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Research Article

Sign-Consensus of Linear Multiagent Systems under a State Observer Protocol

Wei-Lu Diao and Cui-Qin Ma

School of Mathematical Sciences, Qufu Normal University, Qufu 273165, Shandong, China

Correspondence should be addressed to Cui-Qin Ma; cuiqinma@amss.ac.cn

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The sign-consensus problem for linear time-invariant systems under signed digraph is considered. The information of the agents' states is reconstructed, and then, a state observer-type sign-consensus protocol is proposed, whose performance is analyzed using matrix analysis and ordinary differential equation theory. Sufficient conditions for ensuring sign-consensus are given. It is proven that if the adjacency matrix of the signed digraph has strong Perron–Frobenius property or is eventually positive, sign-consensus can be achieved under the proposed protocol. In particular, conventional consensus is a special case of sign-consensus under mild conditions.

1. Introduction

Consensus, as the key to coordination of multiagent systems (MASs), has been investigated extensively in recent years [1–9]. Most existing studies on consensus of MASs usually assume cooperative interactions among the agents, while in many cases, the agents can not only cooperate but also compete with each other, resulting in the coexistence of cooperation and competition in MASs, e.g., the two-party political system and the business alliance of competitors. To describe this scenario, the concept of bipartite consensus is proposed in [10], which means that all agents converge to the same value as required by conventional consensus but with different signs in bipartite consensus. Altafini [10] proves that, under the assumption of signed digraph being strongly connected, single-integrator MASs reach bipartite consensus if and only if the signed digraph is structurally balanced. Following this line, the strongly connected requirement on signed digraph in [10] is relaxed in [11], bipartite consensus under switched signed digraph is investigated in [12], and bipartite consensus for general linear MASs has also been studied in [13]. In [14–18], measurement noise is further considered. As pointed out in [15], the signed digraph being structurally balanced is one of the necessary and sufficient communication conditions to guarantee bipartite consensus regardless of measurement noise. This means that structural balance plays a crucial role in bipartite consensus. Structural balance is fragile and can be easily broken by changing the sign of some edge weight or by adding or deleting an edge in a signed digraph, meaning that structural unbalance can be more often seen in practice.

In fact, studies on structurally unbalanced signed digraph have already been reported in [14, 15, 19–24], etc. In [14, 15], agents converge to zero in mean square under arbitrary initial conditions. In [19], the agents' states are proved to lie in between the polarized values, which are called the interval bipartite consensus. Another topic closely related to structural unbalance is the unanimity of opinion [21]; that is, all agents achieve an agreement or disagreement on a certain subject but with different extents. This concept is further extended in [23] to sign-consensus, meaning that agents reach values with the same sign but different modulus. The concept of sign-consensus is rooted in reality. For example, in a social network, people rarely have the same comment on a topic, but may have the same tendency. In the pioneering work [23], it is shown that, for the linear time-invariant (LTI) MASs, if the graph adjacency matrix is eventually positive, sign-consensus is achieved under a state feedback sign-consensus protocol and a fully distributed sign-consensus protocol, respectively. The fixed signed digraph in [23] is extended in [24] to being switched over time. Sign-consensus protocols are proposed for three types of time-varying graphs, respectively. It is shown that sign-consensus can be reached if the graph adjacency matrix is frequently eventually positive. We notice that both works [23, 24] propose state feedback-type sign-consensus protocols. However, in reality, due to constraints on measurement, it is usually hard to directly measure agents' states, but only the state estimates are available. Hence, it would be more convenient to synthesize consensus protocols based on the agents' state estimates.

With these observations, in this work, we investigate sign-consensus for LTI MASs under signed digraphs. The agents' states are reconstructed, and state observer-type protocols based on them are given. By using tools from matrix analysis and ordinary differential equation theory, the closed-loop system is analyzed. It is shown that if the graph adjacency matrix has strong Perron–Frobenius property or is eventually positive, sign-consensus for LTI MASs can be achieved. Our main contributions are as follows. First, the agent dynamics are extended to be LTI system, not limited to integrators [21, 22]. Second, state observer-type consensus protocols based on state estimates between neighboring agents are proposed, which are more practical than most existing protocols based on the agents' states [23, 24].

