Research Article

Fuzzy Predictive Fault-Tolerant Control for Time-Delay Nonlinear Systems with Partial Actuator Failures

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Received 16 March 2019; Revised 19 July 2019; Accepted 24 July 2019; Published 25 August 2019

Academic Editor: Xianggui Guo

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A fuzzy predictive fault-tolerant control (FPFTC) scheme is proposed for a wide class of discrete-time nonlinear systems with uncertainties, interval time-varying delays, and partial actuator failures as well as unknown disturbances, in which the main opinions focus on the relevant theory of FPFTC based on Takagi-Sugeno (T-S) fuzzy model description of these systems. The T-S fuzzy model represents the discrete-time nonlinear system in the form of the discrete uncertain time-varying delay state space, which is firstly constructed by a set of local linear models and the nonlinear membership functions. The novel improved state space model can be further obtained by extending the output tracking error to the constructed model. Then the fuzzy predictive fault-tolerant control law based on this extended model is designed, which can increase more control degrees of freedom. Utilizing Lyapunov-Krasovskii theory, less conservative delay-range-dependent stable conditions in terms of linear matrix inequality (LMI) constraints are given to ensure the asymptotically robust stability of closed-loop system. Meanwhile, the optimized cost function and H-infinity performance index are introduced to the stable conditions to guarantee the robust performance and antidisturbance capability. The simulation results on the temperature control of a strong nonlinear continuous stirred tank reactor (CSTR) show that the proposed control scheme is feasible and effective.

1. Introduction

With the continuous social and economic progress, living standards of the people have been improved greatly. The demands for higher product quality also increase continuously. In order to satisfy these demands, more strict operation conditions are needed for the manufacturing industry, which is a huge challenge. Under such a context, more efforts should be paid to study the advanced control technique.

Industrial production is the backbone of many of the world's economies [1] and plays an important role in daily life of people. However, most industrial processes have nonlinear characteristics. The traditional control methods have been unable to achieve the goal of higher product quality in industrial production [2]. As a result, a great number of advanced control technologies [2–7] have been proposed over the past decades. Among them, fuzzy model-based approaches that were pointed out clearly in [8] are very effective and feasible to deal with the system nonlinearities. However, the focus is put

on the control methods based on Takagi-Sugeno (T-S) fuzzy model [9, 10]. In T-S fuzzy model, a set of local linear models are weighted by nonlinear membership functions in terms of IF-THEN rules to approximate a large class of nonlinear processes well [11]. In virtue of this, many mature linear theories are able to be applied fully to the stability analysis and control synthesis of nonlinear processes [12–16], which will make great progress in the advanced control theory.

Due to the increasing demands for industrial products, the scale of industrial production is growing rapidly, which makes the industrial equipment operated under more complex environments. With the long-running industrial production, the failure may occur. If a failure cannot be coped with instantaneously in such environments through the suitable corrective action, it will make the control performance deteriorate and even expose the equipment and personnel to serious damage. Thus, fault-tolerant systems [17] and advanced process control algorithms [18] were studied to deal with failure using two soft computing approaches, i.e.,

fuzzy systems and neural networks. Especially, the study of fault-tolerant control (FTC) based on T-S fuzzy model for a class of nonlinear processes with failure [19–24] has attracted a lot of interests over the last few decades. However, the effect of time delay is not fully considered in these references.

Time delay usually occurs in many industrial processes, such as petrochemical furnace, continuous stirred tank reactor, biochemical fermentation process, injection molding batch process, fractionating tower, rolling process, cement grinding system, industrial flotation process and so on. It is well known that time delay is one of the major sources of the instability and performance degradation of system. From the perspective of the term of time delay, it can be divided into two categorizers: constant delay and time-varying delay. The interval time-varying delay is more common, i.e., the delay varies in the upper and lower bounds of the interval and the lower bound is not limited to be zero, which has been identified from many practical industrial processes. But only a few results of FTC for T-S fuzzy system with failure and interval time-varying delay simultaneously can be seen [25–29] during the recent years. However, these methods do not consider the uncertainties or external disturbances that are very important in practical industrial application. Meanwhile, most of them are active FTC. Their main ideas are to apply the proposed observer-based fault estimation (FE) method to obtain the online accurate fault information by solving linear matrix inequality (LMI) conditions and then the fault information is integrated to the design of FTC. Though these methods have made some progress, FE methods need to meet the observer matching condition, which is very difficult for many practical processes. In addition, the obtained LMI-based observer gains are unable to restrain the derivatives of faults effectively within the fault error dynamics. Compared to the active FTC, the passive FTC does not need the accurate fault information, which is simple to implement. The passive FTC can also guarantee the system stability and expected performance within the admissible range of faults. Therefore, the passive FTC is studied for the industrial processes with the partial actuator faults in our work.

There are some researches [30–34] for the industrial process with the partial actuator faults. Based on a two-dimensional Fornasini-Marchisni model, an iterative learning FTC method for batch processes was proposed by Wang et al. [30] in early stage. Wang et al. further extended his result and proposed robust delay dependent iterative learning FTC for batch processes with state delay and actuator failures [31] and robust iterative learning FTC for multiphase batch processes with uncertainties [32], respectively. In addition, Tao et al. [33] proposed a nonminimal state space model predictive control method for linear systems with partial actuator failure; Zhang et al. [34] proposed FTC scheme for flexible spacecraft, which can guarantee the closed-loop system to be asymptotically stable with a H-infinity performance and partial loss of actuator effectiveness; Zhang et al. [35, 36] proposed a linear quadratic FTC method for industrial processes with partial actuator faults and unknown disturbance to ensure the stability and optimal performance; Shi et al. [37] proposed a robust constrained model predictive

fault-tolerant control method for uncertain time-delay system with unknown disturbances and partial actuator failures. However, the above methods do not consider the nonlinearity or the time delay.

In fact, the robust performance for the practical industrial applications is vital because it is always expected to control the industrial processes with uncertainty and time delay using the designed control system. As a result, the optimal performance for control system is also an open topic. Model predictive control (MPC) [38–40] is a selection to improve the control performance of system because it can achieve receding optimization in real-time. It is recognized as the only advanced control method which has made a significant impact on industrial control engineering [41]. For the industrial processes with time-varying delay and actuator faults, the study of FTC integrated with MPC is effective and feasible.

Motivated by the aforementioned references, a fuzzy predictive fault-tolerant control (FPFTC) scheme is proposed, which is suitable for discrete-time nonlinear systems with partial actuator failures, interval time-varying delays, uncertainties and unknown disturbances. To the best of our knowledge, there is no result for our work in this paper. The main contributions of this paper are summarized as follows.

(1) With the local-sector nonlinearity approach, a T-S fuzzy model is established by weighting a set of local linear models using the nonlinear membership functions, which can approximate a large class of nonlinear industrial processes. Then the output tracking error is extended to T-S fuzzy model. Therefore, the performance of the convergence and tracking can be improved for the design of control law based on this extended fuzzy model due to independent adjustment for the process state variables and output tracking error.

(2) Without using some redundant free-weighting matrices and cross terms with differential inequality, less conservative delay-range-dependent stable conditions in terms of linear matrix inequality (LMI) constraints are further given to ensure the closed-loop system to be robustly asymptotically stable.

(3) In order to guarantee the robust performance and overcome any unknown bounded disturbances, the optimized cost function and H_∞ performance index are introduced into the derivation of stability for proposed FPFTC, respectively.

(4) The simulation results on the temperature control of a strong nonlinear continuous stirred tank reactor (CSTR) demonstrate that the proposed control scheme performs excellent tracking and antidisturbance capabilities.

The paper is organized as follows: A process description is shown in Section 2. Section 3 details the fuzzy predictive fault-tolerant control method. Section 4 presents a case study in a CSTR. Conclusions are summarized in Section 5.

The notations used throughout the paper are quite standard. R^i denotes the i th fuzzy rule. $\mathbb{R}^{n_x}$ is the n_x -dimensional, Euclidean space and $\mathbb{R}^{n \times n}$ are the set of $n \times n$ real matrices. $\Delta^{iT}(k)$ denotes the transpose of $\Delta^i(k)$. $*$ is used as an ellipsis for the terms that are implied by symmetry.

2. Process Description

A discrete-time nonlinear system with uncertainties, interval time-varying delay, partial actuator failures, and unknown disturbances is described by a great many of T-S fuzzy rules as follows.

Rule R^i . If $Z_1(k)$ is M_1^i and $Z_2(k)$ is $M_2^i, \dots, Z_q(k)$ is M_q^i then

$$\begin{aligned} x(k+1) &= A^i(k)x(k) + A_d^i(k)x(k-d(k)) \\ &\quad + B^i\alpha u(k) + w(k) \\ y(k) &= C^i x(k) \end{aligned} \quad (1)$$

where $x(k) \in \mathbb{R}^{n_x}, u(k) \in \mathbb{R}^{n_u}, y(k) \in \mathbb{R}^{n_y}, w(k)$ are the state, input, output and unknown external disturbances at the discrete-time k , respectively. $Z_1(k), \dots, Z_q(k)$ are the premise variables. M_h^i ($i = 1, 2, \dots, l, h = 1, 2, \dots, q$) is the h th fuzzy set for the i th fuzzy rule. $d(k)$ denotes the time-varying delay depending on discrete-time k which satisfies

$$d_m \leq d(k) \leq d_M \quad (2)$$

in which d_M and d_m are the upper and down bounds of delay, respectively. $[A^i(k) \ A_d^i(k)] \in \Omega, \Omega$ is uncertainty set. $A^i(k) = A^i + \Delta_a^i(k), A_d^i(k) = A_d^i + \Delta_d^i(k), A^i, A_d^i, B^i$ and C^i are the constant matrices corresponding to appropriate

dimensions for the i th fuzzy rule, and $\Delta_a^i(k), \Delta_d^i(k)$ are uncertain perturbations at discrete-time k that follows

$$[\Delta_a^i(k) \ \Delta_d^i(k)] = N^i \Delta^i(k) [H^i \ H_d^i] \quad (3)$$

with

$$\Delta^{iT}(k) \Delta^i(k) \leq I. \quad (4)$$

where N^i, H^i and H_d^i are known constant matrices with appropriate dimensions. $\Delta^i(k)$ are uncertainties depending on discrete-time k . α stands for the actuator fault that cannot be avoided in actual engineering applications. Therefore, the predefined $u(k)$ cannot be obtained, which can be expressed as $u^F(k) = \alpha u(k)$ or $u^F(k) = u_\alpha$. $\alpha = 0$ is the complete actuator fault and u_α refers to the stuck fault of actuator. For the aforementioned two faults, the system is unable to be controlled. The appropriate measures must be done to deal with the two faults. $\alpha > 0$ is the partial actuator fault that is here studied in this work. α is unknown but assumed to vary within a known range, i.e., $0 \leq \underline{\alpha} \leq \alpha \leq \bar{\alpha}$ where $\underline{\alpha} \leq 1$ and $\bar{\alpha} \geq 1$ are known scalars.

Then the nonlinear system can be transformed into the following form of discrete-time-varying delay state space by weighting a great many of local linear submodels (1):

$$\sum_{\text{T-S-delay}} : \begin{cases} x(k+1) = \sum_{i=1}^l \bar{h}^i(x(k)) (A^i x(k) + A_d^i x(k-d(k)) + \sum_{i=1}^l \bar{h}^i(x(k)) B^i \alpha u(k) + w(k) \\ y(k) = C^i x(k) \end{cases} \quad (5)$$

where $\bar{h}^i(x(k)) = M^i(x(k)) / \sum_{i=1}^l M^i(x(k))$ and $\sum_{i=1}^l \bar{h}^i(x(k)) = 1$. $M^i(x(k)) = \prod_{h=1}^q M_h^i$, in which $M^i(x(k))$ is the membership function for the i th fuzzy rule.

