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Research Article

Fuzzy Emergency Model and Robust Emergency Strategy of Supply Chain System under Random Supply Disruptions

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For random distributors under supply disruptions caused by emergency incidents, a fuzzy emergency model and a robust emergency strategy of the supply chain system are studied. First, for a kind of supply chain system composed of a strategic manufacturer, a backup manufacturer, and multiple distributors, the basic emergency models, including the inventory models and a total cost model, are constructed under random supply disruptions. Then, based on the Takagi-Sugeno fuzzy system, the basic emergency models of the supply chain system are converted into a discrete switching model, which can realize soft switching among the basic emergency models. Furthermore, according to the different inventory levels, the strategic manufacturer's production strategies and the distributors' ordering strategies are designed to reduce the inventory costs of the node enterprises in supply chain system. Second, by defining a discrete piecewise Lyapunov function in each maximal overlapped-rules group, a new fuzzy robust emergency strategy for the supply chain system is proposed through the principle of parallel distributed compensation. This emergency strategy can not only restore the impaired supply chain to the normal operation state but also keep the total cost of the supply chain at a low level and guarantee the robust stability of the emergency supply chain system. Finally, the simulation results illustrate the effectiveness of the proposed fuzzy robust emergency strategy of the supply chain system.

1. Introduction

Because it is difficult to predict emergency incidents including human-caused accidents (e.g., forest fires, terrorist attacks, and production accidents) and acts of God (e.g., earthquakes, hurricanes, tsunamis, and tornadoes), the occurrence of an emergency incident will cause great damage to the supply chain system, such as supply disruption. If supply disruption can not be resolved in a timely and effective manner, more serious consequences such as the collapse of supply chain will occur. Therefore, for the supply chain under supply disruption caused by emergency incident, through the integration of internal and external resources of the supply chain, the emergency strategy should be formed to solve the problem of supply disruption and enhance the ability of risk control for supply disruption.

The aim of this paper is to construct an emergency model of supply chain system under random supply disruptions and design a robust emergency strategy in order to realize the distributors suffered supply disruptions can continue to satisfy customers' demands and guarantee the stable operation of the supply chain system. The rest of this paper is organized as follows. Section 2 reviews related literature. The emergency models of the supply chain system under random supply disruptions are constructed in Section 3. Section 4 proposes a fuzzy robust emergency strategy for the supply chain system under random supply disruptions in the form of Theorems, and the proofs of Theorems are in the Appendix. Section 5 provides a simulation example to verify the effectiveness of the proposed fuzzy robust emergency strategy. The conclusions are presented in Section 6.

2. Literature Review

In recent years, scholars and practitioners have devoted study to the emergency management issues of supply chain [1–9]. Fereiduni and Shahanaghi [1] presented a multiperiod model for the blood supply chain in an emergency situation to optimize decisions related to locating blood facilities and distributing blood products after natural disasters. Li [5] set up an evaluation system for responding capacity of the emergency supply chain. Ma [9] developed an emergency financial service supply chain for natural disaster risks to obtain insights into the cooperative and competitive relationship in the government-market-public partnership system.

Supply chain tends to break down when supply disruptions occur. To restrain the adverse impact of supply disruption on the supply chain performance, supply disruption has received considerable managerial attentions [10]. Giri and Bardhan [11] proposed contract mechanisms to coordinate the centralized and decentralized supply chains under supply disruption. Iakovou et al. [12] designed a stochastic inventory decision-making model for supply chain under supply disruptions and evaluated the merit of contingency strategies in managing supply chain uncertainty. Hou and Zhao [13] made a backup agreement with a penalty scheme between retailers and backup suppliers to mitigate the impact of supply disruption. In the context of supply chain system under supply disruptions, He et al. [14] worked out an emergency procurement strategy to manage supply disruption.

Furthermore, to improve the robustness of the emergency supply chain under supply disruptions, Sawik [15] addressed the robust decision-making issue for a customer-driven supply chain under supply disruptions to ensure the equitably efficient supply chain performance. Tang [16] developed robust supply chain strategies that are able to efficiently control the inherent fluctuations and make the supply chain under supply disruptions become more resilient. Bai and Liu [17] built an optimization model involving various possible supply disruptions to make tools available for producers to develop a robust supply chain network design. Exiting literature [18–20] has designed robust supply chain networks against stochastic demands and supply disruptions.

When supply disruptions occur in the supply chain with information sharing, distributors under supply disruptions will adopt corresponding emergency ordering strategies after comprehensively investigating other distributors' inventory levels. Thus, there are various possibilities in emergency ordering strategies. Different emergency strategies will form different basic emergency models, and each basic emergency model in the supply chain system can be regarded as an emergency subsystem. Then, switching activities will occur among emergency subsystems to maintain the total operating cost of the emergency supply chain at an ideal level. Moreover, Disturbance factors involve switching activities and uncertain customers' demands that exacerbate the fluctuations of the variables in the emergency supply chain system.

However, scholars [11–20] did not consider how distributors deliver goods to each other when the supply chain suffers supply disruptions. Furthermore, current studies [11–20] did not explore the existing switching activities among emergency subsystems.

A fuzzy control model has been set up as well as the related robust issue referred to the nonlinear supply chain system with lead times has been handled by Zhang et al. [21]. Benchmarking to the similar model and approach in [21], the main contributions of this study and the significant differences are as follows.

(1) A Kind of Fuzzy Emergency Model of Supply Chain System Is Constructed under Random Supply Disruptions. This paper focuses on the supply chain under supply disruptions rather than the supply chain with lead times in [21] to build a fuzzy emergency model of supply chain. The established fuzzy emergency model realizes soft switching among the different emergency subsystems. Each variable involved in the soft switching process is an asymptotic change rather than a step change.

(2) Distributors' Ordering Strategies Are Developed. This research focuses on distributors' different ordering objects rather than lead time compression in [21] to develop distributors' ordering strategies. Distributors under supply disruptions are able to order goods from the backup manufacturers and/or other distributors depending on each distributor's inventory level.

(3) A Fuzzy Robust Emergency Strategy Is Outlined. Researchers focus on the impact of supply disruptions on supply chain rather than the impact of lead time on supply chain in [21] to develop a fuzzy robust emergency strategy. Under the developed fuzzy robust emergency strategy, both supply disruption issue can be repaired and the robust stability of the emergency supply chain system under supply disruptions is also obtained.

3. Model Construction

3.1. Basic Emergency Models of Supply Chain System under Random Supply Disruptions. This study considers a two-echelon supply chain system composed of a strategic manufacturer, J distributors, and a backup manufacturer. When supply disruptions occur, to continually satisfy the customers' demands, the distributors not supplied by the strategic manufacturer can order goods from the backup manufacturer, other distributors supplied by a strategic manufacturer, or both at the same time.

It assumes that Distributor a represents the distributor that can not be supplied by the strategic manufacturer, $a = 1, 2, \ldots, L$, and Distributor b represents the distributor that can be supplied by the strategic manufacturer, $b = L + 1, L + 2, \ldots, J$. Then, the emergency supply chain system can be shown in Figure 1.

In Figure 1, $s_0(k)$, $s_a(k)$, and $s_b(k)$ are the strategic manufacturer's inventory level, Distributor a's inventory level, and Distributor b's inventory level at period k, respectively, which are the state variables; $v_0(k)$, $v_a(k)$, and $v_b(k)$ are the strategic manufacturer's production, Distributor a's total ordering quantity and Distributor b's total ordering quantity at period

k, respectively, which are the control variables; $w_a(k)$ and $w_b(k)$ are Distributor a's customers' demands and Distributor b's customers' demands at period k, respectively, which are the uncertain variables; g_0 is the strategic manufacturer's production coefficient ($g_0 = 1$ means that the strategic manufacturer will produce normally; $g_0 = 0$ means that the strategic manufacturer will not produce). $g_{1,b}$, $g_{2,a}$ and $l_{b,a}$ represent the ordering coefficients when Distributor b orders goods from the strategic manufacturer, Distributor a orders goods from the backup manufacturer and Distributor a orders goods from Distributor b, respectively. $g_{2,a}, g_{1,b}, l_{b,a} \in [0, 1]$ and $g_{2,a} + l_{b,a} = 1$.

