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Research Article

Lump and Mixed Rogue-Soliton Solutions of the (2 + 1)-Dimensional Mel'nikov System

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Lump wave and line rogue wave of the (2 + 1)-dimensional Mel'nikov system are derived by taking the ansatz as the rational function. By combining a rational function and different exponential functions, mixed solutions between the lump and soliton are derived. These solutions describe the interaction phenomena of the lump-bright soliton with fission and fusion, the half-line rogue wave with a bright soliton, and a rogue wave excited from the bright soliton pair, respectively. Some special concrete interaction solutions are depicted in both analytical and graphical ways.

1. Introduction

In nonlinear science, searching exact solutions plays an important role in physics and engineering [1–5]. Many methods are very powerful to investigate the nonlinear evolution equations, such as the Darboux transformation [2, 6], the inverse scattering transformation [1, 7], and the Hirota bilinear method [8, 9]. Recently, lump waves, rogue waves, and the interaction solutions between the lump and soliton have intensively aroused much attention [10–17]. There are two main ways to deal with this problem. One method is the KP-hierarchy reduction theory, which was established by the Kyoto School in 1980s [18]. Ohta et al. used this method and obtained various rogue wave solutions of the NLS equation [19, 20], the DS equations [21], and the multicomponent Yajima–Oikawa system (YO system) [22]. The other method is the ansatz method, which directly constructed a positive quadratic form by the bilinear form of a PDE. This method was successfully applied into the KP equation [23, 24], the (2 + 1)-dimensional Boussinesq equation [25], the BKP equation [26, 27], the (2 + 1)-dimensional coupled nonlinear partial differential equation [28], and so on. Besides, Yong et al. used this method into the complex system such as the KPI equation with a self-consistent source [29]. It is obvious that rational solution, rogue wave, mixed rogue wave, and stripe wave of PDEs are intuitively derived by the ansatz method.

Rogue waves are large and spontaneous ocean surface waves that occur in the sea and are a threat even to large ships and ocean liners [30]. Besides an optical analog of rogue waves, optical rogue waves were also observed in optical fibers [31–33]. These facts motivate us to seek rogue waves in more (2 + 1)-dimensional equations. In this paper, we will use the direct ansatz method to investigate lump and mixed rogue-soliton solutions of the (2 + 1)-dimensional complex Mel'nikov system:

$$\begin{aligned} 3u_{tt} - 3u_{xy} - \left(3u^2 + u_{xx} + \delta|A|^2\right)_{xx} &= 0, \\ iA_t - uA - A_{xx} &= 0, \end{aligned} \tag{1}$$

where the function u is the real long wave amplitude, A is the complex short wave amplitude, and δ must satisfy the condition $\delta^2 = 1$. The system described (under certain conditions) the interaction of long wave and short wave envelope propagating on the $x - y$ plane at an angle to each other [34, 35], and this system was first given in [35] for the interactions of water waves. The investigation of this equation has a great application in plasma physics, solid-state physics, nonlinear optics, and hydrodynamics [36, 37]. When $y = x$, the system can be reduced to the NLS–Boussinesq equation, which is used to describe the nonlinear development of modulational instabilities associated with Langmuir field amplitude coupled to intense electromagnetic waves in dispersive media such as plasma electromagnetic waves in the dispersive medium [38]. In an earlier study, the boomeron-type solution which describes the interaction between long and short waves for Mel'nikov system (1) was obtained [36]. Then, the Panlevé analysis and the dromion solution were obtained by Kumar et al. [37]. In addition, multisoliton solutions were derived via the theory of matrices [34]. Hase et al. have derived the bright- and dark-type solitons from the Wronskian solutions of KP-hierarchy theory [39]. Zhang et al. obtained the general higher-order rogue waves and the hybrid solutions [40, 41]. Furthermore, they researched in the multicomponent Mel'nikov system to get the general N-dark solitons and the bright-dark mixed N-solitons through the KP-hierarchy reduction technique [42, 43]. Later, Sun et al. investigated the semirational solutions of the Mel'nikov system with the aid of the bilinear method and the KP-hierarchy reduction method [44]. However, to the best of our knowledge, there are no reports on rational solution, rogue wave, mixed rogue wave, and stripe waves by using the ansatz technique.

This paper is organized as follows. In Section 2, the rational solution and the line rogue wave are derived by using the ansatz technique based on the Hirota bilinear method. Section 3 is devoted to find the mixed solutions between the lump and one bright soliton. By combining a rational function and one exponential function, the lump-bright soliton interaction with fission and fusion phenomena, and the half-line rogue wave with a bright soliton are obtained. In Section 4, a rogue wave excited from the bright soliton pair is given by introducing a rational-exponential function. The last section contains conclusions and discussions.

2. The Rational Solution

In this section, we will give the rational solution of Mel'nikov system (1) based on the Hirota bilinear method. Firstly, using the following dependent variable transformation,

$$A = \frac{\rho \exp(ibt) g}{f}, \qquad u = 2(\ln(f))_{xx} - b, \tag{2}$$

Mel'nikov system (1) is transformed into the bilinear form:

$$\left(D_x^4 + D_x D_y - 3D_t^2 - 6bD_x^2\right) f\cdot f - \delta\rho^2\left(f^2 - gg^*\right) = 0, \qquad \left(D_x^2 - iD_t\right) g\cdot f = 0, \tag{3}$$

where b and ρ are arbitrary constants, f is a real function, g is a complex function, g^* is a complex conjugate function of g, and the operator D is defined by

$$D_x^l D_y^n D_t^m f(x,y,t)\cdot g(x',y',t') = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial x'}\right)^l \left(\frac{\partial}{\partial y} - \frac{\partial}{\partial y'}\right)^n \left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)^m f(x,y,t)\cdot g(x',y',t')\Big|_{x=x',y=y',t=t'}. \tag{4}$$

