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Research Article

Finite Time Synchronization for Fractional Order Sprott C Systems with Hidden Attractors

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Fractional order systems have a wider range of applications. Hidden attractors are a peculiar phenomenon in nonlinear systems. In this paper, we construct a fractional-order chaotic system with hidden attractors based on the Sprott C system. According to the Adomain decomposition method, we numerically simulate from several algorithms and study the dynamic characteristics of the system through bifurcation diagram, phase diagram, spectral entropy, and C_0 complexity. The results of spectral entropy and C_0 complexity simulations show that the system is highly complex. In order to apply such research results to engineering practice, for such fractional-order chaotic systems with hidden attractors, we design a controller to synchronize according to the finite-time stability theory. The simulation results show that the synchronization time is short and the robustness is stable. This paper lays the foundation for the study of fractional order systems with hidden attractors.

1. Introduction

Since Lorenz proposed the first chaotic system [1] in 1963, many chaotic systems [2–6] have been proposed successively. Nonlinear systems have also been used in many fields such as image encryption, secure communication, and UAV navigation.

Chaotic attractors include self-excited attractors and hidden attractors. The self-excited attractor is mainly caused by the unstable equilibrium point, while the hidden attractor is mainly caused by the existence of infinite equilibrium points and disjoint with the unstable equilibrium point. In the last half century people used Routh-Hurwitz criterion and Shilnikov theorem to determine the stability of equilibrium point and then determine whether the system has attractors or chaos. For the hidden attractor, the stable equilibrium point does not mean that the system is stable, which means that the previous judgment method cannot complete the judgment work. Since Leonov and kuznetsov [7] discovered the first Chua hidden attractor, many achievements [8–11] have been made. At present, the researches on hidden attractors are mostly of integer order and few of fractional order.

With the in-depth research, people found the applicable range of the fractional order system is bigger than integer order more [12], especially secure communication. Due to the fact that difficulty of the fractional order synchronization is higher than the integer order system, for the fractional order synchronous research started later than integer order. In 2003, Li Chunguang [13] realized the synchronization of fractional chaos system for the first time. After that, many synchronization methods [14, 15] of fractional-order chaotic systems have been proposed.

In this paper we combine the two hotspots of the fractional system and the hidden attractor. We construct a new chaotic system with hidden attractors based on Sprott C system by adding tiny perturbations. The spectral entropy and C_0 complexity are the newest chaotic system characterization indicators, and we use these indicators to analyze the complex characteristics of new system. Then we realize chaotic synchronization on the basis of fractional finite time stability theory. These properties have significant application value to the area of secure communication and image encryption.

2. Dynamic Analysis

2.1. Fractional Differential Equation. Since fractional differential equations were proposed, many differential definitions have been proposed [16]. There are Grunwald-Letnikov (G-L) definition, Riemann-Liouville (R-L) definition, Caputo definition, etc., the most commonly used being G-L and R-L definition. Caputo definition is suitable for describing initial value problems of differential equations. In this paper, the definition of Caputo fractional-order differential equation is used to solve the chaos analysis of Sprott C system.

The expression of derivative of Caputo type is as follows:

$$ {}_{a}^{C}D_{t}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{a}^{t} \frac{f^{(n)}(\tau)\, d\tau}{(t-\tau)^{\alpha-n+1}} \tag{1} $$

C indicates that this is defined as the Caputo-type fractional order definition and n − 1 < α < n, q is the order of differential operator, and $\Gamma(\bullet)$ is Gamma function. Caputo differentiation involves the following properties.

Theorem 1 (see [16]).

$$ {}_{a}^{C}D_{t}^{q} x^{u} = \frac{\Gamma(u+1)}{\Gamma(u+1-q)} x^{u-q} {}_{a}^{C}D_{t}^{q} x \tag{2} $$

Common differential equations can be described as follows:

$$ {}_{a}^{C}D_{t}^{q} x(t) = Ax(t) \tag{3} $$

The general solution of the above differential equation is

$$ \mathrm{x}(\mathrm{t}) = \mathrm{x}(0)\, \mathrm{E}_{q}\left(At^{q}\right) \tag{4} $$

