

Research Article

Based on Grid Reference Frame for SINS/CNS Integrated Navigation System in the Polar Regions

Song Lijun ^(b),¹ Zhao Wanliang,² Cheng Yuxiang,² and Chen Xiaozhen³

¹School of Information & Control Engineering, Xi'an University of Architecture and Technology, Xi'an 710055, China
 ²Shanghai Aerospace Control Technology Institute, Shanghai 201109, China
 ³Beijing Aerospace Control Instrument Research Institute, Beijing 100039, China

Correspondence should be addressed to Song Lijun; songlijun9071@sina.com

Received 6 August 2018; Accepted 25 October 2018; Published 1 January 2019

Guest Editor: Junpei Zhong

Copyright © 2019 Song Lijun et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

As the inertial navigation system cannot meet the precision requirements of global navigation in the special geographical environment of the Polar Regions, this paper presents Strapdown Inertial Navigation System (SINS)/Celestial Navigation System (CNS) integrated navigation system of airborne based on Grid Reference Frame (GRF) and the simulation is carried out. The result of simulation shows that the SINS/CNS integrated navigation system is superior to the single subsystem in precision and performance, which not only effectively inhibits the error caused by gyro drift but also corrects the navigation parameters of system without delay. Comparing the simulation in the middle and low latitudes and in the Polar Regions, the precision of SINS/CNS integrated navigation system is the same in the middle and low latitudes and in the Polar Regions.

1. Introduction

With the development of science and technology, the change of economy, military situation, energy, shipping, scientific research, and military value of Polar Regions are becoming more and more prominent and the activities of countries in the Polar Regions are also becoming more and more frequently. However, the problems of existing navigation devices are low reliability and poor security in the Polar Regions because there are few inhabitants around the place, unique topographic and geomorphic conditions.

At present, the arrangement of algorithm and the analysis of error are focused on the research of airborne navigation in the Polar Regions. There is not established complete theoretical system which can support the actual navigation system of the Polar Regions, there are many problems which urgently need to be solved. SINS is considered as the first choice of self-contained navigational aids of polar navigation, but SINS has the limitation of accumulating error with time. It is difficult to only rely on SINS to complete the high precision and long navigation of the airborne. Therefore, it is urgent to introduce the combination of external reference information and inertial navigation system for data fusion in polar navigation [1, 2].

2. The SINS/CNS Integrated Navigation System in the Middle and Low Latitudes

The CNS is a navigation which based on indestructible natural celestial bodies. The star sensor is often used to detect the stellar map to determine the attitude of the aircraft, relative to the inertial coordinate system, and the result of attitude is very high precision. However, the update rate of CNS is very low, and it cannot provide real-time information about velocity and position of aircraft. By the high precision attitude of CNS, SINS/CNS integrated navigation system revises the error of SINS with time to improve the measurement precision of integrated navigation system [3, 4].

2.1. The Principle of SINS/CNS Integrated Navigation System. According to the different installation methods of star sensor and inertial device, the working mode of SINS/CNS integrated navigation system can be used as strapdown mode. The



FIGURE 1: The structure of SINS/CNS integrated navigation system.

three-axis attitude of aircraft is given by SINS in SINS/CNS integrated navigation system, and the star sensor output the transformation matrix of inertial coordinate system, relative to the star sensor coordinate system. The process of combinatorial is as follows.

The first, the transformation matrix of inertial coordinate system relative to the star sensor coordinate system, which is calculated by the position and attitude of SINS. The second, the transformation matrix of SINS and the transformation matrix of star sensor output, which is subtracted as the measurement sent to Kalman filter for information fusion to obtain the optimal estimation value of integrated navigation system. The last, the optimal estimation value of integrated navigation system, which is used to adjust the error of SINS; it is the result of integrated navigation system [5, 6]. The structure chart of SINS/CNS integrated navigation system is shown in Figure 1.

