Research Article

Dynamic Behaviors and the Equivalent Realization of a Novel Fractional-Order Memristor-Based Chaotic Circuit

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This paper proposed a novel fractional-order memristor-based chaotic circuit. A memristive diode bridge cascaded with a fractional-order RL filter constitutes the generalized fractional-order memristor. The mathematical model of the proposed fractional-order chaotic circuit is established by extending the nonlinear capacitor and inductor in the memristive chaotic circuit to the fractional order. Detailed theoretical analysis and numerical simulations are carried out on the dynamic behavior of the proposed circuit by investigating the stability of equilibrium points and the influence of circuit parameters on bifurcations. The results show that the order of the fractional-order circuit has a great influence on the dynamical behavior of the system. The system may exhibit complicated nonlinear dynamic behavior such as bifurcation and chaos with the change of the order. The equivalent circuits of the fractional-order inductor and capacitor are also given in the paper, and the parameters of the equivalent circuits are solved by an undetermined coefficient method. Circuit simulations of the equivalent fractional-order memristive chaotic circuit are carried out in order to validate the correctness of numerical simulations and the practicability of using the integer-order equivalent circuit to substitute the fractional-order element.

1. Introduction

Since memristor was first postulated by Leon O. Chua in 1971 [1], the research on it has been a hot topic in research. Memristor is a nonlinear passive two-terminal component and represents the relationship between a charge and flux [2]. As the fourth fundamental circuit element besides the resistor, inductor, and capacitor, memristor has many unique features that other elements do not have. It can remember its history resistance so be known as a contraction of “memory resistor.” The resistance of memristor depends on the magnitude, direction, and duration of the voltage applied to it [3]. In recent years, memristor has been widely used in many fields such as resistive random access memory (RRAM) [4, 5], neural networks [6–8], signal processing [9], chaotic and control systems [10], and sample recognition [7, 11].

The fingerprint of the memristor is that the loci in the voltage-current plane exhibit a pinched hysteresis loop which always passes through the origin when driven by any periodic input voltage source or current source [12]. Based on the above statement, a class of diode bridge circuits can be described as generalized memristors because they have been proved to satisfy the fingerprint of a memristor [13–19]. It is proved that an elementary electronic circuit consisting of a full-wave rectifier and a second-order RLC filter has memory properties [13]. Removing the resistor, another diode bridge circuit cascaded with a second-order filter containing an inductor and a capacitor can also constitute a generalized memristor [14]. Replacing the second-order filter with a first-order parallel RC filter or a series first-order RL filter, a newly generalized memristor is realized, which reduces to a nonlinear fundamental element [15–17]. Recently, an improved memristive diode bridge circuit was put forward comprising

four diodes and an inductor, which has a much simpler circuit realization [18, 19].

Due to the nonlinear characteristics of the memristor, chaotic oscillations can easily emerge. Replacing the Chua's diode in the canonical Chua's oscillator with a memristor, a memristor-based chaotic circuit was implemented in 2008 [20], which is known as the first combination of memristor and chaotic circuit. Since then, the study of memristor chaotic circuits has been widely undertaken [21–27]. A simple memristor chaotic circuit is proposed in paper [23], in which the inductor and capacitors are connected in series and one capacitor cascaded with a memristor. In paper [24], a memristor, an inductor, a capacitor, and a linear negative resistor constitute a chaotic circuit, and paper [25] puts forward a simplest chaotic circuit which has only three circuit elements: a linear passive inductor, a linear passive capacitor, and a nonlinear active memristor.

On the other hand, fractional calculus, a more than 300-year-old mathematical subject, can describe a real object more accurately than the classical “integer” methods [28]. In addition, due to the introduction of extra adjustable parameters, the fractional-order model will increase the degree of freedom. What is worth mentioning is that the principle of fractional calculus is based on the memory property of the fractional-order integral or derivative [29], so the connection between fractional calculus and memristor is straightforward, the memristor can be extended to fractional order as well. In paper [30], a fractional-order memristor model was applied to the Chua's oscillator for the first time, which constructed a fractional-order memristor-based chaotic circuit. Paper [31] proposed a fractional-order generalized memristor which consists of four diodes cascaded with a fractional-order parallel RC filter (the order is between 0 and 1) and established a fractional-order chaotic circuit by replacing the traditional Chua's diode with the proposed memristor. The fractional-order capacitor and inductor are implemented by the corresponding unit circuits. Not only numerical simulations were carried out but also equivalent circuits were realized in the paper.

