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Research Article

Differentiating the Personalized Information of the Physician-Patient Communication for the Chronic Obstructive Pulmonary Disease with General Probabilistic Vector Linguistic Terms

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To accurately determine the follow-up therapeutic schedules for the chronic obstructive pulmonary disease (COPD) patients, this paper aims to develop the analysis tools for the linguistic evaluation to improve the quality of the physician-patient communication. Firstly, we define the general probabilistic vector linguistic term (GPVLT), which is effective to depict people's judgements from different sources. Then, we establish the multigranularity linguistic space and discuss the different forms of the probabilistic vector linguistic units (PVLUs) in it. Later on, we propose the nondirectional and the directional potentials of PVLUs, which can grasp the fuzziness and the development direction of the linguistic evaluations, respectively. Last but not least, the cases about the physician-patient communication for COPD and some comparisons with the other related methods are provided to illustrate the effectiveness and practicability of the PVLUs' potentials.

1. Introduction

Chronic obstructive pulmonary disease (COPD) [1], a life-threatening disease of the lungs, reduces the quality of life and causes premature death. With the deterioration of the environment in the 21st century, which is caused by construction dust, vehicle exhaust, and fumes, more and more people suffer from the COPD [2]. The related literature [3] and the Authoritative Medical Science Communication Network Platform release the fundamental knowledge about COPD, such as the symptoms, classifications, and stages. For this incurable disease, appropriate treatment can slow down its progress. Physician-patient communication, which can help patients develop self-management, is a new strongly recommended way to improve the clinical treatments of COPD [3, 4]. In this case, the quality and orientation of the physician-patient communication are considered to be two significant aspects of the care and treatment of COPD [5, 6].

As an important component of the medical service, the physician-patient communication establishes a special interpersonal relationship between the medical care providers and the receivers. Physician-patient relationship, which is regarded as a type of complex multiattribute problems of social security system [7, 8], has been investigated from different angles in the past, such as the effects of the treatment [5, 8–10], the communication techniques [10], and so on. From the above researches, we may safely obtain the conclusions that (i) language is the most usual and available way in the physician-patient communication process for COPD [3, 7, 9] and (ii) good communication between the doctors and patients has positive impacts on the diagnosis and the treatment of COPD [4, 10]. It follows that distinguishing the quality and change trends of the linguistic evaluations accurately are very important for the physician-patient communication process.

Fuzzy linguistic methods [11], which link the gap between the linguistic fuzziness and the numerical absoluteness, are the straightforward and suitable techniques to handle the decision-making problems with qualitative information. Linguistic evaluation scales (LESs) [11] are the products of fuzzy linguistic methods that each element is the combination symbol of a linguistic descriptor and a real number. Over the past few decades, many kinds of the LESs were presented and investigated, such as the subscript-symmetric LESs [12], the symmetrically and nonuniformly distributed LESs [13], the nonsymmetrically distributed LESs [14, 15], the multiplicative LESs [16], etc. When we use the LESs to evaluate objects and make a decision, the different kinds of linguistic term sets were proposed, like the interval hesitant linguistic term set [17], the hesitant linguistic term set [18], the extended linguistic term set [19], the hesitant fuzzy uncertain linguistic set [20], the dual hesitant fuzzy linguistic set [21], the probabilistic linguistic term set [22, 23], the linguistic 2-tuple model [24], the virtual linguistic model [25, 26], and so on. Moreover, to cope with the specific decision-making problems, some tools, such as the linguistic measures [27], the linguistic integrations [28], the linguistic preference relations [29–31], and the linguistic decision matrices [32, 33], were proposed to process the evaluation information. All the above researches with linguistic evaluations provide the effective procedures to portray the qualitative decision-making problems.

However, in the practical physician-patient communication process of COPD, the same linguistic evaluation may indicate the different meanings for different doctors and patients. For example, suppose that a doctor has an individual LES $S_{\text{doctor}} = \{s_{-1} =$ "the most serious," $s_{-0.7} =$ "serious," $s_{-0.5} =$ "more serious," $s_{-0.2} =$ "slight serious," $s_0 =$ "neutral," $s_{0.2} =$ "slight normal," $s_{0.5} =$ "more normal," $s_{0.7} =$ "normal," $s_1 =$ "the most normal"$\}$ and a COPD patient has an individual LES $S_{\text{patient}} = \{s_{-1} =$ "the most serious," $s_{-0.7} =$ "serious," $s_{-0.4} =$ "slight serious," $s_0 =$ "neutral," $s_{0.4} =$ "slight normal," $s_{0.7} =$ "normal," $s_1 =$ "the most normal"$\}$. The doctor gives "slight serious" as the diagnosis for the patient to explain that the illness is just a slight deviation from "neutral." But the patient may interpret it as close to "serious" based on his individual LES. Thus, the physician-patient communication of COPD is a multigranularity linguistic decision-making [34–37], and there is a gap between the doctor and patient's understandings of the same linguistic evaluation "slight serious." If they fail to properly recognize and deal with the gap in the communication process, it easily leads to poor communication quality or arising conflict.

Multigranularity linguistic decision-making is a kind of linguistic decision-making problem that a group of experts are invited to evaluate the same object together based on several different LESs. In the existing results of multigranularity linguistic evaluation, the LESs are assigned into the different hierarchies according to the characteristic granularities of them [34–37]. In the multigranularity linguistic decision-making process, there is an exploitation phase that should be taken before getting the final alternative solution. Although these results provided good ways to assign several LESs with different granularities in a linguistic evaluation process, they are hard to assign different LESs with different distributions. Besides, to fill up the above gap in the physician-patient communication of COPD, the expression of linguistic evaluation should be able to describe and distinguish the meanings of the same linguistic evaluation from different LESs with different distributions. Although the above-mentioned various linguistic terms can address linguistic evaluations well in decision-making, they are lacking in distinguishing the same linguistic evaluation for different experts based on different individual LESs. To address this issue, in 2016, Zhai et al. [38] introduced a probabilistic linguistic vector expression that can be directly driven from people's own individual LESs. Through a numerical illustration of a personal hospital selection-recommender system, the probabilistic vector expression model of linguistic term has been verified that it can distinguish the different semantics of linguistic terms with the same numerical symbols better than the numerical symbol [38]. In 2017, Li et al. [39] proposed a personalized individual semantics model, which is driven based on a consistency optimization model of linguistic preference relation, by means of interval numerical scales and a 2-tuple linguistic model [24]. Furthermore, this model was extended to the hesitant fuzzy linguistic information environment [40] and was applied in a consensus process of the large-scale linguistic group decision-making [41]. Considering that the probabilistic vector expression contains the probability description of linguistic term and the numerical example in reference [38] is not related to the linguistic preference relation with the consistency problem and the consensus process, we choose the probabilistic vector linguistic term (PVLT) [38] as the research basis of this paper. This paper is the follow-up study of reference [38], which continues the study perspective of hierarchies in people's cognitive conscious that proposed the PVLT in reference [38] and aims to improve the effectiveness of PVLT in algebraic calculations.

PVLT [38] melts the different LESs with different distributions into a plane rectangular coordinate system and expresses them by vectors. It is a convenient technique to distinguish the same linguistic evaluation from different LESs in the multigranularity linguistic decision-making. PVLT can simultaneously take the values and the directions of the linguistic evaluation into account, based on which we can analyze the quality of physician-patient communication in more detail. For example, the values and the change directions of the linguistic evaluation are both important for doctors to determine the follow-up therapeutic schedules for each COPD patient. All linguistic evaluations in a time period (from the start time point to the terminal time point) make up an information flow. Based on PVLT, we can grasp the development direction of the linguistic evaluation flow, which can help us accurately determine the follow-up communication strategies for the treatments of COPD. It conforms to the needed techniques in the physician-patient communication of COPD treatment process.

Even though the PVLT can comprehensively reflects people's judgements based on their own individual LESs, it still has the limitation that the ordinate of the vector in the PVLT falls in the interval $(-\infty, +\infty)$. It may be inconvenient in the integral process of linguistic evaluations [38]. To improve the computational performance of the ordinate of the vector in PVLT, this paper proposes the general PVLT (GPVLT) to limit the ordinate of the vector in PVLT on the interval $[0, 1]$. This way of defining GPVLT can reflect people's different modes of giving linguistic evaluations in decision-making. As a result, the paper aims to define the GPVLT to improve the computational capability of the ordinate of the vector in PVLT and to develop the tools based on GPVLT to handle the physician-patient communication problems for COPD. The main contributions of this paper are listed below:

(1) We introduce the individual vector linguistic system to describe the individual characteristic relations among the domain of discussion, the membership functions and the LES. Through the individual vector linguistic system, all numerical linguistic evaluations can be transformed into the vectors.

(2) We define the multigranularity vector linguistic space (MGVLS), in which the vector linguistic evaluation has the statistic properties, to deal with the multigranularity decision-making problems. In the MGVLS, we extend the PVLT into the GPVLT.

(3) We study several forms of the probabilistic vector linguistic units (PVLUs), which are different kinds of combinations of the GPVLTs. The nondirectional and the directional potentials of PVLUs are proposed.

(4) We apply the potentials of the PVLUs into the physician-patient communication for COPD to illustrate their effectiveness and practicality. Some comparisons are illustrated to show the advantages of the new proposed methods, where some drawbacks of them are also indicated.

The rest of this paper is organized as follows: Section 2 briefly reviews some related concepts of the PVLT. Section 3 defines the individual vector LESs, the MGVLS and the GPVLT. We also present the PVLU and discuss the different forms of it in this section. Section 4 introduces two kinds of potentials for different forms of the PVLUs, i.e., the nondirectional and the directional potentials. The two potentials are applied to deal with the physician-patient communication for COPD in Section 5, which manifests the effectiveness and the applicability of the potentials. Section 6 first compares the new proposed methods with the other related methods and then discusses the drawbacks and the advantages of them. Finally, Section 7 draws some conclusions of the paper and indicates the relevant further studies.

2. Preliminaries

In this section, we review some basic operations of the PVLTs.

Definition 1 (see [38]). Let $\{S_k\}_{k=1}^{N} = \{S_k \mid k = 1, 2, \ldots, N; N \in N^+\}$ be a set of LESs. When a group of experts choose the linguistic terms from the LES S_k to evaluate the objects, the following steps can be used to transform all the selected linguistic evaluations into the normalized vector linguistic terms:

Let $s_\alpha \in [s_{\alpha^\tau}, s_{\alpha^{\tau+1}}]$ be a linguistic term [42] of S_k, where s_{α^τ} and $s_{\alpha^{\tau+1}}$ are the τ − th and $\tau + 1$ − th linguistic terms of S_k, respectively. Here, we take s_α as an example to show the transformation steps:

(1) Normalizing the linguistic term s_α to the normalized linguistic term $s_{\overleftrightarrow{\alpha}}$ by the equation $\overleftrightarrow{\alpha} = (\alpha - \min\{s_\beta \mid s_\beta \in S_k\})/(\max\{s_\beta \mid s_\beta \in S_k\} - \min\{s_\beta \mid s_\beta \in S_k\})$.

(2) Calculating the relative change ratio $rr_{s_{\overleftrightarrow{\alpha}}}$ of the normalized linguistic term $s_{\overleftrightarrow{\alpha}}$ by $rr_{s_{\overleftrightarrow{\alpha}}} = ((\alpha - \alpha^\tau)/(\alpha^{\tau+1} - \alpha)) - 1$.

(3) Obtaining the absolute change ratio $ar_{s_{\overleftrightarrow{\alpha}}}$ of $s_{\overleftrightarrow{\alpha}}$ by $ar_{s_{\overleftrightarrow{\alpha}}} = \sum_{k=1}^{\tau} rr_{s_{\overleftrightarrow{\alpha}k}} + ((\alpha - \alpha^\tau)/(\alpha^{\tau+1} - \alpha)) - 1$.

(4) Transforming the normalized linguistic term $s_{\overleftrightarrow{\alpha}}$ into a vector. In the plane rectangular coordinate system $s_{\overleftrightarrow{\alpha}} - O - ar_{s_{\overleftrightarrow{\alpha}}}$, the abscissa axis indicates the linguistic evaluation $s_{\overleftrightarrow{\alpha}}$ and the vertical axis indicates the absolute change rate of $s_{\overleftrightarrow{\alpha}}$. Let $\vec{i}$ be the unit vector of the abscissa axis and $\vec{j}$ be the unit vector of the vertical axis, then the linguistic evaluation $s_{\overleftrightarrow{\alpha}}$ can be transformed into the normalized vector linguistic term $\vec{s}_{\overleftrightarrow{\alpha}} = s_{\overleftrightarrow{\alpha}}\vec{i} + ar_{s_{\overleftrightarrow{\alpha}}}\vec{j}$.

Additionally, if we consider the statistic property of $\vec{s}_{\overleftrightarrow{\alpha}}$ (denoted as $p_{\vec{s}_{\overleftrightarrow{\alpha}}}$), then the PVLT can be obtained, which is $(\vec{s}_{\overleftrightarrow{\alpha}}, p_{\vec{s}_{\overleftrightarrow{\alpha}}}) = (s_{\overleftrightarrow{\alpha}}\vec{i} + ar_{s_{\overleftrightarrow{\alpha}}}\vec{j}, p_{\vec{s}_{\overleftrightarrow{\alpha}}})$.

Note that, in the above normalization and transformation processes of the linguistic term s_α, all the calculations are applied for the subscript α, where the corresponding linguistic word is unchanged.

3. The GPVLT and the Forms of the PVLUs in the MGVLS

In this section, we first define the individual linguistic system to describe the individual characteristic relations among the domain of discussion, the membership functions, the LESs, and the interval $[0, 1]$. Due to that different people may have different expertise and knowledge levels, in the first section, we also propose the multigranularity linguistic space to simultaneously analyze the experts' linguistic evaluations based on the individual LESs. Through the transformation steps of Definition 1, each linguistic evaluation in the multigranularity linguistic space can be transformed into the normalized vector based on the individual LES. Thus, the MGVLS is proposed in the second section, based on which the concept of GPVLT is presented in the third section. Last but not least, we study the different forms of the PVLUs.

3.1. The Individual Linguistic System and the Multigranularity Linguistic Space

3.1.1. The Individual Linguistic System. Linguistic evaluation, a popular way to describe the evaluations and the opinions of the experts, is hard to be quantified for its fuzziness of the words. To handle this qualitative information in the decision-making problems, the LES is utilized to link the linguistic evaluations and the real numbers. Let $X = \{x\}$ be the domain of discourse and LEs be the linguistic evaluations. Based on a set of membership functions $\{\mu(x) : X \longrightarrow \text{LEs}\}$, we can evaluate each $x \in X$ by a set of linguistic evaluations with the membership degrees. Then the normalized LES translates the linguistic evaluations into the values in the interval $[0, 1]$ by the membership functions $\{\mu(S) : [0, 1] \longrightarrow \text{LEs}\}$. Therefore, the normalized LES can be regarded as the distribution of the linguistic evaluations on $[0, 1]$.

For example, for an issue "selecting the appropriate retirement age to alleviate the employment pressure," we choose the interval $[0, 100]$ as the domain of discourse X and assign $L = \{l_1 = \text{"young,"}\ l_2 = \text{"appropriate,"}\ l_3 = \text{"old"}\}$ as the set of the linguistic evaluations. As shown in Figure 1, the relation between X and L can be described by $\{\mu(x) : X \longrightarrow L\}$ in the plane rectangular coordinate system $X - O - L$, which corresponds to people's linguistic evaluation procedure.

