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Research Article

Optimal Topology of Multilayer Urban Traffic Networks

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Previous urban traffic network-based studies have been based mostly on single-layer networks. Based on their shortcomings, starting from the perspective of a multilayer urban traffic network, this paper takes the different anticongestion abilities and network characteristics of various network structures under the condition of traffic congestion as the research object. Then, a comparative experiment is performed via simulation, and the optimal multilayer urban traffic network topology is obtained under different conditions. It is found that these scale-free related multilayer networks have relatively strong ability to support more traffic flows and have higher anticongestion abilities, regardless of whether it is a lower-layer or upper-layer network. The research results are helpful to deepen our understanding of the characteristics of traffic network structures, help scholars further cognize the structural properties of multilayer urban traffic networks, practically help urban traffic network planners to further optimize the urban traffic network, and broaden the study of multilayer traffic networks.

1. Introduction

With the critical role of urban traffic networks in the urban economy, the characteristics and properties of urban traffic network topology have recently gained increasing attention [1–12]. Scholars have presented in-depth discussions of the process of traffic planning and design from the perspectives of network topology, traffic flow, traffic congestion, network evolution, cascade failure, and network optimization [13–20]. Nevertheless, few studies on optimal network topology consider which network structure can bear more traffic flow, how to achieve a more efficient network structure, or which network can have the greatest anticongestion ability. Since the study of Wu et al. [21], related studies have received increased focus. The Gastner–Newman model assumes that traffic on a network moves with free speeds without considering traffic impedance [22]. However, in real-world situations, network impedance is widely used to assess the ability of networks. For example, Wu et al. [21] uses three types of networks (random, small-world, and scale-free) to determine which network structure suffers most from traffic congestion. Their work has shown that when the traffic flow is low, the random network has relatively strong ability to support considerable of traffic flow, but the scale-free network can support much more traffic flow as the total traffic flow increase. Later, considering the mechanisms of dynamical network evolution and rewiring links, Sun et al. [23] generated community-correlated scale-free transportation networks that can support higher traffic flow.

However, these studies are mainly based on planar networks. Recently, with the study of urban traffic networks and the missing part of the complex coupling mechanisms between different traffic modes, progressively more scholars have paid attention to multilayer urban traffic networks [24–30]. From the review of Wu et al. [20], which comprehensively discussed recent studies on multilayer network topology, we can clearly see that only a few studies have concentrated on the multilayer urban traffic network topology and traffic congestion problems, which means which type of multilayer network works best under different conditions has rarely been considered. Yue et al. [31] analyzed the traffic dynamics on layered complex networks and found that the "physical layer is much more important to the network capacity of two-layer complex networks than the logical layer." In this case, the two-layer complex networks are random network (E) on E, E on scale-free network (S), S on E, and S on S. Later, Zhang et al. [32], Tan et al. [16], and Li et al. [33] used the S on S type of multilayer network to illustrate the attributes and properties of coupling networks. Furthermore, the majority of these studies are based on a few types of network topology and ignore the comparison with other topologies.

Based on the shortcomings of the above research, starting from the perspective of a multilayer urban traffic network, this paper takes different anticongestion abilities and network characteristics of various network structures under the condition of traffic congestion as the research object and conducts a comparative simulation experiment to determine the optimal multilayer urban traffic network topology under different conditions. The research results are helpful in deepening our understanding of the characteristics of traffic network structures and can help scholars to further cognize the structural properties of multilayer urban traffic networks, thereby helping urban traffic network planners to further optimize urban traffic networks.

In this paper, we first review related studies and then propose urban single-layer and multilayer traffic network representation methods. Next, the optimal topology of single-layer urban traffic networks is presented. Then, the optimal topology of multilayer urban traffic networks under different conditions is discussed. Perspectives and conclusions are given at the end of this paper.

2. Methodology

For urban traffic networks, two methods are generally used to represent the network topology: single-layer and multilayer network representation methods. The single-layer network representation method is used to represent street networks or rail networks, while the multilayer representation method is used to represent the coupling of multilayer urban traffic networks. The related network topology structures are introduced. Later, the coupling methods of different networks are illustrated. Then, the measurement method of congestion factor is proposed, and the combination of these methods and the novelty of this research are discussed.

