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Research Article

Fault Tree Interval Analysis of Complex Systems Based on Universal Grey Operation

Feng Zhang, [1] **Shiwang Tan**, [1] **Leilei Zhang**, [2] **Yameng Wang**, [1] **and Yang Gao** [1]

[1] *School of Mechanics, Civil Engineering and Architecture, Northwestern Polytechnical University, Xi'an 710129, China*
[2] *No. 203 Research Institute of China Ordnance Industries, Xi'an 710065, China*

Correspondence should be addressed to Feng Zhang; nwpuwindy@nwpu.edu.cn

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The objective of this study is to propose a new operation method based on the universal grey number to overcome the shortcomings of typical interval operation in solving system fault trees. First, the failure probability ranges of the bottom events are described according to the conversion rules between the interval number and universal grey number. A more accurate system reliability calculation is then obtained based on the logical relationship between the AND gates and OR gates of a fault tree and universal grey number arithmetic. Then, considering an aircraft landing gear retraction system as an example, the failure probability range of the top event is obtained through universal grey operation. Next, the reliability of the aircraft landing gear retraction system is evaluated despite insufficient statistical information describing failures. The example demonstrates that the proposed method provides many advantages in resolving the system reliability problem despite poor information, yielding benefits for the function of the interval operation, and overcoming the drawback of solution interval enlargement under different orders of interval operation.

1. Introduction

The fault tree analysis (FTA) method is typically applied as the main method in the reliability analysis of large systems [1–3]. A fault tree is a logical block diagram composed of a top event (outcome), intermediate events, and bottom events and is used to describe the internal functional logical relationship between events. The logical relationships between the components of a system and their events are obtained based on the operating principle and fault mechanisms of the system. The top-level failure probability of the system can be obtained through the logical relationship between event layers and through data operations using the failure probability statistics of the underlying components of the system. Then, a system-level reliability evaluation can be performed [4–7]. In recent studies, researchers have made major achievements in theoretical system and engineering reliability analyses based on fault trees. The FTA method based on a probability model has seen wide application in several systems engineering fields such as aviation, aerospace, and nuclear power [8–12].

The traditional fault tree analysis method is based on a probability model; when there is a large set of failure samples and other sufficient statistical information describing the evaluated parts of a system, the uncertainty of bottom events can be quantified independently [5, 12]. However, when only small sample sets are available in an engineering analysis case, the statistics describing component failure are insufficient to accurately estimate failure distribution [13, 14]. It is thus difficult to determine the failure probability of components in many kinds of complex systems, such as landing gear retraction systems and large-scale space-borne antennae. This limits the application of the probabilistic model-based fault tree method in complex engineering applications.

Based on the fuzzy set theory, the fuzzy fault tree describes the probability of event occurrence using various fuzzy numbers, addressing the difficulties associated with precisely measuring the probability of base event occurrence due to the complexity of the environment and incomplete data [15–18]. Ding and Lisianski regarded the performance rate and corresponding probability of an event as fuzzy values and developed a reliability evaluation technique for a multistate system using the fuzzy universal generation function [19]. Li et al. introduced random fuzzy variables and proposed a hybrid universal generation function [20]. Liu and Huang proposed a fuzzy continuous-time Markov model with a finite discrete state and used it to evaluate the fuzzy state probability of multistate elements at any time [21]. However, in the fuzzy fault tree, the determination of the fuzzy value, fuzzy variable, and fuzzy state probability is highly subjective.

The interval domain is an important model in non-probability theory: the shape of an interval domain represents the degree to which events occur in an interval model, while the size of the interval domain signifies the volatility or degree of deviation of an uncertain event. To establish an interval model, only the boundaries of an event set are required, not its internal distribution. This results in significant independence from the data compared to a conventional probability model [22–27]. However, it should be pointed out that the power exponentiation of an interval number will lead to the expansion of the interval and that different orders of operation performed on the same interval numbers can provide different expansion intervals [28, 29].

