Hindawi
Complexity
Volume 2019, Article ID 1510257, 22 pages
https://doi.org/10.1155/2019/1510257

Research Article

A Novel Power-Driven Grey Model with Whale Optimization Algorithm and Its Application in Forecasting the Residential Energy Consumption in China

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Received 23 June 2019; Revised 6 September 2019; Accepted 16 September 2019; Published 6 November 2019

Academic Editor: Mohammad Hassan Khooban

Along with the improvement of Chinese people's living standard, the proportion of residential energy consumption in total energy consumption is rapidly increasing in China year by year. Accurately forecasting the residential energy consumption is conducive to making energy programming and supply plan for the administrative departments or energy companies. By improving the grey action quantity of traditional grey model with an exponential time term, a novel power-driven grey model is proposed to forecast energy consumption as reference data for decision makers. The nonlinear parameter of power-driven grey action quantity is a crucial factor to influence the prediction precision. To promote the prediction accuracy of the power-driven grey model, whale optimization algorithm is adopted to seek for the optimal value of the nonlinear parameter. Two validations on real-world datasets are conducted, and the results indicate that the power-driven grey model has significant advantages on the aspect of prediction performance compared with the other seven classical grey prediction methods. Finally, the power-driven grey model is applied in forecasting the total residential energy and the thermal energy consumption of China.

1. Introduction

The residential energy, which includes the energy consumed by urban and rural residents and public facilities, accounts for a large percentage of total energy and continues to expand in China [1]. Meeting people's residential energy demand is always an important part of the energy supply in China. Therefore, it is significantly crucial to predict the energy consumption accurately for energy programming and supply plan of governments or energy companies. Numerous studies have been conducted to predict total energy consumption or other various energy consumptions, e.g., natural gas consumption [2], oil consumption [3], electricity consumption [4], nuclear energy consumption [5], wind energy and renewable energy consumption prediction [6], and so on. For obtaining better prediction result, lots of conventional statistical models and machine learning models were adopted to predict energy consumption, such as ridge regression [7], autoregressive integrated moving average model (ARIMA) [8], support vector regression (SVR) [9], and artificial neural network (ANN) [10]. Unfortunately, machine learning models often need enough training samples to construct models, while the aforementioned statistical models require more available and reliable historical data. There are still difficulties to solve the prediction problems with poor information or small samples. Therefore, the grey prediction method (GM) becomes one of the inevitable choices to handle these problems.

The grey prediction theory was initially put forward to study the prediction problem with inadequate information or small samples by Deng in 1980s [11]. Traditional GM(1,1) model has exhibited excellent ability for homogeneous exponential datasets in the aspect of prediction. However, it cannot always provide a satisfactory result for inhomogeneous exponential sequence. For tackling the challenge, many scholars have engaged in optimization of the traditional grey prediction model. Wang et al. extended the existing grey model by using the exponential preprocessing method and applied it to forecast Beijing's tertiary industry [12]. Xu et al. optimized the initial value of time response function to boost the stability of grey model and predict China's electricity consumption [13]. Wang et al. analyzed the general analytic solution of the grey model's whitening equation and presented an improved grey model by optimizing the initial condition which consisted of the first and last items of the accumulated generating sequence [14]. Meanwhile, optimization of the background value is another crucial aspect to boost the forecasting power of the classical grey prediction methods. Wang et al. utilized the finite integral of the accumulative generating sequence within the interval $[k-1,k]$ as background value to estimate the parameters of the grey model by the least-squares method [15]. Integrating the optimization of background value and the triangular whitening weight function, Ye et al. established a modified Grey-Markov model to handle the fluctuating sequences [16]. Zeng and Li modified the multivariate grey model based on dynamic background-value coefficient whose optimum value was sought out by PSO [17]. Chang et al. designed an adaptive grey prediction model to deal with the non-equigap sequence by optimizing background value coefficient [18]. The hybrid optimization is also an effective method to increase the grey model's prediction performance. Li et al. enhanced the accuracy and application fields of the classical grey model by improving the grey model with the joint optimization of the initial condition and background values [19]. Besides, some scholars made efforts to enhance the adaptation ability of the grey model. Zeng et al. designed a grey predictive framework with a series of various grey structures which can intelligently select the most suitable model to predict the electricity consumption [20]. With the sum of weighted first-order accumulative generating values as an initial condition, Ding designed a self-adapting grey model called NSGM(1,1) to enhance the adaptation ability for various original sequences. Also, the tunable weighted parameters of NSGM(1,1) are automatically sought out by using the ant lion optimizer [21]. Zeng et al. optimized the structure compatibility of a multivariable grey model with adding a random term, a linear term, and a dependent variable lag term [22]. Besides, the fractional-order accumulation is also significantly valid for increasing the prediction capacity of the grey model [23]. Ma et al. presented a fractional time delayed grey model to boost the precision and applicability of the traditional fractional grey model [24]. At the same time, Ma et al. proposed an unbiased fractional discrete grey model (FDGM) in which the order was intelligently sought out by using the grey wolf optimizer to deal with the multivariate time series [25]. By eliminating the inconsistency between its grey difference equation for modeling and discrete function for forecasting, Ma and Liu designed an improved GMC(1,n) model to promote the accuracy of the classical GM(1,n) with convolution integral [26]. The optimization of the accumulated operator is also an effective measure to enhance the ability of the grey prediction model. Ma et al. presented a new fractional accumulated operator and designed a comfortable grey prediction model which obtained better accuracy than the classical fractional grey model [27]. Meanwhile, the optimization of the grey action quantity is also an important method used to boost the prediction performance and applicability of the grey model. Shaikh et al. utilized the GVM(1, 1) [28] and NGBM(1, 1) [29] models to handle the prediction problem of China's natural gas demand with the characteristics of S-shaped data [30]. Li et al. proposed a full-order time power grey model to increase the structure adaptability of the grey model and adopted it to forecast the production of clean energy [31]. The grey action quantity affects the prediction performance and the applicability of the grey model. More details of the grey models with optimization of the grey action quantity is presented in Section 2. However, these improved grey models cannot solve all prediction problems. It is indispensable to continue improving the grey model and extending the application range of the grey model.

Therefore, a novel grey model is proposed to predict the residential energy consumption of China in this paper. There are two aspects of contribution as follows: (1) A novel power-driven grey model is proposed by optimizing the grey action quantity of traditional GM(1,1) model with an exponential term of time. The nonlinear parameter of the exponential term is determined by the whale optimization algorithm (WOA) to promote prediction accuracy. (2) The GM(1,1,$e^{\alpha t}$) model is used to predict China's total residential energy and thermal energy consumption, in which the prediction performance is significantly superior to the other seven contrast grey models.

The rest of the paper is organized as follows. Firstly, an overview of the traditional GM(1,1) model and its extension models is introduced in Section 2. Then, the power-driven grey model is proposed by substituting an exponential time grey action quantity for the constant grey input of traditional GM(1,1) in Section 3. In Section 4, a nonlinear programming problem with equality constraint is established to seek the optimum value of the nonlinear parameter by using the whale optimization algorithm. Meanwhile, the overall algorithm flowchart of GM(1,1,$e^{\alpha t}$) is presented. In Section 5, the validations of GM(1,1,$e^{\alpha t}$) is performed on two real-world datasets. Compared with the other seven existing grey prediction methods, the predicted results show that the proposed model has the most excellent prediction accuracy. In Section 6, the power-driven grey model is used to forecast China's total residential energy and residential thermal energy consumption. At last, the conclusions are drawn in Section 7.

2. GM(1,1) Model and Its Extension Models

The grey prediction method is one of the most popular and effective models to deal with time series prediction. It has been widely employed in many application areas and has obtained remarkable achievement in prediction problems with small samples. To boost the prediction accuracy of the classical grey prediction model, scholars have conducted numerous studies, such as improving the grey accumulation generation, optimizing grey background value, optimizing the initial condition, and so on. This section presents a survey of the classic grey prediction model and its extension models with optimizing grey action quantity.

Definition 1. Let $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$ be the raw sequence. The first-order accumulated sequence generated from the raw sequence is defined as

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right), \tag{1}$$

where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$. The background value sequence generated from consecutive neighbors of first-order accumulated sequence is defined as

$$Z^{(1)} = \left(z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)\right), \tag{2}$$

where $z^{(1)}(k) = 0.5\left(x^{(1)}(k) + x^{(1)}(k-1)\right)$.

Then, the definition of the classical grey model for dealing with univariate time series prediction is presented as follows. The equation

$$x^{(0)}(k) + az^{(1)}(k) = b, \tag{3}$$

is the definition equation of GM(1,1), in which a and b are development coefficient and grey input of the grey model.

The differential equation

$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + ax^{(1)}(t) = b, \tag{4}$$

is the whitening equation of GM(1,1).

In order to perform a prediction task, the key issue is to resolve the optimal value of linear parameters a and b. By using the least-squares method, the optimal parameters $\widehat{u} = [\widehat{a}, \widehat{b}]^T$ can be calculated as follows:

$$\widehat{u} = [\widehat{a}, \widehat{b}]^T = \left(A^T A\right)^{-1} A^T Y, \tag{5}$$

where

$$A = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, \qquad Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}. \tag{6}$$

Once the linear parameters of the grey model are estimated, the time response function of the grey model can be obtained by solving the whitening equation (4). By substituting the initial condition into the solution of the whitening equation (4), the discrete time response sequence of the grey model is formulated as

$$\widehat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{\widehat{b}}{\widehat{a}}\right)e^{-\widehat{a}(k-1)} + \frac{\widehat{b}}{\widehat{a}}. \tag{7}$$

However, the sequence is not the final predicted result of the grey model. By using inverse accumulated generating operator, the restored value of the grey model is calculated as

$$\widehat{x}^{(0)}(k) = \widehat{x}^{(1)}(k) - \widehat{x}^{(1)}(k-1). \tag{8}$$

Then, the sequence produced by the grey model is represented as

$$\widehat{x}^{(0)}(k) = \left(x^{(0)}(1) - \frac{\widehat{b}}{\widehat{a}}\right)\left(1 - e^{\widehat{a}}\right)e^{-\widehat{a}(k-1)}, \tag{9}$$

where $k = 2, 3, \ldots, n$, and $\widehat{x}^{(0)}(1) = x^{(0)}(1)$.

By analyzing the above stored sequence (9), it can be noticed that the GM(1,1) model has an ideal performance for the univariate time series with homogeneous exponential characteristics. Nevertheless, there are many time series data with nonhomogeneous exponential characteristics. Many improved grey models have been studied and designed to enhance the prediction accuracy for the nonhomogeneous exponential data sequence. One of the boosting strategies is to optimize the grey action quantity. A series of grey models with optimization of grey action quantity were proposed as follows.

By replacing the grey input b of the original GM(1,1) model with the term bk, the NGM model [32] can be obtained with the following formula:

$$x^{(0)}(k) + az^{(1)}(k) = bk. \tag{10}$$

By replacing the grey input b of the original GM(1,1) model with the term $bk + c$, the SAIGM model [20] can be obtained with the following formula:

$$x^{(0)}(k) + az^{(1)}(k) = bk + c. \tag{11}$$

By replacing the grey input b of the whitening equation of the original GM(1,1) model with the term $b(x^{(1)}(t))^n$, the whitening equation of the NGBM model [29] can be obtained as follow:

$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + ax^{(1)}(t) = b\left(x^{(1)}(t)\right)^n. \tag{12}$$

When $n = 2$, the NGBM model can be degenerated into the GVM(1,1) [28] model.

By replacing the grey input b of the original GM(1,1) model with the term $\sum_{i=1}^{h} b_i k^i$, the FOTP-GM(1,1,k) model [31] can be obtained with the following equation:

$$x^{(0)}(k) + az^{(1)}(k) = \sum_{i=1}^{h} b_i k^i, \quad h \geq 1. \tag{13}$$

By replacing the grey input b of the whitening equation of the original GM(1,1) model with the term $bt^{\alpha} + c$, GM(1,1,t^{α}) is obtained with the whitening equation:

$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + ax^{(1)}(t) = bt^{\alpha} + c. \tag{14}$$

By replacing the grey input b of the original GM(1,1) model with the term $\sum_{j=2}^{N} b_j x_j^{(1)}(k)$, the GM(1,$N$) model [33] is obtained with the following form:

$$x_1^{(0)}(k) + az_1^{(1)}(k) = \sum_{j=2}^{N} b_j x_j^{(1)}(k). \tag{15}$$

These research results show that the performance and accuracy of these grey models are significantly improved as well as the application range is also expanded into more fields by improving the grey action quality.

3. The New Proposed Power-Driven Grey Model

Obviously, the optimization of the grey action quantity is an effective means to increase the performance and applicability of the grey model from the previous section. This section proposes a novel power-driven grey model in which a natural exponential function of time is considered as the grey action quantity.