2.2. The State Space Mode of SINS/CNS Integrated Navigation System in the Middle and Low Latitudes. It is S coordinate system as the measuring coordinate system of star sensor; the output of star sensor is attitude matrix C_i^s which is the star sensor coordinate system relative to the inertial coordinate system. Due to the high precision of star sensor, the error of star sensor can be ignored as long as the installation error of star sensor is strictly calibrated [7, 8]. The transformation matrix of star sensor can be obtained as follows:

$$\widehat{\mathbf{C}}_{CNS}^{S} = \mathbf{C}_{i}^{s} + \mathbf{C}^{w} \tag{1}$$

The attitude error \mathbf{C}^{w} can be considered as Gauss white noise.

$$\mathbf{C}^{w} = \begin{bmatrix} C_{11}^{w} & C_{12}^{w} & C_{13}^{w} \\ C_{21}^{w} & C_{22}^{w} & C_{23}^{w} \\ C_{31}^{w} & C_{32}^{w} & C_{33}^{w} \end{bmatrix}$$
(2)

The various \mathbf{C}^{w} are satisfied

$$E\left[C_{ij}^{w}(t)\right] = 0$$

$$E\left[C_{ij}^{w}(t) \quad C_{ij}^{w}(\tau)\right] = q_{ij}^{w}\delta(t-\tau) \qquad (3)$$

$$i, j = 1, 2, 3.$$

The transformation matrix from the measurement coordinate system of star sensor S to body coordinate system b is C_b^s , the attitude matrix of SINS is C_{nc}^b , the position matrix

of SINS is C_{ec}^{n} , and the transformation matrix with inertial coordinate system relative to terrestrial coordinate system is C_{i}^{e} ; then the transformation matrix calculated by the SINS is

$$\widehat{\mathbf{C}}_{SINS}^{S} = \mathbf{C}_{b}^{s} \mathbf{C}_{nc}^{b} \mathbf{C}_{ec}^{n} \mathbf{C}_{i}^{e}$$
(4)

where $\mathbf{C}_{nc}^{b} = \mathbf{C}_{n}^{b}[\mathbf{I} + \boldsymbol{\varphi}^{n} \times], \mathbf{C}_{ec}^{n} = \mathbf{C}_{e}^{n}[\mathbf{I} - (\delta P \times)].$ So

$$\widehat{\mathbf{C}}_{SINS}^{S} = \mathbf{C}_{b}^{s} \mathbf{C}_{nc}^{b} \mathbf{C}_{ec}^{n} \mathbf{C}_{i}^{e}$$

$$= \mathbf{C}_{b}^{s} \mathbf{C}_{n}^{b} \left[\mathbf{I} + \boldsymbol{\phi} \times \right] \mathbf{C}_{e}^{n} \left[\mathbf{I} - (\delta \mathbf{P} \times) \right] \mathbf{C}_{i}^{e}$$

$$= \mathbf{C}_{b}^{s} \mathbf{C}_{n}^{b} \mathbf{C}_{e}^{n} \mathbf{C}_{i}^{e} + \mathbf{C}_{b}^{s} \mathbf{C}_{n}^{b} \left(\boldsymbol{\phi} \times \right) \mathbf{C}_{e}^{n} \mathbf{C}_{i}^{e}$$

$$- \mathbf{C}_{s}^{s} \mathbf{C}^{b} \left(\delta \mathbf{P} \times \right) \mathbf{C}^{n} \mathbf{C}_{e}^{e}$$
(5)

where

$$\begin{bmatrix} \phi \times \end{bmatrix} = \begin{bmatrix} 0 & -\delta \phi_U & \delta \phi_N \\ \delta \phi_U & 0 & -\delta \phi_E \\ -\delta \phi_N & \delta \phi_E & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -\delta \lambda \sin L & \delta \lambda \cos L \\ \delta \lambda \sin L & 0 & \delta L \\ -\delta \lambda \cos L & -\delta L & 0 \end{bmatrix}$$
(6)

When the difference of attitude angles measured by SINS and CNS is very small, the cross-coupled term of transformation matrix can be approximated to zero. The measurement of SINS/CNS integrated navigation system is that the transformation matrix of star sensor subtracts from the transformation matrix calculated by SINS; it is

$$\mathbf{Z}_{SINS/CNS} = \widehat{\mathbf{C}}_{SINS}^{S} - \widehat{\mathbf{C}}_{CNS}^{S}$$
$$= \mathbf{C}_{b}^{s} \mathbf{C}_{n}^{b} \left(\boldsymbol{\phi} \times\right) \mathbf{C}_{e}^{n} \mathbf{C}_{i}^{e} - \mathbf{C}_{b}^{s} \mathbf{C}_{n}^{b} \left(\delta \mathbf{P} \times\right) \mathbf{C}_{e}^{n} \mathbf{C}_{i}^{e} \qquad (7)$$
$$- \mathbf{C}^{w}$$