In order to obtain the rational solution of Mel'nikov system (1), we assume f and g have the following function:

$$f = \xi_1^2 + \xi_2^2 + a_9, \qquad g = (b_0 + ic_0) + (b_1 + ic_1)\xi_1 + (b_2 + ic_2)\xi_2 + (b_3 + ic_3)\xi_1^2 + (b_4 + ic_4)\xi_2^2, \tag{5}$$

with

$$\xi_1 = a_1 x + a_2 y + a_3 t + a_4, \qquad \xi_2 = a_5 x + a_6 y + a_7 t + a_8, \tag{6}$$

where $a_i (i = 1, 2, \ldots, 9)$, $b_i (i = 0, 1, \ldots, 4)$, and $c_i (i = 0, 1, \ldots, 4)$ are arbitrary real parameters. Inserting (5) into (3) and balancing different powers of x, y, and t, the relations among arbitrary constants read

$$\begin{aligned} c_4 &= c_3, \\ b_4 &= b_3, \\ c_3 &= -kb_3, \\ b_1 &= kc_1, \\ b_2 &= kc_2, \\ c_0 &= -kb_0, \\ a_2 &= \frac{2b_0b_3k^2 + c_1^2k^2 + 24a_1^2b + 12a_3^2 - 12a_7^2 + 2b_0b_3 + c_1^2 - 2a_9}{4a_1}, \\ a_5 &= 0, \\ a_6 &= \frac{c_1c_2k^2 + 24a_3a_7 + c_1c_2}{4a_1}, \\ a_9 &= \frac{a_1^4}{a_7^2}, \\ b_0 &= \frac{a_1^4 b_3 (a_3^2 - 3a_7^2)}{a_7^2 (a_3^2 + a_7^2)}, \\ c_1 &= -\frac{4a_1^2 b_3 a_3}{a_3^2 + a_7^2}, \\ c_2 &= \frac{4a_1^2 b_3 a_7}{a_3^2 + a_7^2}, \end{aligned} \tag{7}$$

where k is an arbitrary real constant which needs to satisfy the following condition:

$$b_3^2\left(1+k^2\right)-1=0, \tag{8}$$

and the other parameters need to satisfy the restricting condition $a_7\left(a_3^2+a_7^2\right)\neq 0$.

Actually, the functions ξ_1 and ξ_2 are linearly independent. Meanwhile, it is readily observed that at any given time t, the lump solution $u \longrightarrow 0$ when $x^2+y^2 \longrightarrow \infty$. To catch the moving path of the lump wave, the critical point is just calculating the first derivative $(f_x, f_y)=0$. The exact moving path of the lump waves is written as

$$\begin{aligned} x &= \frac{(a_2a_7-a_3a_6)t+(a_2a_8-a_4a_6)}{a_1a_6},\\ y &= \frac{a_7t}{a_6}-\frac{a_8}{a_6}, \end{aligned} \tag{9}$$

which can describe the traveling path of the lump waves along a straight line:

$$y=-\frac{a_1a_7x}{a_2a_7-a_3a_6}+\frac{a_4a_7-a_3a_8}{a_2a_7-a_3a_6}, \tag{10}$$

with a_2 and a_6 satisfying (7). By selecting different kinds of these parameters, we can derive various kinds of structures for Mel'nikov system (1). Here, we obtain two kinds of structures of the lump wave and rogue wave when taking the parameters $a_1=5/3$, $a_3=1/3$, $a_4=-1$, $a_7=-1$, $a_8=0$, $b_3=1/2$, and $b=1$ and $a_1=5/3$, $a_3=0$, $a_4=-1$, $a_7=-1$, $a_8=0$, $b_3=1/2$, and $b=1$.

When $t=0$, their spatial structures and propagations of the lump waves of u and $|A|^2$ are described in Figure 1. The spatial structure of the lump waves of u and $|A|^2$ are plotted in Figures 1(a) and 1(c). Figures 1(b) and 1(d) display the contour plots of the lump waves of u and $|A|^2$ at $t=-40, 0, 38$. The relevant moving direction of the lump waves u and $|A|^2$ are $y=-(5x/9)+(1/3)$. When $a_3=0$, the propagation of the line rogue waves are plotted in Figure 2. The spatial structures of the rogue wave propagation at different times $t=-20,-5,0,5,20$ are described in Figure 2, respectively. It is shown that the line rogue wave occurs from a constant background with a line profile, reaches a peak around time $t=0$, and finally retreats back to a constant background again. Because the functions A and u are handled in the same way, to simplify the calculation, we will only analyze the function u while the function A will not be discussed in the following parts.

3. LUMP Interacting with One Soliton Solution

Interaction solutions between the lump and soliton can be constructed by combining the rational functions and exponential functions. Firstly, we take the functions f and g as the ansatz with the following rational-exponential function:

$$\begin{aligned} f &= \xi_1^2+\xi_2^2+a_9+m_0\exp(\eta),\\ g &= (b_0+ic_0)+(b_1+ic_1)\xi_1+(b_2+ic_2)\xi_2+(b_3+ic_3)\xi_1^2\\ &\quad +(b_4+ic_4)\xi_2^2+(m_1+im_2)\exp(\eta), \end{aligned} \tag{11}$$

with

$$\begin{aligned} \xi_1 &= a_1x+a_2y+a_3t+a_4,\\ \xi_2 &= a_5x+a_6y+a_7t+a_8,\\ \eta &= k_1x+k_2y+k_3t, \end{aligned} \tag{12}$$

where the parameters $a_i\,(i=1,2,\ldots,9)$, $b_i\,(i=0,1,\ldots,4)$, $c_i\,(i=0,1,\ldots,4)$, and $m_i\,(i=0,1,2)$ are determined real parameters. By substituting (11) into (3) and vanishing the different powers of the variables x, y, and t, we obtain a series of algebraic equations on the undetermined parameters. Then, two sets of constraining relations are obtained by solving these equations.