3.1. The Power-Driven Grey Model

Definition 2. Assume that $X^{(0)}, X^{(1)}, Z^{(1)}$ are defined as the same in Definition 1. The differential equation

$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + ax^{(1)}(t) = be^{\alpha t} + c, \tag{16}$$

is defined as the whitening equation of the power-driven grey model (GM(1,1,$e^{\alpha t}$)). The parameter a denotes the development coefficient. The term $be^{\alpha t} + c$ denotes the power-driven grey input in which the coefficient α is a tunable parameter.

By integrating the both sides of whitening equation (16) within $[k-1, k]$, the discrete formulation of the GM(1,1,$e^{\alpha t}$) model can be represented as follows.

Definition 3. The grey differential equation

$$x^{(0)}(k) + az^{(1)}(k) = b\left(\frac{e^{\alpha} - 1}{\alpha}\right)e^{\alpha(k-1)} + c, \tag{17}$$

is called the discrete form of the GM(1,1,$e^{\alpha t}$) model.

3.2. Parameter Estimation of the Power-Driven Grey Model.

For the traditional GM(1,1) model, the parameters a and b can be directly estimated by using the least-squares method because they are linear parameters. From Definition 2, it can be clearly noticed that the parameters a, b, and c of the power-driven grey model are linear parameters while the parameter α is a nonlinear parameter. It is difficult to estimate the nonlinear parameter by using the least-squares method directly. A two-stage strategy is adopted to gain the optimum parameters of the proposed model. In the first stage, the equality equation between the linear parameters and nonlinear parameter is obtained by using the least-squares method under the hypothetical condition that the nonlinear parameter α is given. Then, the optimal nonlinear parameter α is determined by solving an established nonlinear programming problem with equality constraint by using an intelligence algorithm (e.g., whale optimization algorithm [34]). In the second stage, the linear parameters are determined by the least-squares method after seeking out the optimum value of the nonlinear parameter. The process of determining the nonlinear parameter is presented in Section 4, while the linear parameters are estimated as follows.

Assuming the nonlinear parameter α is given, the parameters $\hat{u} = (\hat{a}, \hat{b}, \hat{c})^T$ of the power-driven grey model can be determined by employing the least-square method, and it satisfies

$$\hat{u} = (\hat{a}, \hat{b}, \hat{c})^T = \left(B^T B\right)^{-1} B^T Y, \tag{18}$$

where

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(\nu) \end{bmatrix}, \quad B = \begin{bmatrix} -z^{(1)}(2) & \beta e^{2\alpha} & 1 \\ -z^{(1)}(3) & \beta e^{3\alpha} & 1 \\ \vdots & \vdots & \vdots \\ -z^{(1)}(\nu) & \beta e^{\nu\alpha} & 1 \end{bmatrix}, \tag{19}$$

in which $\beta = \alpha^{-1}(e^{\alpha} - 1)$ and ν denotes the number of samples used for constructing model. The detailed proof process of the linear parameter estimation is omitted here because it is similar to the classical GM(1,1) model.

3.3. The Time Response Function and Restored Response Sequence.

After the parameters of the proposed grey model are determined, the time response and restored value sequence can be obtained as follows.

Theorem 1. *The time response function of the power-driven grey model is defined as*

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{\hat{b}}{\hat{a} + \alpha}e^{\alpha} - \frac{\hat{c}}{\hat{a}}\right)e^{-\hat{a}(k-1)} + \frac{\hat{b}}{\hat{a} + \alpha}e^{\alpha k} + \frac{\hat{c}}{\hat{a}}. \tag{20}$$

The restored response sequence of the power-driven grey model can be obtained by

$$\hat{x}^{(0)}(k) = \left(1 - e^{\hat{a}}\right)\left(x^{(0)}(1) - \frac{\hat{b}}{\hat{a} + \alpha}e^{\alpha} - \frac{\hat{c}}{\hat{a}}\right)e^{-\hat{a}(k-1)} + \frac{\hat{b}(1 - e^{\alpha})}{\hat{a} + \alpha}e^{\alpha k}. \tag{21}$$

Proof. Assume that $\mu(t)$ is an arbitrary function and satisfies

$$\mu'(t) = a\mu(t). \tag{22}$$

Multiply both sides of equation (16) by $\mu(t)$ and obtain

$$\mu(t)\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t}+\mu(t)ax^{(1)}(t)=\mu(t)\left(be^{\alpha t}+c\right). \tag{23}$$

Equation (22) is substituted into equation (23) to get the following formula:

$$\mu(t)\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t}+\mu'(t)x^{(1)}(t)=\mu(t)\left(be^{\alpha t}+c\right). \tag{24}$$

Rearrange equation (24) and obtain

$$\left(\mu(t)x^{(1)}(t)\right)'=\mu(t)\left(be^{\alpha t}+c\right). \tag{25}$$

Integrate both sides of equation (25) and obtain

$$x^{(1)}(t)=\frac{\int\mu(t)\left(be^{\alpha t}+c\right)\mathrm{d}t-C}{\mu(t)}. \tag{26}$$

Solving equation (22), the solution is obtained as

$$\mu(t)=ke^{at}, \tag{27}$$

where k is an arbitrary real number. Substitute equation (27) into equation (26) and obtain

$$x^{(1)}(t)=Ce^{-at}+\frac{b}{a+\alpha}e^{\alpha t}+\frac{c}{a}. \tag{28}$$

Substituting the initial condition and the estimated parameters calculated by (18) into equation (28), the time response function is obtained as

$$\widehat{x}^{(1)}(k)=\left(x^{(0)}(1)-\frac{\widehat{b}}{\widehat{a}+\alpha}e^{\alpha}-\frac{\widehat{c}}{\widehat{a}}\right)e^{-\widehat{a}(k-1)}+\frac{\widehat{b}}{\widehat{a}+\alpha}e^{\alpha k}+\frac{\widehat{c}}{\widehat{a}}. \tag{29}$$

By using inverse accumulation generating operator, the stored value can be calculated as follows:

$$\widehat{x}^{(0)}(k)=\widehat{x}^{(1)}(k)-\widehat{x}^{(1)}(k-1). \tag{30}$$

Then, the stored value is obtained as

$$\begin{aligned}\widehat{x}^{(0)}(k)=&\left(1-e^{\widehat{a}}\right)\left(x^{(0)}(1)-\frac{\widehat{b}}{\widehat{a}+\alpha}e^{\alpha}-\frac{\widehat{c}}{\widehat{a}}\right)e^{-\widehat{a}(k-1)}\\&+\frac{\widehat{b}\left(1-e^{\alpha}\right)}{\widehat{a}+\alpha}e^{\alpha k}.\end{aligned} \tag{31}$$

This completes the proof. □

According to Maclaurin's formula, the expansion of the term $e^{\alpha t}$ can be obtained as

$$e^{at}=1+\alpha t+\frac{(\alpha t)^2}{2!}+\frac{(\alpha t)^3}{3!}+\cdots+\frac{(\alpha t)^n}{n!}+R_n, \tag{32}$$

where R_n is known as the error term. If some higher-order term of equation (32) is ignored, the power-driven grey model can be degenerated into other existing grey models. If the higher-order terms other than first-order terms are ignored, the term $e^{at}=1+\alpha t$ can be obtained. Then, the GM(1,1,$e^{\alpha t}$) model can be degenerated into the grey SAIGM [20] with whitening equation (11). In a similar way, the GM(1,1,$e^{\alpha t}$) model can be degenerated to a kind of FOTP-GM(1,1,k) model [31] with special whitening equation (13) in which the parameter $b_i=b\alpha^i/i!$. When $b\alpha=0$, the GM(1,1,$e^{\alpha t}$) model can be degenerated into the traditional grey model with whitening equation (4).

4. Determining the Nonlinear Parameter of the Power-Driven Grey Model with Whale Optimization Algorithm

From the previous section, the linear parameters of the proposed model are determined by using the least-squares approach under the assumption that the nonlinear parameter is given. However, the nonlinear parameter cannot be directly calculated by the ordinary least-squares method because it is an exponential coefficient of grey action quantity. In fact, the nonlinear parameter α plays an indispensable role in promoting the prediction performance of the power-driven grey model. In this section, an intelligent nature-inspired optimization method called whale optimization algorithm is employed to seek for the optimal value of nonlinear parameter α.

4.1. Constructing the Optimization Problem for the Power-Driven Grey Model. Actually, an optimum value of nonlinear parameter α can make the power-driven grey model obtain the best prediction performance because the parameter not only directly affects the grey action quantity but also can control the development coefficient. Therefore, an optimization problem with constraint is built to obtain the optimum value of α, in which the objective function is to minimize the fit error of the power-driven grey prediction model. The equality constraints of the optimization problem are formulated in the previous modeling process. Mathematically, the optimization problem for seeking out optimal nonlinear parameter is formulated as follows:

$$\min_{\alpha}\frac{1}{n-v}\sum_{m=v+1}^{n}\left|\frac{x^{(0)}(m)-\widehat{x}^{(0)}(m)}{x^{(0)}(m)}\right|\times 100\%$$

$$\text{s.t.}\begin{cases}Y=\begin{bmatrix}x^{(0)}(2)\\x^{(0)}(3)\\\vdots\\x^{(0)}(v)\end{bmatrix},\\[2ex] B=\begin{bmatrix}-z^{(1)}(2)&\beta e^{2\alpha}&1\\-z^{(1)}(3)&\beta e^{3\alpha}&1\\\vdots&\vdots&\vdots\\-z^{(1)}(v)&\beta e^{v\alpha}&1\end{bmatrix},\\[2ex] \widehat{x}^{(1)}(m)=\left(x^{(0)}(1)-\frac{\widehat{b}}{\widehat{a}+\alpha}e^{\alpha}-\frac{\widehat{c}}{\widehat{a}}\right)e^{-\widehat{a}(m-1)}+\frac{\widehat{b}}{\widehat{a}+\alpha}e^{\alpha m}+\frac{\widehat{c}}{\widehat{a}},\\[2ex] \widehat{x}^{(0)}(m)=\widehat{x}^{(1)}(m)-\widehat{x}^{(1)}(m-1),\quad m=2,3,\ldots,v,\ldots,n,\\[1ex] \widehat{u}=\left(\widehat{a},\widehat{b},\widehat{c}\right)^T=\left(B^TB\right)^{-1}B^TY.\end{cases} \tag{33}$$

Input: The raw data $X^{(0)}$, lower and upper bound of α.
Output: The optimal value of the nonlinear parameter α.

Initialize the maximum number of iterations T and the number of humpback whales;
Initialize the locations $\vec{P}$ of the humpback population;
Compute the fitness of each humpback by equation (38);
Determine the best candidate $\vec{P}^*$ based on fitness of each whale agent;
for $k = 1; k < T; k = k + 1$ **do**
  for *each humpback whale* **do**
    Update the parameters r, p, l, β;
    if $\xi < 0.5$ **then**
      if $|C| < 1$ **then**
        Update the location of each humpback by equation (36);
      else
        Determine $\vec{P}_r$ by randomly choosing a whale;
        Update the location of each humpback by equation (37);
      end
    else
      Update the location of each humpback by equation (36);
    end
    Compute the fitness of each humpback by equation (38);
  end
  Update $\vec{P}^*$ if a better solution exists;
end
return the optimum value $\vec{P}^*$;

Algorithm 1: Algorithm of WOA to search for the nonlinear parameter α of the power-driven grey model.