In the above formula, Mittag-Leffter function is

$$ \mathrm{E}_{q}(x) = \sum_{i=0}^{\infty} \frac{x^{i}}{\Gamma(qi+1)} \tag{5} $$

2.2. System Model. The Sprott C system was discovered by J. C. Sprott [17] through computer exhaustion. It consists of five elements, two of which are one of the simplest nonlinear systems and are easier to implement in an application. The Sprott C system has two equilibrium points and is symmetrical, with a pair of conjugate virtual roots at the equilibrium point. By adding a small perturbation term, the pure virtual root is transformed into a pair of conjugate eigenvalues with negative real parts and does not affect its chaotic characteristics. The Sprott C system appears chaotic and the corresponding feature data is stable after the above transformation, which is the hidden attractor. Liu [18] analyzed the stability and coupling synchronization problems of integer-order Sprott b and Sprott C systems and analyzed them by phase space trajectory and circuit simulation. Based on this, this paper extends it to fractional order, constructs hidden attractors, and studies its stability and synchronization problems. Sprott C original system is

$$ \begin{aligned} \dot{x} &= yz \\ \dot{y} &= x - y \\ \dot{z} &= 1 - x^{2} \end{aligned} \tag{6} $$

The modified fractional system is

$$ \begin{aligned} \frac{d^{q}x}{dt^{q}} &= yz + a \\ \frac{d^{q}y}{dt^{q}} &= x - y \\ \frac{d^{q}z}{dt^{q}} &= 1 - x^{2} \end{aligned} \tag{7} $$

where q is the order of nonlinear system and $q \in (0,1]$ and a is the disturbance parameter; let a=0.001 and the right side of (7) is equal to zero, so

$$ \begin{aligned} yz + a &= 0 \\ x - y &= 0 \\ 1 - x^{2} &= 0 \end{aligned} \tag{8} $$

By calculating system (8) we can found that system (7) only has two equilibria $(\pm 1, \pm 1, \mp \mathrm{a})$. The above two equilibrium points are symmetric, so we only discuss the properties of one of them.

2.3. Hidden Attractor. The Jacobean matrix for the equilibrium point A $(1, 1, -0.001)$ is

$$ J(A) = \begin{pmatrix} 0 & -0.001 & 1 \\ 1 & -1 & 0 \\ -2 & 0 & 0 \end{pmatrix} \tag{9} $$

As we all know that the characteristic polynomial $\det(\lambda \boldsymbol{E} - \mathbf{J}) = 0$, there are

$$ \begin{aligned} \lambda_{1} &= -0.9997, \\ \lambda_{2,3} &= -0.0002 \pm 1.4144i \end{aligned} \tag{10} $$

The Jacobean for the equilibrium point B $(-1, -1, 0.001)$ is

$$ J(B) = \begin{pmatrix} 0 & 0.001 & -1 \\ 1 & -1 & 0 \\ 2 & 0 & 0 \end{pmatrix} \tag{11} $$

The eigenvalues are the same as the equilibrium point A. They are

$$ \begin{aligned} \lambda_{1} &= -0.9997, \\ \lambda_{2,3} &= -0.0002 \pm 1.4144i \end{aligned} \tag{12} $$

It can be inferred from (10) and (12) that the real parts of all eigenvalues are negative. So the equilibrium points A and B are both stable when a= 0.001. However, the results of numerical simulation contradict the above theoretical analysis.

3. Solution and Simulation

3.1. Nonlinear Multiplier Sub Decomposition. Based on the improved Adomian algorithm [19], we decompose the nonlinear term of (7) into the following form:

$$\begin{aligned} A_1^0 &= y^0 z^0 \\ A_1^1 &= y^1 z^0 + y^0 z^1 \\ A_1^2 &= y^2 z^0 + y^1 z^1 + y^0 z^2 \\ A_1^3 &= y^3 z^0 + y^2 z^1 + y^1 z^2 + y^0 z^3 \\ A_1^4 &= y^4 z^0 + y^3 z^1 + y^2 z^2 + y^1 z^3 + y^0 z^4 \\ A_1^5 &= y^5 z^0 + y^4 z^1 + y^3 z^2 + y^2 z^3 + y^1 z^4 + y^0 z^5 \end{aligned} \quad (13)$$

$$\begin{aligned} A_3^0 &= -x^0 x^0 \\ A_3^1 &= -2x^1 x^0 \\ A_3^2 &= -2x^2 x^0 - x^1 x^1 \\ A_3^3 &= -2x^3 x^0 - 2x^2 x^1 \\ A_3^4 &= -2x^4 x^0 - 2x^3 x^1 - x^2 x^2 \\ A_3^5 &= -2x^5 x^0 - 2x^4 x^1 - 2x^3 x^2 \end{aligned} \quad (14)$$

Now, let

$$\begin{aligned} x_0 &= x(t_0) = c_1^0 \\ y_0 &= y(t_0) = c_2^0 \\ z_0 &= z(t_0) = c_3^0 \end{aligned} \quad (15)$$

Then

$$\begin{aligned} x_1 &= \left(c_2^0 c_3^0 + a\right) \frac{(t - t_0)^q}{q} \\ y_1 &= \left(c_1^0 - c_2^0\right) \frac{(t - t_0)^q}{q} \\ z_1 &= \left(1 - c_1^0 c_1^0\right) \frac{(t - t_0)^q}{q} \end{aligned} \quad (16)$$