As Figure 1 shows, the basic inventory status models and the total cost of the emergency supply chain system can be expressed as

$$\begin{aligned} s_0(k+1) &= s_0(k) + g_0 v_0(k) - \sum_{b=L+1}^{J} g_{1,b} v_b(k) \\ s_a(k+1) &= s_a(k) + g_{2,a} v_a(k) + \sum_{b=L+1}^{J} l_{b,a} v_a(k) \\ &\quad - w_a(k) \\ s_b(k+1) &= s_b(k) + g_{1,b} v_b(k) - \sum_{a=1}^{L} l_{b,a} v_a(k) - w_b(k), \end{aligned} \tag{1}$$

$$\begin{aligned} C(k) = {} & c_{n0} s_0(k) + \sum_{a=1}^{L} c_{na} s_a(k) + \sum_{b=L+1}^{J} c_{nb} s_b(k) \\ & + c_{r0} g_0 v_0(k) + \sum_{a=1}^{L} c_{oa} g_{2,a} v_a(k) \\ & + \sum_{b=L+1}^{J} c_{ob} g_{1,b} v_b(k) \\ & + \sum_{a=1}^{L} \sum_{b=L+1}^{J} c_{ba} l_{b,a} v_a(k) \end{aligned} \tag{2}$$

where $C(k)$ is the total cost of the emergency supply chain system; c_{n0}, c_{na}, and c_{nb} are the unit inventory costs of the strategic manufacturer, Distributor a, and Distributor b, respectively; c_{r0} is the strategic manufacturer's unit production cost; c_{oa} is the unit emergency ordering cost when Distributor a orders goods from the backup manufacturer; c_{ob} is the unit ordering cost when Distributor b orders goods from the strategic manufacturer; and c_{ba} is the unit emergency ordering cost when Distributor a orders goods from Distributor b.

Based on system (1) and equation (2), we design the strategic manufacturer's production strategies and the distributors' ordering strategies as follows. (1) If each distributor's inventory level is more than the expected inventory, the strategic manufacturer will not produce in period k; if not, the strategic manufacturer will produce in period k. (2) If Distributor a's inventory level is less than the safety inventory, Distributor a will order goods from both the backup manufacturer and Distributor b, whose inventory level is more than the expected inventory; if Distributor a's inventory level is more than the safety inventory and less than the expected inventory, Distributor a will order goods from Distributor b, whose inventory level is more than the expected inventory; if Distributor a's inventory level is more than the expected inventory, Distributor a will stop ordering goods. (3) If Distributor b's inventory level is less than the expected inventory, Distributor b will order goods from the strategic manufacturer, while Distributor b will not deliver goods to Distributor a. If Distributor b's inventory level is more than the expect inventory, Distributor b will stop ordering goods, while Distributor b will deliver goods to Distributor a, who needs to order goods from Distributor b.

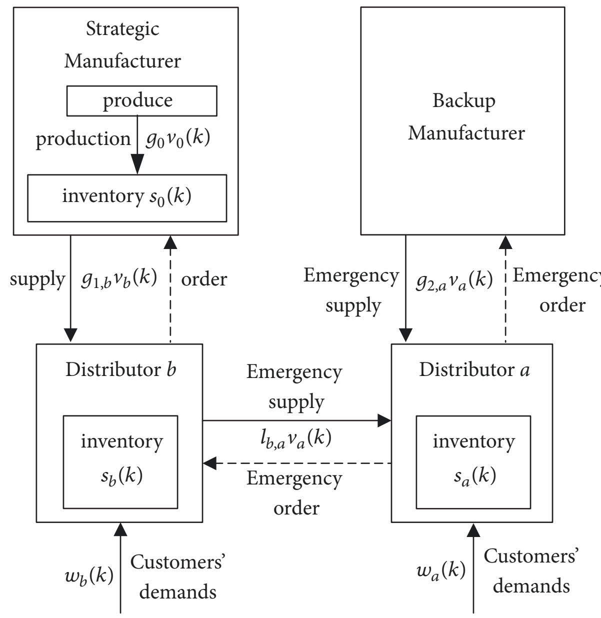

Figure 1: Emergency supply chain system.

Because the strategic manufacturer and the distributors will adopt different production or ordering strategies according to their different inventory statuses, the emergency supply chain includes multiple subsystems. Then, we express the ith emergency subsystem of the supply chain in matrix form as follows:

$$\begin{aligned} \boldsymbol{I}(k+1) &= \boldsymbol{S}_i \boldsymbol{I}(k) + \boldsymbol{R}_i \boldsymbol{O}(k) + \boldsymbol{R}_{wi} \boldsymbol{W}(k) \\ \boldsymbol{C}(k) &= \boldsymbol{T}_i \boldsymbol{I}(k) + \boldsymbol{H}_i \boldsymbol{O}(k), \end{aligned} \tag{3}$$

where $\boldsymbol{I}^{\mathrm{T}}(k) = [s_0(k), s_1(k), \ldots, s_a(k), \ldots s_L(k), s_{L+1}(k), \ldots, s_b(k), \ldots s_J(k)]_{1\times(J+1)}$; $\boldsymbol{O}^{\mathrm{T}}(k) = [v_0(k), v_1(k), \ldots, v_a(k), \ldots v_L(k), v_{L+1}(k), \ldots, v_b(k), \ldots v_J(k)]_{1\times(J+1)}$; $\boldsymbol{W}^{\mathrm{T}}(k) = [0, w_1(k), \ldots, w_a(k), \ldots w_L(k), w_{L+1}(k), \ldots, w_b(k), \ldots w_J(k)]_{1\times(J+1)}$; $\boldsymbol{S}_i = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}_{(J+1)\times(J+1)}$ is the inventory status coefficient matrix; $\boldsymbol{R}_i = \begin{bmatrix} g_0 & \boldsymbol{0} & \boldsymbol{A}_1 \\ \boldsymbol{0} & \boldsymbol{A}_2 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_1 & \boldsymbol{B}_2 \end{bmatrix}_{(J+1)\times(J+1)}$ $(\boldsymbol{A}_1 = [-g_{1,L+1}, \ldots, -g_{1,b}, \ldots, -g_{1,J}],$

$\boldsymbol{A}_2 = \text{diag}[g_{2,1} + \sum_{b=L+1}^{J} l_{b,1}, \ldots, g_{2,a} + \sum_{b=L+1}^{J} l_{b,a}, \ldots, g_{2,L} + \sum_{b=L+1}^{J} l_{b,L}]$, $\boldsymbol{B}_1 = \begin{bmatrix} -l_{L+1,1} & \cdots & -l_{L+1,a} & \cdots & -l_{L+1,L} \\ \vdots & & \vdots & & \vdots \\ -l_{b,1} & \cdots & -l_{b,a} & \cdots & -l_{b,L} \\ \vdots & & \vdots & & \vdots \\ -l_{J,1} & \cdots & -l_{J,a} & \cdots & -l_{J,L} \end{bmatrix}$, $\boldsymbol{B}_2 = \text{diag}[g_{1,L+1}, \ldots, g_{1,b}, \ldots, g_{1,J}])$ is the production and ordering coefficient matrix; $\boldsymbol{T}_i = [c_{n0}, c_{n1}, \ldots, c_{na}, \ldots c_{nL}, c_{n(L+1)}, \ldots, c_{nb}, \ldots c_{nJ}]_{1\times(J+1)}$ is the inventory cost coefficient matrix; $\boldsymbol{H}_i = [c_{r0}, c_{o1}g_{2,1} + \sum_{b=L+1}^{J} c_{b,1}l_{b,1}, \ldots, c_{oa}g_{2,a} + \sum_{b=L+1}^{J} c_{b,a}l_{b,a}, \ldots c_{oL}g_{2,L} + \sum_{b=L+1}^{J} c_{b,L}l_{b,L}, c_{o(L+1)}g_{1,L+1}, \ldots, c_{ob}g_{1,b}, \ldots c_{oJ}g_{1,J}]_{1\times(J+1)}$ is the cost coefficient matrix of the production and ordering.