This paper is organized as follows. Section 2 briefly introduces the fundamental of fractional calculus and constructs a generalized memristor based on a fractional-order series RL filter and then establishes a fractional-order memristive chaotic circuit. In Section 3, detailed theoretical analysis and numerical simulations are carried out on the dynamic behavior of the proposed circuit by investigating the stability of equilibrium points and the influence of system parameters on bifurcations. In Section 4, the equivalent circuits of fractional-order inductor and capacitor are implemented, and circuit simulations of the realized fractional-order memristive chaotic circuit are carried out. Section 5 concludes this article with some additional remarks.

2. Fractional-Order Memristive Chaotic Circuit

2.1. Fundamentals of Fractional Calculus. Fractional calculus, as a generalization of integration and differentiation to integer order, can more accurately characterize the properties

of actual objects. The fundamental integral-differential operator ${}_n D_t^\lambda$ is defined as [32, 33]

$${}_n D_t^\lambda = \begin{cases} \frac{d^\lambda}{dt^\lambda}, & \lambda > 0, \\ 1, & \lambda = 0, \\ \int_n^t (d\tau)^{-\lambda}, & \lambda < 0, \end{cases} \quad (1)$$

where n and t are the bounds of the operation and λ is the order.

There are three equivalent definitions used for the general fractional differentiation known as the Grünwald-Letnikov (GL) definition, the Riemann-Liouville (RL) definition, and the Caputo definition. The Caputo definition is more convenient for initial condition problems because its physical meaning is clear [34–36]. In this work, we will use the differential conformal transformation based on Caputo's derivatives. The Caputo definition can be written as

$${}_n^C D_t^\lambda f(t) = \frac{1}{\Gamma(m-\lambda)} \int_n^t \frac{f^{(m)}(\tau)}{(t-\tau)^{\lambda-m+1}} d\tau, \quad m-1 < \lambda < m, \quad (2)$$

where $\Gamma(\cdot)$ is the gamma function and λ is usually greater than zero [29]. The main advantage of Caputo's approach is that the initial conditions for fractional differential equations with Caputo derivatives are in the same form as for integer-order differential equations, and the Laplace transform of the Caputo derivative allows utilization of initial values of classical integer-order derivatives with known physical interpretations [33]. The Laplace transform of the λ order Caputo differential operator is

$$L\left\{{}_0^C D_t^\lambda f(t)\right\} = s^\lambda F(s) - \sum_{k=0}^{m-1} s^k \left[\frac{d^{\lambda-1-k} f(t)}{dt^{\lambda-1-k}} \right]_{t=0}, \quad (3)$$

where m is an integer and $\lambda \in (m-1, m)$. In this paper, we consider the zero initial conditions, so Laplace transform of Caputo's fractional derivative reduces to [37]

$$L\left\{{}_0^C D_t^\lambda f(t)\right\} = s^\lambda F(s). \quad (4)$$

In this case, the three well-known definitions of fractional derivatives are equivalent [38].

