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*Research Article*

# Adaptive Neural Tracking Control of Robotic Manipulators with Guaranteed NN Weight Convergence

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Although adaptive control for robotic manipulators has been widely studied, most of them require the acceleration signals of the joints, which are usually difficult to measure directly. Although neural networks (NNs) have been used to approximate the unknown nonlinear dynamics in the robotic systems, the conventional adaptive laws for updating the NN weights cannot guarantee that the obtained NN weights converge to their ideal values, which could degrade the tracking control response. To address these two issues, a new adaptive algorithm with the extracted NN weights error is incorporated into adaptive control, where a novel leakage term is superimposed on the gradient method. By using the Lyapunov approach, the convergence of both the tracking error and the estimation error can be guaranteed simultaneously. In addition, two auxiliary functions are introduced to reformulate the robotic model for designing the adaptive law, and a filter operation is used to avoid measuring the acceleration signals. Comparisons to other well-recognized adaptive laws are given, and extensive simulations based on a 2-DOF SCARA robotic system are given to verify the effectiveness of the proposed control strategy.

## 1. Introduction

Robotic manipulators have been widely used to operate some special or repetitive tasks, and accurate modeling and control of robotic manipulators could promote their practical applications. However, nonlinear time-varying dynamics and coupling properties in the robotic systems usually make them a nonlinear multi-input-multi-output (MIMO) system, which is also taken as a preferable experimental platform for researchers to verify different modeling and control algorithms [1]. Hence, those facts stimulate significant research interests from both industrial and academic communities to study the modeling and control of robotic manipulators. However, with the rapid and diverse applications of robotic technology, the working environments of robots are becoming more complex and even harsh such that the requirements for robust and flexible control strategies are becoming increasingly demanding, especially in the case with a changing environment, high-velocity motion, and varying load [2].

In the past decades, many studies on the control design for robotic manipulators have been reported in the literature, such as [1, 3–7]. According to these studies, the tracking control system for robotic manipulators is usually composed of the following elements: a desired path specifies the ideal response of the controlled system; a numerical model describes the robot dynamics and its interactions with surroundings; a well-designed controller is used to generate appropriate control signals to drive the actuators to create the required torques. Then, in this way, the robotic manipulator can follow the desired trajectory. In order to use adaptive control techniques, the linear parameterization of a robot with nonlinear dynamics was derived [3] since the 1980s. Subsequently, computed torque-based adaptive control [1, 3] was also proposed to guarantee the global error convergence. However, the modeling uncertainties existing in the robot manipulators are not considered. In [8–10], adaptive tracking control was proposed for trajectory tracking of robotic systems with uncertain kinematics and dynamics. However, these results are usually based on the assumption that uncertainties can be expressed in a linear parameterization form. In this sense, the aforementioned methods are mainly model-based control. Although they can obtain a satisfactory control performance in theory, the requirements for an accurate mathematical model limit their applications [11]. Moreover, in most of the conventional adaptive control schemes, the joint acceleration signals are assumed to be available to design the controller [1]. However, the measurement of joint accelerations is not practically sensible since it is generally sensitive to the external disturbance and noise [12].

To relax the requirements of the robot model knowledge, neural networks (NNs) [2, 13–15] have been used as an effective tool to approximate the nonlinear uncertainties. As it is shown in [16–18], any continuous functions can be uniformly approximated by a NN with an arbitrary degree of accuracy. In this framework, an adaptive law should be designed to update the unknown NN weights and to retain the system stability. There have been many adaptation methods for updating the NN weights, such as the gradient algorithm, least square (LS), $\sigma$-modification, and projection algorithm [19]. Although these methods can achieve fair tracking performance, only a few of them focus on the learning performance and convergence of the estimated NN weights. The authors of [20] pointed out that with inaccurate and slow convergence of the estimated parameters, the effects induced by the unknown robot dynamics cannot be suppressed in transient time, which may degrade the control performance. To guarantee the parameter convergence, the authors of [21, 22] proposed a composite learning for parameter estimation. Recently, the authors of [6, 23] proposed a novel adaptive law, in which both the tracking control and parameter estimation convergence can be guaranteed simultaneously, while the linearly parameterized form should be imposed again.

In this paper, an adaptive neural control based on a radial basis function neural network (RBFNN) will be proposed for robotic manipulators to achieve guaranteed tracking control and estimation. Firstly, since the measurement of joint accelerations is sensitive to the external noise, we aim to avoid using the acceleration signals directly by reformulating the robotic model. Hence, two auxiliary variables are first designed to reconstruct the robotic model. Then, a low-pass filter is applied to the reformulated model such that the joint acceleration can be avoided in the design of adaptive laws. Moreover, to relax the requirement of system knowledge, a RBFNN is employed to approximate the lumped unknown nonlinear dynamics. To retain the control and estimation convergence, the adaptive parameter estimation proposed in [6] is further tailored and incorporated into an adaptive neural control design. In this new framework, both the tracking error and the NN weight estimation error are used in the adaptive law, and thus, the convergence of the tracking error and weight estimation error can be guaranteed simultaneously. In addition, comparisons to other well-recognized adaptive laws are presented in this paper, which show that better estimation performance with the proposed algorithm can be obtained. Numerical simulations based on a 2-DOF SCARA robot are provided to show the efficiency of the proposed methods.

This paper is organized as follows: The mathematical model of robotic manipulators and a brief introduction of the RBFNN are given in Section 2. An adaptive neural control design based on RBFNN and the stability and robustness analysis are presented in Section 3. Section 4 provides comparisons to other adaptive algorithms. Section 5 shows simulation results based on a 2-DOF SCARA robot. Conclusions of this paper are given in Section 6.

## 2. Problem Formulation and Preliminaries

*2.1. Robot Manipulator Dynamics.* In this paper, we consider robotic manipulators where a set of $n$ rigid bodies are connected in series with the final arm being fixed to the ground. The model of such an $n$-degrees of freedom (DOF) nonlinear robot manipulator can be expressed as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau, \tag{1}$$

where $q, \dot{q}$, and $\ddot{q} \in \mathbb{R}^n$ are the robot joint position, velocity, and acceleration, respectively; $n$ is the number of DOF; $\tau \in \mathbb{R}^n$ is the control torque vector; $M(q) \in \mathbb{R}^{n\times n}$ is the inertia matrix; $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ denotes the lumped Coriolis/centripetal torque, viscous and nonlinear damping; and $G(q) \in R^n$ represents the gravitational effect.

The problem to be addressed in this paper is to design a proper adaptive control such that the output $q$ of robotic manipulator (1) can track a given reference $q_d$, while the convergence of the adaptive laws can be achieved.

According to [1], the following two properties used in the subsequent control design should be presented:

*Property 1.* The matrix $M(q) \in R^{n\times n}$ is symmetric and positive definite.