3.2. Takagi-Sugeno Fuzzy Emergency Model of Supply Chain System under Random Supply Disruptions. To reduce the total cost of the emergency supply chain system, switching activities will occur among the different emergency subsystems of the supply chain. However, the switching activities may cause fluctuations in the emergency supply chain system. The Takagi-Sugeno fuzzy control model can restrain the fluctuations caused by the switching activities and the customers' demands. Therefore, based on the ith emergency subsystem (3), the Takagi-Sugeno fuzzy emergency model of the supply chain system can be described by following fuzzy if-then rules:

R_i: if $s_0(k)$ is M_0^i, $s_1(k)$ is M_1^i,..., $s_j(k)$ is M_j^i, ..., and $s_n(k)$ is M_n^i, then

$$\begin{aligned} \boldsymbol{I}(k+1) &= \boldsymbol{S}_i\boldsymbol{I}(k) + \boldsymbol{R}_i\boldsymbol{O}(k) + \boldsymbol{R}_{wi}\boldsymbol{W}(k) \\ \boldsymbol{C}(k) &= \boldsymbol{T}_i\boldsymbol{I}(k) + \boldsymbol{H}_i\boldsymbol{O}(k) \\ \boldsymbol{I}(k) &= \boldsymbol{\varphi}(k), \end{aligned} \tag{4}$$

where R_i $(i = 1, 2, \ldots, r)$ is the ith fuzzy rule, and r is the number of if-then rules; M_j^i $(j = 0, 1, \ldots, n)$ is the fuzzy set; $\boldsymbol{\varphi}(k)$ is the initial condition of the emergency supply chain system; $k \in \{0, 1, \ldots, N\}$, N is the number of periods. The fuzzy system (4) is a discrete switching model, which describes the different emergency supply chain system in different periods through switching activities among the emergency subsystems.

By singleton fuzzification, product inference and center-average defuzzification, system (4) is inferred as follows:

$$\begin{aligned} \boldsymbol{I}(k+1) &= \sum_{i=1}^{r} h_i(\boldsymbol{I}(k))\left[\boldsymbol{S}_i\boldsymbol{I}(k) + \boldsymbol{R}_i\boldsymbol{O}(k) + \boldsymbol{R}_{wi}\boldsymbol{W}(k)\right] \\ \boldsymbol{C}(k) &= \sum_{i=1}^{r} h_i(\boldsymbol{I}(k))\left[\boldsymbol{T}_i\boldsymbol{I}(k) + \boldsymbol{H}_i\boldsymbol{O}(k)\right], \end{aligned} \tag{5}$$

where $h_i(\boldsymbol{I}(k)) = \mu_i(\boldsymbol{I}(k))/\sum_{i=1}^{r}\mu_i(\boldsymbol{I}(k))$, $\mu_i(\boldsymbol{I}(k)) = \prod_{j=1}^{n} M_j^i(s_j(k))$; $M_j^i(s_j(k))$ is the grade of membership of $s_j(k)$ in the fuzzy set M_j^i. Because $\mu_i(\boldsymbol{I}(k)) \geq 0$, $h_i(\boldsymbol{I}(k)) \geq 0$, and $\sum_{i=1}^{r} h_i(\boldsymbol{I}(k)) = 1$. For simplicity, we abbreviate $h_i(\boldsymbol{I}(k))$ to h_i.

4. Fuzzy Robust Emergency Strategy of Supply Chain System

Before proposing the fuzzy robust emergency strategy of the supply chain system, a parameter γ is introduced to describe the restraint extent of the disturbance factors, which can be expressed as follows:

$$\frac{\left\|\sum_{i=1}^{r} h_i(\boldsymbol{I}(k))\left[\boldsymbol{T}_i\boldsymbol{I}(k) + \boldsymbol{H}_i\boldsymbol{O}(k)\right]\right\|_2}{\|\boldsymbol{W}(k)\|_2} \leq \gamma \tag{6}$$

where $\|\cdot\|_2$ is $l_2 \in [0, \infty)$. Inequality (6) describes the system gain characteristic from the customers' demands to the total cost of the emergency supply chain system. The smaller the parameter γ is, the better the performance of the emergency supply chain system will be.

Based on the principle of parallel distributed compensation, the design of a fuzzy controller synthesizes a local feedback controller for each emergency subsystem of the supply chain. Then, the local inventory feedback controller is formulated as follows:

Controller Rule K^i: if $s_0(k)$ is M_0^i, $s_1(k)$ is M_1^i, ..., $s_j(k)$ is M_j^i,..., and $s_n(k)$ is M_n^i, then

$$\boldsymbol{O}(k) = -\boldsymbol{K}_i\boldsymbol{I}(k), \quad i = 1, 2, \ldots, r, \tag{7}$$

where $\boldsymbol{K}_i$ denotes the inventory feedback constant gain matrix.

The inventory feedback controller of the global emergency supply chain system can be expressed as

$$\boldsymbol{O}(k) = -\sum_{i=1}^{r} h_i\boldsymbol{K}_i\boldsymbol{I}(k). \tag{8}$$

Then, the fuzzy emergency supply chain system can be written as

$$\begin{aligned} \boldsymbol{I}(k+1) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j\left[\boldsymbol{E}_{ij}\boldsymbol{I}(k) + \boldsymbol{R}_{wi}\boldsymbol{W}(k)\right] \\ \boldsymbol{C}(k) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \boldsymbol{F}_{ij}\boldsymbol{I}(k). \end{aligned} \tag{9}$$

where $\boldsymbol{E}_{ij} = \boldsymbol{S}_i - \boldsymbol{R}_i\boldsymbol{K}_j$, $\boldsymbol{F}_{ij} = \boldsymbol{T}_i - \boldsymbol{H}_i\boldsymbol{K}_j$.

Moreover, system (9) can be further expressed as follows:

$$\begin{aligned} \boldsymbol{I}(k+1) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \overline{\boldsymbol{E}}_{ij}\overline{\boldsymbol{I}}(k) \\ \boldsymbol{C}(k) &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \overline{\boldsymbol{F}}_{ij}\overline{\boldsymbol{I}}(k), \end{aligned} \tag{10}$$

where $\overline{\boldsymbol{E}}_{ij} = [\boldsymbol{E}_{ij} \ \ \boldsymbol{R}_{wi}]$, $\overline{\boldsymbol{F}}_{ij} = [\boldsymbol{F}_{ij} \ \ \boldsymbol{0}]$, and $\overline{\boldsymbol{I}}(k) = [\boldsymbol{I}(k) \ \ \boldsymbol{W}(k)]^{\mathrm{T}}$.

The following definitions, property, and lemma as the preparation of the proof for the subsequent Theorem 6 are introduced before further analysis.