The right plane rectangular coordinate system $S - O - L$ in Figure 1 describes the relations among the linguistic evaluations, the normalized LES S, and the interval $[0, 1]$, which corresponds to people's numerical evaluation criteria. For instance, a person illustrated by Figure 1 thinks value 0 is equal to "young," 0.5 is equal to "appropriate" and 1 is equal to "old." Therefore, the membership levels of 0 to "young," 0.5 to "appropriate," and 1 to "old" are all 1, where the normalized LES $S^1 = \{s_0 = \text{"young,"}\ s_{0.5} = \text{"appropriate,"}\ s_1 = \text{"old"}\}$ expresses this strong correspondences. In addition, the membership levels of the relations between all other values of $[0, 1]$ and the linguistic evaluations are less than 1, which can be seen as the weak correspondences. If we denote $[0, 1]$ as the interval $[s_0, s_1]$, then all the strong and weak correspondences between L and $[0, 1]$ can be interpreted by $\{\mu(S) : [s_0, s_1] \longrightarrow L\}$.

Definition 2. Let X be the domain of discourse, $L = \{l_k \mid k = 1, 2, \ldots, N; N \in N^+\}$ be a set of the linguistic evaluations, $S = \{s_\alpha^\tau \mid \alpha \in [0, 1]; \tau = 1, 2, \ldots, N, N \in N^+\}$ be the normalized LES with respect to L, $\{\mu(x) : X \longrightarrow L\}$, and $\{\mu(S) : [s_0, s_1] \longrightarrow L\}$ be two sets of the membership functions, then we call the system built by the above components as an individual linguistic system, denoted by $\Omega(X, \{\mu(x)\}, L, \{\mu(S)\}, S)$.

For the individual linguistic system $\Omega(X, \{\mu(x)\}, L, \{\mu(S)\}, S)$, there are two plane rectangular coordinate systems $X - O - L$ and $S - O - L$, which correspond to people's linguistic evaluation procedures and numerical evaluation criteria, respectively.

By Figure 1, we find that (i) X is divided by the linguistic evaluations L based on the membership functions $\{\mu_t(x) : X \longrightarrow L \mid t = 1, 2, \ldots, N_1; N_1 \in N^+\}$ and (ii) the mapping between L and S is one to one; (iii) the interval $[s_0, s_1]$ is divided by the normalized LES S based on the membership function set $\{u_t(S) : [s_0, s_1] \longrightarrow L \mid t = 1, 2, \ldots, N_2; N \in N^+\}$. Let a positive integer N be the number of subregions of X divided by $\{\mu(x) : X \longrightarrow L\}$, then we can deduce that the numbers of L, S, and the segmentations of $[s_0, s_1]$ are equal to N. This reflects the uniformity of the individual linguistic system $\Omega(X, \{\mu(x)\}, L, \{\mu(S)\}, S)$ with respect to N, which is demonstrated in Figure 2.

3.1.2. The Multigranularity Linguistic Space. Let us continue with the problem of "selecting the appropriate retirement age for alleviating the employment pressure." Another person may assign age 60 as the "appropriate" with the membership level 1, and his/her normalized LES may be $S^2 = \{s_0 = \text{"young,"}\ s_{0.3} = \text{"slightly young,"}\ s_{0.5} = \text{"appropriate,"}\ s_{0.75} = \text{"slightly old,"}\ s_1 = \text{"old"}\}$. Thus, from the normalized LESs S^1 and S^2, it is obvious that people's evaluation thinking and judgement criteria are different. Denoting the individual linguistic systems provided by the two persons as $\Omega^1 = \Omega(X, \{\mu^1(x)\}, L^1, \{\mu^1(S)\}, S^1)$, and $\Omega^2 = \Omega(X, \{\mu^2(x)\}, L^2, \{\mu^2(S)\}, S^2)$, respectively, and gathering the two individual linguistic systems together, we can obtain an information space (as shown in Figure 3).

In Figure 3, the individual linguistic systems Ω^1 and Ω^2 are two different linguistic evaluation sources, where the numbers of S^1 and S^2 are 3 and 5, respectively. For any given linguistic evaluation, it comes from Ω^1 or Ω^2. Then the status-parallel components Ω^1 and Ω^2 develop a greater information space.

Definition 3. Let $\Omega^\tau = \Omega(X, \{\mu^\tau(x)\}, L^\tau, \{\mu^\tau(S)\}, S^\tau)$, $\tau = 1, 2, \ldots, N$ be N individual linguistic systems for the same domain of discourse X, we call $\{\Omega^\tau\}_{\tau=1}^N = \{\Omega(X, \{\mu^\tau(x)\}, L^\tau, \{\mu^\tau(S)\}, S^\tau) \mid \tau = 1, 2, \ldots, N\}$ a multigranularity linguistic space constructed by the individual linguistic systems Ω^1, Ω^2, ..., and Ω^N.

The multigranularity linguistic space $\{\Omega^\tau\}_{\tau=1}^N$ is the largest collection of the linguistic evaluations got from different individual linguistic systems $\Omega^1, \Omega^2, \ldots$, and Ω^N. Let s_α be a linguistic term of the multigranularity linguistic space. Adding the statistic property of it into s_α, we get the probabilistic linguistic term (s_α, p_{s_α}). When we use any (s_α, p_{s_α}) from the multigranularity linguistic space to make a decision, the fundamental work is to analyze the meanings of s_α. Taking the multigranularity linguistic space illustrated by Figure 3 as an example, we deeply discuss the meanings of $s_{0.4}$. It is associated with "appropriate" with the membership level 0.65 in Ω^1, while it is associated with "appropriate" with the membership level 0.45 in Ω^2.

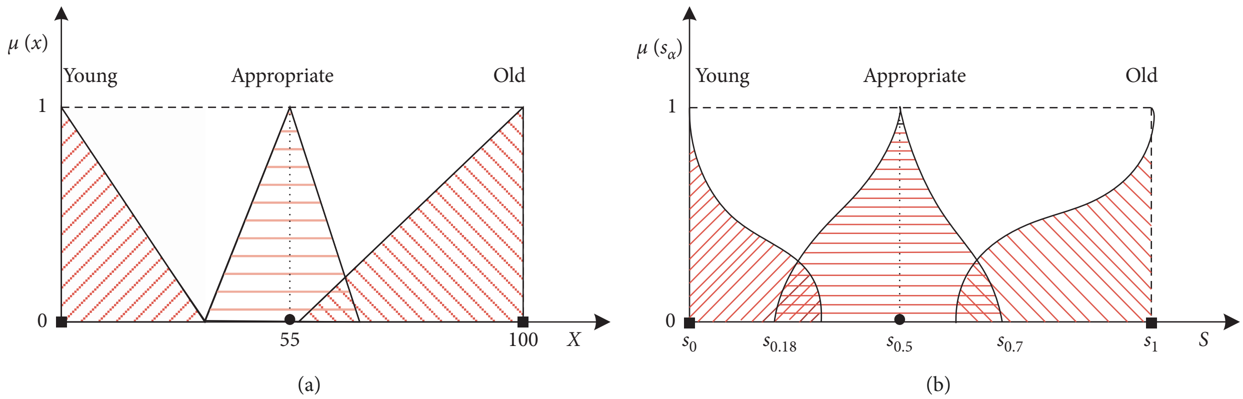


Figure 1: An example of the individual linguistic system $\Omega(X, \{\mu(x)\}, L, \{\mu(S)\}, S)$.

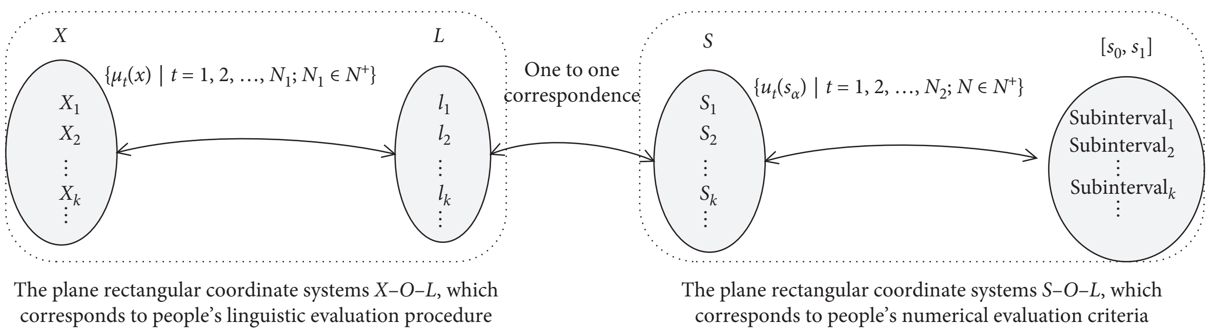


Figure 2: The relations among X, L, S, and the interval $[s_0, s_1]$.

Therefore, when several people give a symbol like $s_{0.4}$ to express their evaluations, it is hard to distinguish the specific meaning of each people.

3.2. The MGVLS. By Definition 1, we transform the above LESs S^1 and S^2 into the normalized vector forms $\vec{S}^1$ and $\vec{S}^2$, where $\vec{S}^1 = \{s_0 \vec{i} - \vec{j} =$ "young," $s_{0.5} \vec{i} - \vec{j} =$ "appropriate," $s_1 \vec{i} + \infty \vec{j} =$ "old"$\}$ and $\vec{S}^2 = \{s_0 \vec{i} - \vec{j} =$ "young," $s_{0.3} \vec{i} - 0.5 \vec{j} =$ "slightly young," $s_{0.5} \vec{i} - 0.7 \vec{j} =$ "appropriate," $s_{0.75} \vec{i} - 0.7 \vec{j} =$ "slightly old," $s_1 \vec{i} + \infty \vec{j} =$ "old"$\}$. Based on $\vec{S}^1$, $s_{0.4}$ is transformed into $s_{0.4} \vec{i} + 2 \vec{j}$. Similarly, based on $\vec{S}^2$, $s_{0.4}$ is transformed into $s_{0.4} \vec{i} - 0.5 \vec{j}$. By this way, it is easy to distinguish the meanings of $s_{0.4}$ that comes from Ω^1 and Ω^2, respectively.

Definition 4. Let $\Omega(X, \{\mu(x)\}, L, \{\mu(S)\}, S)$ be any individual linguistic system. If any probabilistic linguistic term (s_α, p_{s_α}) of Ω is transformed into the PVLT $(\vec{s}_\alpha, p_{\vec{s}_\alpha})$, then Ω can be converted into the individual vector linguistic system $\vec{\Omega}(X, \{\mu(x)\}, L, \{\mu(\vec{S})\}, \vec{S})$.

Based on Definitions 3 and 4, we naturally get the concept of the MGVLS as follows:

Definition 5. Let $\vec{\Omega}^\tau = \vec{\Omega}(X, \{\mu^\tau(x)\}, L^\tau, \{\mu^\tau(\vec{S})\}, \vec{S}^\tau)$ $(\tau = 1, 2, \ldots, N)$ be N individual vector linguistic systems for the same domain of discourse X, then the individual vector linguistic systems $\vec{\Omega}^1, \vec{\Omega}^2, \ldots,$ and $\vec{\Omega}^N$ construct a MGVLS, denoted by $\{\vec{\Omega}^\tau\}_{\tau=1}^N = \{\vec{\Omega}(X, \{\mu^\tau(x)\}, L^\tau, \{\mu^\tau(\vec{S})\}, \vec{S}^\tau) \mid \tau = 1, 2, \ldots, N\}$.

Example 1. Let $\vec{\Omega}^1$ and $\vec{\Omega}^2$ be the corresponding individual vector linguistic systems of $\vec{S}^1 = \{s_0 \vec{i} - \vec{j} =$ "poor," $s_{0.5} \vec{i} =$ "neutral," $s_1 \vec{i} + \infty \vec{j} =$ "good"$\}$ and $\vec{S}^2 = \{s_0 \vec{i} - \vec{j} =$ "poor," $s_{0.3} \vec{i} + 0.5 \vec{j} =$ "slightly poor," $s_{0.5} \vec{i} - 0.2 \vec{j} =$ "neutral," $s_{0.75} \vec{i} =$ "slightly good," $s_1 \vec{i} + \infty \vec{j} =$ "good"$\}$. Then the part of people's numerical evaluation criteria of the MGVLS $\{\vec{\Omega}^\tau\}_{\tau=1}^2$ is the infinite plane area $\{s_0 \le s_\alpha \le s_1, -\infty \le r_{s_\alpha \vec{i}} \le +\infty\}$ depicted in Figure 4.

Comparing the expression of people's numerical evaluation criteria in the multigranularity linguistic space (refer to Figure 3) and the MGVLS in Figure 4, we get that the linguistic evaluations without probabilities in the multigranularity linguistic space are unidimensional, whereas the ones in the MGVLS are dimensional. Therefore, the MGVLS is a stretched form of the multigranularity linguistic space. In the MGVLS, the linguistic evaluation can be distinguished subtler.

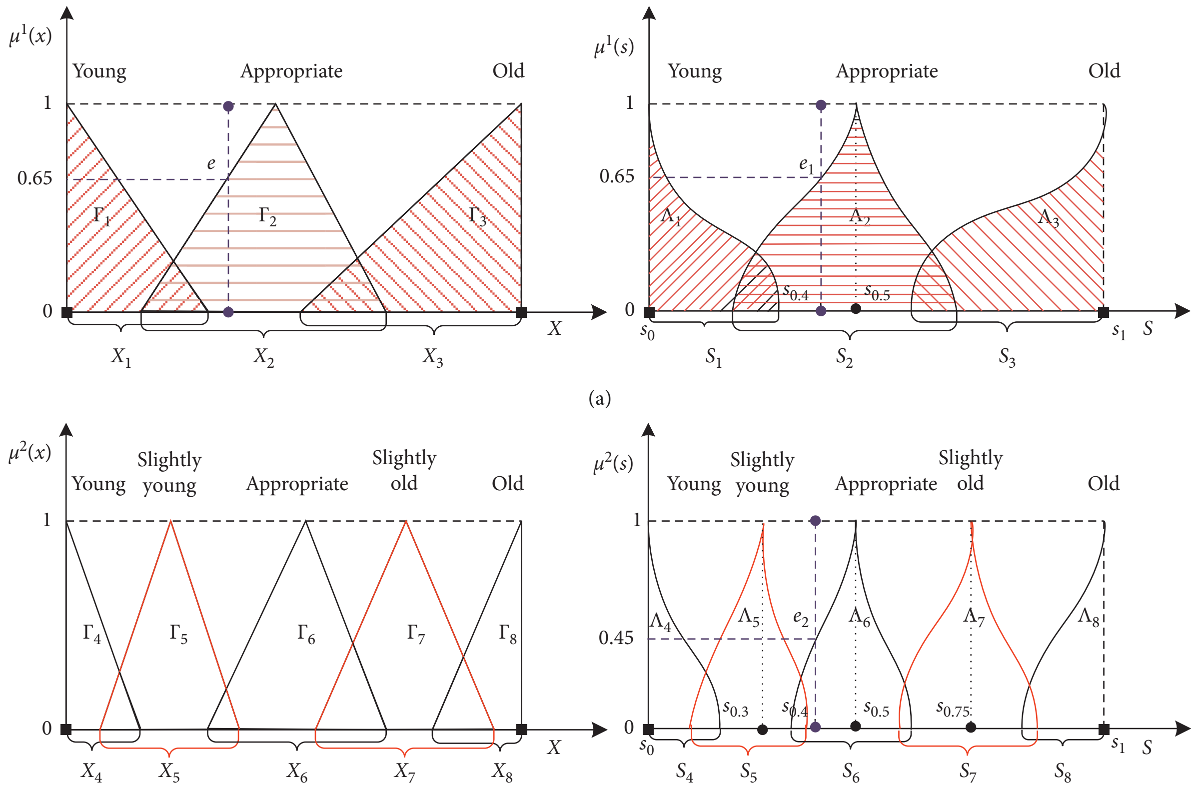


FIGURE 3: An example of an information space constructed with Ω^1 and Ω^2. (a) Ω^1 with $S^1 = \{s_0 = \text{young}, s_{0.5} = \text{appropriate}, s_1 = \text{old}\}$. (b) Ω^2 with $S^2 = \{s_0 = \text{young}, s_{0.3} = \text{slightly young}, s_{0.5} = \text{appropriate}, s_{0.75} = \text{slightly old}, s_1 = \text{old}\}$.

