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Research Article

Delay-Range-Dependent Robust Constrained Model Predictive Control for Industrial Processes with Uncertainties and Unknown Disturbances

Huiyuan Shi,[1,2] Ping Li,[1,2] Limin Wang,[1,3] Chengli Su,[2] Jingxian Yu,[2] and Jiangtao Cao[2]

[1]*School of Automation, Northwestern Polytechnical University, Xian 710072, China*
[2]*School of Information and Control Engineering, Liaoning Shihua University, Fushun 113001, China*
[3]*School of Mathematics and Statistics, Hainan Normal University, Haikou 571158, China*

Correspondence should be addressed to Ping Li; liping@lnpu.edu.cn and Chengli Su; sclwind@sina.com

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A delay-range-dependent robust constrained model predictive control is proposed for discrete-time system with uncertainties and unknown disturbances. The dynamic characteristic of the discrete-time system is established as a new extended state space model in which state variables and output tracking error are integrated and regulated independently. It is used as the design of control law of system, which cannot only guarantee the convergence and tracking performance but also offer more degrees of freedom for designed controller. Unlike the traditional robust model predictive control (RMPC), the novel, less conservative, and more simplified delay-range-dependent stable conditions are derived by linear matrix inequality (LMI) theory and some relaxed technologies, which make use of the information of the upper and lower bounds of the time-varying delay. Meanwhile, the H_∞ performance index is introduced in the RMPC controller design, which can reject any unknown bounded disturbances. As a result, the design controller has better abilities of both tracking and disturbance rejection. The control results on the liquid level of tank system show that the proposed control method is effective and feasible.

1. Introduction

As the rapid development of the modern industry, there are increasing demands for higher product quality and system performance. These demands are in need of more system reliability and safety. The traditional control methods such as PID [1] can gurantee the system reliability and safety to achieve the smooth and effective operation in industrial production. However, due to the multivariable characteristic, multiconstraints, and large time delay for many industrial processes, the conventional approaches are incapable and insufficient.

Therefore, the advanced control technologies [2–8] have attracted more and more attention in the past few decades. Among them, model predictive control (MPC) is recognized as the most efficient and application potential advanced process control methodology, which has acquired huge economic profits on thousands of industrial control systems all over the world [9]. In general, MPC can be divided into two categories as follows.

First is the study of control algorithm and then to prove the stability of proposed method. Initially, a few industrial predictive control algorithms were proposed based on industrial demands, which are composed of model predictive heuristic control (MPHC) [10], dynamic matrix control (DMC) [11], generalized predictive control (GPC) [12], and predictive functional control (PFC) [13]. Since then, in order to obtain a better control performance, many improved methods about MPC have been exploited in various fields [14–19], especially in industrial processes [18–24]. But these methods need more tests to determine the control parameters in practical application. Meanwhile, the quantitative analysis of them also encountered insurmountable bottlenecks.

Second is the research about designing the control approach based on the premise of the stability of system. Taking advantage of optimal control theory, Lyapunov analysis method, linear matrix inequality (LMI) theory, invariant set, and other mature theories, theoretical studies about MPC have achieved a new breakthrough and obtained abundant results, in which the researches of robust model predictive control (RMPC) get more attention. RMPC combines the advantages of robust control and MPC, which can improve poor control performance because of considering the uncertainties of model. It was discussed by a host of researchers from various fields [25–30].

Time delay usually occurs in industrial processes, which may lead to the poor performance and instability of system. Hence, some researches about the stability of systems with time delay were presented [7, 8, 31–35], in which RMPC with time delay has been extensively reported in the past several years [36–46]. In [39–41], some designed approaches about MPC with time-delay-free were proposed, which are delay-independent. They have more conservative results than the delay-dependent ones. References [37, 45, 46] proposed the delay-dependent robust MPC for discrete-time delay system with input constraints by LMI technology. But the time delay of these methods is constant, which still exist some limits. For this reason, limited results [36, 38, 42–44] with respect to delay-dependent RMPC for system with time-varying delay were presented. Bououden et al. [36] investigated the problem of RMPC for active suspension systems with time-varying delays and input constraints. Li et al. [42] studied the RMPC for discrete-time systems with time-varying delay and input constraint, but they did not take advantage of the information of the upper and down bounds of time-varying delay to structure Lyapunov function, which increases some conservative property in stability analysis. Liu and Zhang [43] proposed a delay-dependent RMPC algorithm for linear parameter-varying systems with polytypic description. Lombardi et al. [44] solved the robust control design problem for linear time invariant systems affected by variable feedback delay. Franzè et al. [38] dealt with networked control system problem subject to time-varying delays and data losses. Unfortunately, in aforementioned results, performance of disturbance has not been considered, which is important in practical industrial application. Meanwhile, the derivation of stability for them is conservative in some way. Different from these present researches, the main contributions of this paper are summarized as follows.

(1) The novel extended state space model with state delay and uncertainties is used to describe the dynamic characteristic of the discrete-time system. It is used as the design of the control law, which not only guarantees the convergence and tracking performance but also offers more degrees of freedom for designed controller.

(2) A differential approach is used to construct the Lyapunov-Krasovskii function candidate without some redundant free-weighting matrices that takes advantage of the information of the lower and upper bounds of the time-varying delay. It can avert the bounding and model transformation techniques for cross terms with differential inequality. The novel, less conservative, and more simplified delay-range-dependent stable conditions are given under the time-varying state delay, uncertainties, unknown disturbances, input constraints, and output constraints during the derivation of stability.

(3) The H_∞ performance index is introduced in the derivation of stability for proposed RMPC which can reject any unknown bounded disturbances.

(4) The control results on the liquid level of tank system show that the proposed control method has better abilities of both tracking and disturbance rejection.

The paper is organized as follows: a problem formulation is presented in Section 2. Section 3 details the novel control strategy. Section 4 presents a case study in a tank system. Conclusions are presented in Section 5.

2. Problem Formulation

A practical industrial process, such as injection molding [47], is often represented as the following discrete-time state space form with state delay and uncertainties.

$$\begin{aligned} x(k+1) &= \mathbf{A}(k)x(k) + \mathbf{A}_d(k)x(k-d(k)) + \mathbf{B}(k)u(k) + w(k), \\ y(k) &= \mathbf{C}x(k), \end{aligned} \tag{1}$$

where $x(k) \in \mathbf{R}^{n_x}$, $u(k) \in \mathbf{R}^{n_u}$, $y(k) \in \mathbf{R}^{n_y}$, and $w(k)$ are the state, input, output, and unknown external disturbance at discrete time $k.d(k)$ denotes the discrete time-varying delay depending on discrete-time k which satisfies

$$d_m \le d(k) \le d_M, \tag{2}$$

where d_M and d_m are the upper and down bounds of delay, respectively. $[\mathbf{A}(k)\,\mathbf{A}_d(k)\,\mathbf{B}(k)] \in \Omega$, Ω is uncertainty set. $\mathbf{A}(k) = \mathbf{A} + \Delta_a(k)$, $\mathbf{A}_d(k) = \mathbf{A}_d + \Delta_d(k)$, $\mathbf{B}(k) = \mathbf{B} + \Delta_b(k)$, $\mathbf{A}$, $\mathbf{A}_d$, $\mathbf{B}$, and $\mathbf{C}$ are the constant matrices corresponding to appropriate dimensions, and $\Delta_a(k)$,$\Delta_d(k)$, and $\Delta_b(k)$ are uncertain perturbations at discrete-time k that follows

$$[\,\Delta_a(k) \quad \Delta_d(k) \quad \Delta_b(k)\,] = \mathbf{N}\Delta(k)[\,\mathbf{H} \quad \mathbf{H}_d \quad \mathbf{H}_b\,], \tag{3}$$

with

$$\Delta^{\mathrm{T}}(k)\Delta(k) \le I, \tag{4}$$

where $\mathbf{N}, \mathbf{H}, \mathbf{H}_d$, and $\mathbf{H}_b$ are known constant matrices with appropriate dimensions. $\Delta(k)$ are uncertainties depending on discrete-time k.

In practice, the controller is operated at or near such constraints for the input and output variables in order to get the most efficient or profitable operations in many industrial cases. The corresponding constraints for system (1) can be viewed as

$$\|u(k)\| \le u_M,$$
$$\|y(k)\| \le y_M, \tag{5}$$

where u_M and y_M denote the bounds for the input and output variables, respectively. Therefore, for the system (1) with the constraints (5) and uncertainty set Ω, the control goal of MPC problem can be obtained by minimizing the following robust cost function:

$$\min_{\Delta u(k+i|k), i\ge 0} \max_{\begin{bmatrix} A(k+i) & A_d(k+i) & B(k) \end{bmatrix} \in \Omega, i\ge 0} J_\infty,$$
$$J_\infty(k) = \sum_{i=0}^{\infty} \Big[(x(k+i\,|\,k))^{\mathrm{T}} \mathbf{Q} (x(k+i\,|\,k)) + u(k+i\,|\,k)^{\mathrm{T}} \mathbf{R} u(k+i\,|\,k) \Big],$$
$$\text{subject to} \quad \begin{cases} \|u(k+i\,|\,k)\| \le u_M, \\ \|y(k+i\,|\,k)\| \le y_M, \end{cases} \tag{6}$$

where $u(k+i\,|\,k)$ and $y(k+i\,|\,k)$ denote the predictive input and output for discrete-time $k+i$ made at discrete-time k. $\mathbf{Q}$ and $\mathbf{R}$ are the corresponding weighting matrices of tracking error and control input, respectively.

The control problem is to derive the control law of the following controller design in order to ensure the expected control performance under the case of uncertainties, time-varying delay, unknown disturbances, and the input and output constraints.

Remark 1. Equation (6) is a "min-max" optimization problem. The "max" is to search the largest or "worst-case" value of J_∞ within the uncertainty set Ω. The "min" is to search the current and future control variable to minimize the worst-case value. The "min-max" problem is not tractable under the finite horizon for MPC method. In general, the traditional strategy of RMPC with time-varying delay [36, 38, 42–44] is to design the state feedback control law $u(k+i\,|\,k) = Kx(k+i\,|\,k)$ that minimizes a "worst-case" infinite horizon performance index (6). Using linear matrix inequality (LMI) theory, the infinite horizon optimization problem can be converted into a convex optimization with LMI constraints. Then, based on MPC principle, only the current control law $u(k\,|\,k) = Kx(k\,|\,k)$ is implemented at discrete-time k and the optimization problem is repeated by the new state information at discrete-time $k+1$.