*Property 2.* The matrix $\dot{M}(q) - 2C(q,\dot{q})$ is skew-symmetric; that is,

$$x^T\left[\dot{M}(q) - 2C(q,\dot{q})\right], \quad x = 0,\ \forall x, q, \dot{q} \in \mathbb{R}^n. \tag{2}$$

*2.2. RBF Neural Network.* Neural networks (NNs) have been widely used as a function approximator for unknown nonlinearities due to their elegant approximation abilities [24]. In this paper, a linearly parameterized radial basis function NN (RBFNN) is employed to approximate a continuous function $f(X)$: $\mathbb{R}^l \longrightarrow \mathbb{R}$ over a compact set $\Omega_z$ as

$$f(z) = \sum_{i=1}^{N} w_i^* \psi_i(z) + \varepsilon_i(z) = W^{*T}\psi(z) + \varepsilon, \tag{3}$$

where $z = [z_1, z_2, \cdots, z_l]^T \in \Omega_z \in \mathbb{R}^l$ is the input vector of NN; $N$ is the number of the NN nodes; $\varepsilon \in \mathbb{R}^N$ is the approximation error; and $W^* = [w_1^*, w_2^*, \cdots, w_N^*]^T \in \mathbb{R}^N$ is the ideal NN weights. As shown in [25], the ideal NN weights $W^*$ are defined as

$$W^* = \arg\min_{W\in R^N}\left\{\sup_{z\in\Omega_z}\left|f(z) - W^{*T}\psi(z)\right|\right\}, \tag{4}$$

where $\psi(z) = [\psi_1(z), \psi_2(z), \cdots, \psi_N(z)]^T$ is a nonlinear function vector, the components of which can be presented as

$$\psi_i(z) = \exp\left[-\frac{(z-\varsigma_i)^T(z-\varsigma_i)}{\rho_i^2}\right], \quad i = 1, 2, \ldots, N, \tag{5}$$

where $\varsigma_i = [\varsigma_{i1}, \varsigma_{i2}, \cdots, \varsigma_{il}]^T \in \mathbb{R}^l$ denotes the center of the $i$th basis function and $\rho_i$ is the $i$th variance.

The following assumptions and definition will be used:

*Assumption 1.* The ideal NN weights $W^*$ are bounded such that $\|W^*\| \le \bar{W}$ holds for a positive constant $\bar{W}$, and the approximation error is bounded by $\|\varepsilon\| \le \varepsilon^*$ for a positive constant $\varepsilon^*$.

*Assumption 2.* The reference signal $q_d$ and their derivative $\dot{q}_d$ are smooth and bounded.

*Definition 1* (see [19]). A vector or matrix function $\psi(t)$ is persistently excited (PE) if there exist constants $T > 0, \varepsilon > 0$ such that $\int_t^{t+T} \psi(\tau)\psi^T(\tau)d\tau \ge \zeta I$, $\forall t \ge 0$.

## 3. Adaptive Control Design Based on RBFNN

In this section, a constructive control design will be presented for a robotic manipulator (1), which has two salient features in comparison to the existing results. On the one hand, a new filter operation and proper mathematical developments are introduced to avoid measuring the joint accelerations directly, which is preferable for applications. On the other hand, a novel adaptive law is developed to update the NN weights online, which can guarantee the exponential convergence of the estimated weights.

*3.1. Adaptive Controller Design.* As shown in (1), the studied robotic manipulator is modeled using Lagrange-Euler functions, which usually contain the acceleration signal $\ddot{q}$. This implies that the acceleration signal needs to be available to design the controller. However, the direct measurement or estimation of acceleration signals is generally difficult due to the induced noise. To avoid using the acceleration signal directly, we first define an error variable $r(q, \dot{q})$ as

$$r(q, \dot{q}) = \dot{e} + \Lambda e, \tag{6}$$

where $e = q - q_d$ and $q_d$ is the desired reference of the robot joint; $\Lambda \in \mathbb{R}^{n\times n}$ is a positive diagonal matrix.

According to Assumption 2 and the defined variable $r$ in (6), we have $\dot{e} = r - \Lambda e = \dot{q} - \dot{q}_d$ and $\ddot{e} = \dot{r} - \Lambda\dot{e} = \ddot{q} - \ddot{q}_d$. Then, the velocity and acceleration can be expressed as

$$\begin{aligned} \dot{q} &= r - \Lambda e + \dot{q}_d, \\ \ddot{q} &= \dot{r} - \Lambda\dot{e} + \ddot{q}_d. \end{aligned} \tag{7}$$

Substituting (6) and (7) into the controlled system (1), we have

$$M\dot{r} + Cr = M(\Lambda\dot{e} - \ddot{q}_d) + C(\Lambda e - \dot{q}_d) - G + \tau. \tag{8}$$

However, the design of control law $\tau$ could not be achieved based on (8) because of the nonlinear dynamics induced by $M(q)(\Lambda\dot{e} - \ddot{q}_d) + C(q, \dot{q})(\Lambda e - \dot{q}_d) - G(q)$. To accommodate this issue, an RBFNN is employed to approximate these lumped nonlinearities. Thus, system (8) can be described as

$$M(q)\dot{r} + C(q, \dot{q})r = W^{*T}\psi(z) + \varepsilon + \tau, \tag{9}$$

where $W^*$ is the ideal NN weights; $\psi(z)$ is the basis function with the input vector; $z = [q^T, \dot{q}^T, q_d^T, \dot{q}_d^T, \ddot{q}_d^T]^T$; and $\varepsilon$ is the approximation error, which are defined in (3), (4), and (5).

In order to achieve the tracking control, an adaptive neural controller is designed as

$$\tau = -\widehat{W}^T\psi - Kr, \tag{10}$$

where $K > 0$ is a feedback gain matrix and $\widehat{W}$ is the estimated NN weights of the unknown ideal weights $W^*$, which will be online updated based on the designed adaptive law.

Substituting (10) into (8), the closed-loop control error dynamics with RBFNN can be derived as

$$M(q)\dot{r} + C(q, \dot{q})r + Kr = \tilde{W}^T\psi + \varepsilon, \tag{11}$$

where $\tilde{W} = W^* - \widehat{W}$ is the NN weight error.

The problem to be solved is to design an adaptive law, which can obtain the estimated RBFNN weights $\widehat{W}$ with guaranteed convergence for implementing the proposed control (10).

*Remark 1.* Most of existing adaptive laws for updating $\widehat{W}$ are designed based on the gradient methods on the tracking error dynamics (11), e.g., [1, 18, 26, 27] and the references therein. However, the use of robust leakage terms ($\sigma$-modification and projection algorithm) in this framework makes it difficult to retain the convergence of the NN weights. Hence, we will develop a new adaptive algorithm, which guarantees the convergence of the estimated NN weights $\widehat{W}$, while retaining the robustness.