The main objective in this work is to design a fault-tolerant controller for the T-S fuzzy system (5) with the partial actuator failures so that the measurement of output is able to track the desired set point $c(k)$. In view of this, (6) is defined as follows:

$$\begin{aligned} \beta &= \frac{\bar{\alpha} + \alpha}{2}, \\ \beta_0 &= \frac{\bar{\alpha} - \alpha}{\bar{\alpha} + \alpha} \end{aligned} \quad (6)$$

From (6), we can see that there has an unknown matrix α_0 such as

$$\alpha = (I + \alpha_0) \beta \quad (7)$$

with

$$|\alpha_0| \leq \beta_0 \leq I. \quad (8)$$

3. Fuzzy Predictive Fault-Tolerant Control

3.1. Novel Extended T-S Fuzzy System. Pre- and postmultiplying (5) by back shift operator, the T-S fuzzy model $\sum_{\text{T-S-delay}}$ can be transformed into the following incremental state space form:

$$\begin{aligned} \Delta x(k+1) &= \sum_{i=1}^l \bar{h}^i(x(k)) (A^i(k) \Delta x(k) + A_d^i(k) \Delta x(k-d(k)) \\ &\quad + \sum_{i=1}^l \bar{h}^i(x(k)) B^i \alpha \Delta u(k) + \bar{w}(k) \\ \Delta y(k) &= C^i \Delta x(k) \end{aligned} \quad (9)$$

where $\Delta = 1 - q^{-1}, \bar{w}(k) = \sum_{i=1}^l \sigma(\bar{h}^i) [A_d^i(k)x(k-1-d(k-1)) + A^i(k) \cdot x(k-1) + B^i \alpha u(k-1)] + \sum_{i=1}^l \bar{h}^i(x(k-1)) [(\Delta_a^i(k) - \Delta_a^i(k-1))x(k-1) + (\Delta_d^i(k) - \Delta_d^i(k-1))x(k-1-d(k-1))] +$

$\Delta w(k), \sigma(\bar{h}^i) = \bar{h}^i(x(k)) - \bar{h}^i(x(k-1))$. Defining the set point as $c(k)$, then the output tracking error is

$$e(k) = y(k) - c(k) \quad (10)$$

The following form is presented by synthesizing (6) and (7):

$$e(k+1) = e(k) + C \left(\sum_{i=1}^l \bar{h}^i(x(k)) (A^i(k) \Delta x(k) \right.$$

$$+ \sum_{i=1}^l A_d^i(k) \Delta x(k-d(k)) + \sum_{i=1}^l \bar{h}^i(x(k)) B^i \alpha \Delta u(k) + \bar{w}(k) \Big) \quad (11)$$

By extending the tracking error to T-S fuzzy model (9), the novel extended T-S fuzzy model can be expressed as follows:

$$\sum_{\text{E-T-S-delay}} : \begin{cases} \bar{x}_1(k+1) = \sum_{i=1}^l \bar{h}^i(x(k)) \bar{A}^i(k) \bar{x}_1(k) + \sum_{i=1}^l \bar{h}^i(x(k)) \bar{A}_d^i(k) \cdot \bar{x}_1(k-d(k)) + \sum_{i=1}^l \bar{h}^i(x(k)) \bar{B}^i \alpha \Delta u(k) + \bar{G} \bar{w}(k) \\ \Delta y(k) = \bar{C} \bar{x}_1(k) \\ z(k) = e(k) = \bar{E} \bar{x}_1(k) \end{cases} \quad (12)$$

where

$$\begin{aligned} \bar{x}_1(k) &= \begin{bmatrix} \Delta x(k) \\ e(k) \end{bmatrix}, \\ \bar{x}_1(k-d(k)) &= \begin{bmatrix} \Delta x(k-d(k)) \\ e(k-d(k)) \end{bmatrix}, \\ \bar{A}^i(k) &= \begin{bmatrix} A^i + \Delta_a^i(k) & 0 \\ CA^i + C\Delta_a^i(k) & I \end{bmatrix} = \bar{A}^i + \bar{\Delta}_a^i, \\ \bar{A}^i &= \begin{bmatrix} A^i & 0 \\ CA^i & I \end{bmatrix}, \\ \bar{\Delta}_a^i(k) &= \bar{N}^i \Delta^i(k) \cdot \bar{H}^i, \\ \bar{A}_d^i(k) &= \begin{bmatrix} A_d^i + \Delta_d^i(k) & 0 \\ CA_d^i + C\Delta_d^i(k) & 0 \end{bmatrix} = \bar{A}_d^i + \bar{\Delta}_d^i, \\ \bar{A}_d^i &= \begin{bmatrix} A_d^i & 0 \\ CA_d^i & 0 \end{bmatrix}, \\ \bar{\Delta}_d^i(k) &= \bar{N}^i \Delta^i(k) \bar{H}_d^i, \\ \bar{B}^i &= \begin{bmatrix} B^i \\ CB^i \end{bmatrix}, \\ \bar{N}^i &= \begin{bmatrix} N^i \\ CN^i \end{bmatrix}, \\ \bar{H}^i &= \begin{bmatrix} H^i & 0 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \bar{H}_d^i &= \begin{bmatrix} H_d^i & 0 \end{bmatrix}, \\ \bar{G} &= \begin{bmatrix} I \\ C \end{bmatrix}, \\ \bar{C} &= \begin{bmatrix} C & 0 \end{bmatrix}, \\ \bar{E} &= \begin{bmatrix} 0 & I \end{bmatrix}. \end{aligned} \quad (13)$$

Remark 1. From (12), the output error is introduced to the state variables, which can increase more control degrees of freedom. Meanwhile, the control performance with respect to convergence and tracking will be improved by the following design of control law because this extended fuzzy system $\sum_{\text{E-T-S-delay}}$ can independently adjust the process state variables and output tracking error. Hence, using the parallel distributed compensation (PDC) method, the control law of system is designed as follows:

$$\Delta u^i(k) = \bar{K}^i \bar{x}_1(k) = \bar{K}^i \begin{bmatrix} \Delta x(k) \\ e(k) \end{bmatrix} \quad (14)$$

$$\Delta u(k) = \sum_{i=1}^l \bar{h}^i \bar{K}^i \begin{bmatrix} \Delta x(k) \\ e(k) \end{bmatrix} \quad (15)$$

where $\bar{K}^i$ is the constant gain that can be solved by the subsequent theorems. Substituting (15) to (12), the extended T-S fuzzy closed-loop system can be obtained as follows:

$$\sum_{\text{E-T-S-delay-C}} : \begin{cases} \bar{x}_1(k+1) = \sum_{i=1}^l \sum_{j=1}^l h^i(x(k)) h^j(x(k)) \bar{A}^{ij}(k) \bar{x}_1(k) + \sum_{i=1}^l h^i(x(k)) \bar{A}_d^i(k) \bar{x}_1(k-d(k)) + \bar{G}\bar{w}(k) \\ \Delta y(k) = \bar{C}\bar{x}_1(k) \\ z(k) = e(k) = \bar{E}\bar{x}_1(k) \end{cases} \quad (16)$$

where $\bar{A}^{ij}(k) = \bar{A}^i(k) + \bar{B}^i \alpha \bar{K}^j$ ($i, j \leq l$).

Definition 2 (robust MPC problem). Considering fuzzy closed-loop system $\sum_{\text{E-T-S-delay-C}}$ with uncertainty set Ω , and the state measurement at discrete-time k as $\bar{x}_1(k)$, the robust MPC problem is feasible if the ‘min-max’ optimization problem

$$\begin{aligned} \min_{\Delta u(k+p|k), p \geq 0} \max_{J_\infty} & \left[A^i(k+p) A_d^i(k+p) \right]_{\in \Omega, p \geq 0} \\ \bar{J}_\infty(k) = \sum_{p=0}^{\infty} & \left[(\bar{x}_1(k+p|k))^T \bar{Q}_1 (\bar{x}_1(k+p|k)) \right. \\ & \left. + \Delta u(k+p|k)^T \bar{R}_1 \Delta u(k+p|k) \right] \end{aligned} \quad (17)$$

is solvable.

Definition 3. The fuzzy closed-loop system $\sum_{\text{E-T-S-delay-C}}$ has H_∞ performance, if there exists a scalar $\gamma > 0$ for any $\bar{w}(k)$ and the following conditions hold:

- (1) the resulting fuzzy closed-loop system $\sum_{\text{E-T-S-delay-C}}$ with $\bar{w}(k) = 0$ is asymptotically stable;
- (2) the system output $z(k)$ satisfies $\|z\|_2 \leq \gamma \|\bar{w}\|_2$ under the zero initial condition.

3.2. Main Theorems. The main objective of this section is to solve the predictive fault-tolerant law that ensures the robust stability of fuzzy closed-loop system. The main lemmas and theorems are presented as follows.

Lemma 4 ((Schur complements lemma) [42]). Let W, L and V be matrices of appropriate dimensions in which W, V are real matrices, then for

$$L^T V L - W < 0 \quad (18)$$

if and only if such that

$$\begin{aligned} & \begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix} < 0 \\ \text{or} & \begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0 \end{aligned} \quad (19)$$

Lemma 5 (see [43]). For any vector $\bar{\delta}(k) \in \mathbb{R}^n$, two positive integers κ_0, κ_1 , and matrix $0 < \bar{R} \in \mathbb{R}^{n \times n}$, the following inequality holds:

$$\begin{aligned} & -(\kappa_1 - \kappa_0 + 1) \sum_{t=\kappa_0}^{\kappa_1} \bar{\delta}^T(k) \bar{R} \bar{\delta}(k) \\ & \leq - \sum_{t=\kappa_0}^{\kappa_1} \bar{\delta}^T(k) \bar{R} \sum_{t=\kappa_0}^{\kappa_1} \bar{\delta}(k) \end{aligned} \quad (20)$$

Lemma 6 (see [44]). Let D, F, E , and M be real matrices of appropriate dimensions with satisfying $M = M^T$, then for all $F^T F \leq I$,

$$M + DFE + E^T F^T D^T < 0 \quad (21)$$

if and only if there exists $\varepsilon > 0$ such that

$$M + \varepsilon^{-1} D D^T + \varepsilon E^T E < 0 \quad (22)$$

Remark 7. Based on the Definition 3, the following Theorems 8 and 11 are given for the fuzzy closed-loop system $\sum_{\text{E-T-S-delay-C}}$ with disturbance and without disturbance, respectively.

Theorem 8. Given some scalars $\theta > 0$, $0 \leq d_m \leq d_M$, the delay-range-dependent sufficient conditions for the proposed controller that guarantee the fuzzy closed-loop system $\sum_{\text{E-T-S-delay-C}}$ with $\bar{w}(k) = 0$ to be asymptotically stable and controllable are that there exists symmetric positive matrices $\bar{P}_1, \bar{T}_1, \bar{M}_1, \bar{G}_1, \bar{L}_1, \bar{S}_1, \bar{S}_2, \bar{M}_3, \bar{M}_4, \bar{X}_1, \bar{X}_2 \in \mathbb{R}^{(n_x+n_e) \times (n_x+n_e)}$, matrices $\bar{Y}_1^i, \bar{Y}_1^j \in \mathbb{R}^{n_u \times (n_x+n_e)}$, and positive scalars $\bar{\varepsilon}_i, \bar{\varepsilon}_j$, such that the following LMI holds:

$$\begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & \bar{\Pi}_{14} \\ * & \bar{\Pi}_{22} & 0 & 0 \\ * & * & \bar{\Pi}_{33} & 0 \\ * & * & * & \bar{\Pi}_{44} \end{bmatrix} < 0 \quad (23)$$

$$\begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & \bar{\Pi}_{14} \\ * & \bar{\Pi}_{22} & 0 & 0 \\ * & * & \bar{\Pi}_{33} & 0 \\ * & * & * & \bar{\Pi}_{44} \end{bmatrix} < 0 \quad (24)$$

$$\begin{bmatrix} -1 & \bar{x}_l^T(k|k) \\ \bar{x}_l(k|k) & -\bar{\phi}_l \end{bmatrix} \leq 0 \quad (25)$$

$$\begin{bmatrix} -\phi & \bar{L}_1 \\ * & -\psi_l \end{bmatrix} \leq 0 \quad (26)$$

and the robust state-feedback fault-tolerant controller gains are given by $\bar{K}^i = \bar{Y}_1^i \bar{L}_1^{-1}$, $\bar{K}^j = \bar{Y}_1^j \bar{L}_1^{-1}$.
where

$$\bar{\Pi}_{11} = \begin{bmatrix} \hat{\phi}_1 & 0 & \bar{L}_1 \\ * & -\bar{S}_1 & 0 \\ * & * & -\bar{M}_4 - \bar{X}_1 \end{bmatrix},$$

