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Research Article

Consensus for Mixed-Order Multiagent Systems over Jointly Connected Topologies via Impulse Control

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Because of the complexity of the environment, the dynamics of agents in the same system may be different. That is, the dynamics of some agents may be first ordered, and the others may be second ordered, even high ordered. In addition, the network topologies of systems are always varying over time. Because of these facts, this paper studies the consensus problem of the mixed-order multiagent networks over the jointly connected topologies. By adopting the impulse control technique, some control protocols are proposed based on the information of the agents themselves and their neighbors. Several simulation results are given to verify the correctness of the theoretical results.

1. Introduction

Consensus of multiple dynamic agents is an interesting topic [1–20]. Most of the consensus results are on the homogeneous dynamics. However, because of the various restrictions and the complexity of environments or the different task divisions, the dynamics of different agents in the same system may be different. That is, in a system, the dynamics of some agents are first ordered, and the others may be second ordered, even high ordered, which is called a mixed-order multiagent system or a heterogeneous multiagent system [20–23]. In addition, most of the consensus results on multiagent systems required the network to be connected, even undirected connected. However, practically, the communication links may be broken or disconnected because of the obstacle among agents or the change of the agents themselves. That is to say, the topology of the network could not maintain being connected all the time. Just the opposite, it is usually varying along with the time [24–28]. To reduce the information exchange capacity of agents, this work adopts the impulse control technique, which only uses the information at the impulse instants [10, 29–32], that is, exerting control only at the impulse instant and no control at any other time. The main idea of impulsive control is to drive the state variable of the controlled system instantaneously to some value which is determined by an impulsive control law at each impulsive instant. It is more reasonable to perform this state change within a period of time [10]. Liu et al. [10] propose the pulse-modulated intermittent control. They found that consensus significantly relies on the sampling period, the control gains, the digraph, and the pulse function and gave some necessary and sufficient conditions to ensure the consensus of the controlled system. The impulse control method not only can avoid the abrupt changes between the agents' states but also can greatly reduce the amount of information transferring. Moreover, the continuous time control protocols may lead to chattering because the neighbor relations might change abruptly with the changing of agents' states. Because of all the above problems, this paper studies the consensus problem of the mixed-order multiagent networks over the jointly connected topologies via impulse control technique. For all we know, there are no related topic's results till now.

The rest of this work is organized as follows. The necessary knowledge used in the work is presented in Section 2. The problems and impulse-control protocols for the given multiagent systems are proposed in Section 3. Some simulation results are given in Section 4 to show the feasibility of the control technique. Section 5 summarizes the main ideas and methods of the work.

2. Preliminaries and Some Necessary Lemmas

2.1. Preliminaries. In this section, some necessary notations and the knowledge of graph theory are given.

R^n: n-dimensional real column vector set;

I_n: n-dimensional identity matrix;

$\mathbf{1} = [1, 1, \ldots, 1]^T$: a column vector with all elements being 1 and an appropriate dimension;

$\mathbf{0}$: a zero vector or a zero matrix with an appropriate dimension;

M_n: the set of all real n-dimensional matrices. For a network, $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ denotes the weighted graph, where $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ is the node set, $i \in \Upsilon$ is the ith node, $\Upsilon = \{1, 2, \ldots, n\}$ is the node index set of $\mathcal{G}$, and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. Throughout this paper, the elements of $\mathcal{E}$ denote the communication links between agents. Ordered pair (v_i, v_j) represents a edge in $\mathcal{G}$; $(v_i, v_j) \in \mathcal{E}$ if and only if the ith agent can directly receive the jth agent's information. $\mathcal{N}_i = \{v_j \in \mathcal{V} \mid (v_j, v_i) \in \mathcal{E}\}$ denotes the neighbor agents set of the ith agent. For the weighted directed graph $\mathcal{G}$, $\mathcal{A} = [a_{ij}]$ is the weighted adjacency matrix, and $a_{ij} \geq 0$, $\forall i, j \in \Upsilon$. More specifically, if $(v_j, v_i) \in \mathcal{E}$, $a_{ij} > 0$, otherwise $a_{ij} = 0$, and $a_{ii} = 0$ for all $i \in \Upsilon$. If there is a sequence $(v_i, v_{k_1}), (v_{k_1}, v_{k_2}), \ldots, (v_{k_l}, v_j)$ from two different nodes v_i and v_j, it is said there is a path between the nodes v_i and v_j. If there is a path between any two different nodes, the graph is called connected. $\mathcal{D} = \mathrm{diag}\{\sum_{j=1}^{n} a_{ij},\ i = 1, \ldots, n\}$ is the degree matrix of graph $\mathcal{G}$, and $\mathcal{L} = \mathcal{D} - \mathcal{A} = [l_{ij}]_{n\times n}$ is the Laplacian matrix of graph $\mathcal{G}$. From the definition of $\mathcal{L}$, one can find that all the row sums of $\mathcal{L}$ are zero, and $\mathcal{L}$ has a right eigenvector $\mathbf{1}_n$ with the zero eigenvalue. If there is a node in a digraph, which satisfies the fact that there is a directed path from this node to any other node, the digraph is called containing a spanning tree.