From the above analysis, the acceleration $\ddot{q}$ is embedded in the error signal $\dot{r}$. To make adaptive law independent of the joint acceleration, two auxiliaries are further defined as

$$\begin{aligned} F_1(z) &= Mr, \\ F_2(z) &= -\dot{M}r + Cr. \end{aligned} \tag{12}$$

Hence, (8) can be further written as

$$\dot{F}_1(z) + F_2(z) = W^{*T}\psi(z) + \varepsilon + \tau. \tag{13}$$

It should be noted that the introduced term $\dot{F}_1(z)$ contains the information of the joint acceleration $\ddot{q}$; hence, it cannot be used directly to design the adaptive law. In this case, inspired by [6], a filter operation is applied on system (9) to eliminate the acceleration. We define the filtered variables $F_{1f}, F_{2f}, \psi_f$, and $\tau_f$ with respect to $F_1, F_2, \psi$, and $\tau$ as

$$\begin{cases} k\dot{F}_{1f} + F_{1f} = F_1, & F_1(0) = 0, \\ k\dot{F}_{2f} + F_{2f} = F_2, & F_2(0) = 0, \\ k\dot{\psi}_f + \psi_f = \psi, & \psi(0) = 0, \\ k\dot{\tau}_f + \tau_f = \tau, & \tau(0) = 0, \end{cases} \tag{14}$$

where $k > 0$ is a filtered parameter.

Then, applying a filter $1/(ks+1)$ on system (9), we have

$$\frac{s}{ks+1}[F_1] + \frac{1}{ks+1}[F_2] = \frac{1}{ks+1}\left[W^{*T}\psi\right] + \frac{1}{ks+1}[\varepsilon] + \frac{1}{ks+1}[\tau], \tag{15}$$

where $s$ is the Laplace operator. From the first equation of (14), we can obtain $\dot{F}_{1f} = (F_1 - F_{1f})/k$ such that (15) can be rewritten as

$$W^{*T}\psi_f = \frac{F_1 - F_{1f}}{k} + F_{2f} - \varepsilon_f - \tau_f, \tag{16}$$

where $\varepsilon_f$ is the filtered version of NN error $\varepsilon$ by using the same filter operation as (14). Since the NN approximation error $\varepsilon$ is bounded, its filtered version $\varepsilon_f$ is also bounded, i.e., $\|\varepsilon_f\| \le \varepsilon^*$.

Through the above filter operations, only the variable $F_1$ with $q, \dot{q}$ and its filtered version $F_{1f}$ is used in (16), where the acceleration $\ddot{q}$ can be avoided. Thus, we can design an adaptive law based on (16) rather than the original tracking error (11). Hence, the rest of this section will present a novel adaptive algorithm based on (16), where a new leakage term containing the estimation error is introduced to achieve a guaranteed convergence of NN weights.

By using the above-filtered variables $F_{1f}, F_{2f}, \psi_f$, and $\tau_f$, two auxiliary matrices $P \in \mathbb{R}^{N\times N}$ and $Q \in \mathbb{R}^{N\times n}$ are defined as

$$\begin{cases} \dot{P} = -lP + \psi_f\psi_f^T, & P(0) = 0, \\ \dot{Q} = -lQ + \psi_f\left[\dfrac{F_1 - F_{1f}}{k} + F_{2f} - \tau_f\right]^T, & Q(0) = 0, \end{cases} \tag{17}$$

where $l > 0$ is a positive constant serving as the forgetting factor to retain the boundedness of $P$ and $Q$.

The solution of matrix equation (17) is derived as

$$\begin{cases} P(t) = \displaystyle\int_0^t e^{-l(t-\tau)}\psi_f(r)\psi_f^T(r)dr, \\ Q(t) = \displaystyle\int_0^t e^{-l(t-\tau)}\psi_f(r)\left[\frac{F_1 - F_{1f}}{k} + F_{2f} - \tau_f\right]^T dr. \end{cases} \tag{18}$$

Then, by using the obtained auxiliary matrices $P$ and $Q$, we further define an auxiliary matrix $H$ as

$$H = P\widehat{W} - Q. \tag{19}$$

It is shown in (19) that the matrix $H$ can be online calculated based on the auxiliary matrices $P$ and $Q$ by using the filtered variables $F_{1f}, F_{2f}, \psi_f$, and $\tau_f$ and the estimated NN weights $\widehat{W}$. Hence, it can be used to design the following adaptive law.

Now, an adaptive law with $H$ being a new leakage term is given as

$$\dot{\widehat{W}} = \Gamma\left(\psi r^T - \kappa H\right), \tag{20}$$

where $\Gamma \in \mathbb{R}^{N\times N}$ is a positive diagonal matrix and $\kappa > 0$ is a positive parameter.

To show the merit of the proposed adaptive law (20), we fist present the following fact.

**Lemma 1.** *The defined variable in (19) is equivalent to*

$$H = -P\widetilde{W} + \varphi, \tag{21}$$

*where $\varphi(t) = -\int_0^t e^{-l(t-\tau)}\psi_f(r)\varepsilon_f^T dr$ is a bounded residual error and $\widetilde{W} = W^* - \widehat{W}$ is the estimation error.*

*Proof 1.* By substituting (16) into the second equation of (18), we can verify the fact $Q = PW^* + \varphi$. Then, by substituting it into (19), the fact shown in (21) can be verified. Moreover, since the RBFNN regressor $\psi$ is Lipschitz and the error $\varepsilon$ is bounded, the residual error $\varphi$ is also bounded, i.e., $\|\varphi\| \le \varpi$ for a positive constant $\varpi > 0$.

As shown in (21), one may find that the explicit formulation of the unknown NN weight estimation error $\widetilde{W}$ is embedded in the matrix $H$, which can be online calculated based on the known or measurable robotic dynamics. Specially, when the approximation error is null, i.e., $\varepsilon = 0$, we can verify that $H = -P\widetilde{W}$ holds. In this case, $H$ contains the NN weight error to be minimized. Hence, the use of $H$ in the adaptive law can help to guarantee the convergence of the RBFNN weights $\widehat{W}$ to the ideal unknown value $W^*$. This will be proved in the next subsection.

*Remark 2.* In the proposed adaptive law (20), the first term $\psi r^T$ is the conventional gradient term, and the second part $\kappa H$ is the newly introduced leakage term containing the weight error $\widetilde{W}$. With the help of this new leakage term, a quadratic term of the NN weights error $\tilde{W}$ will be included in the derivative of the Lyapunov function as shown in the next subsection, such that the convergence of tracking error and NN weights error can be guaranteed simultaneously. This is clearly different to other existing adaptive laws.

*Remark 3.* Although some solutions have been reported to avoid using the acceleration signals $\ddot{q}$ (e.g., [1]) or to use differentiators in the design of adaptive laws, we provide an alternative feasible method by reformulating the robotic model with new variables $F_1$ and $F_2$. In this case, the acceleration signals are not required and the extra differentiator is not required, which leads to a simplified control implementation.

*3.2. Stability and Convergence Analysis.* Before proving the convergence of the closed-loop system, we first show the positive definiteness of the introduced matrix $P$. For the sake of simplicity, we define $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ as the minimum and maximum eigenvalues, respectively, of the corresponding matrices. Then, we can present the following lemma.