$$\bar{\Pi}_{12} = \begin{bmatrix} \bar{L}_1 \bar{A}^{iT}(k) + \bar{Y}_1^{iT} \beta \bar{B}^{iT} & \bar{L}_1 \bar{A}^{iT}(k) + \bar{Y}_1^{iT} \beta \bar{B}^{iT} - \bar{L}_1 \\ \bar{S}_1 \bar{A}_d^{iT}(k) & \bar{S}_1 \bar{A}_d^{iT}(k) \\ 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{12} = \begin{bmatrix} \bar{L}_1 \bar{N}^{ij} + \bar{H}^{ij} & \bar{L}_1 \bar{N}^{ij} + \bar{H}^{ij} - \bar{L}_1 \\ \bar{S}_1 \bar{A}_d^{iT}(k) & \bar{S}_1 \bar{A}_d^{iT}(k) \\ 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{13} = \begin{bmatrix} \bar{Y}_1^{iT} \bar{R}_1^{1/2} & \bar{L}_1 \bar{Q}_1^{1/2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{13} = \begin{bmatrix} \frac{\bar{Y}_1^{iT} + \bar{Y}_1^{jT}}{2} \bar{R}_1^{1/2} & \bar{L}_1 \bar{Q}_1^{1/2} \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{14} = \begin{bmatrix} \bar{Y}_1^{iT} \beta & \bar{Y}_1^{jT} \beta \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{14} = \begin{bmatrix} \frac{1}{2} \bar{Y}_1^{jT} \beta & \frac{1}{2} \bar{Y}_1^{iT} \beta & \frac{1}{2} \bar{Y}_1^{jT} \beta & \frac{1}{2} \bar{Y}_1^{iT} \beta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{22} = \text{diag} \begin{bmatrix} -\bar{L}_1 + \bar{\varepsilon}_i \bar{B}^i \beta_0^2 \bar{B}^{iT} \\ -\bar{D}_2^{-2} \bar{X}_1 + \bar{\varepsilon}_i \bar{B}^i \beta_0^2 \bar{B}^{iT} \end{bmatrix},$$

$$\bar{\Pi}_{22} = \text{diag} \begin{bmatrix} -\bar{L}_1 + \bar{\varepsilon}_i \bar{B}^i \beta_0^2 \bar{B}^{iT} + \bar{\varepsilon}_j \bar{B}^j \beta_0^2 \bar{B}^{jT} \\ -\bar{D}_2^{-2} \bar{X}_1 + \bar{\varepsilon}_i \bar{B}^i \beta_0^2 \bar{B}^{iT} + \bar{\varepsilon}_j \bar{B}^j \beta_0^2 \bar{B}^{jT} \end{bmatrix},$$

$$\bar{\Pi}_{33} = -\text{diag} [\theta I \quad \theta I],$$

$$\bar{\Pi}_{44} = -\text{diag} [\bar{\varepsilon}_i I \quad \bar{\varepsilon}_i I],$$

$$\bar{\Pi}_{44} = -\text{diag} [\bar{\varepsilon}_i I \quad \bar{\varepsilon}_j I \quad \bar{\varepsilon}_i I \quad \bar{\varepsilon}_j I], \quad (27)$$

$\hat{\phi}_1 = -\bar{L}_1 + \bar{M}_3 + \bar{D}_1 \bar{S}_2 + \bar{S}_2 - \bar{X}_2$, $\bar{D}_1 = (d_M - d_m + 1)I$, $\bar{D}_2 = d_M I$, $\bar{N}^{ij} = (\bar{A}^{iT}(k) + \bar{A}^{jT}(k))/2$, $\bar{H}^{ij} = (\bar{Y}_1^{jT} \beta \bar{B}^{iT} + \bar{Y}_1^{iT} \beta \bar{B}^{jT})/2$, and $*$ denotes the transposed elements in the symmetric position.

Proof. To ensure robust stability of the fuzzy closed-loop system $\Sigma_{E-T-S\text{-delay-C}}$ with $\bar{w}(k) = 0$, letting that $\bar{x}_1(k)$ satisfies the following robust stability constraint:

$$\begin{aligned} & V(\bar{x}_1(k+i+1|k)) - V(\bar{x}_1(k+i|k)) \\ & \leq -[(\bar{x}_1(k+i|k))^T \bar{Q}_1 (\bar{x}_1(k+i|k)) \\ & \quad + \Delta u(k+i|k)^T \bar{R}_1 \Delta u(k+i|k)] \end{aligned} \quad (28)$$

Summing up both sides of (28) from $i=0$ to ∞ and needing that $V(\bar{x}_1(\infty)) = 0$ or $\bar{x}_1(\infty) = 0$, it has

$$\bar{J}_\infty(k) \leq V(\bar{x}_1(k)) \leq \theta \quad (29)$$

where θ is the upper bound of $\bar{J}_\infty(k)$. For notational simplicity, define $\bar{x}_{1d}(k+p) = \bar{x}_1(k+p-d(k))$, $\bar{x}_{1d_M}(k+p) = \bar{x}_1(k+p-d_M)$, $\bar{\delta}_1(k+p) = \bar{x}_1(k+p+1) - \bar{x}_1(k+p)$, $\bar{\varphi}_1(k+p) = [\bar{x}_1^T(k+p) \quad \bar{x}_{1d}^T(k+p) \quad \bar{x}_{1d_M}^T(k+p)]^T$, and the following Lyapunov-Krasovskii function candidate is established:

$$V(\bar{x}_1(k+p)) = \sum_{i=1}^5 V_i(\bar{x}_1(k+p)) \quad (30)$$

where

$$\begin{aligned}
V_1(\bar{x}_1(k+p)) &= \bar{x}_1^T(k+p) \bar{P}_1 \bar{x}_1(k+p) \\
&= \bar{x}_1^T(k+p) \theta \bar{L}_1^{-1} \bar{x}_1(k+p), \\
V_2(\bar{x}_1(k+p)) &= \sum_{r=k-d(k)}^{k-1} \bar{x}_1^T(r+p) \bar{T}_1 \bar{x}_1(r+p) \\
&= \sum_{r=k-d(k)}^{k-1} \bar{x}_1^T(r+p) \theta \bar{S}_1^{-1} \bar{x}_1(r+p), \\
V_3(\bar{x}_1(k+p)) &= \sum_{r=k-d_M}^{k-1} \bar{x}_1^T(r+p) \bar{M}_1 \bar{x}_1(r+p) \\
&= \sum_{r=k-d_M}^{k-1} \bar{x}_1^T(r+p) \theta \bar{M}_2^{-1} \bar{x}_1(r+p), \\
V_4(\bar{x}_1(k+p)) &= \sum_{s=-d_M}^{-d_m} \sum_{r=k+s}^{k-1} \bar{x}_1^T(r+p) \bar{T}_1 \bar{x}_1(r+p) \\
&= \sum_{s=-d_M}^{-d_m} \sum_{r=k+s}^{k-1} \bar{x}_1^T(r+p) \theta \bar{S}_1^{-1} \bar{x}_1(r+p), \\
V_5(\bar{x}_1(k+p)) &= d_M \sum_{s=-d_M}^{-1} \sum_{r=k+s}^{k-1} \bar{\delta}_1^T(r+p) \bar{G}_1 \bar{\delta}_1(r+p) \\
&= d_M \sum_{s=-d_M}^{-1} \sum_{r=k+s}^{k-1} \bar{\delta}_1^T(r+p) \theta \bar{X}_1^{-1} \bar{\delta}_1(r+p),
\end{aligned} \tag{31}$$

$\bar{P}_1, \bar{T}_1, \bar{M}_1, \bar{M}_2$, and $\bar{G}_1$ are positive definite matrices. Define $\bar{\xi}(k+p) = \begin{bmatrix} \bar{x}_1(k+p)^T & \bar{x}_1(k+p-d(k))^T & \dots \\ \bar{x}_1(k+p-d_M)^T & \dots & \bar{\delta}_1(k+p-1)^T \end{bmatrix}$, $\bar{\psi}_1 = \text{diag}[\bar{P}_1 \ \bar{T}_1 \ \dots \ \bar{M}_1 \ \dots \ d_M \bar{G}_1]$, $\bar{\Pi}_1^{-1} = \text{diag}[\bar{L}_1^{-1} \ \bar{S}_1^{-1} \ \dots \ \bar{M}_2^{-1} \ \dots \ d_M \bar{X}_1^{-1}]$, (30) can be written as follows:

$$\begin{aligned}
V(\bar{x}_1(k+p)) &= \bar{\xi}^T(k+p) \bar{\psi}_1 \bar{\xi}(k+p) \\
&= \bar{\xi}^T(k+p) \theta \bar{\Pi}_1^{-1} \bar{\xi}(k+p)
\end{aligned} \tag{32}$$

Then

$$\begin{aligned}
\Delta V(\bar{x}_1(k+p)) &= V(\bar{x}_1(k+p+1)) \\
&\quad - V(\bar{x}_1(k+p)) \\
&= \sum_{i=1}^5 \Delta V_i(\bar{x}_1(k+p))
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
\Delta V_1(\bar{x}_1(k+p)) &= \bar{x}_1^T(k+p+1) \theta \bar{L}_1^{-1} \bar{x}_1(k+p+1) \\
&\quad - \bar{x}_1^T(k+p) \theta \bar{L}_1^{-1} \bar{x}_1(k+p), \\
\Delta V_2(\bar{x}_1(k+p)) &= \sum_{r=k+1-d(k)}^k \bar{x}_1^T(r+p) \theta \bar{S}_1^{-1} \bar{x}_1(r+p) \\
&\quad - \sum_{r=k-d(k)}^{k-1} \bar{x}_1^T(r+p) \theta \bar{S}_1^{-1} \bar{x}_1(r+p) \\
&\leq \bar{x}_1^T(k+p) \theta \bar{S}_1^{-1} \bar{x}_1(k+p) \\
&\quad - \bar{x}_{1d}^T(k+p) \theta \bar{S}_1^{-1} \bar{x}_{1d}(k+p) \\
&\quad + \sum_{r=k-d_M}^{k-d_m} \bar{x}_1^T(r+p) \theta \bar{S}_1^{-1} \bar{x}_1(r+p), \\
\Delta V_3(\bar{x}_1(k+p)) &= \bar{x}_1^T(k+p) \theta \bar{M}_2^{-1} \bar{x}_1(k+p) \\
&\quad - \bar{x}_{1d_M}^T(k+p) \theta \bar{M}_2^{-1} \bar{x}_{1d_M}(k+p), \\
\Delta V_4(\bar{x}_1(k+p)) &< (d_M - d_m + 1) \bar{x}_1^T(k+p) \theta \bar{S}_1^{-1} \bar{x}_1(k+p) \\
&\quad - \sum_{r=k-d_M}^{k-d_m} \bar{x}_1^T(r+p) \theta \bar{S}_1^{-1} \bar{x}_1(r+p), \\
\Delta V_5(\bar{x}_1(k+p)) &= d_M^2 \bar{\delta}_1^T(k+p) \theta \bar{X}_1^{-1} \bar{\delta}_1(k+p) \\
&\quad - d_M \sum_{r=k-d_M}^{k-1} \bar{\delta}_1^T(r+p) \theta \bar{X}_1^{-1} \bar{\delta}_1(r+p).
\end{aligned} \tag{34}$$

Using Lemma 5, the following expression is presented:

$$\begin{aligned}
\Delta V_5(\bar{x}_1(k+p)) &\leq d_M^2 \bar{\delta}_1^T(k+p) \theta \bar{X}_1^{-1} \bar{\delta}_1(k+p) \\
&\quad - \sum_{r=k-d_M}^{k-1} \bar{\delta}_1^T(r+p) \theta \bar{X}_1^{-1} \sum_{r=k-d_M}^{k-1} \bar{\delta}_1(r+p) \\
&= d_M^2 (\bar{x}_1(k+p+1) - \bar{x}_1(k+p))^T \theta \bar{X}_1^{-1} \\
&\quad \cdot (\bar{x}_1(k+p+1) - \bar{x}_1(k+p)) \\
&\quad - (\bar{x}_1(k+p) - \bar{x}_{1d_M}(k+p))^T \\
&\quad \cdot \theta \bar{X}_1^{-1} (\bar{x}_1(k+p) - \bar{x}_{1d_M}(k+p))
\end{aligned} \tag{35}$$