For graphs $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_m$, which have the same node set $\mathcal{V}$, their connection is called the union graph $\mathcal{G}_{1-m}$, whose node set is $\mathcal{V}$, edge set is the union edge sets of all graphs in the collections, and the connected weight between agent i and agent j is the sum of a_{ij} of the connection graphs $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_m$. Graphs $\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_m$ are called jointly connected, if their union graph $\mathcal{G}_{1-m}$ contains a spanning tree.

Matrix $\mathcal{B} = [b_{ij}] \in R^{n\times r}$ is nonnegative if all its elements b_{ij} are nonnegative. For a nonnegative matrix $\mathcal{B} = [b_{ij}] \in R^{n\times r}$, if it satisfies $\mathcal{B}1_r = 1_n$, then it is called (row) stochastic. A stochastic matrix $\mathcal{B} \in R^{n\times n}$ is said to be indecomposable and aperiodic (SIA) if $\lim_{m\longrightarrow\infty} \mathcal{B}^m = 1_n f^T$, where $f \in R^n$ is a constant vector.

Several lemmas are given in the following for further analysis.

Lemma 1 (see [9]). *For a nonnegative matrix $\mathcal{H} = [h_{ij}] \in M_n$, if its row sums are the same positive constant, which is given by $\mu > 0$, then μ is an eigenvalue of $\mathcal{H}$ with the eigenvector $\mathbf{1}$, and μ is also the spectral radius of matrix $\mathcal{H}$, i.e., $\rho(\mathcal{H}) = \mu$. Eigenvalue μ of matrix $\mathcal{H}$ has algebraic multiplicity equal to one, iff the graph of $\mathcal{H}$ has a spanning tree. If the graph of matrix $\mathcal{H}$ has a spanning tree and all the diagonal element $h_{ii} > 0$, $i = 1, 2, \ldots, n$, then μ is the unique eigenvalue of matrix $\mathcal{H}$ with the maximum modulus.*

Lemma 2 (see [9]). *For a stochastic matrix $\mathcal{S} = [s_{ij}] \in M_n$, if $\mathcal{S}$ has an eigenvalue $\lambda = 1$ with the associate algebraic multiplicity equal to one, and all the other eigenvalues satisfy $|\lambda| < 1$, then $\mathcal{S}$ is SIA. That is, there exists a constant vector α satisfing $\mathcal{S}^T\alpha = \alpha$ and $1^T\alpha = 1$, such that $\lim_{m\longrightarrow\infty} \mathcal{S}^m = 1\alpha^T$.*

3. Main Results

Consider a system with N agents. Suppose that the system consists of n_1 $(0 < n_1 < N)$ first-order agents and $(N - n_1)$ second-order agents. In general, assume the first n_1 agents are first ordered and the other $(N-n_1)$ agents are second ordered. For the first-order agents, their dynamics are described as

$$\dot{p}_i(t) = u_i(t), \quad i = 1, 2, \ldots, n_1 \tag{1}$$

And the dynamics for the second-order agents are given as

$$\begin{aligned} \dot{p}_i(t) &= q_i(t), \\ \dot{q}_i(t) &= u_i(t), \\ & \qquad i = n_1 + 1, n_1 + 2, \ldots, N \end{aligned} \tag{2}$$

where $p_i(t) \in R$ is the position state, $q_i(t) \in R$ is the velocity state, and $u_i(t) \in R$ is the control input of agent i, respectively. Denote vector $p(t) = [p_1(t), \ldots, p_N(t)]^T$, $q(t) = [q_{n_1+1}(t), \ldots, q_N(t)]^T$. Obviously, $p(t)$ and $q(t)$ are N-dimensional and $(N - n_1)$-dimensional column vectors, respectively. Considering each agent as a node in a network, the information flow between neighboring agents of systems (1)-(2) can be seen as a network graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$.