1.1. Organization. The state observer-type consensus protocol is proposed in Section 2. The main results with respect to sign-consensus are given in Section 3. A simulation example is given to verify the proposed theoretical results in Section 4. The paper is concluded in Section 5.

1.2. Notations and Preliminaries. For a matrix or vector A, if all its elements are positive, A is said to be positive, denoted as $A > 0$. For $A \in \mathbb{R}^{n\times n}$, $\lambda_i(A)\,(i = 1,\dots,n)$ is the eigenvalue of A, $\mathrm{Re}(\lambda_i(A))$ is the real part of $\lambda_i(A)$, and $\rho(A)$ is the spectral radius, which is the smallest positive real number satisfying $\rho(A) \geq |\lambda_i(A)|, \forall i = 1,\dots,n$. $A \in \mathbb{R}^{n\times n}$ has the strong Perron–Frobenius property if $\rho(A)$ is a simple positive eigenvalue of A, and its corresponding right eigenvector $v_r > 0$. $A \in \mathbb{R}^{n\times n}$ is eventually positive if $\exists l_0 \in \mathbb{N}$ satisfying $A^l > 0$, $\forall l \geq l_0$. For a given vector $v = (v_1,\dots,v_n)^T \in \mathbb{R}^n$, $\mathrm{sign}(v) = (\mathrm{sign}(v_1),\dots,\mathrm{sign}(v_n))^T$, where $\mathrm{sign}(\,)$ is the sign function. $1_n = (1,\dots,1)^T \in \mathbb{R}^n$, $\otimes$ is the Kronecker product.

$\mathcal{G} = (\mathcal{V},\varepsilon,\mathcal{F})$ is a signed digraph with $\mathcal{V} = \{1,\dots,N\}$, $\varepsilon \subset \mathcal{V}\times\mathcal{V}$, and the adjacency matrix $\mathcal{F} = (f_{ij}) \in \mathbb{R}^{N\times N}$. $\mathcal{N}_i = \{(j,i) \in \mathcal{E}\}$. Generally, $\mathcal{F} = (f_{ij}) \in \mathbb{R}^{N\times N}$ is defined by $f_{ii} = 0$, $f_{ij} \neq 0 \Longleftrightarrow (j,i) \in \varepsilon$; otherwise, $f_{ij} = 0$. Denote $D_{\mathcal{F}} = \rho(\mathcal{F})I_N - \mathcal{F}$ with $\rho(\mathcal{F})$ being the spectral radius of $\mathcal{F}$.

Lemma 1 (see [23]). *Assume $\mathcal{G} = (\mathcal{V},\varepsilon,\mathcal{F})$ is a signed digraph. If $\mathcal{F}$ is eventually positive, then $\mathcal{G}$ is structurally unbalanced.*

Lemma 2 (see [25]). *For $A \in \mathbb{R}^{n\times n}$, A has the strong Perron–Frobenius property $\Longleftrightarrow$ A is eventually positive.*

2. Problem Formulation

Consider an MAS with N agents with each agent being described by

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i, \\ y_i &= Cx_i, \quad \forall i = 1,\dots,N, \end{aligned} \tag{1}$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^m$, and $y_i \in \mathbb{R}^q$ are the state, input, and output, respectively. It is assumed that (A,B,C) is controllable and observable.

The communication topology among agents is represented by a signed digraph $\mathcal{G} = (\mathcal{V},\varepsilon,\mathcal{F})$, where $\mathcal{V} = \{1,\dots,N\}$ and $\mathcal{F} = (f_{ij})_{N\times N}$. This paper is to synthesize a state observer-type control for each agent such that agents in (1) reach sign-consensus. Firstly, we give the definition of sign-consensus for the system in (1).