3. New Control Strategy

3.1. Novel State Space Model with State Delay and Uncertainties. Pre- and postmultiplying (1) by back shift operator Δ, we can obtain as follows:

$$\Delta x(k+1) = \mathbf{A}(k)\Delta x(k) + \mathbf{A}_d(k)\Delta x(k-d(k)) + \mathbf{B}(k)\Delta u(k) + \bar{w}(k),$$
$$\Delta y(k) = \mathbf{C}\Delta x(k), \tag{7}$$

where $\Delta = 1 - q^{-1}$, $\bar{w}(k) = (\Delta_a(k)) - \Delta_a(k-1)x(k-1) + (\Delta_d(k)) - \Delta_d(k-1)x(k-1-d(k-1)) + (\Delta_b(k) - \Delta_b(k-1))u(k-1) + \Delta w(k)$.

Define the set-point as $c(k)$, then the tracking error is

$$e(k) = y(k) - c(k). \tag{8}$$

Using (7) and (8), we can obtain

$$e(k+1) = e(k) + \mathbf{C}(\mathbf{A}(k)\Delta x(k)) + \mathbf{C}(\mathbf{A}_d(k))\Delta x(k-d(k)) + \mathbf{C}(\mathbf{B}(k))\Delta u(k) + \mathbf{C}\bar{w}(k). \tag{9}$$

The novel state space model with state delay and uncertainties is presented by the combination of tracking error and incremental state variable as follows:

$$\bar{x}_1(k+1) = \bar{A}(k)\bar{x}_1(k) + \bar{A}_d(k)\bar{x}_1(k-d(k)) + \bar{B}(k)\Delta u(k) + \bar{\mathbf{G}}\bar{w}(k),$$
$$\Delta y(k) = \bar{C}\bar{x}_1(k),$$
$$z(k) = e(k) = \bar{E}\bar{x}_1(k) \tag{10}$$

where

$$\bar{x}_1(k) = \begin{bmatrix} \Delta x(k) \\ e(k) \end{bmatrix},$$
$$\bar{x}_1(k-d(k)) = \begin{bmatrix} \Delta x(k-d(k)) \\ e(k-d(k)) \end{bmatrix},$$
$$\bar{A}(k) = \begin{bmatrix} \mathbf{A} + \Delta_a(k) & 0 \\ \mathbf{CA} + \mathbf{C}\Delta_a(k) & I \end{bmatrix} = \bar{A} + \bar{\Delta}_a,$$
$$\bar{A} = \begin{bmatrix} \mathbf{A} & 0 \\ \mathbf{CA} & I \end{bmatrix},$$
$$\bar{\Delta}_d(k) = \bar{N}\Delta(k)\bar{H}_d,$$
$$\bar{A}_d(k) = \begin{bmatrix} \mathbf{A}_d + \Delta_d(k) & 0 \\ \mathbf{CA}_d + \mathbf{C}\Delta_d(k) & 0 \end{bmatrix} = \bar{A}_d + \bar{\Delta}_d,$$
$$\bar{A}_d = \begin{bmatrix} \mathbf{A}_d & 0 \\ \mathbf{CA}_d & 0 \end{bmatrix},$$
$$\bar{\Delta}_d(k) = \bar{N}\Delta(k)\bar{H}_d,$$
$$\bar{B}(k) = \begin{bmatrix} \mathbf{B} + \Delta_b(k) \\ \mathbf{CB} + \mathbf{C}\Delta_b(k) \end{bmatrix} = \bar{B} + \bar{\Delta}_b,$$
$$\bar{B} = \begin{bmatrix} \mathbf{B} \\ \mathbf{CB} \end{bmatrix},$$
$$\bar{\Delta}_b(k) = \bar{N}\Delta(k)\bar{H}_b,$$
$$\bar{N} = \begin{bmatrix} \mathbf{N} \\ \mathbf{CN} \end{bmatrix},$$
$$\bar{H} = \begin{bmatrix} \mathbf{H} & 0 \end{bmatrix},$$

$$\begin{aligned}\bar{H}_d &= [\,\mathbf{H}_d \quad 0\,],\\ \bar{H}_b &= [\,\mathbf{H}_b \quad 0\,],\\ \bar{\mathbf{G}} &= \begin{bmatrix} I \\ \mathbf{C} \end{bmatrix},\\ \bar{C} &= [\,\mathbf{C} \quad 0\,],\\ \bar{E} &= [\,0 \quad I\,].\end{aligned} \tag{11}$$

Remark 2. Equation (10) is the novel state space model including the state variables and the output tracking error variable of process, which actually can regulate the dynamic response of the process state and output tracking error separately. This feature will be used as the design of the proposed controller that not only guarantees the convergence and tracking performance but also offers more degrees compared with conventional control approach.

Therefore, based on the above analysis, the control law is designed as follows:

$$\Delta u(k) = \bar{K}\bar{x}_1(k) = \bar{K}\begin{bmatrix} \Delta x(k) \\ e(k) \end{bmatrix}, \tag{12}$$

where $\bar{K}$ is the constant gain of proposed controller which can be calculated by the following design. Then, (10) can be transformed as

$$\begin{aligned}\bar{x}_1(k+1) &= \widehat{A}\,(k)\bar{x}_1(k) + \bar{A}_d(k)\bar{x}_1(k-d(k)) + \bar{\mathbf{G}}\bar{w}(k),\\ \Delta y(k) &= \bar{C}\bar{x}(k),\\ z(k) &= e(k) = \bar{E}\bar{x}(k),\end{aligned} \tag{13}$$

where $A(k) = \bar{A}(\widehat{k}) + \bar{B}(k)\bar{K}$.

Definition 1 (Robust MPC problem). Given the discrete-time system (13) with uncertainties and state measurement at discrete-time k, $\bar{x}(k)$, the robust MPC problem is feasible if the optimization problem

$$\begin{aligned}&\min_{\Delta u(k+i)} \max \bar{J}_\infty,\\ &\bar{J}_\infty(k) = \sum_{i=0}^{\infty}\Big[(\bar{x}_1(k+i\mid k))^{\mathrm{T}}\bar{\mathbf{Q}}_1(\bar{x}_1(k+i\mid k))\\ &\qquad + \Delta u(k+i\mid k)^{\mathrm{T}}\bar{\mathbf{R}}_1\Delta u(k+i\mid k)\Big],\\ &\text{subject to}\quad \begin{cases}\|\Delta u(k+i\mid k)\| \le \Delta u_M,\\ \|\Delta y(k+i)\| \le \Delta y_M,\end{cases}\end{aligned} \tag{14}$$

is solvable.

Remark 3. $\bar{x}_1(k+i\mid k)$ is the extended state variable for discrete-time $k+i$ made at discrete-time k. $\bar{Q}_1$ is the corresponding weighting matrices of extended state variable. Assume that the extended state variable including incremental state variables and tracking error at discrete-time k can be measured in real-time. It is easy to achieve such assumption because the tracking error is calculated in practice and the incremental state variables also can be obtained by selecting the measured input and output variables and their past values as state variables. Therefore, it can satisfy the control requirement to use the state feedback control (12) in this controller design.

Definition 2. The uncertain discrete-time system (13) is robust H_∞ control, if there exists a scalar $\gamma > 0$ for any $\bar{w}(k)$ and the following conditions hold for all admissible uncertainties.

(1) The resulting discrete-time close-loop system (13) with $\bar{w}(k) = 0$ is asymptotically stable.

(2) The system output $z(k)$ satisfies $\|z\| \le \gamma\|\bar{w}\|$ under the zero initial condition.

Remark 4. To reject the unknown bounded disturbances $\bar{w}(k)$ including internal and external disturbances, the robust H_∞ performance γ is introduced in this paper. Smaller values of γ demonstrate better rejection performance or smaller sensitivity against the disturbances.

3.2. Robust Constrained Controller Design. The main objective of this section is to design robust constrained control law $u(k\mid k)$ that guarantees the robust stability of system (13). The main theorem and corollary are summarized as follows.

Lemma 1 (Schur complements lemma [48].) *Let* $\mathbf{W}, \mathbf{L}$, *and* $\mathbf{V}$ *be matrices of appropriate dimensions in which* $\mathbf{W}$ *and* $\mathbf{V}$ *are real matrices, then for*

$$\mathbf{L}^{\mathrm{T}}\mathbf{V}\mathbf{L} - \mathbf{W} < 0, \tag{15}$$

if and only if

$$\begin{bmatrix} -\mathbf{W} & \mathbf{L}^{\mathrm{T}} \\ \mathbf{L} & -\mathbf{V}^{-1} \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\mathbf{V}^{-1} & \mathbf{L} \\ \mathbf{L}^{\mathrm{T}} & -\mathbf{W} \end{bmatrix} < 0. \tag{16}$$

Lemma 2 (see [49]). *For any vector* $\bar{\delta}(k) \in \mathbf{R}^n$, *two positive integers* κ_0, κ_1, *and matrix* $0 < \bar{\mathbf{R}} \in \mathbf{R}^{n\times n}$, *the following inequality holds*

$$-(\kappa_1 - \kappa_0 + 1)\sum_{t=\kappa_0}^{\kappa_1}\bar{\delta}^{\mathrm{T}}(k)\bar{\mathbf{R}}\bar{\delta}(k) \le -\sum_{t=\kappa_0}^{\kappa_1}\bar{\delta}^{\mathrm{T}}(k)\bar{\mathbf{R}}\sum_{t=\kappa_0}^{\kappa_1}\bar{\delta}(k). \tag{17}$$

Lemma 3 (see [50]). *Let* $\mathbf{D}, \mathbf{F}, \mathbf{E}$, *and* $\mathbf{M}$ *be real matrices of appropriate dimensions with satisfying* $\mathbf{M} = \mathbf{M}^{\mathrm{T}}$, *then for all* $\mathbf{F}^{\mathrm{T}}\mathbf{F} \le I$,

$$\mathbf{M} + \mathbf{D}\mathbf{F}\mathbf{E} + \mathbf{E}^{\mathrm{T}}\mathbf{F}^{\mathrm{T}}\mathbf{D}^{\mathrm{T}} < 0, \tag{18}$$

if and only if there exists $\varepsilon > 0$ *such that*

$$\mathbf{M} + \varepsilon^{-1}\mathbf{D}\mathbf{D}^{\mathrm{T}} + \varepsilon\mathbf{E}^{\mathrm{T}}\mathbf{E} < 0. \tag{19}$$