Definition 3. Systems (1)-(2) are said to achieve the stationary consensus if, for any initial states, the trajectories of (1)-(2) satisfy

$$\begin{aligned} \lim_{t\longrightarrow\infty} \left\| p_i(t) - p_j(t) \right\| &= 0, \quad \forall i, j = 1, 2, \ldots, N \\ \lim_{t\longrightarrow\infty} q_i(t) &= 0, \quad \forall i = n_1 + 1, \ldots, N \end{aligned} \tag{3}$$

In the robot fault-tolerant control and hybrid robot formation environment, the stationary consensus in Definition 3, that is, $\lim_{t\longrightarrow\infty}\| p_i(t) - p_j(t)\| = 0$, $\forall i, j = 1, 2, \ldots, N$, and $\lim_{t\longrightarrow\infty} q_i(t) = 0$, $\forall i = n_1 + 1, n_1 + 2, \ldots, N$, means that

the position states of the robots tend to be the same and the velocity states tend to be zero with the development of the time.

For systems (1)-(2), adopt the following impulse control algorithm for the first-order agents

$$u_i(t) = \sum_{k=1}^{\infty}\left[-c_1\sum_{j\in\mathcal{N}_i(t)} l_{ij}(t)\left(p_j(t)-p_i(t)\right)\right]\delta(t-t_k), \quad i=1,2,\ldots,n_1 \tag{4}$$

and the impulse consensus algorithm for the second-order agents is presented as

$$\begin{aligned} u_i(t) = &-c_1\sum_{k=1}^{\infty}\sum_{j\in\mathcal{N}_i(t)} l_{ij}(t)\left[p_j(t)-p_i(t)\right]\delta(t-t_k) \\ &- c_2 q_i(t)\delta(t-t_k), \quad i=n_1+1,n_1+2,\ldots,N \end{aligned} \tag{5}$$

where $l_{ij}(t)$ is the element of Laplacian matrix $\mathscr{L}$, constants $c_1>0$, $c_2>0$ are control gains, and function $\delta(\cdot)$ is defined as $\delta(t)=\left\{\begin{smallmatrix}1, & t=t_k,\\ 0, & t\neq t_k,\end{smallmatrix}\right.$, where $t_{k+1}=t_k+h$, t_k is the sample instant, $k=0,1,2,\ldots$, and $h>0$ is the sample period. Under protocols (4)-(5), systems (1)-(2) are equivalent to

$$\begin{aligned} &\dot{p}_i(t)=0, \quad t\in(t_k,t_{k+1}],\ k=0,1,2,\ldots \\ &\Delta p_i(t_k) = p_i(t_k^+)-p_i(t_k) = u_i(t_k) \\ &\quad = -c_1\sum_{j\in\mathcal{N}_i(t_k)} l_{ij}(t_k)p_j(t_k), \end{aligned} \tag{6}$$

$$\begin{aligned} &\dot{p}_i(t)=q_i(t), \\ &\dot{q}_i(t)=0, \\ &\qquad t\in(t_k,t_{k+1}],\ k=0,1,2,\ldots \\ &\Delta q_i(t_k) = q_i(t_k^+)-q_i(t_k) = u_i(t_k) \\ &\quad = -c_1\sum_{j\in\mathcal{N}_i(t_k)} l_{ij}(t_k)p_j(t_k) - c_2 q_i(t_k) \end{aligned} \tag{7}$$

where $p_i(t_k^+)=\lim_{t\longrightarrow t_k^+} p_i(t)$, $q_i(t_k^+)=\lim_{t\longrightarrow t_k^+} q_i(t)$. Moreover, $p_i(t)$ and $q_i(t)$ are left continuous at $t=t_k$, i.e.,

$$\begin{aligned} &\lim_{t\longrightarrow t_k^-} p_i(t) = p_i(t_k), \\ &\lim_{t\longrightarrow t_k^-} q_i(t) = q_i(t_k). \end{aligned} \tag{8}$$