Definition 1 (see [24]). The system in (1) is said to achieve sign-consensus if there exists a consensus protocol $\{u_i, i = 1,\dots,N\}$ such that for any given initial states $x_i(0)\,(i = 1,\dots,N)$,

$$\lim_{t\longrightarrow\infty}\left(\mathrm{sign}(x_i(t)) - \mathrm{sign}(x_j(t))\right) = 0, \quad i, j \in \{1,\dots,N\}. \tag{2}$$

From Definition 1, given any initial value, the states of agents in (1) converge to values with the same sign but different modulus.

Let $\overline{x}_i$ and $\overline{y}_i$ be the estimates of agent i's state and output, respectively. Denote $\widehat{y}_i = y_i - \overline{y}_i = Cx_i - C\overline{x}_i$ as agent i's output estimate error. In practice, it is usually hard to measure the agent's state. Hence, it would be more convenient to design consensus protocols based on the estimates of the agents' states. The useful information for agent i is synthesized by other agents' state estimates as

$$\overline{\delta_i} = -\sigma_i\overline{x}_i + \sum_{j\in N_i} f_{ij}\overline{x}_j, \quad i = 1,\dots,N, \tag{3}$$

where $\sigma_i > 0$ denotes the degradation rate of the system. A state observer-type consensus protocol is given as follows:

$$\begin{aligned} u_i &= aH\overline{\delta_i} = aH\left(-\sigma_i\overline{x}_i + \sum_{j\in N_i} f_{ij}\overline{x}_j\right), \\ \dot{\overline{x}}_i &= A\overline{x}_i + Bu_i - aG\widehat{y}_i, \quad i = 1,\dots,N, \end{aligned} \tag{4}$$

where

(i) $a \in \mathbb{R}$ satisfies $a \geq 1/(2\min_{i\in\Delta}\mathrm{Re}(\lambda_i(D_{\mathcal{F}})))$ with $\Delta = \{i \mid \mathrm{Re}(\lambda_i(D_{\mathcal{F}})) > 0, i = 1,\dots,N\}$;

(ii) H is designed as $H = R^{-1}B^TP$, where P is the positive definite solution of the algebraic Riccati equation

$$A^TP + PA + Q - PBR^{-1}B^TP = 0, \tag{5}$$

where Q and R are positive definite matrices;

(iii) Since (A,C) is observable, G can be chosen such that $A + aGC$ is Hurwitz.

Next, we will demonstrate that the system in (1) achieves sign-consensus under the protocol in (4).

3. Main Results

Theorem 1. *Suppose that $\mathcal{F}$ has the strong Perron–Frobenius property. Then, the system in (1) can reach sign-consensus under the protocol in (4) with $\sigma_i = \rho(\mathcal{F})$ ($\forall i = 1, \ldots, N$). Moreover, the state estimate error will converge to 0, i.e., $\lim_{t\longrightarrow\infty}(x_i - \overline{x}_i) = 0$ and $\forall i = 1, \ldots, N$.*

Proof. Define $X = [x_1^T, \ldots, x_N^T]^T$ and $\overline{X} = [\overline{x}_1^T, \ldots, \overline{x}_N^T]^T$. Applying the protocol in (4) to the system in (1), we obtain

$$\dot{X} = (I_N \otimes A)X - (aD_{\mathcal{F}} \otimes BH)\overline{X}. \tag{6}$$

By direct calculation, we obtain

$$\dot{\overline{X}} = (I_N \otimes A - aD_{\mathcal{F}} \otimes BH)\overline{X} - (aI_N \otimes GC)\widehat{X}, \tag{7}$$

where $\widehat{X} = X - \overline{X}$. Clearly,

$$\dot{\widehat{X}} = [I_N \otimes (A + aGC)]\widehat{X}. \tag{8}$$

Since G is chosen such that $A + aGC$ is Hurwitz, $I_N \otimes (A + aGC)$ is Hurwitz. Immediately, one has

$$\begin{gathered}\lim_{t\longrightarrow\infty} \widehat{X} = 0, \\ \text{i.e., } \lim_{t\longrightarrow\infty} (x_i - \overline{x}_i) = 0, \quad i = 1, \ldots, N.\end{gathered} \tag{9}$$