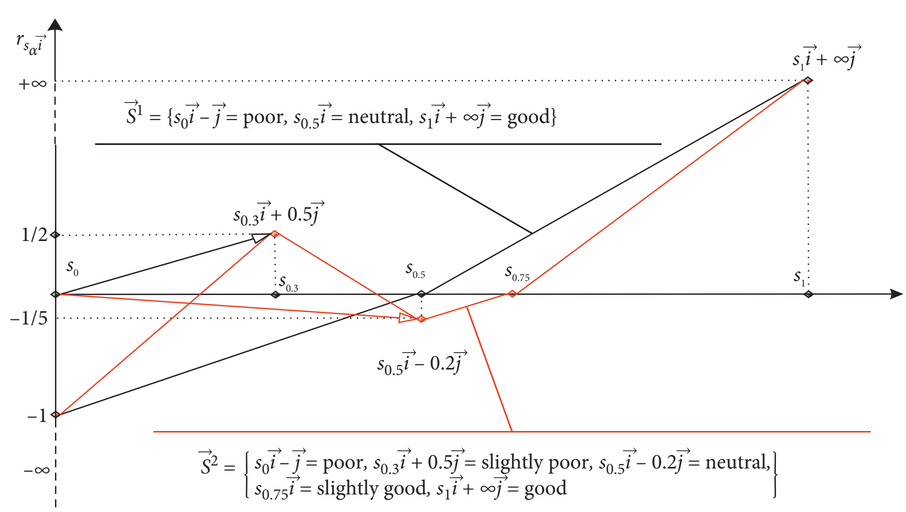


FIGURE 4: The part of people's numerical evaluation criteria of MGVLS $\{\vec{\Omega}^\tau\}_{\tau=1}^2$ derived by $\vec{\Omega}^1$ and $\vec{\Omega}^2$.

3.3. The GPVLT. Let $(\vec{s}_\alpha, p_{\vec{s}_\alpha})$ be any PVLT of the MGVLS $\{\vec{\Omega}^\tau\}_{\tau=1}^2$ as shown in Figure 4. Noticing the ordinate $r_{\vec{s_\alpha i}}$ of $\vec{s}_\alpha$, we find that it monotonically increases while s_α moves from s_{α^τ} to $s_{\alpha^{\tau+1}}$, where s_{α^τ} and $s_{\alpha^{\tau+1}}$ are the τ – th and $\tau + 1$ – th linguistic terms of the normalized LES S, respectively. The monotonicity of $r_{\vec{s_\alpha i}}$ accords with the left side membership function $\mu(s_\alpha)$. In such a case, $r_{\vec{s_\alpha i}}$ can be regarded as the rough expression of $\mu(s_\alpha)$. Due to the complexity in the membership function determination process, the PVLT is definitely a sensible choice to deal with the probabilistic vector linguistic evaluations in the MGVLS. However, these exist some problems in the simulations with the PVLT: (i) $r_{s_\alpha \vec{i}}$ cannot express the right side of $\mu(s_\alpha)$ and (ii) the values of $r_{s_\alpha \vec{i}}$ are all located in $(-\infty, +\infty)$, but the values of the membership function $\mu(s_\alpha)$ are limited in $[0,1]$. If we regard $r_{s_\alpha \vec{i}}$ as the simulation of $\mu(s_\alpha)$, the ranges of $r_{s_\alpha \vec{i}}$ and $\mu(s_\alpha)$ are different. Furthermore, ∞ is not able to be handled in the specific computational process. For example, the infinity ∞ can change the convergence and divergence of the integral. To solve these two issues, we present the concept of the GPVLT below.

Definition 6. Let $\Omega = \Omega(X, \{\mu(x)\}, L, \{\mu(S)\}, S)$ be any individual linguistic system, where $S = \{s_{\alpha^\tau} \mid s_{\alpha^\tau} \in [s_0, s_1], \tau = 1, 2, \ldots, N, s_{\alpha^1} = s_0, s_{\alpha^N} = s_1, N \in N^+\}$ $s_{\alpha^1} = s_0, s_{\alpha^N} = s_1, N \in N^+\}$ is a normalized LES and s_{α^τ} is the $\tau-$th linguistic term of the normalized LES. For any probabilistic linguistic term (s_α, p_{s_α}) of Ω, if we transform it into $(\vec{s}_\alpha, p_{\vec{s}_\alpha})$, where $\vec{s}_\alpha = s_\alpha \vec{i} + r(s_\alpha)\vec{j}$, in which $r(s_\alpha) : [s_0, s_1] \longrightarrow R$ is a function that reflects the membership functions $\{\mu(S)\}$, then $(\vec{s}_\alpha, p_{\vec{s}_\alpha})$ is the GPVLT with respect to (s_α, p_{s_α}).

Remark 1. Although Definition 6 does not restrict the specific characteristic of $r(s_\alpha)$, we usually select the functions for $r(s_\alpha)$ according to the following criteria.

For any $s_{\alpha^\tau} \in S$, which is the target linguistic term, then,

(i) $r(s_\alpha)$ monotonically increases, if $s_{\alpha^{\tau-1}} < s_\alpha < s_{\alpha^\tau}$, $\tau = 2, 3, \ldots, N$

(ii) $r(s_\alpha)$ monotonically decreases, if $s_{\alpha^\tau} < s_\alpha < s_{\alpha^{\tau+1}}$, $\tau = 1, 2, \ldots, N-1$

The PVLT presented by Definition 1 is a particular case of the GPVLT.

For the second normalized LES S^2 in Figure 3, according to Definition 6 and Remark 1, we can assign

$$
r_1(s_\alpha) = \begin{cases} \dfrac{\alpha - \alpha^{\tau-1}}{\alpha^\tau - \alpha^{\tau-1}}, & \alpha^{\tau-1} < \alpha < \alpha^\tau, \tau - 1 \geq 0; \\ 1, & \alpha = \alpha^\tau; \\ \dfrac{\alpha^{\tau+1} - \alpha}{\alpha^{\tau+1} - \alpha^\tau}, & \alpha^\tau < \alpha < \alpha^{\tau+1}, \tau + 1 \leq 5, \end{cases}
$$
$$
\text{or } r_2(s_\alpha) = \begin{cases} \dfrac{(\alpha - \alpha^{\tau-1}) \cdot ((4/\pi) \cdot \arctan(1/(\alpha^\tau - \alpha)) - 1)}{\alpha^\tau - \alpha^{\tau-1}}, & \alpha^{\tau-1} < \alpha < \alpha^\tau, \tau - 1 \geq 0; \\ 1, & \alpha = \alpha^\tau; \\ \dfrac{(\alpha^{\tau+1} - \alpha) \cdot ((4/\pi) \cdot \arctan(1/(\alpha^\tau - \alpha)) - 1)}{\alpha^{\tau+1} - \alpha^\tau}, & \alpha^\tau < \alpha < \alpha^{\tau+1}, \tau + 1 \leq 5, \end{cases} \tag{1}
$$

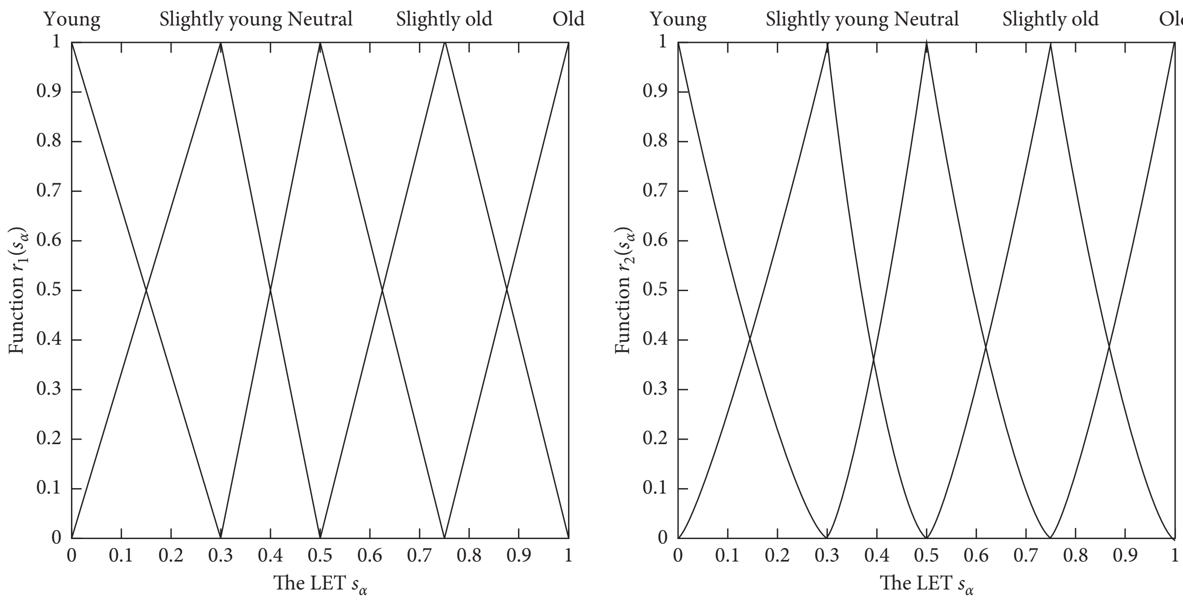
as the coefficient function of $\vec{j}$ within $(\vec{s}_\alpha = s_\alpha \vec{i} + r(s_\alpha)\vec{j}, p_{\vec{s}_\alpha})$, where $\tau = 1, 2, 3, 4, 5$, $s_{\alpha^1} = s_0$ and $s_{\alpha^5} = s_1$. Functions $r_1(s_\alpha)$ and $r_2(s_\alpha)$ can be seen in Figure 5.

As shown in Figure 5, the monotonicity and ranges of $r_1(s_\alpha)$ and $r_2(s_\alpha)$ conform to the membership functions $\{\mu^2(S)\}$. Therefore, $r_1(s_\alpha)$ or $r_2(s_\alpha)$ is more reasonable and precise to be the coefficient function of $\vec{j}$ compared with the one obtained by Definition 1.

As stated in Section 3.2, the part of people's numerical evaluation criteria of any MGVLS can be expressed by a plane area $A : \{s_0 \leq s_\alpha \leq s_1, a \leq r(s_\alpha) \leq b\}$ in a plane rectangular coordinate system $S - O - r(s_\alpha)$, where $[a, b]$ is the range of $r(s_\alpha)$. Each point D in the area A corresponds to a vector $\vec{s}_\alpha = s_\alpha \vec{i} + r(s_\alpha)\vec{j}$. If more than one person provides the point D to express the linguistic evaluations, then the statistic property of the point D can be presented as $p_{\vec{s}_\alpha} = p_D$, i.e., GPVLT (D, p_D). In the practical linguistic evaluation process, the experts only need to provide the LESs and the linguistic evaluations, which is as easy as the existing linguistic decision-making methods. By calculating the frequency of the occurrence of the linguistic terms, their statistics properties can be obtained.

The GPVLTs are the fundamental components of the MGVLS $\{\vec{\Omega}^\tau\}_{\tau=1}^N$. Considering that the corresponding strength of the linguistic evaluations and the values in $[s_0, s_1]$, we give the following definition to classify the GPVLTs:

Definition 7. Let $\{\vec{\Omega}^\tau\}_{\tau=1}^N = \{\vec{\Omega}(X, \{\mu^\tau(x)\}, L^\tau, \{\mu^\tau(\vec{S})\}, \vec{S}^\tau) \mid \tau = 1, 2, \ldots, N\}$ be a MGVLS and $(\vec{s}_\alpha, p_{\vec{s}_\alpha})$ be any GPVLT in $\{\vec{\Omega}^\tau\}_{\tau=1}^N$. If $\vec{s}_\alpha$ is an element of $\vec{S}^\tau$, then we call $(\vec{s}_\alpha, p_{\vec{s}_\alpha})$ the explicit GPVLT. Otherwise, we call it the implicit GPVLT.



(a)

(b)

Figure 5: Two examples of $r(s_\alpha)$ within Definition 6.

According to Definition 7, for any explicit GPVLT $(\vec{s}_\alpha, p_{\vec{s}_\alpha})$, the ordinate coefficient of $\vec{s}_\alpha$ is the maximum of $r(s_\alpha)$, which means that the linguistic evaluation (described by $\vec{s}_\alpha$) strongly corresponds to the value α. For example, in the MGVLS of Example 1, $(s_{0.75}\vec{i}, p_{s_{0.75}\vec{i}})$ is an explicit GPVLT, whereas $(s_{0.76}\vec{i}, p_{s_{0.76}\vec{i}})$ is an implicit PVLT. Then the explicit-implicit property of the GPVLTs is the location property in the plane area $A: \{s_0 \le s_\alpha \le s_1, a \le r(s_\alpha) \le b\}$. Based on the understandings with respect to $\vec{s}_\alpha$, $r(s_\alpha)$, and $p_{\vec{s}_\alpha}$, we can compare them by a score function:

Definition 8. The score function of a GPVLT $(s_\alpha\vec{i} + r(s_\alpha)\vec{j}, p_{\vec{s}_\alpha})$ expressed as $\mathrm{SC}(\vec{s}_\alpha, p_{\vec{s}_\alpha}) = \alpha \cdot r(s_\alpha) \cdot p_{\vec{s}_\alpha}$ was proposed to rank any two GPVLTs $(s_{\alpha_1}\vec{i} + r(s_{\alpha_1})\vec{j}, p_{\vec{s}_{\alpha_1}})$ and $(s_{\alpha_2}\vec{i} + r(s_{\alpha_2})\vec{j}, p_{\vec{s}_{\alpha_2}})$ in a MGVLS.

(i) If $\mathrm{SC}(\vec{s}_{\alpha_1}, p_{\vec{s}_{\alpha_1}}) > \mathrm{SC}(\vec{s}_{\alpha_2}, p_{\vec{s}_{\alpha_2}})$, then $(s_{\alpha_1}\vec{i} + r(s_{\alpha_1})\vec{j}, p_{\vec{s}_{\alpha_1}}) \succ (s_{\alpha_2}\vec{i} + r(s_{\alpha_2})\vec{j}, p_{\vec{s}_{\alpha_2}})$

(ii) If $\mathrm{SC}(\vec{s}_{\alpha_1}, p_{\vec{s}_{\alpha_1}}) < \mathrm{SC}(\vec{s}_{\alpha_2}, p_{\vec{s}_{\alpha_2}})$, then $(s_{\alpha_1}\vec{i} + r(s_{\alpha_1})\vec{j}, p_{\vec{s}_{\alpha_1}}) \prec (s_{\alpha_2}\vec{i} + r(s_{\alpha_2})\vec{j}, p_{\vec{s}_{\alpha_2}})$

(iii) If $\mathrm{SC}(\vec{s}_{\alpha_1}, p_{\vec{s}_{\alpha_1}}) = \mathrm{SC}(\vec{s}_{\alpha_2}, p_{\vec{s}_{\alpha_2}})$, then $(s_{\alpha_1}\vec{i} + r(s_{\alpha_1})\vec{j}, p_{\vec{s}_{\alpha_1}}) = (s_{\alpha_2}\vec{i} + r(s_{\alpha_2})\vec{j}, p_{\vec{s}_{\alpha_2}})$

where "$\succ$" indicates "superior to" and "$\prec$" indicates "inferior to."

The score function of GPVLT is a real mapping from $[s_0, s_1] \times [\min\{r(s_\alpha)\}, \max\{r(s_\alpha)\}] \times [0,1]$ to the interval $[0,1]$. Besides, it is an increasing function of s_α, $r(s_\alpha)$, and $p_{\vec{s}_\alpha}$, respectively.