 $\delta\lambda$ and δL are ascensional difference and declination difference; it is a small angle of arc-second, so it can be ignored the influence by the longitude and latitude, $[\delta P \times] = 0_{3\times3}$; it can be simplified as follows:

$$\mathbf{Z}_{SINS/CNS} = \widehat{\mathbf{C}}_{SINS}^{S} - \widehat{\mathbf{C}}_{CNS}^{S} = \mathbf{C}_{b}^{s} \mathbf{C}_{n}^{b} (\boldsymbol{\phi} \times) \mathbf{C}_{e}^{n} \mathbf{C}_{i}^{e} - \mathbf{C}^{w}$$
(8)

Appointment

$$\mathbf{C}_{b}^{s}\mathbf{C}_{n}^{b} = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix},$$

$$\mathbf{C}_{e}^{n}\mathbf{C}_{i}^{e} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{bmatrix}$$
(9)

It can be obtained

$$\hat{\mathbf{Z}}_{SINS/CNS} = \begin{bmatrix} Z_{11} \\ Z_{12} \\ Z_{13} \\ Z_{21} \\ Z_{22} \\ Z_{23} \\ Z_{33} \end{bmatrix} \\
= \begin{bmatrix} s_{13}n_{21} - s_{12}n_{31} & s_{11}n_{31} - s_{13}n_{11} & s_{12}n_{11} - s_{11}n_{21} \\ s_{13}n_{22} - s_{12}n_{32} & s_{11}n_{32} - s_{13}n_{12} & s_{12}n_{12} - s_{11}n_{22} \\ s_{13}n_{23} - s_{12}n_{33} & s_{11}n_{33} - s_{13}n_{13} & s_{12}n_{13} - s_{11}n_{23} \\ s_{23}n_{21} - s_{22}n_{31} & s_{21}n_{31} - s_{23}n_{11} & s_{22}n_{11} - s_{21}n_{21} \\ s_{23}n_{23} - s_{12}n_{33} & s_{11}n_{33} - s_{13}n_{13} & s_{22}n_{13} - s_{21}n_{21} \\ s_{23}n_{23} - s_{22}n_{33} & s_{21}n_{33} - s_{23}n_{13} & s_{22}n_{13} - s_{21}n_{22} \\ s_{33}n_{21} - s_{32}n_{31} & s_{31}n_{31} - s_{33}n_{11} & s_{32}n_{11} - s_{31}n_{21} \\ s_{33}n_{22} - s_{32}n_{33} & s_{31}n_{33} - s_{33}n_{13} & s_{32}n_{13} - s_{31}n_{23} \\ s_{33}n_{23} - s_{32}n_{33} & s_{31}n_{33} - s_{33}n_{13} & s_{32}n_{13} - s_{31}n_{23} \\ \end{bmatrix} \begin{bmatrix} \delta\phi_E \\ \delta\phi_N \\ \delta\phi_U \end{bmatrix} (10)$$

$$\begin{bmatrix} C_{12} \\ C_{13}^{w} \\ C_{21}^{w} \\ C_{22}^{w} \\ C_{23}^{w} \\ C_{31}^{w} \\ C_{32}^{w} \\ C_{33}^{w} \\ C_{33}^{w} \end{bmatrix} = H_{SINS/CNS} X_{INS} - V_{CNS}$$

2.3. The Simulation and Analysis of SINS/CNS Integrated Navigation System in the Middle and Low Latitudes. The simulation time of SINS/CNS integrated navigation system in the middle and low latitudes is 600s, the time of attitude updating, velocity updating, and position updating are 20ms, and the sampling period of inertial sensor is 10ms [9]. In the simulation, the parameters are set as follows:

The initial error of SINS is as follows.

The initial error of attitude is $[0.5' \ 0.5' \ 20']$, the initial error of velocity is $[0.01 \ 0.01 \ 0.01]$ m/s, and the initial error of position is $[20 \ 20 \ 20]m$.

The parameters of inertial sensor are as follows.

- (1) The random constant drift of gyroscope is $0.01^{\circ}/h$, the random walk coefficient of gyroscope is $0.001^{\circ}/\sqrt{h}$, and the error of scale factor is 30ppm.
- (2) The random constant drift of accelerometer is 40ug and the random walk coefficient of accelerometer is $5 \text{ug}\sqrt{s}$.