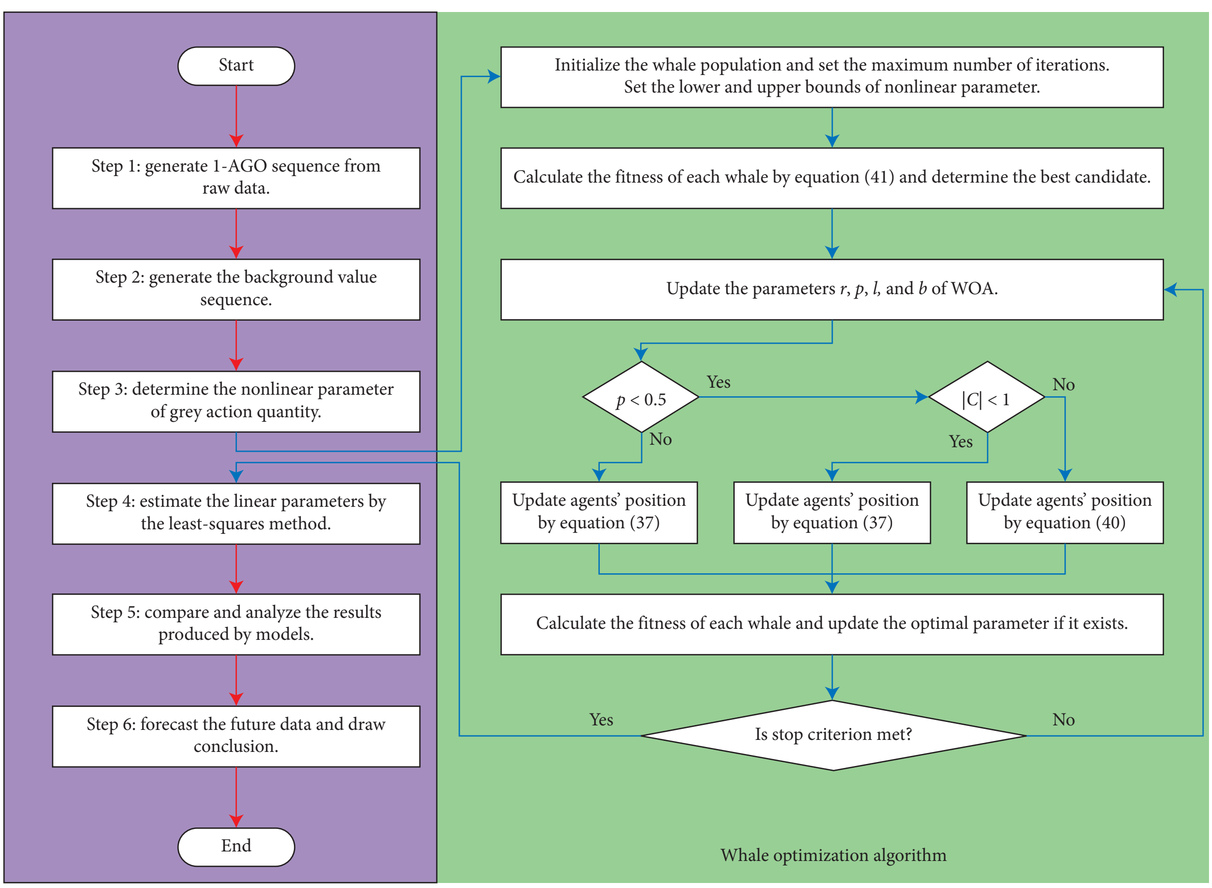

Figure 1: The flowchart of the power-driven grey model.

Table 1: The definition of the comparative grey models used.

No.	Name	Model	Reference
1	GM(1,1)	$(\mathrm{d}x^{(1)}(t)/\mathrm{d}t) + ax^{(1)}(t) = b$	[11]
2	ARGM	$x^{(0)}(k) = ax^{(0)}(k-1) + b$	[43]
3	DGM	$x^{(1)}(k) = ax^{(1)}(k-1) + b$	[44]
4	NGM	$x^{(0)}(k) + az^{(1)}(k) = bk$	[32]
5	NIGM	$\sum_{i=k}^{n} x^{(0)}(i) = a\sum_{i=k}^{n} x^{(0)}(i-1) + (n-k+1)b$	[43]
6	FGM(1,1)	$(\mathrm{d}x^{(r)}(t)/\mathrm{d}t) + ax^{(r)}(t) = b$	[45]
7	SAIGM	$x^{(0)}(k) + az^{(1)}(k) = bk + c$	[20]

Table 2: The definition of performance metrics used.

No.	Name	Formula	Reference
1	RMSE	$((1/v)\sum_{m=1}^{v}(x^{(0)}(m) - \widehat{x}^{(0)}(m))^2)^{0.5}$	[24]
2	MAE	$(1/v)\sum_{m=1}^{v}\lvert x^{(0)}(m) - \widehat{x}^{(0)}(m)\rvert$	[24]
3	NRMSE	$(1/\overline{x})\sqrt{(1/v)\sum_{m=1}^{v}(x^{(0)}(m) - \widehat{x}^{(0)}(m))^2} \times 100$	[24]
4	MAPE	$(1/v)\sum_{m=1}^{v}(\lvert x^{(0)}(m) - \widehat{x}^{(0)}(m)\rvert)(\lvert x^{(0)}(m)\rvert)^{-1} \times 100$	[24]
5	RMSPE	$((1/v)\sum_{m=1}^{v}((x^{(0)}(m) - \widehat{x}^{(0)}(m))(x^{(0)}(m))^{-1})^2)^{0.5} \times 100$	[24]
6	MSE	$1/v\sum_{m=1}^{v}(x^{(0)}(m) - \widehat{x}^{(0)}(m))^2$	[24]
7	IA	$1 - \sum_{m=1}^{v}(x^{(0)}(m) - \widehat{x}^{(0)}(m))^2(\sum_{m=1}^{v}(\lvert x^{(0)}(m) - \overline{x}\rvert + \lvert\widehat{x}^{(0)}(m) - \overline{x}\rvert)^2)^{-1}$	[5]
8	U1	$(\sum_{m=1}^{v}(x^{(0)}(m) - \widehat{x}^{(0)}(m))^2)^{0.5}((\sum_{m=1}^{v}x^{(0)}(m)^2)^{0.5} + (\sum_{m=1}^{v}\widehat{x}^{(0)}(m)^2)^{0.5})^{-1}$	[46]
9	U2	$(\sum_{m=1}^{v}(x^{(0)}(m) - \widehat{x}^{(0)}(m))^2)^{0.5}(\sum_{m=1}^{v}x^{(0)}(m)^2)^{-0.5}$	[46]

Table 3: Lewis' criterion for model evaluation.

MAPE (%)	Prediction performance
< 10	Excellent
10 – 20	Good
20 – 50	Reasonable
> 50	Incorrect

Table 4: The original data of China's natural gas consumption (NGC) (2005–2015).

Year	NGC
2005	467.63
2006	561.41
2007	705.23
2008	812.94
2009	895.2
2010	1069.41
2011	1305.3
2012	1463
2013	1705.37
2014	1868.94
2015	1931.75

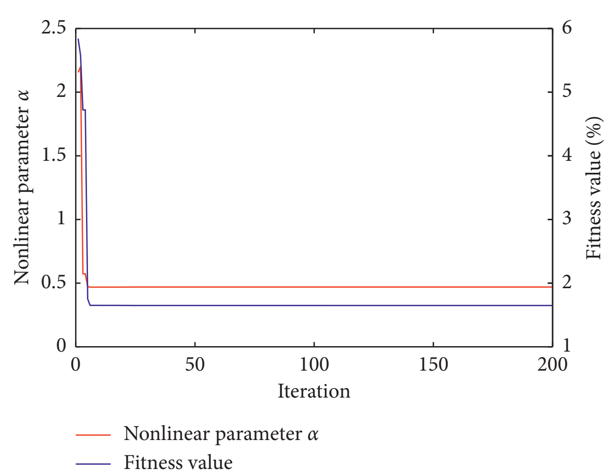

Figure 2: Convergence curve of WOA in Example A.

In this paper, a different strategy which is similar to nest cross validation in machine learning [35] is utilized to seek the optimal value of the coefficient α.. During the simulation stage, the samples are partitioned into two subsets. The first set, including the first v samples (from 1 to v samples), is used for establishing the equality constraint equation (19) between the linear parameters and the nonlinear parameter. The second set, including the last $n - v$ samples (from $v + 1$ to n samples), is utilized to compute the fitness value of the established optimization problem. The value of v satisfies $v = n - 2$ in Sections 5 and 6. The optimum value of nonlinear parameter α is sought out by solving the optimization problem equation (33). In the meanwhile, the linear parameters are also obtained when the optimal value α is substituted into equation (19). This strategy has been utilized to search for the optimum order of the fractional grey prediction model [25, 36].

4.2. Whale Optimization Algorithm. Motivated by the social behavior of humpback group, an intelligent nature-based optimization approach called whale optimization algorithm (WOA) was originated by Mirijalili and Lewis in 2016 [34]. In recent years, WOA has been widely employed to settle the optimization problems in many fields such as image retrieval [37], classification [38], bioinformatics [39], feature selection

Table 5: Fitted and predicted results of various grey models in Example A.

Year	Raw data	GM(1,1)	Error (%)	ARGM	Error (%)	DGM	Error (%)	NGM	Error (%)	NIGM	Error (%)	FGM(1,1)	Error (%)	SAIGM	Error (%)	GM(1,1,e^{at})	Error (%)
2005	467.63	467.63	0	467.63	0	467.63	0	467.63	0	469.7263546	0.448293434	467.63	$2.43E-14$	467.63	0	467.63	0
2006	561.41	581.0296492	3.494709602	570.8747479	$1.69E+00$	582.5467989	3.764948767	326.4241266	−41.85637473	566.9866424	0.993327942	561.41	$-1.44E-09$	578.1643099	2.984326951	585.984921	4.377357191
2007	705.23	678.504639	−3.789595023	685.1861578	−2.842170945	680.4246572	−3.517340834	541.2409827	−23.25326735	676.4109333	−4.086477709	675.7856868	−4.175136232	678.9353041	−3.728527697	692.1766157	−1.850940014
2008	812.94	792.3322774	−2.534962309	811.7504501	−0.146326902	794.7476753	−2.237843466	735.3614988	−9.542955343	799.5205363	−1.650732361	798.9734068	−1.718034933	795.5276016	−2.141904496	814.9110141	0.242455057
2009	895.2	925.2559256	3.357453712	951.8809942	6.331657078	928.2789222	3.695143228	910.7796456	1.740353617	938.0270257	4.784073466	935.7082618	4.525051583	930.4251873	3.934895813	955.0483743	6.685475232
2010	1069.41	1080.479178	1.035073373	1107.031938	3.518008797	1084.24571	1.387279884	1069.297286	−0.010539815	1093.856036	2.28593671	1089.675641	1.895030039	1086.502038	1.598268044	1112.224417	$4.00E+00$
2011	1305.3	1261.743072	−3.336928487	1278.813297	−2.029165926	1266.417594	−2.978809925	1212.542685	−7.106206627	1269.174034	−2.767637051	1264.347874	−3.137372698	1267.083353	−2.927805624	1283.782232	−1.648492178
2012	1463	1473.416252	0.711978939	1469.007663	0.410640007	1479.197481	1.107141546	1341.987232	−8.271549391	1466.418438	0.233659495	1463.349784	0.023908691	1476.016395	0.88970577	1463	$2.62E-08$
2013	1705.37	1720.600254	0.89307624	1679.588701	−1.511771576	1727.72804	1.311037472	1458.96056	−14.44903103	1688.331509	−0.999108165	1690.650448	−0.863129502	1717.752459	0.726086333	1636.180316	−4.057165562
2014	1868.94	2009.252464	7.507595978	1912.74163	2.343661633	2018.015997	7.976499897	1564.664195	−16.28066204	1937.998469	3.695060789	1950.695202	4.374415544	1997.4417	6.875646112	1777.904354	−4.870977476
2015	1931.75	2346.329692	21.4613533	2170.885899	12.37923639	2357.077313	22.01772034	1660.183907	−14.05803509	2218.8904	14.86426298	2248.518858	16.3980255	2321.042867	20.15234203	1843.333931	−4.576993327

Table 6: Evaluation result of different grey models in Example A.

Fitting	GM(1,1)	ARGM	DGM	NGM	NIGM	FGM(1,1)	SAIGM	GM(1,1,$e^{\alpha t}$)
RMSE	23.86274134	27.06135354	23.66839461	118.1206638	24.48123827	24.4715338	23.55366157	28.84466814
MAE	20.25637699	19.68692977	20.89112313	88.25197924	19.59112421	18.18583986	20.50145901	20.47248493
NRMSE	0.007664991	0.024109321	0.00130884	0.262191084	$3.09E-16$	0.009028807	$6.38E-05$	0.036768004
MAPE	2.28258768	2.120482298	2.336063456	11.47265586	2.156267271	1.934316772	2.275679299	2.351034331
RMSPE	2.668940276	2.90890819	2.674700195	17.69775476	2.656105278	2.605146741	2.617187635	3.280388261
MSE	569.4304242	732.3168555	560.1929035	13952.49123	599.3310274	598.8559665	554.7749735	832.01488
IA	0.998657193	0.998291523	0.998685903	0.969110255	0.998594898	0.998594002	0.998699542	0.998042174
U1	0.01235891	0.013937773	0.012237389	0.063448569	0.012659895	0.012673831	0.012179981	0.014834273
U2	0.024676152	0.027983796	0.024475181	0.122147051	0.025315732	0.025305697	0.024356537	0.029827898
Prediction	**GM(1,1)**	**ARGM**	**DGM**	**NGM**	**NIGM**	**FGM(1,1)**	**SAIGM**	**GM(1,1,$e^{\alpha t}$)**
RMSE	252.8476943	141.1491367	260.5294979	275.1057498	170.7912875	189.0706106	236.7945303	**83.450789**
MAE	190.0408036	102.9062759	198.9204499	274.0837795	124.4124534	137.7478703	176.7256754	**82.8804665**
NRMSE	0.179344392	0.08089408	0.187724249	0.258657024	0.106690266	0.120734047	0.166778703	**0.07821556**
MAPE	9.954008505	5.411556534	10.4350859	14.92924272	6.519477311	7.211856848	9.251358159	**4.50171212**
RMSPE	13.13711368	7.326292761	13.54158261	14.96064621	8.861865967	9.811145772	12.30065852	**4.5142693**
MSE	63931.95652	19923.07878	67875.61925	75683.17359	29169.6639	35747.69578	56071.64958	**6964.03418**
IA	0.593522143	0.786941885	0.580405823	0.462920793	0.731260314	0.699839537	0.616307649	**0.83617379**
U1	0.06517862	0.037446493	0.067003457	0.080883402	0.044965721	0.049568808	0.061267078	**0.02322958**
U2	0.137579314	0.076801972	0.141759132	0.149690352	0.092930838	0.102876971	0.128844478	**0.04540719**

[40], image processing [41], and so on. Meanwhile, it is also effective to solve the optimization problems like training multilayer perceptron neural network which involves a complex nonlinear optimization problem [42] and is more complicated than problem (33). This paper adopts the WOA algorithm to solve the nonlinear optimization problem (33). The main idea and model of WOA are mathematically described as follows.