Theorem 1. *Given some scalars $\theta > 0$, $0 \le d_m \le d_M$, the delay-range-dependent sufficient condition for the proposed controller that ensures the uncertain discrete-time closed system (13) with $\bar{w}(k) = 0$ to be asymptotically controllable under the input and output constraints is that there exist symmetric positive matrices $\bar{\mathbf{P}}_1, \bar{\mathbf{T}}_1, \bar{\mathbf{M}}_1, \bar{\mathbf{G}}_1, \bar{\mathbf{L}}_1, \bar{\mathbf{S}}_1, \bar{\mathbf{S}}_2, \bar{\mathbf{M}}_3, \bar{\mathbf{M}}_4, \bar{\mathbf{X}}_1, \bar{\mathbf{X}}_2 \in \mathbf{R}^{(n_x+n_e)}$, and $\bar{\mathbf{Y}}_1 \in \mathbf{R}^{n_u\times(n_x+n_e)}$, such that the following LMI holds*

$$\begin{bmatrix} \widehat{\varphi}_1 & 0 & \bar{\mathbf{L}}_1 & \bar{\mathbf{L}}_1\bar{A}^{\mathrm{T}}(k) + \bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{B}^{\mathrm{T}}(k) & \bar{\mathbf{L}}_1\bar{A}^{\mathrm{T}}(k) + \bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{B}^{\mathrm{T}}(k) - \bar{\mathbf{L}}_1 & \bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{\mathbf{R}}_1^{1/2} & \bar{\mathbf{L}}_1\bar{\mathbf{Q}}_1^{1/2} \\ * & -\bar{\mathbf{S}} & 0 & \bar{\mathbf{S}}_1\bar{A}_d^{\mathrm{T}}(k) & \bar{\mathbf{S}}_1\bar{A}_d^{\mathrm{T}}(k) & 0 & 0 \\ * & * & -\bar{\mathbf{M}}_4 - \bar{\mathbf{X}}_1 & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{\mathbf{L}}_1 & 0 & 0 & 0 \\ * & * & * & * & -\bar{D}_2^{-2}\bar{\mathbf{X}}_1 & 0 & 0 \\ * & * & * & * & * & -\theta I & 0 \\ * & * & * & * & * & 0 & -\theta I \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} -1 & \bar{x}_l^{\mathrm{T}}(k \mid k) \\ \bar{x}_l(k \mid k) & -\bar{\phi}_l \end{bmatrix} \le 0, \tag{21}$$

$$\begin{bmatrix} -\Delta u_M^2 & \bar{\mathbf{Y}}_1 \\ \bar{\mathbf{Y}}_1^{\mathrm{T}} & -\bar{\phi}_l \end{bmatrix} \le 0, \tag{22}$$

$$\begin{bmatrix} -\Delta y_M^2\bar{\phi}_l^{-1} & \bar{C} \\ \bar{C}^{\mathrm{T}} & -I \end{bmatrix} \le 0, \tag{23}$$

and the robust state-feedback controller gains are given by $\bar{K} = \bar{\mathbf{Y}}_1\bar{\mathbf{L}}_1^{-1}$, where $\varphi_1 = -\bar{\mathbf{L}}_1 + \bar{\mathbf{M}}_3 + \bar{D}_1\widehat{\bar{\mathbf{S}}}_2 + \bar{\mathbf{S}}_2 - \bar{\mathbf{X}}_2$, $\bar{D}_1 = (d_M - d_m)I$, $\bar{D}_2 = d_M I$, and $*$ denotes the transposed elements in the symmetric position.

Proof 1. To ensure robust stability of discrete-time close-loop system (13) with $\bar{w}(k) = 0$, letting that $\bar{x}_1(k)$ satisfies the following robust stability constraint:

$$\begin{aligned} &\mathbf{V}(\bar{x}_1(k+i+1 \mid k)) - \mathbf{V}(\bar{x}_1(k+i \mid k)) \\ &\quad \le -\Big[(\bar{x}_1(k+i \mid k))^{\mathrm{T}}\bar{\mathbf{Q}}_1(\bar{x}_1(k+i \mid k)) \\ &\qquad + \Delta u(k+i \mid k)^{\mathrm{T}}\bar{\mathbf{R}}_1\Delta u(k+i \mid k)\Big]. \end{aligned} \tag{24}$$

Summing up both sides of (24) from $i = 0$ to ∞ and needing that $\mathbf{V}(\bar{x}_1(\infty)) = 0$ or $\bar{x}_1(\infty) = 0$, it has

$$\bar{J}_\infty(k) \le \mathbf{V}(\bar{x}_1(k)) \le \theta, \tag{25}$$

where θ is the upper bound of $\bar{J}_\infty(k)$. We construct the following Lyapunov-Krasovskii function candidate.

$$\mathbf{V}(\bar{x}_1(k+i)) = \sum_{i=1}^{5}\mathbf{V}_i(\bar{x}_1(k+i)), \tag{26}$$

where for simplicity, define the following notations:

$$\begin{aligned} \bar{x}_{1d}(k+i) &= \bar{x}_1(k+i-d(k)), \\ \bar{x}_{1d_M}(k+i) &= \bar{x}_1(k+i-d_M), \\ \bar{\delta}_1(k+i) &= \bar{x}_1(k+i+1) - \bar{x}_1(k+i), \\ \bar{\phi}_1(k+i) &= \begin{bmatrix} \bar{x}_1^{\mathrm{T}}(k+i) & \bar{x}_{1d}^{\mathrm{T}}(k+i) & \bar{x}_{1d_M}^{\mathrm{T}}(k+i) \end{bmatrix}^{\mathrm{T}}, \end{aligned}$$

$$\mathbf{V}_1(\bar{x}_1(k+i)) = \bar{x}_1^{\mathrm{T}}(k+i)\bar{\mathbf{P}}_1\bar{x}_1(k+i) = \bar{x}_1^{\mathrm{T}}(k+i)\theta\bar{\mathbf{L}}_1^{-1}\bar{x}_1(k+i),$$

$$\begin{aligned} \mathbf{V}_2(\bar{x}_1(k+i)) &= \sum_{r=k-d(k)}^{k-1}\bar{x}_1^{\mathrm{T}}(r+i)\bar{\mathbf{T}}_1\bar{x}_1(r+i) \\ &= \sum_{r=k-d(k)}^{k-1}\bar{x}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{S}}_1^{-1}\bar{x}_1(r+i), \end{aligned}$$

$$\begin{aligned} \mathbf{V}_3(\bar{x}_1(k+i)) &= \sum_{r=k-d_M}^{k-1}\bar{x}_1^{\mathrm{T}}(r+i)\bar{\mathbf{M}}_1\bar{x}_1(r+i) \\ &= \sum_{r=k-d_M}^{k-1}\bar{x}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{M}}_2^{-1}\bar{x}_1(r+i), \end{aligned}$$

$$\begin{aligned} \mathbf{V}_4(\bar{x}_1(k+i)) &= \sum_{s=-d_M}^{-d_m}\sum_{r=k+s}^{k-1}\bar{x}_1^{\mathrm{T}}(r+i)\bar{\mathbf{T}}_1\bar{x}_1(r+i) \\ &= \sum_{s=-d_M}^{-d_m}\sum_{r=k+s}^{k-1}\bar{x}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{S}}_1^{-1}\bar{x}_1(r+i), \end{aligned}$$

$$\begin{aligned}\mathbf{V}_5(\bar{x}_1(k+i)) &= d_M \sum_{s=-d_M}^{-1} \sum_{r=k+s}^{k-1} \bar{\delta}_1^{\mathrm{T}}(r+i)\bar{\mathbf{G}}_1\bar{\delta}_1(r+i) \\ &= d_M \sum_{s=-d_M}^{-1} \sum_{r=k+s}^{k-1} \bar{\delta}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{X}}_1^{-1}\bar{\delta}_1(r+i),\end{aligned} \tag{27}$$

$\bar{\mathbf{P}}_1, \bar{\mathbf{T}}_1, \bar{\mathbf{M}}_1, \bar{\mathbf{M}}_2$, and $\bar{\mathbf{G}}_1$ are positive definite matrices. Let

$$\begin{aligned}\bar{\xi}(k+i) &= \begin{bmatrix} \bar{x}_1(k+i)^{\mathrm{T}} & \bar{x}_1(k+i-d(k))^{\mathrm{T}} & \cdots \\ \bar{x}_1(k+i-d_M)^{\mathrm{T}} & \cdots & \bar{\delta}_1(k+i-1)^{\mathrm{T}} \end{bmatrix} \bar{\psi}_1 \\ &= \mathrm{diag}\begin{bmatrix} \bar{\mathbf{P}}_1 & \bar{\mathbf{T}}_1 & \cdots & \bar{\mathbf{M}}_1 & \cdots & d_M\bar{\mathbf{G}}_1 \end{bmatrix} \bar{\Pi}_1^{-1} \\ &= \mathrm{diag}\begin{bmatrix} \bar{\mathbf{L}}_1^{-1} & \bar{\mathbf{S}}_1^{-1} & \cdots & \bar{\mathbf{M}}_2^{-1} & \cdots & d_M\bar{\mathbf{X}}_1^{-1} \end{bmatrix}.\end{aligned} \tag{28}$$

We can obtain

$$\mathbf{V}(\bar{x}_1(k+i)) = \bar{\xi}^{\mathrm{T}}(k+i)\bar{\psi}_1\bar{\xi}(k+i) = \bar{\xi}^{\mathrm{T}}(k+i)\theta\bar{\Pi}_1^{-1}\bar{\xi}(k+i), \tag{29}$$

