Supplementary Material: ***Finding of Universal Formulas***

In this supplementary document we provided the details derivation of the *three universal formulas* regarding exponential trajectories of state $x_n(t)$, time period t_{fn} , and minimum pulse amplitude V_A, which are required to analyze the movement of the state trajectories on the piecewise linear (PWL) dynamic route map (DRM) of the proposed 6-lobe Chua corsage memristor.

The *power-off-plot* in Fig. 2 (of the manuscript) shows that the 6-lobe Chua Corsage Memristor has *4-stable equilibrium memory states* at $v = 0V$. Conceptually, the simplest way to switch the memory states of the 6-lobe corsage memristor is to apply a square pulse with an appropriate pulse amplitude V_A, and pulse width Δw. For a successful switching between the memory states of the proposed corsage memristor, the square pulse should have a minimum pulse width for an appropriate pulse amplitude, V_A. Any square pulse with less than the minimum pulse width $\mathbf{\Delta t_m}$ and an appropriate pulse amplitude V_A results in switching failure.

The analysis of the switching kinetics of proposed corsage memristor can be represented through its *dynamic route map (DRM)* where each straight-line segments of the *dx/dt vs. X* route map can be transformed into an exponential function $x_n(t)$. The complete solution $\boldsymbol{x(t)}$ of the state movement trajectories are the summation of each exponential trajectories ($\sum x_n(t)$) of the DRM line segments joined at the various breakpoints.

To derive the universal formulas, for the exponential trajectories $x_n(t)$, time period t_{fn} for the trajectories to move from a starting point to terminal point with a minimum pulse width $\mathbf{\Delta t_m}$, pulse amplitude $\boldsymbol{V_A}$, we studied the case where $x(t)$ moves from Q_1 equilibrium point to Q_5 as shown in the below Fig. 1 and 2.

To switch from equilibrium state Q_1 to Q_5 let's assume we provided an input pulse $v(t)$ with pulse width $t = \Delta w$ and pulse amplitude $V = V_A$ at $x = X_{Q1} = 3$. The applied square pulse with $V = V_A$ V is equivalent to translating the red curve $f(x)$ upwards by V_A units, as shown by the blue curve ($f(x) + V_A$) in Fig. 1. To illustrate the movement of the state variable x along the $f(x)+V_A$ we need to transform the *dx/dt vs. x* route map (purple color segment) into the *x(t) vs. t* trajectorial map by computing the following state equation (1) for conditions $x < 6$ and $v = V_A$ where the resultant equation is shown in (2)

$$\frac{dx}{dt} = 33 - x + |x-6| - |x-12| + |x-20| - |x-30| + |x-42| - |x-56| + v\,, \tag{1}$$

and

$$\frac{dx}{dt} = 3 - x + v\,, \tag{2}$$

where eq. (2) can be factories as below

$$\frac{1}{(3-x+v)}\frac{dx}{dt} = 1 \rightarrow \int \frac{-1}{(3-x+v)}\, d(3-x+v) = \int 1\, dt \rightarrow -\ln(3-x+v) = t + c_1 \rightarrow 3 - x + v = e^{-(t+c_1)}\,,$$

and eventually results in

$$x_1(t) = 3 + v - e^{-(t+c_1)}\,, \tag{3}$$

where $x_1(t)$ represents the state variable of 1st segment (Purple color segment) of Piecewise linear (PWL) DRM. If we replace the value of the equilibrium point Q_1 then we can rewrite (3) as following

$$x_1(t) = X_{Q_1} + v - e^{-(t+c_1)}, \tag{4}$$

where c_1 is a constant.

At $t = t_{01}$, $x_1(t_{01}) = 3$, and $v = 0$ and the constant c_1 can be computed from eq. (4) as

$$X_{Q_1} + v - x_1(t_{01}) = e^{-(t_{01}+c_1)} \rightarrow \ln\left(Q_1 + v - x_1(t_{01})\right) = -\left(t_{01} + c_1\right)$$

and

$$c_1 = -\left[\ln\left(Q_1 + v - x_1(t_{01})\right) + t_{01}\right]. \tag{5}$$

So, the trajectory of $x_1(t)$ can be represented by

$$x_1(t) = Q_1 + v - e^{-t} \cdot e^{-c_1} \rightarrow x_1(t) = Q_1 + v - e^{-t} \cdot e^{-\left\{-\left[\ln\left(Q_1+v-x_1(t_{01})\right)+t_{01}\right]\right\}} \rightarrow x_1(t) = Q_1 + v - e^{-t} \cdot e^{t_{01}}\left(Q_1 + v - x_1(t_{01})\right),$$

and eventually as

$$\boxed{x_1(t) = Q_1 + v\left(1 - e^{-(t-t_{01})}\right) - \left(Q_1 - x_1(t_{01})\right)e^{-(t-t_{01})}}. \tag{6}$$