Case 1

$$\begin{aligned} c_4 &= c_3,\\ b_4 &= b_3,\\ b_3 &= -1,\\ c_3 &= 0,\\ m_1 &= 0,\\ m_2 &= m_0,\\ a_5 &= 0,\\ k_2 &= \frac{2a_3k_1^4+a_2k_1^3+a_3}{a_3k_1},\\ k_3 &= k_1^2,\\ b_2 &= 0,\\ b_1 &= 0,\\ c_2 &= \frac{2a_3}{k_1^2},\\ c_1 &= \frac{2\left(2a_1k_1-a_4\right)}{k_1^2},\\ c_0 &= 0,\\ b_0 &= \frac{a_3^2}{k_1^4},\\ b &= \frac{a_2k_1}{6a_3},\\ a_9 &= \frac{a_3^2}{k_1^4},\\ a_7 &= -a_3,\\ a_6 &= \frac{-a_3\left(6k_1^4-1\right)}{k_1^3},\\ a_1 &= \frac{a_3}{k_1}, \end{aligned} \tag{13}$$

with the condition $k_1a_3\neq 0$.

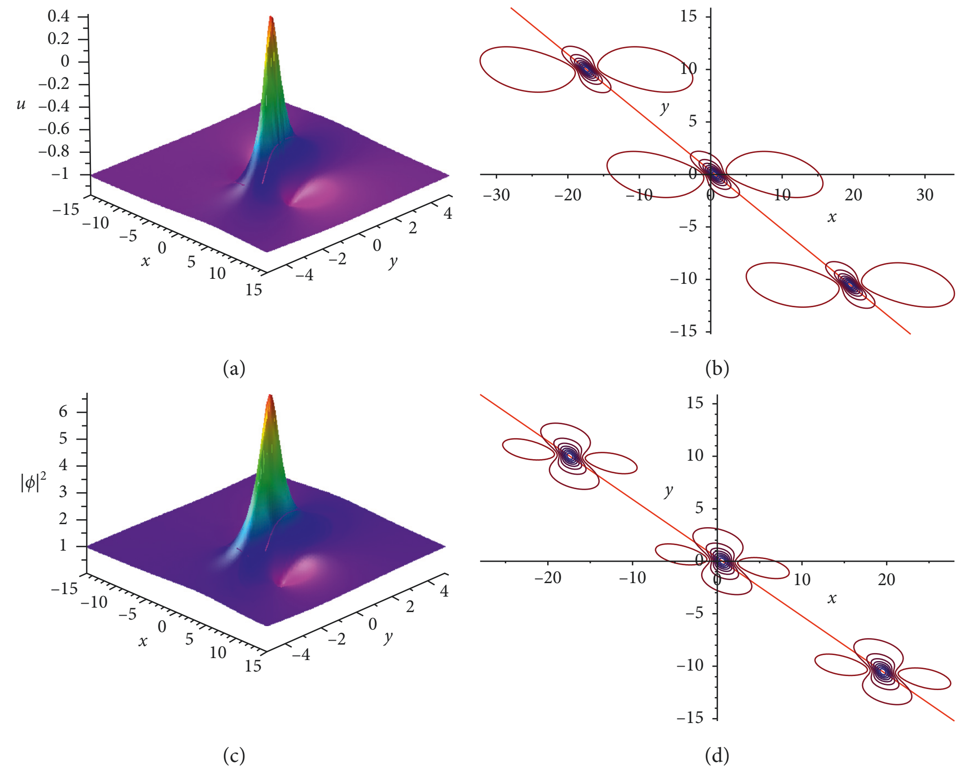

FIGURE 1: Lump solution (2) with (5) and (7) taking parameters $a_1 = 5/3$, $a_3 = 1/3$, $a_4 = -1$, $a_7 = -1$, $a_8 = 0$, $b_3 = 1/2$, and $b = 1$. (a) The lump wave of u at $t = 0$. (b) The contour plot of u at $t = -40, 0, 38$, and red line is the relevant moving direction $y = -(5x/9) + (1/3)$. (c) The lump wave of $|A|^2$ at $t = 0$. (d) The contour plot of $|A|^2$ at $t = -40, 0, 38$, and red line is the relevant moving direction $y = -(5x/9) + (1/3)$.