2.2. Fractional-Order Memristor Model. Paper [17] has demonstrated that a diode bridge circuit cascaded with an inductor and resistor in series satisfied the definition of a generalized memristor, whose equivalent circuit only contains six fundamental elements, which is depicted in Figure 1. Assuming the voltage across the generalized

FIGURE 1: Generalized memristor and its circuit realization: (a) memristive diode bridge with series RL filter and (b) memristor symbol representation.

memristor is u_m , the current flowing through the generalized memristor is i_m , and the current passing through the inductor L_m is i_L , the mathematical model of the proposed generalized memristor circuit is established as

$$\begin{cases} \frac{di_L}{dt} = \frac{u_m}{L_m} - \frac{R_m i_L}{L_m} - \frac{1}{\rho L_m} \ln \left(\frac{(2I_S + i_L)e^{\rho u_m}}{2I_S \cosh(\rho u_m)} \right), \\ i_m = (2I_S + i_L) \tanh(\rho u_m), \end{cases} \quad (5)$$

where I_S and ρ are diode parameters and $\rho = 1/(2nV_T)$. I_S , n , and V_T indicate the reverse saturation current, emission coefficient, and the thermal voltage of the diode, respectively. In this paper, diodes 1N4148 are used for circuit simulations, and parameters of the diodes are presented as $I_S = 2.682$ nA, $n = 1.836$, and $V_T = 25$ mV.

As nonlinear elements, inductors and capacitors can be extended to fractional order from the point of view of fractional calculus [29]. As a result, a fractional-order inductor-based generalized memristive circuit is realized by replacing the integer-order inductor L_m with a fractional-order inductor L_m^λ and whose mathematical model can be expressed as

$$\begin{cases} \frac{d^\lambda i_L}{dt^\lambda} = \frac{u_m}{L_m^\lambda} - \frac{R_m i_L}{L_m^\lambda} - \frac{1}{\rho L_m^\lambda} \ln \left(\frac{(2I_S + i_L)e^{\rho u_m}}{2I_S \cosh(\rho u_m)} \right), \\ i_m = (2I_S + i_L) \tanh(\rho u_m). \end{cases} \quad (6)$$

In order to verify whether the proposed fractional-order generalized memristor satisfied the three fingerprints of a memristor [39], numerical simulations are carried out using MATLAB software. The input voltage is given as a sinusoidal voltage $u_m = A_m \sin(2\pi f t)$, and the parameters of the inductor and the resistor are 280 mH and 580 Ω , respectively. Setting the fractional order λ equals 0.95, the relationship curves of the memristor input voltage and input current versus different frequencies are indicated in Figure 2(a), and the relationship curves with different amplitudes are demonstrated in Figure 2(b). The amplitude of input voltage in Figure 2(a) is set as $A_m = 4$ V, and the frequency of input

voltage is $f = 500$ Hz in Figure 2(b). Moreover, the hysteresis loops with different orders are shown in Figure 2(c), in which the amplitude and frequency are set as $A_m = 4$ V and $f = 500$ Hz, respectively. It can be seen that the hysteresis loops pinched at the origin and the area of the pinched hysteresis loops decrease monotonically as the input voltage frequency increase when the frequency is greater than the critical value. Besides, the pinched hysteresis loop will shrink to a nonlinear single-valued function when the frequency tends to infinity. The areas of pinched hysteresis loops increase as the input voltage amplitude increase as well, as depicted in Figure 2(b). In addition, the pinched hysteresis loops indicated in Figure 2(c) demonstrate that the order of the fractional-order inductor can affect the dynamic characteristics of the memristor. It can be seen that the pinched hysteresis loops with different amplitudes are similar to each other (see Figure 2(b)), but the frequency and the order can bring about an important impact on the memristive characteristics (see Figures 2(a) and 2(c)).

2.3. Fractional-Order Memristor-Based Chaotic Circuit. After the realization of the fractional-order generalized memristor, we consider applying the proposed first-order generalized memristor to a classical Chua's circuit. By replacing the traditional Chua's oscillator with a fractional-order inductor-based memristor, a fractional-order memristive chaotic circuit is established, which is indicated in Figure 3. Since the practical inductor and capacitor can be extended to fractional order to better express its electricity characteristics, the capacitance and inductance used in this paper are all fractional order. Therefore, the circuit is composed of six elements which are a fractional-order inductor L^λ , two fractional-order capacitors C_1^λ and C_2^λ , a fractional-order memristor M^λ , a linear passive resistor, and a negative conductance. Because the proposed fractional-order memristive Chua's circuit has four dynamic elements, there are four corresponding state variables, which are the voltages across the fractional-order capacitors C_1^λ and C_2^λ v_1 and v_2 , the current flowing through the fractional-order inductor L^λ i_3 . The last one is the current passing through the fractional-order inductor L_m^λ in the fractional-order generalized memristor, M^λ i_L , as shown in (6), which can express the dynamic characteristics of the fractional-order generalized memristor M^λ . Utilizing Kirchhoff voltage law (KVL) and Kirchhoff current law (KCL), the characteristic equation of the fractional-order memristive Chua's circuit can be derived as