**Lemma 2** (see [28, 29]). *The matrix $P$ defined in (17) is positive definite (i.e., $\lambda_{\min}(P) > \sigma > 0$) if the RBFNN regressor $\psi$ in (9) is persistently excited (PE). On the other hand, the positive definiteness of $P$ also implies that $\psi$ is PE.*

*Proof 2.* Please refer to [28, 29] for the detailed proof.

The stability of the closed-loop system and the convergence of the tracking and estimation errors can be given as

**Theorem 1.** *Consider robotic manipulator (1) with controller (10) and adaptive law (20), if the RBFNN regressor $\psi(z)$ is PE (i.e. $\lambda_{\min}(P(t)) > \sigma > 0, t > 0$); then, all signals in the closed-loop system are bounded, and the error variable $r$, tracking error $e$, and the NN weights error $\tilde{W}$ all converge to a small compact set around zero.*

*Proof 3.* Select a Lyapunov function as

$$V_1 = \frac{1}{2} r^T M r + \frac{1}{2} tr\left\{\tilde{W}^T \Gamma^{-1} \tilde{W}\right\}. \tag{22}$$

Based on Property 2 shown in Section 2, we can calculate its derivative $\dot{V}_1$ along (11) and (20) as

$$\begin{aligned}
\dot{V}_1 &= r^T M \dot{r} + \frac{1}{2} r^T \dot{M} r + tr\left\{\tilde{W}^T \Gamma^{-1} \dot{\tilde{W}}\right\} \\
&= r^T\left(-Cr - Kr + \tilde{W}^T \psi + \varepsilon\right) + \frac{1}{2} r^T \dot{M} r \\
&\quad - tr\left\{\tilde{W}^T \Gamma^{-1}\left(\Gamma \psi r^T + \kappa \Gamma P \tilde{W} - \kappa \Gamma \varphi\right)\right\} \\
&= \frac{1}{2} r^T\left(\dot{M} - 2C\right) r - r^T K r + r^T \varepsilon - \kappa tr\left\{\tilde{W} P \tilde{W}^T\right\} + \kappa tr\left\{\tilde{W}^T \varphi\right\} \\
&\le -\lambda_{\min}(K)\|r\|^2 - \kappa \lambda_{\min}(P)\left\|\tilde{W}\right\|^2 + r^T \varepsilon + \kappa tr\left\{\tilde{W}^T \varphi\right\}.
\end{aligned} \tag{23}$$

By using Young's inequality [30] for the last two terms in (23), we have

$$\begin{aligned}
\dot{V}_1 \le &-\left(\lambda_{\min}(K) - \frac{1}{2\eta_1}\right)\|r\|^2 - \left(\kappa \lambda_{\min}(P) - \frac{\kappa}{2\eta_2}\right)\left\|\tilde{W}\right\|^2 \\
&+ \frac{\eta_1}{2} \varepsilon^{*2} + \frac{\kappa \eta_2}{2} \varpi^2 \le -\alpha_1 V_1 + \beta_1,
\end{aligned} \tag{24}$$

where $\alpha_1 = \min\{(2\lambda_{\min}(K) - (1/\eta_1))/\lambda_{\max}(M), (2\kappa\sigma - (\kappa/2\eta_2))/\lambda_{\max}(\Gamma^{-1})\}$, and $\beta_1 = (\eta_1/2)\varepsilon^{*2} + (\kappa\eta_2/2)\varpi^2$ are all positive constants since the constants $\eta_1$ and $\eta_2$ can be designed to satisfy the condition $\eta_1 \ge (1/2\lambda_{\min}(K))$ and $\eta_2 > 0$. Consequently, based on the Lyapunov stability theory, we can obtain that the error variable $r$ and the NN weight error $\tilde{W}$ are bounded. From the (6), we can verify that the tracking error $e$ is also bounded. In addition, since the regressor $\psi$ is bounded, we know that the control signal $\tau$ is also bounded based on (10).

Furthermore, the solution of inequality (24) satisfies $V(t) \le (V(0) - (\beta_1/\alpha_1))e^{-\alpha_1 t} + (\beta_1/\alpha_1)$. Hence, the control error $r$ and the NN weights error $\tilde{W}$ will converge to a compact set around zero defined by $\Omega_w := L\{\tilde{W}, r \mid \|\tilde{W}\| \le \sqrt{(V(0)e^{-\alpha_1 t} + (\beta_1/\alpha_1))/\lambda_{\min}(P)}, \|r\| \le \sqrt{V(0)e^{-\alpha_1 t} + (\beta_1/\alpha_1)}\}$, whose size depends on the RBFNN approximation error $\varepsilon^*$, the persistent excitation level $\sigma$, and the learning gain $\Gamma$. Consequently, based on (6), the tracking error $e$ will also converge to a compact set around zero. Hence, the system output $q$ will converge to a neighborhood around the desired reference $q_d$. This completes the proof.

*Remark 4.* The *online* validation of the PE condition through directly checking the regressor vector $\psi(z)$ is generally difficult in particular for nonlinear robotic systems, because the definition of PE is indeed an integral within a shifting time interval. However, according to Lemma 2, the PE condition of the RBFNN regressor $\psi(z)$ is equivalent to the positive definiteness of the introduced auxiliary matrix $P$, such that testing for the positive definiteness of $P$ (e.g., calculate the minimum eigenvalue or rank of matrix $P$) allows to numerically verify the PE condition *online*. Thus, this new adaptive algorithm provides a feasible method for *online* verification of PE condition. Specifically, as shown in [25], the RBFNN-based control system with periodical reference signal allows the PE condition partially fulfilled.

*3.3. Robust Analysis against Exogenous Disturbance.* In this subsection, we further study the robustness of the proposed adaptive control methods for robotic systems with bounded exogenous disturbance. For this purpose, the robotic manipulator (1) can be represented as

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = \tau + d(t), \tag{25}$$

where $d(t)$ denotes a bounded exogenous disturbance, i.e., $\|d(t)\| \le \bar{\lambda}$, $\bar{\lambda} > 0$.

The proposed control (10) is used, while the tracking error (11) is modified as

$$M(q)\dot{r} + C(q,\dot{q})r + Kr = \tilde{W}^T\psi + \varepsilon + d. \tag{26}$$

Moreover, to analyze the convergence of the proposed adaptive law (20), (9) can be replaced by

$$\dot{F}_1(z) + F_2(z) = W^{*T}\psi(z) + \varepsilon + \tau + d. \tag{27}$$

In applying the same filter operations given in (14) on (27) and defining the same auxiliary matrices $P$ and $Q$ in (17), then the matrix $H$ in (19) used to drive the adaptive law (20) can be given as

$$H = -P\tilde{W} - \Psi, \tag{28}$$

where $\Psi(t) = \int_0^t e^{-l(t-\tau)}\psi_f(r)(\varepsilon_f^T + d_f^T)dr$ is a modified residual error and $d_f$ is the filtered version of disturbance $d$ given by $k\dot{d}_f + d_f = d$, $d_f(0) = 0$. Since $d$ is bounded, the filtered version $d_f$ is also bounded. Hence, we know that the residual term $\Psi$ is bounded because of the boundedness of $\psi_f$, $\varepsilon_f$, and $d_f$; i.e., $\|\Psi\| \le \vartheta$ holds for a constant $\vartheta > 0$.