Let all variables be equal to the corresponding coefficient values:

$$\begin{aligned} c_1^1 &= c_2^0 c_3^0 + a \\ c_2^1 &= c_1^0 - c_2^0 \\ c_3^1 &= 1 - c_1^0 c_1^0 \end{aligned} \quad (17)$$

We can find out all the coefficients by using the above method. They are

$$\begin{aligned} c_1^2 &= c_2^0 c_3^1 + c_2^1 c_3^0 + a \\ c_2^2 &= c_1^1 - c_2^1 \\ c_3^2 &= 1 - 2c_1^1 c_1^0 \\ c_1^3 &= c_2^0 c_3^2 + c_2^2 c_3^0 + c_2^2 c_3^2 + a \\ c_2^3 &= c_1^2 - c_2^2 \\ c_3^3 &= 1 - 2c_1^0 c_1^2 - c_1^1 c_1^1 \\ c_1^4 &= c_2^3 c_3^0 + c_2^2 c_3^1 + c_2^1 c_3^2 + a \\ c_2^4 &= c_1^3 - c_2^3 \\ c_3^4 &= 1 - 2c_1^3 c_1^0 - c_1^2 c_1^1 \\ c_1^5 &= c_2^4 c_3^0 + c_2^3 c_3^1 + c_2^2 c_3^2 + c_2^1 c_3^3 + c_2^0 c_3^4 + a \\ c_2^5 &= c_1^4 - c_2^4 \\ c_3^5 &= 1 - 2c_1^4 c_1^0 - 2c_1^3 c_1^1 - c_1^2 c_1^2 \\ c_1^6 &= c_2^5 c_3^0 + c_2^4 c_3^1 + c_2^3 c_3^2 + c_2^2 c_3^3 + c_2^1 c_3^4 + c_2^0 c_3^5 + a \\ c_2^6 &= c_1^5 - c_2^5 \\ c_3^6 &= 1 - 2c_1^5 c_1^0 - 2c_1^4 c_1^1 - 2c_1^3 c_1^2 \end{aligned} \quad (18)$$

Based on the above decomposed coefficients, the equations of the nonlinear system can be combined as follows:

$$x_m(x) = \sum_{n=0}^{6} c_n^m \frac{(t - t_0)^{nq}}{n! q^n} \quad (19)$$

3.2. Numerical Simulation. Bifurcation diagrams are often used in dynamic analysis to observe system characteristics. However, the bifurcation diagram can only show the case where an independent variable changes with one parameter. At present, most studies only use the bifurcation diagram to judge the type of system classification. We use the order parameter q in system (7) as the variable of bifurcation graph and take the initial value as $[x(0), y(0), z(0)] = (0.1, 0.2, 0.3)$ to get the numerical change of state variable y. As shown in Figure 1, the bifurcation diagram is drawn with the maximum value method.

It is obvious that system (7) is chaotic in [0.988, 1.000] and stable in [0.980, 0.988]. The dynamic characteristics of system (7) are very sensitive to the change of order q. When $q < 0.988$, system (7) is in a stable state. When $q > 0.988$, system (7) enters chaos state through period-doubling bifurcation. However, due to the existence of hidden attractors in the system, the bifurcation diagram also loses its absolute accuracy.

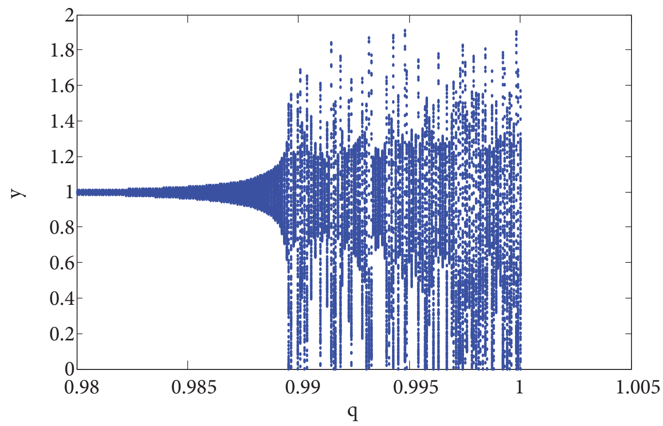

Figure 1: Bifurcation diagram with the change of q.