Example 2. Let $\{\Omega^\tau\}_{\tau=1}^2 = \{\Omega(X, \{\mu^\tau(x)\}, L^\tau, \{\mu^\tau(S)\}, S^\tau) \mid \tau = 1,2\}$ be a multigranularity linguistic space, where $S^1 = \{s_0 =$ "none," $s_{0.3} =$ "slightly low," $s_{0.5} =$ "neutral," $s_{0.75} =$ "slightly high," $s_1 =$ "high"$\}$ and $S^2 = \{s_0 =$ "none," $s_{0.28} =$ "very low," $s_{0.42} =$ "low," $s_{0.5} =$ "neutral," $s_{0.58} =$ "high," $s_{0.72} =$ "very high," $s_1 =$ "perfect"$\}$.

For the given linguistic evaluation $s_{0.65}$, we analyze the meanings of it in Ω^1 and Ω^2, respectively. By using $r(s_\alpha)$, $r_1(s_\alpha)$, and $r_2(s_\alpha)$ shown in Definition 1 and equation (1), we can translate $s_{0.65}$ and the elements in S^1, S^2 into the GPVLTs. Assume that there is no repetition of the GPVLT, and the probability of each GPVLT is equal to 1.

All the transformed results listed in Table 1 are the implicit GPVLTs of $\{\Omega^\tau\}_{\tau=1}^2$; hence, we use the nearby principle to select the linguistic evaluation for $s_{0.65}$. According to the comparisons of the abscissa of $\vec{s}_\alpha$, all transformed results are between "neutral" and "slightly high" in Ω^1. But from the ordinate of $\vec{s}_\alpha$, we find that $(s_{0.65}\vec{i} + (3/5)\vec{j}, 1)$ is the highest. By using the Euclidean measure of the plane rectangular coordinate system, we can calculate all the distances between the transformed GPVLTs and their adjacent explicit GPVLTs. For the given $r_1(s_\alpha)$, the distance between $(s_{0.65}\vec{i} + (3/5)\vec{j}, 1)$ and "slightly high" is less than the distance between $(s_{0.65}\vec{i} + (2/5)\vec{j}, 1)$ and "neutral"; then we should translate $s_{0.65}$ into "slightly high" for Ω^1 when the ordinate coefficient function of $\vec{s}_\alpha$ is $r_1(s_\alpha)$. In the same manner, we can get other translated results. Although we get the different distances between the transformed GPVLTs and their adjacent explicit GPVLTs through $r_1(s_\alpha)$ and $r_2(s_\alpha)$, respectively, the selections are same. Moreover, if we transform $s_{0.65}$ by the $r(s_\alpha)$ presented in Definition 1, we cannot select the better one between "high" and "very high" for $s_{0.65}$ in Ω^2.

Compared to the above nearby principle, we can use the score function of GPVLT to easily make a choice between two adjacent explicit GPVLTs. For $(s_{0.65}\vec{i} + (2/5)\vec{j}, 1)$ and $(s_{0.65}\vec{i} + (3/5)\vec{j}, 1)$ in Ω^1, which are transformed based on $r_1(s_\alpha)$, the difference between them is the ordinates of $\vec{j}$. Considering that the ordinates of $\vec{j}$ in the adjacent explicit GPVLTs are 1 ($\max\{r_1(s_\alpha)\} = 1$) and the score function is the increasing function of $r_1(s_\alpha)$, we should translate $s_{0.65}$ into "slightly high" for Ω^1. It is obvious that we can get the same selections if we use the score function of GPVLT to compare the transformed GPVLTs in Table 1.

3.4. The Forms of the PVLU

Case 1. As the point e_1 in Figure 3, for $x \in X$, there exists a discrete single-point $s_{0.4}$ to express the evaluation of it. Whether we transform $s_{0.4}$ into $(s_{0.4}\vec{i} + 2\vec{j}, 1)$ based on Definition 1 or translate $s_{0.4}$ into $(s_{0.4}\vec{i} + 0.8\vec{j}, 1)$ by $r_1(s_\alpha)$ given in equation (1), the results are both the discretely single GPVLTs. In this case, we call the GPVLT the discrete single probabilistic vector linguistic unit (DSPVLU) and denote it by $(s_\alpha\vec{i} + r_{s_\alpha\vec{i}}\vec{j}, p_{s_\alpha\vec{i} + r_{s_\alpha\vec{i}}\vec{j}})$.

Case 2. As the individual linguistic system in Figure 6, the element $x \in X$ can be evaluated by "neutral" with the membership level 0.65 and "slightly good" with the membership level 0.25 simultaneously.

Translating "neutral" and "slightly good" into the interval $[s_0, s_1]$, we can obtain the linguistic terms s_{α_1} and s_{α_2}. The set $\{s_{\alpha_1}, s_{\alpha_2}\}$ expresses the linguistic evaluation with respect to x. Since s_{α_1} and s_{α_2} are two discrete points, $\{(\vec{s}_{\alpha_1}, p_{\vec{s}_{\alpha_1}}), (\vec{s}_{\alpha_2}, p_{\vec{s}_{\alpha_2}})\}$ is a set with two discrete transformed GPVLTs. Generalizing the number of the elements of $\{(\vec{s}_{\alpha_1}, p_{\vec{s}_{\alpha_1}}), (\vec{s}_{\alpha_2}, p_{\vec{s}_{\alpha_2}})\}$ to N, we get a discrete multiple probabilistic vector linguistic unit (DMPVLU) and denote it by $\{(\vec{s}_{\alpha_k}, p_{\vec{s}_{\alpha_k}})\}_{k=1}^{N} = \{(\vec{s}_{\alpha_k}, p_{\vec{s}_{\alpha_k}}) \mid k = 1, 2, \ldots, N;\ N \in N^+;\ \sum_{k=1}^{N} p_{\vec{s}_{\alpha_k}} \leq 1\}$.

Remark 2. The DMPVLU can be regarded as a collection of the DSPVLUs. But the DSPVLU and the DMPVLU have different meanings for people's judgement thinking, where the DSPVLU indicates no hesitance and the DMPVLU indicates the hesitance in the judgements. Therefore, if we take any DSPVLU for a unit of the probabilistic vector linguistic evaluation, then the probability in $(\vec{s}_\alpha, p_{\vec{s}_\alpha})$ reduces to 1. Moreover, if we take any DMPVLU $\{(\vec{s}_{\alpha_k}, p_{\vec{s}_{\alpha_k}})\}_{k=1}^{N}$ for a unit of the probabilistic vector linguistic evaluation, then the probabilities in it should be normalized by $\overleftrightarrow{p}_{\vec{s}_{\alpha_k}} = p_{\vec{s}_{\alpha_k}} / \sum_{k=1}^{N} p_{\vec{s}_{\alpha_k}}$. Generally, the probability of any DSPVLU used below defaults to 1, and all probabilities in any DMPVLU used below are normalized.

Case 3. In Figure 6, "$s_{0.5}$ = neutral" and "$s_{0.75}$ = slightly good" are two consecutive elements in the normalized LES S. The interval $[s_{\alpha_1}, s_{\alpha_2}] \subset [s_{0.5}, s_{0.75}]$ interprets people's fuzziness between the judgements with "neutral" and "slightly good" [42]. If we transform all values in $[s_{\alpha_1}, s_{\alpha_2}]$ into the GPVLTs, then we can obtain a continuous single-interval probabilistic vector linguistic unit $([\vec{s}_{\alpha_1}, \vec{s}_{\alpha_2}], p(\vec{s}_\alpha))$, where $s_{\alpha_1} \leq s_\alpha \leq s_{\alpha_2}$ and $p(\vec{s}_\alpha)$ is the probability function of $\vec{s}_\alpha$. In such a situation, we call $([\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}], p(\vec{s}_\alpha))$ the continuous single-interval probabilistic vector linguistic unit (CSIPVLU), where $s_{\alpha_a} \leq s_\alpha \leq s_{\alpha_b}$, $[\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}] \subseteq [s_{\alpha^\tau}, s_{\alpha^{\tau+1}}]$, and s_{α^τ} and $s_{\alpha^{\tau+1}}$ are two consecutive elements in the normalized LES S, $p(\vec{s}_\alpha) : [\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}] \longrightarrow [0, 1]$ is the probability function of $\vec{s}_\alpha$.

Remark 3. Firstly, let $([\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}], p(\vec{s}_\alpha))$ be any CSIPVLU, then the vector interval $[\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}]$ is originated from $[s_{\alpha_a}, s_{\alpha_b}]$. Considering that the process of transforming the linguistic term s_α into the normalized vector linguistic term $\vec{s}_\alpha$ does not change the probability of s_α, we know that the probabilistic distribution in $[s_{\alpha_a}, s_{\alpha_b}]$ is the same as $[\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}]$, i.e., $p(s_\alpha) : [s_{\alpha_a}, s_{\alpha_b}] \longrightarrow [0, 1]$ is identical to $p(\vec{s}_\alpha) : [\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}] \longrightarrow [0, 1]$. Secondly, the numbers of the elements in the DSPVLU and the DMPVLU are both positive integers, whereas the number of the elements in the CSIPVLU is uncountable. Hence, the CSIPVLU can be regarded as the extended form of the DSPVLU and the DMPVLU. Thirdly, any CSIPVLU $([\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}], p(\vec{s}_\alpha))$ can be normalized by $\overleftrightarrow{p}(\vec{s}_\alpha) = p(\vec{s}_\alpha) / \int_{\vec{s}_{\alpha_a}}^{\vec{s}_{\alpha_b}} p(\vec{s}_\alpha)$, where $\int_{\vec{s}_{\alpha_a}}^{\vec{s}_{\alpha_b}} p(\vec{s}_\alpha)$ is the definite integral of $p(\vec{s}_\alpha)$ on the interval $[\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}]$. In general, all CSIPVLUs used below are normalized.

Case 4. For the CSIPVLU $([\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}], p(\vec{s}_\alpha))$ depicted in Case 3, the condition $[\vec{s}_{\alpha_a}, \vec{s}_{\alpha_b}] \subseteq [s_{\alpha^\tau}, s_{\alpha^{\tau+1}}]$ and s_{α^τ} and $s_{\alpha^{\tau+1}}$ are two consecutive linguistic terms in the normalized LES" is somewhat rigorous. In the practical situations, it is difficult to reach it for the flexibility of the linguistic expressions and the fuzziness of the judgements. When the interval $[s_{\alpha_a}, s_{\alpha_b}]$ falls on $[s_{\alpha^\tau}, s_{\alpha^{\tau+k}}] \subseteq [s_0, s_1]$, where s_{α^τ} is the τ − th element of the LES S and $k \geq 2$, we can divide $[s_{\alpha_a}, s_{\alpha_b}]$ into a sequence of the CSIPVLUs $([\vec{s}_{\alpha_a}, \vec{s}_{\alpha^{\tau+1}}], p(s_\alpha))$, $([\vec{s}_{\alpha^{\tau+1}}, \vec{s}_{\alpha^{\tau+2}}], p(s_\alpha)), \ldots, ([\vec{s}_{\alpha^{\tau+k-1}}, \vec{s}_{\alpha_b}], p(s_\alpha))$. The divided sequence of the CSIPVLUs $\{([\vec{s}_{\alpha_k}, \vec{s}_{\alpha_{k+1}}], p_k(\vec{s}_\alpha))\}_{k=1}^{N}$ is called as the continuous multi-intervals probabilistic vector linguistic unit (CMIPVLU), where $p_k(\vec{s}_\alpha)$ is a probabilistic distribution on $[\vec{s}_{\alpha_k}, \vec{s}_{\alpha_{k+1}}]$.

Remark 4. The probabilities in any CMIPVLU $\{([\vec{s}_{\alpha_k}, \vec{s}_{\alpha_{k+1}}], p_k(\vec{s}_\alpha))\}_{k=1}^{N}$ can be normalized by $\overleftrightarrow{p}_k(\vec{s}_\alpha) = p_k(\vec{s}_\alpha) / \sum_{k=1}^{N} p_k(\vec{s}_\alpha)$. Generally, all CMIPVLUs used below are normalized.

Table 1: The transformed results of $s_{0.65}$.

The coefficient function of $\vec{j}$	Source	The transformed GPVLTs	The adjacent explicit GPVLTs	The score of each transformed GPVLT	The Euclidean distance between the transformed LVET and the adjacent explicit GPVLTs	The meanings of $s_{0.65}$
r	Ω^1	$(s_{0.65}\vec{i} - (1/5)\vec{j}, 1)$	$(s_{0.5}\vec{i} + \vec{j}, 1)$ neutral	0.130	1.209	Not select
r_1		$(s_{0.65}\vec{i} + (2/5)\vec{j}, 1)$		0.260	0.618	Not select
r_2		$(s_{0.65}\vec{i} + (81/250)\vec{j}, 1)$		0.211	0.692	Not select
r		$(s_{0.65}\vec{i} - (1/5)\vec{j}, 1)$	$(s_{0.75}\vec{i} + \vec{j}, 1)$ slightly high	0.130	1.204	Select
r_1		$(s_{0.65}\vec{i} + (3/5)\vec{j}, 1)$		0.390	0.412	Select
r_2		$(s_{0.65}\vec{i} + (131/250)\vec{j}, 1)$		0.341	0.486	Select
r	Ω^2	$(s_{0.65}\vec{i} + (37/195)\vec{j}, 1)$	$(s_{0.58}\vec{i} + \vec{j}, 1)$ high	0.123	0.813	—
r_1		$(s_{0.65}\vec{i} + (13/25)\vec{j}, 1)$		0.338	0.485	Select
r_2		$(s_{0.65}\vec{i} + (129/250)\vec{j}, 1)$		0.335	0.489	Select
r		$(s_{0.65}\vec{i} + (37/195)\vec{j}, 1)$	$(s_{0.72}\vec{i} + \vec{j}, 1)$ very high	0.123	0.813	—
r_1		$(s_{0.65}\vec{i} + (12/25)\vec{j}, 1)$		0.312	0.525	Not select
r_2		$(s_{0.65}\vec{i} + (109/250)\vec{j}, 1)$		0.283	0.568	Not select

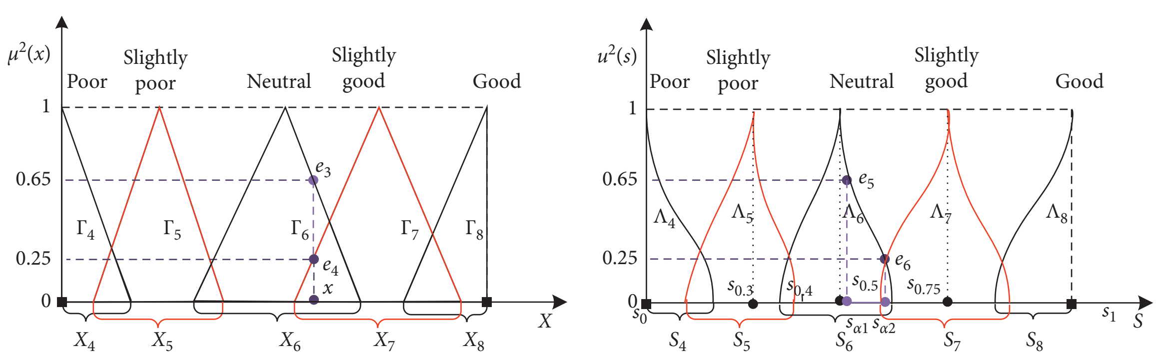


Figure 6: An example of the individual linguistic system. $S = \{s_0 = \text{poor}, s_{0.3} = \text{slightly poor}, s_{0.5} = \text{neutral}, s_{0.75} = \text{slightly good}, s_1 = \text{good}\}$.