Then, we have the following corollary.

**Corollary 1.** *Considering the robotic manipulator (25) subject to bounded disturbances, the controller (10) and adaptive law (20) are adopted. If the RBFNN regressor $\psi(z)$ is PE (i.e., $\lambda_{\min}(P(t)) > \sigma > 0, t > 0$), the closed-loop system is stable. Moreover, the error variable $r$ and the NN weight error $\tilde{W}$ are bounded; converge to a compact set around zero, of which the bound depends on the disturbance bound.*

*Proof 4.* The proof of Corollary 1 can be carried out by selecting the same Lyapunov function $V_1 = (1/2)r^T Mr + (1/2)tr\{\tilde{W}^T\Gamma^{-1}\tilde{W}\}$ and calculating its derivative along (26) and (20) with (28). The detailed proof is similar to that of Theorem 1 and thus will not be repeated again. The main difference is that the disturbance is involved in the error dynamics (26) and also the residual error $\Psi$ in (28). Hence, the ultimate bound determined by the constant $\beta_1$ in the Lyapunov function (24) will be affected by the bound of disturbance $\bar{\lambda}$, apart from the NN approximation error $\varepsilon^*$.

*Remark 5.* In practical control implementation, there may be also measurement noise and modeling uncertainties, which could be lumped into the NN approximation error $\varepsilon^*$. Hence, (27) can be further replaced as

$$\dot{F}_1(z) + F_2(z) = W^{*T}\psi(z) + \tau + \Delta, \tag{29}$$

where $\Delta = \varepsilon + d \in \mathbb{R}^n$ denotes the lumped disturbance containing the NN approximation error $\varepsilon$ and the effect of disturbance and measurement noise $d$. Then, the robustness of the proposed control and adaptive law can be analyzed similarly. The only difference is that the residual term $\Psi$ in (28) is modified as $\Psi(t) = \int_0^t e^{-l(t-\tau)}\psi_f(r)\Delta_f^T dr$, where $\Delta_f$ is the filtered version of $\Delta$ given by $k\dot{\Delta}_f + \Delta_f = \Delta$, $\Delta_f(0) = 0$.

## 4. Comparison to Other Adaptive Laws

To show the advantages and superior performance of this new adaptation over classical methods, we will compare it with a widely recognized gradient-based adaptive algorithm and the $\sigma$-modification method [31].

*4.1. Gradient Method.* The classical gradient-based adaptive law is solely driven by the control error $r$, which is given by

$$\dot{\hat{W}} = \Gamma\psi r^T. \tag{30}$$

Considering the fact $\dot{\tilde{W}} = -\dot{\hat{W}}$, the estimation error of (30) is derived as

$$\dot{\tilde{W}} = -\Gamma\psi r^T. \tag{31}$$

Then, selecting $V_G = (1/2)r^T Mr + (1/2)tr\{\tilde{W}^T\Gamma^{-1}\tilde{W}\}$, we have its derivative along (11) and (31) as

$$\begin{aligned}\dot{V}_G &= r^T M\dot{r} + \frac{1}{2}r^T\dot{M}r + tr\left\{\tilde{W}^T\Gamma^{-1}\dot{\tilde{W}}\right\} \\ &= \frac{1}{2}r^T\left(\dot{M} - 2C\right)r - r^T Kr + r^T\varepsilon \le -\alpha_2\|r\|^2 + \beta_2,\end{aligned} \tag{32}$$

where $\alpha_2 = \lambda_{\min}(K) - (1/2\eta_3)$ and $\beta_2 = (\eta_3/2)\varepsilon^{*2}$, with $\eta_3 > (1/2\lambda_{\min}(K))$.

Based on the Lyapunov stability theory, the control error $r$ will converge to a compact set around zero. However, the convergence of the NN weights error $\tilde{W}$ cannot be guaranteed, especially in the absence of PE condition. More specifically, when the system is subject to disturbance and uncertainties, the online obtained NN weights $\hat{W}$ may even be unbounded; i.e., it could lead to parameter drift and bursting phenomena [32]. Hence, the convergence and robustness of the gradient algorithm (30) are questionable.

*4.2. σ-Modification [31].* To guarantee the boundedness of the estimated NN weights, a leakage term $\sigma\hat{W}$ is added in the gradient algorithm, leading to the following σ-modification method:

$$\dot{\hat{W}} = \Gamma\left(\psi r^T - \kappa\hat{W}\right). \tag{33}$$

Then, its estimation error dynamics can be given as

$$\dot{\tilde{W}} = -\Gamma\psi r^T + \kappa\Gamma\hat{W} = -\Gamma\psi r^T + \kappa\Gamma W^* - \kappa\Gamma\tilde{W}. \tag{34}$$

We select the Lyapunov function as $V_\sigma = (1/2)r^T M r + (1/2)tr\{\tilde{W}^T \Gamma^{-1} \tilde{W}\}$, and its derivative along (11) and (34) can be derived as

$$\begin{aligned} \dot{V}_\sigma &= r^T M \dot{r} + \frac{1}{2} r^T \dot{M} r + tr\left\{\tilde{W}^T \Gamma^{-1} \dot{\tilde{W}}\right\} \\ &= -r^T K r - \kappa tr\left\{\tilde{W}^T \tilde{W}\right\} + r^T \varepsilon + \kappa tr\left\{\tilde{W}^T W\right\} \\ &\le -\left(\lambda_{\min}(K) - \frac{1}{2\eta_4}\right)\|r\|^2 - \left(\kappa - \frac{\kappa}{2\eta_5}\right)\|\tilde{W}\|^2 + \frac{\eta_4}{2}\varepsilon^{*2} \\ &\quad + \frac{\kappa\eta_5}{2}\bar{W}^2 \le -\alpha_3 V_\sigma + \beta_3, \end{aligned} \tag{35}$$

where the constants $\alpha_3 = \min\{(2\lambda_{\min}(K) - (1/\eta_4))/\lambda_{\max}(M), (2\kappa - (\kappa/2\eta_5))/\lambda_{\max}(\Gamma^{-1})\}$ and $\beta_3 = (\eta_4/2)\varepsilon^{*2} + (\kappa\eta_5/2)\bar{W}^2$ are positive by setting the design parameters $\eta_4 > (1/2\lambda_{\min}(K))$ and $\eta_5 > (1/2)$. Based on the Lyapunov theory, we can obtain that the error variable $r$ and the estimation error $\tilde{W}$ are all bounded; i.e., the $\sigma$-modification method has guaranteed robustness.

