

Research Article

Mechanism for Measuring System Complexity Applying Sensitivity Analysis

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This work proposes a complexity metric which maps internal connections of the system and its relationship with the environment through the application of sensitivity analysis. The proposed methodology presents (i) system complexity metric, (ii) system sensitivity metric, and (iii) two models as case studies. Based on the system dynamics, the complexity metric maps the internal connections through the states of the system and the metric of sensitivity evaluates the contribution of each parameter to the output variability. The models are simulated in order to quantify the complexity and the sensitivity and to analyze the behavior of the systems leading to the assumption that the system complexity is closely linked to the most sensitive parameters. As findings from results, it may be observed that systems may exhibit high performance as a result of optimized configurations given by their natural complexity.

1. Introduction

The scientific and technological advances of the second half of the twentieth century have generated significant changes in the dynamics of human civilization. The creation of electronic systems and their network structure revolutionized communication systems, modifying the social and economic relations in the world. The systems became more integrated and interdependent, consequently, more complex; since the network structure was not restricted to the computational systems, it is embedded in the human relationships.

Bar-Yam [1] argues that this increase in complexity is directly related to the increasing interdependence of the global economic and social systems, as well as political instabilities. According to Bar-Yam [1], the interdependence is characterized by the network control structure, which considers lateral interactions and transfers decision making to teams due to the high complexity of collective behavior.

The network structure assigns a prominent role to the interactions that, in turn, are responsible for the holistic approach in the study of systems [2–4]. For centuries, scientists tried to explain the whole by its parts, making successive divisions in search of the smallest structure that characterized each system and, ultimately, all systems. According to Bak [5], physicists have been reductionist in considering that the world could be understood in terms of the properties of simple building blocks. Although they have been successful in some cases, the complexity of the systems requires a global analysis instead.

Likewise, in engineering studies, researchers have realized that subdividing systems to analyze them may cause significant losses in the internal structure of the original system [6]. Considering the use of computational tools, it is preferable to carry out the study throughout the system, modeling it in terms of inputs and outputs to simulate its behavior. Another relevant aspect is the number of parameters with their respective variabilities, which may

constitute a bottleneck in understanding the systems. In order to reduce the number of variables, several studies use sensitivity analysis for fixing nonessential parameters, since they generate low impact on the output of the system [7, 8].

This holistic approach to systems is based on a philosophical assertion that the whole is more than the sum of the parts. According to Simon [9], in complex systems, this statement means that the properties of the whole cannot be easily inferred from the properties of the parts and their interaction laws. For this reason, complexity has arisen as a unifying feature of our world, regardless of the scale and of the kind of system in analysis [1, 10].

According to Holland [11], the term complexity has assumed such importance that now designates a scientific field with many branches. Some authors consider that the science of the 21st Century is the science of complexity [12]. However, there is no consensus on the quantitative definition of complexity. None of the various measures of complexity is universally accepted by scientists, nor they are practical [13].

Lloyd [14] argues that the measures of complexity are developed to respond to questions about the system with respect to (i) difficulty of description, (ii) difficulty of creation, or (iii) degree of organization. In these categories, the complexity has been approached in different ways, such as entropy [15, 16], statistics [17, 18], fractal dimension [19, 20], algorithmic information content [21, 22], dynamic depth [23], tracking performance [24], and connections [25], among many other forms.

Based on the fractal dimension as measure for self-similar objects, Balaban et al. [26] proposes a metric for quantifying emergence and self-organisation extending fractal dimension to a function, since most of the fractal-like objects have multiple scaling rates. Thus the multifractal analysis investigates the statistical scaling laws of complex fragmented geometrical objects as bacteria aggregates. Balaban et al. [26] observe the evolution of the spatial arrangement of *Enterobacter cloacae* aggregates and apply multifractal analysis to calculate dynamics changes in emergence and self-organisation within the bacterial population. As experimental results, the emergence degree decreases as aggregates populate the plate while the self-organisation degree increases.

Given the relevance of geometrical and computational frameworks, Joosten et al. [27] define the space-time diagrams using small Turing machines with a one-way infinite tape as a computational model and translate these diagrams to fractal dimension. The results from this work have shown that there is a strong relation between the fractal dimension of the Turing machine used and its runtime complexity.

Among the complexity metrics, fractal dimension is frequently applied to the analysis of textures, shapes, and network structures [20, 28, 29]. However, when the detailed system dynamics is available, other metrics may be more effective, such as those based on interactions, for instance, the metrics proposed by Koorehdavoudi and Bogdan [2] for quantifying complexity from spatiotemporal interactions, which estimates the free energy landscape of the states and distinguishes between stable and transition states. This framework was applied to three natural groups: swimming bacteria, flying pigeons, and ants. The analysis has shown that

the collective group has had lower energy level and higher degree of complexity at stable states compared to transition ones.

Regarding the connections, the complexity may be measured from the evolution of the system over time, considering the active connections in each state. The connections depend mainly on (i) the transition from one state to another due to the occurrence of events and (ii) the change of the input parameters that lead to variation in the system output [25, 30]. Through sensitivity analysis, the effect of a given input is measured on the output, assessing how the uncertainties in the parameters affect the uncertainty in the system response [31].

Sensitivity analysis is relevant to the study of complexity because certain variables may eventually emerge and have a significant impact on the system. Even if the variables are hidden, the relevance of each one may be defined previously by means of its sensitivity and so anticipate strategies if such variables emerge. According to Holland [11], emergence characterizes complex systems and helps distinguish these systems from others; however this characteristic has no sharp demarcation. To define the system as complex is still a subjective effort.

Here, we focus on the degree of system complexity as a measure that comprises mechanisms related to internal and external interactions of the system. Thus considering (i) the increase in complexity, (ii) the holistic approach to systems, (iii) complexity as a unifying variable, and (iv) the absence of a practical and representative quantitative definition of complexity, we propose a complexity metric based on connections, which may be weighted according to the relevance of each one. This metric is applicable to any system that may be modeled and simulated from its input parameters and output variables.