Definition 1 (see [22]). A cluster of fuzzy sets $\{\mathrm{F}_j^u, u = 1, 2, \ldots, q_j\}$ are said to be a standard fuzzy partition (SFP) in the universe X if each F_j^u is a normal fuzzy set and F_j^u ($u = 1, 2, \ldots, q_j$) are full-overlapped in the universe X. q_j is said to be the number of fuzzy partitions of the jth input variable on X.

Definition 2 (see [22]). For a given fuzzy system, an overlapped-rules group with the largest amount of rules is said to be a maximal overlapped-rules group (MORG).

Definition 3 (see [23]). Given a scalar $\gamma > 0$, the discrete switched system (10) is said to be robustly stable with the disturbance attenuation level γ constraint under the H_∞ norm if the following conditions are satisfied:

(1) When $\boldsymbol{W}(k) \equiv \mathbf{0}$, the fuzzy system (10) is asymptotically stable.

(2) When $\boldsymbol{W}(k) \neq \mathbf{0}$, under the condition of the initial value of zero, any uncertain customers' demands meet $\|C(k)\|_2^2 \leq \gamma \|\boldsymbol{W}(k)\|_2^2$.

Property 4 (see [22]). If the input variables of a fuzzy system adopt SFPs, then all the rules in an overlapped-rules group must be included in an MORG.

Lemma 5 (see [21]). *For any real matrices* $\boldsymbol{X}_{ij}$ ($1 \leq i, j \leq n$) *and* $\boldsymbol{S} > \mathbf{0}$ *with appropriate dimensions, the following inequality holds:*

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} h_i h_j h_k h_l \boldsymbol{X}_{ij}^{\mathrm{T}} \boldsymbol{S} \boldsymbol{X}_{kl} \leq \sum_{i=1}^{n}\sum_{j=1}^{n} h_i h_j \boldsymbol{X}_{ij}^{\mathrm{T}} \boldsymbol{S} \boldsymbol{X}_{ij}. \quad (11)$$

The fuzzy robust emergency strategy for the supply chain system will be given in the form of Theorem 6.

Theorem 6. *For a given scalar $\gamma > 0$, if there exist local common positive definite matrices $\boldsymbol{P}_c$ in $\boldsymbol{G}_c$ satisfying*

$$\begin{bmatrix} -\overline{\overline{\boldsymbol{P}}} & * & * \\ \overline{\boldsymbol{E}}_{ii} & -\boldsymbol{P}_c^{-1} & * \\ \overline{\boldsymbol{F}}_{ii} & \mathbf{0} & -\boldsymbol{I} \end{bmatrix} < \mathbf{0}, \quad i \in B_c, \quad (12)$$

$$\begin{bmatrix} -4\overline{\overline{\boldsymbol{P}}} & * & * \\ 2\overline{\overline{\boldsymbol{E}}}_{ij} & -\boldsymbol{P}_c^{-1} & * \\ 2\overline{\overline{\boldsymbol{F}}}_{ij} & \mathbf{0} & -\boldsymbol{I} \end{bmatrix} < \mathbf{0}, \quad i < j,\ i, j \in B_c, \quad (13)$$

then the fuzzy emergency supply chain system (10) with SFP inputs is robustly asymptotically stable and the H_∞ norm is less than a given bound γ, where B_c is the set of the rule numbers included in $\boldsymbol{G}_c$, $\boldsymbol{G}_c$ denotes the cth MORG, $c = 1, 2, \ldots, \prod_{j=1}^{n}(m_j - 1)$, m_j is the number of fuzzy partitions of the jth input variable, $\overline{\overline{\boldsymbol{P}}} = \left[\begin{smallmatrix} \boldsymbol{P}_c & * \\ \mathbf{0} & \gamma^2 \boldsymbol{I} \end{smallmatrix}\right]$, $\overline{\overline{\boldsymbol{E}}}_{ij} = (\overline{\boldsymbol{E}}_{ij} + \overline{\boldsymbol{E}}_{ji})/2$, and $\overline{\overline{\boldsymbol{F}}}_{ij} = (\overline{\boldsymbol{F}}_{ij} + \overline{\boldsymbol{F}}_{ji})/2$.

When designing the actual H_∞ controller, we will transform Theorem 6 into Theorem 7, in which the inequalities can be solved as linear matrix inequalities (LMIs).

Theorem 7. *For the fuzzy emergency supply chain system (10) with SFP inputs, if there exist a given scalar $\gamma > 0$, local common positive definite matrices $\boldsymbol{P}_c$, and matrices $\boldsymbol{K}_{ic}$, $\boldsymbol{K}_{jc}$ in $\boldsymbol{G}_c$ satisfying*

$$\begin{bmatrix} -\boldsymbol{P}_c & * & * & * \\ \mathbf{0} & -\gamma^2 \boldsymbol{I} & * & * \\ \boldsymbol{S}_i - \boldsymbol{R}_i \boldsymbol{K}_{ic} & \boldsymbol{R}_{wi} & -\boldsymbol{P}_c & * \\ \boldsymbol{T}_i - \boldsymbol{H}_i \boldsymbol{K}_{ic} & \mathbf{0} & \mathbf{0} & -\boldsymbol{I} \end{bmatrix} < \mathbf{0}, \quad i \in B_c, \quad (14)$$

$$\begin{bmatrix} -4\boldsymbol{P}_c & * & * & * \\ \mathbf{0} & -\gamma^2 \boldsymbol{I} & * & * \\ \boldsymbol{S}_i - \boldsymbol{R}_i \boldsymbol{K}_{jc} + \boldsymbol{S}_j - \boldsymbol{R}_j \boldsymbol{K}_{ic} & \boldsymbol{R}_{wi} + \boldsymbol{R}_{wj} & -\boldsymbol{P}_c & * \\ \boldsymbol{T}_i - \boldsymbol{H}_i \boldsymbol{K}_{jc} + \boldsymbol{T}_j - \boldsymbol{H}_j \boldsymbol{K}_{ic} & \mathbf{0} & \mathbf{0} & -\boldsymbol{I} \end{bmatrix} < \mathbf{0}, \quad i < j,\ i, j \in B_c, \quad (15)$$

then the fuzzy emergency supply chain system (10) is robustly asymptotically stable under the performance γ, where B_c is the set of the rule numbers included in $\boldsymbol{G}_c$, $\boldsymbol{G}_c$ denotes the cth MORG, $c = 1, 2, \ldots, \prod_{j=1}^{n}(m_j - 1)$, and m_j is the number of the fuzzy partitions of the jth input variable.

5. Simulation Analysis

This study chooses a supply chain composed of one strategic manufacturer and two distributors in the section steel industry to verify the control effect of the fuzzy robust emergency strategy proposed in Section 4. It assumes that Distributor 1 can not order section steels from the strategic manufacturer under an emergency incident, but Distributor 2 can be supplied by the strategic manufacturer. For the emergency supply chain under supply disruption, it assumes there is a backup manufacturer.

The sketch map of the fuzzy membership functions of the distributors is shown in Figure 2.

In Figure 2, $s_1(k)$ denotes Distributor 1's inventory level and $s_2(k)$ denotes Distributor 2's inventory level. Suppose $s_1(k)$ and $s_2(k)$ can be measured, then the fuzzy partitions of $s_1(k)$ and $s_2(k)$ are $F_1^m(s_1(k))$ ($m = 1, 2$) and $F_2^p(s_2(k))$ ($p = 1, 2$), respectively, and conform to the conditions of SFP. Q_{0s} and Q_{1s} are Distributor 1's safety inventory and expected inventory, respectively. Q_{0t} and Q_{1t} are Distributor 2's safety inventory and expected inventory, respectively. Let $M_1^1 = M_1^2 = F_1^1$, $M_1^3 = M_1^4 = F_1^2$, $M_2^1 = M_2^3 = F_2^1$, $M_2^2 = M_2^4 = F_2^2$, $Q_{0s} = 10$, $Q_{1s} = 50$, $Q_{0t} = 15$ and $Q_{1t} = 60$ ($\times 10^5$ ton).