Then,

$$\begin{aligned}\Delta\mathbf{V}(\bar{x}_1(k+i)) &= \mathbf{V}(\bar{x}_1(k+i+1)) - \mathbf{V}(\bar{x}_1(k+i)) \\ &= \sum_{i=1}^{5} \Delta\mathbf{V}_i(\bar{x}_1(k+i)),\end{aligned} \tag{30}$$

where

$$\begin{aligned}\Delta\mathbf{V}_1(\bar{x}_1(k+i)) &= \bar{x}_1^{\mathrm{T}}(k+i+1)\theta\bar{\mathbf{L}}_1^{-1}\bar{x}_1(k+i+1) \\ &\quad - \bar{x}_1^{\mathrm{T}}(k+i)\theta\bar{\mathbf{L}}_1^{-1}\bar{x}_1(k+i), \\ \Delta\mathbf{V}_2(\bar{x}_1(k+i)) &= \sum_{r=k+1-d(k)}^{k} \bar{x}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{S}}_1^{-1}\bar{x}_1(r+i) \\ &\quad - \sum_{r=k-d(k)}^{k-1} \bar{x}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{S}}_1^{-1}\bar{x}_1(r+i) \\ &\le \bar{x}_1^{\mathrm{T}}(k+i)\theta\bar{\mathbf{S}}_1^{-1}\bar{x}_1(k+i) - \bar{x}_{1d}^{\mathrm{T}}(k+i)\theta\bar{\mathbf{S}}_1^{-1}\bar{x}_{1d}(k+i) \\ &\quad + \sum_{r=k+1-d_M}^{k-d_m} \bar{x}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{S}}_1^{-1}\bar{x}_1(r+i), \\ \Delta\mathbf{V}_3(\bar{x}_1(k+i)) &= \bar{x}_1^{\mathrm{T}}(k+i)\theta\bar{\mathbf{M}}_2^{-1}\bar{x}_1(k+i) - \bar{x}_{1d_M}^{\mathrm{T}}(k+i)\theta\bar{\mathbf{M}}_2^{-1}\bar{x}_{1d_M}(k+i), \\ \Delta\mathbf{V}_4(\bar{x}_1(k+i)) &= (d_M - d_m)\bar{x}_1^{\mathrm{T}}(k+i)\theta\bar{\mathbf{S}}_1^{-1}\bar{x}_1(k+i) \\ &\quad - \sum_{r=k+1-d_M}^{k-d_m} \bar{x}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{S}}_1^{-1}\bar{x}_1(r+i), \\ \Delta\mathbf{V}_5(\bar{x}_1(k+i)) &= d_M^2\bar{\delta}_1^{\mathrm{T}}(k+i)\theta\bar{\mathbf{X}}_1^{-1}\bar{\delta}_1(k+i) \\ &\quad - d_M \sum_{r=k-d_M}^{k-1} \bar{\delta}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{X}}_1^{-1}\bar{\delta}_1(r+i).\end{aligned} \tag{31}$$

Using Lemma 2, it has

$$\begin{aligned}\Delta\mathbf{V}_5(\bar{x}_1(k+i)) &\le d_M^2\bar{\delta}_1^{\mathrm{T}}(k+i)\theta\bar{\mathbf{X}}_1^{-1}\bar{\delta}_1(k+i) \\ &\quad - \sum_{r=k-d_M}^{k-1} \bar{\delta}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{X}}_1^{-1} \sum_{r=k-d_M}^{k-1} \bar{\delta}_1(r+i) \\ &= d_M^2(\bar{x}_1(k+i+1) - \bar{x}_1(k+i))^{\mathrm{T}}\theta\bar{\mathbf{X}}_1^{-1} \\ &\quad \cdot (\bar{x}_1(k+i+1) - \bar{x}_1(k+i)) - (\bar{x}_1(k+i) \\ &\quad - \bar{x}_{1d_M}(k+i))^{\mathrm{T}}\theta\bar{\mathbf{X}}_1^{-1}(\bar{x}_1(k+i) - \bar{x}_{1d_M}(k+i)).\end{aligned} \tag{32}$$

From (24), it can obtain

$$\theta^{-1}\Delta\mathbf{V}(\bar{x}_1(k+i\mid k)) + \theta^{-1}\bar{J}_i(k) \le 0, \tag{33}$$

where

$$\begin{aligned}\bar{J}_i(k) &= (\bar{x}_1(k+i\mid k))^{\mathrm{T}}\bar{\mathbf{Q}}_1(\bar{x}_1(k+i\mid k)) \\ &\quad + \Delta u(k+i\mid k)^{\mathrm{T}}\bar{\mathbf{R}}_1\Delta u(k+i\mid k).\end{aligned} \tag{34}$$

Based on (30), (32), and (33), it has

$$\begin{aligned}&\theta^{-1}\Delta\mathbf{V}(\bar{x}_1(k+i)) + \theta^{-1}\bar{J}_i(k) \\ &\quad \le \bar{\phi}_1^{\mathrm{T}}(k)\bar{\Phi}_1\bar{\phi}_1(k), \\ &\bar{\Phi}_1 = \begin{bmatrix} \bar{\varphi}_1 & 0 & \bar{\mathbf{X}}_1^{-1} \\ * & -\bar{\mathbf{S}}_1^{-1} & 0 \\ * & 0 & -\bar{\mathbf{M}}_2^{-1} - \bar{\mathbf{X}}_1^{-1} \end{bmatrix} \\ &\quad + \bar{\Lambda}_1^{\mathrm{T}}\bar{\mathbf{L}}_1^{-1}\bar{\Lambda}_1 + \bar{\Lambda}_2^{\mathrm{T}}\bar{D}_2^2\bar{\mathbf{X}}_1^{-1}\bar{\Lambda}_2 + \bar{\lambda}_1^{\mathrm{T}}\theta^{-1}\bar{\lambda}_1 \\ &\quad + \bar{\lambda}_2^{\mathrm{T}}\theta^{-1}\bar{\lambda}_2,\end{aligned} \tag{35}$$

where $\bar{\varphi}_1 = -\bar{\mathbf{L}}_1^{-1} + \bar{\mathbf{M}}_2^{-1} + \bar{D}_1\bar{\mathbf{S}}_1^{-1} + \bar{\mathbf{S}}_1^{-1} - \bar{\mathbf{X}}_1^{-1}$, $\bar{\Lambda}_1 = \begin{bmatrix} \widehat{A}(k) & \bar{A}_d(k) & 0 \end{bmatrix}$, $\bar{\Lambda}_2 = \begin{bmatrix} \widehat{A}(k) - I & \bar{A}_d(k) & 0 \end{bmatrix}$, $\bar{\lambda}_1 = \begin{bmatrix} \bar{\mathbf{Q}}_1^{1/2} & 0 & 0 \end{bmatrix}$, and $\bar{\lambda}_2 = \begin{bmatrix} \bar{\mathbf{R}}_1^{1/2}\bar{\mathbf{Y}}_1\bar{\mathbf{L}}_1^{-1} & 0 & 0 \end{bmatrix}$. Using Lemma 1, we can obtain the following LMI condition by letting $\bar{\Phi}_1 < 0$.

$$\begin{bmatrix} \bar{\varphi}_1 & 0 & \bar{\mathbf{X}}_1^{-1} & \widehat{A}^{\mathrm{T}}(k) & \widehat{A}^{\mathrm{T}}(k) - I & \bar{\mathbf{L}}_1^{-1}\bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{\mathbf{R}}_1^{1/2} & \bar{\mathbf{Q}}_1^{1/2} \\ * & -\bar{\mathbf{S}}_1^{-1} & 0 & \bar{A}_d^{\mathrm{T}}(k) & \bar{A}_d^{\mathrm{T}}(k) & 0 & 0 \\ * & * & -\bar{\mathbf{M}}_2^{-1} - \bar{\mathbf{X}}_1^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & -\bar{\mathbf{L}}_1 & 0 & 0 & 0 \\ * & * & * & * & -\bar{D}_2^{-2}\bar{\mathbf{X}}_1 & 0 & 0 \\ * & * & * & * & * & -\theta I & 0 \\ * & * & * & * & * & * & -\theta I \end{bmatrix} < 0. \tag{36}$$

In order to obtain delay-range-dependent sufficient (20), we only need to pre- and postmultiply LMI (36) by diag $\begin{bmatrix} \bar{\mathbf{L}}_1 & \bar{\mathbf{S}}_1 & \bar{\mathbf{X}}_1 & I & I & I & I \end{bmatrix}$ and let $\bar{\mathbf{L}}_1\bar{\mathbf{M}}_2^{-1}\bar{\mathbf{L}}_1 = \bar{\mathbf{M}}_3$, $\bar{\mathbf{L}}_1\bar{\mathbf{S}}_1^{-1}\bar{\mathbf{L}}_1 = \bar{\mathbf{S}}_2$, $\bar{\mathbf{L}}_1\bar{\mathbf{X}}_1^{-1}\bar{\mathbf{L}}_1 = \bar{\mathbf{X}}_2$, $\bar{\mathbf{X}}_1\bar{\mathbf{M}}_2^{-1}\bar{\mathbf{X}}_1 = \bar{\mathbf{M}}_4$, $\bar{K} = \bar{\mathbf{Y}}_1\bar{\mathbf{L}}_1^{-1}$.

Therefore, if the sufficient (20) holds, the asymptotical stability of the discrete-time close-loop system (13) with

$\bar{w}(k)=0$ will be guaranteed. Taking the maximum value of $\bar{x}_l(k)=\max\left(\bar{x}_1(r)\quad \bar{\delta}_1(r)\right)$, $r\in(k-d_M,k)$, it has

$$\mathbf{V}(\bar{x}_1(k))\le \bar{x}_l^{\mathrm{T}}(k)\overline{\psi}_l\bar{x}_l(k)\le\theta, \tag{37}$$

where $\overline{\psi}_l=\bar{\mathbf{P}}_1+\bar{d}_M\bar{\mathbf{T}}_1+d_M\bar{\mathbf{M}}_1+(d_m+d_M/2)(d_M-d_m+1)\bar{\mathbf{T}}_1+d_M^2\cdot(1+d_M/2)\bar{\mathbf{G}}_1$. Letting $\overline{\phi}_l=\theta\overline{\psi}_l^{-1}$, it can obtain the sufficient (21) using Lemma 1.

For the input constraint in (14), it has

$$\begin{aligned}\|\Delta u(k+i)\mid k\|^2&=\left\|\bar{\mathbf{Y}}_1\bar{\mathbf{L}}_1^{-1}\bar{x}_1(k+i\mid k)\right\|^2\\&=\left\|\bar{\mathbf{Y}}_1\theta^{-1}\bar{\mathbf{P}}_1\bar{x}_1(k+i\mid k)\right\|^2\\&\le\left\|\bar{\mathbf{Y}}_1\theta^{-1}\overline{\varphi}_l\bar{x}_l(k+i\mid k)\right\|^2\\&=\left\|\bar{\mathbf{Y}}_1\overline{\phi}_l^{-1}\bar{x}_l(k+i\mid k)\right\|^2\le\bar{\mathbf{Y}}_1\overline{\phi}_l^{-1}\bar{\mathbf{Y}}_1^{\mathrm{T}},\end{aligned} \tag{38}$$

which is less than Δu_M by (22).

For the output constraint in (14), it has $\|\Delta y(k+i)\|^2=\|\bar{C}\bar{x}_1(k+i\mid k)\|^2$ which is less than $\Delta y_M^2\bar{x}_1(k+i\mid k)\overline{\phi}_l^{-1}\bar{x}_1^{\mathrm{T}}(k+i\mid k)$ by (23). Taking the maximum value of $\bar{x}_l(k+i)=\max(\bar{x}_1(k+i))$, it has $\Delta y_M^2\bar{x}_1(k+i)\overline{\phi}_l^{-1}\bar{x}_1^{\mathrm{T}}(k+i)\le\Delta y_M^2\bar{x}_l(k+i)\overline{\phi}_l^{-1}\bar{x}_l^{\mathrm{T}}(k+i)\le\Delta y_M^2$.