The proposed metric covers a wide range of systems in the physical world. Using this metric, it is possible (i) to say how complex a particular system is or how much more complex one system is than another, (ii) to use the complexity in the objective function of optimization process, in order to minimize it, or as a constraint, in order not to exceed the value defined as a reference, and (iii) to support decision making.

In order to apply the proposed metrics, Section 2 presents complexity metric developed by adapting the Second Law of Thermodynamics. Our proposed methodology is presented in Section 3, where the connection-based complexity and sensitivity metrics are defined and two systems are modelled as case study. The complexity and sensitivity of different models are analyzed in Section 4, leading to the proposal to include the sensitivity index in the systems complexity metric as a factor of relevance of each connection (Section 5). By including this factor, the complexity metric will consider the descriptive and organizational aspects of the systems, verified by the number of connections and the relevance of each connection, respectively.

2. Metric of System Complexity

Several metrics to calculate the complexity have been developed based on the size of the system, entropy, information,

cost, hierarchy, organization, and other criteria [14]. The complexity measures are used to compare systems or different configurations of the same system [13, 32]. In some cases, these measures are dimensionless, allowing to compare one value to another one measured in the same system or in different systems, as long as the nature of them allows comparison [32, 33].

Some complexity metrics are proposed based on the information entropy [32, 34–37]. Considering the system connections, Paiva [33] presents the modeling of Shannon [38] adapted by Lemes [35] to measure the system complexity using

$$\Gamma(\nu) = - \sum_{i,j=1}^{|\nu|} p(\chi)_{i,j} \log_2 p(\chi)_{i,j} \quad (1)$$

where $\Gamma(\nu)$ is the complexity of the system connections equivalent to the entropy in information exchange, ν is the set of connections between elements of the system, $|\nu|$ is the total number of connections, $p(\chi)$, $\forall \chi \in \nu \mid i \neq j$ is the frequency with which the connections between elements i and j occur, where p is given by $\alpha/|\nu|$, in which α is the number of connections between the elements i and j .

3. Methodology

3.1. Proposed Metric of System Complexity. Based on the Paiva's [33] modeling, the proposed method measures the complexity of real systems using expression (2). This metric considers the connections regardless of information exchange, observing their probabilities according to expression (3).

$$\psi(c) = - \sum_{i=1}^{\rho} P(c_i) \cdot \log_2 P(c_i) \quad (2)$$

$$P(c) = \frac{1}{n_e \cdot (n_r + n_q)} \quad (3)$$

where $\psi(c)$ is the system complexity based on connections, $P(c_i)$ is the probability of occurrence of the connection c_i between two elements, and ρ is the number of active connections at the instant t , expressed by (4). The variables n_e , n_r , and n_q correspond to the number of entities, resources, and queues at the instant t , respectively. These variables are components of the system according to the discrete-event modeling, which is applied to the systems investigated in the case studies. In this kind of model, the dynamics of the system is known with respect to the interaction between its components, which enables its modeling in terms of connections.

$$\rho = \sum_{i=1}^k n_{c_i} \cdot n_{\epsilon_i} \quad (4)$$

where k is the number of entities states, n_c is the number of active connections per entity in each state, and n_e is the number of entities in each state.

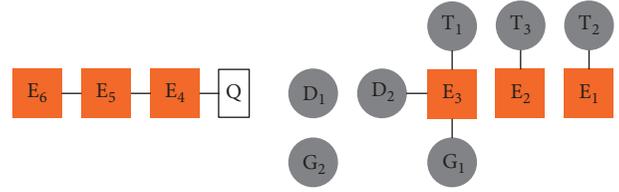


FIGURE 1: Representation of the connections established by the entities E_1 to E_6 .

In (3), n_e may be generalized to the function $e(\xi)$, seeing that some constraints may rule out the connection between entities and resources or queues. In this function, ξ corresponds to the constraints, which lead to the decrease in the number of relationship possibilities. The function $e(\xi)$ may assume values in the interval $1 \leq e(\xi) \leq n_e$.

Expressions (3) and (4) are useful in the context of discrete event systems, which consist of a class of dynamic systems that depend on the occurrence of events to the state change, i.e., new set of values of the attributes at a given instant [30]. The concept of queue is commonly used in discrete-event modeling, since the entities often need to share the system resources. In these cases, the entities have to wait in queue in order to use certain resources, which provide them some service or something they need [39].

The proposed metric maps the active connections related to entities, resources, and queues at any given time t , expressed through the relationship matrix M . Figure 1 shows a system configuration with the entities E_1 to E_6 (in orange) and the resources $D_1, D_2, T_1, T_2, T_3, G_1$, and G_2 (in gray).

In Figure 1, there are 8 active connections in the system: (i) E_1 and T_2 , (ii) E_2 and T_3 , (iii) E_3 and T_1 , (iv) E_3 and D_2 , (v) E_3 and G_1 , (vi) the queue Q and E_4 , (vii) E_4 and E_5 , and (viii) E_5 and E_6 . No entity is connected to the resources D_1 and G_2 ; hence they are idle.

3.2. Proposed Metric of Sensitivity Analysis. In this work, a quantitative analysis is proposed of the curves that express the impact generated by the changes of the input parameters. These changes are carried out from the base values of the parameters, which may be defined as optimized solution or as the best bet for parameters. While one parameter is changed, the others are held at their base values and the output is measured. This approach is known as one-at-a-time measures.

The system output is given by the function $y = f(x_1, x_2, \dots, x_m)$. When y is the output corresponding to the base values of the parameters, $\beta = y$. A base axis is defined parallel to the axis of the abscissa in the graph, presenting constant value in the ordinate corresponding to the output β , as represented by the dashed line in Figure 2.