The main idea of WOA is to imitate the predation behaviors of humpback group, for example, bubble-net feeding for catching fish. When the humpback whales catch fish, they usually encircle the fish school whose position is considered as the current best candidate target. Then, these whales update their positions based on the candidate target. Mathematically, the encircling behavior is represented as follows:

$$
\begin{aligned}
\vec{D} &= \left|2\vec{r}\cdot\vec{P}^{*}(i) - \vec{P}(i)\right|,\\
\vec{C} &= 2f(i)\cdot\vec{r} - f(i),\\
f(i) &= 2 - \frac{2i}{T},\\
\vec{P}(i+1) &= \vec{P}^{*}(i) - \vec{C}\cdot\vec{D},
\end{aligned}
\tag{34}
$$

where $\vec{P}(i)$ denotes the current position of the humpback whale, $\vec{P}^{*}(i)$ denotes the best current position of the humpback whale, the vector $\vec{r}$ is randomly generated in the interval $[0,1]$, and T denotes the maximum number of iterations. Furthermore, humpback whales move in spirals when they catch the prey. To simulate the helix-shaped movement, the spiral updating position is represented as follows:

$$
\vec{P}(i+1) = \left|\vec{P}^{*}(i) - \vec{P}(i)\right|\cdot e^{\beta l}\cdot\cos(2\pi l) + \vec{P}^{*}(i),
\tag{35}
$$

where the coefficient l is a stochastic number in the interval $[-1,1]$ and β is an arbitrary constant which determines the shape of the spiral movement. However, encircling and spiral moving behaviors happen simultaneously in the real world. For keeping it simple in this model, the entire predation movement of humpbacks is mathematically represented as follows:

$$
\vec{P}(i+1) = \begin{cases} \vec{P}^{*}(i) - \left(2f(i)\cdot\vec{r} - f(i)\right)\cdot\vec{D}, & \text{if } \xi < 0.5\,(a),\\ \left|\vec{P}^{*}(i) - \vec{P}(i)\right|\cdot e^{\beta l}\cdot\cos(2\pi l) + \vec{P}^{*}(i), & \text{if } \xi \geq 0.5\,(b), \end{cases}
\tag{36}
$$

where ξ is a probability to choose a movement strategy from encircling and spiral moving behaviors. When the norm of $\vec{C}$ is greater than 1, the position of all whales is updated based on the position of a whale randomly selected, not on optimal ones. Mathematically, the model can be formulated as follows:

$$
\vec{P}(i+1) = \vec{P}_r(i) - \vec{C}\cdot\left|2\vec{r}\cdot\vec{P}_r(i) - \vec{P}(i)\right|,
\tag{37}
$$

where $\vec{P}_r$ is the position of a whale randomly chosen from humpback group. Based on the principle of humpback's predation behavior, it is iterative to update the position of each whale until the stop criteria are met.

4.3. Implementation of WOA for Searching the Optimal Nonlinear Parameter. In the nonlinear programming problem (33), the main purpose is to find out the optimum value of the nonlinear parameter to obtain the highest performance of the proposed grey model. From section 4.2, it can be noticed that the original WOA is initially designed for unconstrained optimization and cannot directly solve the optimization problem with constraint. Therefore, the original WOA needs to be revised based on equation (33). Primarily, the fitness function needs to be established to calculate the

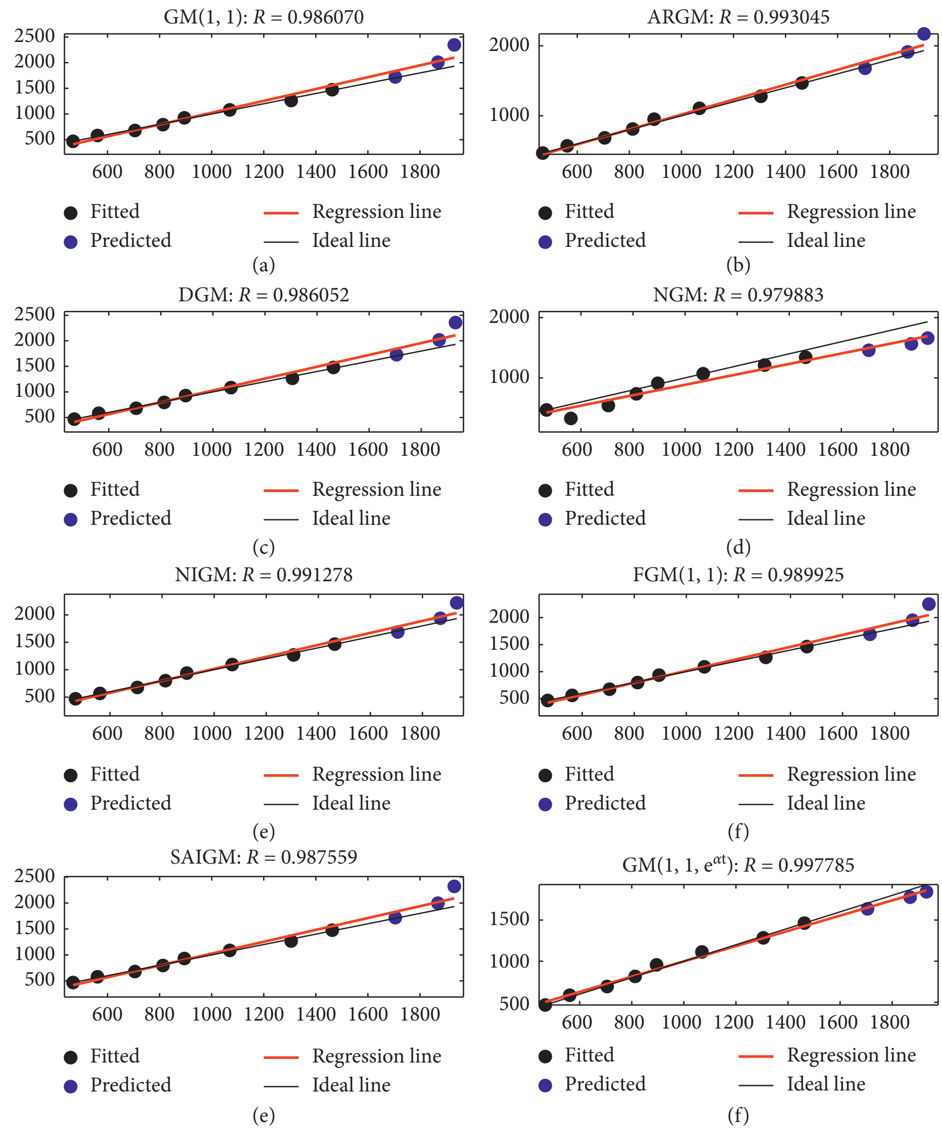

Figure 3: Comparison of the true value and the produced value by different grey models in Example A.

Table 7: The raw sequence of China's total energy consumption (TEC) (2008–2018).

Year	TEC
2008	320611
2009	336126
2010	360648
2011	387043
2012	402138
2013	416913
2014	425806
2015	429905
2016	435819
2017	448529.1
2018	464000

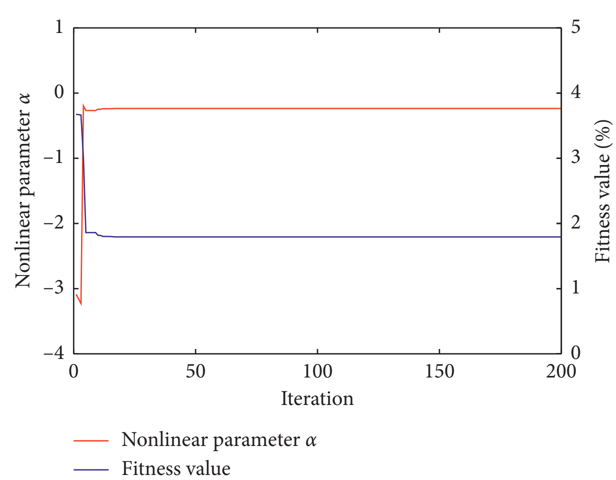

Figure 4: Convergence curve of WOA in Example B.

Table 8: Fitted and predicted results of various grey models in Example B.

Year	Real data	GM(1,1)	Error (%)	ARGM	Error (%)	DGM	Error (%)	NGM	Error (%)	NIGM	Error (%)	FGM(1,1)	Error (%)	SAIGM	Error (%)	GM(1,1,e^{at})	Error (%)
2008	320611	320611	0	320611	0	320611	0	320611	0	311679.0155	−2.785925775	320611	0	320611	0	320611	0
2009	336126	349189.7029	3.886549367	344430.113	$2.47E+00$	349311.5014	3.922785319	195251.0697	−41.91134582	343554.0921	2.209912975	336126	$-6.09E-09$	334656.6702	−0.437136603	302216.2049	−10.08841777
2010	360648	363180.4997	0.70220817	364871.1079	1.17097776	363285.296	0.731265934	324553.7721	−10.00815973	368260.8022	2.110867706	362542.7915	0.525385288	363846.1266	0.886772319	336515.6299	−6.691391627
2011	387043	377731.8583	−2.405712472	382413.0823	−1.196228253	377818.0957	−2.383431369	378605.0893	−2.18009644	387411.2353	0.095140671	384858.2814	−0.564464039	385905.0726	−0.294005419	363964.4755	−5.962780493
2012	402138	392866.2383	−2.305616897	397467.1865	−1.161495191	392932.263	−2.289198485	401199.7052	−0.233326558	402254.9392	0.029079382	402143.1792	0.001287911	402575.3761	0.108762691	386004.0797	−4.012035752
2013	416913	408606.9995	−1.992262303	410386.2597	−1.565492142	408651.0547	−1.981695294	410644.7418	−1.503493099	413760.4517	−0.756164554	414991.5431	−0.46087718	415173.395	−0.417258508	403772.8105	$-3.15E+00$
2014	425806	424978.4373	−0.194352063	421473.1004	−1.017575986	424998.6581	−0.189603218	414592.9707	−2.633365728	422678.4964	−0.734490251	424176.7152	−0.382635476	424693.9224	−0.261170013	418169.7514	−1.793363307
2015	429905	442005.8207	2.814766209	430987.5627	0.251814401	442000.2281	2.813465333	416243.4155	−3.177814742	429590.9676	−0.073046936	430405.4018	0.116398218	431888.7395	0.461436707	429905.0001	$1.97E-08$
2016	435819	459715.4311	5.483109076	439152.6474	0.764915571	459681.9259	5.475421199	416933.337	−4.333373018	434948.8973	−0.199647723	434274.3904	−0.354415389	437325.9795	0.345781049	439539.3936	0.853655668
2017	448529.1	478134.6031	6.600575768	446159.7284	−0.528253708	478070.9591	6.58638629	417221.7391	−6.980006624	439101.8855	−2.101806668	436274.3346	−2.732211883	441434.99	−1.581638728	447515.8839	−0.225897516
2018	464000	497291.7661	7.174949597	452173.0382	−2.548914172	497195.624	7.154229303	417342.2974	−10.05553935	442320.9107	−4.672217527	436804.2529	−5.861152385	444540.236	−4.193914654	454184.3177	−2.115448779

Table 9: Evaluation result of different grey models in Example B.