From (28), it can obtain

$$\theta^{-1} \Delta V(\bar{x}_1(k+p|k)) + \theta^{-1} \bar{J}_p(k) \leq 0 \quad (36)$$

where

$$\begin{aligned} \bar{J}_p(k) &= (\bar{x}_1(k+p|k))^T \bar{Q}_1 (\bar{x}_1(k+p|k)) \\ &\quad + \Delta u(k+p|k)^T \bar{R}_1 \Delta u(k+p|k). \end{aligned} \quad (37)$$

Synthesizing (33)-(36), it has

where

$$\begin{aligned} \bar{\phi}_1 &= -\bar{L}_1^{-1} + \bar{M}_2^{-1} + \bar{D}_1 \bar{S}_1^{-1} + \bar{S}_1^{-1} - \bar{X}_1^{-1}, \\ \bar{\Lambda}_1 &= \left[\sum_{i=1}^l \sum_{j=1}^l \bar{h}^i(x(k)) \bar{h}^j(x(k)) \bar{A}^{ij}(k) \quad \sum_{i=1}^l \bar{h}^i(x(k)) \bar{A}_d^i(k) \quad 0 \right], \\ \bar{\Lambda}_2 &= \left[\sum_{i=1}^l \sum_{j=1}^l \bar{h}^i(x(k)) \bar{h}^j(x(k)) \bar{A}^{ij}(k) - I \quad \sum_{i=1}^l \bar{h}^i(x(k)) \bar{A}_d^i(k) \quad 0 \right], \\ \bar{\lambda}_1 &= [\bar{Q}_1^{1/2} \quad 0 \quad 0], \\ \bar{\lambda}_2 &= \left[\sum_{i=1}^l \bar{h}^i(x(k)) \bar{R}_1^{1/2} \bar{Y}_1^i \bar{L}_1^{-1} \quad 0 \quad 0 \right]. \end{aligned} \quad (40)$$

Using Lemma 4, the condition $\bar{\Phi}_1 < 0$ holds and is equivalent to the following:

$$\begin{aligned} &\begin{bmatrix} \bar{\phi}_1 & 0 & \bar{X}_1^{-1} & \bar{A}^{iT}(k) & \bar{A}^{iT}(k) - I & \bar{L}_1^{-1} \bar{Y}_1^{iT} \bar{R}_1^{1/2} & \bar{Q}_1^{1/2} \\ * & -\bar{S}_1^{-1} & 0 & \bar{A}_d^{iT}(k) & \bar{A}_d^{iT}(k) & 0 & 0 \\ * & * & -\bar{M}_2^{-1} - \bar{X}_1^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{L}_1 & 0 & 0 & 0 \\ * & * & * & * & -\bar{D}_2^{-2} \bar{X}_1 & 0 & 0 \\ * & * & * & * & * & -\theta I & 0 \\ * & * & * & * & * & * & -\theta I \end{bmatrix} < 0 \\ &\begin{bmatrix} \bar{\phi}_1 & 0 & \bar{X}_1^{-1} & \bar{N}^{ij}(k) & \bar{N}^{ij}(k) - I & \bar{L}_1^{-1} \frac{\bar{Y}_1^{iT} + \bar{Y}_1^{jT}}{2} \bar{R}_1^{1/2} & \bar{Q}_1^{1/2} \\ * & -\bar{S}_1^{-1} & 0 & \bar{A}_d^{iT}(k) & \bar{A}_d^{iT}(k) & 0 & 0 \\ * & * & -\bar{M}_2^{-1} - \bar{X}_1^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{L}_1 & 0 & 0 & 0 \\ * & * & * & * & -\bar{D}_2^{-2} \bar{X}_1 & 0 & 0 \\ * & * & * & * & * & -\theta I & 0 \\ * & * & * & * & * & * & -\theta I \end{bmatrix} < 0 \end{aligned} \quad (41)$$

where $\tilde{N}^{ij}(k) = (\tilde{A}^{ijT}(k) + \tilde{A}^{jiT}(k))/2$.

Thanks to $\tilde{A}^{ij}(k) = \bar{A}^i(k) + \bar{B}^i \alpha \bar{K}^j$, $\alpha = (I + \alpha_0)\beta$, $|\alpha_0| \leq \beta_0 \leq I$, (41) can be transformed into the following LMIs using Lemmas 6 and 4:

$$\begin{bmatrix}
 \bar{\phi}_1 & 0 & \bar{X}_1^{-1} & \bar{A}^{iT}(k) + \bar{K}^{iT} \bar{\beta} \bar{B}^{iT} & \bar{A}^{iT}(k) + \bar{K}^{iT} \bar{\beta} \bar{B}^{iT} - I & \bar{L}_1^{-1} \bar{Y}_1^{iT} \bar{R}_1^{1/2} & \bar{Q}_1^{1/2} & \bar{K}^{iT} \bar{\beta} & \bar{K}^{iT} \bar{\beta} \\
 * & -\bar{S}_1^{-1} & 0 & \bar{A}_d^{iT}(k) & \bar{A}_d^{iT}(k) & 0 & 0 & 0 & 0 \\
 * & * & -\bar{M}_2^{-1} - \bar{X}_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\
 * & * & * & -\bar{L}_1 + \bar{\varepsilon}_i \bar{B}^i \beta_0^2 \bar{B}^{iT} & 0 & 0 & 0 & 0 & 0 \\
 * & * & * & * & -\bar{D}_2^{-2} \bar{X}_1 + \bar{\varepsilon}_i \bar{B}^i \beta_0^2 \bar{B}^{iT} & 0 & 0 & 0 & 0 \\
 * & * & * & * & * & -\theta I & 0 & 0 & 0 \\
 * & * & * & * & * & * & -\theta I & 0 & 0 \\
 * & * & * & * & * & * & * & -\bar{\varepsilon}_i I & 0 \\
 * & * & * & * & * & * & * & * & -\bar{\varepsilon}_i I
 \end{bmatrix} < 0$$

$$\begin{bmatrix}
 \bar{\phi}_1 & 0 & \bar{X}_1^{-1} & \bar{N}^{ij} + \bar{L}_1^{-1} \bar{H}^{ij} & \bar{N}^{ij} + \bar{L}_1^{-1} \bar{H}^{ij} - I & \bar{L}_1^{-1} \bar{Y}_1^{iT} + \bar{Y}_1^{iT} \bar{R}_1^{1/2} & \bar{Q}_1^{1/2} & \frac{1}{2} \bar{K}^{iT} \bar{\beta} & \frac{1}{2} \bar{K}^{iT} \bar{\beta} & \frac{1}{2} \bar{K}^{iT} \bar{\beta} & \frac{1}{2} \bar{K}^{iT} \bar{\beta} \\
 * & -\bar{S}_1^{-1} & 0 & \bar{A}_d^{iT}(k) & \bar{A}_d^{iT}(k) & 0 & 0 & 0 & 0 & 0 & 0 \\
 * & * & -\bar{M}_2^{-1} - \bar{X}_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 * & * & * & -\bar{L}_1 + \bar{\varepsilon}_i \bar{B}^i \beta_0^2 \bar{B}^{iT} + \bar{\varepsilon}_j \bar{B}^j \beta_0^2 \bar{B}^{iT} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 * & * & * & * & -\bar{D}_2^{-2} \bar{X}_1 + \bar{\varepsilon}_i \bar{B}^i \beta_0^2 \bar{B}^{iT} + \bar{\varepsilon}_j \bar{B}^j \beta_0^2 \bar{B}^{iT} & 0 & 0 & 0 & 0 & 0 & 0 \\
 * & * & * & * & * & -\theta I & 0 & 0 & 0 & 0 & 0 \\
 * & * & * & * & * & * & -\theta I & 0 & 0 & 0 & 0 \\
 * & * & * & * & * & * & * & -\bar{\varepsilon}_i I & 0 & 0 & 0 \\
 * & * & * & * & * & * & * & * & -\bar{\varepsilon}_j I & 0 & 0 \\
 * & * & * & * & * & * & * & * & * & -\bar{\varepsilon}_j I & 0 \\
 * & * & * & * & * & * & * & * & * & * & -\bar{\varepsilon}_j I
 \end{bmatrix} \quad (42)$$

< 0

where $\bar{N}^{ij} = (\bar{A}^{iT}(k) + \bar{A}^{jT}(k))/2$ and $\bar{H}^{ij} = (\bar{Y}_1^{iT} \bar{\beta} \bar{B}^{iT} + \bar{Y}_1^{jT} \bar{\beta} \bar{B}^{jT})/2$.

The delay-range-dependent sufficient conditions (23) and (24) can be obtained by using $\text{diag}[\bar{L}_1 \ \bar{S}_1 \ \bar{X}_1 \ I \ I \ I \ I]$ to pre- and postmultiply (42) and letting $\bar{L}_1 \bar{M}_2^{-1} \bar{L}_1 = \bar{M}_3$, $\bar{L}_1 \bar{S}_1^{-1} \cdot \bar{L}_1 = \bar{S}_2$, $\bar{L}_1 \bar{X}_1^{-1} \bar{L}_1 = \bar{X}_2$, $\bar{X}_1 \bar{M}_2^{-1} \bar{X}_1 = \bar{M}_4$, $\bar{K}^i = \bar{Y}_1^i \bar{L}_1^{-1}$, $\bar{K}^j = \bar{Y}_1^j \bar{L}_1^{-1}$.

Moreover, to obtain the invariant set of the fuzzy closed-loop system $\sum_{E-T-S\text{-delay-C}}$, taking the maximum value of $\bar{x}_i(k) = \max(\bar{x}_i(r) \ \bar{\delta}_i(r))$, $r \in (k - d_M, k)$, one has

$$V(\bar{x}_1(k)) \leq \bar{x}_1^T(k) \bar{\psi}_i \bar{x}_1(k) \leq \theta \quad (43)$$

where $\bar{\psi}_i = \bar{P}_1 + \bar{d}_M \bar{T}_1 + \bar{d}_M \bar{M}_1 + ((d_m + d_M)/2)(d_M - d_m + 1) \bar{T}_1 + \bar{d}_M^2 ((1 + d_M)/2) \bar{G}_1$. Letting $\bar{\phi}_1 = \theta \bar{\psi}_i^{-1}$, it can obtain the sufficient condition (25) using the Lemma 4.

In order to ensure the matrix $\bar{\psi}_i$ to be invertible, letting $\psi_i = \bar{L}_1 \bar{\psi}_i \bar{L}_1$, $\bar{\psi}_i^{-1} = \bar{L}_1 \psi_i^{-1} \bar{L}_1$, $\bar{\psi}_i^{-1} < \theta^{-1} \bar{\phi} = \phi$, we can obtain $\bar{L}_1 \psi_i^{-1} \bar{L}_1 < \phi$, i.e., $-\phi + \bar{L}_1 \psi_i^{-1} \bar{L}_1 < 0$. Then, by the Lemma 4, the sufficient condition (26) can be obtained.

This completes the proof of Theorem 8. $\square$

Remark 9. Different from the traditional methods [45–47], a differential approach is applied to build up the Lyapunov-Krasovskii function candidate without some redundant free-weighting matrices. This constructed Lyapunov-Krasovskii function can take use of the information of the lower and upper bounds of the interval time-varying delay. Meanwhile, the bounding and model transformation techniques for cross terms using differential inequality are averted during the derivation of the obtained stable conditions (23) and (24).

Remark 10. Theorem 8 provides the sufficient conditions, together with the expression of the constant gains $\bar{K}^i, \bar{K}^j$, for the asymptotic stability of the fuzzy closed-loop system $\sum_{E-T-S\text{-delay-C}}$ with $\bar{w}(k) = 0$ that depends on the upper and lower bounds of the time-varying delay. First, Lyapunov-Krasovskii function $V(\bar{x}_1(k + p))$ is used, then the constant gains $\bar{K}^i, \bar{K}^j$ are acquired letting $\theta^{-1} \Delta V(\bar{x}_1(k + p | k)) + \theta^{-1} \bar{J}_p(k) \leq 0$ based on the expression of (28); second, it is converted into LMIs shown in (23) and (24). In addition, the invariant set (25) and condition (26) are acquired through some relax technologies. Therefore, by solving these LMIs in the Theorem 8, the corresponding control gains $\bar{K}^i, \bar{K}^j$ can be obtained.