Therefore, the input and output constraints in (14) are guaranteed by (22) and (23). This completes Proof 1.

Remark 5. Different from traditional method [51–53], a differential approach is employed to construct the Lyapunov-Krasovskii function candidate without some redundant free-weighting matrices that take advantage of the information of the lower and upper bounds of the time-varying delay. The novel and less conservative delay-range-dependent stable condition (20) that averts the bounding and model transformation techniques for cross terms using differential inequality is derived. In addition, the more simplified input and output constraint conditions (22) and (23) and invariant set (21) are also derived using some relax technique. If the time-varying delay reduces to constant delay, the following Corollary 1 is given. Therefore, it is more comprehensive to take into account the delay in this paper.

Corollary 1. *Given some scalars $\theta>0$, $0\le d\le\bar{d}_1$, the delay-dependent sufficient conditions for the proposed controller that ensures the uncertain discrete-time closed system (13) with $\bar{w}(k)=0$ to be asymptotically controllable under the input and output constraints are that there exist symmetric positive matrices $\bar{\mathbf{P}}_1$, $\bar{\mathbf{T}}_1$, $\bar{\mathbf{G}}_1$, $\bar{\mathbf{L}}_1$, $\bar{\mathbf{S}}_1$, $\bar{\mathbf{S}}_2$, $\bar{\mathbf{X}}_1$, $\bar{\mathbf{X}}_2$, $\bar{\mathbf{X}}_3$, $\in\mathbf{R}^{(n_x+n_e)}$, and $\bar{\mathbf{Y}}_1\in\mathbf{R}^{n_u\times(n_x+n_e)}$, such that the following LMI holds*

$$\begin{bmatrix}\tilde{\varphi}_1 & \bar{\mathbf{L}}_1 & \bar{\mathbf{L}}_1\bar{A}^{\mathrm{T}}(k)+\bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{B}^{\mathrm{T}}(k) & \bar{\mathbf{L}}_1\bar{A}^{\mathrm{T}}(k)+\bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{B}^{\mathrm{T}}(k)-\bar{\mathbf{L}}_1 & \bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{\mathbf{R}}_1^{1/2} & \bar{\mathbf{L}}_1\bar{\mathbf{Q}}_1^{1/2}\\ * & -\bar{\mathbf{X}}_3-\bar{\mathbf{X}}_1 & \bar{\mathbf{X}}_1\bar{A}_d^{\mathrm{T}}(k) & \bar{\mathbf{X}}_1\bar{A}_d^{\mathrm{T}}(k) & 0 & 0\\ * & * & -\bar{\mathbf{L}}_1 & 0 & 0 & 0\\ * & * & * & -\bar{D}^{-2}\bar{\mathbf{X}}_1 & 0 & 0\\ * & * & * & * & -\theta I & 0\\ * & * & * & * & 0 & -\theta I\\ * & * & * & * & & \end{bmatrix},$$
$$\begin{bmatrix}-1 & \bar{x}_l^{\mathrm{T}}(k\mid k)\\ \bar{x}_l(k\mid k) & -\tilde{\phi}_l\end{bmatrix}\le 0,$$
$$\begin{bmatrix}-\Delta u_M^2 & \bar{\mathbf{Y}}_1\\ \bar{\mathbf{Y}}_1^{\mathrm{T}} & -\tilde{\phi}_l\end{bmatrix}\le 0,$$
$$\begin{bmatrix}-\Delta y_M^2\tilde{\phi}_l^{-1} & \bar{C}\\ \bar{C}^{\mathrm{T}} & -I\end{bmatrix}\le 0, \tag{39}$$

where $\tilde{\varphi}_1=-\bar{\mathbf{L}}_1+\bar{\mathbf{S}}_2-\bar{\mathbf{X}}_2$, $\bar{\mathbf{X}}_1S_1^{-1}\bar{\mathbf{X}}_1=\bar{\mathbf{X}}_3$, $\bar{D}=\bar{d}_1I$, $\tilde{\phi}_l=\theta\tilde{\psi}_l^{-1}$, $\tilde{\psi}_l=\bar{\mathbf{P}}_1+\bar{d}_1\bar{\mathbf{T}}_1+d_M^2((1+d_M)/2)\bar{G}_1$.

Proof 2. Choose the following Lyapunov function candidate.

$$\mathbf{V}(\bar{x}_1(k+i))=\sum_{i=1}^{3}\mathbf{V}_i(\bar{x}_1(k+i)), \tag{40}$$

where

$$\begin{aligned}
\mathbf{V}_1(\bar{x}_1(k+i)) &= \bar{x}_1^{\mathrm{T}}(k+i)\bar{\mathbf{P}}_1\bar{x}_1(k+i) \\
&= \bar{x}_1^{\mathrm{T}}(k+i)\theta\bar{\mathbf{L}}_1^{-1}\bar{x}_1(k+i), \\
\mathbf{V}_2(\bar{x}_1(k+i)) &= \sum_{r=k-\bar{d}_1}^{k-1} \bar{x}_1^{\mathrm{T}}(r+i)\bar{\mathbf{T}}_1\bar{x}_1(r+i) \\
&= \sum_{r=k-\bar{d}_1}^{k-1} \bar{x}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{S}}_1^{-1}\bar{x}_1(r+i), \\
\mathbf{V}_3(\bar{x}_1(k+i)) &= \bar{d}_1 \sum_{s=-\bar{d}_1}^{-1} \sum_{r=k+s}^{k-1} \bar{\delta}_1^{\mathrm{T}}(r+i)\bar{\mathbf{G}}_1\bar{\delta}_1(r+i) \\
&= \bar{d}_1 \sum_{s=-\bar{d}_1}^{-1} \sum_{r=k+s}^{k-1} \bar{\delta}_1^{\mathrm{T}}(r+i)\theta\bar{\mathbf{X}}_1^{-1}\bar{\delta}_1(r+i),
\end{aligned} \tag{41}$$

Then, Corollary 1 is derived similarly to Proof 1.

Theorem 2. *Given some scalars $\gamma > 0$, $\theta > 0$, and $0 \le d_m \le d_M$, the delay-range-dependent sufficient conditions for the proposed controller that ensure the uncertain discrete-time closed system (13) with $\bar{w}(k) \neq 0$ to be asymptotically controllable and to be a H_∞ performance less than γ under the input and output constraints are that there exist symmetric positive matrices $\bar{\mathbf{P}}_1, \bar{\mathbf{T}}_1, \bar{\mathbf{M}}_1, \bar{\mathbf{G}}_1, \bar{\mathbf{L}}_1, \bar{\mathbf{S}}_1, \bar{\mathbf{S}}_2, \bar{\mathbf{M}}_3, \bar{\mathbf{M}}_4, \bar{\mathbf{X}}_1, \bar{\mathbf{X}}_2 \in \mathbf{R}^{(n_x+n_e)}$, and $\bar{\mathbf{Y}}_1 \in \mathbf{R}^{n_u\times(n_x+n_e)}$ and positive scalars $\bar{\varepsilon}_1, \bar{\varepsilon}_2$, such that the following LMI holds*

$$\begin{bmatrix}
\overline{\cap}_{11} & \overline{\cap}_{12} & \overline{\cap}_{13} & \overline{\cap}_{14} & \overline{\cap}_{15} & \overline{\cap}_{16} & \overline{\cap}_{17} & \overline{\cap}_{18} & \overline{\cap}_{19} & \overline{\cap}_{1,10} \\
* & \overline{\cap}_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & \overline{\cap}_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \overline{\cap}_{44} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & \overline{\cap}_{55} & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & \overline{\cap}_{66} & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & \overline{\cap}_{77} & 0 & 0 & 0 \\
* & * & * & * & * & * & * & \overline{\cap}_{88} & 0 & 0 \\
* & * & * & * & * & * & * & * & \overline{\cap}_{99} & 0 \\
* & * & * & * & * & * & * & * & * & \overline{\cap}_{10,10}
\end{bmatrix} < 0, \tag{42}$$

$$\begin{bmatrix} -1 & \bar{x}_l^{\mathrm{T}}(k\mid k) \\ \bar{x}_l(k\mid k) & -\bar{\phi}_l \end{bmatrix} \le 0, \tag{43}$$

$$\begin{bmatrix} -\Delta u_M^2 & \bar{\mathbf{Y}}_1 \\ \bar{\mathbf{Y}}_1^{\mathrm{T}} & -\bar{\phi}_l \end{bmatrix} \le 0, \tag{44}$$

$$\begin{bmatrix} -\Delta y_M^2\bar{\phi}_l^{-1} & \bar{C} \\ \bar{C}^{\mathrm{T}} & -I \end{bmatrix} \le 0, \tag{45}$$

and the robust state-feedback H_∞ controller gains are given by $\bar{K} = \bar{\mathbf{Y}}_1\bar{\mathbf{L}}_1^{-1}$, where

$$\begin{aligned}
\overline{\cap}_{11} &= \begin{bmatrix} \widehat{\varphi}_1 & 0 & \bar{\mathbf{L}}_1 & 0 \\ 0 & -\bar{\mathbf{S}}_1 & 0 & 0 \\ \bar{\mathbf{L}}_1 & 0 & -\bar{\mathbf{M}}_4 - \bar{\mathbf{X}}_1 & 0 \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}, \\
\overline{\cap}_{12} &= \begin{bmatrix} \bar{\mathbf{L}}_1\bar{A}^{\mathrm{T}} + \bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{B}^{\mathrm{T}} \\ \bar{\mathbf{S}}_1\bar{A}_d \\ 0 \\ \bar{\mathbf{G}}^{\mathrm{T}} \end{bmatrix}, \\
\overline{\cap}_{13} &= \begin{bmatrix} \bar{\mathbf{L}}_1\bar{A}^{\mathrm{T}} + \bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{B}^{\mathrm{T}} \text{-} \bar{\mathbf{L}}_1 \\ \bar{\mathbf{S}}_1\bar{A}_d \\ 0 \\ \bar{\mathbf{G}}^{\mathrm{T}} \end{bmatrix}, \\
\overline{\cap}_{14} &= \begin{bmatrix} \bar{\mathbf{L}}_1\bar{E}^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\
\overline{\cap}_{15} &= \begin{bmatrix} \bar{\mathbf{Y}}_1\bar{\mathbf{R}}_1^{1/2} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\
\overline{\cap}_{16} &= \begin{bmatrix} \bar{\mathbf{L}}_1\bar{\mathbf{Q}}_1^{1/2} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\
\overline{\cap}_{17} = \overline{\cap}_{19} &= \begin{bmatrix} \bar{\mathbf{L}}_1\bar{H}^{\mathrm{T}} \\ \bar{\mathbf{S}}_1\bar{H}_d^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix}, \\
\overline{\cap}_{18} = \overline{\cap}_{1,10} &= \begin{bmatrix} \bar{\mathbf{Y}}_1\bar{H}_b^{\mathrm{T}} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\
\overline{\cap}_{22} &= \left[-\bar{\mathbf{L}}_1 + \bar{\varepsilon}_1\bar{N}\bar{N}^{\mathrm{T}} + \bar{\varepsilon}_2\bar{N}\bar{N}^{\mathrm{T}}\right], \\
\overline{\cap}_{44} &= -I, \\
\overline{\cap}_{55} = \overline{\cap}_{66} &= -\theta I, \\
\overline{\cap}_{33} &= \left[-\bar{\mathbf{X}}_1\bar{D}_2^{-2} + \bar{\varepsilon}_1\bar{N}\bar{N}^{\mathrm{T}} + \bar{\varepsilon}_2\bar{N}\bar{N}^{\mathrm{T}}\right], \\
\overline{\cap}_{77} = \overline{\cap}_{99} &= -\bar{\varepsilon}_1 I, \\
\overline{\cap}_{88} = \overline{\cap}_{10,10} &= -\bar{\varepsilon}_2 I.
\end{aligned} \tag{46}$$