By assumption, $\mathcal{F}$ has the strong Perron–Frobenius property. This implies that $\rho(\mathcal{F})$ is a simple positive eigenvalue of $\mathcal{F}$, and its corresponding right eigenvector $v_r > 0$. Noting that $D_{\mathcal{F}} = \rho(\mathcal{F})I_N - \mathcal{F}$. Hence, $\lambda_i(D_{\mathcal{F}}) = \rho(\mathcal{F}) - \lambda_i(\mathcal{F})$, $i = 1, \ldots, N$. Clearly, 0 is a simple eigenvalue of $D_{\mathcal{F}}$ with corresponding right eigenvector v_r. For simplicity, we assume $\lambda_1(D_{\mathcal{F}}) = 0$. Then, $\mathrm{Re}(\lambda_i(D_{\mathcal{F}})) > 0, i = 2, \ldots, N$. There must exist a nonsingular matrix $\mathcal{S} = [v_r | S_2] \in \mathbb{R}^{N\times N}$ with $S_2 \in \mathbb{R}^{N\times(N-1)}$ such that

$$\mathcal{S}^{-1} D_{\mathcal{F}} \mathcal{S} = J = \begin{bmatrix} 0 & 0 \\ 0 & J_{N-1} \end{bmatrix}, \tag{10}$$

where $J_{N-1} \in \mathbb{R}^{(N-1)\times(N-1)}$ is a Jordan block with $\lambda_i(D_{\mathcal{F}})(i = 2, \ldots, N)$ on its diagonal. Let $W(t) = (\mathcal{S}^{-1} \otimes I_n) X \triangleq [W_1^T(t), \ldots, W_N^T(t)]^T = [W_1^T(t) \,|\, \Omega^T(t)]^T$ and $\widehat{W}(t) = (\mathcal{S}^{-1} \otimes I_n)\widehat{X} \triangleq [\widehat{W}_1^T(t) [\widehat{\Omega}^T](t)]^T$. Then, by (6)–(8), we have

$$\dot{W}(t) = (I_N \otimes A - aJ \otimes BH)W(t) + (aJ \otimes BH)\widehat{W}(t), \tag{11}$$

or equivalently,

$$\begin{aligned}&\dot{W}_1(t) = AW_1(t), \\ &\dot{\Omega}(t) = (I_{N-1} \otimes A - aJ_{N-1} \otimes BH)\Omega(t) + (aJ_{N-1} \otimes BH)\widehat{\Omega}(t).\end{aligned} \tag{12}$$

Obviously, $I_{N-1} \otimes A - aJ_{N-1} \otimes BH$ is an upper triangle block matrix with diagonal blocks $A - a\lambda_i(D_{\mathcal{F}})BH$, $i = 2, \ldots, N$. Notice that $a \geq 1/(2\min_{i\in\Delta}\mathrm{Re}(\lambda_i(D_{\mathcal{F}})))$ and $H = R^{-1}B^T P$, where P is positive definite solution of (5). Then, by adopting the similar arguments as in [26], we obtain that $A - a\lambda_i(D_{\mathcal{F}})BH\,(i = 2, \ldots, N)$ is Hurwitz. By (9), we know that $\lim_{t\longrightarrow\infty}\widehat{W}(t) = \lim_{t\longrightarrow\infty}(\mathcal{S}^{-1} \otimes I_n)\widehat{X} = 0$, and hence, $\lim_{t\longrightarrow\infty}\widehat{\Omega}(t) = 0$. According to (12) and the ordinary differential equation theory, we obtain

$$\begin{aligned}&\lim_{t\longrightarrow\infty} W_1(t) = \lim_{t\longrightarrow\infty} e^{At} W_1(0), \\ &\lim_{t\longrightarrow\infty} \Omega(t) = 0.\end{aligned} \tag{13}$$