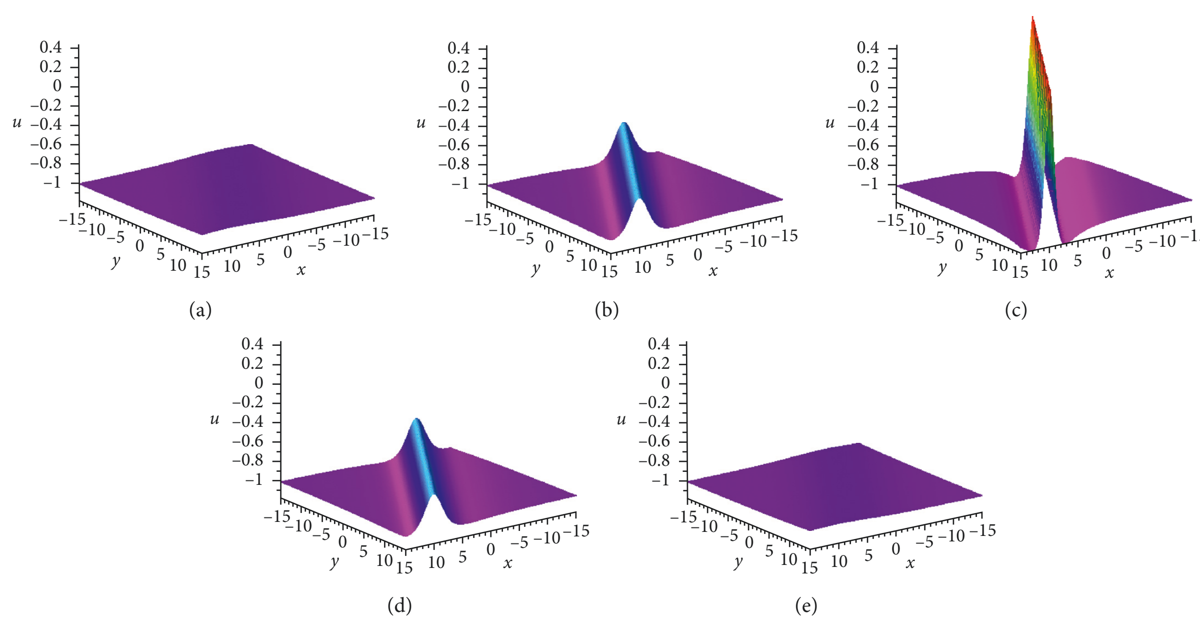

FIGURE 2: The propagations of line rogue wave (2) with (5) and (7) taking the parameters $a_1 = 5/3$, $a_3 = 0$, $a_4 = -1$, $a_7 = -1$, $a_8 = 0$, $b_3 = 1/2$, and $b = 1$ at different times (a) $t = -20$, (b) $t = -5$, (c) $t = 0$, (d) $t = 5$, and (e) $t = 20$.

It is known that the stripe soliton is expressed by the exponential function and the lump wave is a type of rational function. Thus, the solution of interaction (2) with (11) and (13) describes the interaction between a lump and one bright soliton. For this situation, the propagation process contains two kinds of phenomena: fusion and fission. As shown in Figure 3, the propagations of the interaction solution mixed lump-bright soliton are plotted. Figures 3(a)–3(c) represent

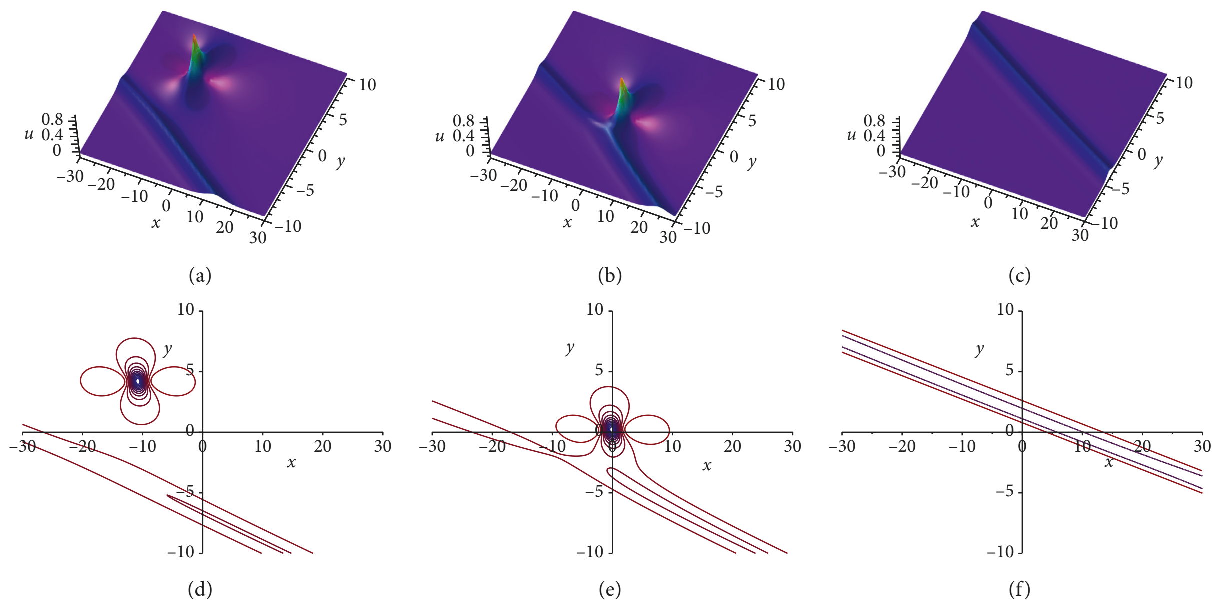

FIGURE 3: The propagations of the interaction solution between a lump wave and bright soliton (2) with (11) and (13) taking the parameters $m_0 = 1/5$, $k_1 = -(1/2)$, $a_2 = 1/2$, $a_3 = -(3/2)$, $a_4 = 1/4$, and $a_8 = -(3/2)$ at different times (a) $t = -20$, (b) $t = 0$, and (c) $t = 60$. The contour plots at different times (d) $t = -20$, (e) $t = 0$, and (f) $t = 60$.

the solution at different times $t = -20, 0, 60$, respectively. Figures 3(d)–3(f) describe the homologous contour plots at time $t = -20, 0, 60$. It is shown that one bright soliton and one lump fuse into one bright soliton gradually, which represents the fusion process. While the parameter $k_1 = 3/4$, the interaction solution between one lump and one bright soliton is plotted in Figure 4, which exhibits the fission process that one bright soliton splits into one bright soliton and one lump conversely.

Case 2

$$
\begin{aligned}
c_4 &= c_3,\\
b_4 &= b_3,\\
b_3 &= -1,\\
c_3 &= 0,\\
m_1 &= 0,\\
m_2 &= m_0,\\
a_5 &= 0,\\
c_1 &= \frac{2(2a_1k_1 - a_4)}{k_1^2},\\
b_2 &= 0,\\
b_1 &= 0,\\
k_3 &= k_1^2,\\
k_2 &= \frac{4\sqrt[4]{6}}{3} + \frac{\sqrt{6}a_2}{6a_3},\\
k_1 &= 6^{-(1/4)},\\
c_2 &= 2\sqrt{6}a_3,\\
c_0 &= 0,\\
b_0 &= 6a_3^2,\\
b &= \frac{6^{-(5/4)}a_2}{a_3},\\
a_9 &= 6a_3^2,\\
a_7 &= -a_3,\\
a_6 &= 0,\\
a_1 &= 6^{1/4}a_3,
\end{aligned}
\tag{14}
$$

which need to satisfy the condition $k_1 a_3 \neq 0$.