The universal grey number provides the function of the interval operation and overcomes the drawback associated with traditional interval operation, i.e., the change in solution interval with order of operation [29]. Some scholars have gradually introduced and successfully applied the universal grey operation to structural reliability research [30–33]. Luo introduced the grey range transformation into the process of model building to eliminate the incomparability of different dimensions and achieved an effective risk assessment of the ice plug phenomenon [30]. Jin et al. proposed a generalized Rayleigh quotient method based on generalized grey mathematics to represent the interval parameters in uncertain structures using generalized grey numbers [31]. Liu et al. considered the uncertainty of the interval arithmetic for the structural, nonprobabilistic reliability calculation of nonlinear systems, using the universal grey number instead of the interval parameters to overcome the impact of interval arithmetic uncertainty on reliability results [32].

Based on the advantages of the universal grey number method, a new method for solving the reliability of the top event of a fault tree is proposed in this paper to overcome the shortcomings of the existing nonprobabilistic reliability method of interval operation. The proposed method complies with the conversion rule between the interval number and universal number, and the four arithmetic operations of the universal grey number.

2. Interval Analysis of Simple System Fault Trees

2.1. Four Arithmetic Operations of Traditional Interval Analysis. Let '$*$' represent a real binary operation on the set of real numbers, $* \in \{+,-,\cdot,/\}$. For $[x] = [\underline{x},\overline{x}] \in I(R)$ and $[y] = [\underline{y},\overline{y}] \in I(R)$, the binary operation on interval set $I(R)$ is defined as follows:

$$[x] * [y] = \{z \mid z = x * y,\ x \in [x],\ y \in [y]\} \qquad (1)$$

The four arithmetic operation rules can then be derived as follows [27]:

$$\begin{aligned}
[x] + [y] &= [\underline{x} + \underline{y}, \overline{x} + \overline{y}] \\
[x] - [y] &= [\underline{x} - \overline{y}, \overline{x} - \underline{y}] \\
[x] \cdot [y] &= \left[\min\left(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\,\overline{y}\right), \max\left(\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\underline{y}, \overline{x}\,\overline{y}\right)\right] \\
\frac{[x]}{[y]} &= [\underline{x}, \overline{x}] \cdot \left[\frac{1}{\overline{y}}, \frac{1}{\underline{y}}\right] \quad (0 \notin [y])
\end{aligned} \qquad (2)$$

It can be seen in (2) that the calculation of the interval number provides an extremely wide range due to the influence of interval expansion. This is the primary drawback of the interval method.

2.2. Four Arithmetic Operations of Interval Analysis Based on the Universal Grey Operator. Setting the domain as $U = R$ (the set of real numbers), the universal grey number set in R is denoted by $g(R)$. Calling an element in $g(R)$ the universal grey number, $g = (x, [\underline{\mu}, \overline{\mu}])$, $x \in R$, and $\underline{\mu}, \overline{\mu} \in R$, where x is the observed value and $[\underline{\mu}, \overline{\mu}]$ is the grey information portion of x.

The corresponding four arithmetic operation rules are accordingly [27]

$$g_1 + g_2 = \left(x_1 + x_2, \left[\frac{x_1\underline{\mu_1} + x_2\underline{\mu_2}}{x_1 + x_2}, \frac{x_1\overline{\mu_1} + x_2\overline{\mu_2}}{x_1 + x_2}\right]\right) \qquad (3)$$

$$g_1 - g_2 = \left(x_1 - x_2, \left[\frac{x_1\underline{\mu_1} - x_2\underline{\mu_2}}{x_1 - x_2}, \frac{x_1\overline{\mu_1} - x_2\overline{\mu_2}}{x_1 - x_2}\right]\right) \qquad (4)$$

$$g_1 \times g_2 = \left(x_1 x_2, \left[\underline{\mu_1}\,\underline{\mu_2}, \overline{\mu_1}\,\overline{\mu_2}\right]\right) \qquad (5)$$

$$\frac{g_1}{g_2} = \left(\frac{x_1}{x_2}, \left[\frac{\underline{\mu_1}}{\underline{\mu_2}}, \frac{\overline{\mu_1}}{\overline{\mu_2}}\right]\right) \qquad (6)$$

In practical applications, the universal grey number and interval number can be interchanged with each other via conversion. For the grey number $g = (x, [\underline{\mu}, \overline{\mu}])$, the corresponding interval number is in the form of $g = [x\underline{\mu}, x\overline{\mu}]$. The interval number $g = [\underline{x}, \overline{x}]$ can be uniformly expressed as $g = (\overline{x}, [\underline{x}/\overline{x}, 1])$ for ease of operation.