Proof 3. To build the H_∞ performance of the uncertain discrete-time closed-loop system (13) with zero initial condition, we define the following performance index.

$$J = \sum_{k=0}^{\infty}\left[z^{\mathrm{T}}(k)z(k) - \gamma^2\bar{w}^{\mathrm{T}}(k)\bar{w}(k)\right]. \tag{47}$$

Then, for any $\bar{w}(k) \in l_2[0,\infty]$ with nonzero, it is obvious that

$$J \le \sum_{k=0}^{\infty}\left[z^{\mathrm{T}}(k)z(k) - \gamma^2\bar{w}^{\mathrm{T}}(k)\bar{w}(k) + \Delta\mathbf{V}(\bar{x}_1(k))\right], \tag{48}$$

yet

$$\begin{aligned}
&z^{\mathrm{T}}(k)z(k) - \gamma^2\bar{w}^{\mathrm{T}}(k)\bar{w}(k) + \Delta\mathbf{V}(\bar{x}_1(k)) \\
&= \begin{bmatrix}\bar{\phi}_1(k)\\ \bar{w}(k)\end{bmatrix}^{\mathrm{T}}\left\{\begin{bmatrix}\bar{\varphi}_1 & 0 & \bar{\mathbf{X}}_1^{-1} & 0\\ * & -\bar{\mathbf{S}}_1^{-1} & 0 & 0\\ * & * & -\bar{\mathbf{M}}_2^{-1}-\bar{\mathbf{X}}_1^{-1} & 0\\ * & * & * & -\gamma^2\end{bmatrix}\right. \\
&\quad + \begin{bmatrix}\bar{\Lambda}_1^{\mathrm{T}}\\ \bar{\mathbf{G}}^{\mathrm{T}}\end{bmatrix}\bar{\mathbf{L}}_1^{-1}\begin{bmatrix}\bar{\Lambda}_1 & \bar{\mathbf{G}}\end{bmatrix}\bar{\Lambda}_1 + \begin{bmatrix}\bar{\Lambda}_2^{\mathrm{T}}\\ \bar{\mathbf{G}}^{\mathrm{T}}\end{bmatrix}\bar{D}_2^2\bar{\mathbf{X}}_1^{-1}\begin{bmatrix}\bar{\Lambda}_2 & \bar{\mathbf{G}}\end{bmatrix}\begin{bmatrix}\bar{E}^{\mathrm{T}}\\ 0\\ 0\\ 0\end{bmatrix} \\
&\quad \cdot\begin{bmatrix}\bar{E} & 0 & 0 & 0\end{bmatrix} + \begin{bmatrix}\bar{\lambda}_1 & 0\end{bmatrix}^{\mathrm{T}}\theta^{-1}\begin{bmatrix}\bar{\lambda}_1 & 0\end{bmatrix} \\
&\quad \left. + \begin{bmatrix}\bar{\lambda}_2 & 0\end{bmatrix}^{\mathrm{T}}\theta^{-1}\begin{bmatrix}\bar{\lambda}_2 & 0\end{bmatrix}\right\}\begin{bmatrix}\bar{\phi}_1(k)\\ \bar{w}(k)\end{bmatrix}.
\end{aligned} \tag{49}$$

Based on Proof 1 and the delay-range-dependent sufficient condition (42), we can obtain that

$$\begin{aligned}
&\begin{bmatrix}\bar{\varphi}_1 & 0 & \bar{\mathbf{X}}_1^{-1} & 0\\ * & -\bar{\mathbf{S}}_1^{-1} & 0 & 0\\ * & * & -\bar{\mathbf{M}}_2^{-1}-\bar{\mathbf{X}}_1^{-1} & 0\\ * & * & * & -\gamma^2\end{bmatrix} + \begin{bmatrix}\bar{\Lambda}_1^{\mathrm{T}}\\ \bar{\mathbf{G}}^{\mathrm{T}}\end{bmatrix}\bar{\mathbf{L}}_1^{-1}\begin{bmatrix}\bar{\Lambda}_1 & \bar{\mathbf{G}}\end{bmatrix} \\
&\quad + \begin{bmatrix}\bar{\Lambda}_2^{\mathrm{T}}\\ \bar{\mathbf{G}}^{\mathrm{T}}\end{bmatrix}D_2^2\bar{\mathbf{X}}_1^{-1}\begin{bmatrix}\bar{\Lambda}_2 & \bar{\mathbf{G}}\end{bmatrix} + \begin{bmatrix}\bar{E}^{\mathrm{T}}\\ 0\\ 0\\ 0\end{bmatrix}\begin{bmatrix}\bar{E} & 0 & 0 & 0\end{bmatrix} \\
&\quad + \begin{bmatrix}\bar{\lambda}_1 & 0\end{bmatrix}^{\mathrm{T}}\theta^{-1}\begin{bmatrix}\bar{\lambda}_1 & 0\end{bmatrix} + \begin{bmatrix}\bar{\lambda}_2 & 0\end{bmatrix}^{\mathrm{T}}\theta^{-1}\begin{bmatrix}\bar{\lambda}_2 & 0\end{bmatrix}_2 < 0.
\end{aligned} \tag{50}$$

Therefore, the H_∞ performance index $\|z\| \le \gamma\|\bar{w}\|$ is guaranteed. This completes Proof 3.

Remark 6. Theorems 1 and 2 are sufficient conditions for designed proposed controller that robustly stabilizes uncertain discrete-time closed system (13), which can guarantee the H_∞ and optimization performances. Similarly, the special case for constant delay is given in Corollary 2.

Corollary 2. *Given some scalars $\gamma > 0$, $\theta > 0$, and $0 \le d \le \bar{d}_1$, the delay-dependent sufficient conditions for the proposed controller that ensures the uncertain discrete-time closed system (13) with $\bar{w}(k) \ne 0$ to be asymptotically controllable and to be a H_∞ performance less than γ under the input and output constraints are that there exist symmetric positive matrices $\bar{\mathbf{P}}_1$, $\bar{\mathbf{T}}_1$, $\bar{\mathbf{G}}_1$, $\bar{\mathbf{L}}_1$, $\bar{\mathbf{S}}_1$, $\bar{\mathbf{S}}_2$, $\bar{\mathbf{X}}_1$, $\bar{\mathbf{X}}_2$, $\bar{\mathbf{X}}_3$, $\in \mathbf{R}^{(n_x+n_e)}$ and $\bar{\mathbf{Y}}_1 \in \mathbf{R}^{n_u\times(n_x+n_e)}$ and positive scalars $\bar{\varepsilon}_1, \bar{\varepsilon}_2$, such that the following LMI holds*

$$\begin{bmatrix}
\tilde{\cap}_{11} & \tilde{\cap}_{12} & \tilde{\cap}_{13} & \tilde{\cap}_{14} & \tilde{\cap}_{15} & \tilde{\cap}_{16} & \tilde{\cap}_{17} & \tilde{\cap}_{18} & \tilde{\cap}_{19} & \tilde{\cap}_{1,10}\\
* & \overline{\cap}_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & \overline{\cap}_{33} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & \overline{\cap}_{44} & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & \overline{\cap}_{55} & 0 & 0 & 0 & 0 & 0\\
* & * & * & * & * & \overline{\cap}_{66} & 0 & 0 & 0 & 0\\
* & * & * & * & * & * & \overline{\cap}_{77} & 0 & 0 & 0\\
* & * & * & * & * & * & * & \overline{\cap}_{88} & 0 & 0\\
* & * & * & * & * & * & * & * & \overline{\cap}_{99} & 0\\
* & * & * & * & * & * & * & * & * & \overline{\cap}_{10,10}
\end{bmatrix} < 0,$$

$$\begin{bmatrix}-1 & \bar{x}_l^{\mathrm{T}}(k\,|\,k)\\ \bar{x}_l(k\,|\,k) & -\tilde{\phi}_l\end{bmatrix} \le 0,$$

$$\begin{bmatrix}-\Delta u_M^2 & \bar{\mathbf{Y}}_1\\ \bar{\mathbf{Y}}_1^{\mathrm{T}} & -\tilde{\phi}_l\end{bmatrix} \le 0,$$

$$\begin{bmatrix}-\Delta y_M^2\tilde{\phi}_l^{-1} & \bar{C}\\ \bar{C}^{\mathrm{T}} & -I\end{bmatrix} \le 0, \tag{51}$$

and the robust state-feedback H_∞ controller gains are given by $\bar{K} = \bar{\mathbf{Y}}_1\bar{\mathbf{L}}_1$, where

$$\tilde{\cap}_{11} = \begin{bmatrix}\tilde{\varphi}_1 & \bar{\mathbf{L}}_1 & 0\\ * & -\bar{\mathbf{X}}_3-\bar{\mathbf{X}}_1 & 0\\ * & * & -\gamma^2 I\end{bmatrix},$$

$$\tilde{\cap}_{12} = \begin{bmatrix}\bar{\mathbf{L}}_1\bar{A} + \bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{B}^{\mathrm{T}}\\ \bar{\mathbf{X}}_1\bar{A}_d\\ \bar{\mathbf{G}}^{\mathrm{T}}\end{bmatrix},$$