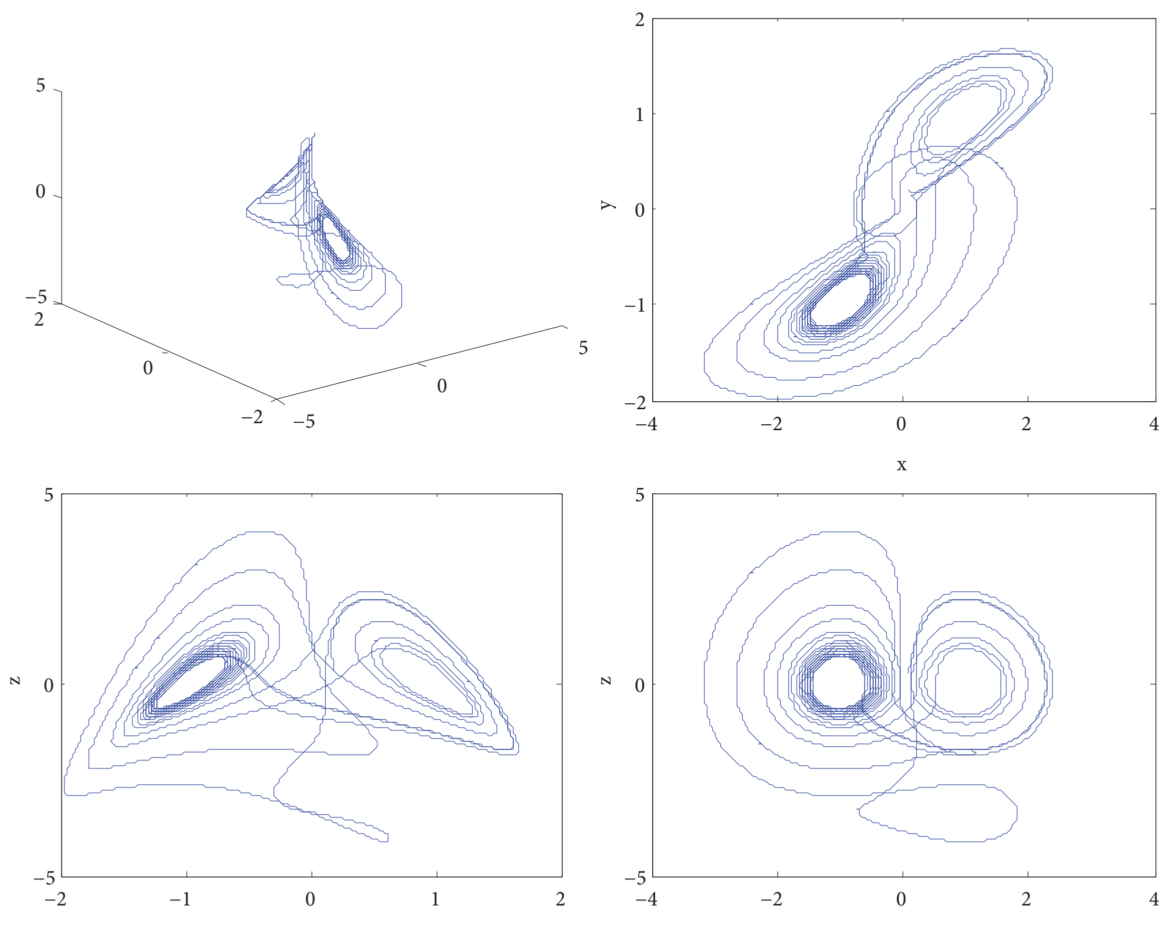

Figure 2: Phase diagram of system (7).

Simulation of chaotic image and time domain image will help us to analyze its dynamic features. Take order q=0.99, and the initial point is

$$[x(0), y(0), z(0)] = [0.1, 0.2, 0.3] \tag{20}$$

The Douglas-Jones method was adopted for numerical simulation. Step size $h = 0.01$, simulation duration $T = 100s$, and chaotic images and time-domain images were obtained. As shown in Figures 2 and 3, the system is chaotic.

System (7) is chaotic and shows the characteristics of double vortexes. According to Shilnikov's theorem [20], a necessary and sufficient condition for a system to chaos is that it should have at least one unstable point. However, according to (10) and (12), the two equilibrium points of system (7)

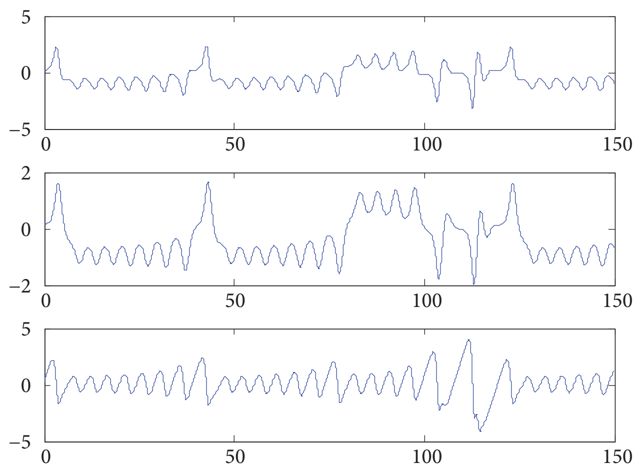

FIGURE 3: Time series diagram of system (7).