In fact, compared with (31), it is shown in (34) that a forgetting factor term $-\kappa\Gamma\tilde{W}$ is involved, such that the estimation error dynamics in (34) are bounded-input-bounded-output (BIBO) stable. Hence, the $\sigma$-modification can guarantee the boundedness of the control error $r$ and the NN weight error $\tilde{W}$. However, since a pure damping term $\kappa\hat{W}$ is added to (33), the estimated NN weights $\hat{W}$ will stay in a neighborhood of a preselected value only but will not converge to their true values, because the upper bound $\bar{W}$ of the unknown NN weights $W^*$ is involved in the error bound $\beta_3$ in (35) and thus $\beta_3 \ne 0$ even when the NN weight error and tracking error are null (i.e., $\varepsilon^* = r = 0$). This can also be observed from the estimation error (34) represented as a transfer function as $\tilde{W} = (1/(s + \kappa\Gamma))[-\Gamma\psi r^T + \kappa\Gamma W^*]$.

*4.3. Proposed Adaptive Law.* With the help of the new leakage term $\kappa H$, the NN weight error of the proposed adaptive law (20) can be given as

$$\dot{\tilde{W}} = -\Gamma\psi r^T + \kappa\Gamma H = -\Gamma\psi r^T - \kappa\Gamma P\tilde{W} + \kappa\Gamma\varphi. \tag{36}$$

As shown in (36), a forgetting factor $-\kappa\Gamma P\tilde{W}$ stemming from the new leakage term $\kappa H$ is involved to retain its BIBO stability, and thus the boundedness of the NN weight error can be guaranteed. In addition, the estimation error dynamics (36) can be further rewritten as a transfer function as $\tilde{W} = (1/(s + \kappa\Gamma P))[-\Gamma\psi r^T + \kappa\Gamma\varphi]$. Consequently, the bound of the NN weight error mainly depends on the residual error $\varphi$, which is a function of the NN approximation error $\varepsilon$ as shown in (21). Consequently, the NN weight error will converge to zero as long as the control error and the NN approximation error are null (i.e., $\varepsilon^* = r = 0$). This implies that the proposed adaptive law achieves better estimation performance compared with the other two adaptive laws, while retaining the same robustness as the $\sigma$-modification method.

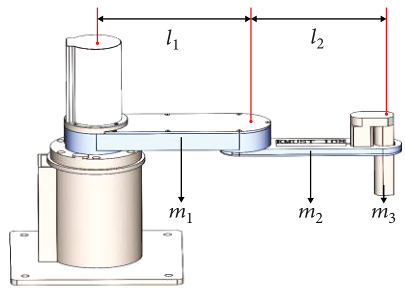



Figure 1: 2-DOF SCARA robot structure.

*Remark 6.* Compared to the conventional adaptive laws, the proposed adaptive law (20) uses filter operations (14) to extract the information of the NN weight error. In this framework, the filter coefficient $k$ (14) should be set small to retain fast convergence. On the other hand, the constant $k$ defines the bandwidth of the filter $1/(ks + 1)$. Hence, the robustness and transient convergence should be managed when we set this constant. The forgetting factor $l$ in (17) aims to guarantee the boundness of the auxiliary matrices $P$ and $Q$; thus, $l$ cannot be set too large. The constant $\kappa$ in (20) affects the convergence of the NN weight error $\tilde{W}$; in general, $\kappa$ should not be set too large since the residual term $\kappa\Gamma\varphi$ may be large with a large $\kappa$. Finally, the selection of the learning gain $\Gamma$ in (20), the diagonal matrix $\Lambda$ in (6), and the feedback gain matrix $K$ in (10) should be considered the tradeoff between the convergence of the tracking error and the robustness of the closed-loop system.

## 5. Simulation

In this paper, a 2-DOF SCARA robot designed in our lab is used to show the validity of the proposed control schemes. The diagram of the 2-DOF SCARA robot can be found in Figure 1, and its schematic diagram can be found in Figure 2.

According to [33], the kinetic energy $E$ and the potential energy $Z$ of the 2-DOF SCARA robot can be given as

$$\begin{aligned} E &= \left(\frac{1}{8}m_1 + \frac{1}{2}m_2 + \frac{1}{2}m_3\right) l_1^2 \dot{q}_1^2 + \left(\frac{1}{8}m_2 + \frac{1}{2}m_3\right) l_2^2 (\dot{q}_1 + \dot{q}_2)^2 \\ &\quad + \left(\frac{1}{2}m_2 + m_3\right) l_1 l_2 \left(\dot{q}_1^2 + \dot{q}_1\dot{q}_2\right) \cos(q_2), \end{aligned} \tag{37}$$

where $m_1 = 2.3Kg$, $m_2 = 0.8Kg$, $m_3 = 0.6Kg$, $l_1 = 0.25m$, and $l_2 = 0.25m$, since the 2-DOF SCARA robot is moving in the horizontal plane such that the potential energy $Z = 0$. Based on [1], the Lagrange kinetic equation of this robotic system is given as $(d/dt)(\partial L/\partial\dot{q}) - (\partial L/\partial q) = \tau$, where $L = E - Z$ is the Lagrangian.

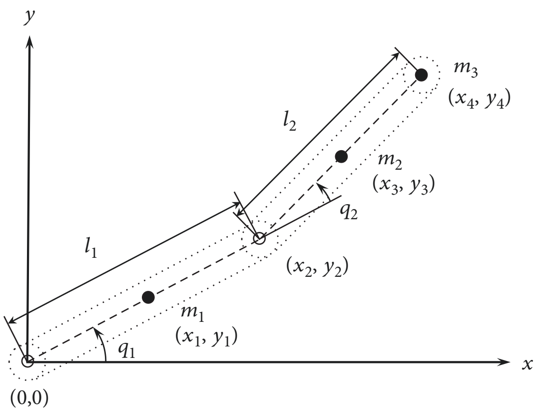



Figure 2: Schematic diagram of the 2-DOF SCARA robot.

Then, the dynamics of the 2-DOF SCARA robot are described by

$$\tau = \underbrace{\begin{bmatrix} al_1^2 + bl_2^2 + 2cl_1 l_2 \cos(q_2) & bl_2^2 + cl_1 l_2 \cos(q_2) \\ cl_2^2 + cl_1 l_2 \cos(q_2) & bl_2^2 \end{bmatrix}}_{M(q)} \ddot{q} + \underbrace{\begin{bmatrix} -2cl_1 l_2 \dot{q}_2 \sin(q_2) \; cl_1 l_2 \dot{q}_2 \sin(q_2) \\ l_1 l_2 \dot{q}_1 \sin(q_2) \end{bmatrix}}_{C(q,\dot{q})}, \tag{38}$$

where $a = (1/4)m_1 + m_2 + m_3$, $b = (1/4)m_2 + m_3$, and $c = (1/2)m_2 + m_3$.