The time period needed for the $x_1(t)$ trajectories to move from the starting point to the terminal point of the segment can be computed by replacing $t = t_{f1}$ in (6) where $x_1(t_{f1}) = [(X_{Q1} + X_{Q2})/2] = 6$, and $v \neq 0$.

$$x_1(t_{f1}) = X_{Q_1} + v\left(1 - e^{-(t_{f1}-t_{01})}\right) - \left(X_{Q_1} - x_1(t_{01})\right)e^{-(t_{f1}-t_{01})}$$

$$x_1(t_{f1}) - X_{Q_1} - v = -\left(v + \left(X_{Q_1} - x_1(t_{01})\right)\right)e^{-(t_{f1}-t_{01})}$$

$$\frac{\left(Q_1 + v - x_1(t_{f1})\right)}{\left(v + \left(Q_1 - x_1(t_{01})\right)\right)} = e^{-(t_{f1}-t_{01})},$$

and eventually

$$\boxed{t_{f1} = t_{01} - \ln\left[\frac{\left(Q_1 + v - x_1(t_{f1})\right)}{\left(v + \left(Q_1 - x_1(t_{01})\right)\right)}\right]}. \tag{7}$$

For the positive slope DRM segment (***Fluorescent green segment***) the state equation in (1) can be rewrite as follow for state variable condition $6 \leq x < 12$ where $v = V_A$,

$$\frac{dx}{dt} = -9 + x + v, \tag{8}$$

and can be factories as

$$\frac{1}{(-9+x+v)}\frac{dx}{dt} = 1 \rightarrow \int \frac{1}{(-9+x+v)}\, d(-9+x+v) = \int 1\, dt \rightarrow \ln(-9+x+v) = t + c_2 \rightarrow x(t) = 9 - v + e^{(t+c_2)},$$

and the resultant state variable is

$$x(t) = 9 - v + e^{(t+c_2)}. \tag{9}$$

If we replace the value of the equilibrium point Q_2 then we can rewrite (9) as following

$$x_2(t) = X_{Q_2} - v + e^{(t+c_2)}, \tag{10}$$

where $x_2(t)$ represents the state variable of 2nd segment (***Fluorescent green segment***) of PWL DRM where c_2 is the time constant for 2nd segment of the DRM. The time constant c_2 can be computed similar to c_1 at $t = t_{02} = t_{f1}$ and $x_2(t_{02}) = x_2(t_{f1}) = [(X_{Q1} + X_{Q2})/2] = 6$ and $v \neq 0$.

$$x_2(t_{02}) = X_{Q_2} - v + e^{(t_{02}+c_2)} \rightarrow \ln\left(x_2(t_{02}) - X_{Q_2} + v\right) = \left(t_{02} + c_2\right),$$

and

$$c_2 = \ln\left(x(t_{02}) - X_{Q_2} + v\right) - t_{02}.$$

So, the trajectory of $x_1(t)$ can be represented by

$$x_2(t) = X_{Q_2} - v + e^t \cdot e^{c_2} \rightarrow x_2(t) = X_{Q_2} - v + e^t \cdot e^{\left[\ln\left(x_2(t_{02}) - X_{Q_2} + v\right) - t_{02}\right]} \rightarrow x_2(t) = X_{Q_2} - v + e^{(t-t_{02})}\left(x_2(t_{02}) - X_{Q_2} + v\right),$$

and eventually,

$$\boxed{x_2(t) = X_{Q_2} - v\left(1 - e^{(t-t_{02})}\right) - \left(X_{Q_2} - x_2(t_{02})\right)e^{(t-t_{02})}}. \tag{11}$$

Similarly the time period of $x_2(t)$ trajectories can be computed by replacing $t = t_{f2}$ in (11) where $x_2(t_{f1}) = [(X_{Q2} + X_{Q3})/2] = 12$, and $v \neq 0$.

