)e_{yi}^{\mathrm{T}}(t)e_{yi}(t)\Big], \tag{34}$$

which is similar to the inequality (27). The remainder of the argument is analogous to that of Theorem 1, and it is omitted here.

It follows from the proof of Theorem 1 that the functions $\gamma_{xi}(t)$ and $\gamma_{yi}(t)$ are designed for the disturbances $\Delta_x(t)$ and $\Delta_y(t)$. Therefore, we obtain the following corollary without considering the disturbances.

Corollary 2. *Consider the system (4)–(7) under Hypothesis 1. If the disturbances $\Delta_x(t) = \Delta_y(t) = 0$, choose the parameter updating laws (14) and the adaptive controllers*

$$\begin{cases} u_{xi}(t) = -\eta_{xi}(t)e_{xi}(t) + \sum_{j=1}^{N}p_{ij}(t)\Gamma\bar{y}_j(t), \\ \dot{\eta}_{xi}(t) = k_{xi}e_{xi}^{\mathrm{T}}(t)e_{xi}(t), \quad k_{xi} > 0, \\ u_{yi}(t) = -\eta_{yi}(t)e_{yi}(t), \\ \dot{\eta}_{yi}(t) = k_{yi}e_{yi}^{\mathrm{T}}(t)e_{yi}(t), \quad k_{yi} > 0, \\ \dot{p}_{ij}(t) = -e_{xi}^{\mathrm{T}}(t)\Gamma\bar{y}_j(t), \end{cases} \tag{35}$$

where $i, j = 1, 2, \ldots, N$; the three statements of Theorem 1 still hold.

Proof. Choose the following Lyapunov function:

$$W(t) = V_1(t) + V_2(t) + W_3(t), \tag{36}$$

where $V_1(t)$ and $V_2(t)$ are defined in the proof of Theorem 1, and

$$W_3(t) = \frac{1}{2}\sum_{i=1}^{N}\left[k_{xi}^{-1}(\eta_{xi}(t) - \eta_{xi}^*)^2 + k_{yi}^{-1}\left(\eta_{yi}(t) - \eta_{yi}^*\right)^2\right]. \tag{37}$$

Analogous to the proof of Theorem 1, it follows from Lemma 1 that

$$\begin{aligned}\dot{V}_1(t) \le{}& \sum_{p=1}^{2}\sum_{j=1}^{N}\tilde{\alpha}_p^{\mathrm{T}}(t) f_p\left(\bar{x}_j(t)\right) e_{xj}(t) \\ &+ \sum_{j=1}^{N}\tilde{\beta}^{\mathrm{T}}(t) g^{\mathrm{T}}\left(\bar{y}_j(t)\right) e_{yj}(t) \\ &+ \sum_{i=1}^{N}\left[(L + l - \eta_{xi}(t))\|e_{xi}(t)\|^2\right. \\ &\left.+ \left(L + l - \eta_{yi}(t)\right)\|e_{yi}(t)\|^2\right],\end{aligned} \tag{38}$$

and

$$\begin{aligned}\dot{W}_3(t) ={}& \sum_{i=1}^{N}\left[(\eta_{xi}(t) - \eta_{xi}^*)\|e_{xi}(t)\|^2\right. \\ &\left.+ \left(\eta_{yi}(t) - \eta_{yi}^*\right)\|e_{yi}(t)\|^2\right].\end{aligned} \tag{39}$$

Thus, one has

$$\begin{aligned}\dot{W}(t) \le{}& \sum_{i=1}^{N}\left[(L + l - \eta_{xi}^*) e_{xi}^{\mathrm{T}}(t) e_{xi}(t)\right. \\ &\left.+ \left(L + l - \eta_{yi}^*\right) e_{yi}^{\mathrm{T}}(t) e_{yi}(t)\right],\end{aligned} \tag{40}$$

which is similar to the inequality (27). The remainder of the argument is analogous to that of Theorem 1, and it is omitted here.

3. A Numerical Example

In this section, a special case of the system (4)–(7) is given to illustrate the effectiveness of the three statements of Theorem 1 one by one.

For clarity, we choose the coupling matrices as follows, the row sums of which are equal to 5:

$$\begin{aligned}C &= \begin{pmatrix} 3 & 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 3 & 2 \\ 2 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}, \\ \bar{C} &= \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 3 \\ 3 & 2 & 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 & 3 & 2 \end{pmatrix}.\end{aligned} \tag{41}$$

The topology structure corresponding to the above coupling matrices is shown in Figure 2. For instance, the element $\bar{c}_{21} = 3$ implies that there is a coupling from the oscillator $\bar{y}_2$ to the oscillator $\bar{x}_1$, and the weight of this coupling is 3; the element $c_{11} = 3$ implies that there is a coupling from the oscillator y_1 to the oscillator x_1, and the weight of this coupling is 3.