4. The Potentials for the PVLUs

According to the different forms of PVLUs discussed in Section 3.4, in this section, we introduce two types of the potentials for PVLUs.

4.1. Nondirectional Potentials of the PVLUs

4.1.1. Nondirectional Potential of the DSPVLU.

Definition 9. Let $(\vec{s}_\alpha, p_{\vec{s}_\alpha}) = (s_\alpha \vec{i} + r(s_\alpha)\vec{j}, p_{\vec{s}_\alpha})$ be a DSPVLU, then its isolated point potential can be obtained by

$$\mathrm{IPP}_{(\vec{s}_\alpha, p_{\vec{s}_\alpha})} = p_{\vec{s}_\alpha} \cdot \left| s_\alpha \vec{i} + r(s_\alpha)\vec{j} \right| = p_{\vec{s}_\alpha} \cdot \sqrt{\alpha^2 + r^2(s_\alpha)}. \quad (2)$$

The isolated point potential of the DSPVLU, which is a real increasing function of s_α, $r(s_\alpha)$, and $p_{\vec{s}_\alpha}$, is derived from the norm of vector in mathematical theory. Usually, the $p_{\vec{s}_\alpha}$ in equation (2) reduces to 1.

4.1.2. Nondirectional Potential of the DMPVLU.

As stated in Remark 2, the DMPVLU can be regarded as a series of DSPVLUs. Based on equation (2), we define the multipoints potential of the DMPVLU as follows:

Definition 10. Let $\{(\vec{s}_{\alpha_k}, p_{\vec{s}_{\alpha_k}})\}_{k=1}^N = \{(\vec{s}_{\alpha_k}, p_{\vec{s}_{\alpha_k}})|,\ k = 1, 2, \ldots, N; N \in N^+; \sum_{k=1}^N p_{\vec{s}_{\alpha_k}} \leq 1\}$ be a DMPVLU, then its multiple points potential can be given by

$$\begin{aligned}
\mathrm{MPP}_{\left\{(\vec{s}_{\alpha_k}, p_{\vec{s}_{\alpha_k}})\right\}_{k=1}^N} &= \sum_{k=1}^N \left(p_{\vec{s}_{\alpha_k}} \cdot \left| \vec{s}_{\alpha_k} \right| \right) \\
&= \sum_{k=1}^N \left(p_{\vec{s}_{\alpha_k}} \cdot \left| s_{\alpha_k} \vec{i} + r(s_{\alpha_k})\vec{j} \right| \right) \\
&= \sum_{k=1}^N \left(p_{\vec{s}_{\alpha_k}} \cdot \sqrt{\alpha_k^2 + r^2(s_{\alpha_k})} \right).
\end{aligned} \quad (3)$$

Similar to Definition 9, the multiple points potential is a real increasing function of s_{α_k}, $r(s_{\alpha_k})$, and $p_{\vec{s}_{\alpha_k}}$.

4.1.3. Nondirectional Potential of the CSIPVLU. Let $([\vec{s}_a, \vec{s}_b], p(\vec{s}))$ be any CSIPVLU of the MGVLS. If $D_0 = \vec{s}_a$ and $D_n = \vec{s}_b$, then $([\vec{s}_a, \vec{s}_b], p(\vec{s}))$ is equal to $([D_0, D_n], p(\vec{s}))$. As described in Section 3.3, when we transform every value of $[s_a, s_b]$ to $[\vec{s}_a, \vec{s}_b]$, a continuous finite curve $C: \begin{cases} s_\alpha(\alpha) = \alpha, \\ r(s_\alpha) = r(\alpha), \end{cases} \alpha \in [a, b]$ is obtained.

In Figure 7, the finite curve C can be regarded as the locus of the point $D = \vec{s}_\alpha = s_\alpha \vec{i} + r(s_\alpha)\vec{j}$ moving from D_0 to D_n, where $p(\vec{s}) : [D_0, D_n] \longrightarrow [0, 1]$ is the probability function of D. Taking $p(\vec{s})$ as the density function of the finite curve C, we can calculate the weight of C by the following four steps:

Step 1 (subdividing curve C into a set of curve segments). Insert a set of dots $\{(D_k, p_{D_k})\}_{k=0}^N$ to subdivide the curve C into n curve segments $\{C_k\}_{k=1}^N = \{C_k \mid k = 1, 2, \cdots, n; n \in N^+\}$, and denote the length of the $k-$th curve segment by $\Delta a_{D_{k-1}D_k}$ for $k = 1, 2, \ldots, n$.

Step 2 (taking dots from every curve segment). Take any point from each curve segment C_k, denoted as (ξ_k, η_k), and then obtain a set of dots $\{(\xi_k, \eta_k) \mid k = 1, 2, \ldots, n; n \in N^+\}$.

Step 3 (taking approximations)

(1) Approximate the arc-length of each C_k. Replace C_k by the line segment L_k, where the dots D_{k-1} and D_k are the endpoints of L_k. In the plane rectangular coordinate system $S - O - r(s_\alpha)$, the length of L_k can be described by $\Delta l_k = \sqrt{(\Delta s_{\alpha_k})^2 + (\Delta r(s_{\alpha_k}))^2}$, where $\Delta s_{\alpha_k} = \alpha_k - \alpha_{k-1}$ and $\Delta r(s_{\alpha_k}) = r(s_{\alpha_k}) - r(s_{\alpha_k})$. So, the arc-length of C_k can be approximated by Δl_k, i.e., $\Delta a_k = \Delta a_{D_{k-1}D_k} \approx \sqrt{(\Delta s_{\alpha_k})^2 + \Delta r^2(s_{\alpha_k})}$.

(2) Approximate the weight of C_k. Utilize $p(\xi_k \vec{i} + \eta_k \vec{j})$ to represent the density of C_k, then the weight of C_k, denoted as ΔW_k, can be approximated by $p(\xi_k \vec{i} + \eta_k \vec{j}) \cdot \Delta l_k$.

(3) Approximate the weight of the curve C. Adding all the approximations of ΔW_k together, we get the approximation of the weight of the curve C as follows:

$$W = \sum_{k=1}^N \Delta W_k = \sum_{k=1}^N \left[p(\vec{s}) \cdot \Delta a_k\right] \approx \sum_{k=1}^N \left[p\left(\xi_k \vec{i} + \eta_k \vec{j}\right) \cdot \Delta l_k\right]. \tag{4}$$

Step 4 (calculating the limit of $\sum_{k=1}^N [p(\xi_k \vec{i} + \eta_k \vec{j}) \cdot \Delta l_k]$). Let a be the maximum value of Δa_k, where Δa_k is the arc-length of the curve segment C_k, then we can subdivide C infinitely when a approaches to 0. The limit value of $\sum_{k=1}^N [p(\xi_k \vec{i} + \eta_k \vec{j}) \cdot \Delta l_k]$ with $a \longrightarrow 0$ is the accurate value of W.

Definition 11. If the limit of $\sum_{k=1}^N [p(\xi_k \vec{i} + \eta_k \vec{j}) \cdot \Delta l_k]$ exists when a approaches to 0, then we call it the nondirectional curve potential of the CSIPVLU $([\vec{s}_a, \vec{s}_b], p(\vec{s}))$ and denote it by $\int_C p(\vec{s}) \cdot \mathrm{d}a$, where $C: \begin{cases} s_\alpha(\alpha) = \alpha, \\ r(s_\alpha) = r(\alpha), \end{cases} \alpha \in [a, b]$ is the integral curve, $p(\vec{s}) : [D_0, D_n] \longrightarrow [0, 1]$ is the integrand function, da is the arc-length integral infinitesimal.

Obviously, $\mathrm{NDCP}_{([\vec{s}_a, \vec{s}_b], p(\vec{s}))} = \int_C p(\vec{s})\mathrm{d}a = \int_a^b p(\vec{s}) \cdot \sqrt{1 + (r'(\alpha))^2}\mathrm{d}\alpha$, which is the first curvilinear integral of $p(\vec{s}) : [D_0, D_n] \longrightarrow [0, 1]$ on the curve C. The nondirectional curve potential is a quantity property of the CSIPVLU, and its physical significance is the weight of C.

Remark 5. In Definition 11, the segmentation process of the curve C and the way of selecting dots from $\{C_k\}_{k=1}^N$ are arbitrary. Particularly, in $\mathrm{NDCP}_{([\vec{s}_a, \vec{s}_b], p(\vec{s}))} = \int_a^b p(\vec{s}) \cdot \sqrt{1 + (r'(\alpha))^2}\mathrm{d}\alpha$, to guarantee the meaningfulness of the arc-length, the lower integral limit a must be less than the upper integral limit b.

Until now, it can be easy to deduce that the values of the nondirectional curve potentials will be different if the probability distribution $p(\vec{s}) : [D_0, D_n] \longrightarrow [0, 1]$ changes, even though the integral curve C remains the same, which manifests the significance of the statistical properties of the vector linguistic evaluation.

4.1.4. Nondirectional Potential of the CMIPVLU. For any CMIPVLU $\{([\vec{s}_{\alpha_k}, \vec{s}_{\alpha_{k+1}}], p(\vec{s}_\alpha))\}_{k=1}^N$, we can get its nondirectional curve potential based on the properties of the first curvilinear integral and Definition 11.

Definition 12. Let $\{([\vec{s}_{\alpha_k}, \vec{s}_{\alpha_{k+1}}], p(\vec{s}_\alpha))\}_{k=1}^N$ be a CMIPVLU, then its nondirectional curve potential can be obtained by

$$\mathrm{NDCP}_{\{([\vec{s}_{\alpha_k}, \vec{s}_{\alpha_{k+1}}], p(\vec{s}_\alpha))\}_{k=1}^N} = \sum_{k=1}^N \mathrm{NDCP}_{([\vec{s}_{\alpha_k}, \vec{s}_{\alpha_{k+1}}], p(\vec{s}_\alpha))}. \tag{5}$$

4.2. Directional Potentials of the PVLU

To enhance the practicability and effectiveness of the tools to investigate the directional linguistic evaluation in physician-patient communication, this subsection introduces the directional potentials for the PVLUs.

4.2.1. Directional Potential of the DSPVLU

Definition 13. Let $(D_k, p_{D_k}) = (\vec{s}_{\alpha_k} \vec{i} + r(\alpha_k)\vec{j}, p_{\vec{s}_{\alpha_k}})$ be a DSPVLU, then its directional potential is

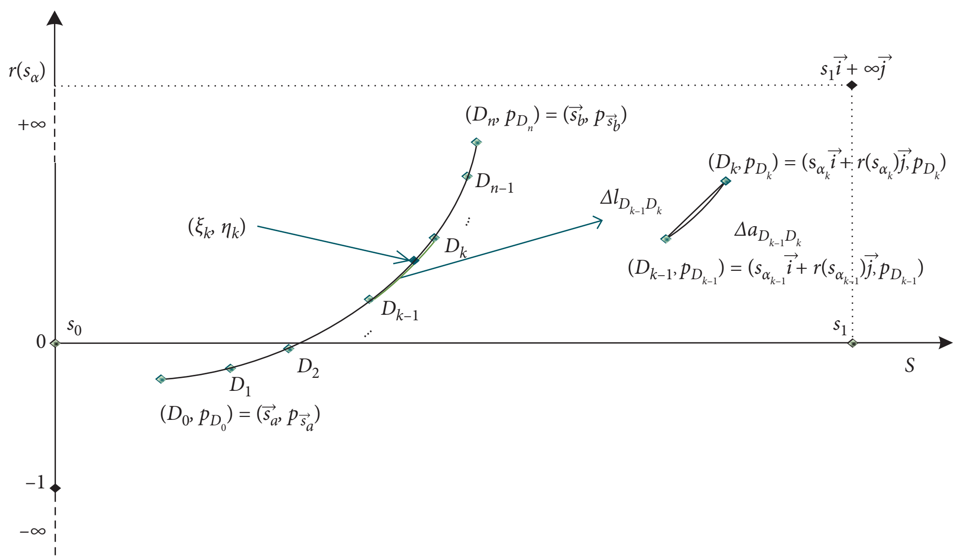


Figure 7: The visual representation of the CSIPVLU $([D_0, D_n], p(\vec{s}))$.

$$\begin{aligned}\mathrm{DP}_{\left(D_k, p_{D_k}\right)} &= p_{\vec{s}_{\alpha_k}} \cdot \left(\vec{s}_{\alpha_k}\vec{i} + r(\alpha_k)\vec{j}\right) \\ &= p_{\vec{s}_{\alpha_k}} \cdot \vec{s}_{\alpha_k}\vec{i} + p_{\vec{s}_{\alpha_k}} \cdot r(\alpha_k)\vec{j}.\end{aligned} \tag{6}$$

This equation obtains a directional vector which contains the potential of the DSPVLU.

4.2.2. Directional Potential of the DMPVLU. Furthermore, Definition 13 can be generalized to the DMPVLU that contains more than one point.

Definition 14. Let $\{(D_k, p_{\vec{s}_{\alpha_k}})\}_{k=1}^{N}$ be a DMPVLU, then its directional potential can be defined by

$$\mathrm{DP}_{\left\{\left(D_k, p_{\vec{s}_{\alpha_k}}\right)\right\}k=1^N} = \sum_{k=2}^{N}\left(p_{D_k}\cdot \alpha_k \cdot \vec{i} + p_{D_k}\cdot r\left(s_{\alpha_k}\right)\vec{j}\right). \tag{7}$$

4.2.3. Directional Potential of the CSIPVLU. For a CSIPVLU $([\vec{s}_a, \vec{s}_b], p(\vec{s})) = ([D_0, D_n], p(\vec{s}))$, there exist two trajectories that vary from D_0 to D_n and D_n to D_0, whose paths are denoted as $([\vec{s}_a \longrightarrow \vec{s}_b], p(\vec{s})) = ([D_0 \longrightarrow D_n], p(\vec{s}))$ and $([\vec{s}_b \longrightarrow \vec{s}_a], p(\vec{s})) = ([D_n \longrightarrow D_0], p(\vec{s}))$, respectively. As shown in Figure 8, for the path $([D_0 \longrightarrow D_n], p(\vec{s}))$, we can get its corresponding directional curve $\vec{C}: \begin{cases} s_\alpha(\alpha) = \alpha, \\ r(s_\alpha) = r(\alpha), \end{cases} \alpha : a \longrightarrow b$, where $D_0 = s_{\alpha_a}\vec{i} + r(s_{\alpha_a})\vec{j}$ is the starting point and $D_n = s_{\alpha_b}\vec{i} + r(s_{\alpha_b})\vec{j}$ is the terminal point. Based on the probability function $p(\vec{s}) : [\vec{s}_b \longrightarrow \vec{s}_a] \longrightarrow [0,1]$, we can establish a directional function as $\vec{F}(\vec{s}) = p(\vec{s})\cdot \alpha\vec{i} + p(\vec{s})\cdot r(\alpha)\ \vec{j} = F_s\vec{i} + F_r\vec{j}$, where $\alpha \in [a,b]$. If we regard it as the variable force on $\vec{C}$, then the work of $\vec{F}(\vec{s})$ along with $\vec{C}$ can be calculated by the following four steps:

Step 1 (subdividing $\vec{C}$ into an ordered set of directional curve segments). Insert a sequence of ordered dots $\{(D_k, p_{D_k})\}_{k=0}^{N}$ into the directional curve, then $\vec{C}$ can be subdivided into a sequence of ordered directional curve segments $\{\vec{C}_k\}_{k=1}^{N} = \{\vec{C}_k \mid k = 1, 2, \cdots, n; n \in N^+\}$. For the k-th directional curve segment, the dot D_{k-1} is the starting point and D_k is the terminal point.

Step 2 (taking a sequence of ordered dots). Take any point in each directional curve segment $\vec{C}_k$, and denote it as (s_{ξ_k}, η_k), then the dots of $\{(s_{\xi_k}, \eta_k) \mid k = 1, 2, \cdots, n; n \in N^+\}$ successively fall on the directional curve $\vec{C}$.