$$x_2(t_{f2}) = Q_2 - v\left(1 - e^{(t_{f2}-t_{02})}\right) - \left(Q_2 - x_2(t_{02})\right)e^{(t_{f2}-t_{02})},$$

$$x_2(t_{f2}) - Q_2 + v = \left(v - \left(Q_2 - x_2(t_{02})\right)\right)e^{(t_{f2}-t_{02})},$$

$$\frac{\left(x(t_{f2}) - Q_2 + v\right)}{v - \left(Q_2 - x(t_{02})\right)} = e^{(t_{f2}-t_{02})},$$

and finally,

$$\boxed{t_{f2} = t_{02} + \ln\left[\frac{\left(Q_2 - v - x(t_{f2})\right)}{\left(-v + \left(Q_2 - x(t_{02})\right)\right)}\right]}. \tag{12}$$

Similarly the exponential trajectories, and the time period of 3rd segment (magenta color segment), 4th segment (burgundy color segment), 5th segment (cyan color segment), and finally 6th segment (green color segment) can be represented as

$$x_3(t) = X_{Q_3} + v(t)\left(1 - e^{-(t-t_{03})}\right) - \left(X_{Q_3} - x_3(t_{03})\right)e^{-(t-t_{03})}$$
$$t_{f3} = t_{03} - \ln\left[\frac{\left(X_{Q_3} + v(t) - x_3(t_{f3})\right)}{\left(v(t) + \left(X_{Q_3} - x_3(t_{03})\right)\right)}\right] \quad , \tag{13}$$

$$x_4(t) = X_{Q_4} - v(t)\left(1 - e^{(t-t_{04})}\right) - \left(X_{Q_4} - x_4(t_{04})\right)e^{(t-t_{04})}$$
$$t_{f4} = t_{04} + \ln\left[\frac{\left(X_{Q_4} - v(t) - x_4(t_{f4})\right)}{\left(-v(t) + \left(X_{Q_4} - x_4(t_{04})\right)\right)}\right] \quad , \tag{14}$$

$$\left. \begin{array}{l} x_{4wv}(t) = Q_4 - v(t)\left(1 - e^{(t-t_{04wv})}\right) - \left(Q_4 - x_{4wv}(t_{04wv})\right)e^{(t-t_{04wv})} \\ t_{f4wv} = t_{04wv} + \ln\left[\dfrac{\left(Q_4 - v(t) - x_{4wv}(t_{f4wv})\right)}{\left(-v(t) + \left(Q_4 - x_{4wv}(t_{04})\right)\right)}\right] \end{array} \right|_{v(t)=0} \quad , \tag{15}$$

where $x_{4w}(t)$ represent the segment where v = V_A ⟶ to v = 0.

$$\left. \begin{array}{l} x_5(t) = Q_5 + v(t)\left(1 - e^{-(t-t_{05})}\right) - \left(Q_5 - x(t_{05})\right)e^{-(t-t_{05})} \\ t_{f5} = t_{05} - \ln\left[\dfrac{\left(Q_5 + v(t) - x_5(t_{f5})\right)}{\left(v(t) + \left(Q_5 - x_5(t_{05})\right)\right)}\right] \end{array} \right|_{v(t)=0} \tag{16}$$

So the **universal formulas** for the exponential movement of the state trajectories of an line segment can be represented as

$$x_n(t) = Q_n - m \cdot v(t)\left(1 - e^{m(t-t_{0n})}\right) - \left(Q_n - x(t_{0n})\right)e^{m(t-t_{0n})} \quad , \tag{17}$$

where,

$$m = \operatorname{sgn}\left(\frac{(dx/dt)_{start} - (dx/dt)_{terminal}}{x_{start} - x_{terminal}}\right) . \tag{18}$$

The time needed for the trajectories to move from starting point to the terminal point of a segment,

$$t_{fn} = t_{0n} + \frac{1}{m}\ln\left[\frac{\left(Q_n - mv(t) - x(t_{fn})\right)}{\left(-mv(t) + \left(Q_n - x(t_{0n})\right)\right)}\right] . \tag{19}$$

The appropriate pulse amplitude V_A is computed by replacing $t = t_{fn}$ in (17) and substituting the value of t_{fn} from (18) to (17) where the resultant equation is shown as follow

$$V_A > Q_{(n-1)} - x_n\left(t_{0(n-1)}\right) , \tag{20}$$

where $Q_{(n-1)}$ and $x(t_{0(n-1)})$ represent the immediate before equilibrium point and the initial state of the resultant memory state Q_n.