Based on the previous results [6], we set the parameters of the drive network (4) as $K = 1$, $n = 4$, $k = 1$, $\alpha_1 = (\nu_1, \nu_2, \nu_3)^{\mathrm{T}} = (0.1,0.1,0.1)^{\mathrm{T}}$, $\alpha_2 = (\delta_1, \delta_2, \delta_3)^{\mathrm{T}} = (0.35,0.35,0.35)^{\mathrm{T}}$, and $\beta = (\beta_1, \beta_2, \beta_3)^{\mathrm{T}} = (0.28,1,0.2)^{\mathrm{T}}$. The initial values $\overline{\alpha}_1(0)$, $\overline{\alpha}_2(0)$, and $\overline{\beta}(0)$ are selected randomly in $[0, 1] \times [0, 1] \times [0, 1]$, the initial values $x_i(0)$ and $\bar{x}_i(0)$ are selected randomly in $[0, 1] \times [0, 1] \times [0, 1]$, the initial values $y_i(0)$ and $\bar{y}_i(0)$ are selected randomly in $[0, 1] \times [0, 1]$, and the disturbances $\Delta_x(t) = (\cos t, -\sin t, \sin t)^{\mathrm{T}}$ and $\Delta_y(t) = (-\cos t, 2\sin t, -3\sin t)^{\mathrm{T}}$. Then, with the help of Matlab, the following figures are provided to verify the effectiveness of the obtained theoretical results.

Figures 3 and 4 are presented to verify the validity of the parameter laws (14). As can be seen, the parameter updating laws (14) are in a good agreement with the actual value of the corresponding parameters; i.e.,

$$\begin{aligned}&\lim_{t\to\infty}\bar{\nu}_1(t) = \lim_{t\to\infty}\bar{\nu}_2(t) = \lim_{t\to\infty}\bar{\nu}_3(t) = 0.1, \\ &\lim_{t\to\infty}\overline{\delta}_1(t) = \lim_{t\to\infty}\overline{\delta}_2(t) = \lim_{t\to\infty}\overline{\delta}_3(t) = 0.1, \\ &\lim_{t\to\infty}\overline{\beta}_1(t) = 0.28, \\ &\lim_{t\to\infty}\overline{\beta}_2(t) = 1, \\ &\lim_{t\to\infty}\overline{\beta}_3(t) = 0.2.\end{aligned} \tag{42}$$