The parameters of star sensor are as follows.

The horizontal measuring precision of angle is 10'', the azimuth measuring precision of angle is 20'', and the error of installation which between CNS and SINS is $[3', 3', 3']^{T}$.

These parameters are the same as in the Polar Regions. The simulation results are showed in Figure 2.

(1) The attitude error of SINS/CNS in the middle and low latitudes is showed as Figure 2(a). The attitude of CNS is considered as observation information in the SINS/CNS integrated navigation system, the convergence rate of attitude is quickened, the measurement accuracy of SINS/CNS integrated navigation system is improved effectively, and, after a few seconds, the attitude error is up to 0.15'.

(2) The velocity error and the position error of SINS/CNS in the middle and low latitudes are showed as Figures 2(b) and 2(c). They are divergent in the SINS/CNS integrated navigation system, because CNS was unable to provide real-time information of the velocity and position of aircraft.

(3) The constant drift of gyroscope and accelerometer of SINS/CNS in the middle and low latitude are showed as Figures 2(d) and 2(e). The horizontal attitude error is the main error of integrated navigation system; the influence of azimuth attitude error is relatively weak. The constant drift of gyroscope directly affects attitude error in the attitude information which is given by the inertial navigation system. It is better of the constant drift of gyroscope, and it is divergent of the constant drift of accelerometer in the SINS/CNS integrated navigation system.

3. The Algorithm of SINS/CNS Integrated Navigation System Based on Grid Reference Frame

In order to avoid the convergence of azimuth reference line at the poles, all the longitudes are redefined which are parallel to the Greenwich meridian [10, 11]. In this way, the azimuth attitude is relative to the Greenwich meridian and its parallel line, which is grid navigation. The grid navigation and the grid coordinate system have been described in the author's literature [12], so that the author don not bore you with unnecessary details.

The celestial navigation is an old method of navigation and position, and the irreplaceable of celestial navigation is determined by the autonomy of celestial navigation. Even in the present age, the advanced development of radio navigation system and the accuracy and timeliness of ship positioning have been well solved, the celestial navigation is still unwavering in the navigation. As early as 1989, Guo Honggui has proposed the celestial navigation of Polar Regions in the literature [13].



(a) The attitude error of SINS/CNS in the middle and low latitudes





(c) The position error of SINS/CNS in the middle and low latitude



FIGURE 2

3.1. The Relationship of the Grid Orientation and the True North. It is very difficult to navigate from the true north because of the convergence of polar meridians. The north azimuth inertial navigation system is adopted in the middle and low latitude, so the true north is thought as azimuth reference in the celestial navigation system. To overcome the difficult of convergence of polar meridians, the grid coordinate system is adopted in the inertial navigate system and the grid north is thought as azimuth reference in the celestial navigation system [13, 14].

Furthermore, the position of grid inertial navigation system can be directly matched with polar chart, and it can be realized by superposing grid lines on chart of Polar Regions. Because the grid line is parallel to the Greenwich meridian, the angle between the grid north (GN) and the true north (TN) is determined by the longitude of location and the convergence factor of chart [15-17].

The Northern Hemisphere is as follows:

GN = TN + longitude (W) * the convergence factor ofchart

GN = TN - longitude (E) * the convergence factor ofchart

The Southern Hemisphere is as follows:

GN = TN –longitude (W)* the convergence factor of chart

GN = TN +longitude (E)* the convergence factor of chart

3.2. The State Space Mode of SINS/CNS Integrated Navigation System in the Polar Regions. Because the star sensor optical axis is fixedly connected with aircraft, the state variable contained the installation deviation angle between the SINS and the star sensor. It selects the state variable of SINS/CNS as follows. The misalignment angles of $\boldsymbol{\phi} = [\boldsymbol{\phi}_{G_E} \ \boldsymbol{\phi}_{G_N} \ \boldsymbol{\phi}_{G_U}]^T$ are the misalignment angle of grid eastward, the misalignment angle of grid northward, and the misalignment angle of grid azimuth. The velocity errors of $\delta \boldsymbol{v}^G = [\delta v_{G_E} \ \delta v_{G_N} \ \delta v_{G_U}]^T$ are the velocity error of grid eastward, the velocity error of grid northward, and the velocity error of grid vertical. The position errors of $\delta \mathbf{R}^e = [\delta x \ \delta y \ \delta z]^T$ are the position error of x-axis, the position error of y-axis, and the position error of z-axis. The constant drift of gyroscope $\boldsymbol{\varepsilon} = [\varepsilon_x \ \varepsilon_y \ \varepsilon_z]^T$ is the constant drift of x-axis, the constant drift of y-axis, and the constant drift of z-axis. The constant drift of accelerometer $\nabla = [\nabla_x \quad \nabla_y \quad \nabla_z]^T$ is the constant drift of x-axis, the constant drift of y-axis, and the constant drift of z-axis. The installation deviation angles of $\boldsymbol{\mu} = [\mu_x \ \mu_y \ \mu_z]^T$ are the installation deviation angle of x-axis, the installation deviation angle of yaxis, and the installation deviation angle of z-axis. The system state vector of SINS/CNS is