Since $X = (\mathcal{S} \otimes I_n)W(t) = (v_r \otimes I_n)W_1(t) + (S_2 \otimes I_n)\Omega(t)$,

$$\lim_{t\longrightarrow\infty} X(t) = (v_r \otimes I_n) \lim_{t\longrightarrow\infty} e^{At} W_1(0), \tag{14}$$

or equivalently,

$$\lim_{t\longrightarrow\infty} x_i(t) = v_{r_i} \lim_{t\longrightarrow\infty} e^{At} W_1(0), \tag{15}$$

where v_{r_i} is the ith component of v_r. Note that $v_{r_i} > 0$, $\forall i = 1, \ldots, N$. Thus, by Definition 1, the system in (1) can achieve sign-consensus under the protocol in (4). □

Remark 1. In Theorem 1, sign-consensus is achieved based on a state observer-type protocol. This is different from protocols in [23, 24] where state feedback protocols are proposed. Communication topology $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{F})$ is directed, and it is assumed that $\mathcal{F}$ has the strong Perron–Frobenius property. Such an assumption together with Lemma 1 implies that $\mathcal{G}$ is structurally unbalanced.

Theorem 2. *For the system in (1), suppose that $\mathcal{F}$ is eventually positive. Then, the system in (1) can achieve sign-consensus under the protocol in (4) with $\sigma_i = \rho(\mathcal{F})$ ($\forall i = 1, \ldots, N$).*

Proof. By assumption, $\mathcal{F}$ is eventually positive. This together with Lemma 2 gives that $\mathcal{F}$ has the strong Perron–Frobenius property. Applying the same procedure as in Theorem 1, one sees that the system in (1) can achieve sign-consensus under the protocol in (4).

From (14), we know that v_r is vital to the collective behavior of MASs. In particular, if $v_r = 1_n$, then conventional consensus is achieved. □

Corollary 1. *For the system in (1), suppose that $\mathcal{F}$ has the strong Perron–Frobenius property. Then, the system in (1) can achieve conventional consensus under the protocol in (4) with $v_r = 1_n$ and $\sigma_i = \rho(\mathcal{F})(\forall i = 1, \ldots, N)$.*

Remark 2. In Corollary 1, if the requirement on $\mathcal{F}$ is changed to being eventually positive, Corollary 1 still holds.

4. Simulation

Example 1. Consider the system in (1) with 6 agents, where

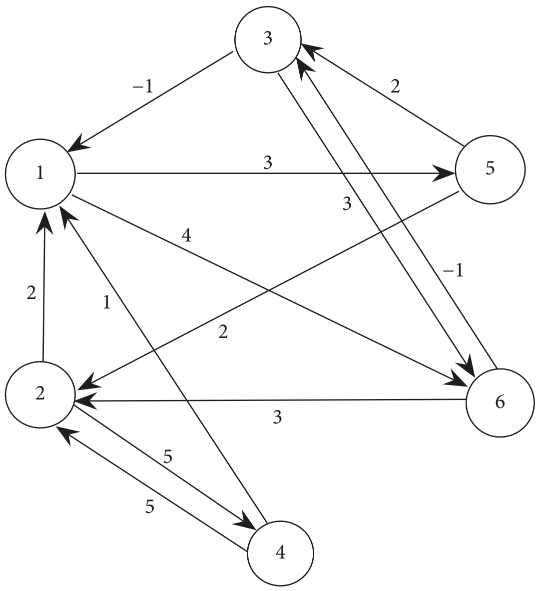

Figure 1: Communication topology among 6 agents.