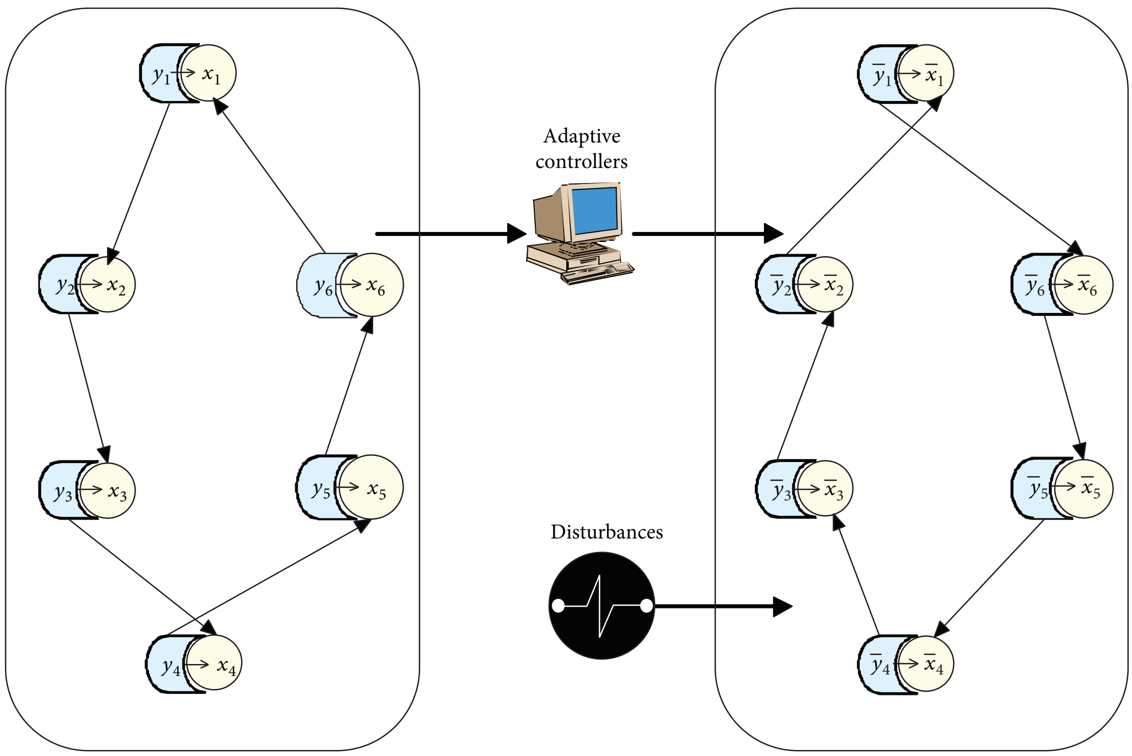

Figure 2: The topology structure of a special case of the network (4)–(7) with $N = 6$.

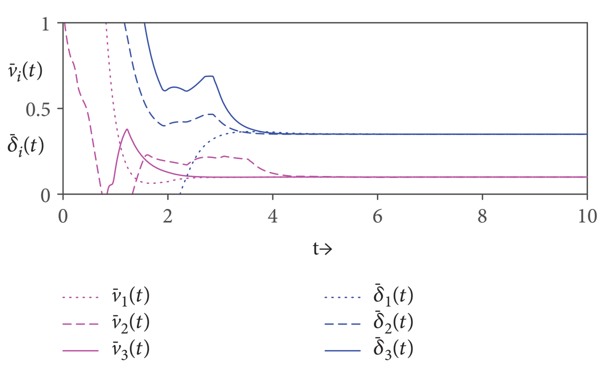

Figure 3: Parameter identification of the system (4)–(7): $\lim_{t\to\infty} \overline{\alpha}_1(t) = \alpha_1$ and $\lim_{t\to\infty}\overline{\alpha}_2(t) = \alpha_2$.

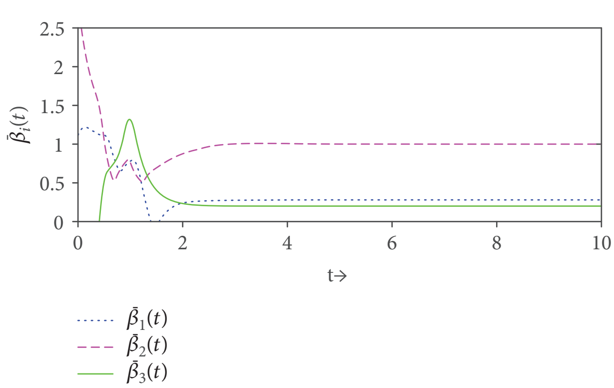

Figure 4: Parameter identification of the system (4)–(7): $\lim_{t\to\infty}\overline{\beta}(t) = \beta$.

Therefore, the first item of Theorem 1 is verified by Figures 3 and 4.

Figure 5 depicts the time evolutions of the control gains $\eta_{xi}(t)$ of the adaptive controllers (15). As can be seen, for each adaptive control gain $\eta_{xi}(t)$, there exists a positive constant η^*_{xi} such that

$$\lim_{t\to\infty} \eta_{xi}(t) = \eta^*_{xi},\ i = 1, 2, \dots, 6. \tag{43}$$

The time evolutions of $\eta_{yi}(t), \gamma_{xi}(t), \gamma_{yi}(t)$ are all similar to those of $\eta_{xi}(t), i = 1, 2 \cdots, 6$, and we omitted them here. The time evolutions of the topology updating laws $p_{ij}(t)$ in the adaptive controllers (15) are plotted in Figure 6, which verifies that

$$\lim_{t\to\infty} \left(p_{ij}(t)\right)_{6\times 6} = \left(c_{ij} - \bar{c}_{ij}\right)_{6\times 6} = \begin{pmatrix} 1 & 2 & 0 & 0 & 0 & -3 \\ -3 & 1 & 2 & 0 & 0 & 0 \\ 0 & -3 & 1 & 2 & 0 & 0 \\ 0 & 0 & -3 & 1 & 2 & 0 \\ 0 & 0 & 0 & -3 & 1 & 2 \\ 2 & 0 & 0 & 0 & -3 & 1 \end{pmatrix}. \tag{44}$$

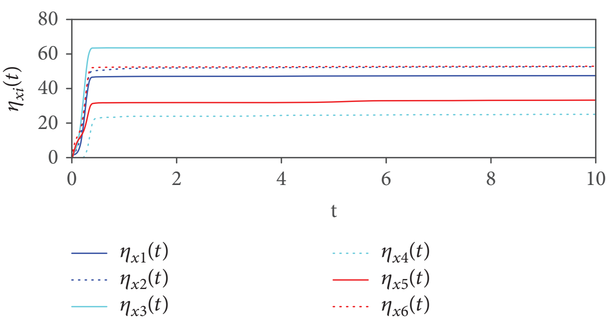

Figure 5: The convergence of the control gains $\eta_{xi}(t)$ of the adaptive controllers (15): $\lim_{t\to\infty}\eta_{xi}(t) = \eta^*_{xi}$.