Figure 2 shows there exists one MORG called G_1 (including R_1, R_2, R_3 and R_4). Under the different fuzzy rules, the strategic manufacturer's production strategies and distributors' ordering strategies are outlined as follows: R_1: The strategic manufacturer produces section steels normally, Distributor 1 orders section steels from the backup manufacturer,

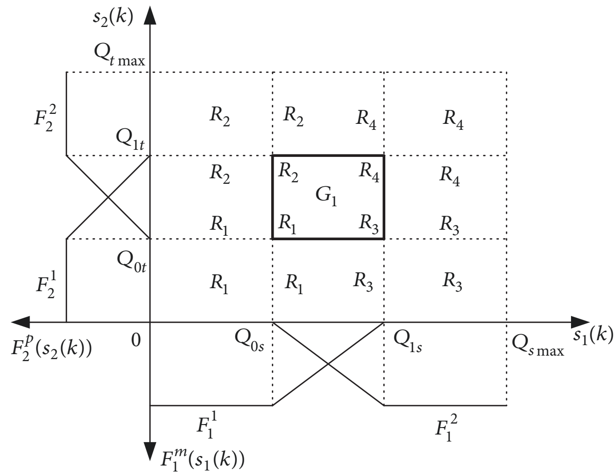

Figure 2: The sketch map of the fuzzy membership functions.

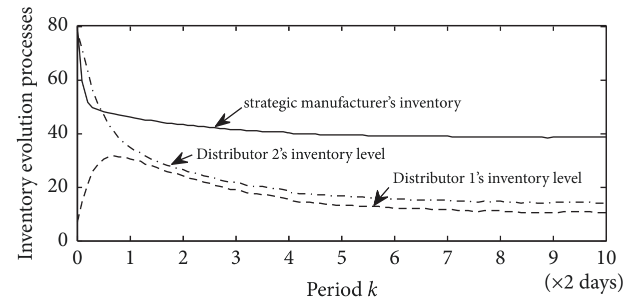

Figure 3: Evolution processes of state variables under the first initial inventory statuses (×10^5 ton).

and Distributor 2 orders section steels from the strategic manufacturer; R_2: The strategic manufacturer produces section steels normally, Distributor 1 orders section steels from both the backup manufacturer and Distributor 2, and Distributor 2 does not order section steels; R_3: The strategic manufacturer produces section steels normally, Distributor 1 does not order section steels, and Distributor 2 orders section steels from the strategic manufacturer; R_4: The strategic manufacturer does not produce section steels, Distributor 1 and Distributor 2 does not order section steels.

Based on the statistical data from the Chinese steel industry, the strategic manufacturer's production strategies, and the distributors' ordering strategies, the parameters are set as follows: $c_{n0} = 0.4$, $c_{n1} = 0.8$, $c_{n2} = 0.6$, $c_{r0} = 1.5$, $c_{o1} = 3.2$, $c_{o2} = 4.5$, $c_{21} = 0.6$ ($\times 10^4$ Yuan per ton), $\boldsymbol{S}_i = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $(i = 1, 2, 3, 4)$, $\boldsymbol{R}_1 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\boldsymbol{R}_2 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & -0.7 & 1 \end{bmatrix}$, $\boldsymbol{R}_3 = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\boldsymbol{R}_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\boldsymbol{T}_i = [0.4 \;\; 0.8 \;\; 0.6]$ $(i = 1, 2, 3, 4)$, $\boldsymbol{H}_1 = [1.5 \;\; 4.5 \;\; 3.2]$, $\boldsymbol{H}_2 = [1.5 \;\; 1.77 \;\; 3.2]$, $\boldsymbol{H}_3 = [1.5 \;\; 0 \;\; 3.2]$, $\boldsymbol{H}_4 = [0 \;\; 0 \;\; 0]$, $\gamma = 0.5$.

As Theorem 7 shows, (14) and (15) are solved by using the feasp solver in LMI Toolbox of MATLAB; it concludes that the emergency supply chain system under supply disruption is robustly stable because the following results meet the conditions of Theorem 7:

$$\boldsymbol{P}_1 = \begin{bmatrix} 1.3404 & 0.3346 & 0.3108 \\ 0.3346 & 4.7340 & 0.4067 \\ 0.3108 & 0.4067 & 4.1003 \end{bmatrix},$$

$$\boldsymbol{K}_{11} = \begin{bmatrix} 0.6575 & -0.2234 & 0.0548 \\ -0.0993 & 0.1771 & -0.1067 \\ -0.0827 & -0.0920 & 0.1818 \end{bmatrix},$$

$$\boldsymbol{K}_{21} = \begin{bmatrix} 0.5949 & -0.1105 & -0.0120 \\ -0.0901 & 0.1577 & -0.0977 \\ -0.1483 & 0.0249 & 0.1111 \end{bmatrix},$$

$$\boldsymbol{K}_{31} = \begin{bmatrix} 0.5220 & 0.0180 & -0.0921 \\ -0.1282 & 0.2283 & -0.1391 \\ -0.1479 & 0.0233 & 0.1126 \end{bmatrix},$$

$$\boldsymbol{K}_{41} = \begin{bmatrix} 1.1108 & -0.0702 & -0.0571 \\ -0.0928 & 0.1529 & -0.0685 \\ -0.2727 & 0.0177 & 0.1697 \end{bmatrix}. \quad (16)$$

In the emergency supply chain system, two simulation tests will be executed to verify the control effect of restraining the fluctuations caused by the switching activities and customers' demands. Because the simulation results are denoted by the actual values, that is, the sum of the deviation values and the normal values, the normal values are set as $\vec{s}_0(k) = 60$, $\vec{s}_1(k) = 75$, and $\vec{s}_2(k) = 73 (\times 10^5$ ton); $\vec{v}_0(k) = 32$, $\vec{v}_1(k) = 15$, and $\vec{v}_2(k) = 18$ ($\times 10^5$ ton). Let $w_1(k), w_2(k) \sim N(30, 0.85^2)$.

It assumes that the first initial inventory statuses are $s_0(0) = 20$, $s_1(0) = -68$, and $s_2(0) = 5$ ($\times 10^5$ ton). According to the initial inventory statuses and the developed emergency strategies, the strategic manufacturer will produce section steels, Distributor 1 will order 70 percent of the total quantity of section steels from Distributor 2 and 30 percent of the total quantity of section steels from the backup manufacturer, and Distributor 2 will not order section steels in the beginning. Then, by utilizing the fuzzy robust emergency strategy, Figures 3–5 show the simulation results under the first initial inventory statuses.

On the other hand, it assumes that the second initial inventory statuses are $s_0(0) = 20$, $s_1(0) = -70$, and $s_2(0) = -63$ ($\times 10^5$ ton). According to the initial inventory statuses and the developed emergency strategies, the strategic manufacturer will produce section steels, Distributor 1 will order section steels only from the backup manufacturer, and Distributor 2 will order section steels from the strategic manufacturer. Then, by utilizing the fuzzy robust emergency strategy, Figures 6–8 show the simulation results are under the second initial inventory statuses.

Figures 3 and 6 show that the emergency ordering strategies can make Distributor 1 under supply disruption continue to satisfy the customers' demands before the impaired supply

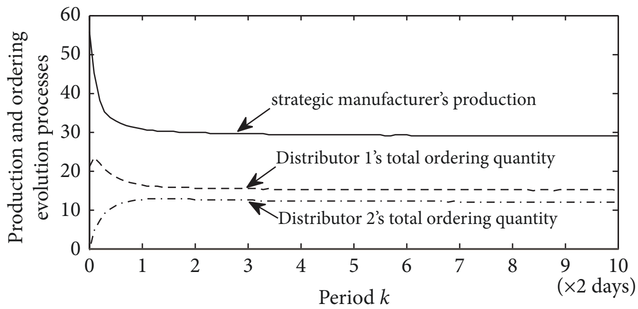

Figure 4: Evolution processes of control variables under the first initial inventory statuses ($\times 10^5$ ton).