$$\begin{aligned} \boldsymbol{X}_{C} &= \left[\boldsymbol{\phi}_{G_{E}}, \boldsymbol{\phi}_{G_{N}}, \boldsymbol{\phi}_{G_{U}}, \delta \boldsymbol{\nu}_{G_{E}}, \delta \boldsymbol{\nu}_{G_{N}}, \delta \boldsymbol{\nu}_{G_{U}}, \delta \boldsymbol{x}, \delta \boldsymbol{y}, \delta \boldsymbol{z}, \boldsymbol{\varepsilon}_{\boldsymbol{x}}, \boldsymbol{\varepsilon}_{\boldsymbol{y}}, \right. \\ & \left. \boldsymbol{\varepsilon}_{z}, \nabla_{\boldsymbol{x}}, \nabla_{\boldsymbol{y}}, \nabla_{\boldsymbol{z}}, \boldsymbol{\mu}_{\boldsymbol{x}}, \boldsymbol{\mu}_{\boldsymbol{y}}, \boldsymbol{\mu}_{\boldsymbol{z}} \right]^{T} \end{aligned} \tag{11}$$

Based on the error state equation of SINS and the system state vector of SINS/CNS integrated navigation system, the



(a) The three-dimensional trajectory of the SINS/CNS in the Polar Regions



(b) The position trajectory of the SINS/CNS in the Polar (c) The attitude trajectory of the SINS/CNS in the Polar Regions

Figure 3

state equation of SINS/CNS integrated navigation system can be received as follows:

$$\begin{bmatrix} \dot{\phi} \\ \delta \dot{v}^{G} \\ \delta \dot{R}^{e} \\ \dot{\epsilon} \\ \dot{\bar{v}} \\ \dot{\mu} \end{bmatrix}$$

$$= \begin{bmatrix} -\left(\omega_{iG}^{G}\times\right) C_{wGv} C_{13} - C_{b}^{G} 0_{3} \\ \left(f^{G}\times\right) C_{22} C_{23} 0_{3} C_{b}^{G} \\ 0_{3} C_{G}^{e} C_{33} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} 0_{3} \\ 0_{3} 0_{3} \\ 0_{3} 0_{3} \\ 0_{3} 0_{3} \\ 0_{3}$$

$$+ \begin{bmatrix} -C_b^G \varepsilon_w^b \\ C_b^G \nabla_w^b \\ \mathbf{0}_{3,1} \\ \mathbf{0}_{3,1} \\ \mathbf{0}_{3,1} \\ \mathbf{0}_{3,1} \end{bmatrix}$$

where

$$C_{13} = C_{weR} + C_{wGR}$$

$$C_{22} = v^{G} \times C_{wGv} - \left[\left(2\omega_{ie}^{G} + \omega_{eG}^{G} \right) \times \right]$$

$$C_{23} = v^{G} \times \left(2C_{weR} + C_{wGR} \right)$$

$$C_{33} = -C_{G}^{e} \left(v^{G} \times \right) C_{R}$$
(13)

(12)

The measurements of SINS/CNS integrated navigation system in the Polar Regions are the difference between