In the simulations, the initial joint position $q(0)$ and the initial velocity $\dot{q}(0)$ are set as $q(0) = [0.2\; 0.3]^T$ and $\dot{q}(0) = [0\; 0]^T$, respectively. The desired trajectories to be tracked are $q_{d1} = 0.5 \sin(t)$ and $q_{d2} = \cos(0.5t)$. The number of the NN node is set as $N = 16$, and the initial RBFNN weights are set as $\widehat{W}(0) = \text{zeros}(16, 2)$. The center $\varsigma_i$ in (5) is uniformly distributed in $[1, -1]$, and the variance $\rho_i^2$ is set as 50. The other parameters used in the control and the adaptive law are $\Lambda = 5I_2$, $k = 0.0001$, $l = 90$, $\kappa = 1$, $K = 5I_2$, and $\Gamma = 100I_{16}$.

Figures 3–6 show the tracking performance of robot joint position and velocity. As it is shown in these simulation results, the adaptive control (10) with both the proposed adaptive law (20) and $\sigma$-modification method can achieve satisfactory tracking performance. However, as it can be seen from Figures 4 and 6, smaller steady-state errors can be achieved by using the proposed adaptive law (20) in comparison with that of the $\sigma$-modification method. This result validates the analysis in Theorem 1 and the discussions in Section 3; that is, a quadratic term of the NN weights error $\widetilde{W}$ induced by the proposed leakage term $H$ is incorporated into the derivative of the Lyapunov function, such that the convergence of both the tracking error and the NN weights error can be guaranteed simultaneously. It can be also observed from Figure 7 that the NN weights $\widehat{W}$ updated by the proposed adaptive law (20) can achieve convergence after a short transient stage,

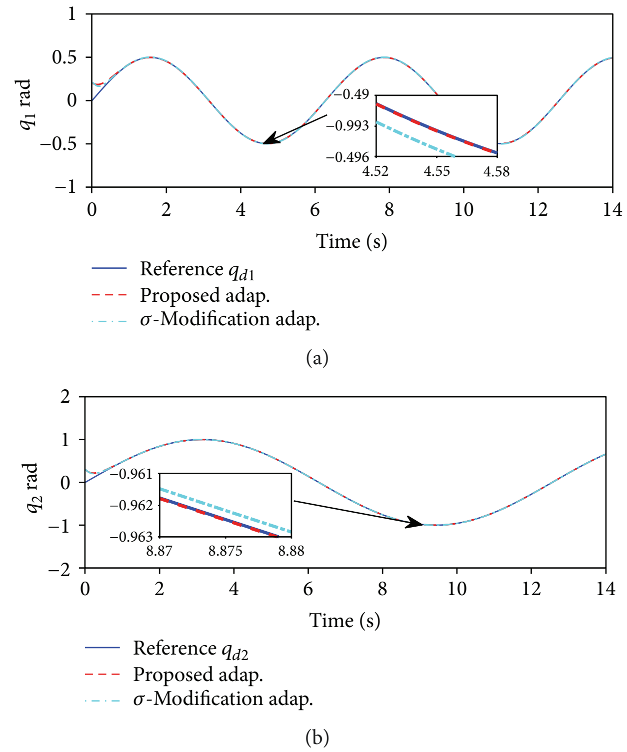



Figure 3: Tracking performance of robotic joint position $q_1$ and $q_2$.

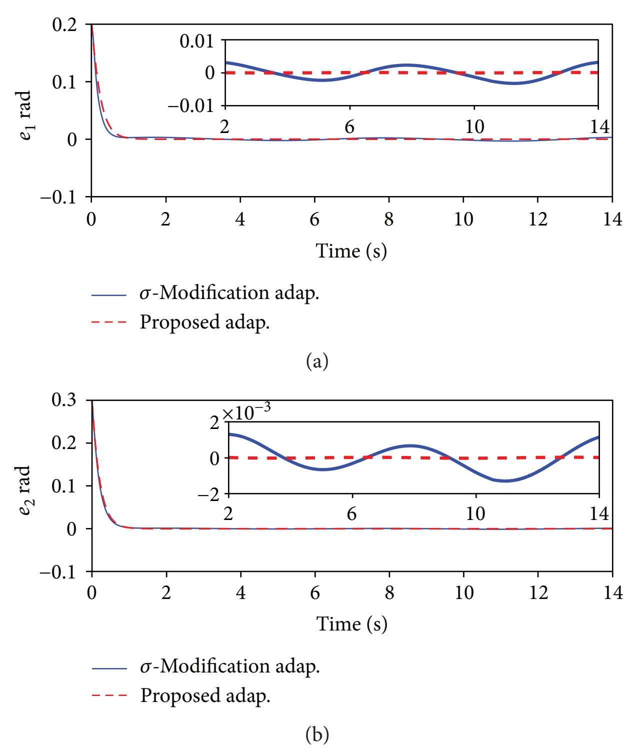
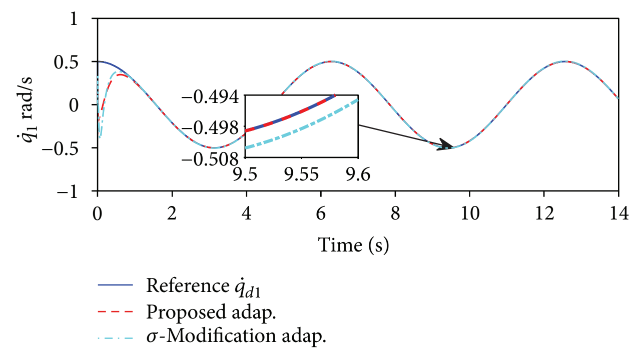
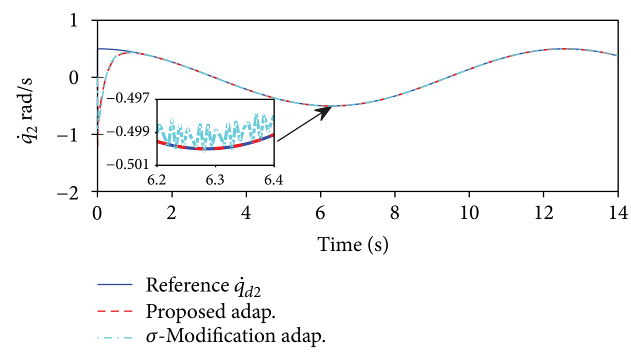



Figure 4: Tracking error of robotic joint position $q_1$ and $q_2$.