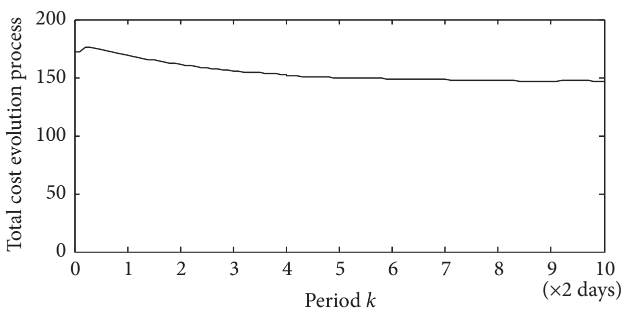

Figure 5: Evolution process of total cost under the first initial inventory statuses ($\times 10^7$ yuan).

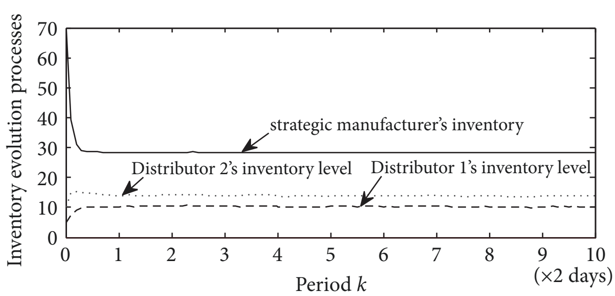

Figure 6: Evolution processes of state variables under the second initial inventory statuses ($\times 10^5$ ton).

chain is restored to the normal state. Figures 3–8 indicate that the 'strategic manufacturer's inventory level and production', 'Distributor 1's inventory level and total ordering quantity', 'Distributor 2's inventory level and total ordering quantity', and the 'total cost of the supply chain system' fluctuate at a small scale. Therefore, the fuzzy robust emergency strategy can restrain the impacts of the supply disruption, the switching activities among subsystems, and the uncertain customers' demands on the supply chain system. In practice, this paper provides a new way for managers to design a robust emergency strategy to restore the impaired supply chain to the normal operation state under the low cost.

6. Conclusions

This study has put forward a fuzzy emergency model and developed a robust emergency strategy to manage the supply

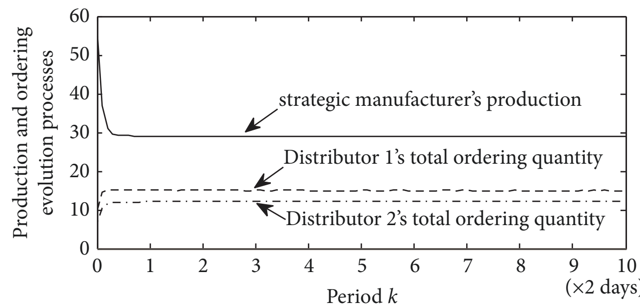

Figure 7: Evolution processes of control variables under the second initial inventory statuses ($\times 10^5$ ton).

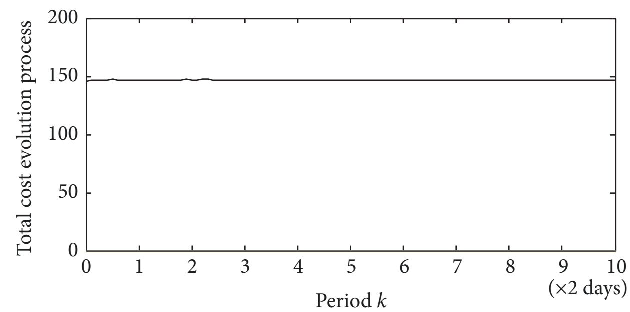

Figure 8: Evolution process of total cost under the second initial inventory statuses ($\times 10^7$ yuan).

chain under random supply disruptions. First, basic emergency models of a supply chain system under random supply disruptions are constructed, where distributors under supply disruptions can order goods from the backup manufacturer or/and other distributors. According to the inventory levels, the priority is given to distributors under supply disruptions to order from distributors supplied by the strategic manufacturer. Based on the distributors' safety inventory and expected inventory, basic models are transformed into a Takagi-Sugeno fuzzy emergency model with multiple emergency subsystems. In addition, the fuzzy emergency model is a kind of soft switching model. Furthermore, a robust emergency strategy is developed for the Takagi-Sugeno fuzzy emergency model. If the local common positive definite matrices are found in each MORG, the Takagi-Sugeno fuzzy system is robustly stable. Thus, the fuzzy robust emergency strategy is less conservative and difficult than other common fuzzy robust strategies. Finally, simulation results show that the supply disruption issue is successfully resolved by the fuzzy emergency model, and the robust emergency strategy also restrains the fluctuations effectively caused by switching activities among emergency subsystems and uncertain customers' demands. Thus, the developed fuzzy robust emergency strategy can guarantee the supply chain system under random supply disruptions to be robustly stable and keep the total cost of the emergency supply chain system on an ideal level.

In future research, the impacts of the distributors' ordering lead times and the strategic manufacturer's production lead times on the emergency supply chain system will be explored. The robust operation issue will also be considered for converting between the normal supply chain system and the emergency supply chain system.

Appendix

A. Proof of Theorem 6

Assume that the fuzzy system (10) contains f overlapped-rules groups, v_d $(d = 1, 2, \ldots, f)$ is the operating region of the dth overlapped-rules group and $L_d = \{$the rule numbers included in the dth overlapped-rules group$\}$.

First, we will show that the fuzzy system (10) is robustly asymptotically stable with state input variables $\boldsymbol{I}(k)$ and $\boldsymbol{I}(k+1)$ in the same overlapped-rules group. Then, the local model of the dth overlapped-rules group can be described as

$$\begin{aligned}\boldsymbol{I}(k+1) &= \sum_{i\in L_d}\sum_{j\in L_d} h_i h_j \overline{\boldsymbol{E}}_{ij}\overline{\boldsymbol{I}}(k) \\ \boldsymbol{C}(k) &= \sum_{i\in L_d}\sum_{j\in L_d} h_i h_j \overline{\boldsymbol{F}}_{ij}\overline{\boldsymbol{I}}(k),\end{aligned} \tag{A.1}$$

For system (A.1), we define the following discrete Lyapunov function:

$$V_d(\boldsymbol{I}(k)) = \boldsymbol{I}^{\mathrm{T}}(k)\boldsymbol{P}_c\boldsymbol{I}(k). \tag{A.2}$$