Fitting	GM(1,1)	ARGM	DGM	NGM	NIGM	FGM(1,1)	SAIGM	GM(1,1,$e^{\alpha t}$)
RMSE	8410.121727	4906.499126	8410.286218	51928.17151	5158.480682	1367.489689	1660.996872	18561.19627
MAE	6926.686251	4221.269336	6927.244243	27186.02944	3881.517201	1016.979092	1384.772743	14753.88106
NRMSE	1.79E−05	0.006017121	0.000384048	0.199776443	5.88E−16	0.003063485	1.47E−04	0.108418844
MAPE	1.787683435	1.104264936	1.788930619	7.705950265	1.099328531	0.256381015	0.358317783	3.9624714
RMSPE	2.197483638	1.313602202	2.200080172	15.3330731	1.509443367	0.34767392	0.437458618	5.139941157
MSE	70730147.47	24073733.67	70732914.28	2696534997	26609922.95	1870028.049	2758910.609	344518006.9
IA	0.987856201	0.995717785	0.987850143	0.775004734	0.995552338	0.999687628	0.99954467	0.95091572
U1	0.010871273	0.00635032	0.010870745	0.069089208	0.006667366	0.001768468	0.002146699	0.024435002
U2	0.021739295	0.012682793	0.02173972	0.134228952	0.013334139	0.003534819	0.004293505	0.047978772
Prediction	GM(1,1)	ARGM	DGM	NGM	NIGM	FGM(1,1)	SAIGM	GM(1,1,$e^{\alpha t}$)
RMSE	29188.27919	7225.052506	29121.06984	34223.58019	13657.87156	17245.03553	11989.99302	**6088.66592**
MAE	28931.23345	5843.326918	28866.80301	32283.57548	10658.80219	13665.04068	9353.617817	**4849.76408**
NRMSE	0.111492795	0.013953907	0.111244499	0.124411774	0.041076011	0.052661204	0.03217455	**0.00913139**
MAPE	6.419544814	1.280694484	6.405345597	7.122972999	2.324557306	2.982593219	2.04044481	**1.06500067**
RMSPE	6.457863097	1.566430259	6.443180338	7.496943508	2.960127649	3.739148714	2.595513162	**1.32349051**
MSE	851955642.2	52201383.72	848036708.6	1171253441	186537455.5	297391250.5	143759932.6	**37071852.7**
IA	0.521410498	0.823769072	0.52205901	0.349931321	0.596904989	0.495386509	0.615535006	**0.88095424**
U1	0.031445311	0.008068562	0.031375108	0.039484363	0.015373596	0.019477492	0.013461203	**0.00678995**
U2	0.064920974	0.016070062	0.064771486	0.07612056	0.030378027	0.038356646	0.026668308	**0.0135425**

fitness of each whale agent. According to equation (33), the fitness function can be represented as follows:

$$\text{fitness} = \frac{1}{n-v}\sum_{m=v+1}^{n}\left|\frac{x^{(0)}(m)-\hat{x}^{(0)}(m)}{x^{(0)}(md)}\right| \times 100\%. \tag{38}$$

The revised WOA is presented in detail in Algorithm 1.

4.4. Modeling Procedure of the Power-Driven Grey Model. Based on the modeling process of the power-driven grey model and WOA for seeking out the optimal nonlinear parameter, the overall computational steps of GM(1,1,$e^{\alpha t}$) with WOA is depicted in the flowchart shown in Figure 1. In the proposed model, the critical issue is to search optimal nonlinear parameter α and estimate the linear parameters to construct the model for achieving a better prediction performance. Firstly, the equality equation of the optimization problem is formulated under the assumption that the nonlinear parameter is given. Then, the optimal nonlinear parameter is sought out through solving the optimization problem by nature-inspired optimization algorithm WOA, and the estimated linear parameters are calculated by the least-squares method. Finally, the power-driven grey model with optimal parameters is used to forecast the future value in case study.

5. Validation of the Power-Driven Grey Model

In this section, the validations are performed to examine the forecasting superiority of the proposed GM(1,1,$e^{\alpha t}$) model through two real-world examples.

5.1. Contrast Grey Models and Performance Criteria. To illustrate the advantages of the power-driven grey model with WOA in the aspect of prediction performance, the numerical validation study is conducted on two real-world data sequences. A series of classic existing grey models listed in Table 1 are used to compare with the power-driven grey model. The power-driven grey model and the contrastive grey models are all realized by MATLAB. Then, all validation experiments and case studies are performed on the MATLAB platform 2019a.

Nine evaluation criteria tabulated in Table 2 are adopted to evaluate the prediction ability of the aforementioned grey prediction models. Meanwhile, Lewis' criteria [47]shown in Table 3 are also adopted to illustrate the prediction power of grey models.

For seeking out the optimal nonlinear parameter of the proposed model, the necessary parameters of WOA are set to the same values in validation experiments and applications as follows. The population size of the humpbacks is set to 30. Maximum iteration is set to 200. The minimum value and maximum value of the nonlinear parameter are set to −10 and 10, respectively.

5.2. Example A: Predicting the Natural Gas Consumption of China. In this section, the validation experiment is to study and analyze China's natural gas consumption (NGC). The original sequence of the natural gas consumption during 2005–2015 is listed in Table 4, which was collected from China's National Bureau of Statistics. To construct a power-driven grey model and validate its superiority, the dataset is partitioned into two subdatasets, including training set and test set. The test set, including the natural gas consumption in the last 3 years, is employed to check the prediction performance of grey models. The training set, including the natural gas consumption from 2005 to 2012, is utilized to build models of the eight grey models.

For achieving better prediction accuracy, the WOA method is used to seek the optimum value of the proposed model's nonlinear parameter. Figure 2 shows the

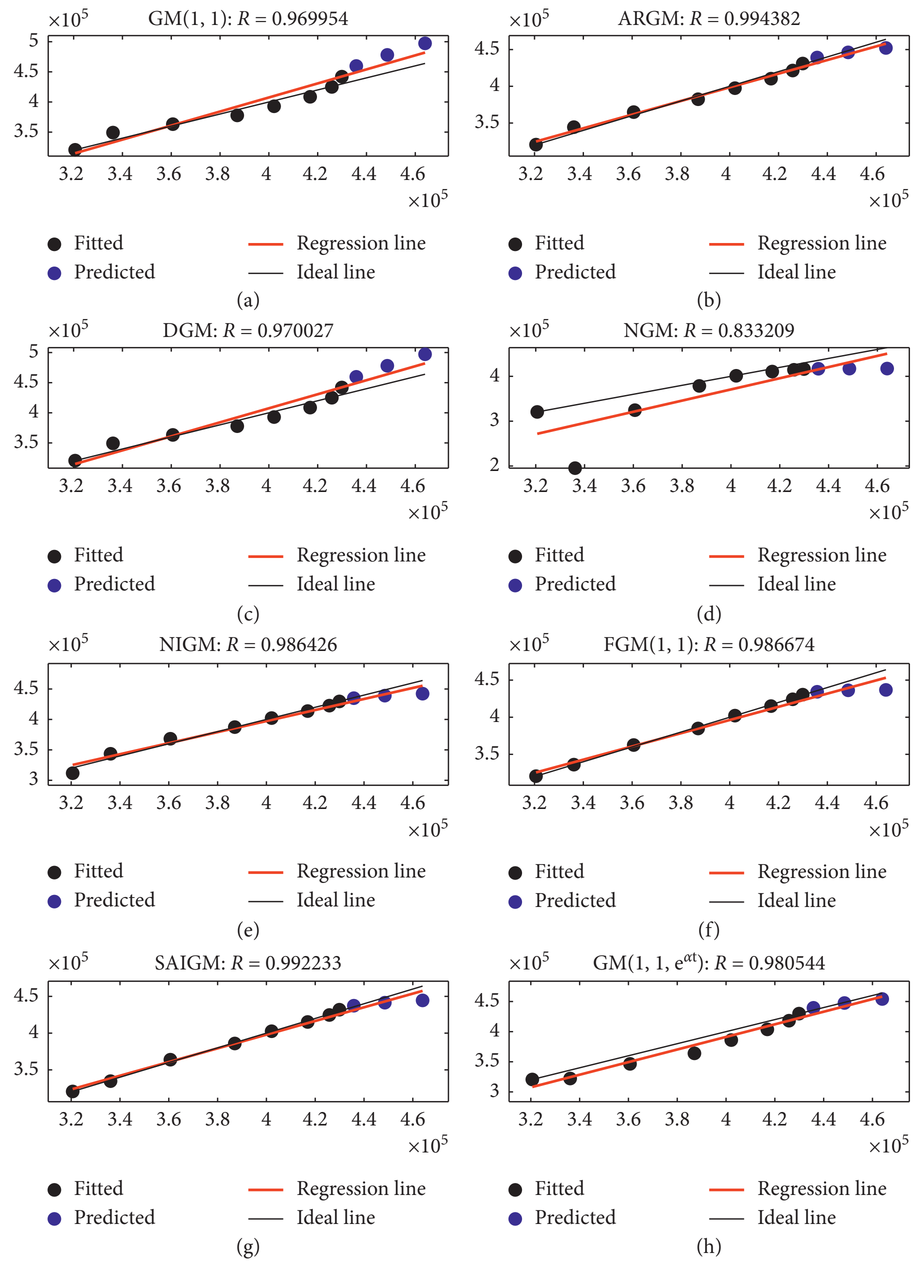

FIGURE 5: Comparison of the true value and the produced value by different grey models in Example B.

TABLE 10: Raw sequence of China's total residential energy consumption (TREC) (2005–2015).

Year	TREC
2005	27573
2006	27765
2007	30814
2008	31898
2009	33843
2010	36470
2011	39584
2012	42306
2013	45531
2014	47212
2015	50099

convergence curve. It can be noticed that the fitness function converges to a constant after dozens of iterations. Then, the linear parameters a, b, and c and nonlinear parameter α are obtained and are equal to −0.175843, −4.869699, 463.613598, and 0.469429, respectively. The order of FGM(1,1) is equal to 0.862399 obtained by the whale optimization algorithm. All established models are used to predict the consumption from 2013 to 2015. For each grey model, the produced results and their absolute percentage error (APE) [12] are tabulated in Table 5. The evaluation values of the eight models are given in Table 6. Based on Lewis' criteria, GM(1,1,$e^{\alpha t}$), GM(1,1), ARGM, SAIGM, NIGM, and FGM models exhibit excellent prediction performance while the other grey models only show good prediction performance.

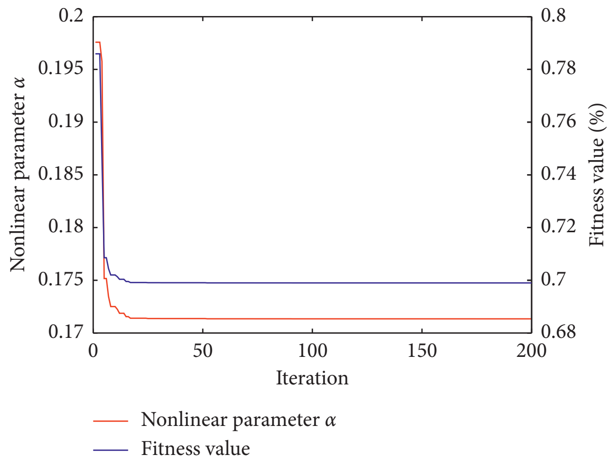

Figure 6: Convergence curve of WOA in Case 1.

Furthermore, the GM(1,1,$e^{\alpha t}$) model obtains the lowest MAPE of prediction. From Figure 3, it can be noticed that the results produced by GM(1,1,$e^{\alpha t}$) are more approximate to real values than those of the other seven contrast models. Overall, the proposed model can more accurately predict natural gas consumption, though its fitted performance is not better than the other comparative models.

5.3. Example B: Predicting the Total Energy Consumption of China. In this example, China's total energy consumption from 2008 to 2018 is employed to examine the accuracy of the grey prediction models. The raw sequence is listed in Table 7, which was collected from China's National Bureau of Statistics. To construct models and validate the prediction performance, the raw dataset is broken into two groups, including training set and test set. The training set, including the consumption from 2008 to 2015, is used to build models of the proposed model and other contrast grey models. The test set containing the rest digits is utilized to test the prediction accuracy of all models.

During the stage of constructing the new proposed model, the WOA optimizer is utilized to seek the optimum value of power-driven grey model's nonlinear parameter. From Figure 4, which shows convergence curve of WOA, it can be noticed that the fitness function converges to a constant rapidly after dozens of iterations. Then, the linear parameters a, b, and c and nonlinear parameter α are obtained and are equal to −0.003666, −224002.105686, 458089.69852, and −0.235980, respectively. The order of FGM(1,1) is equal to 0.209972 obtained by the whale optimization algorithm. All established models are employed to forecast the energy consumption from 2016 to 2018 in China. For each grey model, the raw data, fitted data, predicted data, and their absolute percentage error (APE) are tabulated in Table 8. The evaluation results of the eight models are given in Table 9. Based on Lewis' criteria, the GM(1,1,$e^{\alpha t}$) model shows an excellent forecasting ability. Although other grey models also exhibit excellent prediction performances, the proposed model has the lowest value of MAPE. It can be noticed that the forecasted values of the GM(1,1,$e^{\alpha t}$) model are more approximate to the actual values than those of the other seven comparative grey models in Figure 5. Overall, the power-driven grey model has the highest prediction performance of the total energy consumption though it is not the best one for fitted performance compared with the other contrast grey models.