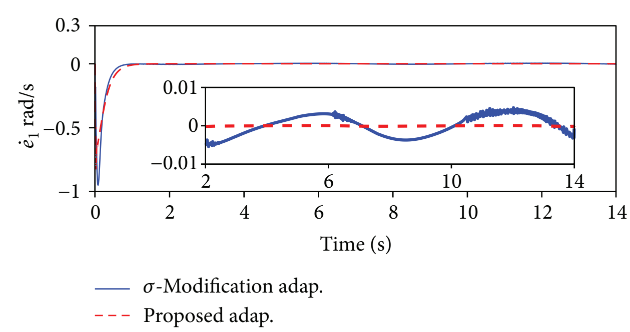



(a)

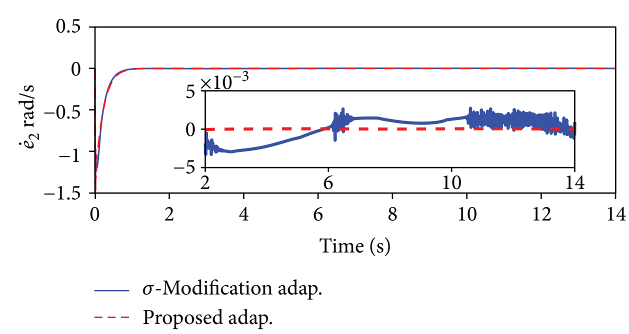



(b)

Figure 5: Tracking performance of robotic joint velocity $\dot{q}_1$ and $\dot{q}_2$.




(a)




(b)

Figure 6: Tracking error of robotic joint velocity $\dot{q}_1$ and $\dot{q}_2$.

while the NN weights with $\sigma$-modification have fair oscillations even when the tracking errors reach the steady state. Therefore, the proposed adaptive neural control with

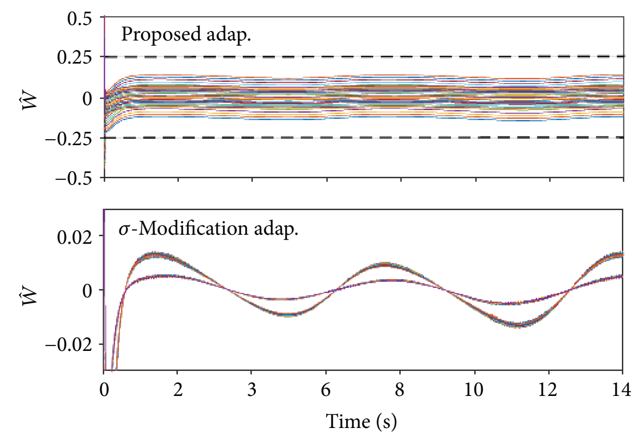
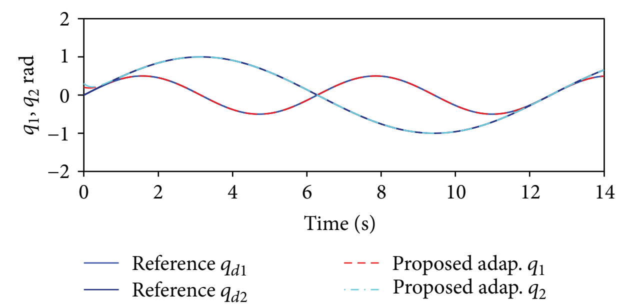
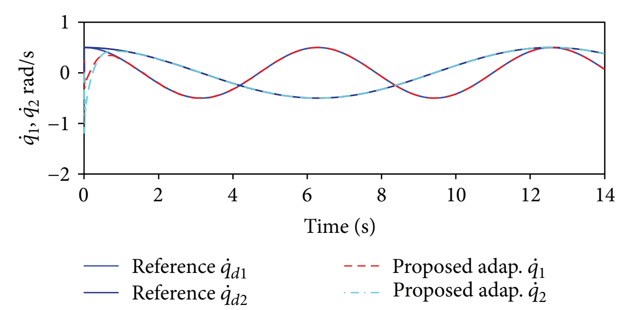



Figure 7: The online obtained weight of RBFNN.




(a)




(b)

Figure 8: Tracking performance of the proposed adaptive law with bounded exogenous disturbances.

a new adaptive algorithm can improve both the transient and steady-state tracking performances.

Moreover, to test the robustness of the proposed method, the bounded exogenous disturbance signals $d(t) = [0.5 \sin(16t)e^{-0.2t} + 0.03\,; \cos(8t)e^{-0.4t} + 0.02]$ are added in the measured position signals of joint 1 and joint 2, respectively. Simulation results given in Figures 8 and 9 show that the proposed adaptive neural control is robust against the modeling uncertainties and disturbances as that achieved by using the $\sigma$-modification scheme. From these simulation results, one can claim that the proposed adaptive neural control for robot manipulators can achieve better convergence performance and comparative robustness against the external disturbances.

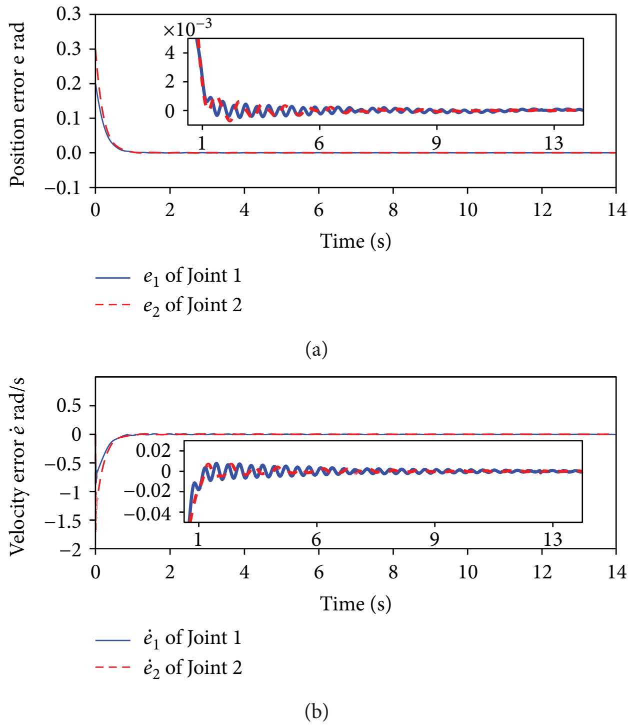



Figure 9: Tracking error of the proposed adaptive law with bounded exogenous disturbances.

## 6. Conclusions

In this paper, we introduce a new adaptive algorithm to achieve the guaranteed estimation convergence and improve the tracking performance of adaptive neural control for robot manipulators. Two auxiliary functions are first introduced to reconstruct the robotic model such that the acceleration signals are not used in the control implementation and online learning. The unknown nonlinear system dynamics can be effectively approximated by using a RBFNN, where a novel leakage term is superimposed on the gradient-based adaptive law, which is used to online update the RBFNN weights. Through rigorous stability analysis based on the Lyapunov theory, it is proved that the proposed adaptive control could guarantee the convergence of both the tracking error and the NN weight error simultaneously. Therefore, improved tracking control performance can be achieved in comparison to other adaptive methods. In addition, it is shown that even in the presence of bounded disturbances, the proposed algorithm can retain the robustness. Simulations based on a 2-DOF SCARA robotic manipulator are given to validate the efficiency of the proposed adaptive neural control. A future work will focus on relaxing the required PE condition for the adaptive control design.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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