$$\begin{cases} \frac{d^\lambda v_1}{dt^\lambda} = \frac{v_2 - v_1}{RC_1^\lambda} - \frac{i_3}{C_1^\lambda}, \\ \frac{d^\lambda v_2}{dt^\lambda} = \frac{v_2(RG - 1)}{RC_2^\lambda} + \frac{v_1}{RC_2^\lambda} - \frac{(2I_S + i_L) \tanh(\rho v_2)}{C_2^\lambda}, \\ \frac{d^\lambda i_3}{dt^\lambda} = \frac{v_1}{L^\lambda}, \\ \frac{d^\lambda i_L}{dt^\lambda} = \frac{v_2}{L_m^\lambda} - \frac{R_m i_L}{L_m^\lambda} - \frac{1}{\rho L_m^\lambda} \ln \left(\frac{(2I_S + i_L)e^{\rho v_2}}{2I_S \cosh(\rho v_2)} \right). \end{cases} \quad (7)$$

FIGURE 2: The hysteresis loops of the fractional-order memristor with different sinusoidal input voltages: (a) $A_m = 4$ V and $\lambda = 0.95$ with different frequencies, (b) $f = 500$ Hz and $\lambda = 0.95$ with different amplitudes, and (c) $A_m = 4$ V and $f = 500$ Hz with different orders.

FIGURE 3: Fractional-order memristive Chua's circuit.

The model of the fractional-order memristive Chua's circuit is established in MATLAB, and simulations are carried out in numerical. Set capacitance $C_1^\lambda = 100$ nF, capacitance

$C_2^\lambda = 10$ nF, inductance $L^\lambda = 12$ mH, resistance $R = 2$ k Ω , and conductance $G = 0.6667$ mS, and the parameters of the fractional-order generalized memristor are the same as that mentioned above. When the fractional order λ is taken as 0.98 and the initial values of the state variables are selected as $v_1 = 0$ V, $v_2 = 0.01$ V, $i_3 = 0$ A, and $i_L = 0$ A, the phase diagrams of the fractional-order memristive Chua's circuit are exhibited in Figure 4.

3. Chaotic Dynamic Behavior of the Fractional-Order Memristive Chua's Circuit

3.1. Equilibrium Points and Stability. In this section, equilibrium points and eigenvalues of the corresponding Jacobian

FIGURE 4: Phase diagrams of the fractional-order memristive Chua's circuit. Phase diagrams in the (a) $v_2(t)$ – $v_1(t)$ plane, (b) $v_2(t)$ – $i_3(t)$ plane, (c) $v_2(t)$ – $i_L(t)$ plane, and (d) $v_2(t)$ – $i_m(t)$ plane.

matrix are calculated to qualitatively analyze the dynamic behavior of the proposed fractional-order memristive chaotic circuit. The equilibrium points of such a circuit can be obtained by solving the following equations:

$$\begin{cases} v_1 = 0, \\ v_2(RG - 1) = R(2I_S + i_L) \tanh(\rho v_2), \\ v_2 = R_m i_L + \frac{1}{\rho} \ln \left(\frac{(2I_S + i_L)e^{\rho v_2}}{2I_S \cosh(\rho v_2)} \right), \\ v_2 = i_3 R. \end{cases} \quad (8)$$