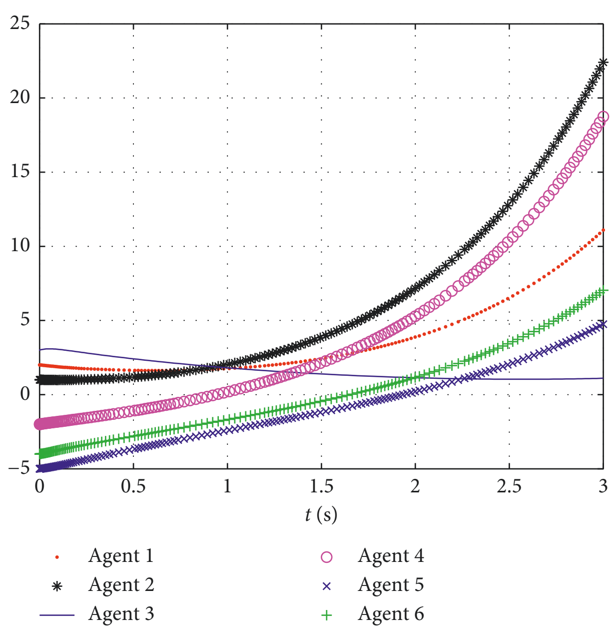

Figure 2: State trajectories of x_{i1} for 6 agents.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \tag{16}$$
$$C = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}.$$

Obviously, (A, B, C) is controllable and observable. Assume that interactions among the 6 agents are represented by $\mathcal{G} = (\mathcal{V}, \varepsilon, \mathcal{F})$, where $\mathcal{V} = \{1, \ldots, 6\}$ and $\mathcal{F} = (f_{ij}) \in \mathbb{R}^{6\times 6}$ with $f_{12} = f_{25} = f_{35} = 2$, $f_{13} = f_{36} = -1$, $f_{14} = 1$, $f_{24} = f_{42} = 5$, $f_{26} = f_{51} = f_{63} = 3$, and $f_{61} = 4$. From Figure 1,

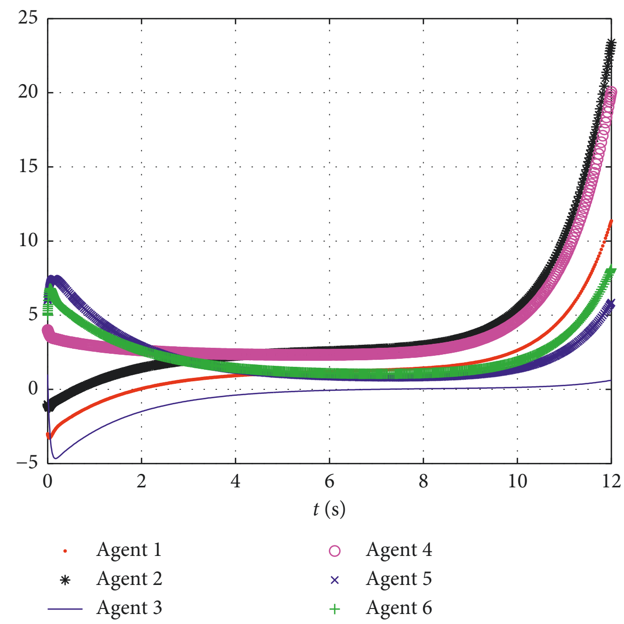

Figure 3: State trajectories of x_{i2} for 6 agents.

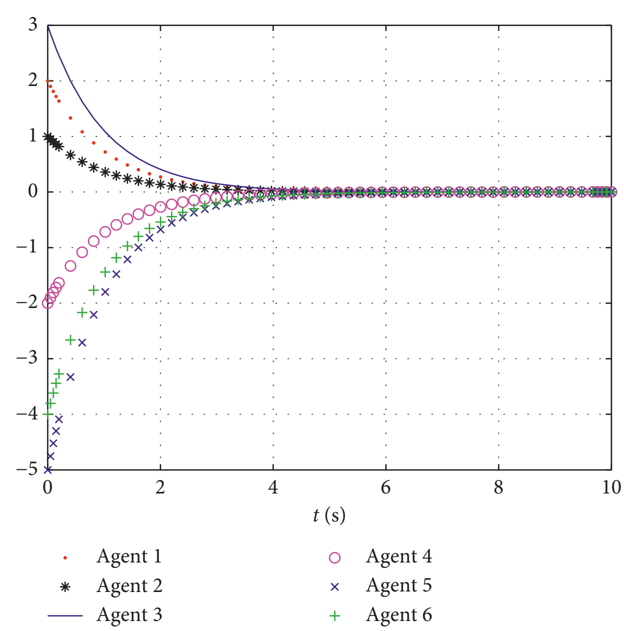

Figure 4: State estimate error trajectories of x_{i1}.