(a) The misalignment angle of SINS/CNS in the Polar Regions



(c) The position error of SINS/CNS in the Polar Regions



(e) The constant drift of accelerometer of SINS/CNS in the Polar Regions

the transformation matrix of SINS and the transformation matrix of star sensor, so the measurement of the SINS/CNS integrated navigation system in the Polar Regions is shown as

$$Z_{CNS} = \tilde{\boldsymbol{u}}^{p} - \tilde{\boldsymbol{u}}^{c}$$
$$= \left[(\boldsymbol{u}^{c} \times) \quad \boldsymbol{0}_{3} \quad \boldsymbol{u}^{c} \times \boldsymbol{M}_{p} \quad \boldsymbol{0}_{3} \quad \boldsymbol{0}_{3} \quad \boldsymbol{C}_{b}^{P} \left(\boldsymbol{u}^{c} \times \right) \right] \begin{bmatrix} \boldsymbol{\phi} \\ \delta \boldsymbol{v}^{G} \\ \delta \boldsymbol{R}^{e} \\ \boldsymbol{\varepsilon} \\ \nabla \\ \boldsymbol{\mu} \end{bmatrix}$$



(b) The velocity error of SINS/CNS in the Polar Regions



(d) The constant drift of gyroscope of SINS/CNS in the Polar Regions



(f) The error of installation deviation angle of SINS/CNS in the Polar Regions

Figure 4

$$\delta u^{\nu}$$
 (14)

where

+

$$\boldsymbol{M}_{p} = \begin{bmatrix} -1 & 0 & 0\\ 0 & \cos L & 0\\ 0 & \sin L & 0 \end{bmatrix}$$
(15)

3.3. The Trajectory of SINS/CNS Integrated Navigation System in the Polar Regions. It is the 6h in the simulation of SINS/CNS integrated navigation system when the aircraft is in the Polar Regions; the trajectory is shown in Figure 3.

Complexity

The parameters of trajectory are the starting point which is $[45^{\circ}N \ 108^{\circ}E \ 500m]$ and culmination of latitude which is $89.26^{\circ}N$.

The parameters of velocity are the initial velocity which is 0m/s and the top velocity which is 310m/s.

3.4. The Simulation of SINS/CNS Integrated Navigation System in the Polar Regions. In the simulation, the parameters of device are the same as the middle and low latitudes. The results of simulation are shown in Figure 4.

(1) The misalignment angle of SINS/CNS integrated navigation system in the Polar Regions is showed in Figure 4(a); they are convergent to 0.5' in 100s and basically the same to the attitude error of SINS/CNS integrated navigation system in the middle and low latitude.

(2) The velocity error and the position error of SINS/CNS integrated navigation system in the Polar Regions are showed in Figures 4(b) and 4(c); the error of velocity is Schuler period oscillation; the maximum amplitude is not more than 0.5m/s. The error of position is the same as the error of velocity; it is also Schuler period oscillation, and the maximum amplitude is not more than 30m.

(3) The constant drift of gyroscope and accelerometer of SINS/CNS integrated navigation system in the Polar Regions is showed in Figures 4(d) and 4(e); the amplitude error of velocity and position also decreases when the constant drift of gyroscope and accelerometer is estimated. The amplitude of velocity and position error also decreases.

(4) The error of installation deviation angle of SINS/CNS integrated navigation system in the Polar Regions is showed in Figure 4(f), the error of installation deviation angle is in accordance with the misalignment angle, and the error of installation deviation angle of SINS/CNS integrated navigation system in the Polar Regions converges to 0.3'.

4. Summary

The SINS/CNS integrated navigation system in the Polar Regions based on the Grid Reference Frame are the choreography of inertial mechanics and overcome the problem which is the difficulties in positioning and orientation of airborne caused by meridian convergence. In the paper, combining with the error of SINS and the state variable of SINS/CNS integrated navigation system in the Polar Regions which based on the Grid Reference Frame, the state space mode of SINS/CNS integrated navigation system in the Polar Regions is established. The simulation results showed that the SINS/CNS integrated navigation system in the Polar Regions could repress the error caused by the constant drift of gyroscope and correct the navigation parameters of SINS/CNS integrated navigation system. With the same precision of inertial components, the precision of SINS/CNS integrated navigation system in the Polar Regions and in the middle and low latitudes is consistent.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Disclosure

The research area is detection and information technology.

Conflicts of Interest

We declare that we do not have any commercial or associative interest that represents conflicts of interest in connection with the work submitted.

Acknowledgments

This work is partially supported by the School Foundation Research Fund of 2018 and the National Natural Science Foundation of China Project no. 51678470.