From (6)-(7) one can obtain

$$p_i(t_{k+1}) = p_i(t_k) - c_1\sum_{j\in\mathcal{N}_i(t_k)} l_{ij}(t_k)p_j(t_k), \quad i=1,2,\ldots,n_1, \tag{9}$$

$$\begin{aligned} p_i(t_{k+1}) &= p_i(t_k) + (1-c_2)q_i(t_k)h \\ &\quad - c_1 h\sum_{j\in\mathcal{N}_i(t_k)} l_{ij}(t_k)p_j(t_k), \\ q_i(t_{k+1}) &= -c_1\sum_{j\in\mathcal{N}_i(t_k)} l_{ij}(t_k)p_j(t_k) + (1-c_2)q_i(t_k), \\ &\qquad i=n_1+1,n_1+2,\ldots,N \end{aligned} \tag{10}$$

Denote $y_i(t_k)=p_i(t_k)+q_i(t_k)h, i=n_1+1,n_1+2,\ldots,N$, $\tilde{p}_i(k)=p_i(t_k)$, $\tilde{y}_i(k)=y_i(t_k), l_{ij}(k)=l_{ij}(t_k)$, $\forall i,j\in\Upsilon$. Then (9) and (10) can be written as

$$\tilde{p}_i(k+1) = \tilde{p}_i(k) - c_1\sum_{j\in\mathcal{N}_i(k)} l_{ij}(k)\tilde{p}_j(k), \quad i=1,2,\ldots,n_1, \tag{11}$$

$$\begin{aligned} \tilde{p}_i(k+1) &= c_2\tilde{p}_i(k) + (1-c_2)\tilde{y}_i(k) \\ &\quad - c_1 h\sum_{j\in\mathcal{N}_i(k)} l_{ij}(k)\tilde{p}_j(k), \\ \tilde{y}_i(k+1) &= (2c_2-1)\tilde{p}_i(k) + 2(1-c_2)\tilde{y}_i(k) \\ &\quad - 2c_1 h\sum_{j\in\mathcal{N}_i(k)} l_{ij}(k)\tilde{p}_j(k), \\ &\qquad i=n_1+1,n_1+2,\ldots,N \end{aligned} \tag{12}$$

Let

$$\begin{aligned} \tilde{P}_1(k) &= \left(\tilde{p}_1(k),\tilde{p}_2(k),\ldots,\tilde{p}_{n_1}(k)\right)^T \\ \tilde{P}_2(k) &= \left(\tilde{p}_{n_1+1}(k),\tilde{p}_{n_1+2}(k),\ldots,\tilde{p}_N(k)\right)^T \\ \tilde{Y}(k) &= \left(\tilde{y}_{n_1+1}(k),\tilde{y}_{n_1+2}(k),\ldots,\tilde{y}_N(k)\right)^T, \\ L &= \begin{bmatrix} L_{ff} & L_{fs} \\ L_{sf} & L_{ss} \end{bmatrix} \end{aligned} \tag{13}$$

where

$$L_{ff} = \begin{bmatrix} l_{11} & \cdots & l_{1n_1} \\ \vdots & \vdots & \vdots \\ l_{n_1 1} & \cdots & l_{n_1 n_1} \end{bmatrix}$$
$$L_{fs} = \begin{bmatrix} l_{1(n_1+1)} & \cdots & l_{1N} \\ \vdots & \vdots & \vdots \\ l_{n_1(n_1+1)} & \cdots & l_{n_1 N} \end{bmatrix}$$
$$L_{sf} = \begin{bmatrix} l_{(n_1+1)1} & \cdots & l_{(n_1+1)n_1} \\ \vdots & \vdots & \vdots \\ l_{N1} & \cdots & l_{Nn_1} \end{bmatrix} \tag{14}$$
$$L_{ss} = \begin{bmatrix} l_{(n_1+1)(n_1+1)} & \cdots & l_{(n_1+1)N} \\ \vdots & \vdots & \vdots \\ l_{N(n_1+1)} & \cdots & l_{NN} \end{bmatrix}$$