Then, based on Lemma 5, we can obtain $\Delta V_d(\boldsymbol{I}(k))$ as follows:

$$\begin{aligned}\Delta V_d(\boldsymbol{I}(k)) &= V_d(\boldsymbol{I}(k+1)) - V_d(\boldsymbol{I}(k)) = \boldsymbol{I}^{\mathrm{T}}(k+1) \\ &\cdot \boldsymbol{P}_c\boldsymbol{I}(k+1) - \boldsymbol{I}^{\mathrm{T}}(k)\boldsymbol{P}_c\boldsymbol{I}(k) \\ &= \sum_{i\in L_d}\sum_{j\in L_d} h_i h_j \sum_{p\in L_d}\sum_{q\in L_d} h_p h_q \\ &\cdot \left[\overline{\boldsymbol{I}}^{\mathrm{T}}(k)\overline{\boldsymbol{E}}_{ij}^{\mathrm{T}}\boldsymbol{P}_c\overline{\boldsymbol{E}}_{pq}\overline{\boldsymbol{I}}(k) - \boldsymbol{I}^{\mathrm{T}}(k)\boldsymbol{P}_c\boldsymbol{I}(k)\right] \\ &= \sum_{i\in L_d}\sum_{j\in L_d} h_i h_j \sum_{p\in L_d}\sum_{q\in L_d} h_p h_q \overline{\boldsymbol{I}}^{\mathrm{T}}(k) \\ &\cdot \left(\overline{\boldsymbol{E}}_{ij}^{\mathrm{T}}\boldsymbol{P}_c\overline{\boldsymbol{E}}_{pq} - \overline{\boldsymbol{P}}\right)\overline{\boldsymbol{I}}(k) \\ &= \sum_{i\in L_d}\sum_{j\in L_d} h_i h_j \sum_{p\in L_d}\sum_{q\in L_d} h_p h_q \overline{\boldsymbol{I}}^{\mathrm{T}}(k) \\ &\cdot \left[\left(\frac{\overline{\boldsymbol{E}}_{ij} + \overline{\boldsymbol{E}}_{ji}}{2}\right)^{\mathrm{T}}\boldsymbol{P}_c\left(\frac{\overline{\boldsymbol{E}}_{pq} + \overline{\boldsymbol{E}}_{qp}}{2}\right) - \overline{\boldsymbol{P}}\right]\overline{\boldsymbol{I}}(k) \\ &= \sum_{i\in L_d}\sum_{j\in L_d} h_i h_j \sum_{p\in L_d}\sum_{q\in L_d} h_p h_q \overline{\boldsymbol{I}}^{\mathrm{T}}(k) \\ &\cdot \left(\overline{\overline{\boldsymbol{E}}}_{ij}^{\mathrm{T}}\boldsymbol{P}_c\overline{\overline{\boldsymbol{E}}}_{pq} - \overline{\boldsymbol{P}}\right)\overline{\boldsymbol{I}}(k) \le \sum_{i\in L_d}\sum_{j\in L_d} h_i h_j \overline{\boldsymbol{I}}^{\mathrm{T}}(k) \\ &\cdot \left(\overline{\overline{\boldsymbol{E}}}_{ij}^{\mathrm{T}}\boldsymbol{P}_c\overline{\overline{\boldsymbol{E}}}_{pq} - \overline{\boldsymbol{P}}\right)\overline{\boldsymbol{I}}(k),\end{aligned} \tag{A.3}$$

where $\overline{\boldsymbol{P}} = \left[\begin{smallmatrix}\boldsymbol{P}_c & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0}\end{smallmatrix}\right]$ and $\overline{\overline{\boldsymbol{E}}}_{pq} = (\overline{\boldsymbol{E}}_{pq} + \overline{\boldsymbol{E}}_{qp})/2$.

$\Delta V_d(\boldsymbol{I}(k))$ can be further expressed as

$$\begin{aligned}&\Delta V_d(\boldsymbol{I}(k)) \\ &\quad \le \sum_{i=j, i\in L_d} h_i^2\overline{\boldsymbol{I}}^{\mathrm{T}}(k)\left[\overline{\boldsymbol{E}}_{ii}^{\mathrm{T}}\boldsymbol{P}_c\overline{\boldsymbol{E}}_{ii} - \overline{\boldsymbol{P}}\right]\overline{\boldsymbol{I}}(k) \\ &\qquad + 2\sum_{\substack{i<j \\ i\in L_d, j\in L_d}} h_i h_j \overline{\boldsymbol{I}}^{\mathrm{T}}(k)\left[\overline{\overline{\boldsymbol{E}}}_{ij}^{\mathrm{T}}\boldsymbol{P}_c\overline{\overline{\boldsymbol{E}}}_{ij} - \overline{\boldsymbol{P}}\right]\overline{\boldsymbol{I}}(k).\end{aligned} \tag{A.4}$$

On the one hand, we assume that the customers' demands $\boldsymbol{W}(k) \neq \boldsymbol{0}$ and there is H_∞ performance index function M_1 such that

$$M_1 = \sum_{k=0}^{N-1}\left[\boldsymbol{C}^{\mathrm{T}}(k)\boldsymbol{C}(k) - \gamma^2\boldsymbol{W}^{\mathrm{T}}(k)\boldsymbol{W}(k)\right]. \tag{A.5}$$

The above equation can be rewritten as

$$\begin{aligned}M_1 &= \sum_{k=0}^{N-1}\left[\boldsymbol{C}^{\mathrm{T}}(k)\boldsymbol{C}(k) - \gamma^2\boldsymbol{W}^{\mathrm{T}}(k)\boldsymbol{W}(k)\right. \\ &\quad \left.+ \Delta V_d(\boldsymbol{I}(k))\right] - V_d(\boldsymbol{I}(N)) \le \sum_{k=0}^{N-1}\left[\boldsymbol{C}^{\mathrm{T}}(k)\boldsymbol{C}(k)\right. \\ &\quad \left.- \gamma^2\boldsymbol{W}^{\mathrm{T}}(k)\boldsymbol{W}(k) + \Delta V_d(\boldsymbol{I}(k))\right].\end{aligned} \tag{A.6}$$

Substituting inequation (A.4) into inequation (A.6), we have

$$\begin{aligned}M_1 &\le \sum_{k=0}^{N-1}\left\{\sum_{i=j, i\in L_d} h_i^2\overline{\boldsymbol{I}}^{\mathrm{T}}(k)\left[\overline{\boldsymbol{E}}_{ii}^{\mathrm{T}}\overline{\boldsymbol{P}}_c\overline{\boldsymbol{E}}_{ii} - \overline{\overline{\boldsymbol{P}}} + \overline{\boldsymbol{F}}_{ii}^{\mathrm{T}}\overline{\boldsymbol{F}}_{ii}\right]\right. \\ &\quad \left.\cdot \overline{\boldsymbol{I}}(k)\right\} + 2\sum_{k=0}^{N-1}\left\{\sum_{\substack{i<j, i\in L_d \\ j\in L_d}} h_i h_j \overline{\boldsymbol{I}}^{\mathrm{T}}(k)\right. \\ &\quad \left.\cdot \left[\overline{\overline{\boldsymbol{F}}}_{ij}^{\mathrm{T}}\boldsymbol{P}_c\overline{\boldsymbol{F}}_{ij} - \overline{\overline{\boldsymbol{P}}} + \overline{\overline{\boldsymbol{F}}}_{ij}^{\mathrm{T}}\overline{\overline{\boldsymbol{F}}}_{ij}\right]\overline{\boldsymbol{I}}(k)\right\}.\end{aligned} \tag{A.7}$$

We can obtain $\overline{\boldsymbol{E}}_{ii}^{\mathrm{T}}\boldsymbol{P}_c\overline{\boldsymbol{E}}_{ii} - \overline{\overline{\boldsymbol{P}}} + \overline{\boldsymbol{F}}_{ii}^{\mathrm{T}}\overline{\boldsymbol{F}}_{ii} < \boldsymbol{0}$ and $\overline{\boldsymbol{E}}_{ij}^{\mathrm{T}}\boldsymbol{P}_c\overline{\overline{\boldsymbol{E}}}_{ij} - \overline{\overline{\boldsymbol{P}}} + \overline{\overline{\boldsymbol{F}}}_{ij}^{\mathrm{T}}\overline{\overline{\boldsymbol{F}}}_{ij} < \boldsymbol{0}$, which are equivalent to inequation (12) and inequation (13) by the Schur complement. Then, $M_1 < 0$ can be obtained, i.e., $\boldsymbol{C}^{\mathrm{T}}(k)\boldsymbol{C}(k) < \gamma^2\boldsymbol{W}^{\mathrm{T}}(k)\boldsymbol{W}(k)$; moreover, let $N \longrightarrow +\infty$, then we have $\|\boldsymbol{C}(k)\|_2^2 < \gamma^2\|\boldsymbol{W}(k)\|_2^2$. As a result, the emergency supply chain system (A.1) is proved to be asymptotically stable in the case of $\boldsymbol{W}(k) \neq \boldsymbol{0}$.