$$\tilde{\cap}_{13} = \begin{bmatrix}\bar{\mathbf{L}}_1\bar{A} + \bar{\mathbf{Y}}_1^{\mathrm{T}}\bar{B}^{\mathrm{T}} - \bar{\mathbf{L}}_1\\ \bar{\mathbf{X}}_1\bar{A}_d\\ \bar{\mathbf{G}}^{\mathrm{T}}\end{bmatrix},$$

$$\tilde{\cap}_{14} = \begin{bmatrix}\bar{\mathbf{L}}_1\bar{E}^{\mathrm{T}}\\ 0\\ 0\end{bmatrix},$$

$$\tilde{\cap}_{15} = \begin{bmatrix} \bar{\mathbf{Y}}_1^{\mathrm{T}} \bar{\mathbf{R}}_1^{1/2} \\ 0 \\ 0 \end{bmatrix},$$

$$\tilde{\cap}_{16} = \begin{bmatrix} \bar{\mathbf{L}}_1 \bar{\mathbf{Q}}_1^{1/2} \\ 0 \\ 0 \end{bmatrix},$$

$$\tilde{\cap}_{17} = \tilde{\cap}_{19} = \begin{bmatrix} \bar{\mathbf{L}}_1 \bar{H}^{\mathrm{T}} \\ \bar{\mathbf{X}}_1 \bar{H}_d^{\mathrm{T}} \\ 0 \end{bmatrix},$$

$$\tilde{\cap}_{18} = \tilde{\cap}_{1,10} = \begin{bmatrix} \bar{\mathbf{Y}}_1^{\mathrm{T}} \bar{H}_b^{\mathrm{T}} \\ 0 \\ 0 \end{bmatrix},$$

$$\overline{\cap}_{22} = \left[-\bar{\mathbf{L}}_1 + \bar{\varepsilon}_1 \bar{N} \bar{N}^{\mathrm{T}} + \bar{\varepsilon}_2 \bar{N} \bar{N}^{\mathrm{T}} \right],$$

$$\overline{\cap}_{44} = -I,$$

$$\overline{\cap}_{55} = \overline{\cap}_{66} = -\theta I,$$

$$\overline{\cap}_{33} = \left[-\bar{\mathbf{X}}_1 \bar{D}_2^{-2} + \bar{\varepsilon}_1 \bar{N} \bar{N}^{\mathrm{T}} + \bar{\varepsilon}_2 \bar{N} \bar{N}^{\mathrm{T}} \right],$$

$$\overline{\cap}_{77} = \overline{\cap}_{99} = -\bar{\varepsilon}_1 I,$$

$$\overline{\cap}_{88} = \overline{\cap}_{10,10} = -\bar{\varepsilon}_2 I. \tag{52}$$

The proof is omitted.

Remark 7. Based on the different disturbance cases, the final objective of all of the theorems and corollaries is to use those sufficient conditions to solve the control law (12) for proposed delay-range-dependent robust constrained model predictive controller. Therefore, the control law with the form (12) can be got. Substituting (12) into (1), the better control performance can be achieved for the system with under uncertainties, time-varying delay, unknown disturbance, input constraints, and output constraints.

4. Case Study in a Tank System

4.1. System Description. In this study, the tank system (TTS20) [54] in Figure 1 made by Ingenieurbüro Gurski-Schramm Company is used as a simulation object, because it can imitate a part or whole of many controlled process in practical industry. The principal structure and overall process flow of the TTS20 are shown in Figure 2.

The plant is composed of three plexiglas cylinders T1, T3, and T2 with the cross section A. These are connected serially with each other by cylindrical pipes with the cross section S_n. Located at T2 is the single so-called nominal outflow valve. It also has a circular cross section S_n. The outflowing liquid (usually distilled water) is collected in a reservoir, which supplies the pumps 1 and 2. Here, the circle is closed. $H_{\max}$ denotes the highest possible liquid level. In case the liquid level of T1 or T2 exceeds this value, the corresponding pump will be switched off automatically. Q_1 and

Figure 1: TTS20-tank tank.

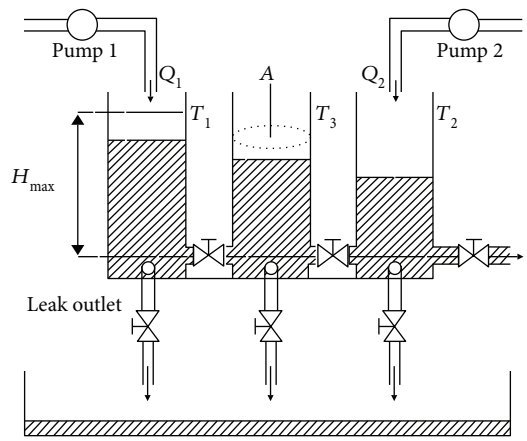

Figure 2: Structure and process flow of TTS20.

Q_2 are the flow rates of the pumps 1 and 2. In addition, for the purpose of simulating leaks, each tank additionally has a circular opening with the cross section S_l and a manually adjustable ball valve in series. Here, the drain valve and the discharge of water leak can describe the fault information of water tank.

The distilled water from reservoir is injected into T1 and T2 by the pump 1 (P1) and pump 2 (P2), respectively. Then, the water from their bottom valve and T3 drain valve is excluded to the reservoir for supply of P1 and P2 and this forms a circuit. Among them, T1, T2, and T3 are measured by three pressure level sensors as a measurement element of the system, and the flow rates of Q_1 and Q_2 are adjusted by a digital controller.

4.2. Process Model. Through the combination of P1, P2, the cylindrical valve, and water tank leakage valve, tank system can be easily converted into a single-input single-output, or multiple-input multiple-output system, or first order, second order, and third order models. The diagram of principle structure to define the variables and parameters of TTS20 is shown in Figure 3. Here, we only open the cylindrical valve between T1 and T3 and the water tank leakage valve of T3. The flow rate of P1 is as the control input of system, and the level of

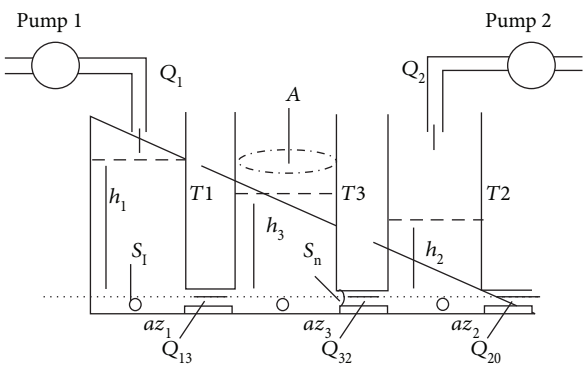

Figure 3: Principle structure to define the variables and parameters of TTS20.

T1 in the liquid level is as the output of system. Therefore, the tank is transformed into a single-input single-output (SISO) model by adjusting the flow rate. The SISO model of the TTS20 is

$$\begin{bmatrix} \dot{h}_1 \\ \dot{h}_3 \end{bmatrix} = \frac{1}{S} \begin{bmatrix} -\mathbf{Q}_{13} \\ -\mathbf{Q}_{13} - \mathbf{Q}_{\text{out}} \end{bmatrix} \mathbf{Q}_{\text{in}}, \qquad y = h_1, \tag{53}$$

where h_1, h_3 chosen as the controlled variables are the height of water level in T1 and T3, respectively, Q_{in} chosen as the manipulated variable of the system is the flow rate of P1, $Q_{13} = az_1 S_n \operatorname{sgn}(h_1 - h_3)\sqrt{2g|h_1 - h_3|}$ is the flow rate from T1 to T3, $Q_{\text{out}} = az_2 S_1 \sqrt{2gh_2}$ is the leakage flow in the bottom of T3, $S_1 = S_n = 5 \times 10^{-5}$ m^2, $S = 0.154$ m^2, $H_{\max} = 0.6$ m, $az_1 = 0.48$, and $az_2 = 0.58$ are the outflow coefficients, and sgn $(\cdot)$ is the symbolic function. The initial values of h_1, h_3 are set as 0. The state variables and the input are

$$x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} h_1(k) \\ h_3(k) \end{bmatrix}, \qquad u(k) = Q_{\text{in}}(k). \tag{54}$$

With local linearization to (53) in operating points 0.33 $H_{\max}$, the SISO discrete-time state space model with state delay, uncertainties and unknown disturbance for TTS20 is

$$x(k+1) = \mathbf{A}(k)x(k) + \mathbf{A}_d(k)x(k - d(k)) + \mathbf{B}(k)u(k) + w(k), \qquad y = \mathbf{C}x(k), \tag{55}$$

where

$$1 \le d(k) \le 3,$$
$$\mathbf{A} = \begin{bmatrix} 0.9850 & 0.0107 \\ 0.0078 & 0.9784 \end{bmatrix},$$
$$\mathbf{B} = \begin{bmatrix} 64.4453 \\ 0.2559 \end{bmatrix},$$
$$\mathbf{A}_d = \begin{bmatrix} 0.1057 & 0.0004 \\ 0.0002 & 0.0807 \end{bmatrix},$$
$$\mathbf{N} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$\mathbf{H} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$
$$\mathbf{H}_d = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix},$$
$$\mathbf{H}_b = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$
$$\mathbf{C} = [1 \quad 0],$$
$$\Delta(k) = \begin{bmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{bmatrix},$$
$$w(k) = (0.0005\Delta_3 \quad 0.0005\Delta_4)^{\mathrm{T}}, \tag{56}$$

in which $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ is a random number within $[-1 \quad 1]$.

Remark 8. In (55), it is assumed that the process has the characteristics of state delay, uncertainties and unknown disturbance. These characteristics are common for many industrial processes. In order to deal these conditions, it is assumed in the simulation case of the tank system.