The proposed method based on Saraiva [40] calculates the area of the polygons formed by the base axis and the curves related to the one-at-a-time measures in order to determine the system sensitivity to variations in the parameters. The coordinates in the graph represent the relation between the inputs and output, enabling the computation of the absolute

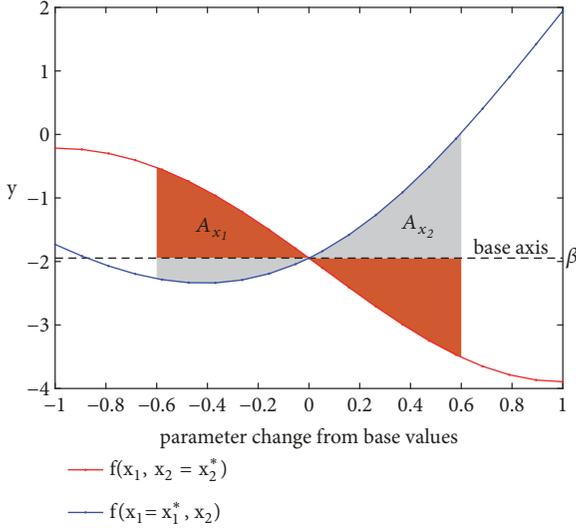


FIGURE 2: Example of area delimitation for the parameters x_1 and x_2 .

difference given by the output value obtained and the value β (for the base case).

This sensitivity metric has mathematical properties of entropy related to the lack of information about system behavior. Likewise the fact that the entropy is maximum when the most uncertain situation is expressed in terms of probabilities [38], the most sensitive parameter is the one that generates greater variability in the model output for the same conditions of analysis. Besides that, similarly to classical mechanics in which the experimenter controls the microstate by manipulating parameters from usual macroscopic ones [41], the sensitivity analysis addresses the internal mechanisms of the system through the manipulation of its inputs and outputs.

In order to determine the parameters sensitivity, the proposed metric requires few one-at-a-time measures (set of parameters values and the respective output value), which may be obtained either by experiments performed in the real system or by simulations performed in the model. In both cases, the system is observed as a whole, leading us to believe this may be an appropriate mechanism to measure complexity. Some researches have showed the relationship between complexity and sensitivity [42–44]; however none of them proposes to measure complexity using sensitivity analysis.

The parameters influence on the model output may be quantified by the sensitivity indices. Here, we define sensitivity index S_{x_i} as the contribution of the area A_{x_i} concerning the parameter x_i in relation to the total area, where m is the number of parameters, given by

$$S_{x_i} = \frac{A_{x_i}}{\sum_{i=1}^m A_{x_i}} \quad (5)$$

Figure 2 presents the polygons used to calculate the sensitivity index for the interval from -60% to 60% of parameter change from base values. This interval represents

an example of area delimitation for applying the proposed metric. One may choose any interval comprising between -100% and 100% of parameter change. The area A_{x_1} (in orange) is defined by the base axis and the curve in red given by variation in parameter x_1 while x_2 is held in its base value x_2^* . The area A_{x_2} (in gray) is defined by the base axis and the curve in blue given by variation in parameter x_2 while x_1 is held in its base value x_1^* . The number of polygons for each parameter is defined by the intersections of the base axis with the curve of the parameter x_i that is varying for interval of interest, as expressed by

$$\delta_{x_i} = \eta_{x_i} + 1 \quad (6)$$

where δ_{x_i} is the number of polygons and η_{x_i} is the number of intersections of the x_i curve with base axis in the analysed interval.

3.3. Model for the Distribution Center Problem. The distribution center of a company consists of the logistics of products delivery. After separating the products, an order of delivery is generated. During the center operation, the following steps are performed: (1) the distribution center generates the orders of delivery; (2) the orders are kept in queue until the resources are available; (3) the truck stays on the dock while the loading process is performed; (4) the truck leaves for delivery, releasing the dock and the group of workers for new loading; (5) the products are transported to their destinations; (6) the truck comes back to the distribution center for new deliveries. In the model, the duration at each stage corresponds to values given by probability distributions.

As typical discrete event system, the distribution center is modelled in terms of entities, queues, and resources. The entities are the orders of delivery, which are waiting in line for the availability of the resources: docks, trucks, and group of workers. The set of discrete states concerning the orders are (i) waiting in the queue, (ii) being loaded, and (iii) being transported. Based on the states, how many resources are being used at every instant of time t is determined.

The distribution center works like an open system, in which new entities may be integrated at any moment; hence the number of entities (orders of delivery) and the demand for resources (docks, trucks, and groups of workers) vary over time. Different performance measures may be chosen for the problem of the distribution center, e.g., the average waiting time in queue, the average time for transportation, or the percentage of use of the system resources.

At this work, the delivery time t_d of orders is considered as performance measure, which is the time spent between the moment that the order is generated and the delivery to the customers is performed. The variable t_d is composed of the sum of the times of (i) waiting in queue, (ii) loading the products into the truck, and (iii) transporting the products from the distribution center to the customers. That system is represented by λ .

The measure for complexity is calculated by (2), in which the active connections are mapped based on the states of the orders. The number of active connections is determined by (4), in which the order in state *waiting in the queue* adds one

connection to the system (with the order ahead of it), the order in state *being loaded* adds three connections (one with the dock, one with the truck, and another with the group of workers), and the order in state *being transported* contributes with one connection to the system (with the truck).

The probability of connection occurrence in the distribution center problem is given by (3), in which the number of entities is equal to the total number of orders in the system at the instant t , considering that each entity (order) may be attended by any system resource or be kept in queue. In this model, there is only one queue and the number of resources is equal to the sum of the number of docks, trucks, and groups of workers. The queue modeling applies the FIFO (first-in-first-out) policy, in which the first entity that arrives in the queue is the first one to be attended and, consequently, to leave the queue.

Considering that there are 2 docks, 3 trucks, and 2 groups of workers into the distribution center, the system configuration may occur at any instant t according to Figure 1. This configuration is expressed by relationship matrix M^λ in (7), in which the columns represent the entities E_1 to E_6 and the rows represent the queue Q , docks D_1 and D_2 , trucks T_1 to T_3 , and group of workers G_1 and G_2 , in this sequence.

$$M^\lambda = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

In (7), the orders E_1 and E_2 are being transported in trucks T_2 and T_3 , respectively, the order E_3 is being carried by the group of workers G_1 into the truck T_1 parked in the dock D_2 , and the orders E_4 , E_5 , and E_6 are waiting in line. The resources dock D_1 and group of workers G_2 are idle; no entity is connected to them. The number of active connections is computed from the matrix M^λ .