2.3. Interval Analysis of Fault Trees Based on Universal Grey Operator for an OR Gate Operator. A fault tree interval analysis based on the universal grey operator is performed by using an OR gate operator with three bottom events as an example. The fault tree is shown in Figure 1.

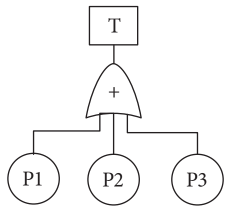

Figure 1: OR gate operator with three bottom events.

The failure function of the top event in relation to the bottom event is given by

$$P_{T1} = 1 - (1 - P_1)(1 - P_2)(1 - P_3) \tag{7}$$

and the other three equivalent forms are given as follows:

$$P_{T2} = 1 - (1 - P_1 - P_2 + P_1P_2)(1 - P_3) \tag{8}$$

$$P_{T3} = 1 - (1 - P_1)(1 - P_2 - P_3 + P_2P_3) \tag{9}$$

$$P_{T4} = P_1 + P_2 + P_3 - P_1P_2 - P_1P_3 - P_2P_3 + P_1P_2P_3 \tag{10}$$

The failure probabilities of the three bottom events are expressed by the interval number as follows:

$$P_1 = [0.1, 0.3] \tag{11}$$

$$P_2 = [0.2, 0.6] \tag{12}$$

$$P_3 = [0.4, 0.8] \tag{13}$$

Substituting the values in (11), (12), and (13) into (7) to (10), the probability interval range of the top event of the fault tree can be obtained as follows:

$$\begin{aligned} P_{T1} &= [1, 1] - [0.7, 0.9] \cdot [0.4, 0.8] \cdot [0.2, 0.6] = [1, 1] \\ &\quad - [0.056, 0.432] = [0.568, 0.944] \end{aligned} \tag{14}$$

$$\begin{aligned} P_{T2} &= [1, 1] - ([1, 1] - [0.1, 0.3] - [0.2, 0.6] \\ &\quad + [0.1, 0.3] \cdot [0.2, 0.6]) \cdot ([1, 1] - [0.4, 0.8]) = [1, 1] \\ &\quad - [0.12, 0.88] \cdot [0.2, 0.6] = [0.472, 0.976] \end{aligned} \tag{15}$$

$$\begin{aligned} P_{T3} &= [1, 1] - ([1, 1] - [0.1, 0.3]) \cdot ([1, 1] - [0.2, 0.6] \\ &\quad - [0.4, 0.8] + [0.2, 0.6] \cdot [0.4, 0.8]) = [1, 1] \\ &\quad - [0.7, 0.9] \cdot [-0.32, 0.88] = [0.208, 1.288] \end{aligned} \tag{16}$$

$$\begin{aligned} P_{T4} &= [0.1, 0.3] + [0.2, 0.6] + [0.4, 0.8] - [0.1, 0.3] \\ &\quad \cdot [0.2, 0.6] - [0.1, 0.3] \cdot [0.4, 0.8] - [0.2, 0.6] \\ &\quad \cdot [0.4, 0.8] + [0.1, 0.3] \cdot [0.2, 0.6] \cdot [0.4, 0.8] \\ &= [-0.192, 1.704] \end{aligned} \tag{17}$$