Obviously, $O_1 = (0, 0, 0, 0)$ is one equilibrium point of the fractional-order memristive Chua's circuit, but the analytic solutions of v_2 and i_L cannot be computed in MATLAB, so we consider using the graphic analytic method to obtain

the other equilibrium points [16]. The relationship between v_2 and i_L can be written as

$$\begin{cases} v_2 = \pm \frac{1}{\rho} \operatorname{acosh} \left(\frac{(2I_S + i_L)e^{\rho R_m i_L}}{2I_S} \right), \\ i_L = \frac{(RG - 1)v_2}{R \tanh(\rho v_2)} - 2I_S. \end{cases} \quad (9)$$

Utilizing (9), we can draw two functions between v_2 and i_L ; then the intersection coordinates of two functions can easily be obtained, as shown in Figure 5. Thus, the other two equilibrium points are obtained as $O_2 = (0, 1.134, 0.000567, 0.000189)$ and $O_3 = (0, -1.134, -0.000567, 0.000189)$.

FIGURE 5: Two functions and their intersection points.

The Jacobian matrix of the characteristic (7) can be described as

$$\begin{bmatrix} -\frac{1}{RC_1^\lambda} & \frac{1}{RC_1^\lambda} & -\frac{1}{C_1^\lambda} & 0 \\ \frac{1}{RC_2^\lambda} & \Lambda_1 & 0 & -\frac{\tanh(\rho v_2)}{C_2^\lambda} \\ \frac{1}{L^\lambda} & 0 & 0 & 0 \\ 0 & \Lambda_2 & 0 & -\frac{R_m}{L_m^\lambda} - \frac{1}{\rho L_m^\lambda (2I_S + i_L)} \end{bmatrix}, \quad (10)$$

where Λ_1 and Λ_2 can be expressed as

$$\Lambda_1 = \frac{RG - 1}{RC_2^\lambda} + \frac{\rho}{C_2^\lambda} (2I_S + i_L) (\tanh^2(\rho v_2) - 1), \quad (11)$$

$$\Lambda_2 = \frac{1}{L_m^\lambda} - \frac{2I_S e^{-\rho v_2} \cosh(\rho v_2)}{\rho L_m^\lambda (2I_S + i_L)} \cdot \left(\frac{\rho(2I_S + i_L) e^{\rho v_2}}{2I_S \cosh(\rho v_2)} - \frac{\rho(2I_S + i_L) e^{\rho v_2} \sinh(\rho v_2)}{2I_S \cosh^2(\rho v_2)} \right). \quad (12)$$

Then the eigenvalues at equilibrium points $O_{1,2,3}$ can be calculated as

$$\begin{aligned} O_1 : \zeta_1 &= -61, 124, 000, \\ \zeta_2 &= 20, 000, \\ \zeta_{3,4} &= -4000 \pm 26, 000i, \\ O_{2,3} : \zeta_{1,2} &= -6106 \pm 26, 054i, \\ \zeta_{3,4} &= 10, 038 \pm 15, 524i. \end{aligned} \quad (13)$$

FIGURE 6: Bifurcation diagram of the fractional-order memristive chaotic circuit with the different order.

It can be seen that equilibrium point O_1 has a positive real root and equilibrium point $O_{2,3}$ have two complex conjugate roots with the positive real part. As a result, the system is unstable and has chaotic attractors.

3.2. Bifurcation with the Changing of Order λ . The fractional-order models can increase the flexibility and degrees of freedom by means of the fractional parameters. In this section, the change of the fractional order λ is applied in the fractional-order memristive Chua's circuit, and the influence of the order on the performance of the fractional-order memristive chaotic circuit is discussed. Taking the same circuit parameters as mentioned above and the initial conditions are set as (0 V, 0.01 V, 0 A, 0 A), the bifurcation diagram of the fractional-order chaotic circuit described in (7) can be obtained, as depicted in Figure 6. It shows that the dynamical behaviors of the system can be divided into three states: periodic state, bifurcation, and chaos. The system has two blocks of chaos around $\lambda = 0.93$ and $\lambda = 0.99$ when the order λ varies from 0.9 to 1. With the increase of the order, the system enters into the chaotic state from the periodic state abruptly when $\lambda > 0.92$. However, the chaotic behavior does not exist for a long time; the system runs back to the periodic state when order λ is just over 0.94. When the order is higher than 0.97, the system enters into chaos once more. Furthermore, the system displays complicated nonlinear dynamic behavior such as bifurcation and chaos when order λ is greater than 0.96. In order to further illustrate the influence of the order on the dynamic behaviors of the system, a comparative analysis is carried out. The phase diagrams of the fractional-order memristive Chua's circuit when order equals to 0.955 and 0.965 are indicated in Figure 7. It can be seen that the circuit is a single cycle when $\lambda = 0.955$, but when $\lambda = 0.965$, the circuit has two or more cycles, bifurcation occurs, which corresponds well with the results in the bifurcation diagram as shown in Figure 6. In addition, it is worth noting that the orbits of state variable $v_1(t)$ with order