Then (11)-(12) can be described as the following form:

$$\phi(k+1) = \begin{bmatrix} \widetilde{P}_1(k+1) \\ \widetilde{P}_2(k+1) \\ \widetilde{Y}(k+1) \end{bmatrix} = M(k) \begin{bmatrix} \widetilde{P}_1(k) \\ \widetilde{P}_2(k) \\ \widetilde{Y}(k) \end{bmatrix} = M(k)\phi(k) \tag{15}$$

That is,

$$\phi(k+1) = M(k)\phi(k) \tag{16}$$

where

$$M(k) = \left[M_1(k) \quad M_2(k) \quad M_3(k)\right] \tag{17}$$

with

$$M_1(k) = \begin{bmatrix} I_{n_1} - c_1 L_{ff}(k) \\ -c_1 h L_{sf}(k) \\ -2c_1 h L_{sf}(k) \end{bmatrix}$$
$$M_2(k) = \begin{bmatrix} -c_1 L_{fs}(k) \\ c_2 I_{N-n_1} - c_1 h L_{ss}(k) \\ (2c_2 - 1) I_{N-n_1} - 2c_1 h L_{ss}(k) \end{bmatrix} \tag{18}$$
$$M_3(k) = \begin{bmatrix} 0 \\ (1-c_2) I_{N-n_1} \\ 2(1-c_2) I_{N-n_1} \end{bmatrix}$$

Theorem 4. *Suppose that the network is fixed and the control gains c_1, c_2 satisfy the following conditions*

(i) $c_1 > 0, 1/2 < c_2 < 1, h > 0$; *(ii)* $c_1 \max_{1\le i\le n_1} \sum_{j\in\mathcal{N}_i} a_{ij} < 1$; *(iii)* $c_1 h \max_{n_1+1\le i\le N} \sum_{j\in\mathcal{N}_i} a_{ij} < c_2 - 1/2$.

Then under protocols (4)-(5), systems (1)-(2) can achieve consensus asymptotically if and only if the topology graph of the network contains a spanning tree.

Proof. Note that if conditions (i)-(iii) hold, matrix $M = [m_{ij}]$ is nonnegative, $m_{ii} > 0$, and all the row sums of M are equal to 1. Then according to Lemma 1, 1 is the eigenvalue of matrix M with algebraic multiplicity being one and the unique eigenvalue of maximum modulus. Hence all the other eigenvalues of M satisfy $|\lambda(M)| < 1$, where $\lambda(M)$ is any eigenvalue of M besides the eigenvalue 1. Then the matrix M is SIA according to Lemma 2. That is, there exists a constant vector $\alpha \in R^{2N-n_1}$ such that $\lim_{k\longrightarrow\infty} M^k = 1\alpha^T$, which implies that $\lim_{k\longrightarrow\infty} \phi(k) = \lim_{k\longrightarrow\infty} M^k\phi(0) = 1\alpha^T\phi(0)$. Hence $\lim_{k\longrightarrow\infty} \widetilde{p}_i(k) = \lim_{k\longrightarrow\infty} \widetilde{y}_i(k) = \alpha^T\phi(0)$. Then one can get $\lim_{k\longrightarrow\infty} p_i(t_k) = \lim_{k\longrightarrow\infty} y_i(t_k) = \alpha^T\phi(0)$ and $\lim_{k\longrightarrow\infty} q_i(t_k) = 0$. That is, systems (1)-(2) achieve consensus asymptotically. This completes the proof. □

Theorem 5. *Suppose that the network topology of systems (1)-(2) is directed and switched jointly connected. And it is jointly connected in each time interval $[t_\varrho, t_{\varrho+1}), \varrho = 0, 1, 2, \ldots$, with $0 < t_{\varrho+1} - t_\varrho \le T_1$ for some constant $T_1 > 0$. If the adjacency weights $a_{ij}(t) \ge 0$ and the control gains c_1, c_2 satisfy*

(i) $c_1 > 0, 1/2 < c_2 < 1, h > 0$;

(ii) $c_1 \max_{1\le i\le n_1} \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t) < 1$;

(iii) $c_1 h \max_{n_1+1\le i\le N} \sum_{j\in\mathcal{N}_i(t)} a_{ij}(t) < c_2 - 1/2$.