3.4. Model for Medical Center Problem. The medical center consists of the processes of medical care and basic procedures. The flow at medical center is as follows: (1) the patients arrive at the medical center; (2) the patients wait in queue for medical assistance; (3) the patients get the medical appointment; (4) after the appointment, some of the patients are released and others are forwarded to (5) perform basic procedures (receiving medication or doing medical exams); (6) after medication or exams, some of the patients are released and others perform new exams or take more medication, returning to step 5, and some of them wait in queue to come back to the physician; (7) after a new medical appointment, some of the patients are directed to further exams or medication, returning to step 5, and the others leave the center.

During the care process, queues of people may be generated in order to wait for (i) medical appointment, (ii) medication, and (iii) medical exams. The set of discrete states concerning patients are (i) waiting in queue, (ii) having medical appointment, (iii) receiving medication, and (iv) doing exams. The resources are used according to the demand.

The medical center is also an open system. It means that the number of entities (patients) and the demand for resources (physicians, nurses, and technicians) vary over time. In this model, the time between arrival and leaving of patients t_p is adopted as the performance measure. The medical center system is represented by ϕ .

The complexity measure is calculated by expression (2), in which the active connections of medical center are mapped based on states. The number of active connections is determined by (4), in which each patient adds one connection, no matter which state he or she is in.

The connection probability in the medical center is given by expression (3), where the number of entities is equal to the total number of patients in the system at the moment t , considering that each patient may be attended by any system resource or be waiting in any queue. The resources are computed by the sum of the number of physicians, nurses, and technicians. There are four queues in the system: (i) queue Q_1 for medical appointment on arrival, (ii) queue Q_2 for medication, (iii) queue Q_3 for examination, and (iv) queue Q_4 for appointment on medical return after basic procedures. The queues Q_1 , Q_2 , Q_3 , and Q_4 are defined according to the FIFO (first-in-first-out) policy, which states that each entity waiting for resource availability is added to the end of the respective queue.

Considering the medical center have 2 physicians, 2 nurses, and 1 technician in its staff, the configuration presented in matrix M^ϕ (8) may occur at any instant. Each column represents the patients P_1 to P_7 and the rows represent the queues Q_1 to Q_4 and the resources: physicians F_1 and F_2 , nurses N_1 and N_2 , and technician C_1 , in this sequence.

$$M^\phi = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

In (8), the patient P_1 is doing exam with the technician C_1 , patients P_2 and P_6 are receiving medication by nurses N_1 and N_2 , respectively, patient P_3 is waiting in queue Q_3 to be examined, P_4 and P_5 are consulting with physicians F_1 and F_2 , respectively, and P_7 is waiting in queue Q_1 to get an appointment.

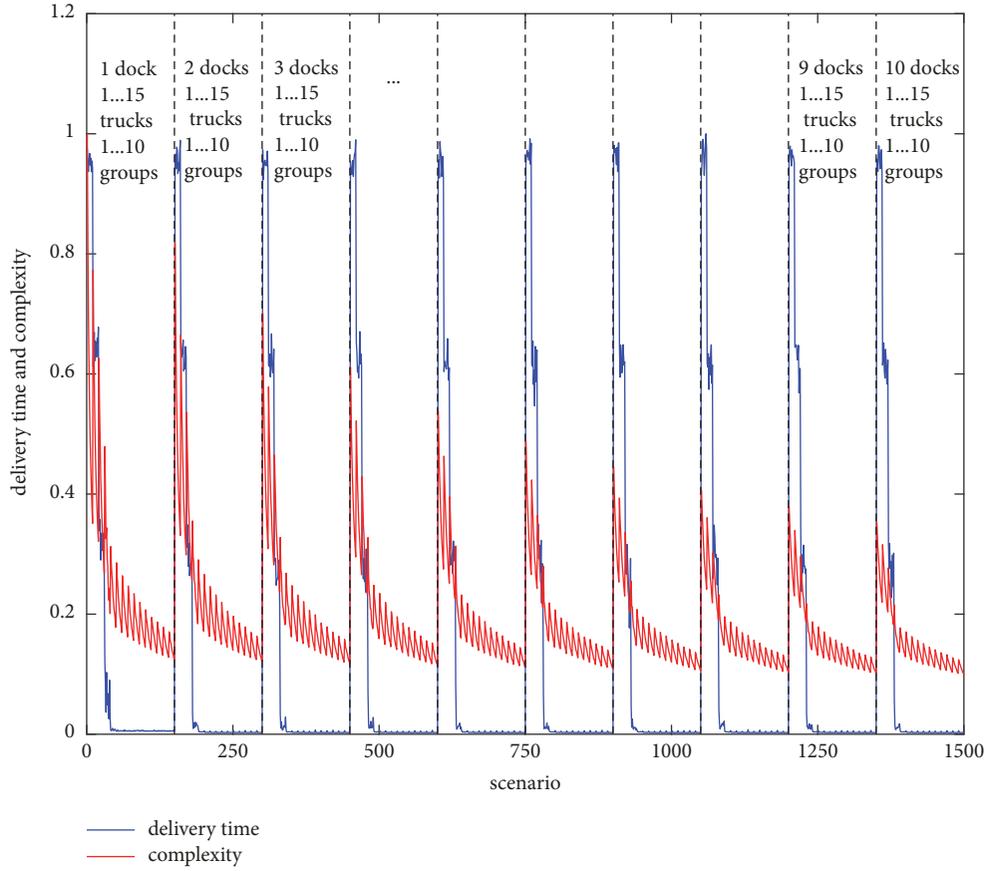


FIGURE 3: Delivery time t_d and complexity $\psi(c)^\lambda$ measures related to scenarios of the distribution center.