On the other hand, if $\boldsymbol{W}(k) \equiv \boldsymbol{0}$, it is obvious that inequation (A.4) is equivalent to the following inequality:

$$\begin{aligned}&\Delta V_d(\boldsymbol{I}(k)) \\ &\quad \le \sum_{i=j, i\in L_d} h_i^2\overline{\boldsymbol{I}}^{\mathrm{T}}(k)\left[\overline{\boldsymbol{E}}_{ii}^{\mathrm{T}}\boldsymbol{P}_c\overline{\boldsymbol{E}}_{ii} - \overline{\overline{\boldsymbol{P}}}\right]\overline{\boldsymbol{I}}(k)\end{aligned}$$

$$+2\sum_{\substack{i<j\\ i\in L_d, j\in L_d}} h_i h_j \overline{\boldsymbol{I}}^{\mathrm{T}}(k)\left[\overline{\overline{\boldsymbol{E}}}_{ij}^{\mathrm{T}} \boldsymbol{P}_c \overline{\overline{\boldsymbol{E}}}_{ij} - \overline{\overline{\boldsymbol{P}}}\right]\overline{\boldsymbol{I}}(k). \tag{A.8}$$

According to inequation (12) and inequation (13), we can obtain $\overline{\boldsymbol{E}}_{ii}^{\mathrm{T}}\boldsymbol{P}_c\overline{\boldsymbol{E}}_{ii} - \overline{\overline{\boldsymbol{P}}} < \mathbf{0}$ and $\overline{\overline{\boldsymbol{E}}}_{ij}^{\mathrm{T}}\boldsymbol{P}_c\overline{\overline{\boldsymbol{E}}}_{ij} - \overline{\overline{\boldsymbol{P}}} < \mathbf{0}$, respectively. Accordingly, we can conclude that $\Delta V_d(\boldsymbol{I}(k)) < 0$. As a result, the state feedback controller guarantees that the local system (A.1) is asymptotically stable in the dth overlapped-rules group.

Second, we will show that the fuzzy system is robustly asymptotically stable for state input variables $\boldsymbol{I}(k)$ and $\boldsymbol{I}(k+1)$ in the different overlapped-rules groups. A characteristic function in any overlapped-rules group is constructed as follows:

$$\lambda_d = \begin{cases} 1, & \boldsymbol{I}(k) \in \nu_d \\ 0, & \boldsymbol{I}(k) \notin \nu_d, \end{cases} \tag{A.9}$$

where $\sum_{d=1}^{f}\lambda_d = 1$, then the global model of the fuzzy system in the input universe of the discourse is described as

$$\begin{aligned}\boldsymbol{I}(k+1) &= \sum_{d=1}^{f}\lambda_d\left[\sum_{i\in L_d}\sum_{j\in L_d} h_i h_j \overline{\boldsymbol{E}}_{ij}\overline{\boldsymbol{I}}(k)\right]\\ \boldsymbol{C}(k) &= \sum_{d=1}^{f}\lambda_d\left[\sum_{i\in L_d}\sum_{j\in L_d} h_i h_j \overline{\boldsymbol{F}}_{ij}\overline{\boldsymbol{I}}(k)\right].\end{aligned} \tag{A.10}$$

After $\boldsymbol{P}_m = \sum_{d=1}^{f}\lambda_d\boldsymbol{P}_c$ is defined, a piecewise Lyapunov function is constructed in the input universe of the discourse as

$$\begin{aligned}V(\boldsymbol{I}(k)) &= \boldsymbol{I}^{\mathrm{T}}(k)\boldsymbol{P}_m\boldsymbol{I}(k)\\ &= \boldsymbol{I}^{\mathrm{T}}(k)\left(\sum_{d=1}^{f}\lambda_d\boldsymbol{P}_c\right)\boldsymbol{I}(k)\\ &= \sum_{d=1}^{f}\lambda_d\boldsymbol{I}^{\mathrm{T}}(k)\boldsymbol{P}_c\boldsymbol{I}(k) = \sum_{d=1}^{f}\lambda_d V_d(\boldsymbol{I}(k)).\end{aligned} \tag{A.11}$$

On the one hand, we assume that the customers' demands $\boldsymbol{W}(k) \neq \mathbf{0}$. For system (A.10), we can obtain $M_2 = \sum_{k=0}^{N-1}\sum_{d=1}^{f}\lambda_d[\boldsymbol{C}^{\mathrm{T}}(k)\boldsymbol{C}(k) - \gamma^2\boldsymbol{W}^{\mathrm{T}}(k)\boldsymbol{W}(k)]$ after the H_∞ performance index function $M_1 = \sum_{k=0}^{N-1}[\boldsymbol{C}^{\mathrm{T}}(k)\boldsymbol{C}(k) - \gamma^2\boldsymbol{W}^{\mathrm{T}}(k)\boldsymbol{W}(k)]$ is considered. Following a similar proof as above, we can obtain $M_2 < 0$, i.e., $\boldsymbol{C}^{\mathrm{T}}(k)\boldsymbol{C}(k) < \gamma^2\boldsymbol{W}^{\mathrm{T}}(k)\boldsymbol{W}(k)$; moreover, let $N \longrightarrow +\infty$, then we have $\|\boldsymbol{C}(k)\|_2^2 < \gamma^2\|\boldsymbol{W}(k)\|_2^2$. As a result, system (A.10) is proved to be asymptotically stable in the case of $\boldsymbol{W}(k) \neq \mathbf{0}$.

On the other hand, if $\boldsymbol{W}(k) \equiv \mathbf{0}$, we have:

$$\begin{aligned}\Delta V(\boldsymbol{I}(k)) &= V(\boldsymbol{I}(k+1)) - V(\boldsymbol{I}(k))\\ &= \sum_{d=1}^{f}\lambda_d V_d(\boldsymbol{I}(k+1)) - \sum_{d=1}^{f}\lambda_d V_d(\boldsymbol{I}(k))\\ &= \sum_{d=1}^{f}\lambda_d\left[V_d(\boldsymbol{I}(k+1)) - V_d(\boldsymbol{I}(k))\right]\\ &= \sum_{d=1}^{f}\lambda_d\Delta V_d(\boldsymbol{I}(k)) < 0.\end{aligned} \tag{A.12}$$

Hence, in any overlapped-rules group, system (A.10) with $\boldsymbol{W}(k) \equiv \mathbf{0}$ is asymptotically stable by the fuzzy controller (8). Therefore, based on property 1, we can conclude that the fuzzy system (10) is robustly asymptotically stable with condition (12) and condition (13) by resorting to find local common positive definite matrices $\boldsymbol{P}_c$ in $\boldsymbol{G}_c$. **Q.E.D.**

B. Proof of Theorem 7

The proof processes of Theorem 7 are similar to those of Theorem 6. Theorem 7 is easily demonstrated by using the Schur complement and matrix transformations. Thus, the proof of Theorem 7 is omitted for the sake of brevity. **Q.E.D.**

Data Availability

The simulation data used to support the findings of this study are included within the manuscript. These data are restricted by the China Iron and Steel Association in order to protect trade secrets. Data can be available from www.chinaisa.org.cn for researchers who meet the criteria for access to confidential data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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