$\mathcal{G}$ is structurally unbalanced. By direct calculation, one has $\mathcal{F}^l > 0$, $\forall l \geq 7$. This implies that $\mathcal{F}$ is eventually positive. Therefore, by Lemma 2, one sees that $\mathcal{F}$ has strong Perron–Frobenius property, and hence, communication conditions in Theorem 1 and Theorem 2 are satisfied.

Note that $\lambda_1(D_{\mathcal{F}}) = 0$, $\lambda_2(D_{\mathcal{F}}) = 10.4347$, $\lambda_3(D_{\mathcal{F}}) = 6.7544 - 1.4418j\,(j^2 = -1)$, $\lambda_4(D_{\mathcal{F}}) = 6.7544 + 1.4418j$, $\lambda_5(D_{\mathcal{F}}) = 5.5158 - 2.1144j$, and $\lambda_6(D_{\mathcal{F}}) = 5.5158 + 2.1144j$. Therefore, $a \geq 1/(2\min_{2\leq i\leq 6}\mathrm{Re}(\lambda_i(D_{\mathcal{F}}))) = 0.0906$, and hence, we assume $a = 1$. Let $G = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$. By calculation, one obtains that $A + aGC$ is Hurwitz. Choose $Q = CC^T$ and

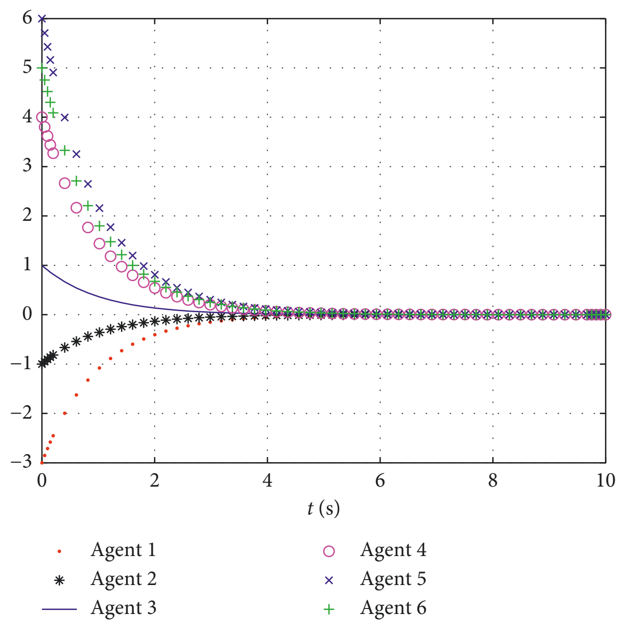

Figure 5: State estimate error trajectories of x_{i2}.

$R = 1$. Then, one can get $P = \begin{bmatrix} 12.3485 & 5.4495 \\ 5.4495 & 3.4495 \end{bmatrix}$ which is the positive definite solution of (5), and $H = R^{-1}B^T P = [5.4495\ 3.4495] \in \mathbb{R}^{1\times 2}$. Applying the protocol in (4) to the system in (1), we obtain that sign-consensus is achieved. The simulation results are given in Figures 2 and 3, where x_{i1} and x_{i2} are the first and second components of agent i, respectively. Moreover, from Figures 4 and 5, we see that state estimate error converges to zero. Thus, the simulation example verifies the validity of Theorem 1 and Theorem 2.

5. Conclusions

The sign-consensus of MASs is investigated where each agent is described by an LTI system. A state observer-type protocol is designed, which is more practical than the usual state feedback protocols. It is shown that if the adjacency matrix has strong Perron–Frobenius property or is eventually positive, then sign-consensus can be achieved based on the proposed protocols.

Data Availability

The MATLAB code used in the example can be obtained from the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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