The failure probability intervals of the three bottom events can then be expressed in terms of universal grey numbers as

$$P_1 = \left(0.3, \left[\frac{1}{3}, 1\right]\right) \tag{18}$$

$$P_2 = \left(0.6, \left[\frac{1}{3}, 1\right]\right) \tag{19}$$

$$P_3 = \left(0.8, \left[\frac{1}{2}, 1\right]\right) \tag{20}$$

The interval operation based on the universal grey operator is performed by substituting (18) to (20) into (7) to (10), and the probability interval ranges of the top event of the fault tree in the four forms are as follows:

$$\begin{aligned} P_{T1} &= (1, [1, 1]) - \left\{(1, [1, 1]) - \left(0.3, \left[\frac{1}{3}, 1\right]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(0.6, \left[\frac{1}{3}, 1\right]\right)\right\} \cdot \left\{(1, [1, 1]) \right. \\ &\quad \left. - \left(0.8, \left[\frac{1}{2}, 1\right]\right)\right\} = (1, [1, 1]) - \left(0.056, \left[\frac{54}{7}, 1\right]\right) \\ &= [0.568, 0.944] \end{aligned} \tag{21}$$

$$\begin{aligned} P_{T2} &= (1, [1, 1]) - \left\{(1, [1, 1]) - \left(0.3, \left[\frac{1}{3}, 1\right]\right) \right. \\ &\quad \left. - \left(0.6, \left[\frac{1}{3}, 1\right]\right) + \left(0.3, \left[\frac{1}{3}, 1\right]\right) \cdot \left(0.6, \left[\frac{1}{3}, 1\right]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(0.8, \left[\frac{1}{2}, 1\right]\right)\right\} = (1, [1, 1]) \\ &\quad - \left(0.056, \left[\frac{2.16}{0.28}, 1\right]\right) = [0.568, 0.944] \end{aligned} \tag{22}$$

$$\begin{aligned} P_{T3} &= (1, [1, 1]) - \left\{(1, [1, 1]) - \left(0.3, \left[\frac{1}{3}, 1\right]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(0.6, \left[\frac{1}{3}, 1\right]\right) - \left(0.8, \left[\frac{1}{2}, 1\right]\right) \right. \\ &\quad \left. + \left(0.6, \left[\frac{1}{3}, 1\right]\right) \cdot \left(0.8, \left[\frac{1}{2}, 1\right]\right)\right\} = [0.568, 0.944] \end{aligned} \tag{23}$$

$$\begin{aligned} P_{T4} &= \left(0.3, \left[\frac{1}{3}, 1\right]\right) + \left(0.6, \left[\frac{1}{3}, 1\right]\right) + \left(0.8, \left[\frac{1}{2}, 1\right]\right) \\ &\quad - \left(0.3, \left[\frac{1}{3}, 1\right]\right) \cdot \left(0.6, \left[\frac{1}{3}, 1\right]\right) - \left(0.3, \left[\frac{1}{3}, 1\right]\right) \\ &\quad \cdot \left(0.8, \left[\frac{1}{2}, 1\right]\right) - \left(0.6, \left[\frac{1}{3}, 1\right]\right) \cdot \left(0.8, \left[\frac{1}{2}, 1\right]\right) \\ &\quad + \left(0.3, \left[\frac{1}{3}, 1\right]\right) \cdot \left(0.6, \left[\frac{1}{3}, 1\right]\right) \cdot \left(0.8, \left[\frac{1}{2}, 1\right]\right) \\ &= [0.568, 0.944] \end{aligned} \tag{24}$$

For the four equivalent forms of the failure function of the fault tree composed of three OR gate bottom events shown in Figure 1, four different failure probability interval values of the top event are obtained using the traditional interval

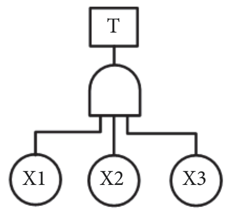

Figure 2: AND gate operator with three bottom events.

computation: [0.568, 0.944], [0.472, 0.976], [0.208, 1.288], and [-0.192, 1.704]. Because (7) has a physical meaning and is not simplified, [0.568, 0.944] is the correct result. Although (8) to (10) are equivalent to (7) in form, different orders of interval computation can result in different expansion degrees of the failure probability with respect to the top event in the fault tree.

The failure probability interval of the top event calculated using the four equivalent forms is [0.568, 0.944] as determined through the universal grey operation, the same value obtained using (7), but without the enlargement or reduction by different degrees when a traditional interval operation is applied using different orders of operation. This indicates that an interval analysis combined with grey operation can overcome the drawbacks of traditional interval operations.