4. Results

4.1. Case Study 1: Distribution Center. The simulation of the distribution center model presented in Section 3.3 generated orders of delivery with probability distribution $Z_1^\lambda \sim \exp(120)$, the groups of workers loaded the products in the truck during time $Z_2^\lambda \sim N(100, 30)$, and the truck spent time $Z_3^\lambda \sim U(120, 240)$ for transporting the products to customers and $Z_4^\lambda \sim U(120, 240)$ for returning to the distribution center.

The number of resources used in the simulation ranged from 1 to 10 for docks, 1 to 15 for trucks, and 1 to 10 for groups of workers, compounding 1,500 scenarios. The simulation was performed for 180 days by scenario, considering 24 daily hours of operation and that each order of delivery corresponds to the truckload. In the simulated scenarios, complexity $\psi(c)^\lambda$ and delivery time t_d were calculated.

The normalized values of t_d and $\psi(c)^\lambda$ for all scenarios are presented in Figure 3. As the simulation is the combination of resources docks, trucks, and groups of workers, in this order, in Figure 3, the peaks of t_d (in blue) correspond to the combinations in which there was only one truck. At every change in the number of docks, one truck was used during 10 scenarios, leading to the highest values of t_d and $\psi(c)^\lambda$ (in red). Furthermore the oscillations that occur between values 0.1 and 0.3 of $\psi(c)^\lambda$ are derived from changes in the

number of trucks, indicating greater sensitivity in this system parameter.

Figure 3 shows that whenever the number of truck is > 6 , values of t_d approach zero. In the 1,500 scenarios, the lowest complexity is $\psi(c)^\lambda = 0.2768$, which corresponds to lowest delivery time and to scenario with the greatest number of available resources, as shown in Table 1. The highest delivery time is $t_d \approx 340\times$ bigger than the lowest time and it corresponds to the greatest complexity and lowest number of resources.

The complexity measure is a way of seeing the system as a whole. The connections mapping concerning the states expresses the system configuration. Thus the relation R_c^λ between performance measure and complexity expressed by (9) may lead to some findings.

$$R_c^\lambda = \frac{t_d}{\psi(c)^\lambda} \quad (9)$$

The lowest (in bold) and greatest (in italic) values of t_d , $\psi(c)^\lambda$ and normalized R_c^λ are shown in Table 2. The lowest value of R_c^λ refers to $t_d = 3.15 \cdot 10^2$, only 1.12 \times the lowest t_d , whereas the number of resources was approximately 72% lower than the scenario of lowest t_d . The system performance was 300 \times greater than the worst case and only 0.11 \times lower than in the best case. This shows that the use of resources was

TABLE 1: Scenarios related to the lowest and greatest values of t_d , $\psi(c)^\lambda$, and R_c^λ .

t_d	$\psi(c)^\lambda$	R_c^λ	dock	truck	group of workers
$2.81 \cdot 10^2$	0.277	0.007	10	15	10
$3.15 \cdot 10^2$	0.686	0.003	2	6	2
$9.46 \cdot 10^4$	2.778	0.222	1	1	1
$9.46 \cdot 10^4$	0.617	1	10	1	10

TABLE 2: Scenarios related to the lowest and greatest values of t_p , $\psi(c)^\phi$, and R_c^ϕ .

t_p	$\psi(c)^\phi$	R_c^ϕ	physician	technician	nurse
$5.34 \cdot 10^1$	0.252	0.082	8	10	13
$5.59 \cdot 10^1$	0.241	0.089	8	10	15
$5.82 \cdot 10^1$	0.359	0.062	7	7	5
$1.16 \cdot 10^3$	0.493	0.908	3	3	15
$1.03 \cdot 10^3$	<i>0.811</i>	0.490	3	2	5
$1.06 \cdot 10^3$	0.410	1	3	8	15

optimized, since the ratio value R_c^λ indicates the lowest cost in terms of complexity for each minute of permanence of the entities in the system.

4.2. Case Study 2: Medical Center. The model of the medical center presented in Section 3.4 was simulated following the flow in which the patients (1) arrive at the medical center in time with probability distribution $Z_1^\phi \sim \exp(5)$; (2) wait in queue Q_1 for medical appointment; (3) are attended by physician for a time $Z_2^\phi \sim N(15, 5)$; (4) after the appointment, 1% of patients are released and 99% are forwarded to (5) perform basic procedures, wherein (5a) 40% of them wait in queue Q_2 in order to receive medication and (5b) 60% of them wait in queue Q_3 for doing medical exams; (6) after medication or exams, 10% of patients are released, 40%, perform new exams or take more medication, returning to step 5, and 50% wait in queue Q_4 to return to the physician; (7) after a new appointment, 40% of patients are directed to further exams or medication, returning to step 5, and the other 60% leave the medical center. The medication time lasts $Z_3^\phi \sim N(20, 5)$, the duration of exams is $Z_4^\phi \sim N(15, 5)$, and the return to the physician in new appointment lasts $Z_5^\phi \sim N(10, 5)$.

The number of physicians, technicians, and nurses used in the simulation ranged from 3 to 8, 2 to 10, and 5 to 15, respectively, compounding 594 different scenarios. The simulation was performed for 180 days by scenario, considering 24 daily hours of operation. The medical care time t_p in minutes spent within the medical center by patient and complexity $\psi(c)^\phi$ of the system was calculated for all scenarios.

The lowest complexity obtained from all scenarios $\psi(c)^\phi = 0.2407$ occurred when the greatest number of resources were used, i.e., 8 physicians, 10 technicians, and 15 nurses. The greatest complexity $\psi(c)^\phi = 0.8105$ occurred when there was the lowest number of resources, i.e., 3 physicians, 2 technicians, and 5 nurses, therefore lengthy waiting times.

Figure 4 presents the percentage values of resource utilization and the measure of normalized complexity for all scenarios. It may be observed that in the 200 initial scenarios, where there were less than 5 physicians, the utilization of this resource got values between 0.9 and 1. The peaks of complexity occurred whenever the number of physicians was increased but the number of nurses and the number of technicians were minimum for analyzed scenarios, causing increase in the size of the queues Q_2 and Q_3 .