Theorem 11. Given some scalars $\gamma > 0$, $\theta > 0$, $0 \leq d_m \leq d_M$, the delay-range-dependent sufficient conditions for the proposed controller that ensure the fuzzy closed-loop system $\Sigma_{E-T-S\text{-delay-C}}$ with $\bar{w}(k) \neq 0$ to be asymptotically controllable and to be a H_∞ performance less than γ is that there exists the symmetric positive matrices $\bar{P}_1, \bar{T}_1, \bar{M}_1, \bar{G}_1, \bar{L}_1, \bar{S}_1, \bar{S}_2, \bar{M}_3, \bar{M}_4, \bar{X}_1, \bar{X}_2 \in \mathbb{R}^{(n_x+n_e) \times (n_x+n_e)}$, matrices $\bar{Y}_1^i, \bar{Y}_1^j \in \mathbb{R}^{n_e \times (n_x+n_e)}$, and positive scalars $\bar{\varepsilon}_1^i, \bar{\varepsilon}_2^i, \bar{\varepsilon}_1^j, \bar{\varepsilon}_2^j$, such that the following LMI holds:

$$\begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & \bar{\Pi}_{14} \\ * & \bar{\Pi}_{22} & 0 & 0 \\ * & * & \bar{\Pi}_{33} & 0 \\ * & * & * & \bar{\Pi}_{44} \end{bmatrix} < 0 \quad (44)$$

$$\begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & \bar{\Pi}_{13} & \bar{\Pi}_{14} \\ * & \bar{\Pi}_{22} & 0 & 0 \\ * & * & \bar{\Pi}_{33} & 0 \\ * & * & * & \bar{\Pi}_{44} \end{bmatrix} < 0 \quad (45)$$

$$\begin{bmatrix} -1 & \bar{x}_l^T(k|k) \\ \bar{x}_l(k|k) & -\bar{\phi}_l \end{bmatrix} \leq 0 \quad (46)$$

$$\begin{bmatrix} -\phi & \bar{L}_1 \\ * & -\psi_l \end{bmatrix} \leq 0 \quad (47)$$

and the robust state-feedback H_∞ controller gains are given by $\bar{K}^i = \bar{Y}_1^i \bar{L}_1^{-1}$, $\bar{K}^j = \bar{Y}_1^j \bar{L}_1^{-1}$.
where

$$\begin{aligned} \bar{\Pi}_{11} &= \begin{bmatrix} \bar{\Pi}_{11} & 0 \\ 0 & -\gamma^2 I \end{bmatrix}, \\ \bar{\Pi}_{12} &= \begin{bmatrix} \bar{\Pi}_{12} & \bar{\Pi}_1 \\ \bar{\Pi}_2 & 0 \end{bmatrix}, \\ \bar{\Pi}_1 &= \begin{bmatrix} \bar{L}_1 \bar{E}^T \\ 0 \\ 0 \end{bmatrix}, \\ \bar{\Pi}_2 &= [\bar{G}^T \quad \bar{G}^T], \\ \bar{\Pi}_{12} &= \begin{bmatrix} \bar{\Pi}_{12} & \bar{\Pi}_1 \\ \bar{\Pi}_2 & 0 \end{bmatrix}, \\ \bar{\Pi}_{13} &= \begin{bmatrix} \bar{\Pi}_{13} \\ 0 \end{bmatrix}, \\ \bar{\Pi}_{13} &= \begin{bmatrix} \bar{\Pi}_{13} \\ 0 \end{bmatrix}, \\ \bar{\Pi}_{12} &= \begin{bmatrix} \bar{L}_1 \frac{\bar{A}^{iT} + \bar{A}^{jT}}{2} + \bar{H}^{ij} & \bar{L}_1 \frac{\bar{A}^{iT} + \bar{A}^{jT}}{2} + \bar{H}^{ij} - \bar{L}_1 \\ \bar{S}_1 \bar{A}_d^{iT}(k) & \bar{S}_1 \bar{A}_d^{iT}(k) \\ 0 & 0 \end{bmatrix}, \\ \bar{\Pi}_{14} &= \begin{bmatrix} \bar{\Pi}_3 & \bar{\Pi}_{14} \\ 0 & 0 \end{bmatrix}, \\ \bar{\Pi}_{14} &= \begin{bmatrix} \bar{\Pi}_3 & \bar{\Pi}_{14} \\ 0 & 0 \end{bmatrix}, \\ \bar{\Pi}_3 &= \begin{bmatrix} \bar{L}_1 \bar{H}^{iT} & \bar{L}_1 \bar{H}^{jT} \\ \bar{S}_1 \bar{H}_d^{iT} & \bar{S}_1 \bar{H}_d^{jT} \\ 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
\bar{\Pi}_3 &= \frac{1}{2} \begin{bmatrix} \bar{L}_1 \bar{H}^{iT} & \bar{L}_1 \bar{H}^{jT} & \bar{L}_1 \bar{H}^{iT} & \bar{L}_1 \bar{H}^{jT} \\ 2\bar{S}_1 \bar{H}_d^{iT} & 0 & 2\bar{S}_1 \bar{H}_d^{iT} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
\bar{\Pi}_{22} &= \text{diag} \begin{bmatrix} -\bar{L}_1 + \bar{\varepsilon}_1^i \bar{N}^i \bar{N}^{iT} + \bar{\varepsilon}_2^i \bar{B}^i \beta_0^2 \bar{B}^{iT} \\ -\bar{X}_1 \bar{D}_2^{-2} + \bar{\varepsilon}_1^i \bar{N}^i \bar{N}^{iT} + \bar{\varepsilon}_2^i \bar{B}^i \beta_0^2 \bar{B}^{iT} \\ -I \end{bmatrix}, \\
\bar{\Pi}_{33} &= \bar{\Pi}_{33}, \\
\bar{\Pi}_{44} &= -\text{diag} [\bar{\varepsilon}_1^i I \quad \bar{\varepsilon}_1^j I \quad \bar{\varepsilon}_2^i I \quad \bar{\varepsilon}_2^j I], \\
\bar{\Pi}_{22} &= \text{diag} \begin{bmatrix} -\bar{L}_1 + \bar{\varepsilon}_1^i \bar{N}^i \bar{N}^{iT} + \bar{\varepsilon}_1^j \bar{N}^j \bar{N}^{jT} + \bar{\varepsilon}_2^i \bar{B}^i \beta_0^2 \bar{B}^{iT} + \bar{\varepsilon}_2^j \bar{B}^j \beta_0^2 \bar{B}^{jT} \\ -\bar{X}_1 \bar{D}_2^{-2} + \bar{\varepsilon}_1^i \bar{N}^i \bar{N}^{iT} + \bar{\varepsilon}_1^j \bar{N}^j \bar{N}^{jT} + \bar{\varepsilon}_2^i \bar{B}^i \beta_0^2 \bar{B}^{iT} + \bar{\varepsilon}_2^j \bar{B}^j \beta_0^2 \bar{B}^{jT} \\ -I \end{bmatrix}, \\
\bar{\Pi}_{44} &= -\text{diag} [\bar{\varepsilon}_1^i I \quad \bar{\varepsilon}_1^j I \quad \bar{\varepsilon}_1^i I \quad \bar{\varepsilon}_1^j I \quad \bar{\varepsilon}_2^i I \quad \bar{\varepsilon}_2^j I \quad \bar{\varepsilon}_2^i I \quad \bar{\varepsilon}_2^j I].
\end{aligned} \tag{48}$$

Proof. Based on the derivation of Theorem 8, (38) can be rewritten due to $\bar{w}(k) \in l_2[0, \infty]$ as follows:

$$\theta^{-1} \Delta V(\bar{x}_1(k)) + \theta^{-1} \bar{J}(k) \leq \bar{\varphi}_2^T(k)(k) \bar{\Phi}_2 \bar{\varphi}_2(k) \tag{49}$$

where

$$\begin{aligned}
\bar{J}(k) &= \bar{x}_1(k)^T \bar{Q}_1 \bar{x}_1(k) + \Delta u(k)^T \bar{R}_1 \Delta u(k), \\
\bar{\varphi}_2(k) &= [\bar{x}_1^T(k) \quad \bar{x}_{1d}^T(k) \quad \bar{x}_{1d_M}^T(k) \quad \bar{w}(k)]^T,
\end{aligned}$$

$$\begin{aligned}
\bar{\Phi}_2 &= \begin{bmatrix} \bar{\phi}_1 & 0 & \bar{X}_1^{-1} & 0 \\ * & -\bar{S}_1^{-1} & 0 & 0 \\ * & * & -\bar{M}_2^{-1} - \bar{X}_1^{-1} & 0 \\ * & * & * & 0 \end{bmatrix} \\
&+ \begin{bmatrix} \bar{\Lambda}_1^T \\ \bar{G}^T \end{bmatrix} \bar{L}_1^{-1} [\bar{\Lambda}_1 \quad \bar{G}] \bar{\Lambda}_1 \\
&+ \begin{bmatrix} \bar{\Lambda}_2^T \\ \bar{G}^T \end{bmatrix} \bar{D}_2^2 \bar{X}_1^{-1} [\bar{\Lambda}_2 \quad \bar{G}] \\
&+ [\bar{\lambda}_1^T \quad 0]^T \theta^{-1} [\bar{\lambda}_1^T \quad 0] \\
&+ [\bar{\lambda}_2^T \quad 0]^T \theta^{-1} [\bar{\lambda}_2^T \quad 0].
\end{aligned} \tag{50}$$

In order to make the fuzzy closed-loop system $\Sigma_{E-T-S\text{-delay-C}}$ to be the H_∞ performance, the following expression is presented:

$$\begin{aligned}
z^T(k) z(k) - \gamma^2 \bar{w}^T(k) \bar{w}(k) + \theta^{-1} \Delta V(\bar{x}_1(k)) \\
+ \theta^{-1} \bar{J}(k) \leq \bar{\varphi}_2^T(k)(k) \bar{\Phi}_3 \bar{\varphi}_2(k)
\end{aligned} \tag{51}$$

where

$$\begin{aligned}
\bar{\Phi}_3 &= \begin{bmatrix} \bar{\phi}_1 & 0 & \bar{X}_1^{-1} & 0 \\ * & -\bar{S}_1^{-1} & 0 & 0 \\ * & * & -\bar{M}_2^{-1} - \bar{X}_1^{-1} & 0 \\ * & * & * & -\gamma^2 \end{bmatrix} \\
&+ \begin{bmatrix} \bar{\Lambda}_1^T \\ \bar{G}^T \end{bmatrix} \bar{L}_1^{-1} [\bar{\Lambda}_1 \quad \bar{G}] \bar{\Lambda}_1 \\
&+ \begin{bmatrix} \bar{\Lambda}_2^T \\ \bar{G}^T \end{bmatrix} \bar{D}_2^2 \bar{X}_1^{-1} [\bar{\Lambda}_2 \quad \bar{G}] + \begin{bmatrix} \bar{E}^T \\ 0 \\ 0 \\ 0 \end{bmatrix} [\bar{E} \quad 0 \quad 0 \quad 0] \\
&+ [\bar{\lambda}_1^T \quad 0]^T \theta^{-1} [\bar{\lambda}_1^T \quad 0] + [\bar{\lambda}_2^T \quad 0]^T \theta^{-1} [\bar{\lambda}_2^T \quad 0]
\end{aligned} \tag{52}$$

Using Lemma 4, the condition $\bar{\Phi}_3 < 0$ holds and is equivalent to the following:

$$\begin{bmatrix} \bar{\phi}_1 & 0 & \bar{X}_1^{-1} & 0 & \bar{A}^{iT}(k) & \bar{A}^{iT}(k) - I & \bar{E}^T & \bar{L}_1^{-1} \bar{Y}_1^T \bar{R}_1^{1/2} & \bar{Q}_1^{1/2} \\ * & -\bar{S}_1^{-1} & 0 & 0 & \bar{A}_d^{iT}(k) & \bar{A}_d^{iT}(k) & 0 & 0 & 0 \\ * & * & -\bar{M}_2^{-1} - \bar{X}_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 & \bar{G}^T & \bar{G}^T & 0 & 0 & 0 \\ * & * & * & * & -\bar{L}_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{D}_2^{-2} \bar{X}_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -\theta I & 0 \\ * & * & * & * & * & * & * & * & -\theta I \end{bmatrix} < 0 \quad (53)$$

$$\begin{bmatrix} \bar{\phi}_1 & 0 & \bar{X}_1^{-1} & 0 & \bar{N}^{ij}(k) & \bar{N}^{ij}(k) - I & \bar{E}^T & \bar{L}_1^{-1} \frac{\bar{Y}_1^T + \bar{Y}_1^{jT}}{2} \bar{R}_1^{1/2} & \bar{Q}_1^{1/2} \\ * & -\bar{S}_1^{-1} & 0 & 0 & \bar{A}_d^{iT}(k) & \bar{A}_d^{iT}(k) & 0 & 0 & 0 \\ * & * & -\bar{M}_2^{-1} - \bar{X}_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 & \bar{G}^T & \bar{G}^T & 0 & 0 & 0 \\ * & * & * & * & -\bar{L}_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{D}_2^{-2} \bar{X}_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -\theta I & 0 \\ * & * & * & * & * & * & * & * & -\theta I \end{bmatrix} < 0 \quad (54)$$