6. Applications

Residential energy consumption refers to the energy consumption of urban residents, rural residents, and public facilities. The residential energy consumption has been the second largest part of total energy consumption in China [48]. Effective forecasting of the residential energy consumption plays an indispensable role in programming and planning of energy for governments and companies. So, the research studies of forecasting the total residential energy and the residential thermal energy consumption are, respectively, conducted in this section.

6.1. Case 1: Predicting the Total Residential Energy Consumption of China. Original data on total residential energy consumption (2005–2015) were collected from China's National Bureau of Statistics and are tabulated in Table 10. The original data are partitioned into two groups, including training set and test set. The training set, including annual total residential energy consumption between 2005 and 2012, is employed to establish models of the proposed and the other seven contrast grey models. The test set, including the total residential energy consumption in the last three years, is utilized to verify the prediction performance of these grey models.

To get a better prediction performance of the power-driven grey model, it is essential to seek out the optimal parameters of the model. WOA algorithm is employed to seek for the optimal parameter of the proposed model. Furthermore, the linear parameter is estimated by using the least-squares method once the nonlinear parameter is

Table 11: Fitted and predicted results of various grey models in Case 1.

Year	Real data	GM(1,1)	Error (%)	ARGM	Error (%)	DGM	Error (%)	NGM	Error (%)	NIGM	Error (%)	FGM(1,1)	Error (%)	SAIGM	Error (%)	GM(1,1,e^{at})	Error (%)
2005	27573	27573	0	27573	0	27573	0	27573	0	27058.56265	−1.865728606	27573	0	27573	0	27573	0
2006	27765	27978.32853	0.768336125	29005.12306	$4.47E+00$	27991.46972	0.815666186	15193.77532	−45.27723636	28538.6003	2.786242738	28291.15519	1.895030383	28213.87712	1.616701298	28590.2434	2.972243475
2007	30814	29952.57086	−2.795577125	30630.24572	−0.596333737	29966.86358	−2.749193289	26179.55591	−15.04006001	30206.15237	−1.972634606	29924.47954	−2.886741277	29999.16318	−2.644372115	30669.18209	−0.4699744
2008	31898	32066.1222	0.527061881	32474.3775	1.806939309	32081.66352	0.575783803	32092.74897	0.610536628	32084.97616	0.5861689	31901.97945	0.012475549	31981.14675	0.260664469	32845.45897	2.970277048
2009	33843	34328.81263	1.435489247	34567.03309	2.139388033	34345.7076	1.485410853	35275.57717	4.233008816	34201.83887	1.060304543	34142.0597	0.883667805	34181.49939	1.000205036	35111.28887	3.747566316
2010	36470	36751.16589	0.770951158	36941.70473	1.293404786	36769.52816	0.821300144	36988.76246	1.422436136	36586.89902	0.320534735	36622.63513	0.418522422	36624.28035	0.423033577	37455.11423	$2.70E+00$
2011	39584	39344.44832	−0.605173011	39636.3982	0.132372173	39364.40085	−0.554767454	37910.89951	−4.22670899	39274.13608	−0.782800938	39341.62499	−0.612305507	39336.19965	−0.626011396	39860.69337	0.699003063
2012	42306	42120.72123	−0.437949155	42694.24116	0.917697623	42142.39703	−0.386713401	38407.2479	−9.215600867	42301.83456	−0.009845972	42306	$1.14E-08$	42346.91016	0.096700621	42305.99989	$-2.61E-07$
2013	45531	45092.89704	−0.962208079	46164.17331	1.390642227	45116.43995	−0.910500641	38674.41178	−15.05916456	45713.12948	0.400012041	45527.84857	−0.006921508	45689.33182	0.347745099	44761.89622	−1.689187102
2014	47212	48274.79929	2.251121097	50101.72973	6.120752634	48300.3649	2.305271744	38818.21509	−17.77892253	49556.62086	4.966154493	49022.73478	3.835327421	49400.01157	4.634439477	47190.53749	−0.045459867
2015	50099	51681.22697	3.158200705	54569.92965	8.924189408	51708.9835	3.213604062	38895.61849	−22.36248529	53887.06613	7.56116117	52808.95862	5.409207009	53519.52298	6.827527459	49543.45316	−1.108898065

Table 12: Evaluation result of different grey models for fitting and forecasting China's total residential energy consumption in Case 1.

Fitting	GM(1,1)	ARGM	DGM	NGM	NIGM	FGM(1,1)	SAIGM	GM(1,1,$e^{\alpha t}$)
RMSE	390.9308897	592.0396847	390.5257193	4998.407423	433.8786898	393.6339653	366.3717398	729.3183711
MAE	304.3361034	454.5790022	305.3384416	3115.451246	359.0785841	264.2156166	266.0438679	555.9520645
NRMSE	0.001442512	0.034214153	0.000188067	0.215925463	$3.43E-16$	0.001570568	$3.22E-05$	0.04351679
MAPE	0.917567213	1.41907916	0.923604391	10.00319848	1.17303263	0.838592869	0.833461064	1.695028338
RMSPE	1.219447364	1.958309635	1.219282349	17.31812039	1.471344436	1.287221758	1.186116654	2.227857299
MSE	152826.9605	350510.9882	152510.3374	24984076.77	188250.7174	154947.6986	134228.2517	531905.2864
IA	0.998469768	0.996464675	0.998474462	0.832432082	0.998113113	0.99844636	0.998656649	0.994674042
U1	0.005725206	0.008618064	0.005717969	0.07547321	0.006352639	0.005765005	0.005364149	0.010598626
U2	0.011447175	0.01733601	0.011435311	0.146362557	0.012704766	0.011526326	0.010728038	0.021355782
Prediction	**GM(1,1)**	**ARGM**	**DGM**	**NGM**	**NIGM**	**FGM(1,1)**	**SAIGM**	**GM(1,1,$e^{\alpha t}$)**
RMSE	1129.146983	3095.192957	1147.235089	8999.707111	2574.222417	1881.723156	2346.09186	**547.909444**
MAE	1027.709741	2664.610899	1037.636147	8817.91821	2104.938825	1507.948278	1922.28879	**448.704651**
NRMSE	0.02676036	0.096930345	0.027692398	0.320768731	0.076571197	0.054778092	0.069926951	**0.0163225**
MAPE	2.123843293	5.478528089	2.143125482	18.4001908	4.309109234	3.083818646	3.936570679	**0.94784892**
RMSPE	2.307063232	6.299170718	2.343113335	18.64536826	5.227933254	3.828372361	4.768440944	**1.16691584**
MSE	1274972.909	9580219.441	1316148.349	80994728.08	6626621.053	3540882.034	5504147.015	**300204.759**
IA	0.941388626	0.738632316	0.939683635	0.267264603	0.797293581	0.866620632	0.819041097	**0.9797724**
U1	0.011752671	0.031568432	0.011937749	0.104105989	0.026407079	0.019427025	0.024114228	**0.00577616**
U2	0.02369601	0.064954984	0.024075603	0.188865715	0.05402202	0.039489395	0.049234526	**0.0114983**

determined. The convergence curve of WOA is shown in Figure 6. Obviously, it can be noticed that the fitness function converges to a constant after dozens of iterations from the convergence curve. According to the computational steps of the power-driven grey model, the linear parameters a, b, and c and nonlinear parameter α are equal to −0.085448, −1868.619677, 27446.346885, and 0.171344, respectively. The order of FGM(1,1) obtained by the WOA optimizer is equal to 1.047609. By using these established grey models, the fitted and predicted results are produced and are given in Table 11. The evaluation results of the 8 grey models are tabulated in Table 12. Obviously, the prediction performance of the power-driven grey method is much better than those of other seven contrast grey prediction methods; though the fit ability is not the best, it is not the worst. The results produced by each model are also shown in Figure 7 in which the horizontal axis denotes the raw values and the vertical axis denotes the simulated or predicted values. It can be found that the values produced by the GM(1,1,$e^{\alpha t}$) model are almost equal to the real consumption. Meanwhile, the correlation coefficient between the raw values and the produced values of the proposed model is also the highest among the eight prediction methods. Above all, the performance of the power-driven grey model is the best for forecasting the total residential consumption compared with the other seven grey prediction methods.

6.2. Case 2: Predicting the Residential Thermal Energy Consumption. In this case study, the main aim is to forecast China's residential thermal consumption, which is considered as a raw sequence. The raw data listed in Table 13 were collected from China's National Bureau of Statistics from 2005 to 2016. The first seven digits are used to build models of the power-driven grey model and the other seven comparative grey models, respectively. The rest of the digits of raw data are utilized to validate the predicted values.

The results produced by these established grey prediction methods are tabulated in Table 14. The comparisons of various grey prediction methods are plotted in Figure 8, in which the horizontal axis denotes the original value and the vertical axis represents the generated values. From the regression lines in Figure 8, it can be found that the simulated and forecasted consumptions of the power-driven grey prediction method are very close to the actual consumption while the other seven grey prediction methods are worse than the proposed method. Meanwhile, the correlation coefficient between the raw consumptions and the generated consumptions of the proposed model is the highest among the eight grey prediction methods. The fitted and predicted metrics of these models are listed in Table 15. It can be clearly noticed that the prediction accuracy of the proposed prediction model is the best; though the fitting accuracy is not the best, it is not the worst. The convergence curve of WOA during the stage of building power-driven grey model is plotted in Figure 9. The fitness value rapidly stabilizes to a constant after a few iterations. It shows that the WOA method is effective to search for the optimum value of nonlinear parameter. According to the calculation procedures of the power-driven grey model, the linear parameters a, b, and c and nonlinear parameter α are equal to −0.015782, 35273.868015, 13840.161440, and 0.057975, respectively, in this case. The order of FGM(1,1), which is equal to −0.071090, is also calculated by the WOA optimizer. To sum up, the power-driven grey prediction method has better

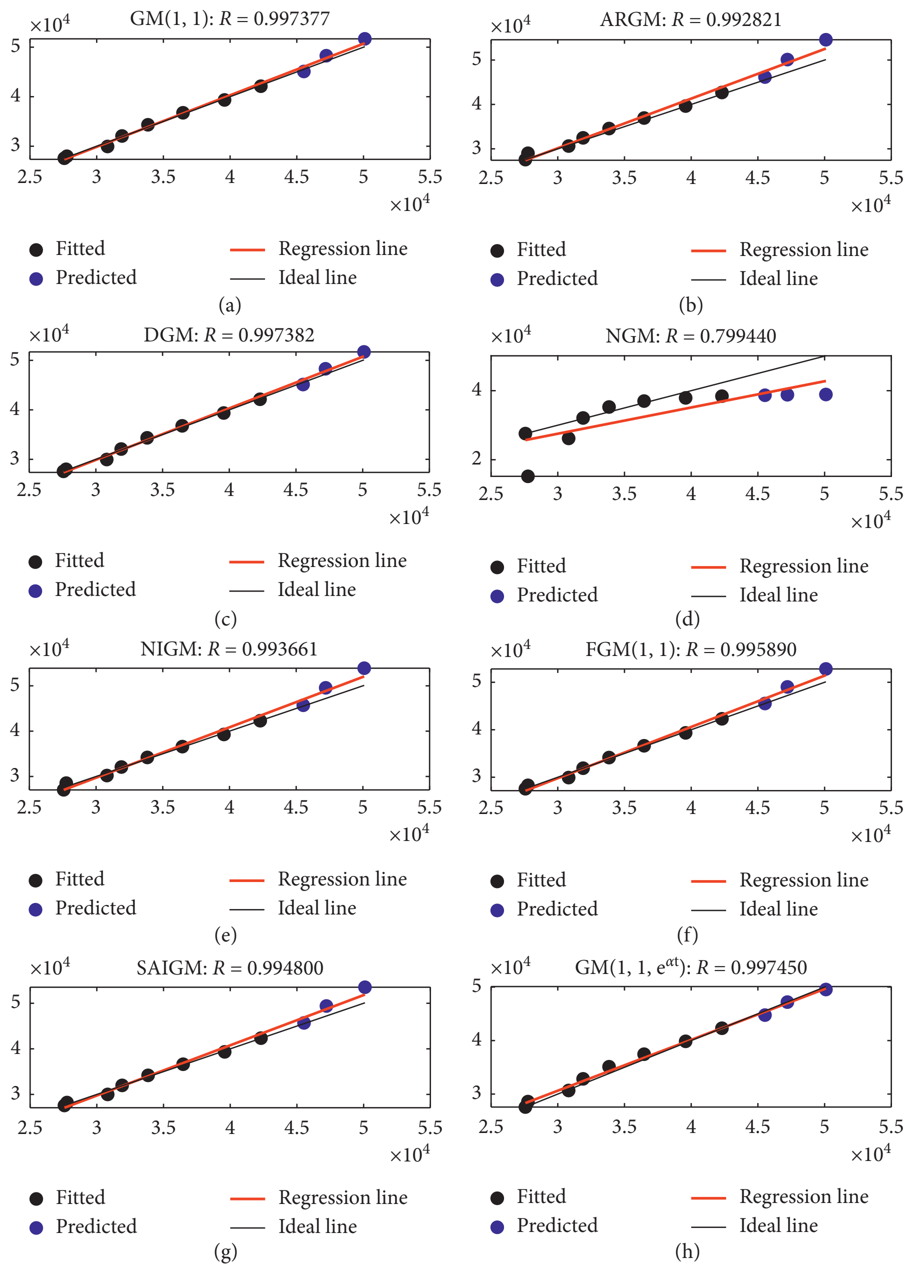

Figure 7: Comparison of the true value and the produced value by different grey models in Case 1.