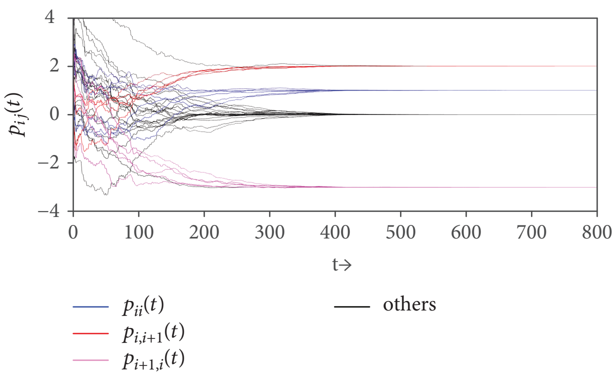

Figure 6: The convergence of the topology updating laws $p_{ij}(t)$ in the adaptive controllers (15): $\lim_{t\to\infty}p_{ij}(t) = c_{ij} - \bar{c}_{ij}$.

In particular, the blue lines indicate that $\lim_{t\to\infty}p_{ii}(t) = 1$, the red lines indicate that $\lim_{t\to\infty}p_{i,i+1}(t) = 2$, the magenta lines indicate that $\lim_{t\to\infty}p_{i+1,i}(t) = -3$, and the black lines indicate that $\lim_{t\to\infty}p_{61}(t) = 2$, $\lim_{t\to\infty}p_{16}(t) = -3$, and $\lim_{t\to\infty}p_{ij}(t) = 0$ for other elements. Therefore, the second item of Theorem 1 is verified by Figures 5 and 6.

Finally, the time evolutions of the outer synchronization errors $e_i(t) = \|e_{xi}(t)\| + \|e_{yi}(t)\|$, where $i = 1, 2, \ldots, 6$ are shown in Figure 7, which shows that the errors converge to zero and outer synchronization is achieved under the adaptive control schemes.

4. Conclusions

Based on Lyapunov stability theory, this paper has discussed the dynamic principles of mammals' circadian rhythms, where the suprachiasmatic nucleus (SCN) of the hypothalamus was modeled by a Goodwin oscillator and the vasoactive intestinal polypeptides (VIP) was modeled by a Van der Pol oscillator. Considering that it is very difficult to get the exact values of the system parameters, this paper has proposed effective parameter updating laws to identify the unknown parameters. The result has been proved based on strict theoretical reduction, and it should have a theoretical advantage over the previous results, which were based on the statistical method or experimental data. Another contribution of this paper is the problem of outer synchronization under adaptive control. Noticing that the coupling manner is different from the widely accepted coupling of the classical complex network, this paper has designed targeted adaptive controllers to synchronize the drive Goodwin oscillator network and the response one. The effectiveness of the obtained results has been verified both theoretically and numerically.

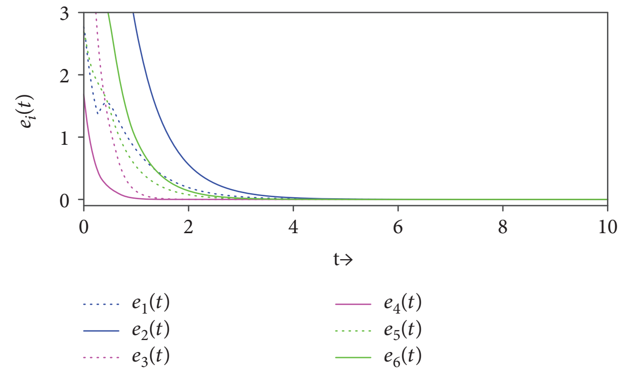

Figure 7: Time evolutions of the outer synchronization errors $e_i(t) = \|e_{xi}(t)\| + \|e_{yi}(t)\|$: $\lim_{t\to\infty}\|e_{xi}(t)\| = \lim_{t\to\infty}\|e_{yi}(t)\| = 0$, $i = 1, 2, \ldots, 6$.

We hope that the results can provide theoretical guidance for biology experiments in spite of the confusing biological applications, and we will continue to study the context of the biological interpretation later. Another possible further work is the identification of unknown time-varying parameters since this paper is only applicable to the identification of given and fixed parameters.

Data Availability

The authors affirm that all data necessary for confirming the conclusions of the article are present in the article.

Conflicts of Interest

The authors declare that there is no conflict of interests regarding the publication of this article.

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