Thanks to $\bar{A}^{ij}(k) = \bar{A}^i(k) + \bar{B}^i \alpha \bar{K}^j, \bar{A}^i(k) = \bar{N}^i \bar{\Delta}^i(k) [\bar{H}^i \bar{H}_d^i], \alpha = (I + \alpha_0) \beta, \Delta^i(k) \Delta^i(k) \leq I |\alpha_0| \leq \beta_0 \leq I$, (53) and (54) can be transformed into the following LMIs using Lemmas 6 and 4:

$$\begin{bmatrix} \bar{\phi}_1 & 0 & \bar{X}_1^{-1} & 0 & \bar{A}^T + \bar{K}^T \beta \bar{B}^T & \bar{A}^T + \bar{K}^T \beta \bar{B}^T - I & \bar{E}^T & \bar{L}_1^{-1} \bar{Y}_1^T \bar{R}_1^{1/2} & \bar{Q}_1^{1/2} & \bar{H}^T & \bar{H}^T & \bar{K}^T \beta & \bar{K}^T \beta \\ * & -\bar{S}_1^{-1} & 0 & 0 & \bar{A}_d^T & \bar{A}_d^T & 0 & 0 & 0 & \bar{H}_d^T & \bar{H}_d^T & 0 & 0 \\ * & * & -\bar{M}_2^{-1} - \bar{X}_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 & \bar{G}^T & \bar{G}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\bar{L}_1 + \bar{\varepsilon}_1^i \bar{N}^i \bar{N}^T + \bar{\varepsilon}_2^j \bar{B}^j \beta_0^2 \bar{B}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\bar{X}_1 \bar{D}_2^{-2} + \bar{\varepsilon}_1^i \bar{N}^i \bar{N}^T + \bar{\varepsilon}_2^j \bar{B}^j \beta_0^2 \bar{B}^T & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\theta I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\theta I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\bar{\varepsilon}_1^i I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \bar{\varepsilon}_1^i I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\bar{\varepsilon}_2^j I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & * & -\bar{\varepsilon}_2^j I \end{bmatrix} < 0 \quad (55)$$

$$\begin{bmatrix}
\bar{\phi}_1 & 0 & \bar{X}_1^{-1} & 0 & \frac{\bar{A}^T + \bar{A}^T}{2} + \bar{L}_1^{-1} \bar{H}^{ij} & \frac{\bar{A}^T + \bar{A}^T}{2} + \bar{L}_1^{-1} \bar{H}^{ij} - I & \bar{E}^T & \bar{L}_1^{-1} \frac{\bar{Y}_1^T + \bar{Y}_1^T}{2} \bar{R}_1^{1/2} & \bar{Q}_1^{1/2} & \frac{1}{2} \bar{H}^T & \frac{1}{2} \bar{H}^T & \frac{1}{2} \bar{H}^T & \frac{1}{2} \bar{H}^T & \frac{1}{2} \bar{K}^T \beta & \frac{1}{2} \bar{K}^T \beta & \frac{1}{2} \bar{K}^T \beta & \frac{1}{2} \bar{K}^T \beta \\
* & -\bar{S}_1^{-1} & 0 & 0 & \bar{A}_d^T & \bar{A}_d^T & 0 & 0 & 0 & \frac{1}{2} \bar{H}_d^T & 0 & \frac{1}{2} \bar{H}_d^T & 0 & 0 & 0 & 0 & 0 \\
* & * & -\bar{M}_2^{-1} - \bar{X}_1^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\gamma^2 & \bar{G}^T & \bar{G}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\bar{L}_1 + \bar{e}_1^j \bar{N}^j \bar{N}^{jT} + \bar{e}_1^j \bar{N}^j \bar{N}^{jT} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
& & & & + \bar{e}_2^j \bar{B}^j \beta_0^2 \bar{B}^{jT} + \bar{e}_2^j \bar{B}^j \beta_0^2 \bar{B}^{jT} & & & & & & & & & & & & & \\
* & * & * & * & * & -\bar{X}_1 \bar{D}_2^{-2} + \bar{e}_1^j \bar{N}^j \bar{N}^{jT} + \bar{e}_1^j \bar{N}^j \bar{N}^{jT} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
& & & & & + \bar{e}_2^j \bar{B}^j \beta_0^2 \bar{B}^{jT} + \bar{e}_2^j \bar{B}^j \beta_0^2 \bar{B}^{jT} & & & & & & & & & & & & \\
* & * & * & * & * & * & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -\theta I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & -\theta I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & -\bar{e}_1^j I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & -\bar{e}_1^j I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & -\bar{e}_1^j I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & -\bar{e}_2^j I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & -\bar{e}_2^j I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & -\bar{e}_2^j I & 0 & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -\bar{e}_2^j I & 0 \\
* & * & * & * & * & * & * & * & * & * & * & * & * & * & * & * & -\bar{e}_2^j I
\end{bmatrix} \quad (56)$$

< 0

The delay-range-dependent sufficient conditions (44) and (45) can be obtained by using $\text{diag}[\bar{L}_1 \ \bar{S}_1 \ \bar{X}_1 \ I \ I \ I \ I]$ to pre- and postmultiply (55) and (56) and letting $\bar{L}_1 \bar{M}_2^{-1} \bar{L}_1 = \bar{M}_3$, $\bar{L}_1 \bar{S}_1^{-1} \bar{L}_1 = \bar{S}_2$, $\bar{L}_1 \bar{X}_1^{-1} \bar{L}_1 = \bar{X}_2$, $\bar{X}_1 \bar{M}_2^{-1} \bar{X}_1 = \bar{M}_4$, $\bar{K}^i = \bar{Y}_1^i \bar{L}_1^{-1}$, $\bar{K}^j = \bar{Y}_1^j \bar{L}_1^{-1}$.

Summing up both sides of (51) from $k=0$ to ∞ , it has

$$\begin{aligned}
& \sum_{k=0}^{\infty} [z^T(k) z(k)] - \sum_{k=0}^{\infty} [\gamma^2 \bar{w}^T(k) \bar{w}(k)] \\
& + \sum_{k=0}^{\infty} [\theta^{-1} \Delta V(\bar{x}_1(k))] + \sum_{k=0}^{\infty} [\theta^{-1} \bar{J}(k)] \\
& = \sum_{k=0}^{\infty} [z^T(k) z(k)] - \sum_{k=0}^{\infty} [\gamma^2 \bar{w}^T(k) \bar{w}(k)] \\
& + \theta^{-1} V(\bar{x}_1(\infty)) - \theta^{-1} V(\bar{x}_1(0)) + \theta^{-1} \bar{J}_{\infty} < 0
\end{aligned} \quad (57)$$

With the zero initial condition, we have

$$\begin{aligned}
V(\bar{x}_1(0)) &= 0, \\
V(\bar{x}_1(\infty)) &\geq 0, \\
\bar{J}_{\infty} &> 0, \\
\sum_{k=0}^{\infty} [z^T(k) z(k)] &< \sum_{k=0}^{\infty} [\gamma^2 \bar{w}^T(k) \bar{w}(k)]
\end{aligned} \quad (58)$$

Therefore, the H_{∞} performance index $\|z\|_2 \leq \gamma \|\bar{w}\|_2$ is guaranteed. This completes the proof of Theorem 11. $\square$

Remark 12. Theorem 11 provides the sufficient conditions, together with the expression of the constant gains $\bar{K}^i$, $\bar{K}^j$, for the asymptotic stability of the fuzzy closed-loop system

$\Sigma_{E-T-S\text{-delay-C}}$ with $\bar{w}(k) \neq 0$ that depends on the upper and lower bounds of the time-varying delay. First, Lyapunov-Krasovskii function $V(\bar{x}_1(k+p))$ is used, then the constant gains $\bar{K}^i$, $\bar{K}^j$ are acquired letting $z^T(k) z(k) - \gamma^2 \bar{w}^T(k) \cdot \bar{w}(k) + \theta^{-1} \Delta V(\bar{x}_1(k)) + \theta^{-1} \bar{J}(k) < 0$; second, it is converted into LMIs shown in (44) and (45). In addition, the invariant set (46) and condition (47) are acquired through some relax technologies. Therefore, by solving these LMIs in the Theorem 11, the corresponding control gains $\bar{K}^i$, $\bar{K}^j$ can be obtained.

Remark 13. Theorem 8 provides LMIs to guarantee the fuzzy closed-loop system $\Sigma_{E-T-S\text{-delay-C}}$ with $\bar{w}(k) = 0$ to be asymptotically stable and controllable. In the derivation of the Theorem 8, we can see that the optimized cost function is introduced to the stable conditions to guarantee the robust performance. On this basis, for the fuzzy closed-loop system $\Sigma_{E-T-S\text{-delay-C}}$ with $\bar{w}(k) \neq 0$, the H-infinity performance index is introduced to the stable conditions of the Theorem 11 to guarantee the antidisturbance capability.

4. Case Study in a CSTR

4.1. System Description. In this section, the nonlinear continuous stirred reactor (CSTR) [48] in Figure 1 is used to verify the control performance of the proposed method. Assume that an exothermic irreversible reaction ($A \rightarrow B$) occurs in the CSTR.

Suppose at the input that the fresh feed of pure A is to be mixed with a recycle stream of unreacted A with recycle flow rate $(1-\lambda)q$ [49]. Let $d(t)$ as the time-varying the output exits the CSTR. It includes the following two nonlinear differential equations:

FIGURE 1: Nonlinear continuous stirred reactor.

$$\begin{aligned} \dot{T}(t) &= \frac{q}{V} [\lambda T_0 + (1 - \lambda) T(t - d(t)) - T(t)] \\ &\quad - \frac{k_0 \Delta H}{\rho C_p} C_A(t) \exp\left(\frac{-E}{RT(t)}\right) \\ &\quad + \frac{UA}{\rho C_p V} [\alpha T_c(t) - T(t)] \end{aligned} \quad (59)$$

$$\begin{aligned} \dot{C}_A(t) &= \frac{1}{V} [\lambda q C_{A0} + (1 - \lambda) q C_A(t - d(t)) - q C_A(t)] \\ &\quad - k_0 C_A(t) \exp\left(\frac{-E}{RT(t)}\right) \end{aligned} \quad (60)$$

where C_A is the concentration of A component when it flows out of the reactor, T is the temperature of the reactor, T_c is the temperature of the coolant stream, and α is an unknown actuator failure. Other process parameters are $q = 100\text{L/min}$, $\lambda = 0.9$, $V = 100\text{L}$, $C_{A0} = 1\text{mol/L}$, $T_0 = 400\text{K}$, $\rho = 1000\text{g/L}$, $C_p = 1\text{J/gK}$, $k_0 = 4.71 \times 10^8 \text{min}^{-1}$, $E/R = 800\text{K}$, $\Delta H = -2 \times 10^5 \text{J/mol}$, $UA = 1 \times 10^5 \text{J/minK}$, and the main control objective is to adjust the temperature of the coolant stream T_c to track the expected value of the temperature T of the reactor.