A and B are all stable, so hidden attractors appear here. According to the above analysis, we can find that system (7) is a fractional-order chaotic system with hidden attractors.

3.3. Complexity Analysis. Complexity is a new quantitative evaluation tool for nonlinear systems. The more complex the dynamic properties of nonlinear systems, the higher the information entropy. In this section, we will analyze the behavioral complexity of system (7) from the perspective of spectral entropy and C_0 complexity, respectively. The first step is to remove the DC information from the chaotic sequence according to the system equation. Formula is as follows:

$$\mathrm{x}\left(\mathrm{n}\right)=x\left(n\right)-\overline{x}\left(\overline{x}=\frac{1}{N}\sum_{n=0}^{N-1}x\left(n\right)\right) \tag{21}$$

Then you take the Fourier transform of the x sequence:

$$\mathrm{X}\left(\mathrm{k}\right)=\sum_{n=0}^{N-1}x\left(n\right)e^{-j(2\pi/N)nk}=\sum_{n=0}^{N-1}x\left(n\right)W_{K}^{nk} \tag{22}$$

The relative power spectrum P_k is

$$\begin{aligned}P_k&=\frac{p_{(k)}}{p_{tot}}=\frac{(1/N)\left|X\left(k\right)\right|^2}{(1/N)\sum_{k=0}^{N/2-1}\left|X\left(k\right)\right|^2}\\&=\frac{\left|X\left(k\right)\right|^2}{\sum_{k=0}^{N/2-1}\left|X\left(k\right)\right|^2}\end{aligned} \tag{23}$$

So, the spectral entropy is

$$\begin{aligned}se&=-\sum_{k=0}^{N/2-1}P_k\ln P_k\\\mathrm{SE}\left(\mathrm{N}\right)&=\frac{se}{\ln\left(N/2\right)}\end{aligned} \tag{24}$$

The simulation result plays in Figure 4, the parameters remain unchanged, and the system complexity is calculated as the order q changes.

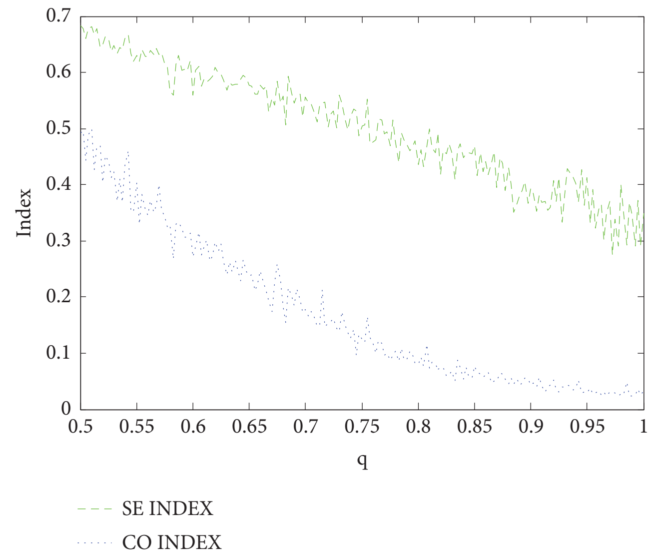

FIGURE 4: SE and C_0 complexity diagrams.

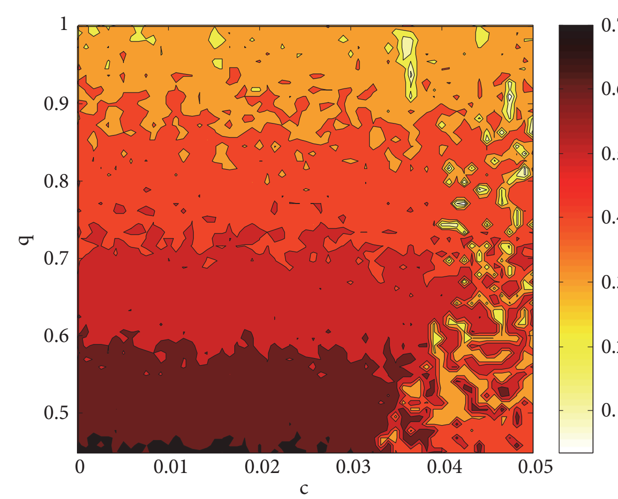

FIGURE 5: SE index.

As shown in Figure 4, both SE complexity and C_0 complexity of the system decrease with the increase of order q. Only one single parameter change does not yet show the overall complexity of the system.