Then under protocols (4)-(5), systems (1)-(2) can achieve consensus asymptotically if and only if the union network topology contains a spanning tree.

Proof. For some constants T_1, T_2 satisfying $0 < T_2 < T_1$, in time interval $[t_\varrho, t_{\varrho+1}), \varrho = 0, 1, 2, \ldots$, with $t_{\varrho+1} - t_\varrho \le T_1, t_0 = 0$, there are several nonoverlapping subintervals $[t_{\varrho_0}, t_{\varrho_1}), [t_{\varrho_1}, t_{\varrho_2}), \ldots, [t_{\varrho_{m_\varrho-1}}, t_{\varrho_{m_\varrho}})$, with $t_{\varrho_0} = t_\varrho, t_{\varrho_{m_\varrho}} = t_{\varrho+1}$, satisfying $t_{\varrho_{k+1}} - t_{\varrho_k} \ge T_2, 0 \le k \le m_\varrho - 1$ for some integer $m_\varrho \ge 0$ and constants $T_1 > 0, T_2 > 0$, such that the topology $\mathcal{G}_\sigma$ switches at t_{ϱ_k} and is invariant during each subinterval $[t_{\varrho_k}, t_{\varrho_{k+1}})$. Obviously, there is at most $\gamma = \lceil T_1/T_2 \rceil$ subintervals in each interval $[t_\varrho, t_{\varrho+1})$. Hence there are at most γ graphs, denoted by $\{\mathcal{G}_{\varrho 1}, \mathcal{G}_{\varrho 2}, \ldots, \mathcal{G}_{\varrho\gamma}\} \subset \overline{\mathcal{G}}$ in each time interval $[t_\varrho, t_{\varrho+1})$. Note that if conditions (i)-(iii) hold, then matrix $M_{\varrho l} = [m_{ij}], l = 1, 2, \ldots, \gamma$, is nonnegative and $m_{ii}(k) > 0$. And all the row sums of matrix $M_{\varrho l}$ are equal to 1. Because the union graph contains a spanning tree in each interval $[t_\varrho, t_{\varrho+1}), \varrho = 0, 1, 2, \ldots$, the union of graph $\{\mathcal{G}_{\varrho 1}, \mathcal{G}_{\varrho 2}, \ldots, \mathcal{G}_{\varrho\gamma}\} \subset \overline{\mathcal{G}}$ contains a spanning tree. Note that matrix $\widetilde{M} = M_{\varrho\gamma} \cdots M_{\varrho 2} M_{\varrho 1}$ is SIA from Lemma 3.9 in [9]. According to Lemma 1, matrix $\widetilde{M}$ has an eigenvalue 1, which is the unique eigenvalue with the maximum modulus of $\widetilde{M}$. Hence all the other eigenvalues of $\widetilde{M}$ satisfy $|\lambda(\widetilde{M})| < 1$, where $\lambda(\widetilde{M})$ refers to any eigenvalue of $\widetilde{M}$ besides the eigenvalue 1. Based on Lemma 2, matrix $\widetilde{M}$ is SIA. The left proof of Theorem 5 is similar to that of Theorem 4. To save space it is omitted here.

From the above analysis, systems (1)-(2) can achieve consensus asymptotically if and only if the union graph of the

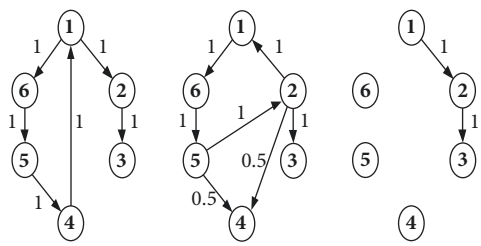

Figure 1: Graph $\mathcal{G}_1$, $\mathcal{G}_2$, and $\mathcal{G}_3$ in sequence.