2.4. Interval Analysis of Fault Trees Based on the Universal Grey Operator for an AND Gate Operator. A fault tree interval analysis based on the universal grey operator is illustrated by using an AND gate operator with three bottom events as an example. The fault tree is shown in Figure 2.

The failure function of the top event related to the bottom events is given by

$$X_T = X_1 \cdot X_2 \cdot X_3 \tag{25}$$

The failure probabilities of the three bottom events are indicated by their interval numbers with the following interval values:

$$X_1 = [0.1, 0.3] \tag{26}$$

$$X_2 = [0.2, 0.6] \tag{27}$$

$$X_3 = [0.4, 0.8] \tag{28}$$

The failure probability of the top event can then be calculated according to the traditional interval operation using:

$$X_T = [0.1, 0.3] \cdot [0.2, 0.6] \cdot [0.4, 0.8] = [0.008, 0.144] \tag{29}$$

For the AND gate operator, the failure probability intervals of the three bottom events can be explained in terms of universal grey numbers as follows:

$$X_1 = \left(0.3, \left[\frac{1}{3}, 1\right]\right) \tag{30}$$

$$X_2 = \left(0.6, \left[\frac{1}{3}, 1\right]\right) \tag{31}$$

$$X_3 = \left(0.8, \left[\frac{1}{2}, 1\right]\right) \tag{32}$$

The probability interval value of the top event of the fault tree as determined by the universal grey operation is thus

$$\begin{aligned} X_T &= \left(0.3, \left[\frac{1}{3}, 1\right]\right) \cdot \left(0.6, \left[\frac{1}{3}, 1\right]\right) \cdot \left(0.8, \left[\frac{1}{2}, 1\right]\right) \\ &= (0.144, [0.0556, 1]) = [0.008, 0.144] \end{aligned} \tag{33}$$

Because the failure function of the three bottom events in the AND gate operator has only one form, the traditional interval analysis and the proposed grey number interval analysis will provide the same solution for the failure probability of the top event. For the AND gate, the traditional interval arithmetic is equivalent to the universal grey operation. In actual engineering, however, complex system fault trees are generally composed of multiple AND gates and OR gates. Therefore, in order to demonstrate the effectiveness of the proposed universal grey operation, it is necessary to perform an interval analysis of a complex system fault tree that is appropriate for universal grey operation.

3. Interval Analysis of Landing Gear Retraction System Fault Tree Based on Universal Grey Operation

3.1. Fault Tree Model of Landing Gear Retraction System. An aircraft landing gear retraction system mainly consists of hydraulic retractable cylinders, shock struts, foldable rear struts, two rotation shafts fixed to the airframe, and other corresponding attachments. When an aircraft extends its landing gear, the lower lock opens first, then hydraulic oil is injected into the actuating cylinder, causing its piston rod to extend outward, pushing the landing gear shock strut to rotate it about the front rotation shaft. When the landing gear is in place, the upper lock is closed and the injection of hydraulic oil stops. When an aircraft retracts its landing gear, the upper lock opens first, then the hydraulic oil is injected into the actuating cylinder causing its piston rod to retract inward, pulling the landing gear shock strut to rotate it about the front rotation shaft. Once the landing gear is stowed, the lower lock is closed and the injection of hydraulic oil stops [34].

In this study, the system fault tree was constructed according to the working principle, composed structure, and fault classification of the landing gear retraction system, as shown in Figure 3.

This paper reviews the failure probability interval range of each bottom event in the landing gear retraction system fault