FIGURE 7: The phase diagrams of the fractional-order memristive Chua's circuit with the different order.

λ do not change smoothly with order λ , which is different from most of other bifurcation diagrams. From the above analysis, we know that different orders will cause different dynamical behaviors of the memristive chaotic circuit, such as bifurcation and chaos, which indicates that the order plays a significant role in the performance of the fractional-order memristive chaotic circuit.

3.3. Bifurcation with the Changing of Inductor L^λ . The bifurcation diagram of $v_2(t)$ with inductor L^λ is depicted in Figure 8.

The orbits painted in red indicate the integer-order system with order $\lambda = 1$, and the orbits painted in blue express the fractional-order system whose order $\lambda = 0.92$. For the integer-order system, as the increase of inductor value, the orbits of the memristive Chua's circuit start from the periodic state and then enter into chaos abruptly. After increasing for a period of time, the system returns back to periodic behavior. The phenomenon is not the same as that in the fractional-order system. The inductance value required for the fractional-order system to enter into chaotic behavior is larger than that of the integer-order system. Moreover, the fractional-order system is still in chaos when the inductance is greater than 33 mH, but the orbits are in a periodic state for the integer-order systems. In order to verify whether the theoretical analysis of the bifurcation diagram is correct, the phase diagrams of the integer-order system and fractional-order system with order $\lambda = 0.92$ are given in Figure 9 when inductance $L^\lambda = 20$ mH and $L^\lambda = 40$ mH, respectively. It can be seen that the results of the phase diagrams well demonstrate the correctness of the bifurcation diagram. The integer-order system is chaotic but the fractional-order system is periodic when inductance value $L^\lambda = 20$ mH. Meanwhile, the integer-order system has a periodic behavior

FIGURE 8: Bifurcation diagrams of the fractional-order memristive chaotic circuit and integer-order memristive chaotic circuit with different inductance L^λ .

but the fractional-order system has a chaotic behavior when inductance value $L^\lambda = 40$ mH.

4. Circuit Realization of the Fractional-Order Memristive Chaotic Circuit

In this section, equivalent circuits are used to simulate the fractional-order memristive chaotic circuit. Since the inductor and capacitor can be extended to fractional order, the Oustaloup filter algorithm is used to obtain the approximate

FIGURE 9: Phase diagrams of the integer-order system and fractional-order system: (a) $L^\lambda = 20$ mH and (b) $L^\lambda = 40$ mH.

transfer function of fractional Laplace transform operator s^λ [40], then simplifying the transfer functions into zero-pole forms and using chain circuits to realize the equivalent circuit of fractional-order inductor and capacitor [41].

4.1. Equivalent Circuit of Fractional-Order Generalized Memristor. The fractional Laplace transform operator of inductor in fractional-order generalized memristor can be physically realized using the RL chain circuits in Figure 10. The transfer function of the fractional-order inductor can be approximately expressed as

$$\frac{1}{L} s^{-\lambda} = \sum_{i=1}^n \frac{1/L_i}{(s + R_i/L_i)} + \frac{1}{R_0}, \quad (14)$$

where λ is the order of the fractional-order inductor. Selecting $L = 280$ mH, $\lambda = 0.95, 0.98$ and $n = 7$, resistance and inductance can be calculated by the method of undetermined coefficients. The parameters of the equivalent chain circuit of fractional-order inductor are listed in Tables 1 and 2. The magnitude and phase of the ideal $1/(Ls^\lambda)$ and 7th order Oustaloup approximation and fractance circuit approximation with $L = 280$ mH and order $\lambda = 0.95, 0.98$ are presented in Figure 11. From the figure, we observe that the approximation is relatively good from 10^{-1} Hz to 10^5 Hz.