The closer the order q is to 1, the lower the information entropy is, and the attachment near zero is at its maximum. When the order q is fixed, the change of the constant a does not greatly change the information entropy of the system.

Conversely, when the constant a is constant, the information entropy and complexity of the system will change as the order q changes. We change the order q and the disturbance a at the same time, so that we can observe all the complexity changes. As shown in Figures 5 and 6, the smaller the parameter a, the greater the complexity of the system. The order q also has the same properties.

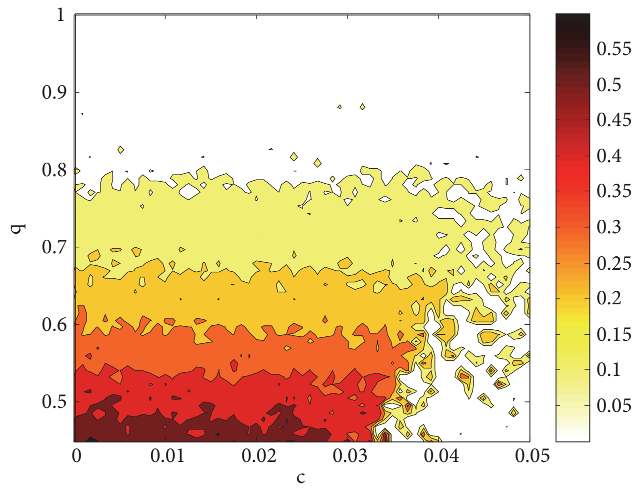

Figure 6: C_0 index.

4. Finite Time Synchronization

4.1. Finite Time Stability Theory. Synchronization of fractional-order systems is an important part of secure communication. At present, many methods have been proposed, such as drive-response method, generalized synchronization method, projection synchronization, etc. Zhao Lingdong [21] proposed a finite-time stability theory for fractional-order systems, which is characterized by fast speed and good robustness. Zheng Guangchao [22] successfully synchronized a chaotic system by using fractional-order finite-time synchronization theory.

Theorem 2. *The general fractional order system satisfies the following conditions:*

$$x\,{}_{\alpha}^{C}D_t^q x^T = \frac{\Gamma(2)}{\Gamma(2+\alpha)^a}\,{}_{\alpha}^{C}D_t^q x\left(x^q\right)^T \le -k\left(xx^T\right)^{\alpha}, \qquad \alpha < \frac{q+q^2}{2}, \; k>0 \tag{25}$$

where variables $x = [x_1, x_2, \dots, x_n]$ and $x_q = [{x_1}^q, {x_2}^q, \dots, {x_n}^q]$. The variable x of the fractional-order system will be close to zero before the time reaches which is the following expression.

$$t = \left[v(0)^{q-2\alpha/(1+q)} \cdot \frac{\Gamma(1+q-2\alpha/(1+q))\,\Gamma(1+q)}{\Gamma(1+2q-2\alpha/(1+q))\,k\Gamma(2+q)}\right]^{1/q}, \qquad v = x\left(x^q\right)^T \tag{26}$$

When a, b>0 and 0<c<1, we can get the following inequality:

$$(a+b)^c \le a^c + b^c \tag{27}$$

Let formula (7) be the drive system; parameter q=0.99; then its corresponding response system is

$$\begin{aligned} \frac{d^q x_1}{dt^q} &= y_1 z_1 + a - u_1 \\ \frac{d^q y_1}{dt^q} &= x_1 - y_1 - u_2 \\ \frac{d^q z_1}{dt^q} &= 1 - {x_1}^2 - u_3 \end{aligned} \tag{28}$$

u_1, u_2, and u_3 are controllers designed based on theorem; the error between drive system and response system is as follows:

$$\begin{aligned} e_1 &= x_1 - x, \\ e_2 &= y_1 - y, \\ e_3 &= z_1 - z \end{aligned} \tag{29}$$

Then, the error system is

$$\begin{aligned} \frac{d^q e_1}{dt^q} &= y_1 e_3 + e_2 z - u_1 \\ \frac{d^q e_2}{dt^q} &= e_1 - e_2 - u_2 \\ \frac{d^q e_3}{dt^q} &= -e_1\left(x + x_1\right) - u_3 \end{aligned} \tag{30}$$

Therefore, the controller is

$$\begin{aligned} u_1 &= -e_3\left(x + x_1\right) + k{e_1}^{\beta} \\ u_2 &= e_1\left(1 + z\right) + k{e_2}^{\beta} \\ u_3 &= y_1 e_1 + k{e_3}^{\beta} \end{aligned} \tag{31}$$

The system will be stable within t(s).

$$t = \left\{\left[e\left(e^q\right)^T\right]^{q-(1+\beta)/(1-q)} \cdot \frac{\Gamma(1+q-(1+\beta)/(1-q))\,\Gamma(1+q)}{\Gamma(1+2q-(1+\beta)/(1-q))\,k\Gamma(2+q)}\right\}^{1/q} \tag{32}$$

where $e = [e_1, e_2, e_3]$ and $e^q = [{e_1}^q, {e_2}^q, {e_3}^q]$.