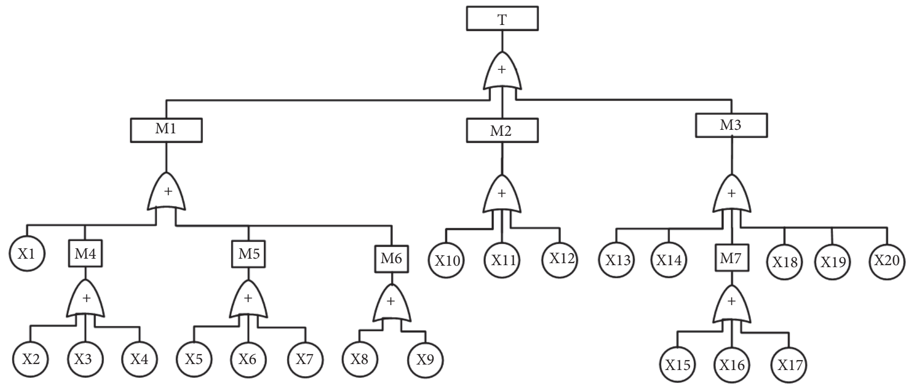

Figure 3: Fault tree of an aircraft landing gear retraction system, where T is a landing gear retraction system fault; M1 is an actuating cylinder fault; M2 is a control system fault; M3 is a hydraulic system fault; M4 is a piston rod fault; M5 is a sealing ring fault; M6 is an end eyelet fault; M7 is a hydraulic pump fault; X1 is a cylinder fracture; X2 is a seized piston rod; X3 is a loose piston rod; X4 is a fractured piston rod; X5 is a mechanically damaged sealing ring; X6 is an aging and cracking sealing ring; X7 is a chemically corroded sealing ring; X8 is an eyelet bolt fracture; X9 is an eyelet rotation; X10 is the failure of the pressure switch; X11 is the failure of the electromagnetic switch; X12 is the failure of the safety valve; X13 is oil leakage from the hydraulic line; X14 is an air lock fault; X15 is a fracture of the pump plunger piston spring; X16 is oil leakage from the pump rotary joint; X17 is a damaged pump motor; X18 is oil contamination; X19 is a broken hard pipe; X20 is a break in the accumulator.

Table 1: Interval of basic event failure probability in the landing gear retraction system fault tree.

Fault code	Interval of basic event failure probability	Fault code	Interval of basic event failure probability
X1	$[2.02\times10^{-5}, 6.40\times10^{-5}]$	X11	$[2.14\times10^{-4}, 8.09\times10^{-4}]$
X2	$[2.62\times10^{-5}, 9.29\times10^{-5}]$	X12	$[4.95\times10^{-4}, 1.44\times10^{-3}]$
X3	$[6.42\times10^{-5}, 1.40\times10^{-4}]$	X13	$[2.62\times10^{-3}, 7.20\times10^{-3}]$
X4	$[2.10\times10^{-6}, 8.85\times10^{-6}]$	X14	$[2.62\times10^{-4}, 8.25\times10^{-4}]$
X5	$[4.60\times10^{-4}, 2.45\times10^{-3}]$	X15	$[4.60\times10^{-5}, 1.85\times10^{-4}]$
X6	$[2.62\times10^{-4}, 8.29\times10^{-4}]$	X16	$[2.44\times10^{-4}, 7.52\times10^{-4}]$
X7	$[2.44\times10^{-4}, 9.72\times10^{-4}]$	X17	$[2.45\times10^{-3}, 7.72\times10^{-3}]$
X8	$[4.60\times10^{-4}, 1.95\times10^{-3}]$	X18	$[2.79\times10^{-3}, 9.85\times10^{-3}]$
X9	$[2.24\times10^{-4}, 7.29\times10^{-4}]$	X19	$[2.12\times10^{-5}, 8.29\times10^{-5}]$
X10	$[1.62\times10^{-5}, 8.29\times10^{-5}]$	X20	$[4.42\times10^{-5}, 1.70\times10^{-4}]$

tree, shown in Table 1. By referring to the logical relationship OR gates in the fault tree, the probabilistic structure function model of the fault tree can be obtained as

$$P_T = 1 - (1 - P_{M1})(1 - P_{M2})(1 - P_{M3}) \tag{34}$$

$$\begin{aligned} P_{M1} &= 1 - (1 - P_{X1})(1 - P_{M4})(1 - P_{M5})(1 - P_{M6}) \\ P_{M2} &= 1 - (1 - P_{X10})(1 - P_{X11})(1 - P_{X12}) \\ P_{M3} &= 1 - (1 - P_{X13})(1 - P_{X14})(1 - P_{M7})(1 - P_{X18}) \\ &\quad \cdot (1 - P_{X19})(1 - P_{X20}) \end{aligned} \tag{35}$$

$$\begin{aligned} P_{M4} &= 1 - (1 - P_{X2})(1 - P_{X3})(1 - P_{X4}) \\ P_{M5} &= 1 - (1 - P_{X5})(1 - P_{X6})(1 - P_{X7}) \\ P_{M6} &= 1 - (1 - P_{X8})(1 - P_{X9}) \\ P_{M7} &= 1 - (1 - P_{X15})(1 - P_{X16})(1 - P_{X17}) \end{aligned} \tag{36}$$