4.2. Equivalent Circuit of Fractional-Order Memristive Chua's Circuit. After realizing the equivalent circuit of fractional-order inductor, we consider using the same method to achieve the equivalent circuit of the fractional-order capacitor. The approximate equivalence of the fractional-order capacitor can be actualized utilizing RC ladder topologies, as shown in Figure 12. The transfer function of the RC ladder network is expressed as

$$\frac{1}{C} s^{-\lambda} = \sum_{i=1}^n \frac{1/C_i}{(s + 1/R_i C_i)} + R_0. \quad (15)$$

FIGURE 10: Equivalent chain circuit of fractional-order inductor.

In consideration of capacitors $C_1^\lambda = 100$ nF and $C_2^\lambda = 10$ nF in the fractional-order memristive Chua's circuit, choosing $\lambda = 0.98$ and $n = 7$, resistance and capacitance can be calculated as well, as listed in Tables 3 and 4. The bode diagram of the ideal $1/(Cs^\lambda)$ and 7th order Oustaloup approximation and fractance circuit approximation with $C^\lambda = 10$ nF and 100 nF and order $\lambda = 0.98$ is indicated in Figure 13. It can be seen that the approximation is pretty good from 10^{-2} Hz to 10^5 Hz. The chain equivalent parameters of fractional-order inductor L^λ in the fractional-order memristive Chua's circuit can be calculated as well, as listed in Tables 5 and 6, and the bode diagram shows a good approximation from 10^{-2} Hz to 10^5 Hz, as shown in Figure 14. Thus, the equivalent circuit of the fractional-order memristive Chua's circuit has been completed as shown in Figure 15, which has four equivalent circuit units, and then the circuit simulation can be carried out on the fractional-order memristive equivalent circuit.

4.3. Circuit Simulations of the Fractional-Order Memristive Equivalent Circuit. In this section, we consider utilizing PSpice to simulate the fractional-order memristive equivalent

TABLE 1: The resistance parameters of the equivalent chain circuit of fractional-order inductor.

λ	R_0 (Ω)	R_1 (Ω)	R_2 (Ω)	R_3 (Ω)	R_4 (Ω)	R_5 (Ω)	R_6 (Ω)	R_7 (Ω)
0.95	1.25e6	7.74e4	964.42	12.14	1.53e-1	1.92e-3	2.52e-6	7.76e-1
0.98	2.03e6	2.55e5	2.77e3	30.39	3.33e-1	3.65e-3	1.78e-6	4.18e-1

TABLE 2: The inductance parameters of the equivalent chain circuit of fractional-order inductor.

λ	L_1 (H)	L_2 (H)	L_3 (H)	L_4 (H)	L_5 (H)	L_6 (H)	L_7 (H)
0.95	0.6896	0.8595	1.0820	1.3621	1.7147	0.4440	1.3688e5
0.98	2.4385	2.6473	2.9025	3.1825	3.4893	0.3367	7.9025e4

FIGURE 11: Bode diagram of Oustaloup approximation and fractance circuit approximation.

FIGURE 12: Equivalent ladder circuit of the fractional-order capacitor.

circuit. Considering order $\lambda = 0.98$, the parameters of resistance R_m and diodes in fractional-order generalized memristor are the same as mentioned above, and fractional-order inductor $L_m = 280$ mH is realized by a chain unit, whose equivalent parameters are listed in Tables 1 and 2. In addition, resistance R and negative conductance $-G$ in the fractional-order memristive chaotic circuit are constant, while fractional-order capacitance $C_1^\lambda = 100$ nF and $C_2^\lambda = 10$ nF and fractional-order inductance $L^\lambda = 12$ mH are implemented by corresponding equivalent circuits, whose parameters are given in Tables 3–6. Thus, an equivalent circuit of the fractional-order memristor-based fractional-

order Chua's circuit is achieved. Figure 16 shows the phase diagrams of the realized fractional-order memristive Chua's circuit by PSpice, and the results well verify the correctness of numerical simulations and the practicability of the fractional-order equivalent circuit.