The solution of interaction (2) with (11) and (14) describes the interaction mixed rogue wave and a bright soliton. The spatial structures of the propagation at different times $t = -30, -10, -3.3, 0, 3.3, 30$ are shown in Figure 5, respectively. It is shown that the half-line rogue wave arises from a constant background, then interacts with a bright soliton, reaches a peak around time $t = -3.3$, and finally retreats to a constant background again.

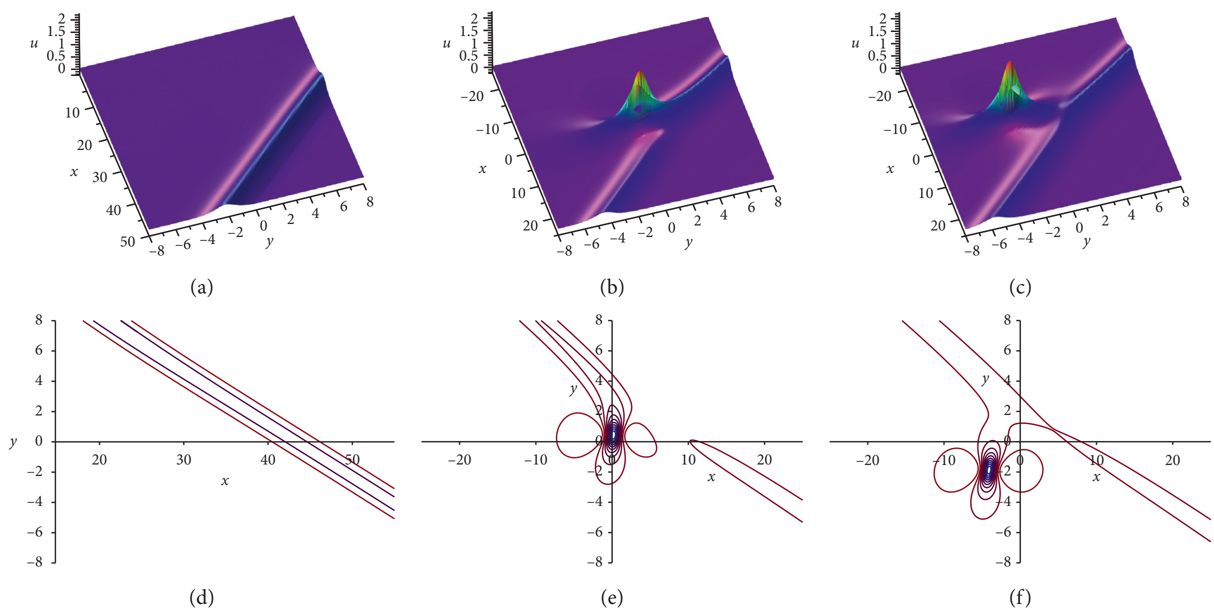

Figure 4: The propagations of the interaction solution between a lump wave and bright soliton (2) with (11) and (13) taking the parameters $m_0 = 1/5$, $k_1 = 3/4$, $a_2 = 1/2$, $a_3 = -(3/2)$, $a_4 = 1/4$, and $a_8 = -(3/2)$ at different times (a) $t = -40$, (b) $t = 0$, and (c) $t = 5$. The contour plots at different times (d) $t = -40$, (e) $t = 0$, and (f) $t = 5$.

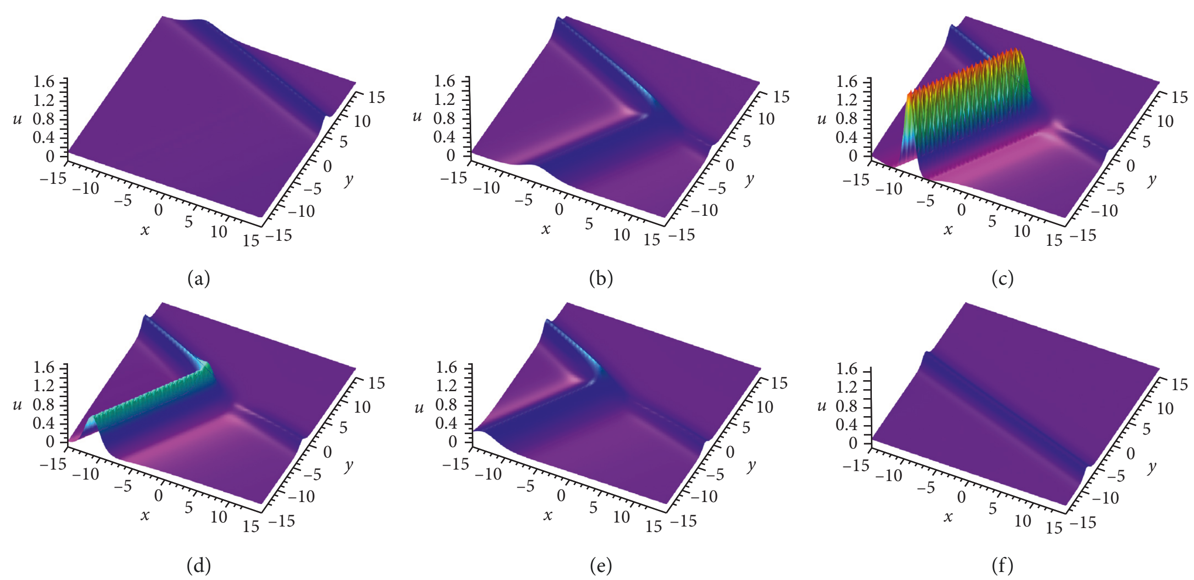

Figure 5: The propagations of the interaction solution between a rogue wave and bright soliton (2) with (11) and (14) taking the parameters $m_0 = 2$, $a_4 = 5/2$, $a_8 = -3$, $k_1 = 1$, $a_2 = -1$, and $a_3 = 3$ at different times (a) $t = -30$, (b) $t = -10$, (c) $t = -3.3$, (d) $t = 0$, (e) $t = 3.3$, and (f) $t = 30$.