2.1. Single-Layer Network Representation Method. This method is widely applied and accepted by scholars [34–37]. The single-layer network representation method is based on the primal approach, as shown in Figure 1, the streets or roads are represented as black lines on the right, the nodes stand for street intersections, and grey spots are buildings. The rail networks also can be represented by the single-layer network representation method.

With this, the urban transportation networks can be represented as different undirected or directed connected networks:

$$G = \langle V, E, W \rangle, \tag{1}$$

where V is the set of nodes and N is the number of nodes when

$$V = \{v_i \mid i \in I \equiv \{1, 2, \ldots, N\}\}, \tag{2}$$

E is the unordered pairs or edges of elements of V and is denoted by e_{ij}, and

$$E = \{e_{ij} = (v_i, v_j) \mid i, j \in I\}, \tag{3}$$

and W is the weight of each edge, the weight can be treated as the traffic flows passed by.

In addition, the number of edges is denoted as M.

The adjacency matrix of the single-layer networks is

$$A = [a_{ij}]_{n\times n}, \tag{4}$$

representing the connection between nodes v_i and v_j, which is defined as

$$a_{ij} = \begin{cases} 1, & (v_i, v_j) \in V, \\ 0, & (v_i, v_j) \notin V, \end{cases} \tag{5}$$

where $a_{ii} = 0$ to remove any self-connections. In addition, $A = [a_{ij}]_{n\times n}$ is symmetrical and nonnegative.

2.2. Multilayer Network Representation Method. The undirected multilayer network (see Figure 2) can be represented as

$$G = \langle G^{\mathrm{U}}, G^{\mathrm{L}} \rangle, \tag{6}$$

as the set of different layers; here, this study uses the superscript U to define the upper-layer network and superscript L to set the lower layer [28, 36].

The rail network and urban street network can be represented as a connected network:

$$\begin{aligned} G^{\mathrm{U}} &= \langle V^{\mathrm{U}}, E^{\mathrm{U}}, W^{\mathrm{U}} \rangle, \\ G^{\mathrm{L}} &= \langle V^{\mathrm{L}}, E^{\mathrm{L}}, W^{\mathrm{L}} \rangle, \\ G^{\mathrm{C}} &= \langle E^{\mathrm{C}}, W^{\mathrm{C}} \rangle, \end{aligned} \tag{7}$$

in its primal weighted (denoted by W in function) representation [37]; red nodes represent rail stations and blue nodes denote road intersections; solid lines represent their connections and dotted lines stand for the coupling links between different layers. The rail network station is connecting with the nearest street network intersection [30].

The multilayer network model of urban traffic networks, the upper layer represents rail network topology, and the lower-layer represents the street network topology.

Similarly, based on the definition of a single-layer network, we have the definition of the multilayer network:

(a) (b)

Figure 1: The single-layer network representation method.

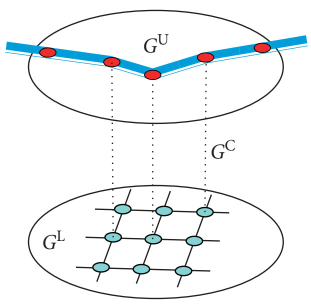


Figure 2: The multilayer network representation method.

$$\begin{aligned} N^{\text{multi}} &= N^{U} + N^{L}, \\ V^{\text{multi}} &= V^{U} + V^{L}, \\ E^{\text{multi}} &= E^{U} + E^{L} + E^{C}. \end{aligned} \tag{8}$$

The adjacency matrix of networks $\text{adj} = [a_{ij}]_{n\times n}$ is symmetrical and nonnegative, the connection between zones i and j is represented, where

$$a_{ij} = \begin{cases} d_{ij} \times W, & (v_i, v_j) \in E, \\ 0, & (v_i, v_j) \notin E, \end{cases} \tag{9}$$

where d_{ij} is the Euclidean distance. Define $a_{ii} = 0$ to theoretically remove any self-connections to exclude the impact of the network element itself. Then, the adjacency matrix of multilayer networks is

$$\text{adj}^{\text{multi}} = \begin{bmatrix} \text{adj}^{U}_{N^U \times N^U} & \text{adj}^{C}_{N^U \times N^L} \\ \text{adj}^{C}_{N^L \times N^U} & \text{adj}^{L}_{N^L \times N^L} \end{bmatrix}. \tag{10}$$

2.3. Related Network Topology Structures. This part provides a brief summary of network models which commonly used in complex network studies, which are regular networks (R), random graphs (E), small-world network (W), scale-free network (S), relative Neighborhood graph (RNG), and Gabriel graph (GG).