References

- L. Brigham, "Polar Ocean Navigation," *Encyclopedia of Earth sciences*, pp. 512–515, 2014.
- [2] Z. Zhao, Y.-B. Liang, and J. Hu, "Artic oil and natural gas potential and exploration and development trend," *Earth Science Frontiers*, vol. 21, no. 3, pp. 47–55, 2014.
- [3] C. Yang, X. Wang, Z. Li, Y. Li, and C. Su, "Teleoperation control based on combination of wave variable and neural networks," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2017.
- [4] G.-L. Yang, L.-F. Wang, and S.-J. Yang, "Online calibration method of missile-borne SINS/CNS integrated navigation system," *Journal of Projectiles, Rockets, Missiles and Guidance*, vol. 34, no. 5, pp. 29–32, 2014.
- [5] B. Zhou, W.-L. Zhao, Y.-J. Rong, X.-M. Hu, and J.-Y. Ma, "Robust Adaptive Kalman Filtering Algorithm for Integrated Navigation Based on MEMS-INS/GNSS," *navigation and control*, vol. 17, no. 4, pp. 14–20, 2018.
- [6] C. Yang, K. Huang, H. Cheng, Y. Li, and C. Su, "Haptic identification by ELM-controlled uncertain manipulator," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 2398–2409, 2017.
- [7] L.-F. Zhou, X.-M. Zhao, Z.-N. Hou, and C.-L. Ding, "CNS/SINS integrated calibration technique based on level damping," *Zhongguo Guanxing Jishu Xuebao/Journal of Chinese Inertial Technology*, vol. 25, no. 5, pp. 561–565, 2017.
- [8] C. Yang, J. Luo, Y. Pan, Z. Liu, and C. Su, "Personalized variable gain control with tremor attenuation for robot teleoperation," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, pp. 1–12, 2017.
- [9] X.-N. Huang, J.-R. Liu, and W. Yang, "An Evaluation Method for the Head Accuracy of Inertial Navigation System in Flight Test," *navigation and control*, vol. 4, pp. 7–12, 2017.
- [10] Q. Zhou, Y. Qin, Q. Fu, and Y. Yue, "Grid mechanization in inertial navigation systems for transpolar aircraft," *Xibei Gongye Daxue Xuebao/Journal of Northwestern Polytechnical University*, vol. 31, no. 2, pp. 210–217, 2013.
- [11] Y. Gao, M. Liu, G. Li, and X. Guang, "Initial alignment for SINS based on pseudo-earth frame in polar regions," *Sensors*, vol. 17, no. 6, 2017.
- [12] L.-J. Song, Z. Li, Q.-W. Fu, and B. He, "Researched of SINS/GPS Integrated Navigation System based on Grid Reference Frame for Transpolar Aircraft," *Journal of System Simulation*, 2018.

- [13] H.-G. Guo and W.-S. Wang, "Celestial navigation in polar Regions," *Journal of Dalian Maritime University*, vol. 15, no. 3, pp. 43–48, 1989.
- [14] V. B. Larin and A. A. Tunik, "On inertial navigation system error correction," *International Applied Mechanics*, vol. 48, no. 2, pp. 213–223, 2012.
- [15] H. Zhen, Y. Wang, and H.-B. Wnag, "Simulation of geomagnetic aided submarine navigation based on EMAG2," *Progress in Geophysics*, vol. 27, no. 4, pp. 1795–1803, 2012.
- [16] Q. Li, Y. Ben, F. Yu, and J. Tan, "Transversal strapdown INS based on reference ellipsoid for vehicle in the Polar Region," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 9, pp. 7791–7795, 2016.
- [17] Y.-Q. Yao, X.-S. Xu, Y. Li, Y.-T. Liu, J. Sun, and J.-W. Tong, "Transverse Navigation under the Ellipsoidal Earth Model and its Performance in both Polar and Non-polar areas," *Journal of Navigation*, vol. 69, no. 2, pp. 335–352, 2016.



Operations Research

International Journal of Mathematics and Mathematical Sciences







Applied Mathematics

Hindawi

Submit your manuscripts at www.hindawi.com



The Scientific World Journal



Journal of Probability and Statistics







International Journal of Engineering Mathematics

Journal of Complex Analysis

International Journal of Stochastic Analysis



Advances in Numerical Analysis



Mathematics



Mathematical Problems in Engineering



Journal of **Function Spaces**



International Journal of **Differential Equations**



Abstract and Applied Analysis



Discrete Dynamics in Nature and Society



Advances in Mathematical Physics