Fig. 1 and Fig. 2 shows the segments of the DRM of 6-lobe corsage memristor and the segment wise trajectorial movement, respectively.

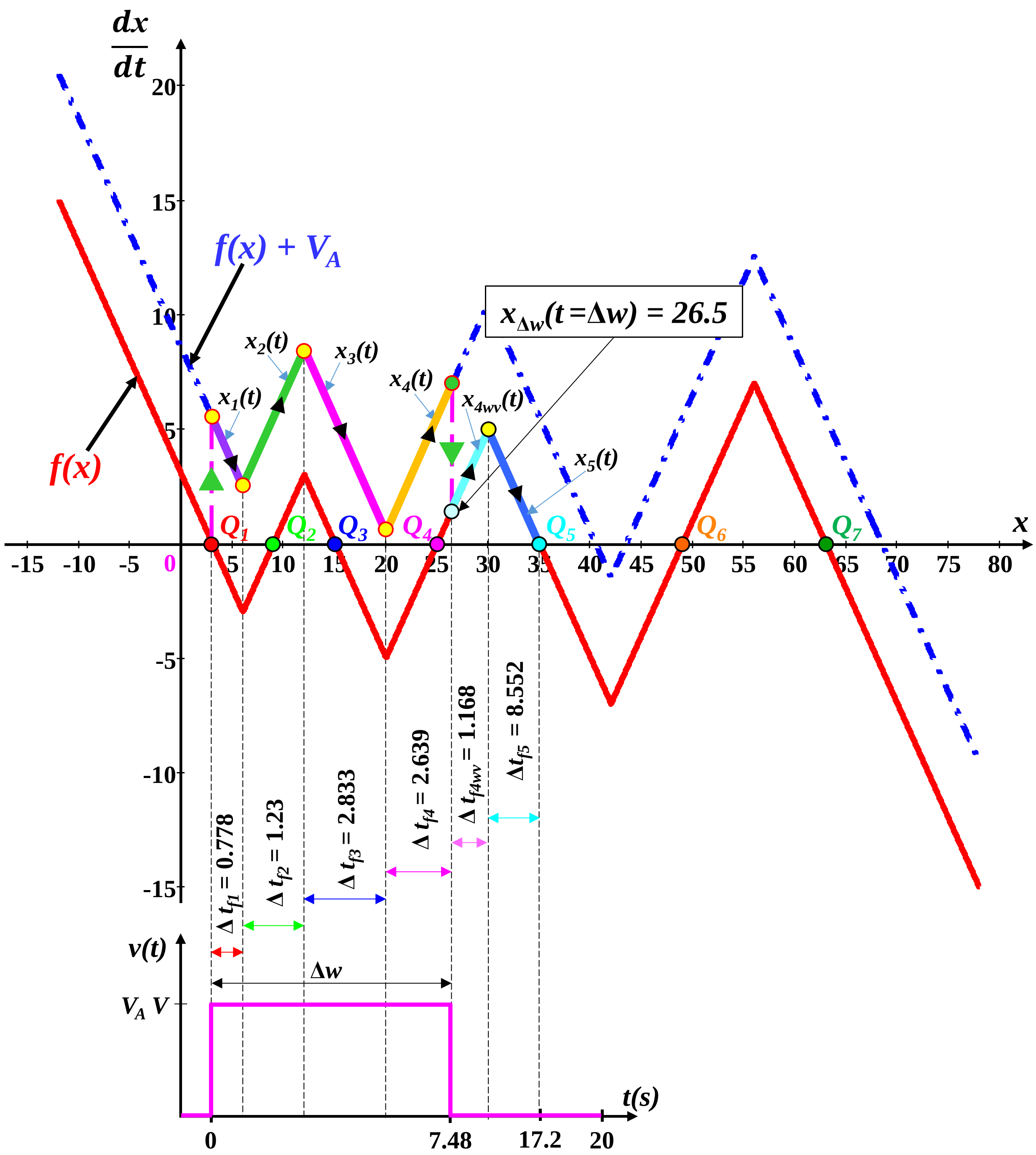

FIG. 1. Dynamic routes of the switching kinetics of 6-lobe Chua corsage memristor. The two magenta-color vertical line segments indicate an instantaneous jump between the red and the blue piecewise-linear plots in the dynamic route map.