3.2. Failure Probability of Top Event of the System Fault Tree under Universal Grey Operation. Depending on the conversion rule of the interval number and universal grey number, the failure probabilities of the bottom events of the landing gear retraction system, expressed using universal grey numbers, are given in Table 2.

The universal grey operation process is first conducted as follows on the lowest level of gates:

$$\begin{aligned} P_{M4} &= 1 - (1 - P_{X2})(1 - P_{X3})(1 - P_{X4}) \\ &= (1, [1, 1]) \\ &\quad - \left\{(1, [1, 1]) - \left(9.29 \times 10^{-5}, [0.282024, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(1.40 \times 10^{-4}, [0.458571, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(8.85 \times 10^{-6}, [0.237288, 1]\right)\right\} \\ &= \left(2.4 \times 10^{-4}, [0.3793, 1]\right) \end{aligned} \tag{37}$$

Table 2: Universal grey representation of basic event failure probability for landing gear retraction system fault tree.

Fault code	Universal grey representation of failure probability	Fault code	Universal grey representation of failure probability
X1	$(6.40 \times 10^{-5}, [0.315625, 1])$	X11	$(8.09 \times 10^{-4}, [0.264524, 1])$
X2	$(9.29 \times 10^{-5}, [0.282024, 1])$	X12	$(1.44 \times 10^{-3}, [0.343750, 1])$
X3	$(1.40 \times 10^{-4}, [0.458571, 1])$	X13	$(7.20 \times 10^{-3}, [0.363889, 1])$
X4	$(8.85 \times 10^{-6}, [0.237288, 1])$	X14	$(8.25 \times 10^{-4}, [0.317576, 1])$
X5	$(2.45 \times 10^{-3}, [0.187755, 1])$	X15	$(1.85 \times 10^{-4}, [0.248649, 1])$
X6	$(8.29 \times 10^{-4}, [0.316043, 1])$	X16	$(7.52 \times 10^{-4}, [0.324468, 1])$
X7	$(9.72 \times 10^{-4}, [0.251029, 1])$	X17	$(7.72 \times 10^{-3}, [0.317358, 1])$
X8	$(1.95 \times 10^{-3}, [0.235897, 1])$	X18	$(9.85 \times 10^{-3}, [0.283249, 1])$
X9	$(7.29 \times 10^{-4}, [0.307270, 1])$	X19	$(8.29 \times 10^{-5}, [0.255730, 1])$
X10	$(8.29 \times 10^{-5}, [0.195416, 1])$	X20	$(1.70 \times 10^{-4}, [0.260000, 1])$

$$\begin{aligned} P_{M5} &= 1 - (1 - P_{X5})(1 - P_{X6})(1 - P_{X7}) \\ &= (1, [1, 1]) \\ &\quad - \left\{(1, [1, 1]) - \left(2.45 \times 10^{-3}, [0.187755, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(8.29 \times 10^{-4}, [0.316043, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(9.72 \times 10^{-4}, [0.251029, 1]\right)\right\} \\ &= \left(4.246 \times 10^{-3}, [0.227505, 1]\right) \end{aligned} \tag{38}$$

$$\begin{aligned} P_{M6} &= 1 - (1 - P_{X8})(1 - P_{X9}) \\ &= (1, [1, 1]) \\ &\quad - \left\{(1, [1, 1]) - \left(1.95 \times 10^{-3}, [0.235897, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(7.29 \times 10^{-4}, [0.307270, 1]\right)\right\} \\ &= \left(2.678 \times 10^{-3}, [0.2555464, 1]\right) \end{aligned} \tag{39}$$

$$\begin{aligned} P_{M7} &= 1 - (1 - P_{X15})(1 - P_{X16})(1 - P_{X17}) \\ &= (1, [1, 1]) \\ &\quad - \left\{(1, [1, 1]) - \left(1.85 \times 10^{-4}, [0.248649, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(7.52 \times 10^{-4}, [0.324468, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(7.72 \times 10^{-3}, [0.317358, 1]\right)\right\} \\ &= \left(8.65 \times 10^{-3}, [0.3167134, 1]\right) \end{aligned} \tag{40}$$