2.3.1. Regular Networks. Regular networks are the most common network patterns in the real urban network system from the point of view of urban morphology, especially the square regular lattice. This pattern is with obvious artificial trace and it is strictly designed by planners as Up Down cities (Figure 3(a)), like the most famous planned city Chandigarh and most of the American cities. Opposite to this, another network pattern which is named Bottom Up cities (Figure 3(c)) is generated with less man-made planning. Between them is Mixed Pattern cities (Figure 3(b)), which partly depends on general planning.

Regular networks are regular for the reason that each node has the same or the nearly the same number of degree values; in a real situation, the degree of most street intersections equals four (Figure 3(a)). Regular networks are highly ordered; particularly, a regular square lattice is a nonrandom network where each node connects to all of its nearest neighbours. Lattices can also be represented as different forms. Nevertheless, sometimes regular square lattice needs to be designed in combination with other form of network structures to pursue better function layouts [40].

2.3.2. Erdos–Renyi Random Graphs. The Erdos–Renyi (ER) random graphs model, also called simply random graphs, was presented by Erdos and Renyi in the 1950s and 1960s. Erdos and Renyi characterized random graphs and shown that many of the properties of such networks can be calculated analytically. The research of random planar graphs of the urban network is rare recently. Eisenstat [41] focused on the shortest paths and a maximum flow of the street network and proposed the Quadtree model. Another work considered the grid network, the static random planar graph, and the growing random planar graph, to analyse the London primal and dual street network in great depth [42].

2.3.3. Watts–Strogatz Small-World Network. In 1998, Duncan J. Watts and Steven Strogatz published in Nature of the first small-world network model, which through a single parameter smoothly interpolates between a random graph and a lattice. Not very later, Newman and Duncan J. Watts presented another model. Their models demonstrated that with the reconnection or addition of only a small number of long-range edges, a regular graph, in which the diameter is proportional to the network size, can be transformed into a

Figure 3: Difference between urban network patterns, from regular to random. (a) Up Down cities. (b) Mixed Pattern cities. (c) Bottom Up cities (source: maps of Mexico, Barcelona, and Toscana).

"small-world" in which the average path length of the network is relatively small, while their clustering coefficient stays large. It has been observed that some of the urban networks exhibit and obey the small-world property [43–46], and Latora and Marchiori [36] analytically proved the whole transportation system of Boston is following this behaviour. Further, for reconnecting the edges, the model is named as WS small-world, and for addition of the edges, the model is named as NW small-world. The existence of the small-world in an urban area is identical intuitive (Figure 4); these urban traffic networks which have overpass or shortcuts or bridges are nature representation of small-world networks.

2.3.4. Barabasi–Albert Scale-free Network. Recent attention in scale-free networks started since 1999, with the efforts of Albert and Barabasi at the University of Notre Dame where they mapped the topology of a portion of the networks, fundamentally based on the research of Watts and Strogatz (small-world model). Scale-free networks are widely observed in natural and human-made systems [47], including street traffic networks ([35, 48–50]). The degree distribution of scale-free network is following the power-law, at least asymptotically, an empirical law formulated by mathematical statistics, which refers to the fact that many types of urban networks can be approximated with the family of power-law probability distributions.

The generation of algorithm (network growth and preferential attachment) is the most important part, which procedure is as follows. At first, the network begins with an initially connected network of m_0 nodes. Then, new nodes are added to the network one at a time. Each new node is connected to $m < m_0$ existing nodes with a probability p that is proportional to the number of links that the existing nodes already have. Formally, the probability $\prod_i$ denotes that the new node is connected to node i which is $\prod_i = k_i/\sum_j k_j$, and the sum is made over all preexisting nodes j. Hub nodes tend to quickly accumulate even more links, while nodes with only a few links are unlikely to be chosen as the destination for a new link. The preferential attachment can be observed everywhere and can be applied in the urban traffic networks modelling as normally for the representation method of dual approach. Many researches have shown that traffic networks are typically and theoretically scale-free [35, 50, 51] with dual approach rather than primal approach, with power-law distribution in log-log plot, and the degree distribution exponent has large effects on the performances of traffic network, and the distribution of urban traffic flows also following scale-free properties [52, 53]. Zhang [54] proved that 50 extracted dual urban traffic networks of USA are following scale-free properties, and Kalapala et al. [55] found that, with the dual representation, the degree distribution of the urban street networks can better fit with the power-law functions with

$$P(k) \sim k^{-\lambda}, \tag{11}$$

while some other researches also pointed out similar properties but with different ranges of λ [42, 56].