Step 3 (approximating the work of $\vec{F}(\vec{s})$ along with the curve segment $\vec{C}_k$)

(1) Approximate the directional curve $\vec{C}_k$. Replace $\vec{C}_k$ by the directional line segment $\vec{L}_k$, where the dots D_{k-1} and D_k are the starting point and the terminal point, respectively. In the plane rectangular coordinate system $S - O - r(s_\alpha)$, $\vec{L}_k$ can be described by the vector $\Delta\vec{L}_k = \Delta s_{\alpha_k}\vec{i} + \Delta r(s_{\alpha_k})\vec{j}$, which means $\Delta\vec{C}_k \approx \Delta s_{\alpha_k}\vec{i} + \Delta r(s_{\alpha_k})\vec{j}$, where $\Delta s_{\alpha_k} = \alpha_k - \alpha_{k-1}$ and $\Delta r(s_{\alpha_k}) = r(s_{\alpha_k}) - r(s_{\alpha_{k-1}})$.

(2) Approximate the work of $\vec{F}(\vec{s})$ on $\vec{C}_k$. Let $\Delta\vec{W}_k$ be the work of $\vec{F}(\vec{s})$ along with $\vec{C}_k$. For each point of $\{(s_{\xi_k}, \eta_k) \mid k = 1, 2, \ldots, n; n \in N^+\}$, we compute the value of $\vec{F}(\vec{s})$ on it and denote the result by $\vec{F}(s_{\xi_k}\vec{i} + \eta_k\vec{j}) = p(s_{\xi_k}\vec{i} + \eta_k\vec{j})\cdot \xi_k\vec{i} + p(s_{\xi_k}\vec{i} + \eta_k\vec{j})\cdot r(s_{\xi_k})\vec{j} = F_s(s_{\xi_k}$

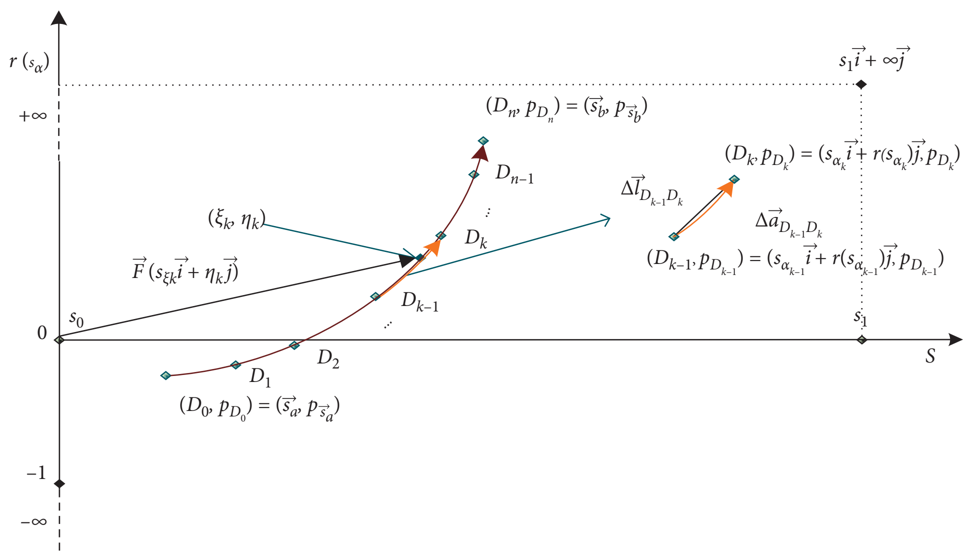


Figure 8: The visual representation of the work of $\vec{F}(\vec{s})$ along with $\vec{C}$.

$\vec{i} + \eta_k\vec{j})\vec{i} + F_r(s_{\xi_k}\vec{i} + \eta_k\vec{j})\vec{j}$. Replace the value of $\vec{F}(\vec{s})$ at every point in $\vec{C}_k$ by $\vec{F}(s_{\xi_k}\vec{i} + \eta_k\vec{j})$, then the work of $\vec{C}_k$, denoted as $\Delta\vec{W}_k$, can be approximated by $F_s(s_{\xi_k}\vec{i} + \eta_k\vec{j}) \cdot \Delta s_{\alpha_k} + F_r(s_{\xi_k}\vec{i} + \eta_k\vec{j}) \cdot \Delta r(s_{\alpha_k})$.

(3) Approximate the work of $\vec{F}(\vec{s})$ on $\vec{C}$. Let $\vec{W}$ be the work of $\vec{F}(\vec{s})$ on $\vec{C}$. Sum all $\Delta\vec{W}_k$, then we can get the approximation of $\vec{W}$:

$$\vec{W} = \sum_{k=1}^{N} \Delta\vec{W}_k \approx \sum_{k=1}^{N}\left(F_s\left(s_{\xi_k}\vec{i} + \eta_k\vec{j}\right) \cdot \Delta s_{\alpha_k} + F_r\left(s_{\xi_k}\vec{i} + \eta_k\vec{j}\right) \cdot \Delta r\left(s_{\alpha_k}\right)\right). \tag{8}$$

Step 4 (calculate the limit of equation (8)). Let $\vec{a}$ be the maximum value of $\Delta\vec{a}_k$, where $\Delta\vec{a}_k$ is the arc-length of the directional curve $\vec{C}_k$, then we can subdivide the directional curve $\vec{C}$ infinitely when $\vec{a}$ approaches to 0. The limit value of $\sum_{k=1}^{N}(F_s(s_{\xi_k}\vec{i} + \eta_k\vec{j}) \cdot \Delta s_{\alpha_k} + F_r\ (s_{\xi_k}\vec{i} + \eta_k\vec{j}) \cdot \Delta r(s_{\alpha_k}))$ with $\vec{a} \longrightarrow 0$ is an accurate value of $\vec{W}$.

Definition 15. If the limit of $\sum_{k=1}^{N}(F_s(s_{\xi_k}\vec{i} + \eta_k\vec{j}) \cdot \Delta s_{\alpha_k} + F_r(s_{\xi_k}\vec{i} + \eta_k\vec{j}) \cdot \Delta r(s_{\alpha_k}))$ exists with $\vec{a} \longrightarrow 0$, then the directional curve potential of the CSIPVLU $([\vec{s}_a \longrightarrow \vec{s}_b], p(\vec{s})) = ([D_0 \longrightarrow D_n], p\ (\vec{s}))$ can be defined as

$$\begin{aligned}
\mathrm{DCP}_{([\vec{s}_a \longrightarrow \vec{s}_b], p(\vec{s}))} &= \int_{\vec{C}} \vec{F}(\vec{s}) \cdot \mathrm{d}\vec{a} = \lim_{\vec{a}\longrightarrow 0}\sum_{k=1}^{N}\left(F_s\left(s_{\xi_k}\vec{i} + \eta_k\vec{j}\right) \cdot \Delta s_{\alpha_k} + F_r\left(s_{\xi_k}\vec{i} + \eta_k\vec{j}\right) \cdot \Delta r\left(s_{\alpha_k}\right)\right) \\
&= \int_{\vec{C}} F_s\left(s_{\xi_k}\vec{i} + \eta_k\vec{j}\right) \cdot \mathrm{d}s_\alpha + F_r\left(s_{\xi_k}\vec{i} + \eta_k\vec{j}\right) \cdot \mathrm{d}r\left(s_\alpha\right) \\
&= \int_{\vec{C}} F_s\left(s_{\xi_k}\vec{i} + \eta_k\vec{j}\right) \cdot \mathrm{d}s_\alpha + \int_{\vec{C}} F_r\left(s_{\xi_k}\vec{i} + \eta_k\vec{j}\right) \cdot \mathrm{d}r\left(s_\alpha\right).
\end{aligned} \tag{9}$$

In this equation, $\int_{\vec{C}} F_s(s_{\xi_k}\vec{i} + \eta_k\vec{j}) \cdot \mathrm{d}s_\alpha$ and $\int_{\vec{C}} F_r(s_{\xi_k}\vec{i} + \eta_k\vec{j}) \cdot \mathrm{d}r(s_\alpha)$ are the components of $\vec{W}$ falling on the abscissa and vertical axes, respectively.

Remark 6. The segmentation process of the directional curve $\vec{C}$ and the way of selecting dots from $\{\vec{C}_k\}_{k=1}^{N}$ are arbitrary. In addition, the directional curve potential of the CSIPVLU $([\vec{s}_a \longrightarrow \vec{s}_b], p(\vec{s}))$ can be calculated by

$$\begin{aligned}\int_{\vec{C}} \vec{F}(\vec{s}) \cdot \mathrm{d}\vec{a} &= \int_{\vec{C}} F_s\left(s_{\xi_k}\vec{i} + \eta_k \vec{j}\right) \cdot \mathrm{d}s_\alpha + \int_{\vec{C}} F_r\left(s_{\xi_k}\vec{i} + \eta_k \vec{j}\right) \cdot \mathrm{d}r(s_\alpha) \\ &= \int_a^b p\left(s_{\xi_k}\vec{i} + \eta_k \vec{j}\right) \cdot \alpha \,\mathrm{d}\alpha + \int_a^b p\left(s_{\xi_k}\vec{i} + \eta_k \vec{j}\right) \cdot r(\alpha)\mathrm{d}r(\alpha).\end{aligned} \tag{10}$$

Since the integral infinitesimal $\Delta\vec{a}$ of $\int_{\vec{C}} \vec{F}(\vec{s}) \cdot \mathrm{d}\vec{a}$ is directional, the integral lower limit a is not required to be less than the integral upper limit b when we calculate $\int_{\vec{C}} \vec{F}(\vec{s}) \cdot \mathrm{d}\vec{a}$.

4.2.4. Directional Potential of the CMIPVLU. For a CMIPVLU $\left\{\left(\left[\vec{s}_{\alpha_k}, \vec{s}_{\alpha_{k+1}}\right], p\left(\vec{s}_\alpha\right)\right)\right\}_{k=1}^N$, we can get its directional curve potential based on the properties of the second curvilinear integral and Definition 15.

Definition 16. Let $\left\{\left(\left[\vec{s}_{\alpha_k} \longrightarrow \vec{s}_{\alpha_{k+1}}\right], p\left(\vec{s}_\alpha\right)\right)\right\}_{k=1}^N$ be a directional CMIPVLU, then its directional curve potential can be obtained by

$$\mathrm{DCP}_{\left\{\left(\left[\vec{s}_{\alpha_k} \longrightarrow \vec{s}_{\alpha_{k+1}}\right], p\left(\vec{s}_\alpha\right)\right)\right\}_{k=1}^N} = \sum_{k=1}^N \mathrm{DCP}_{\left(\left[\vec{s}_{\alpha_k} \longrightarrow \vec{s}_{\alpha_{k+1}}\right], p\left(\vec{s}_\alpha\right)\right)}. \tag{11}$$

5. The Application Examples of the Potentials

In this section, we illustrate two numerical examples to show how to use the potentials of PVLUs to address the linguistic evaluation problems in the physician-patient communication for COPD.

5.1. The Applications of the Nondirectional Curve Potential in Measuring the Fuzziness of the CSIPVLU

Example 3. Let $[s_{0.8}, s_{0.85}] \subset [s_0, s_1]$ be an interval linguistic evaluation, S^1 and S^2 be two normalized LESs illustrated in Example 2, $r_1(s_\alpha)$ and $r_2(s_\alpha)$ be the ordinate coefficient functions given in equation (1). We transform $[s_{0.8}, s_{0.85}]$ into the CSIPVLUs by using $r_1(s_\alpha)$ and $r_2(s_\alpha)$ based on S^1 and S^2, respectively, and calculate the nondirectional curve potentials of the transformed units; the results are listed in Table 2:

By Table 2, U_1 and U_2 are transformed from S^1 with respect to "$s_{0.75}\vec{i} + \vec{j}$ = slightly high" by $r_1(s_\alpha)$ and $r_2(s_\alpha)$, respectively. Referring to Figure 5, we know that $r_2(s_\alpha)$ is twisted more than $r_1(s_\alpha)$, which demonstrates that the judgement thinking reflected by $r_2(s_\alpha)$ is fuzzier than the one reflected by $r_1(s_\alpha)$. The nondirectional curve potentials of U_1 and U_2 present the fuzzy comparisons between U_1 and U_2. The same explanations can be conducted to the fuzzy comparisons between U_3 and U_4.

In addition, U_1 and U_3 are both transformed by $r_1(s_\alpha)$, while U_1 is from S^1 and U_3 is from S^2. Comparing S^1 with S^2, we know that S^2 is subdivided more. Therefore, the fuzziness of U_3 should be less than U_1. The nondirectional curve potentials of U_1 and U_3 show the difference of fuzziness between U_1 and U_3. The comparative results can also be summarized for U_2 and U_4.

5.2. The Applications of the Potentials in Dealing with the Linguistic Evaluation in Physician-Patient Communication for COPD

Nowadays, many major hospitals pay more attention to patients' evaluations and satisfactions to improve the physician-patient relation. On the website of the West China Hospital, there are questionnaires to track and collect the social evaluation opinions and patients' satisfactions for medicine service. In the evaluation systems, the indexes about the quality of physician-patient communications are necessary and important. The ways to obtain patients' evaluations are various, such as questionnaire survey, visiting patients by mails or telephones, and so on.

Suppose that the physician E is a specialist on COPD. His performance is regularly evaluated according to the rules of the hospital. By mail and telephone following up, we get the same linguistic evaluation from patients H_1, H_2 H_3, and H_4 for E, which is $[s_{0.43}, s_{0.54}]$, where $s_{0.43}$ is the earlier information and $s_{0.54}$ is the later information. The values $s_{0.43}$ and $s_{0.54}$ are the any given patients' evaluation information based on the multigranularity linguistic space $\{\Omega^\tau\}_{\tau=1}^2$ illustrated in Example 2, which are used to illustrate the usage of the directional potentials with respect to the CMIPVLUs. Based on the LESs S^1, S^2, and the functions $r_1(s_\alpha)$, and $r_2(s_\alpha)$ mentioned in equation (1), we can transform $[s_{0.43}, s_{0.54}]$ into four CMIPVLUs. By equation (11), we can calculate the abscissa component, the vertical component, and the total

Table 2: The nondirectional potentials of the CSIPVLUs.

CSIPVLUs	Source	NDCP
$U_1 = \left([s_{0.8}\vec{i} + 0.80\vec{j}, s_{0.85}\vec{i} + 0.60\vec{j}], 20\right)$	From S^1 to "$s_{0.75}\vec{i} + \vec{j}$ = slightly high" based on $r_1(s_\alpha)$	4.12
$U_2 = \left([s_{0.8}\vec{i} + 0.75\vec{j}, s_{0.85}\vec{i} + 0.52\vec{j}], 20\right)$	From S^1 to "$s_{0.75}\vec{i} + \vec{j}$ = slightly high" based on $r_2(s_\alpha)$	4.62
$U_3 = \left([s_{0.8}\vec{i} + 0.72\vec{j}, s_{0.85}\vec{i} + 0.54\vec{j}], 20\right)$	From S^2 to "$s_{0.72}\vec{i} + \vec{j}$ = very high" based on $r_1(s_\alpha)$	3.74
$U_4 = \left([s_{0.8}\vec{i} + 0.65\vec{j}, s_{0.85}\vec{i} + 0.45\vec{j}], 20\right)$	From S^2 to "$s_{0.72}\vec{i} + \vec{j}$ = very high" based on $r_2(s_\alpha)$	4.05

value of the directional curve potential for each CMIPVLU, respectively. The results are shown in Table 3.