4. Rogue Wave Excited from the Stripe Soliton Pair

Following the idea of considering the interaction solution between a lump and one bright soliton, we aim at seeking for the interaction mixed lump and a pair of bright solitons. To achieve this aim, we assume f and g as follows:

$$\begin{aligned} f &= \xi_1^2 + \xi_2^2 + a_9 + m_1 \exp(\eta) + m_2 \exp(-\eta), \\ g &= (b_0 + ic_0) + (b_1 + ic_1)\xi_1 + (b_2 + ic_2)\xi_2 + (b_3 + ic_3)\xi_1^2 \\ &\quad + (b_4 + ic_4)\xi_2^2 + m_3 \exp(\eta) + m_4 \exp(-\eta), \end{aligned} \tag{15}$$

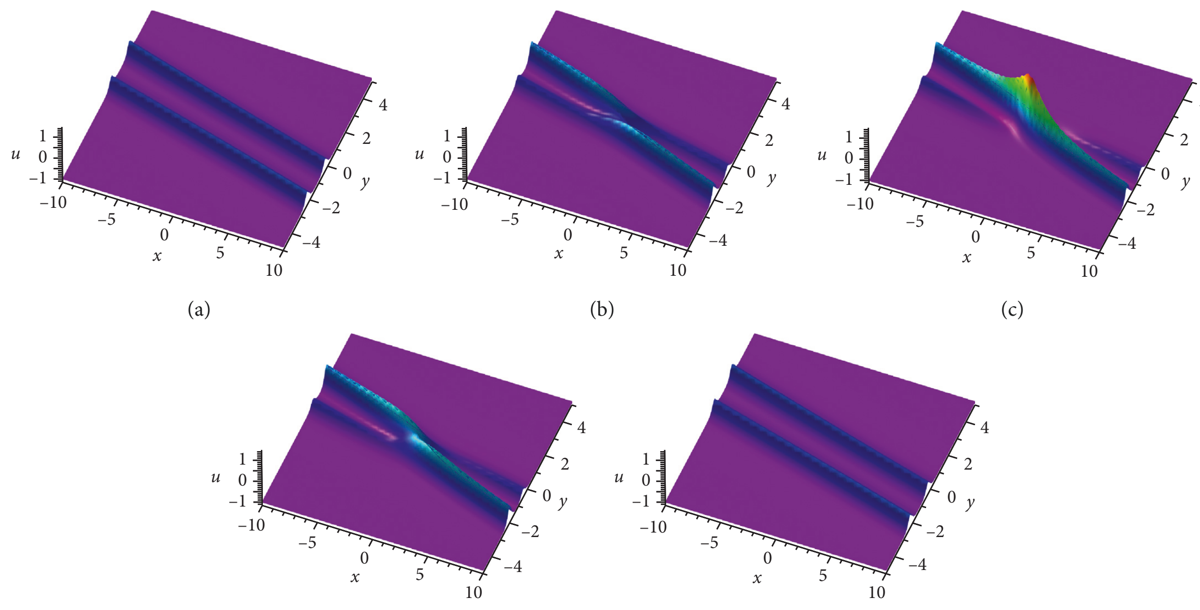

Figure 6: The propagations of the interaction solutions between two bright solitons and rogue wave (2) with (15) and (16) taking the parameters $m_1 = 1$, $m_2 = 3$, $a_4 = 2$, $a_8 = 1$, $k_1 = 0.8$, $a_1 = -3.2$ at different times (a) $t = -30$, (b) $t = -5$, (c) $t = 0$, (d) $t = 5$, and (e) $t = 30$.

with

$$\begin{aligned} \xi_1 &= a_1 x + a_2 y + a_3 t + a_4, \\ \xi_2 &= a_5 x + a_6 y + a_7 t + a_8, \\ \eta &= k_1 x + k_2 y + k_3 t, \end{aligned} \tag{16}$$

where the parameters $a_i\,(i = 1, 2, \ldots, 9)$, $b_i\,(i = 0, 1, \ldots, 4)$, $c_i\,(i = 0, 1, \ldots, 4)$, and $m_i\,(i = 1, 2, 3, 4)$ are determined real parameters. Similarly, after the substitution of (15) into (3), vanishing all the coefficients of the exponential functions and the variables x, y, and t, we obtain a set of algebraic equations. By solving these equations, the relations of parameters are yielded as follows:

$$\begin{aligned} c_4 &= c_3, \\ b_4 &= b_3, \\ b_3 &= -1, \\ c_3 &= 0, \\ m_3 &= m_1, \\ m_4 &= m_2, \\ a_5 &= 0, \\ k_3 &= 0, \\ b_1 &= 0, \\ b_2 &= 0, \\ c_1 &= 0, \\ c_2 &= \frac{-4a_7}{k_1^2}, \\ b_0 &= \frac{-a_9 k_1^4 + 4a_7^2}{k_1^4}, \\ c_0 &= 0, \\ b &= \frac{3k_1^4 + 2}{6k_1^2}, \\ a_2 &= 0, \\ a_3 &= 0, \\ a_6 &= 0, \\ a_7 &= -k_1 a_1, \\ a_9 &= \frac{k_1^4 m_1 m_2 + a_1^4}{k_1^2 a_1^2}, \\ k_2 &= \frac{2\left(k_1^4 + 2\right)}{k_1}, \end{aligned} \tag{17}$$

where $a_1 k_1 \neq 0$ and $a_9 > 0$.