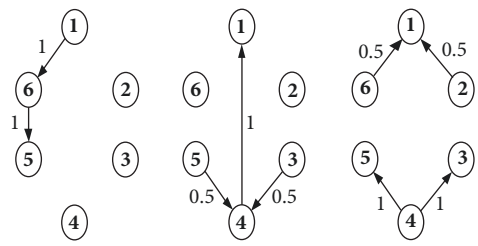

Figure 2: Graph $\mathcal{G}_4$, $\mathcal{G}_5$, and $\mathcal{G}_6$ in sequence.

jointed networks contains a spanning tree. This completes the proof. □

Remark 6. The advantage of the method in this work is adopting the impulse control method to solve the continuous time consensus problems. The impulse control technique requires much less information of the multiple agents than the usual method [29–32]. Accordingly, it greatly reduces the control cost in the engineering applications.

Remark 7. Note that when $n_1 = N$, systems (1)-(2) reduce to the single-order multiagent systems. And when $n_1 = 0$, systems (1)-(2) reduce to the general second-order systems. Hence the first-order and second-order multiagent systems could be regarded as the special cases of the considered mixed-order systems in this work. That is, the results in this paper are the generalization of the existing consensus results of the first-order and second-order multiagent systems.

4. Numerical Simulation Examples

To verify the correctness of the main results, some numerical examples are given in the following.

Example 1. Consider a mixed-order multiagent system containing six agents, that is, agent i, $i = 1, 2, 3, 4, 5, 6$. Without loss of generality, suppose that agents 1 and 2 are first ordered, and the remaining four agents are second ordered. The initial states of the agents are $p(0) = (-12, 12, 22, -22, 50, -50)^T$ and $q(0) = (-25, -11, 11, 25)^T$. The network graphs of the system $\mathcal{G}_k$, $k = 1, 2, 3, 4, 5, 6$, are given in Figures 1 and 2.

From Figures 1 and 2 one can find that graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ contain spanning trees, and the union of graphs $\mathcal{G}_3$, $\mathcal{G}_4$, and $\mathcal{G}_5$ contains a spanning tree, while the union of graphs $\mathcal{G}_3$, $\mathcal{G}_4$, and $\mathcal{G}_6$ contains no spanning tree. Note that, under

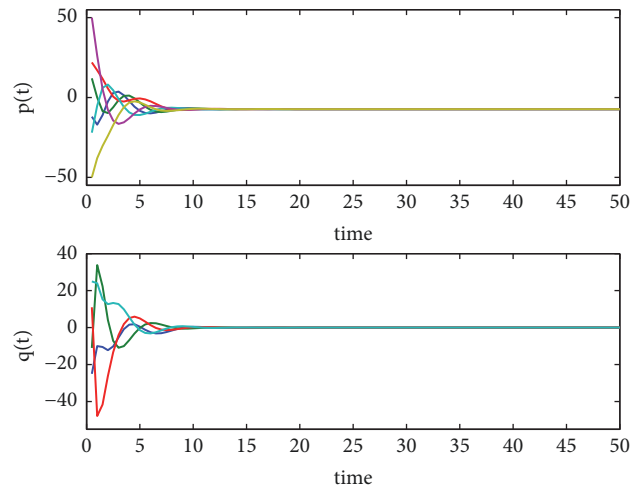

Figure 3: Agents' state trajectories over network $\mathcal{G}_1$.

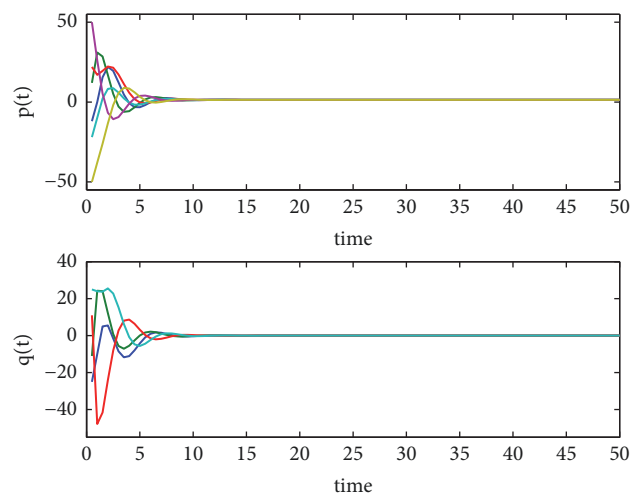

Figure 4: Agents' state trajectories over network $\mathcal{G}_2$.