Table 13: Raw sequence of China's residential thermal energy consumption (RThEC) (2005–2016).

Year	RThEC
2005	52044
2006	56948
2007	57689
2008	62765
2009	67000
2010	67410
2011	70044
2012	77608
2013	81472
2014	86482
2015	93841
2016	98623

accuracy for forecasting the residential thermal energy consumption than the other contrast grey prediction models.

7. Conclusions

A novel power-driven grey prediction method called GM(1,1,$e^{\alpha t}$) is proposed to forecast the total residential energy consumption and the residential thermal energy consumption of China in this paper. The grey input of GM(1,1,$e^{\alpha t}$) is an exponential term which is different from the grey action quantity of the traditional grey model. It plays an imperative role in raising the prediction performance of GM(1,1,$e^{\alpha t}$). The optimal value of nonlinear parameter α is sought out by using

Table 14: Fitted and predicted results of various grey models in Case 2.

Year	Real data	GM(1,1)	Error (%)	ARGM	Error (%)	DGM	Error (%)	NGM	Error (%)	NIGM	Error (%)	FGM(1,1)	Error (%)	SAIGM	Error (%)	GM(1,1,e^{at})	Error (%)
2005	52044	52044	0	52044	0	52044	0	52044	0	51888.19898	−0.299364038	52044	0	52044	0	52044	0
2006	56948	56831.2207	−0.205063046	56445.26181	$-8.83E-01$	56850.51634	−0.171180124	32790.06199	−42.42104728	56047.04046	−1.582074074	54662.18926	−4.013856046	56093.70088	−1.500138938	53564.22871	−5.941861514
2007	57689	59393.10731	2.953955369	60129.28218	4.230064973	59409.97388	2.983192421	53714.75243	−6.88909077	59664.98819	3.425242585	59743.78058	3.561823886	59515.12026	3.165456597	56731.39771	−1.659939135
2008	62765	62070.48086	−1.106538906	63212.9447	0.713685488	62084.66032	−1.083947558	61922.32276	−1.342590999	62812.39016	0.075504119	63668.89356	1.440123579	62613.27866	−0.241729217	60087.13419	−4.266495356
2009	67000	64868.54734	−3.18127262	65794.08529	−1.799872708	64879.76336	−3.164532294	65141.68751	−2.773600732	65550.44495	−2.163515	66447.8307	−0.82413329	65418.71828	−2.360121975	63642.68594	−5.01091651
2010	67410	67792.74748	0.567790358	67954.59627	0.807886474	67800.70427	0.579593939	66404.46187	−1.491675015	67932.39175	0.774946961	68301.59137	1.32263963	67959.09564	0.814561098	67409.97201	$-4.15E-05$
2011	70044	70848.76722	1.148945267	69763.02459	−0.401141304	70853.14838	1.155200125	66899.7767	−4.488925964	70004.54551	−0.056328148	69432.19338	−0.873460429	70259.45423	0.307598404	71401.62276	1.938242767
2012	77608	74042.54885	−4.594179917	71276.74656	−8.157990718	74043.01605	−4.593577918	67094.06064	−13.54749428	71807.19754	−7.474490333	70001.66656	−9.80096568	72342.47139	−6.784775551	75631.02234	−2.547389011
2013	81472	77380.30251	−5.02221314	72543.78834	−10.95862586	77376.49422	−5.026887497	67170.26722	−17.55416927	73375.3989	−9.937894128	70137.08261	−13.91265391	74228.68188	−8.890561319	80112.35362	−1.668851119
2014	86482	80868.5183	−6.490924933	73604.34959	−14.89055573	80860.04835	−6.500718823	67200.15874	−22.29578555	74739.64194	−13.57780586	69937.99929	−19.12999319	75936.6803	−12.19365845	84860.64593	−1.8747879
2015	93841	84513.97888	−9.939174904	74492.07893	−20.61883513	84500.43498	−9.953607718	67211.88349	−28.37684649	75926.45323	−19.09031955	69482.71953	−25.95697027	77483.30439	−17.43128868	89891.82555	−4.208367825
2016	98623	88323.77266	−10.44302783	75235.14151	−23.71440586	88304.71485	−10.46235173	67216.48244	−31.84502353	76958.90932	−21.96657035	68833.09601	−30.20583839	78883.80106	−20.01480278	95222.76925	−3.447705659

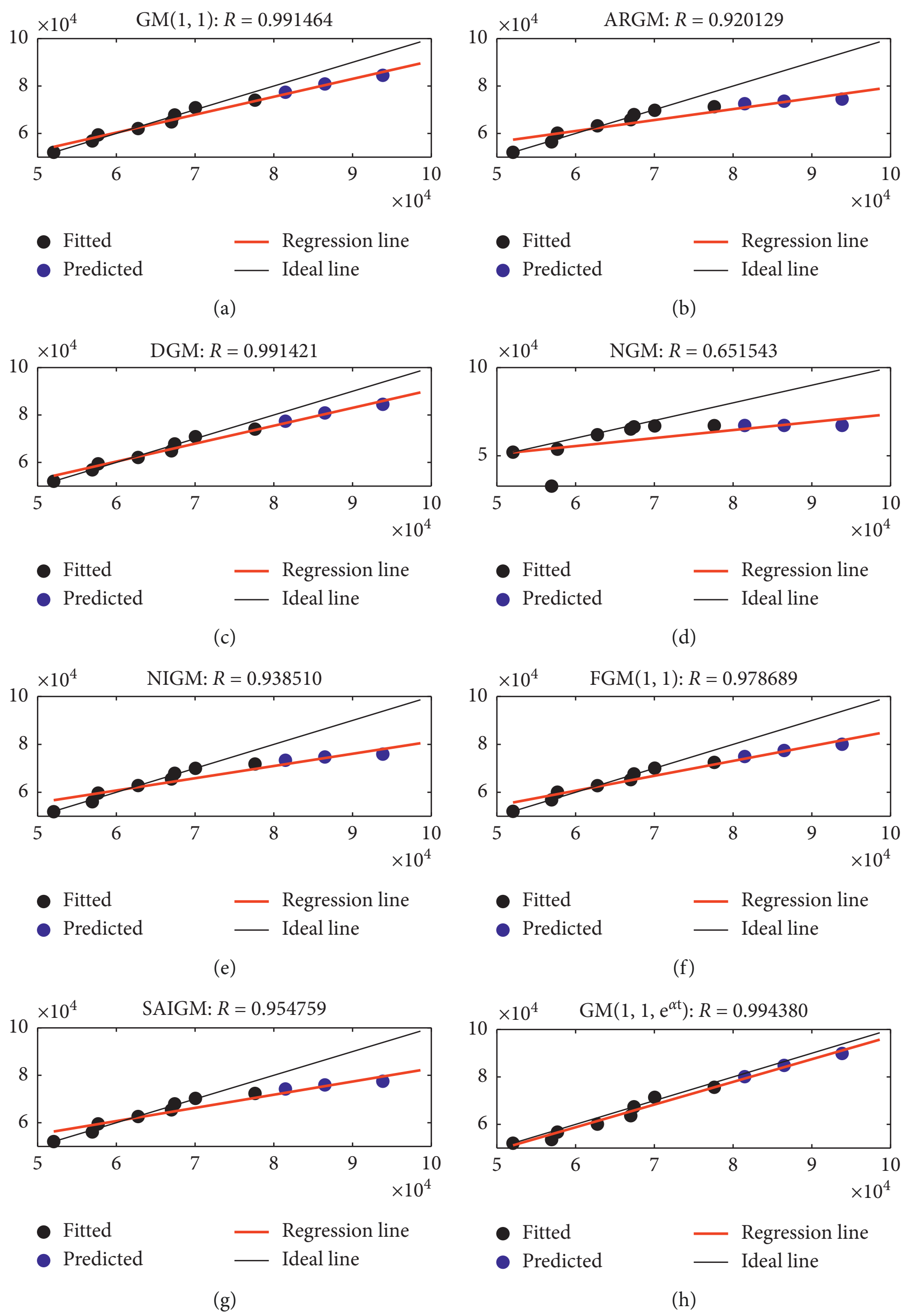

Figure 8: Comparison of the true value and the produced value by different grey models in Case 2.

the WOA algorithm. Compared with seven classical grey prediction methods such as ARGM, DGM, NGM, NIGM, FGM(1,1), GM(1,1), and SAIGM, the proposed grey model obtains a more superior prediction accuracy in validation experiments and case studies. From their fitted and predicted results, it can be clearly noticed that the GM(1,1,$e^{\alpha t}$) model reaches the lowest forecast errors though the fit errors are not the lowest. The main reason for taking higher fitting error is that the strategy similar to cross validation is chosen to build the GM(1,1,$e^{\alpha t}$) model. In fact, the strategy can overcome the overfitting phenomenon of the prediction problem. According to Lewis' criterion of accuracy evaluation, the fit abilities of the proposed grey model are still excellent in all validations and case studies because the MAPEs of fitting are less than 10%. In summary, three conclusions are drawn as follows. Firstly, the improvement of grey action quantity with an exponential term of time is one of the effective methods to improve the prediction accuracy of the grey prediction method. Secondly, the nonlinear parameter of exponential grey action quantity plays a significant role in forecasting future data accurately. Moreover, the heuristic optimization algorithm WOA can be used to seek out the optimal value of the nonlinear parameter effectively. Thirdly, the strategy similar to cross validation can be used to conquer the overfitting problem in prediction task.

Table 15: Evaluation result of different grey models for fitting and forecasting China's residential thermal energy consumption in Case 2.

Fitting	GM(1,1)	ARGM	DGM	NGM	NIGM	FGM(1,1)	SAIGM	GM(1,1,$e^{\alpha t}$)
RMSE	1117.216715	1084.839675	1117.196143	9369.107976	1008.424292	1112.393086	995.4137672	2159.813561
MAE	833.481874	774.6359237	831.2695007	4997.562392	727.3628862	644.5599287	739.7103295	1676.346441
NRMSE	0.000311765	0.008800034	0.000138821	0.21331217	$4.44E-16$	0.005034	$2.05E-05$	0.054996175
MAPE	1.309080795	1.26220759	1.305378066	8.486704395	1.196710704	1.046859636	1.198515175	2.688266118
RMSPE	1.762923227	1.821901401	1.764262819	16.38324835	1.673963545	1.83762531	1.632581987	3.487383104
MSE	1248173.188	1176877.121	1248127.222	87780184.26	1016919.552	1237418.377	990848.5679	4664794.619
IA	0.991453209	0.991823554	0.991452721	0.70140632	0.99308751	0.991283835	0.993269893	0.971417611
U1	0.008969537	0.008695398	0.00896862	0.077822239	0.008095409	0.008923061	0.00799091	0.017509782
U2	0.017936366	0.017416568	0.017936035	0.150416426	0.016189757	0.017858925	0.015980879	0.034674746
Prediction	GM(1,1)	ARGM	DGM	NGM	NIGM	FGM(1,1)	SAIGM	GM(1,1,$e^{\alpha t}$)
RMSE	7127.875052	15537.08007	7138.65228	21828.1977	14334.23205	10884.47247	13027.7043	**2666.27728**
MAE	6579.37576	14174.77901	6588.258311	20426.62949	13043.67981	10068.06587	11830.2122	**2461.54694**
NRMSE	0.167934454	0.361802375	0.168161176	0.521376951	0.332931775	0.256981089	0.301958773	**0.06282945**
MAPE	7.297904145	15.66808266	7.307428738	22.72386382	14.40941604	11.1683294	13.06301736	**2.74950079**
RMSPE	7.69811988	16.70857475	7.709280273	23.69867698	15.40101991	11.76115179	13.9871564	**2.91164244**
MSE	50806602.76	241400857	50960356.37	476470215	205470208.5	118471740.9	169721079.4	**7109034.54**
IA	0.770969645	0.473466789	0.770285171	0.372333584	0.502732671	0.618658116	0.538683921	**0.96778228**
U1	0.042144608	0.096270537	0.042210649	0.140714021	0.088194495	0.065740224	0.079555449	**0.01537902**
U2	0.081048357	0.176666229	0.0811709	0.248200135	0.162989102	0.123763197	0.148133072	**0.03031723**

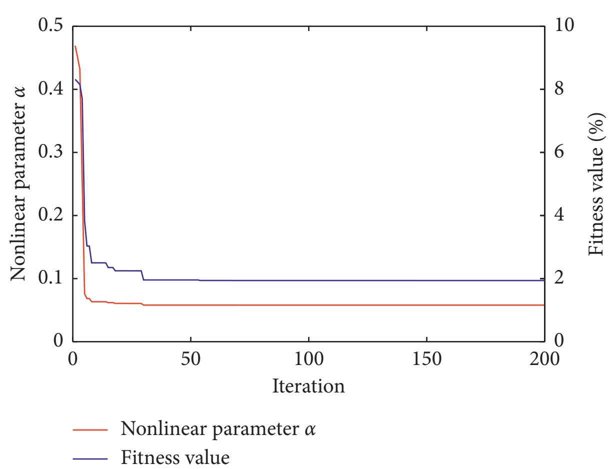

Figure 9: Convergence curve of WOA in Case 2.