The interaction solutions between two bright solitons and a rogue wave are shown in Figure 6 with the parameters selected as $m_1 = 1$, $m_2 = 3$, $k_1 = 0.8$, $a_1 = -3.2$, $a_4 = 2$, and

$a_8 = 1$. Figure 6 represents the spatial structures of propagation for this interaction solution of two bright solitons with a rogue wave at different times $t = -30, -5, 0, 5, 30$, respectively. As can be seen, this solution illustrates the superposition between the two bright solitons and a rogue wave. Also, we find that a rogue wave excited from the bright soliton pair. From the whole evolution, the fundamental rogue wave is the rational wave, and the rogue wave reaches the highest amplitude around time $t = 0$ at $x = 0$ and $y = 0$.

5. Summary and Discussions

In this work, we derive the rational and rational-exponential solutions of the $(2+1)$-dimensional Mel'nikov system by using the direct ansatz method. Based on the bilinear form of the Mel'nikov system, the ansatz form is taken as the rational function, the different linear combinations of the rational function and exponential functions, respectively. Not only can we drive the rational solution, but also we obtain the rogue wave and the interaction solutions. The exact rational solution depicts the lump structure and the line rogue wave by selecting different parameters. Besides, the rational-exponential solutions contain two types of the interaction solutions. One type solution is the interaction solution between a lump wave and a bright soliton. For this situation, the propagation process contains fusion and fission phenomena. The other type solution is the mixed half-line rogue wave and a bright soliton. In addition, by adding two exponential functions to a rational function, a rogue wave excited from the bright soliton pair is derived. It is worth mentioning that there are three fundamental forms of the rogue waves, which are the line wave, half-line wave, and lump-type wave. Also, the peaks of these rogue waves appear at different times when we take different ansatz forms.

Data Availability

The data that support the plots and other findings in this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

[1] C. S. Gardner, J. M. Greene, M. D. Kruskal, and R. M. Miura, "Method for solving the Korteweg-deVries equation," *Physical Review Letters*, vol. 19, no. 19, pp. 1095–1097, 1967.

[2] V. B. Matveev and A. S. Mikhail, *Darboux Transformations and Solitons*, Springer, Berlin, Germany, 1992.

[3] X. Tang, S. Lou, and Y. Zhang, "Localized excitations in $(2+1)$-dimensional systems," *Physical Review E*, vol. 66, no. 4, Article ID 046601, 2002.

[4] J. Lin, X.-W. Jin, X.-L. Gao, and S.-Y. Lou, "Solitons on a periodic wave background of the modified KdV-sine-gordon equation," *Communications in Theoretical Physics*, vol. 70, no. 2, p. 119, 2018.

[5] B. Ren, X.-P. Cheng, and J. Lin, "The $(2+1)$-dimensional Konopelchenko-Dubrovsky equation: nonlocal symmetries and interaction solutions," *Nonlinear Dynamics*, vol. 86, no. 3, pp. 1855–1862, 2016.

[6] Zhaqilao and Z. Qiao, "Darboux transformation and explicit solutions for two integrable equations," *Journal of Mathematical Analysis and Applications*, vol. 380, no. 2, pp. 794–806, 2011.

[7] J.-L. Ji and Z.-N. Zhu, "Soliton solutions of an integrable nonlocal modified Korteweg-de Vries equation through inverse scattering transform," *Journal of Mathematical Analysis and Applications*, vol. 453, no. 2, pp. 973–984, 2017.

[8] R. Hirota, *The Direct Method in Soliton Theory*, Cambridge University Press, Cambridge, UK, 2004.

[9] J. Satsuma, "Hirota bilinear method for nonlinear evolution equations," *Direct and Inverse Methods in Nonlinear Evolution Equations*, Springer, Berlin, Germany, pp. 171–222, 2003.

[10] A.-M. Wazwaz, "Multiple-soliton solutions for the calogero-bogoyavlenskii-Schiff, Jimbo-Miwa and YTSF equations," *Applied Mathematics and Computation*, vol. 203, no. 2, pp. 592–597, 2008.

[11] D. J. Kaup, "The lump solutions and the Bäcklund transformation for the three-dimensional three-wave resonant interaction," *Journal of Mathematical Physics*, vol. 22, no. 6, pp. 1176–1181, 1981.

[12] K. Imai, "Dromion and lump solutions of the Ishimori-I equation," *Progress of Theoretical Physics*, vol. 98, no. 5, pp. 1013–1023, 1997.

[13] J. Satsuma and M. J. Ablowitz, "Two-dimensional lumps in nonlinear dispersive systems," *Journal of Mathematical Physics*, vol. 20, no. 7, pp. 1496–1503, 1979.

[14] I. Ahmed, A. R. Seadawy, and D. Lu, "Mixed lump-solitons, periodic lump and breather soliton solutions for $(2+1)$-dimensional extended Kadomtsev-Petviashvili dynamical equation," *International Journal of Modern Physics B*, vol. 33, no. 5, Article ID 1950019, 2019.

[15] S. Lou and J. Lin, "Rogue waves in nonintegrable KdV-type systems," *Chinese Physics Letters*, vol. 35, no. 5, Article ID 050202, 2018.

[16] B. Ren, J. Lin, and Z. M. Lou, "A new nonlinear equation with lump-soliton, lump-periodic, and lump-periodic-soliton solutions," *Complexity*, vol. 2019, Article ID 4072754, 10 pages, 2019.

[17] Z.-Y. Ma, J.-X. Fei, and J.-C. Chen, "Lump and stripe soliton solutions to the generalized nizhnik-novikov-veselov equation," *Communications in Theoretical Physics*, vol. 70, no. 5, p. 521, 2018.

[18] M. Jimbo and T. Miwa, "Solitons and infinite dimensional lie algebras," *Publications of the Research Institute for Mathematical Sciences*, vol. 19, no. 3, pp. 943–1001, 1983.