In the above table, U_1 and U_2 are both got from S^1 with respect to "$s_{0.5}\vec{i} + \vec{j}$ = neutral," and obtained by $r_1(s_\alpha)$ and $r_2(s_\alpha)$, respectively. Making comparisons between the works of U_1 and U_2, we can find that (i) their abscissa works are always same since the ranges of s_α are $[s_{0.43}, s_{0.54}]$, either U_1 or U_2 and (ii) their vertical works are different due to the integral curves of $\int_{\vec{C}} \vec{F}(\vec{s}) \cdot d\vec{a}$ are different with $r_1(s_\alpha)$ and $r_2(s_\alpha)$.

For U_1, in the left side of "$s_{0.5}\vec{i} + \vec{j}$ = neutral," the abscissa work and the vertical work are both positive, which indicates that either evaluations or satisfactions develop in the positive direction. However, in the right side of "$s_{0.5}\vec{i} + \vec{j}$ = neutral," the abscissa work is positive but the vertical work is negative, which means that the evaluations develop in the positive direction but the satisfactions develop in the negative direction. Moreover, the total work of U_1 is positive, which expresses that the overall work of evaluations and satisfactions is positively effective.

Comparing the works among U_1, U_2, U_3, and U_4, we get the individual meaning of $[s_{0.43}, s_{0.54}]$ for each patient. When they give "$s_{0.5}\vec{i} + \vec{j}$ = neutral" to express the evaluations for the current physician-patient communication, the first patient's positive development is greater than others, and the fourth patient requires more attentions and improvements in the medical treatment process.

6. Comparisons and Discussions

6.1. Comparisons among Different PVLUs. The nondirectional potentials and the directional potentials of the PVLUs introduced in Section 4 can be summarized in the following table.

In Table 4, the sign "√" expresses "existence," which indicates that the corresponding values of the directional potentials of the PVLUs can be obtained. For example, for the CSIPVLU, there are two "√" existing in the columns of the nondirectional curve potential and the directional curve potential, respectively. It means that the nondirectional curve potential and the directional curve potential of the CSIPVLU can be calculated. Additionally, the sign "√**v**" interprets "existence and vector," which manifests that the results of the directional potentials of the PVLUs are vectors. Moreover, different forms of the PVLUs can be compared by the potentials that fall in the same colored area.

Example 4. For several DSPVLUs and DMPVLUs listed in Table 5, their nondirectional potentials and directional potentials can be calculated. The results are shown in Table 5.

In Table 5, we calculate the IPPs of the DSPVLUs, the MPPs of the DMPVLUs, and the DPs of the DSPVLUs and the DMPVLUs. For the nondirectional potentials of the IPP and the MPP, we get an order that $U_1 \succ U_3 \succ U_5 \succ U_4 \succ U_2$. Additionally, for the single vectors DPs, by assigning the probability of them as 1, the order $U_3 \succ U_5 \succ U_1 \succ U_4 \succ U_2$ can be obtained. Obviously, the orders based on the nondirectional and directional potentials are different. The reason is that the IPP and the MPP are the holistic reflection of s_α, $r(s_\alpha)$, and $p_{s_\alpha\vec{i}} + r(s_\alpha)\vec{j}$, while the DP is the component reflection of the s_α with $p_{s_\alpha\vec{i}} + r(s_\alpha)\vec{j}$ and the $r(s_\alpha)$ with $p_{s_\alpha\vec{i}} + r(s_\alpha)\vec{j}$. Thus, in practice, we should choose a desirable potential to cope with the probabilistic vector linguistic evaluation according to the different reflections about the properties of the nondirectional and directional potentials.

6.2. Comparisons between the New Proposed Method and the Other Related Methods

Example 5. This example is to apply the GPVLT in personal hospital selection-recommender system, which has been illustrated with PVLT in Section 5 of reference [38]. A COPD patient consults three experts groups (denoted by e_1, e_2, and e_3) to choose a hospital from two alternatives H_1 and H_2 for the follow-up treatment. Because the experts can use different LESs to give the linguistic evaluations, this problem is a multigranularity linguistic decision-making. Let S_{e_1}, S_{e_2} and S_{e_3} be the experts' LESs, respectively, where $S_{e_1} = \{s_{-4}$ = "extremely low," s_{-3} = "very low," s_{-2} = "low," s_{-1} = "few low," s_0 = "moderate," s_1 = "few high," s_2 = "high," s_3 = "very high," s_4 = "extremely high"$\}$, $S_{e_2} = \{s_0$ = "extremely low," $s_{0.25}$ = "few low," $s_{0.5}$ = "moderate," $s_{0.75}$ = "high," s_1 = "extremely high"$\}$ and $S_{e_3} = \{s_0$ = "extremely low," $s_{0.28}$ = "very low," $s_{0.42}$ = "low," $s_{0.5}$ = "moderate," $s_{0.58}$ = "high," $s_{0.72}$ = "very high," s_1 = "extremely high"$\}$.

Replacing the vector in each PVLT of Tables 6 and 7 in reference [38] by the GPVLT based on $r_2(s_\alpha)$ in equation (1), we can get the DMPVLUs matrices provided by the experts on two hospitals over four attributes (denoted by A_1, A_2, A_3, and A_4) as

Table 3: The directional potentials of the CMIPVLUs.

CMIPVLUs	Source	The work in the left side of $s_{0.5}\vec{i}+\vec{j}$ = neutral		The work in the right side of $s_{0.5}\vec{i}+\vec{j}$ = neutral		The work of CMIPVLU	
		The abscissa work	The vertical work	The abscissa work	The vertical work	The abscissa work	The vertical work
$U_1=\left\{\begin{array}{l}\left(\left[s_{0.43}\vec{i}+0.67\vec{j}\longrightarrow s_{0.5}\vec{i}+\vec{j}\right],(120/13)\right),\\ \left(\left[s_{0.5}\vec{i}+\vec{j}\longrightarrow s_{0.54}\vec{i}+0.83\vec{j}\right],(120/13)\right)\end{array}\right\}$	From S^1 based on $r_1(s_\alpha)$	0.29	2.56	0.2	−1.41	0.49	1.15
		2.85		−1.21		1.64	
$U_2=\left\{\begin{array}{l}\left(\left[s_{0.43}\vec{i}+0.61\vec{j}\longrightarrow s_{0.5}\vec{i}+\vec{j}\right],(120/13)\right),\\ \left(\left[s_{0.5}\vec{i}+\vec{j}\longrightarrow s_{0.54}\vec{i}+0.79\vec{j}\right],(120/13)\right)\end{array}\right\}$	From S^1 based on $r_2(s_\alpha)$	0.29	2.90	0.2	−1.74	0.49	1.16
		3.19		−1.54		1.65	
$U_3=\left\{\begin{array}{l}\left(\left[s_{0.43}\vec{i}+0.2\vec{j}\longrightarrow s_{0.5}\vec{i}+\vec{j}\right],(120/13)\right),\\ \left(\left[s_{0.5}\vec{i}+\vec{j}\longrightarrow s_{0.54}\vec{i}+0.5\vec{j}\right],(120/13)\right)\end{array}\right\}$	From S^2 based on $r_1(s_\alpha)$	0.29	4.43	0.2	−3.46	0.49	0.97
		4.72		−3.26		1.46	
$U_4=\left\{\begin{array}{l}\left(\left[s_{0.43}\vec{i}+0.18\vec{j}\longrightarrow s_{0.5}\vec{i}+\vec{j}\right],(120/13)\right),\\ \left(\left[s_{0.5}\vec{i}+\vec{j}\longrightarrow s_{0.54}\vec{i}+0.47\vec{j}\right],(120/13)\right)\end{array}\right\}$	From S^2 based on $r_2(s_\alpha)$	0.29	4.46	0.2	−3.58	0.49	0.88
		4.75		−3.38		1.37	

Table 4: The summaries of the potentials of the PVLUs.

PVLUs	Potentials				
	The nondirectional potentials			The directional potentials	
	IPP	MPP	NDCP	DP	DCP
DSPVLU	√			√**v**	
DMPVLU		√		√**v**	
CSIPVLU			√		√
CMIPVLU			√		√

IPP = isolated point potential; MPP = multiple point potential; NDCP=nondirectional curve potential; DP = directional potential; DCP = directional curve potential.

Table 5: The potentials of several DSPVLUs and DMPVLUs.

PVLUs	Potentials				
	IPP	MPP	The order based on IPP and MPP	DP	The order based on DP
$U_1 = (s_{0.5}\vec{i} + \vec{j}, 1)$	1.12	—	1	$s_{0.5}\vec{i} + \vec{j}$	3
$U_2 = (s_{0.43}\vec{i} + 0.2\vec{j}, 1)$	0.48	—	5	$s_{0.43}\vec{i} + 0.2\vec{j}$	5
$U_3 = (s_{0.54}\vec{i} + 0.5\vec{j}, 1)$	0.74	—	2	$s_{0.54}\vec{i} + 0.5\vec{j}$	1
$U_4 = \{(s_{0.43}\vec{i} + 0.2\vec{j}, (1/2)), (s_{0.54}\vec{i} + 0.5\vec{j}, (1/2))\}$	—	0.61	4	$s_{0.49}\vec{i} + 0.35\vec{j}$	4
$U_5 = \{(s_{0.43}\vec{i} + 0.2\vec{j}, (1/5)), (s_{0.54}\vec{i} + 0.5\vec{j}, (4/5))\}$	—	0.69	3	$s_{0.52}\vec{i} + 0.44\vec{j}$	2

Denote each element of the above table as $\{(\vec{s}, p)\}_{mn} = \{(\vec{s}, p)_{\sigma_{mn}}\}_{mn}$, where $\sigma_{mn} = 1, 2, \ldots, \#\{(\vec{s}, p)\}_{mn}$ and $\#\{(\vec{s}, p)\}_{mn}$ is the granularity of the GPVLTs in the DMPVLU $\{(\vec{s}, p)\}_{mn}$. Normalizing the probabilities in each element of Table 6 by $\sum_{\sigma_{mn}=1}^{\#\{(\vec{s},p)\}_{mn}} p_{\sigma_{mn}} = 1$ and taking $p_{\sigma_{mn}}$ as the weight of $\vec{s}_{\sigma_{mn}}$, we can aggregate $\{(\vec{s}, p)\}_{mn}$ into a GPVLT by the directional potential of DMPVLU defined in Definition 14. Thus, Table 6 can be rewritten into Table 7 as follows:

Note that the probability of each GPVLT in the above table defaults to 1. Based on the Euclidean distance, we can get the distance between each GPVLT in Table 7 and the adjacent linguistic terms based on each individual LES, denoted by $d(e_{mn}, \vec{\alpha}_{mn})$, where e_{mn} is the element of row m, column n in the above table and $s_{\vec{\alpha}_{mn}}$ is the most adjacent linguistic term from $s_{e_{mn}}$ in the LES S_{e_n}. According to the idea of the value function defined by equation (4) in Step 3 of Algorithm 4.1 in reference [38], we use $\mathrm{EP}(e_{mn}) = \theta_k \lambda_i + w_k v_t - d(e_{mn}, \vec{\alpha}_{mn}) + 1$ as the value function can get the selection sequences (11, 34, 32, 13) for H_1 and (14, 11, 23, 22) for H_2. For each vector in Table 7, take the subscript of the most adjacent linguistic term from $s_{e_{mn}}$ in the LES S_{e_n} as the evaluation value to aggregate them by Step 5 of Algorithm 4.1 in reference [38], we get 1.92 as the final evaluation value of H_1 and get 1.50 as the final evaluation value of H_2. Obviously, we should recommend the first hospital to the patient, which is the same as the result in reference [38].

Comparing the above computing process to the one in reference [38], it is clear that each GPVLT in Table 7 can be explained into a linguistic term based on each individual LES. For example, for the vector $0.52\vec{i} + 0.90\vec{j}$ in the first row and the first column in Table 7, by the Euclidean distance and the nearby principle, it can be explained by the linguistic term "moderate" in the LES S_{e_1}. But in reference [38], the aggregated results of the experts on two hospitals over four attributes are real values with fuzzy degrees, which is hard to explain as a linguistic term in the LES. This is because that the ordinate of the vector in the GPVLT is finite but the one of the vector in the PVLT defined in reference [38] may be infinite. Thus, the result with the GPVLT in this paper helps the decision maker understand computed result more easily than the one based on the PVLT in reference [38], which is one of the advantages of GPVLT over the PVLT introduced in reference [38].

Moreover, as shown in Example 5, all DMPVLUs given by the experts for all attributes can be aggregated by the nondirectional potential and directional potential defined in Definition 10 (the aggregated result is an accurate real number) and Definition 14 (the aggregated result is an accurate vector), respectively. But in reference [38], because of the infinity ordinate of vector in the PVLT, we have to aggregate the experts' linguistic evaluations by the curve fitting method, which is an approximation method with error. In this sense, the GPVLT proposed in this paper improves the computational performance of the ordinate of the PVLT defined in reference [38].

6.3. *Discussions.*

In this subsection, we point out some drawbacks and advantages of the new proposed methods.

Drawbacks:

(1) In practical decision-making problems, it is hard to determine the probability functions of the CSIPVLU and the CMIPVLU.

Table 6: The decision matrix of the experts on two hospitals over four attributes.

		$A_1\,(0.33)$	$A_2\,(0.27)$	$A_3\,(0.25)$	$A_4\,(0.15)$
H_1	$e_1\,(0.45)$	$\{(0.38\vec{i}+\vec{j},0.2),(0.5\vec{i}+\vec{j},0.38),(0.75\vec{i}+\vec{j},0.42)\}$	$\{(0.38\vec{i}+\vec{j},0.15),(0.42\vec{i}+0.63\vec{j},0.36),(0.46\vec{i}+0.63\vec{j},0.29),(0.75\vec{i}+\vec{j},0.2)\}$	$\{(0.46\vec{i}+0.63\vec{j},0.07),(0.5\vec{i}+\vec{j},0.31),(0.72\vec{i}+0.72\vec{j},0.21),(0.85\vec{i}+0.74\vec{j},0.42)\}$	$\{(0.38\vec{i}+\vec{j},0.31),(0.42\vec{i}+0.63\vec{j},0.16),(0.83\vec{i}+\vec{j},0.53)\}$
	$e_2\,(0.35)$	$\{(0.28\vec{i}+0.85\vec{j},0.15),(0.75\vec{i}+\vec{j},0.49),(0.9\vec{i}+0.52\vec{j},0.36)\}$	$\{(0.42\vec{i}+0.61\vec{j},0.4),(0.5\vec{i}+\vec{j},0.13),(0.75\vec{i}+\vec{j},0.47)\}$	$\{(0.13\vec{i}+0.44\vec{j},0.34),(0.42\vec{i}+0.61\vec{j},0.45),(0.95\vec{i}+0.75\vec{j},0.21)\}$	$\{(0.33\vec{i}+0.61\vec{j},0.2),(0.42\vec{i}+0.61\vec{j},0.19),(0.5\vec{i}+\vec{j},0.33),(0.58\vec{i}+0.61\vec{j},0.29)\}$
	$e_3\,(0.20)$	$\{(0.42\vec{i}+\vec{j},0.26),(0.5\vec{i}+\vec{j},0.21),(0.58\vec{i}+\vec{j},0.53)\}$	$\{(0.42\vec{i}+\vec{j},0.2),(0.54\vec{i}+0.47\vec{j},0.35),(0.5\vec{i}+\vec{j},0.27),(0.75\vec{i}+\vec{j},0.18)\}$	$\{(0.5\vec{i}+\vec{j},0.43),(0.58\vec{i}+\vec{j},0.17),(0.75\vec{i}+\vec{j},0.39)\}$	$\{(0.42\vec{i}+\vec{j},0.3),(0.5\vec{i}+\vec{j},0.21),(0.58\vec{i}+\vec{j},0.49)\}$
H_2	$e_1\,(0.45)$	$\{(0.28\vec{i}+0.74\vec{j},0.25),(0.45\vec{i}+0.55\vec{j},0.4),(0.75\vec{i}+\vec{j},0.35)\}$	$\{(0.35\vec{i}+0.74\vec{j},0.17),(0.47\vec{i}+0.72\vec{j},0.22),(0.6\vec{i}+0.74\vec{j},0.4),(0.75\vec{i}+\vec{j},0.2)\}$	$\{(0.38\vec{i}+\vec{j},0.06),(0.47\vec{i}+0.72\vec{j},0.33),(0.57\vec{i}+0.5\vec{j},0.23),(0.65\vec{i}+\vec{j},0.38)\}$	$\{(0.38\vec{i}+\vec{j},0.13),(0.45\vec{i}+0.55\vec{j},0.38),(0.93\vec{i}+0.55\vec{j},0.5)\}$
	$e_2\,(0.35)$	$\{(0.35\vec{i}+0.52\vec{j},0.14),(0.7\vec{i}+0.75\vec{j},0.43),(0.8\vec{i}+0.75\vec{j},0.43)\}$	$\{(0.39\vec{i}+0.48\vec{j},0.26),(0.5\vec{i}+\vec{j},0.39),(0.75\vec{i}+\vec{j},0.36)\}$	$\{(0.13\vec{i}+\vec{j},0.38),(0.66\vec{i}+\vec{j},0.41),(0.9\vec{i}+\vec{j},0.21)\}$	$\{(0.45\vec{i}+0.75\vec{j},0.18),(0.47\vec{i}+0.85\vec{j},0.25),(0.5\vec{i}+\vec{j},0.3),(0.78\vec{i}+0.85\vec{j},0.26)\}$
	$e_3\,(0.20)$	$\{(0.4\vec{i}+0.84\vec{j},0.26),(0.5\vec{i}+\vec{j},0.21),(0.68\vec{i}+0.68\vec{j},0.53)\}$	$\{(0.4\vec{i}+0.84\vec{j},0.23),(0.56\vec{i}+0.73\vec{j},0.31),(0.7\vec{i}+0.84\vec{j},0.28),(0.9\vec{i}+0.56\vec{j},0.19)\}$	$\{(0.5\vec{i}+\vec{j},0.27),(0.7\vec{i}+0.84\vec{j},0.41),(0.75\vec{i}+0.86\vec{j},0.32)\}$	$\{(0.49\vec{i}+0.86\vec{j},0.35),(0.66\vec{i}+0.53\vec{j},0.19),(0.98\vec{i}+0.9\vec{j},0.47)\}$