Prove. According to the error system and the controller, the synchronization error is

$$\begin{aligned} \frac{d^q e_1}{dt^q} &= e_1 z + y_1 e_3 + e_3\left(x + x_1\right) - k{e_1}^{\beta} \\ \frac{d^q e_2}{dt^q} &= -e_1 z - e_2 - k{e_2}^{\beta} \\ \frac{d^q e_3}{dt^q} &= -e_1\left(x + x_1\right) - y_1 e_1 - k{e_3}^{\beta} \end{aligned} \tag{33}$$

By deriving (17),

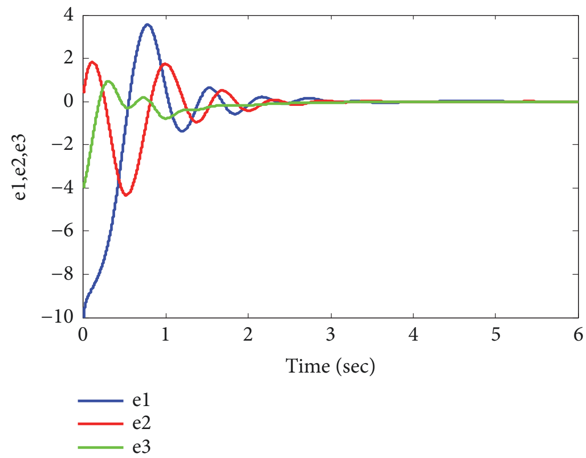

FIGURE 7: Curves of synchronous error.

$$\frac{\Gamma(2)}{\Gamma(2+\alpha)}\frac{dq}{dt}\left[e_1,e_2,e_3\right]\left[{e_1}^q,{e_2}^q,{e_3}^q\right]^T=\left[e_1,e_2,e_3\right]\left[\frac{d^qe_1}{dt^q}\frac{d^qe_2}{dt^q}\frac{d^qe_3}{dt^q}\right]$$
$$=\left(e_1,e_2,e_3\right)\left[e_1z+y_1e_3+e_3\left(x+x_1\right)-k{e_1}^{\beta}-e_1z-e_2-k{e_2}^{\beta}-e_1\left(x+x_1\right)-y_1e_1-k{e_3}^{\beta}\right]=-e_2^2-k{e_1}^{1+\beta}-k{e_2}^{1+\beta}$$
$$-k{e_3}^{1+\beta}\le-k{e_1}^{1+\beta}-k{e_2}^{1+\beta}-k{e_3}^{1+\beta}=-\mathrm{k}\left(e_1^2\right)^{(1+\beta)/2}-\mathrm{k}\left(e_2^2\right)^{(1+\beta)/2}-\mathrm{k}\left(e_3^2\right)^{(1+\beta)/2} \tag{34}$$

Combining the following formula:

$$(a+b)^c\le a^c+b^c \tag{35}$$

There are

$$-\left(e_1^2\right)^{(1+\beta)/2}-\left(e_2^2\right)^{(1+\beta)/2}-\left(e_3^2\right)^{(1+\beta)/2}\le-\left(e_1^2+e_2^2+e_3^2\right)^{(1+\beta)/2} \tag{36}$$

Therefore

$$\begin{aligned}&\frac{\Gamma(2)}{\Gamma(2+\alpha)}\frac{dq}{dt}\left[e_1,e_2,e_3\right]\left[{e_1}^q,{e_2}^q,{e_3}^q\right]^T\\&\le-\mathrm{k}\left(e_1^2+e_2^2+e_3^2\right)^{(1+\beta)/2}\\&=-k\left\{\left[e_1,e_2,e_3\right]\left[{e_1}^q,{e_2}^q,{e_3}^q\right]^T\right\}^{(1+\beta)/2}\\&=-k\left(ee^T\right)^{(1+\beta)/2}\end{aligned} \tag{37}$$

It is obvious that (37) satisfies the conditions in Theorem 2. In other words, the variables e_1, e_2, and e_3 are stable for less than t the error system can be synchronized within a limited time.