[19] Y. Ohta, D.-S. Wang, and J. Yang, "General N-Dark-Dark solitons in the coupled nonlinear schrödinger equations," *Studies in Applied Mathematics*, vol. 127, no. 4, pp. 345–371, 2011.

[20] Y. Ohta and J. Yang, "General high-order rogue waves and their dynamics in the nonlinear Schrödinger equation," *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 468, no. 2142, pp. 1716–1740, 2012.

[21] Y. Ohta and J. Yang, "Rogue waves in the Davey-Stewartson I equation," *Physical Review E*, vol. 86, no. 3, Article ID 036604, 2012.

[22] J. Chen, Y. Chen, B. F. Feng, and K. I. Maruno, "General mixed multi-soliton solutions to one-dimensional multi-component Yajima-Oikawa system," *Journal of the Physical Society of Japan*, vol. 84, no. 7, Article ID 074001, 2015.

[23] W.-X. Ma, "Lump solutions to the Kadomtsev-Petviashvili equation," *Physics Letters A*, vol. 379, no. 36, pp. 1975–1978, 2015.

[24] H.-Q. Zhao and W.-X. Ma, "Mixed lump-kink solutions to the KP equation," *Computers & Mathematics with Applications*, vol. 74, no. 6, pp. 1399–1405, 2017.

[25] H.-C. Ma and A.-P. Deng, "Lump solution of $(2+1)$-dimensional Boussinesq equation," *Communications in Theoretical Physics*, vol. 65, no. 5, pp. 546–552, 2016.

[26] J.-B. Zhang and W.-X. Ma, "Mixed lump-kink solutions to the BKP equation," *Computers & Mathematics with Applications*, vol. 74, no. 3, pp. 591–596, 2017.

[27] J.-Y. Yang, W. X. Ma, and Z. Y. Qin, "Abundant mixed lump-soliton solutions to the BKP equation," *East Asian Journal on Applied Mathematics*, vol. 8, no. 2, pp. 224–232, 2018.

[28] B. Ren, W. X. Ma, and J. Yu, "Characteristics and interactions of solitary and lump waves of a $(2+1)$-dimensional coupled nonlinear partial differential equation," *Nonlinear Dynamics*, vol. 96, no. 1, pp. 717–727, 2019.

[29] X. Yong, W.-X. Ma, Y. Huang, and Y. Liu, "Lump solutions to the Kadomtsev-Petviashvili I equation with a self-consistent source," *Computers & Mathematics with Applications*, vol. 75, no. 9, pp. 3414–3419, 2018.

[30] C. Kharif, E. Pelinovsky, and A. Slunyaev, *Rogue Waves in the Ocean*, Springer Science & Business Media, Berlin, Germany, 2008.

[31] D. R. Solli, C. Ropers, P. Koonath, and B. Jalali, "Optical rogue waves," *Nature*, vol. 450, no. 7172, pp. 1054–1057, 2007.

[32] B. Kibler, J. Fatome, C. Finot et al., "The Peregrine soliton in nonlinear fibre optics," *Nature Physics*, vol. 6, no. 10, pp. 790–795, 2010.

[33] D. Lu, A. R. Seadawy, and I. Ahmed, "Peregrine-like rational solitons and their interaction with kink wave for the resonance nonlinear Schrödinger equation with Kerr law of nonlinearity," *Modern Physics Letters B*, vol. 33, no. 24, Article ID 1950292, 2019.

[34] V. K. Mel'Nikov, "A direct method for deriving a multi-soliton solution for the problem of interaction of waves on the x, y plane," *Communications in Mathematical Physics*, vol. 112, no. 4, pp. 639–652, 1987.

[35] V. K. Mel'Nikov, "On equations for wave interactions," *Letters in Mathematical Physics*, vol. 7, no. 2, pp. 129–136, 1983.

[36] V. K. Mel'nikov, "Reflection of waves in nonlinear integrable systems," *Journal of Mathematical Physics*, vol. 28, no. 11, pp. 2603–2609, 1987.

[37] C. S. Kumar, R. Radha, and M. Lakshmanan, "Exponentially localized solutions of Mel'nikov equation," *Chaos, Solitons & Fractals*, vol. 22, no. 3, pp. 705–712, 2004.

[38] S. V. Singh, N. N. Rao, and P. K. Shukla, "Nonlinearly coupled Langmuir and dust-acoustic waves in a dusty plasma," *Journal of Plasma Physics*, vol. 60, no. 3, pp. 551–567, 1998.

[39] Y. Hase, R. Hirota, Y. Ohta, and J. Satsuma, "Soliton solutions to the mel'nikov equations," *Journal of the Physical Society of Japan*, vol. 58, no. 8, pp. 2713–2720, 1989.

[40] X. Zhang and Y. Chen, "General high-order rogue waves to nonlinear Schrödinger-Boussinesq equation with the dynamical analysis," *Nonlinear Dynamics*, vol. 93, no. 4, pp. 2169–2184, 2018.

[41] X. Zhang, T. Xu, and Y. Chen, "Hybrid solutions to Mel'nikov system," *Nonlinear Dynamics*, vol. 94, no. 4, pp. 2841–2862, 2018.

[42] Z. Han, Y. Chen, and J. Chen, "Bright-dark mixed N-soliton solutions of the multi-component mel'nikov system," *Journal of the Physical Society of Japan*, vol. 86, no. 10, Article ID 104008, 2017.

[43] Z. Han, Y. Chen, and J. Chen, "General N-dark soliton solutions of the multi-component Mel'nikov system," *Journal of the Physical Society of Japan*, vol. 86, no. 7, Article ID 074005, 2017.

[44] B. Sun and Z. Lian, "Rogue waves in the multicomponent Mel'nikov system and multicomponent Schrödinger-Boussinesq system," *Pramana*, vol. 90, no. 2, p. 23, 2018.