As future work, more studies and applications about the strategy of determining the nonlinear parameters in the grey model should be carried out to overcome the overfitting problem in prediction task. Besides, the power-driven grey model can also be employed to solve more problems such as solving the industrial, rural, and urban energy supply and demand prediction problems. At the same time, it is worth studying to extend the power-driven grey model to other more prediction applications.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This study was supported by the National Natural Science Foundation of China (nos. 71901184, 61672136, and 11872323), the Humanities and Social Science Project of Ministry of Education of China (no. 19YJCZH119), the Open Fund (PLN 201710) of State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation (Southwest Petroleum University), and the Doctoral Research Foundation of Southwest University of Science and Technology (no. 16zx7140).

References

[1] Y. Yang, J. Liu, Y. Lin, and Q. Li, *The Impact of Urbanization on China's Residential Energy Consumption*, Vol. 49, Elsevier B.V., Amsterdam, Netherlands, 2019.

[2] B. Zeng and C. Li, "Forecasting the natural gas demand in China using a self-adapting intelligent grey model," *Energy*, vol. 112, pp. 810–825, 2016.

[3] J. Li, R. Wang, J. Wang, and Y. Li, "Analysis and forecasting of the oil consumption in China based on combination models optimized by artificial intelligence algorithms," *Energy*, vol. 144, pp. 243–264, 2018.

[4] J. Wang, P. Du, H. Lu, W. Yang, and T. Niu, "An improved grey model optimized by multi-objective ant lion optimization algorithm for annual electricity consumption forecasting," *Applied Soft Computing*, vol. 72, pp. 321–337, 2018.

[5] W. Wu, X. Ma, B. Zeng, Y. Wang, and W. Cai, "Application of the novel fractional grey model FAGMO(1,1,*k*) to predict China's nuclear energy consumption," *Energy*, vol. 165, pp. 223–234, 2018.

[6] W. Wu, X. Ma, B. Zeng, Y. Wang, and W. Cai, "Forecasting short-term renewable energy consumption of China using a novel fractional nonlinear grey Bernoulli model," *Renewable Energy*, vol. 140, pp. 70–87, 2019.

[7] M. Ma, X. Ma, W. Cai, and W. Cai, "Carbon-dioxide mitigation in the residential building sector: a household scale-

based assessment," *Energy Conversion and Management*, vol. 198, Article ID 111915, 2019.

[8] C. Deb, F. Zhang, J. Yang, S. E. Lee, and K. W. Shah, "A review on time series forecasting techniques for building energy consumption," *Renewable and Sustainable Energy Reviews*, vol. 74, pp. 902–924, 2017.

[9] H. Zhong, J. Wang, H. Jia, Y. Mu, and S. Lv, "Vector field-based support vector regression for building energy consumption prediction," *Applied Energy*, vol. 242, pp. 403–414, 2019.

[10] P. Du, J. Wang, W. Yang, and T. Niu, "A novel hybrid model for short-term wind power forecasting," *Applied Soft Computing*, vol. 80, pp. 93–106, 2019.

[11] D. Ju-Long, "Control problems of grey systems," *Systems & Control Letters*, vol. 1, no. 5, pp. 288–294, 1982.

[12] Q. Wang, L. Liu, S. Wang, J.-Z. Wang, and M. Liu, "Predicting Beijing's tertiary industry with an improved grey model," *Applied Soft Computing*, vol. 57, pp. 482–494, 2017.

[13] N. Xu, Y. Dang, and Y. Gong, "Novel grey prediction model with nonlinear optimized time response method for forecasting of electricity consumption in China," *Energy*, vol. 118, pp. 473–480, 2017.

[14] Y. Wang, Y. Dang, Y. Li, and S. Liu, "An approach to increase prediction precision of GM(1,1) model based on optimization of the initial condition," *Expert Systems with Applications*, vol. 37, no. 8, pp. 5640–5644, 2010.

[15] Z.-X. Wang, Y.-G. Dang, and S.-F. Liu, "Optimization of background value in GM(1,1) model," *Systems Engineering—Theory & Practice*, vol. 28, no. 2, pp. 61–67, 2009.

[16] J. Ye, Y. Dang, and B. Li, "Grey-Markov prediction model based on background value optimization and central-point triangular whitenization weight function," *Communications in Nonlinear Science and Numerical Simulation*, vol. 54, pp. 320–330, 2018.

[17] B. Zeng and C. Li, "Improved multi-variable grey forecasting model with a dynamic background-value coefficient and its application," *Computers & Industrial Engineering*, vol. 118, pp. 278–290, 2018.

[18] C.-J. Chang, D.-C. Li, C.-C. Chen, and C.-S. Chen, "A forecasting model for small non-equigap data sets considering data weights and occurrence possibilities," *Computers & Industrial Engineering*, vol. 67, no. 1, pp. 139–145, 2014.

[19] K. Li, L. Liu, J. Zhai, T. M. Khoshgoftaar, and T. Li, "The improved grey model based on particle swarm optimization algorithm for time series prediction," *Engineering Applications of Artificial Intelligence*, vol. 55, pp. 285–291, 2016.

[20] B. Zeng, W. Meng, and M. Tong, "A self-adaptive intelligence grey predictive model with alterable structure and its application," *Engineering Applications of Artificial Intelligence*, vol. 50, pp. 236–244, 2016.

[21] S. Ding, "A novel self-adapting intelligent grey model for forecasting China's natural-gas demand," *Energy*, vol. 162, pp. 393–407, 2018.

[22] B. Zeng, H. Duan, and Y. Zhou, "A new multivariable grey prediction model with structure compatibility," *Applied Mathematical Modelling*, vol. 75, pp. 385–397, 2019.

[23] L. Wu, S. Liu, L. Yao, S. Yan, and D. Liu, "Grey system model with the fractional order accumulation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 18, no. 7, pp. 1775–1785, 2013.

[24] X. Ma, X. Mei, W. Wu, X. Wu, and B. Zeng, "A novel fractional time delayed grey model with grey wolf optimizer and its applications in forecasting the natural gas and coal consumption in Chongqing China," *Energy*, vol. 178, pp. 487–507, 2019.

[25] X. Ma, M. Xie, W. Wu, B. Zeng, Y. Wang, and X. Wu, "The novel fractional discrete multivariate grey system model and its applications," *Applied Mathematical Modelling*, vol. 70, pp. 402–424, 2019.

[26] X. Ma and Z. Liu, "The GMC(1,*n*) model with optimized parameters and its application," *Journal of Grey System*, vol. 29, no. 4, pp. 122–138, 2017.

[27] X. Ma, W. Wu, B. Zeng, Y. Wang, and X. Wu, "The conformable fractional grey system model," *ISA Transactions*, 2019.

[28] K.-L. Wen and Y.-F. Huang, "The development of grey verhulst toolbox and the analysis of population saturation state in Taiwan-Fukien," in *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, vol. 6, pp. 5007–5012, Hague, Netherlands, 2004.

[29] Z.-X. Wang, K. W. Hipel, Q. Wang, and S.-W. He, "An optimized NGBM(1,1) model for forecasting the qualified discharge rate of industrial wastewater in China," *Applied Mathematical Modelling*, vol. 35, no. 12, pp. 5524–5532, 2011.

[30] F. Shaikh, Q. Ji, P. H. Shaikh, N. H. Mirjat, and M. A. Uqaili, "Forecasting China's natural gas demand based on optimised nonlinear grey models," *Energy*, vol. 140, pp. 941–951, 2017.

[31] S. Li, X. Ma, and C. Yang, "A novel structure-adaptive intelligent grey forecasting model with full-order time power terms and its application," *Computers & Industrial Engineering*, vol. 120, pp. 53–67, 2018.

[32] P.-Y. Chen and H.-M. Yu, "Foundation settlement prediction based on a novel NGM model," *Mathematical Problems in Engineering*, vol. 2014, pp. 1–8, 2014.

[33] J. Ren, "GM(1,*N*) method for the prediction of anaerobic digestion system and sensitivity analysis of influential factors," *Bioresource Technology*, vol. 247, pp. 1258–1261, 2018.

[34] S. Mirjalili and A. Lewis, "The whale optimization algorithm," *Advances in Engineering Software*, vol. 95, pp. 51–67, 2016.

[35] C. Bergmeir and J. M. Benítez, "On the use of cross-validation for time series predictor evaluation," *Information Sciences*, vol. 191, pp. 192–213, 2012.

[36] B. Wei, N. Xie, and A. Hu, "Optimal solution for novel grey polynomial prediction model," *Applied Mathematical Modelling*, vol. 62, pp. 717–727, 2018.

[37] M. A. E. Aziz, A. A. Ewees, and A. E. Hassanien, "Multi-objective whale optimization algorithm for content-based image retrieval," *Multimedia Tools and Applications*, vol. 77, no. 19, pp. 26135–26172, 2018.

[38] A. Tharwat, Y. S. Moemen, and A. E. Hassanien, "Classification of toxicity effects of biotransformed hepatic drugs using whale optimized support vector machines," *Journal of Biomedical Informatics*, vol. 68, pp. 132–149, 2017.

[39] G. Xiong, J. Zhang, D. Shi, and Y. He, "Parameter extraction of solar photovoltaic models using an improved whale optimization algorithm," *Energy Conversion and Management*, vol. 174, pp. 388–405, 2018.

[40] M. M. Mafarja and S. Mirjalili, "Hybrid whale optimization algorithm with simulated annealing for feature selection," *Neurocomputing*, vol. 260, pp. 302–312, 2017.

[41] M. A. E. Aziz, A. A. Ewees, and A. E. Hassanien, "Whale optimization algorithm and moth-flame optimization for multilevel thresholding image segmentation," *Expert Systems with Applications*, vol. 83, pp. 242–256, 2017.

[42] Z. Alameer, M. A. Elaziz, A. A. Ewees, H. Ye, and Z. Jianhua, "Forecasting gold price fluctuations using improved multilayer perceptron neural network and whale optimization algorithm," *Resources Policy*, vol. 61, pp. 250–260, 2019.

[43] L. Wu, S. Liu, H. Chen, and N. Zhang, "Using a novel grey system model to forecast natural gas consumption in China," *Mathematical Problems in Engineering*, vol. 2015, no. 2, pp. 1–7, 2015.

[44] J. Ye, Y. Dang, S. Ding, and Y. Yang, "A novel energy consumption forecasting model combining an optimized DGM(1,1) model with interval grey numbers," *Journal of Cleaner Production*, vol. 229, pp. 256–267, 2019.

[45] L. Wu, S. Liu, Z. Fang, and H. Xu, "Properties of the GM(1,1) with fractional order accumulation," *Applied Mathematics and Computation*, vol. 252, pp. 287–293, 2015.

[46] W. Yang, J. Wang, H. Lu, T. Niu, and P. Du, "Hybrid wind energy forecasting and analysis system based on divide and conquer scheme: a case study in China," *Journal of Cleaner Production*, vol. 222, pp. 942–959, 2019.

[47] Y.-C. Hu, "Electricity consumption prediction using a neural-network-based grey forecasting approach," *Journal of the Operational Research Society*, vol. 68, no. 10, pp. 1259–1264, 2017.

[48] M. Zhang and C. Bai, "Exploring the influencing factors and decoupling state of residential energy consumption in Shandong," *Journal of Cleaner Production*, vol. 194, pp. 253–262, 2018.