4.2. The Numerical Simulation. In this section we will simulate the synchronization process in MATLAB using the Adomian decomposition method. The step size h=0.001, and simulation time T=6S. In order to make it easy to observe the synchronization results, the initial point is

$$\begin{aligned}\left[x(0),y(0),z(0)\right]&=[5,0.2,2]\\\left[x_1(0),y_1(0),z_1(0)\right]&=[-5,0,-2]\end{aligned} \tag{38}$$

Disturbance parameter a=0.002. The parameters of the controller are k=1 and $\beta = 0.8$. The error system synchronization process is shown in Figure 7.

e_1, e_2, and e_3 are the initial errors between the x, y, and z variables, respectively. For observability simulation results, here we take different initial values. As shown in Figure 7, the simulation synchronization time is approximately 4 seconds. After 4 seconds, e_1, e_2, and e_3, all three errors, are equal to 0, which means synchronization is achieved. The maximum initial error e_1 is equal to -10, the smallest is e_2 equal to 0.2, and the synchronization error range is large.

x1 and y1 are the x variables of the drive system and the response system, respectively. x2 and y2 are the y variables of the drive system and the response system, respectively. x3 and y3 are the z variables of the drive system and the response system, respectively. It is not difficult to konw from Figure 8 that the synchronization process of variables is very short and the robustness is well after synchronization. Synchronization under different parameters can be considered in future studies. According to (32), we know that the synchronization stabilization time is only related to the order of the fractional order system. However, the robustness of synchronization has no related theorem or formula that can be inferred to be related to parameter changes.

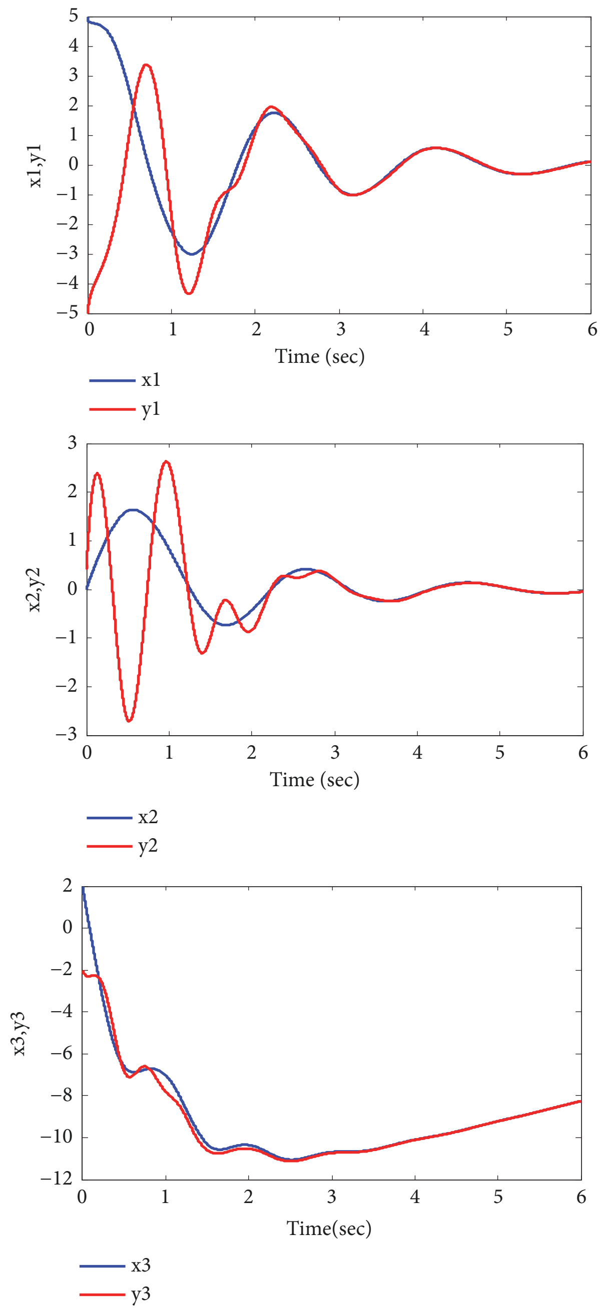

Figure 8: Plots of state variables.

5. Conclusion

This paper verifies that the hidden attractor can be generated without changing the nature of the original system by adding small perturbations. We decompose the nonlinear system by Adomain decomposition method and make up the algorithm to simulate the complexity. The simulation results show that the nonlinear system has higher complexity with the decrease of order q. In order to accelerate the application of the results, a controller is designed based on the finite time stability theory. After adding the controller, the synchronization system achieves synchronization in a short time. The controlled system after synchronization also has good robustness. This paper lays a foundation for the study of fractional systems with hidden attractors. This paper also puts forward some innovative ideas for the dynamic analysis of nonlinear systems.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Ethical Approval

All procedures performed in studies involving human participants were in accordance with the ethical standards of the institutional and/or national research committee and with the 1964 Helsinki declaration and its later amendments or comparable ethical standards. This article does not contain any studies with animals performed by any of the authors.

Consent

Informed consent was obtained from all individual participants included in the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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