Table 7: The aggregated results of the experts on two hospitals over four attributes.

		$A_1\,(0.33)$	$A_2\,(0.27)$	$A_3\,(0.25)$	$A_4\,(0.15)$
H_1	$e_1\,(0.45)$	$0.52\vec{i}+0.90\vec{j}$	$0.44\vec{i}+0.68\vec{j}$	$0.50\vec{i}+0.58\vec{j}$	$0.32\vec{i}+0.48\vec{j}$
	$e_2\,(0.35)$	$0.53\vec{i}+0.58\vec{j}$	$0.31\vec{i}+0.45\vec{j}$	$0.31\vec{i}+0.41\vec{j}$	$0.38\vec{i}+0.59\vec{j}$
	$e_3\,(0.20)$	$0.40\vec{i}-0.76\vec{j}$	$0.54\vec{i}+0.81\vec{j}$	$0.42\vec{i}+0.69\vec{j}$	$0.38\vec{i}+0.73\vec{j}$
H_2	$e_1\,(0.45)$	$0.56\vec{i}+0.83\vec{j}$	$0.50\vec{i}+0.70\vec{j}$	$0.44\vec{i}+0.63\vec{j}$	$0.32\vec{i}+0.29\vec{j}$
	$e_2\,(0.35)$	$0.49\vec{i}+0.50\vec{j}$	$0.39\vec{i}+0.61\vec{j}$	$0.37\vec{i}+0.73\vec{j}$	$0.48\vec{i}+0.76\vec{j}$
	$e_3\,(0.20)$	$0.43\vec{i}+0.60\vec{j}$	$0.61\vec{i}+0.73\vec{j}$	$0.56\vec{i}+0.76\vec{j}$	$0.64\vec{i}+0.70\vec{j}$

(2) The nondirectional potentials cannot be utilized to compare all forms of the PVLUs together since various potentials have different meanings. For example, the multiple point potential of the DMPVLU and the nondirectional curve potential of the CSIPVLU mean diversely.

Advantages:

(1) The GPVLT is more appropriate and comprehensive than the PVLT in portraying people's judgements.

(2) The nondirectional potentials and the directional potentials possess the strong abilities to compare not only the discrete PVLUs, but also the continuous PVLUs.

(3) For an object, we can grasp its directional changes from two angles, i.e., the evaluations and the satisfactions, by the directional curve potential of the continuous PVLUs. It provides the flexibility and effectiveness to let people choose the component works or the aggregated work of the continuous PVLUs according to real problems.

7. Conclusions

In this paper, we have extended the PVLT into the GPVLT to improve the computational performance of the ordinate of the vector in PVLT. Based on GPVLT, we have studied the forms of the PVLUs, i.e., DSPVLUs, DMPVLUs, CSIPVLUs, and CMIPVLUs and proposed the nondirectional potentials and the directional potentials for them. Because the GPVLT can distinguish the directional linguistic evaluation, the new proposed potentials have enriched the theories of the PVLTs to open a new prospect of analyzing and utilizing the linguistic evaluation. Later on, the cases about the physician-patient communication for COPD have been illustrated to demonstrate the effectiveness and practicability of the potentials of PVLUs. Furthermore, we have compared the new proposed methods with other related methods and listed the drawbacks and advantages of the GPVLT and the potentials of the PVLUs.

Moreover, there are some potential directions for the further investigation. For example, we have developed several basic ways to study the linguistic evaluations through the analytical properties of functions, such as the directional potentials of the CSIPVLUs and the CMIPVLUs. Thus, we can develop more tools on these results to promote the application of GPVLT in analyzing the linguistic evaluations for the physician-patient communication of COPD. Secondly, considering the advantages of GPVLT in describing the same linguistic evaluation from different sources in the multigranularity linguistic decision-making, we will continue to study the practical applications of applying the GPVLT in the multigranularity linguistic decision-making methods.

Data Availability

The numerical data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] S. W. Weldam, J. W. J. Lammers, M. J. Heijmans, and M. J. Schuurmans, "Perceived quality of life in chronic obstructive pulmonary disease patients: a cross-sectional study in primary care on the role of illness perceptions," *BMC Family Practice*, vol. 15, no. 1, p. 140, 2014.

[2] J. Vestbo, S. S. Hurd, A. G. Agustí et al., "Global strategy for the diagnosis, management, and prevention of chronic obstructive pulmonary disease: GOLD executive summary," *American Journal of Respiratory & Critical Care Medicine*, vol. 187, no. 4, pp. 532–555, 2013.

[3] M. Ekström, J. Sundh, and K. Larsson, "Patient reported outcome measures in chronic obstructive pulmonary disease—which to use?," *Expert Review of Respiratory Medicine*, vol. 10, no. 3, p. 315, 2016.

[4] D. Zagami, J. Hockenhull, A. Bodger, and K. B. Sriram, "Communication of pulmonary function test results: a survey of patient's preferences," *PLoS One*, vol. 10, no. 5, Article ID e0126617, 2015.

[5] E. M. Buining, M. K. Kooijman, I. C. S. Swinkels, M. F. Pisters, and C. Veenhof, "Exploring physiotherapists' personality traits that may influence treatment outcome in patients with chronic diseases: a cohort study," *BMC Health Services Research*, vol. 15, no. 1, pp. 1–11, 2015.

[6] A. Ninou, E. Guthrie, V. Paika et al., "Illness perceptions of people with long-term conditions are associated with frequent use of the emergency department independent of mental

illness and somatic symptom burden," *Journal of Psychosomatic Research*, vol. 81, pp. 38–45, 2016.
[7] S. Farzadnia and H. Giles, "Patient-provider health interactions: a communication accommodation theory perspective," *International Journal of Society, Culture & Language*, vol. 3, no. 2, pp. 17–34, 2015.
[8] S. A. Keaton and H. Giles, "Subjective health: the roles of communication, language, aging, stereotypes, and culture," *International Journal of Society, Culture & Language*, vol. 4, no. 2, p. 1. in press, 2015.
[9] B. C. M. Boerebach, R. A. Scheepers, R. M. van der Leeuw, M. J. Heineman, O. A. Arah, and K. M. J. M. H. Lombarts, "The impact of clinicians' personality and their interpersonal behaviors on the quality of patient care: a systematic review," *International Journal for Quality in Health Care*, vol. 26, no. 4, pp. 426–481, 2014.
[10] A. M. Hesson, I. Sarinopoulos, R. M. Frankel, and R. C. Smith, "A linguistic study of patient-centered interviewing: emergent interactional effects," *Patient Education and Counseling*, vol. 88, no. 3, pp. 373–380, 2012.
[11] L. A. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning-I," *Information Sciences*, vol. 8, no. 3, pp. 199–249, 1975.
[12] Z. S. Xu, "A method based on linguistic aggregation operators for group decision making with linguistic preference relations," *Information Sciences*, vol. 166, no. 1–4, pp. 19–30, 2004.
[13] Z. Xu, "An interactive approach to multiple attribute group decision making with multigranular uncertain linguistic information," *Group Decision and Negotiation*, vol. 18, no. 2, pp. 119–145, 2009.
[14] F. Herrera and E. Herrera-Viedma, "Linguistic decision analysis: steps for solving decision problems under linguistic information," *Fuzzy Sets and Systems*, vol. 115, no. 1, pp. 67–82, 2000.
[15] V. Torra, "Negation functions based semantics for ordered linguistic labels," *International Journal of Intelligent Systems*, vol. 11, no. 11, pp. 975–988, 1996.
[16] Z. S. Xu, "EOWA and EOWG operators for aggregating linguistic labels based on linguistic preference relations," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 12, no. 6, pp. 791–810, 2004.
[17] J.-Q. Wang, J.-T. Wu, J. Wang, H.-Y. Zhang, and X.-H. Chen, "Interval-valued hesitant fuzzy linguistic sets and their applications in multi-criteria decision-making problems," *Information Sciences*, vol. 288, no. 1, pp. 55–72, 2014.
[18] R. M. Rodriguez, L. Martinez, and F. Herrera, "Hesitant fuzzy linguistic term sets for decision making," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 1, pp. 109–119, 2012.
[19] H. Wang, "Extended hesitant fuzzy linguistic term sets and their aggregation in group decision making," *International Journal of Computational Intelligence Systems*, vol. 8, no. 1, pp. 14–33, 2015.
[20] Z. M. Zhang and C. Wu, "Hesitant fuzzy linguistic aggregation operators and their applications to multiple attribute group decision making," *Journal of Intelligent & Fuzzy Systems*, vol. 26, no. 5, pp. 2185–2202, 2014.
[21] S. H. Yang and Y. B. Ju, "Multi-attribute decision-making method based on dual hesitant fuzzy linguistic variables," *Operations Research and Management Science*, vol. 24, no. 5, pp. 91–96, 2015.
[22] Q. Pang, H. Wang, and Z. Xu, "Probabilistic linguistic term sets in multi-attribute group decision making," *Information Sciences*, vol. 369, pp. 128–143, 2016.
[23] Z. Xu, Y. He, and X. Wang, "An overview of probabilistic-based expressions for qualitative decision-making: techniques, comparisons and developments," *International Journal of Machine Learning and Cybernetics*, vol. 10, no. 6, pp. 1513–1528, 2019.
[24] F. Herrera and L. Martinez, "A 2-tuple fuzzy linguistic representation model for computing with words," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 6, pp. 746–752, 2000.
[25] Z. Xu, "Deviation measures of linguistic preference relations in group decision making," *Omega*, vol. 33, no. 3, pp. 249–254, 2005.
[26] Z. Xu and H. Wang, "On the syntax and semantics of virtual linguistic terms for information fusion in decision making," *Information Fusion*, vol. 34, pp. 43–48, 2017.
[27] H. Liao, Z. Xu, and X.-J. Zeng, "Distance and similarity measures for hesitant fuzzy linguistic term sets and their application in multi-criteria decision making," *Information Sciences*, vol. 271, no. 3, pp. 125–142, 2014.
[28] H. Liao and Z. Xu, "Intuitionistic fuzzy hybrid weighted aggregation operators," *International Journal of Intelligent Systems*, vol. 29, no. 11, pp. 971–993, 2014.
[29] E. Herrera-Iedma, F. Herrera, F. Chiclana, and M. Luque, "Some issues on consistency of fuzzy preference relations," *European Journal of Operational Research*, vol. 154, no. 1, pp. 98–109, 2004.
[30] B. Zhu and Z. Xu, "Consistency measures for hesitant fuzzy linguistic preference relations," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 1, pp. 35–45, 2014.
[31] H. Liao, Z. Xu, X.-J. Zeng, and D.-L. Xu, "An enhanced consensus reaching process in group decision making with intuitionistic fuzzy preference relations," *Information Sciences*, vol. 329, no. C, pp. 274–286, 2016.
[32] J.-Q. Wang, J. Wang, Q.-H. Chen, H.-Y. Zhang, and X.-H. Chen, "An outranking approach for multi-criteria decision-making with hesitant fuzzy linguistic term sets," *Information Sciences*, vol. 280, pp. 338–351, 2014.
[33] Y. Zheng, Z. Xu, Y. He, and H. Liao, "Severity assessment of chronic obstructive pulmonary disease based on hesitant fuzzy linguistic COPRAS method," *Applied Soft Computing*, vol. 69, pp. 60–71, 2018.
[34] F. Herrera and L. Martinez, "A model based on linguistic 2-tuples for dealing with multigranular hierarchical linguistic contexts in multi-expert decision-making," *IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics)*, vol. 31, no. 2, pp. 227–234, 2001.
[35] M. Espinilla, J. Liu, and L. Martínez, "An extended hierarchical linguistic model for decision-making problems," *Computational Intelligence*, vol. 27, no. 3, pp. 489–512, 2011.
[36] Z. Chen and D. Ben-Arieh, "On the fusion of multi-granularity linguistic label sets in group decision making," *Computers & Industrial Engineering*, vol. 51, no. 3, pp. 526–541, 2006.
[37] M. Li, M. Yuan, and Y. Xu, "An approach to task-oriented knowledge recommendation based on multi-granularity fuzzy linguistic method," *Kybernetes*, vol. 44, no. 3, pp. 460–474, 2015.
[38] Y. Zhai, Z. Xu, and H. Liao, "Probabilistic linguistic vector-term set and its application in group decision making with multi-granular linguistic information," *Applied Soft Computing*, vol. 49, pp. 801–816, 2016.
[39] C.-C. Li, Y. Dong, F. Herrera, E. Herrera-Viedma, and L. Martínez, "Personalized individual semantics in computing with words for supporting linguistic group decision making.

An application on consensus reaching," *Information Fusion*, vol. 33, pp. 29–40, 2017.
[40] C.-C. Li, R. M. Rodríguez, L. Martínez, Y. Dong, and F. Herrera, "Personalized individual semantics based on consistency in hesitant linguistic group decision making with comparative linguistic expressions," *Knowledge-Based Systems*, vol. 145, pp. 156–165, 2018.
[41] C.-C. Li, Y. Dong, and F. Herrera, "A consensus model for large-scale linguistic group decision making with a feedback recommendation based on clustered personalized individual semantics and opposing consensus groups," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 2, pp. 221–233, 2019.
[42] Z. Xu, "Uncertain linguistic aggregation operators based approach to multiple attribute group decision making under uncertain linguistic environment," *Information Sciences*, vol. 168, no. 1–4, pp. 171–184, 2004.