2.3.5. Relative Neighborhood Graph and Gabriel Graph. According to the planar restriction with no crossing links, a network has sparse characteristics and the number of neighbor nodes in the generated topology is smaller than a constant. In this context, minimal spanning tree (MST) [57], relative neighborhood graph (RNG), and Gabriel graph (GG) can be introduced for a simple description and can be used to construct planar network topology structures.

The RNG was proposed by Lankford [58] and Toussaint [59]. Ten years later, Jaromczyk and Toussaint [60] provided clear definitions and functions for the RNG and its relatives. Let V be a set of points in a plane. Each pair of nodes p, q (unordered) has its own "lune" as $A_{p,q} = A(p, \delta(p, q)) \cap A(q, \delta(p, q))$, where $A_{p,q}$ is the intersection of the circular region of points p and q, with the radius being the distance between points p and q as $\delta(p, q)$. If there does not exist such a point g in $A_{p,q}$ $(q \neq g)$, point q is called a "relative neighbor" of point p. The RNG is widely used in wireless networks, circuits, navigation, and location [61].

The GG was proposed by Gabriel and Sokal [62]. Let V be a set of points in a plane. For each pair of nodes p, q (unordered), connect the points p, q with the line $E_{p,q}$, and generate a circular region with $E_{p,q}$ as the diameter. If there does not exist such a point g in the circular region, then the GG can be generated.

These graphs are related as $\mathrm{MST} \subseteq \mathrm{RNG} \subseteq \mathrm{GG}$ [61]; generally, the RNG can be created easily using a distributed algorithm, but the accessibility and connectivity are

(a) (b) (c)

Figure 4: The nature representation of small-world networks in urban traffic networks (source: maps of Shanghai, Chongqing, and Chendu).

relatively poorer than those of GG [61, 63]. Referring to Ding et al. [25], we can see that the RNG and GG can be used to represent connected rail networks and road networks.

2.4. Coupling of Different Networks. For coupling of these different structures of networks, a coupling matrix is generated, as shown in Figure 5. The R-*E* coupling structure indicates that the lower-layer network (regular networks) is coupled with the upper-layer network (Erdos–Renyi random graph).

Although GG and RNG can in some way represent real traffic networks, they might not be the best network structures; hence, we test them separately and compare them with various multilayer networks.

Two types of methods can be used to connect the different layers: random connection and complete connection. In the first method, the nodes on the upper layer are randomly chosen to connect with the nearest nodes on the lower layer with probability $p = 0.5$. In the second method, the nodes on the upper layer are completely connected with the nearest nodes on the lower layer. Additionally, there is another construction condition, that is, whether the lower layer and upper layer have the same number of nodes.

The coupled networks are weighted networks, with the traffic capacity determined and the traffic flow assigned and attached.

Clearly, the traffic congestion status will change when the upper layer and lower layer have different traffic capacities. Multilayer networks might have different levels of link capacity; here, we roughly set the link capacity of the upper-layer link to Θ times that of the lower-layer link. We first set $\Theta = 10$; later, its influence will be assessed. We set the tunable parameter Θ to show the influence of the link capacities of different layers. The function can be written as $C^{\text{upper-layer}} = \Theta C^{\text{lower-layer}}$. The measurement of this tunable parameter can help us to determine the capacity ratio between different layers.

2.5. Measurement of Congestion Factor. Consistent with the research of Wu et al. [21] and Sun et al. [23], we set $N = 100$ for the proposed R, E, W, and S and generate the related networks. For R, $\langle k \rangle = 4$. E, W, and S are in line with Wu et al. [21]. For W, we have $p = 0.1$, and for S, we have $\lambda = 2.5$.