protocols (4) and (5) with $c_1 = 0.5$, $c_2 = 0.8$, $h = 0.5$, conditions (i), (ii), and (iii) in Theorems 4 and 5 hold. The state trajectories of the agents over networks $\mathcal{G}_1$ and $\mathcal{G}_2$ are given in Figures 3 and 4, respectively, which show that all the position state trajectories converge together and velocity state trajectories converge to zero. The state trajectories over networks switching among $\mathcal{G}_3$, $\mathcal{G}_4$, and $\mathcal{G}_5$ are given in Figure 5, and the state trajectories over networks switching among $\mathcal{G}_3$, $\mathcal{G}_4$, and $\mathcal{G}_6$ are given in Figure 6. Figure 5 illustrates that the position and velocity state trajectories can converge together when the union network graph contains a spanning tree, while Figure 6 illustrates that the state trajectories cannot converge together when the union network graph contains no spanning tree. The simulation results verify the correctness of the main results in this work.

Remark 8. From the numerical simulation results, which have not been given in the paper because of the limited space, under the conditions in Theorems 4 and 5, the control

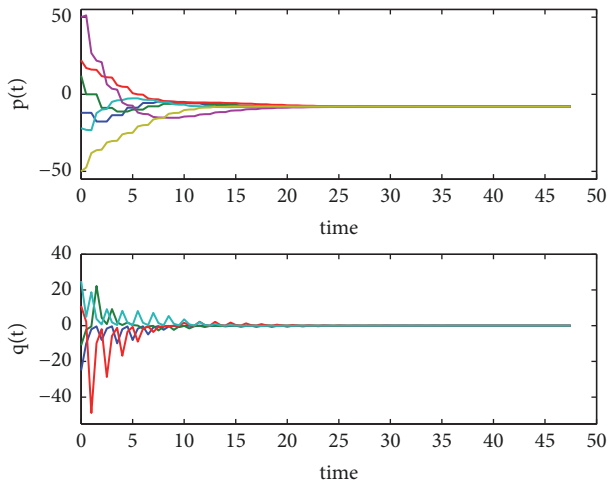

Figure 5: Agents' state trajectories over networks switching among $\mathcal{G}_3$, $\mathcal{G}_4$, and $\mathcal{G}_5$.

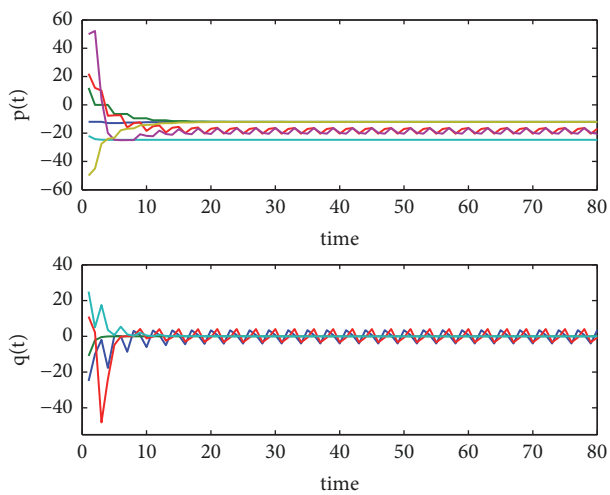

Figure 6: Agents' state trajectories over networks switching among $\mathcal{G}_3$, $\mathcal{G}_4$, and $\mathcal{G}_6$.

gains c_1 and c_2 affect the convergence speed, but it is not monotonically increasing or decreasing as the control gains increase.

5. Conclusions

This work studies the consensus problem for the mixed-order multiagent systems over the jointly connected topologies. By adopting the graph theory, matrix theory, and control theory, impulse consensus protocols are designed and analyzed for the mixed-order multiagent systems. Some simulation examples are given to verify the correctness of the main results and the effectiveness of the control method. However, the related topics of the systems in the turbulent, noisy, or some other uncertainty case have not been considered. The related problems will be studied in the future work.

Data Availability

No data were used to support this study.

Consent

Informed consent was obtained from all individual participants included in the study.

Disclosure

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest. The authors also confirm that the mentioned received funding did not lead to any conflicts of interest regarding the publication of this manuscript.

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