4.2. Modelling Using T-S Fuzzy Method. Substituting the above process parameters to (55) and (56), they can be transformed into the following form:

$$\begin{aligned} \dot{T}(t) &= 360 + 0.1 \times T(t - d(t)) - T(t) + 200 \times 4.71 \\ &\quad \times 10^8 \times \exp\left(\frac{-8000}{T(t)}\right) C_A(t) + \alpha T_c(t) \\ &\quad - T(t) \end{aligned} \quad (61)$$

$$\begin{aligned} \dot{C}_A(t) &= 0.9 + 0.1 \times C_A(t - d(t)) - C_A(t) - 4.71 \\ &\quad \times 10^8 \times \exp\left(\frac{-8000}{T(t)}\right) C_A(t) + T \times \frac{1}{T} \end{aligned} \quad (62)$$

The state space model can be acquired by selecting the state variables as

$$x(t) = [x_1(t) \ x_2(t)]^T = [T \ C_A]^T \quad (63)$$

manipulate variable as

$$u(t) = T_c \quad (64)$$

giving the form

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t - d(t)) + B u(t) \\ y(t) &= Cx(t) \end{aligned} \quad (65)$$

where

$$\begin{aligned} A &= \begin{bmatrix} 360 \times \frac{1}{x_1(t)} - 2 & 200 \times 4.71 \times 10^8 \times \exp\left(\frac{-8000}{x_1(t)}\right) \\ \frac{0.9}{x_1(t)} & -\left(1 + 4.71 \times 10^8 \times \exp\left(\frac{-8000}{x_1(t)}\right)\right) \end{bmatrix}, \\ A_d &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ B &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ C &= [1 \ 0]. \end{aligned} \quad (66)$$

Remark 14. The original nonlinear differential equations (59) and (60) can be expressed by the nonlinear model (65) in the form of state space. Here, the T-S fuzzy strategy is used to linearize the nonlinear system (65) in the discrete time. The specific step is as follows.

Firstly, the premise variables are defined as $Z_1(t) = \exp(-8000/x_1(t))$, $Z_2(t) = 1/x_1(t)$. The matrix A can be transformed into the following form:

$$A = \begin{bmatrix} 360 \times Z_2(t) - 2 & 200 \times 4.71 \times 10^8 \times Z_1(t) \\ 0.9 \times Z_2(t) & -(1 + 4.71 \times 10^8 \times Z_1(t)) \end{bmatrix} \quad (67)$$

Next, calculate the minimum and maximum values of $Z_1(t)$ and $Z_2(t)$ under $x_1(t) \in [250, 500]$. They can be obtained as follows:

$$\begin{aligned} \min_{x_1(t)} Z_1(t) &= \exp\left(\frac{-8000}{250}\right), \\ \max_{x_1(t)} Z_1(t) &= \exp\left(\frac{-8000}{500}\right), \\ \min_{x_1(t)} Z_2(t) &= \frac{1}{500}, \\ \max_{x_1(t)} Z_2(t) &= \frac{1}{250}. \end{aligned} \quad (68)$$

From the maximum and minimum values, $Z_1(t)$ and $Z_2(t)$ are expressed as

$$\begin{aligned} Z_1(t) &= \exp\left(\frac{-8000}{x_1(t)}\right) \\ &= M_1(Z_1(t)) \times \exp\left(\frac{-8000}{250}\right) + M_2(Z_1(t)) \\ &\quad \times \exp\left(\frac{-8000}{500}\right) \\ Z_2(t) &= \exp\left(\frac{-8000}{x_1(t)}\right) \\ &= N_1(Z_2(t)) \times \frac{1}{500} + N_2(Z_2(t)) \times \frac{1}{250} \end{aligned} \quad (69)$$

where $M_1(Z_1(t)) + M_2(Z_1(t)) = 1$, $N_1(Z_2(t)) + N_2(Z_2(t)) = 1$. Therefore, the membership functions can be obtained as follows:

$$\begin{aligned} M_1(Z_1(t)) &= \frac{e^{8000/500} Z_1(t) - 1}{e^{8000/500} - e^{8000/250}}, \\ M_2(Z_1(t)) &= 1 - M_1(Z_1(t)) \\ N_1(Z_2(t)) &= \frac{500 \times (Z_2(t) \times 250 - 1)}{250 - 500}, \\ N_2(Z_2(t)) &= 1 - N_1(Z_2(t)) \end{aligned} \quad (70)$$

The above membership functions are defined as ‘Positive’, ‘Negative’, ‘Big’, and ‘Small’, as shown in Figures 2 and 3.

After discretization in the sample time of 1 seconds, the nonlinear system (65) can be described by the following fuzzy model with uncertainties and unknown disturbances.

Rule 1. IF $Z_1(k)$ is “Negative” and $Z_2(k)$ is “Big”, then

$$\begin{aligned} x(k+1) &= A^1(k)x(k) + A_d^1(k)x(k-d(k)) \\ &\quad + B^1\alpha u(k) + w(k) \end{aligned} \quad (71)$$

Rule 2. IF $Z_1(k)$ is “Negative” and $Z_2(k)$ is “Small”, then

$$\begin{aligned} x(k+1) &= A^2(k)x(k) + A_d^2(k)x(k-d(k)) \\ &\quad + B^2\alpha u(k) + w(k) \end{aligned} \quad (72)$$

Rule 3. IF $Z_1(k)$ is “Positive” and $Z_2(k)$ is “Big”, then

$$\begin{aligned} x(k+1) &= A^3(k)x(k) + A_d^3(k)x(k-d(k)) \\ &\quad + B^3\alpha u(k) + w(k) \end{aligned} \quad (73)$$

Rule 4. IF $Z_1(k)$ is “Positive” and $Z_2(k)$ is “Small”, then

$$\begin{aligned} x(k+1) &= A^4(k)x(k) + A_d^4(k)x(k-d(k)) \\ &\quad + B^4\alpha u(k) + w(k) \end{aligned} \quad (74)$$

Here the output maintains $y(k) = Cx(k)$. After defuzzification, the nonlinear system (65) is expressed as the following linear model:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^4 \hat{h}^i [A^i(k)x(k) + A_d^i(k)x(k-d(k)) \\ &\quad + B^i\alpha u(k)] + w(k) \\ y(k) &= Cx(k) \end{aligned} \quad (75)$$

where

$$\begin{aligned} 1 &\leq d(k) \leq 4, \\ A^1 &= \begin{bmatrix} 0.5712 & 0.0006 \\ 0.0017 & 0.3679 \end{bmatrix}, \\ B^1 &= \begin{bmatrix} 0.7657 \\ 0.0011 \end{bmatrix}, \\ A^2 &= \begin{bmatrix} 0.2780 & 0.0004 \\ 0.0006 & 0.3679 \end{bmatrix}, \\ B^2 &= \begin{bmatrix} 0.5640 \\ 0.0004 \end{bmatrix}, \\ A^3 &= \begin{bmatrix} 1.1409 & 223.3625 \\ 0.0001 & 0.0148 \end{bmatrix}, \\ B^3 &= \begin{bmatrix} 1.0624 \\ 0.00007 \end{bmatrix}, \\ A^4 &= \begin{bmatrix} 0.3956 & 79.0066 \\ 0.00001 & 0.0027 \end{bmatrix}, \\ B^4 &= \begin{bmatrix} 0.6494 \\ 0.00002 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
A_d^1 &= \begin{bmatrix} 0.0766 & 0.000036 \\ 0.000109 & 0.0632 \end{bmatrix}, \\
A_d^2 &= \begin{bmatrix} 0.0564 & 0.000029 \\ 0.00004 & 0.0632 \end{bmatrix}, \\
A_d^3 &= \begin{bmatrix} 0.1062 & 20.4414 \\ 0.000007 & 0.0032 \end{bmatrix}, \\
A_d^4 &= \begin{bmatrix} 0.0649 & 12.6004 \\ 0.0000021 & 0.0023 \end{bmatrix}, \\
N^1 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
H^1 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \\
H_d^1 &= \begin{bmatrix} 0.01 & 0 \\ 0 & 0.03 \end{bmatrix}, \\
N^2 &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
H^2 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \\
H_d^2 &= \begin{bmatrix} 0.02 & 0 \\ 0 & 0.02 \end{bmatrix}, \\
N^3 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \\
H^3 &= \begin{bmatrix} 0.3 & 0 \\ 0 & 0.01 \end{bmatrix}, \\
H_d^3 &= \begin{bmatrix} 0.3 & 0 \\ 0 & 0.001 \end{bmatrix}, \\
N^4 &= \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
H^4 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.001 \end{bmatrix}, \\
H_d^4 &= \begin{bmatrix} 0.02 & 0 \\ 0 & 0.002 \end{bmatrix}, \\
C &= [1 \ 0], \\
\Delta(k) &= \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix}, \\
w(k) &= (0.4\Delta_3 \ 0.4\Delta_4)^T,
\end{aligned} \tag{76}$$

in which $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ are random numbers within $[-1 \ 1]$. There exists an unknown actuator failure α . However, we know that $0.6 = \underline{\alpha} \leq \alpha \leq \bar{\alpha} = 1.2$. Using (6), $\beta = 0.9, \beta_0 = 0.33$ can be obtained. The set-point profile is set as

$$\begin{aligned}
c(k) &= 376 + k \times \frac{(380 - 376)}{120}, \quad 0 < k \leq 120 \\
c(k) &= 370, \quad 120 < k \leq 300 \\
c(k) &= 360 + (k - 300) \times \frac{(360 - 370)}{180}, \quad 300 < k \leq 480 \\
c(k) &= 370, \quad 480 < k \leq 600
\end{aligned} \tag{77}$$

4.3. Simulation Results. The model (75) is used as the proposed method in this section. The control parameters are selected as $\bar{Q}_1 = \text{diag}[15, 6, 1]$, $\bar{R}_1 = 0.15$.

Case 1 (constant fault). In this case, two groups of different constant fault values are considered to evaluate the control performance of the proposed method. The two cases are as follows:

- case 1. $\alpha = 0.6$
- case 2. $\alpha = 0.8$

The control results of the proposed method for the temperature T of CSTR process with such a partial actuator fault are shown in Figures 4 and 5. Figure 4 shows the output response and the effect of control input for the proposed method. It is clearly found that, as the fault value α becomes severe, the control performance of the proposed approach deteriorates. However, the output response can track the set profile as smoothly and quickly as possible. From the figure, it also can be seen that, as the fault value α becomes severe, the control input of the proposed method becomes violent. But a faster control action can be seen by using the proposed method to track the changes of the set-value and overcome the uncertainties, unknown disturbance and constant actuator fault. Figure 5 shows the effect of the output tracking error between the output response and set profile. A significantly different about the control performance can be found in the figure.

Case 2 (random-varying fault). In this case, two groups of different random-varying fault values are further studied to test the control performance of the proposed method. The two cases are as follows:

- case 3. $\alpha = 0.8 + 0.2\Delta_5$
- case 4. $\alpha = 0.8 + 0.1\Delta_5$

where Δ_5 is a random number within $[-1 \ 1]$. Under such partial actuator fault, we'll see that the control performance deteriorates with a little oscillation shown in Figures 6 and 7. From Figure 6, it is also clearly seen that, as the range of the random-varying fault becomes large, the control performance also worsens. However, the proposed method

FIGURE 2: Membership functions $M_1(Z_1(t))$ and $M_2(Z_1(t))$. FIGURE 3: Membership functions $N_1(Z_2(t))$ and $N_2(Z_2(t))$.

FIGURE 4: Output response and control input and under constant fault.

has good robustness and convergence against such random-varying fault, uncertainties and unknown disturbances. From this figure, the faster and smoother input signal can be found using the proposed method. A good control performance can be found in the Figure 7.

5. Conclusion

In this work, a fuzzy predictive fault-tolerant control scheme is reported for a wide class of time-delay nonlinear systems with uncertainties, partial actuator failures, and unknown disturbances. By utilizing the prescribed fuzzy rules, the nonlinear system is constructed by a number of local discrete uncertain time-varying delay state space models and the nonlinear membership functions. The extended state space model is obtained by introducing the output tracking error in order to increase more control degrees of freedom.

By establishing a Lyapunov-Krasovskii function candidate, less conservative delay-range-dependent stable conditions combined with optimized cost function and H-infinity performance index are given in terms of LMI. An industrial case on CSTR shows that the proposed control scheme has better capabilities of both tracking and disturbance rejection. Because of the multiphase characteristic in many industrial processes such as an injection molding process including filling, packing, cooling, and mold open, the traditional control methods are very difficult for such conditions. Recent works have demonstrated that the average dwell method is good at dealing with the switching problem in each phase leading to the motivation of integrating the average dwell method with robust predictive control to achieve a novel control scheme for industrial processes, which will be some interesting issues as our future works. In addition, the utilization of the nonquadratic Lyapunov functions to relax

FIGURE 5: Output tracking error under constant fault.