	R	E	S	W
R	R-R	R-E	R-S	R-W
E	E-R	E-E	E-S	E-W
S	S-R	S-E	S-S	S-W
W	W-R	W-E	W-S	W-W

Figure 5: The coupling matrix of multilayer networks.

We can see from Wu et al. [21] that changing λ has little influence on the performance of S when $\lambda \in [2.2, 3]$; hence, we choose $\lambda = 2.5$.

The link capacity C_a (the maximum possible crossing flows) on link "a" is assigned randomly in a given range [20, 60], and the flows must obey the function

$$f_a > \tau \times C_a. \tag{12}$$

Then, traffic can be defined as congested. Here, τ is a tunable parameter with $\tau \geq 1$. When the tunable parameter τ is larger than 1, the assigned traffic flow is larger than the designed traffic capacity, which might cause traffic congestion. In line with Wu et al. [21], we set the value of τ to 1.5 first for the single-layer networks, which means that when the traffic flow passing one link is 1.5 times its designed traffic capacity, the link will be totally congested. However, in Wu et al.'s study [21] and some related studies, the influence of τ is still unclear. In the research of Wu et al. [21] and Sun et al. [64], they treat $\tau = 1.5$, while in the research of Sun et al. [23] and Maniadakis and Varoutas [65], they treat $\tau = 1$. Hence, in this paper, we expand the interval and choose $\tau \in [1, 2]$ to better assess and illustrate the changing trends of multilayer networks.

Here, f_a is the traffic flow on the corresponding link, and total traffic flow $Q = \Sigma f_a$. If we increase the total traffic flow Q, the traffic flows on each link will increase, and the total number of congested links will increase.

To discuss the congestion effects, a Frank-Wolfe-BPR flow assignment method is introduced, combined with the widely used and well-known Bureau of Public Roads (BPR) function:

$$v_a^c = v_a^f \left[1 + \alpha \left(\frac{f_a}{C_a} \right)^{\beta} \right]. \tag{13}$$

It is used to reflect the relationship of the free flow speed v_a^f and congested speed v_a^c on link a. v_a^c and v_a^f can be converted to travel cost, and α and β are correction factors equal to 0.15 and 4, respectively.

Then, the total traffic flow is increased at each time step. The traffic flows are assigned again by the Frank-Wolfe-BPR method [66, 67], and the travel cost on the congested links is set as infinite. Then, in each iteration, TCC is calculated to represent the number of links with traffic flows exceeding the link capacity:

$$\mathrm{TCC} = \begin{cases} \mathrm{TCC} + 1, & \text{if } f_a > \tau \times C_a, \\ \mathrm{TCC}, & \text{otherwise.} \end{cases} \tag{14}$$

Then, the congestion factor J can be measured as

$$J = \frac{\mathrm{TCC}}{M}, \tag{15}$$

where M is the total number of edges.

2.6. Combination of These Methods and the Novelty of This Research. Recent researches on multilayer traffic network are mainly based on the combination of a small number of network structures, such as the study of Yue et al. [31], Zhang et al. [32], and Tan et al. [16]; they only considered the combination of random networks and scale-free networks but did not take into account other network structures. They are more focused on the relationships between average transmission time and packet generation rate to reflect the structural characteristics of networks. However, their results may not seem intuitive at first and did not take into account the particularity of traffic network impedance. Some earlier studies relate to the optimal network structures, such as Wu et al. [21], Sun et al. [64], Sun et al. [23], and Maniadakis and Varoutas [65], did not consider the situation of multilayer traffic networks, and simply take single-layer network as their research objects, which were not in line with the actual situations. Therefore, our study is relatively novel, which united these two aspects together. It not only considers the coupling of these different networks from the perspective of multilayer traffic network theory but also considers the impedance of these traffic networks. The results are more intuitive and convenient for traffic planners to directly use.

3. Optimal Topology of Single-Layer Urban Traffic Networks

The change in network performance as the total traffic flow of single-layer urban traffic networks increases is shown in Figure 6. It shows that the scale-free network can support much more traffic flow, which means that, for the design of a traffic network, the proposed network structure should mainly obey the scale-free network property. All the average results are from 100 simulation iterations.