In the scenario with the lowest complexity, the time t_p that patients were in medical center was approximately 14.9, 58.7, 71.13, and 74.07 minutes for those who left (1) without examination or medication, (2) after doing exams and/or taking medication, (3) after a new medical appointment, and (4) after all procedures, respectively. For the scenario with the greatest complexity, the times were 774.68, 1032.20, 1191.25 and, 1128.28 minutes, respectively, due to the time in queues.

Considering the relation between performance and complexity measures for medical center, expressed by (10), the lowest (in bold) and greatest (in italic) values of t_p , $\psi(c)^\phi$ and normalized R_c^ϕ are shown in Table 2.

$$R_c^\phi = \frac{t_p}{\psi(c)^\phi} \quad (10)$$

In Table 2, the lowest value of R_c^ϕ equal to 0.062 represents reduced number of resources compared to other scenarios with lowest values of t_p and $\psi(c)^\phi$. Even with fewer resources, the system achieved near-peak performance values when $R_c^\phi = 0.062$. The greatest values of all variables were found when the number of physician was lowest, indicating this parameter as the most sensitive in the system.

4.3. Case Study 3: Sensitivity Analysis. The complexity of the distribution center and medical center was analyzed in Section 4.1 and Section 4.2, respectively. As analysis findings, some parameters were pointed out as more sensitive due to the significant impact generated by their scarcity. In order to

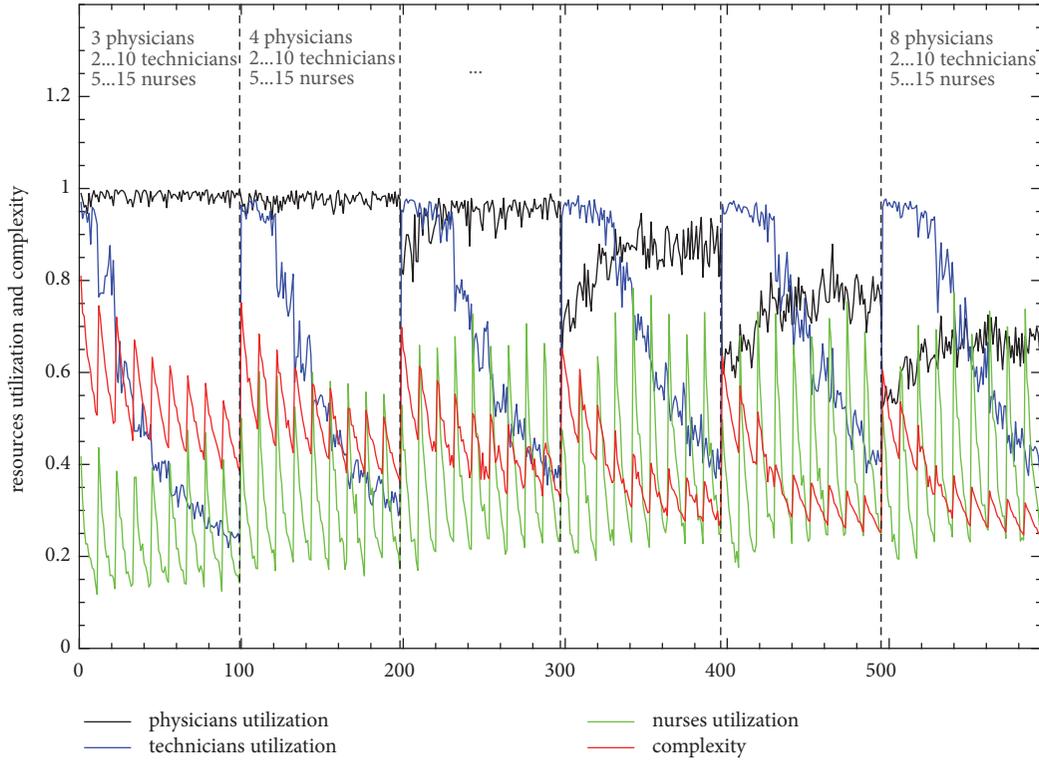


FIGURE 4: Resource utilization and complexity $\psi(c)^\phi$ related to scenarios of the medical center.

TABLE 3: Data for sensitivity analysis of distribution center and medical center.

resource	range	base values
dock	1 – 4	2
truck	1 – 12	6
group of workers	1 – 4	2
physician	3 – 8	7
technician	2 – 10	7
nurse	5 – 10	5

check how sensitive each parameter is, the metric proposed in the Section 3.2 was applied to the models.

In order to obtain the one-at-a-time measures, the configuration given by the lowest value of R_c was assumed as the optimized solution and the base for the local sensitivity analysis, as presented in Table 3. The parameter influence on model output was calculated according to the variability of the parameters. The range chosen for variation expresses values between -100% and 100% from base value of each parameter, shown in Table 3. In both case studies, the complexity was considered as model output for calculating the sensitivity index.

The results obtained by the sensitivity analysis are presented in the following sections. The metric proposed in Section 3.2 was used to calculate the sensitivity indices, which measures how much the area between curve of the parameter variation and base axis represents in relation to the sum of the areas of all parameters. Before applying that method, two

TABLE 4: Sensitivity index for distribution center related to output $\psi(c)^\lambda$.