5. Conclusions

In this paper, a fractional-order memristor-based chaotic circuit is presented. The inductor in generalized memristor and the capacitors and inductors in the memristive chaotic circuit are all fractional order. Firstly, the mathematical model of fractional-order inductor-based generalized memristor is established, and then the characteristic equations of the fractional-order memristive chaotic circuit are derived. After that, theoretical analysis and numerical simulations are carried out such as the analysis of equilibrium point and stability and the influence of the order on the stability of the system. The results show that the fractional-order circuit exhibits different dynamic behaviors such as

TABLE 3: The resistance parameters of the equivalent ladder circuit of the fractional-order capacitor.

C^λ (nF)	R_0 (Ω)	R_1 (Ω)	R_2 (Ω)	R_3 (Ω)	R_4 (Ω)	R_5 (Ω)	R_6 (Ω)	R_7 (Ω)
10	13.80	109.66	1.01e4	9.21e5	8.40e7	7.66e9	1.57e13	6.70e7
100	1.38	10.97	1.01e3	9.21e4	8.40e6	7.66e8	1.57e12	6.70e6

TABLE 4: The capacitance parameters of the equivalent ladder circuit of the fractional-order capacitor.

C^λ (nF)	C_1 (μ F)	C_2 (μ F)	C_3 (μ F)	C_4 (μ F)	C_5 (μ F)	C_6 (μ F)	C_7 (μ F)
10	8.71e-2	9.45e-2	1.04e-1	1.14e-1	1.25e-1	1.20e-2	2.82e3
100	8.71e-1	9.45e-1	1.04	1.14	1.25	1.20e-1	2.82e4

FIGURE 13: Bode diagram of Oustaloup approximation and fractance circuit approximation.

TABLE 5: The resistance parameters of the equivalent chain circuit of fractional-order inductor.

R_0 (Ω)	R_1 (Ω)	R_2 (Ω)	R_3 (Ω)	R_4 (Ω)	R_5 (Ω)	R_6 (Ω)	R_7 (Ω)
8.69e4	1.09e4	118.80	1.30	1.43e-2	1.57e-4	7.63e-8	1.79e-2

TABLE 6: The inductance parameters of the equivalent chain circuit of fractional-order inductor.

L_1 (H)	L_2 (H)	L_3 (H)	L_4 (H)	L_5 (H)	L_6 (H)	L_7 (H)
0.1045	0.1135	0.1244	0.1364	0.1495	0.0144	3.3868e3

bifurcation and chaos with the change of the order, which indicates the importance of order effects on the dynamic behaviors of the system. In order to verify the aforementioned analysis, the equivalent circuit of the fractional-order memristive chaotic circuit is presented. The nonlinear elements inductor and capacitor are approximately equivalent through a unit circuit. Using the method of undetermined coefficients to solve circuit parameters, a fractional-order memristor-based equivalent circuit is constructed in PSpice. The results of circuit simulations well confirm the investigation on the fractional-order memristive chaotic circuit both in theoretical and numerical.

FIGURE 14: Bode diagram of Oustaloup approximation and fractance circuit approximation.

FIGURE 15: The equivalent realization of the fractional-order memristor-based chaotic circuit.

FIGURE 16: Phase diagrams of the realized fractional-order memristive Chua's circuit. Phase diagrams in the (a) $v_2(t)-v_1(t)$ plane, (b) $v_2(t)-i_3(t)$ plane, (c) $v_2(t)-i_L(t)$ plane, and (d) $v_2(t)-i_m(t)$ plane.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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