Allowing the universal grey operation to be conducted as follows on the next highest level of gates,

$$\begin{aligned} P_{M1} &= 1 - (1 - P_{X1})(1 - P_{M4})(1 - P_{M5})(1 - P_{M6}) \\ &= (1, [1, 1]) \\ &\quad - \left\{(1, [1, 1]) - \left(6.40 \times 10^{-5}, [0.315625, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(2.4 \times 10^{-4}, [0.3793, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(4.246 \times 10^{-3}, [0.227505, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(2.678 \times 10^{-3}, [0.2555464, 1]\right)\right\} \\ &= \left(7.2145 \times 10^{-3}, [0.2431464, 1]\right) \end{aligned} \tag{41}$$

$$\begin{aligned} P_{M2} &= 1 - (1 - P_{X10})(1 - P_{X11})(1 - P_{X12}) \\ &= (1, [1, 1]) \\ &\quad - \left\{(1, [1, 1]) - \left(8.29 \times 10^{-5}, [0.195416, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(8.09 \times 10^{-4}, [0.264524, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(1.44 \times 10^{-3}, [0.343750, 1]\right)\right\} \\ &= \left(2.33 \times 10^{-3}, [0.310623, 1]\right) \end{aligned} \tag{42}$$

$$\begin{aligned} P_{M3} &= 1 - (1 - P_{X13})(1 - P_{X14})(1 - P_{M7})(1 - P_{X18}) \\ &\quad \cdot (1 - P_{X19})(1 - P_{X20}) = (1, [1, 1]) \\ &\quad - \left\{(1, [1, 1]) - \left(7.20 \times 10^{-3}, [0.363889, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(8.25 \times 10^{-4}, [0.317576, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(8.65 \times 10^{-3}, [0.3167134, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(9.85 \times 10^{-3}, [0.283249, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(8.29 \times 10^{-5}, [0.255730, 1]\right)\right\} \\ &\quad \cdot \left\{(1, [1, 1]) - \left(1.70 \times 10^{-4}, [0.260000, 1]\right)\right\} \\ &= \left(2.65324 \times 10^{-2}, [0.318568, 1]\right) \end{aligned} \tag{43}$$

Then, for the top event,

$$\begin{aligned} P_T &= 1 - (1 - P_{M1})(1 - P_{M2})(1 - P_{M3}) \\ &= (1, [1, 1]) \\ &\quad - \left\{(1, [1, 1]) - \left(7.2145 \times 10^{-3}, [0.2431464, 1]\right)\right\} \end{aligned}$$

$$\cdot \left\{(1,[1,1]) - \left(2.33\times 10^{-3},[0.310623,1]\right)\right\}$$
$$\cdot \left\{(1,[1,1]) - \left(2.65324\times 10^{-2},[0.318568,1]\right)\right\}$$
$$= (0.0358,[0.3044832,1]) = [0.0109, 0.0358] \tag{44}$$

indicating that the failure probability interval range of the aircraft landing gear retraction system varies between 0.0109 and 0.0358.

4. Conclusions

This paper proposes a new method for system fault tree analysis that overcomes the shortcomings of existing interval operation methods based on the conversion rules between the interval number and the universal grey number and describes the failure probability interval range of the bottom events of a fault tree using the universal numbers. Based on the logical relationship between the AND gates and OR gates of a fault tree and combined with the four arithmetic operations of the universal grey number, a more accurate system reliability calculation result is achieved. The proposed method is demonstrated to be advantageous for interval operation and is shown to overcome the drawbacks of enlarged or reduced solution intervals when using different orders of interval operation.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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