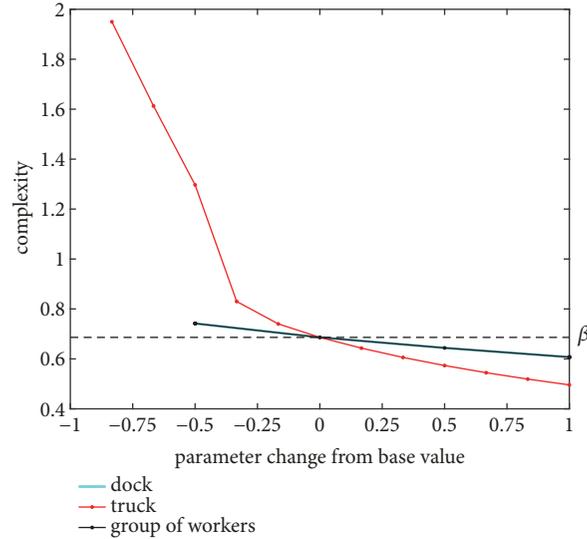
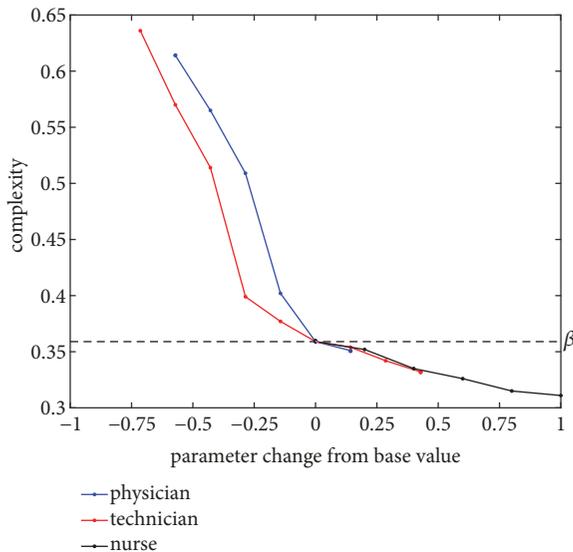
interval	S_{dock}^λ	S_{truck}^λ	S_{group}^λ
$[-1, 1]$	0.09	0.82	0.09
$[-0.5, 1]$	0.18	0.64	0.18

intervals were chosen for analysis. The first interval consists of the variation of the parameters between -100% and 100% and the second one refers to the interval in which all parameters have been changed according to their range presented in Table 3.

4.3.1. Sensitivity Analysis of Distribution Center. The one-at-a-time measures for the distribution center are presented in Figure 5 related to the complexity $\psi(c)^\lambda$ as model output. As observed in the graphs, the lower the number of the resource truck, the greater the impact on the complexity, especially between -100% and -30% .

According to the values presented in Table 4, the resource truck presented highest sensitivity index, equal to 82% for the interval $[-1, 1]$ and 64% for the interval $[-0.5, 1]$, where all parameters may be changed. Values change of that parameter inferior to -30% from base value led to significant impact on the system, corresponding to the scenarios in which there were less than five trucks at the distribution center.

The resources dock and group of workers presented the same values for sensitivity indices, equal to 9% for the interval

FIGURE 5: One-at-a-time measures related to the output $\psi(c)^\lambda$.FIGURE 6: One-at-a-time measures related to the output $\psi(c)^\phi$.

$[-1, 1]$ and 18% for the interval $[-0.5, 1]$. As it may be observed, these resources have low impact on model output because the dynamics of distribution center depends mostly on the resource truck.

The optimized solution for distribution center, i.e., 2 docks, 6 trucks, and 2 groups of workers, may be regarded robust for parameter change between -15% and 15% . In this range, the complexity presented low variation, $0.64 \leq \psi(c)^\lambda \leq 0.74$, which means that the increase or decrease of resources in one unit has not significantly affected the system.

4.3.2. Sensitivity Analysis of Medical Center. Based on the medical center simulation, Figure 6 presents one-at-a-time measures for the complexity $\psi(c)^\phi$ as model output. By

TABLE 5: Sensitivity index for medical center related to the output $\psi(c)^\phi$.

Interval	$S_{physician}^\phi$	$S_{technician}^\phi$	S_{nurse}^ϕ
$[-1, 1]$	0.40	0.46	0.14
$[0, 0.14]$	0.46	0.29	0.25

analysing the graphs, it may be seen that all curves have similar behavior; i.e., as the number of resources increased, the values of $\psi(c)^\phi$ decreased.

The resource physician obtained the higher sensitivity indices as presented in Table 5, equal to 40% related to the interval $[-1, 1]$ and 46% related to $[0, 0.14]$, concerning the region in which all parameters have variation from base values. Regarding these intervals, the resource technician presented sensitivity indices equal to 46% and 29%, respectively, while the resource nurse was the least sensitive parameter, with indices lower than 30%. The lowest indices for resource nurse may be explained by the fact that the range was begun from the base value, which is the optimized value for this resource. So the situation of scarcity of that parameter was not evaluated. In the opposite way, the resource technician presented the worst case of scarcity, which explains its higher sensitivity index related to the interval $[-1, 1]$.

Between -15% and 20% of parameter change, low impact on the output was observed, $0.35 \leq \psi(c)^\phi \leq 0.40$, indicating robustness around 15% of the configuration given by the lowest R_c , i.e., 7 physicians, 7 technicians, and 5 nurses. However when the number of physicians and the number of technicians were minimum for analyzed scenarios, the complexity was almost twice the value β .

5. Discussion

The system complexity measure contributes to the knowledge of the system as a whole. The simulated scenarios for the

distribution center and the medical center have shown that the system becomes less complex as the number of resources increases. However, complexity saturates from a certain number of resources, indicating idleness in the system.

As the complexity $\psi(c)$ is based on the connections, complexity measures may reflect (i) configurations with idle resources, (ii) optimized configurations, in which the system exhibits high performance, or (iii) configurations with scarce resources, hence expressive size of queues. The type of configurations may be distinguished one from another by applying the relation R_c , expressed by

$$R_c = \frac{\text{performance}}{\text{complexity}} \quad (11)$$

The lowest value of R_c has indicated the suitable number of resources for the system. In this configuration, there is a measure that expresses the level of complexity in which the system achieves its goal with high performance. We denote this measure as *natural complexity* of the system, i.e., the proper level of system complexity. By using natural complexity as reference, the complexity decrease may mean that resources are becoming idle and the complexity increase may indicate that the system is overloaded and underperforming, as expressed by

$$\begin{aligned} \text{complexity} \downarrow &\implies \text{idleness} \\ \text{natural complexity} &\implies \text{performance} \uparrow \\ \text{complexity} \uparrow &\implies \text{overload} \end{aligned} \quad (12)$$

In addition to the system overload, the complexity peaks have indicated the most sensitive parameters. The sensitivity analysis has been performed and has confirmed this point. In this paper, the sensitivity analysis contributed to (i) quantifying the influence of the parameters, (ii) understanding relationships between input and output variables, and (iii) checking the robustness of optimized solution. Besides that we here propose that the sensitivity indices are used to quantify the coupling between components of a system, since its inner structure reveals the system relationship with its environment.