4.3. Simulation Results. For the comparison, the model (55) is used as the traditional robust constrained MPC and proposed method in this paper, respectively. The input and output constraints are

$$\begin{cases} |y(k+i \mid k)| \le 0.12, 0 < k < 100, \\ |y(k+i \mid k)| \le 0.23, 100 \le k < 200, \\ |u(k+i \mid k)| \le 0.0005. \end{cases} \tag{57}$$

The set point for two methods is set as

$$\begin{aligned} c(k) &= 0.1, \quad 0 < k \le 100, \\ c(k) &= 0.2, \quad 100 < k < 200. \end{aligned} \tag{58}$$

In order to describe the tracking performance, the following index is used.

$$DT(k) = \sqrt{e^{\mathrm{T}}(k)e(k)}. \tag{59}$$

To analyze the effect of $\bar{\mathbf{Q}}_1$, three groups of different $\bar{\mathbf{Q}}_1$, $\bar{\mathbf{Q}}_1 = \operatorname{diag}[5, 2, 1]$, $\bar{\mathbf{Q}}_1 = \operatorname{diag}[10, 5, 1]$, and $\bar{\mathbf{Q}}_1 = \operatorname{diag}$

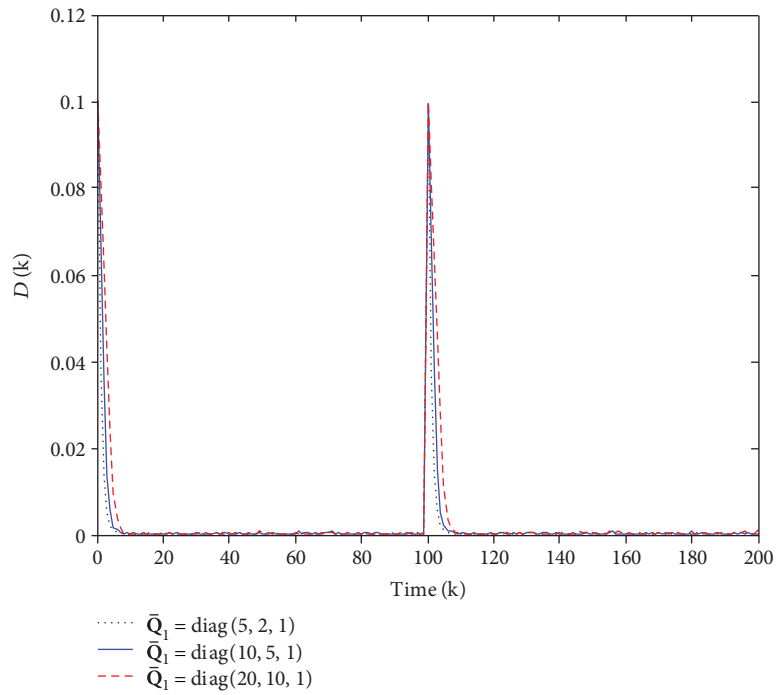

Figure 4: The comparison results of different $\bar{\mathbf{Q}}_1$.

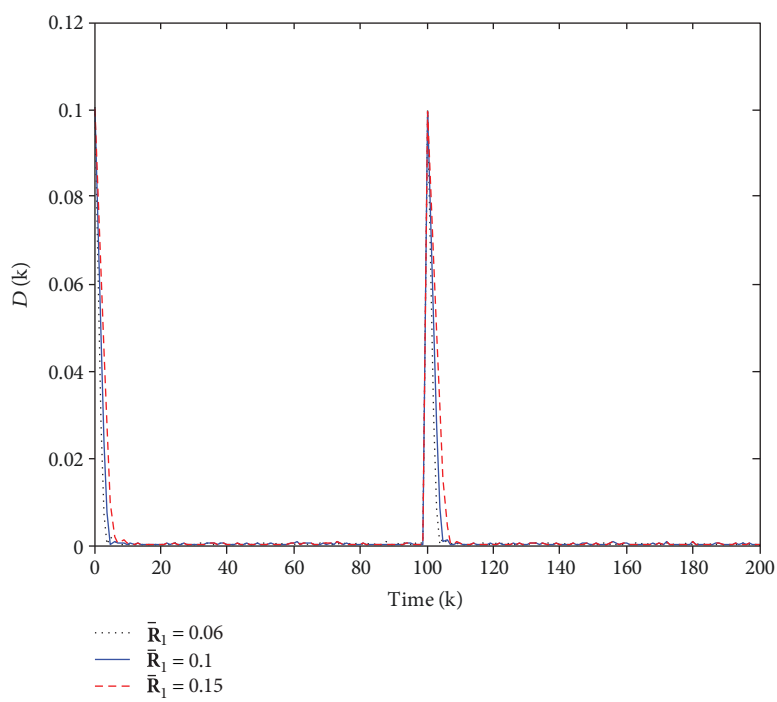

Figure 5: The comparison results of different $\bar{\mathbf{R}}_1$.

[20, 10, 1], are used in the proposed method and the result is shown in Figure 4, in which the weighting matrices $\bar{\mathbf{R}}_1$ is fixed to be 0.1. From Figure 4, it can be seen that the speed of the tracking performance $DT(k)$ is descending for the three values as $\bar{\mathbf{Q}}_1$ becomes large. This indicates that large value of this weighting matrix will result in smoother convergence, but the slower response to process output. The same control effect of $\bar{\mathbf{R}}_1$, three groups of different $\bar{\mathbf{R}}_1$, $\bar{\mathbf{R}}_1 = 0.06$, $\bar{\mathbf{R}}_1 = 0.1$, and $\bar{\mathbf{R}}_1 = 0.15$, are shown in Figure 5, in which the weighting matrix $\bar{\mathbf{Q}}_1 = \text{diag}\,[10, 5, 1]$. Therefore, through repetitive test, the control parameters of the proposed method are chosen as $\bar{\mathbf{Q}}_1 = \text{diag}\,[10, 5, 1]$, $\bar{\mathbf{R}}_1 = 0.1$. And the parameters of the traditional RMPC method are set as $\mathbf{Q} = 1$, $\mathbf{R} = 0.1$.

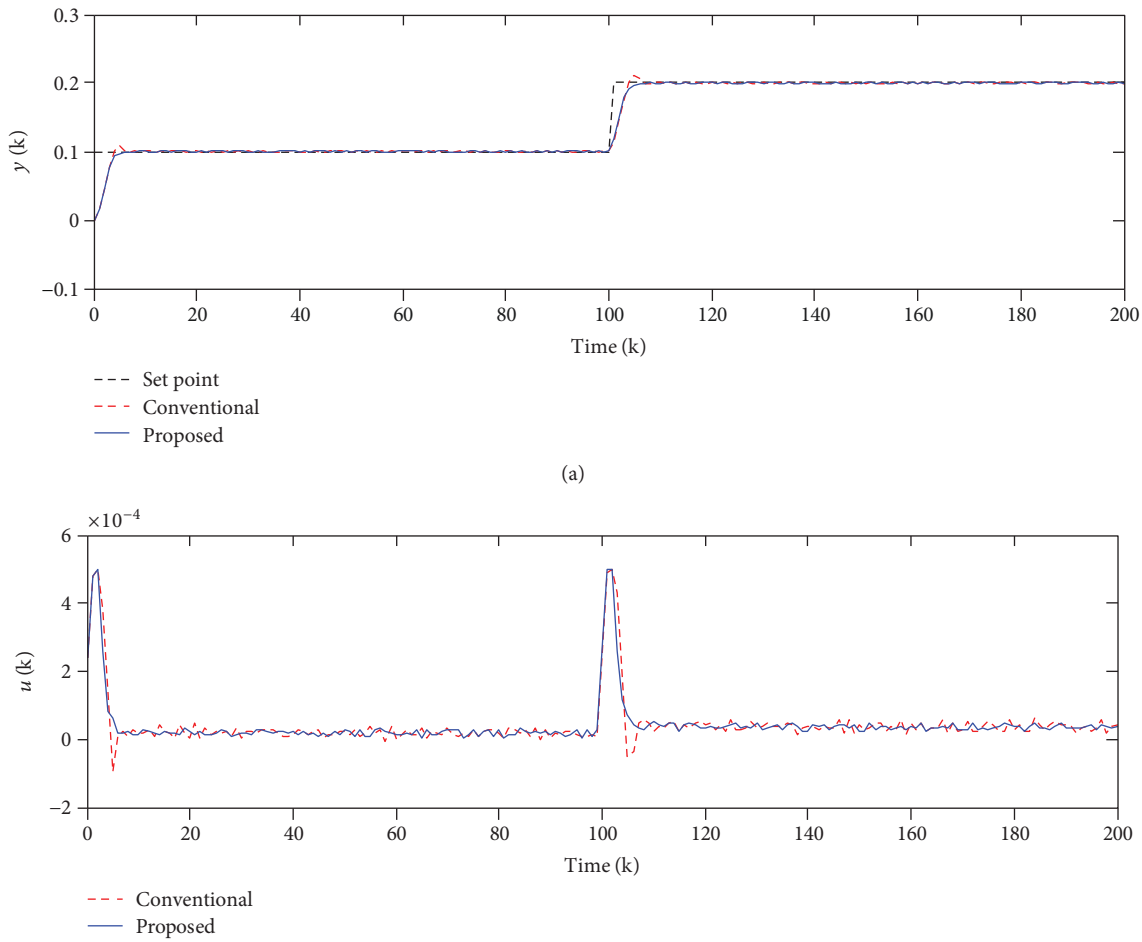

Figure 6: (a) Output response and (b) control input under the proposed and conventional method.

The proposed control method used in water level of the tank system compared with conventional control is shown in Figure 6. It can be seen that better tracking and stronger disturbance rejection are shown as follows. Figure 6(a) shows the output response under the proposed and conventional method. It can find an obvious bigger overshoot under the conventional approach. Although the conventional method can achieve the tracking for the changes of the set point, the proposed approach has better tracking performance. Figure 6(b) shows the effect with regard to control input. From the figure, it can be found that the proposed method has a smoother action to track the changes of the set point and overcome the uncertainties and unknown disturbance. Meanwhile, the output response and control input for both methods can remain within the given conditions of constraints. Simply put, the proposed control method has a few better abilities of both tracking and disturbance rejection under the input and output constraints.

5. Conclusion

A new delay-range-dependent robust constrained MPC is proposed for discrete-time system with uncertainties and unknown disturbances in this work. The system is constructed as a new extended state space model in which state variable and tracking error are integrated and regulated independently, which not only guarantees the convergence and tracking performance but also offers more degrees of freedom for designed controller. By structuring a Lyapunov-Krasovskii function candidate, a novel, less conservative, and more simplified delay-range-dependent stable conditions combined with H_∞ performance index are obtained in terms of LMI. An industrial case study on the liquid level of tank system shows that the proposed control method has better abilities of both tracking and disturbance rejection under the input and output constraints.

Because of the strong nonlinearity in many industrial processes such as polymerization process cannot be described by the simple linear model, how to solve such problem causes a great challenge. Recent works have demonstrated that the Takagi-Sugeno (T-S) model or neural network model is good at addressing nonlinear dynamic leading to the motivation of integrating either of the above modeling method with RMPC to achieve a new RMPC control scheme for nonlinear industrial processes, which will be some interesting issues as our future works.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no competing financial interest.

Acknowledgments

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