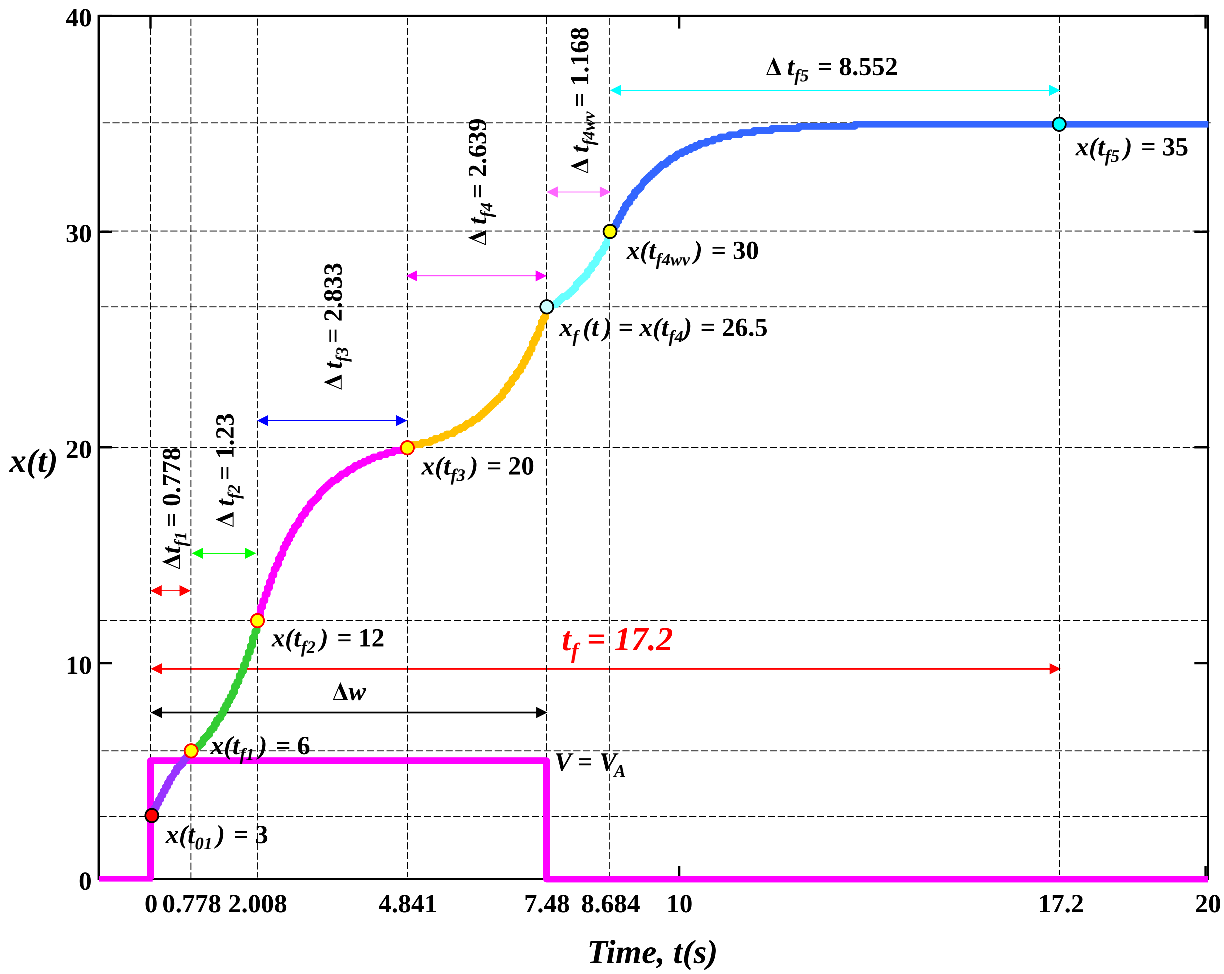

FIG. 2. Movement of the exponential trajectories of *x(t)* with respect to time, *t*.