So far we have measured the complexity based only on system connections regardless of their relevance. In order to make a complexity metric more comprehensive, this work proposes the use of sensitivity analysis in the metric of system complexity by the inclusion of relevance factor of connection γ . The sensitivity indices associated with each connection allows the measure of complexity to become greater expressive, regarding different strengths of coupling in the system.

Thus, we propose updating expression (2) to (13), given by

$$\psi(c, \gamma) = \sum_{\kappa=1}^{\rho} \gamma(c_{\kappa}) - \sum_{\kappa=1}^{\rho} P(c_{\kappa}) \cdot \log_2 P(c_{\kappa}) \quad (13)$$

where $\psi(c, \gamma)$ is the complexity of the system based on weighted connections, ρ is the number of active connections,

$P(c_{\kappa})$ is the probability of occurrence of the connection c_{κ} , and $\gamma(c_{\kappa})$ is the relevance factor of connection c_{κ} , defined based on the sensitivity indices S_{x_i} of the parameters. The results obtained from sensitivity analysis have used complexity measures as model output; however future works must use another measure in order to apply the metric $\psi(c, \gamma)$. We recommend that sensitivity indices are calculated based on system performance as model output.

In this paper, we presented a local sensitivity metric, called method of the area. However the local approach may make the complexity analysis unstable if the parameters base values are in a region of system instability. In order to overcome this problem, the global sensitivity analysis should be performed to comprise regions of instability and stability of the analyzed system. In this way, the impact of the nonlinearity in every region of operation of the system will be considered.

The proposed complexity metric abstracts aspects related to the spatial arrangement of the system, taking into account spatiotemporal interactions, as an alternative to Koorehdavoudi and Bogdan [2] approach. While the complexity metric proposed by Koorehdavoudi and Bogdan [2] is defined as the product between emergence and self-organization, both characteristics build on the missing information definition according to Shannon [38], the metric proposed here captures the spatial arrangement in terms of connections and uses the entropy conception to quantify the influence of the uncertainty inside the system added to the influence of the uncertainty generated by external elements, applying sensitivity analysis.

The sensitivity analysis was chosen because even parameters considered as less important by the uncertainty analysis (i.e., with low variability) may lead to significant changes on the output model due their sensitivity [45]. Thus we consider that the metric using connections weighted by sensitivity indices is able to capture the degree of system complexity as the combination between order and disorder, regularity and randomness, as discussed by Deacon and Koutroufinis [23] and Kurths et al. [46].

For instance, if we apply the proposed metric to a control system of DC motor, whose model presents continuous variables and continuous time, we could observe that when the motor is running, all connections are active and the probabilities of occurrence of all connections are equal to 1; thus the second part of the expression (13) is *zero* because the uncertainty related to the connections does not exist in this case. If any connection is broken, the system does not work. Thus, the uncertainty in the system is attributed to the parameters change (field voltage, control variables, etc.), which may be evaluated through the sensitivity analysis. In addition to operational aspects, the application of the metric based on weighted connections may show observable characteristics in structural terms and in relation with the environment, given by the local or global sensitivity analysis.

This example of control system of DC motor shows that some metrics such as fractal dimension could not be used due the absence of geometrical patterns described in phase space. Even for other types of systems, such as those analysed in the case studies (distribution center and

medical center), the fractal dimension would be not effective, since the arrangements observable during simulation are abstractions made from system operation, besides the fact that they would not probably have self-similarity. Therefore the proposed metric is able to quantify complexity related to system dynamics in several contexts.

The complexity metric $\psi(c, \gamma)$ considers both the difficulty of describing systems and their degree of organization. The configuration of the system in terms of the connections between its parts is the modeling used to describe it. The degree of organization, on the other hand, may be observed by the relevance of each connection, given by the sensitivity measures.

In future works, we intend to apply the proposed complexity metric $\psi(c, \gamma)$ to wide range of systems, both human-made and natural ones. Another issue is to investigate how complexity measures behave for different probabilities distributions used in the models. When we applied the complexity metric based on connections $\psi(c)$, we observed that low complexity was related to scenarios of greater number of resources, since the queue size decreased and the probabilities of occurrence of connections were lower than ones when there were few resources. However the probability distribution of arrival of entities in the systems was modeled as exponential distribution. Further researches could apply other distributions and observe the results.

In this paper, the main contribution is the proposal of integrating several system characteristics (configuration, arrangement, performance, and workload) into the complexity metric $\psi(c, \gamma)$. The use of sensitivity indices as weights for the connections enables combining features of external and internal dynamics of the system. Another important contribution is the conception of *natural complexity of systems*: a new concept that could be investigated in future work in order to make it a reasonable reference to evaluate the systems, since the concept comprises both effectiveness and efficiency.

6. Conclusion

This paper has proposed sensitivity and complexity metrics based on one-at-a-time values and system connections, respectively. It has been observed that complexity may indicate (i) most sensitive parameter, (ii) idleness or overload in the system, and (iii) lowest or greatest number of resources. The relation between performance and complexity has led to scenarios with optimized configuration for meeting the demand. Considering these cases, the paper has established that systems have their proper level of complexity, denoted natural complexity. Regarding the different types of couplings in the system, the use of sensitivity analysis has been proposed in order to determine the relevance factor of connection, contributing to more accurate measurement of the system complexity.

Data Availability

The CSV file with data used to support the findings of this study are available from the corresponding author upon

request. The data are results from the simulation of the models.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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