FIGURE 6: Output response and control input under random-varying fault.

FIGURE 7: Output error under random-varying fault.

the design conditions is another idea to guarantee the closed-loop system to be asymptotically stable and controllable, which may be our future research work.

Data Availability

We cannot offer the related data about our paper because the extended work is study about multiphase control that is stated in conclusion.

Conflicts of Interest

The authors declare no competing financial interest.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grant 61673199 and Natural Science Foundation of Liaoning Province 20180550905 and 2019JH3/10300025.

References

- [1] M. Dotoli, A. Fay, M. Miśkowicz, and C. Seatzu, "Advanced control in factory automation: a survey," *International Journal of Production Research*, vol. 55, no. 5, pp. 1243–1259, 2016.
- [2] H. Shi, C. Su, J. Cao, P. Li, J. Liang, and G. Zhong, "Non-linear adaptive predictive functional control based on the takagi-sugeno model for average cracking outlet temperature of the ethylene cracking furnace," *Industrial & Engineering Chemistry Research*, vol. 54, no. 6, pp. 1849–1860, 2015.
- [3] H. Shi, C. Su, J. Cao, P. Li, Y. Song, and N. Li, "Incremental multivariable predictive functional control and its application in a gas fractionation unit," *Journal of Central South University*, vol. 22, no. 12, pp. 4653–4668, 2015.
- [4] A. Locatelli and N. Schiavoni, "Fault-tolerant pole-placement in double-integrator networks," *IEEE Transactions on Automatic Control*, vol. 57, no. 11, pp. 2912–2917, 2012.
- [5] R. Zhang, S. Wu, and F. Gao, "State Space Model Predictive Control for Advanced Process Operation: A Review of Recent Development, New Results, and Insight," *Industrial & Engineering Chemistry Research*, vol. 56, no. 18, pp. 5360–5394, 2017.
- [6] L. Wang, B. Liu, J. Yu, P. Li, R. Zhang, and F. Gao, "Delay-range-dependent-based hybrid iterative learning fault-tolerant guaranteed cost control for multi-phase batch processes," *Industrial & Engineering Chemistry Research*, vol. 57, no. 8, pp. 2932–2944, 2018.
- [7] L. Wang, Y. Shen, J. Yu, P. Li, R. Zhang, and F. Gao, "Robust iterative learning control for multi-phase batch processes: an average dwell-time method with 2D convergence indexes," *International Journal of Systems Science*, vol. 49, no. 2, pp. 324–343, 2017.
- [8] A. Nguyen, T. Taniguchi, L. Eciolaza, V. Campos, R. Palhares, and M. Sugeno, "Fuzzy control systems: past, present and future," *IEEE Computational Intelligence Magazine*, vol. 14, no. 1, pp. 56–68, 2019.
- [9] M. Klug, E. B. Castelan, V. J. S. Leite, and L. F. P. Silva, "Fuzzy dynamic output feedback control through nonlinear Takagi-Sugeno models," *Fuzzy Sets and Systems*, vol. 263, pp. 92–111, 2015.
- [10] A. Schrodtt and A. Kroll, "On iterative closed-loop identification using affine takagi-sugeno models and controllers," *International Journal of Fuzzy Systems*, vol. 19, no. 6, pp. 1978–1988, 2017.
- [11] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, vol. SMC-15, no. 2, pp. 116–132, 1985.
- [12] L. Zhao, H. Gao, and H. R. Karimi, "Robust stability and stabilization of uncertain T-S fuzzy systems with time-varying delay: an input-output approach," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 5, pp. 883–897, 2013.
- [13] F. O. Souza, V. C. S. Campos, and R. M. Palhares, "On delay-dependent stability conditions for Takagi-Sugeno fuzzy systems," *Journal of The Franklin Institute*, vol. 351, no. 7, pp. 3707–3718, 2014.
- [14] A.-T. Nguyen, M. Dambrine, and J. Lauber, "Simultaneous design of parallel distributed output feedback and anti-windup compensators for constrained Takagi-Sugeno fuzzy systems," *Asian Journal of Control*, vol. 18, no. 5, pp. 1641–1654, 2016.
- [15] B. Ding and H. Pan, "Dynamic output feedback-predictive control of a takagi-sugeno model with bounded disturbance," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 3, pp. 653–667, 2017.
- [16] D. El Hellani, A. El Hajjaji, and R. Ceschi, "Finite frequency Hoo filter design for T-S fuzzy systems: New approach," *Signal Processing*, vol. 143, pp. 191–199, 2018.
- [17] M. Witczak, *Fault Diagnosis and Fault-Tolerant Control Strategies for Non-Linear Systems*, vol. 266, Springer, Switzerland, 2014.
- [18] M. Ławryńczuk, *Computationally Efficient Model Predictive Control Algorithms: A Neural Network Approach*, Springer, Switzerland, 2014.
- [19] Y. Y. Xu, S. C. Tong, and Y. M. Li, "Prescribed performance fuzzy adaptive fault-tolerant control of non-linear systems with actuator faults," *IET Control Theory & Applications*, vol. 8, no. 6, pp. 420–431, 2014.
- [20] I. H. Brahim, M. Chaabane, and D. Mehdi, "Fault-tolerant control for T-S fuzzy descriptor systems with sensor faults: an LMI approach," *International Journal of Fuzzy Systems*, vol. 19, no. 2, pp. 516–527, 2017.
- [21] D. Rotondo, F. Nejjari, and V. Puig, "Fault tolerant control of a proton exchange membrane fuel cell using Takagi-Sugeno virtual actuators," *Journal of Process Control*, vol. 45, pp. 12–29, 2016.
- [22] Y. Lina and L. Chunhui, "Fault diagnosis and sliding mode fault tolerant control for non-gaussian stochastic distribution control systems using T-s Fuzzy model," *Asian Journal of Control*, vol. 19, no. 2, pp. 636–646, 2017.
- [23] L. Cheng, W. Liu, Z. Hou, T. Huang, J. Yu, and M. Tan, "An Adaptive Takagi-Sugeno Fuzzy Model-Based Predictive Controller for Piezoelectric Actuators," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 4, pp. 3048–3058, 2017.
- [24] S. Abdelmalek, A. T. Azar, and D. Dib, "A Novel Actuator Fault-tolerant Control Strategy of DFIG-based Wind Turbines Using Takagi-Sugeno Multiple Models," *International Journal of Control, Automation, and Systems*, vol. 16, no. 3, pp. 1415–1424, 2018.
- [25] H. Gassara, A. El Hajjaji, and M. Chaabane, "Adaptive fault tolerant control design for Takagi-Sugeno fuzzy systems with interval time-varying delay," *Optimal Control Applications and Methods*, vol. 35, no. 5, pp. 609–625, 2014.

- [26] S.-J. Huang and G.-H. Yang, "Fault tolerant controller design for T-S fuzzy systems with time-varying delay and actuator faults: a k-step fault-estimation approach," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 6, pp. 1526–1540, 2014.
- [27] C. Sun, F. Wang, and X. He, "Robust dynamic output feedback fault-tolerant control for Takagi-Sugeno fuzzy systems with interval time-varying delay via improved delay partitioning approach," *Open Mathematics*, vol. 14, no. 1, pp. 986–1006, 2016.
- [28] H. Li, F. You, F. Wang, and S. Guan, "Robust fast adaptive fault estimation and tolerant control for T-S fuzzy systems with interval time-varying delay," *International Journal of Systems Science*, vol. 48, no. 8, pp. 1708–1730, 2017.
- [29] L. Wang, B. Li, J. Yu, R. Zhang, and F. Gao, "Design of fuzzy iterative learning fault-tolerant control for batch processes with time-varying delays," *Optimal Control Applications and Methods*, vol. 39, no. 6, pp. 1887–1903, 2018.
- [30] Y. Wang, J. Shi, D. Zhou, and F. Gao, "Iterative Learning Fault-Tolerant Control for Batch Processes," *Industrial & Engineering Chemistry Research*, vol. 45, no. 26, pp. 9050–9060, 2006.
- [31] L. Wang, S. Mo, D. Zhou, F. Gao, and X. Chen, "Robust delay dependent iterative learning fault-tolerant control for batch processes with state delay and actuator failures," *Journal of Process Control*, vol. 22, no. 7, pp. 1273–1286, 2012.
- [32] L. Wang, L. Sun, J. Yu, R. Zhang, and F. Gao, "Robust iterative learning fault-tolerant control for multiphase batch processes with uncertainties," *Industrial & Engineering Chemistry Research*, vol. 56, no. 36, pp. 10099–10109, 2017.
- [33] J. Tao, Y. Zhu, and Q. Fan, "Improved state space model predictive control design for linear systems with partial actuator failure," *Industrial & Engineering Chemistry Research*, vol. 53, no. 9, pp. 3578–3586, 2014.
- [34] R. Zhang, J. Qiao, T. Li, and L. Guo, "Robust fault-tolerant control for flexible spacecraft against partial actuator failures," *Nonlinear Dynamics*, vol. 76, no. 3, pp. 1753–1760, 2014.
- [35] R. Zhang, R. Lu, A. Xue, and F. Gao, "New Minmax Linear Quadratic Fault-Tolerant Tracking Control for Batch Processes," *IEEE Transactions on Automatic Control*, vol. 61, no. 10, pp. 3045–3051, 2016.
- [36] R. Zhang, F. Gao, and P. D. Christofides, "An improved approach for H_{∞} design of linear quadratic tracking control for chemical processes with partial actuator failure," *Journal of Process Control*, vol. 58, pp. 63–72, 2017.
- [37] H. Shi, P. Li, C. Su, Y. Wang, J. Yu, and J. Cao, "Robust constrained model predictive fault-tolerant control for industrial processes with partial actuator failures and interval time-varying delays," *Journal of Process Control*, vol. 75, pp. 187–203, 2019.
- [38] M. Ławryńczuk, "Nonlinear predictive control of a boiler-turbine unit: a state-space approach with successive on-line model linearisation and quadratic optimization," *Isa T*, vol. 67, pp. 476–495, 2017.
- [39] H. Li, Y. Shi, W. Yan, and F. Liu, "Receding horizon consensus of general linear multi-agent systems with input constraints: An inverse optimality approach," *Automatica*, vol. 91, pp. 10–16, 2018.
- [40] H. Li, W. Yan, and Y. Shi, "Triggering and control codesign in self-triggered model predictive control of constrained systems: with guaranteed performance," *IEEE Transactions on Automatic Control*, vol. 63, no. 11, pp. 4008–4015, 2018.
- [41] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," *Control Engineering Practice*, vol. 11, no. 7, pp. 733–764, 2003.
- [42] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, Pa, USA, 1994.
- [43] J. Qiu, Y. Xia, H. Yang, and J. Zhang, "Robust stabilisation for a class of discrete-time systems with time-varying delays via delta operators," *IET Control Theory & Applications*, vol. 2, no. 1, pp. 87–93, 2008.
- [44] K.-W. Yu and C.-H. Lien, "Stability criteria for uncertain neutral systems with interval time-varying delays," *Chaos, Solitons & Fractals*, vol. 38, no. 3, pp. 650–657, 2008.
- [45] H. Li, B. Chen, Q. Zhou, and C. Lin, "A delay-dependent approach to robust H_{∞} control for uncertain stochastic systems with state and input delays," *Circuits, Systems and Signal Processing*, vol. 28, no. 1, pp. 169–183, 2009.
- [46] T. Liu and F. Gao, "Robust two-dimensional iterative learning control for batch processes with state delay and time-varying uncertainties," *Chemical Engineering Science*, vol. 65, no. 23, pp. 6134–6144, 2010.
- [47] H. Li, Q. Zhou, B. Chen, and H. Liu, "Parameter-dependent robust stability for uncertain Markovian jump systems with time delay," *Journal of The Franklin Institute*, vol. 348, no. 4, pp. 738–748, 2011.
- [48] J. Morningred, B. E. Paden, D. E. Seborg, and D. A. Mellichamp, "An adaptive nonlinear predictive controller," *Chemical Engineering Science*, vol. 47, no. 4, pp. 755–762, 1992.
- [49] Y. Cao and P. M. Frank, "Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 2, pp. 200–211, 2000.

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