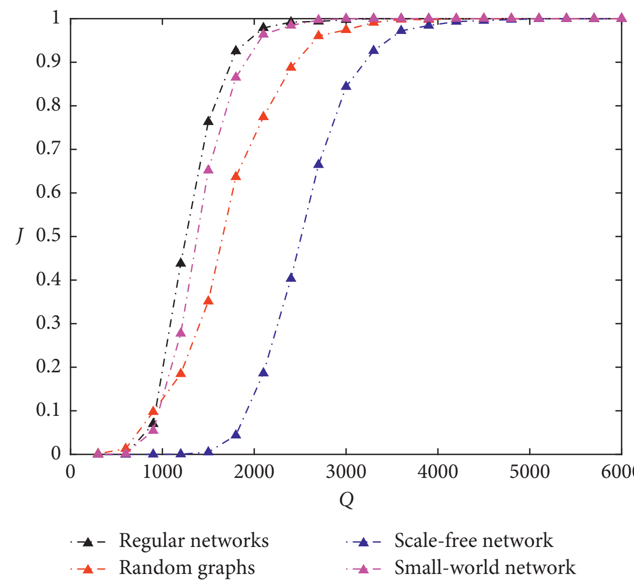


Figure 6: Network performance as the total traffic flow increases.

4. Optimal Topology of Multilayer Urban Traffic Networks

In line with the creation of these single-layer networks, we have the basic network structures of upper-layer and lower-layer networks. With the coupling of these different network structures, we further consider different network construction conditions. In this section, the optimal topology of multilayer urban traffic networks under different conditions will be discussed.

4.1. Lower Layer and Upper Layer Have the Same Number of Nodes. Using the same multilayer network generation methods, we set the lower layer and upper layer to have $N^{\mathrm{L}} = N^{\mathrm{U}} = 100$ and then connect the nodes of the lower layer and upper layer with the strategies presented in Section 2.4.

The link capacity C_a is set randomly in the given range [20, 60]. Q is initially set to increase until approximately 4×10^4, at which point different groups are clearly separated and results get stable.

If different layers are randomly connected, as shown in Figure 7, these scale-free related multilayer networks have relatively strong ability to support more traffic flow, regardless of whether it is a lower-layer or upper-layer network. The first group is *S-S*, which has the greatest anticongestion ability initially. The second group has 3 different network topologies, *R-S*, *E-S*, and *W-S*, which are networks with *S* as the upper-layer network. The next group includes networks with *S* as the lower-layer network, e.g., *S-R*, *S-W*, and *S-E*. Clearly, GG-RNG, RNG-GG, RNG-RNG, and GG-GG have relatively weak anticongestion ability; they belong to the fourth group. The remaining networks all belong to the fifth group. When the total traffic flow is small, all the networks can support more traffic flow. As the total traffic flow increases, differences emerge and the networks can be divided into groups.

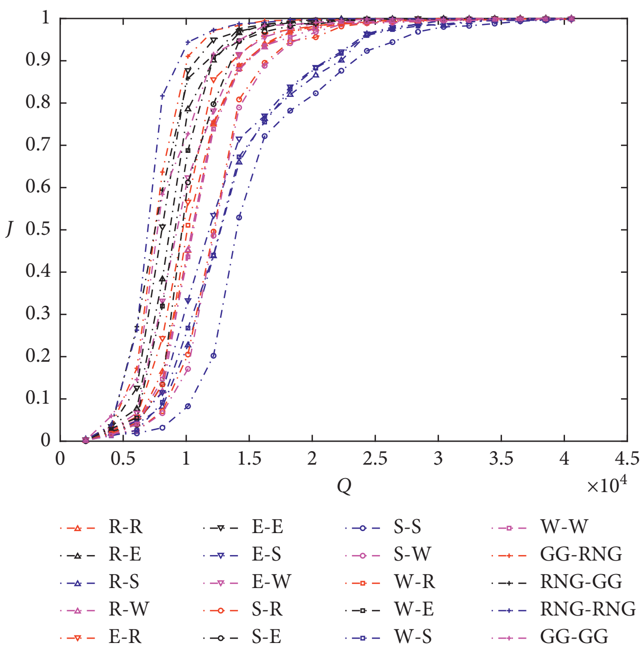


Figure 7: Randomly coupled multilayer network performance as the total traffic flow increases.

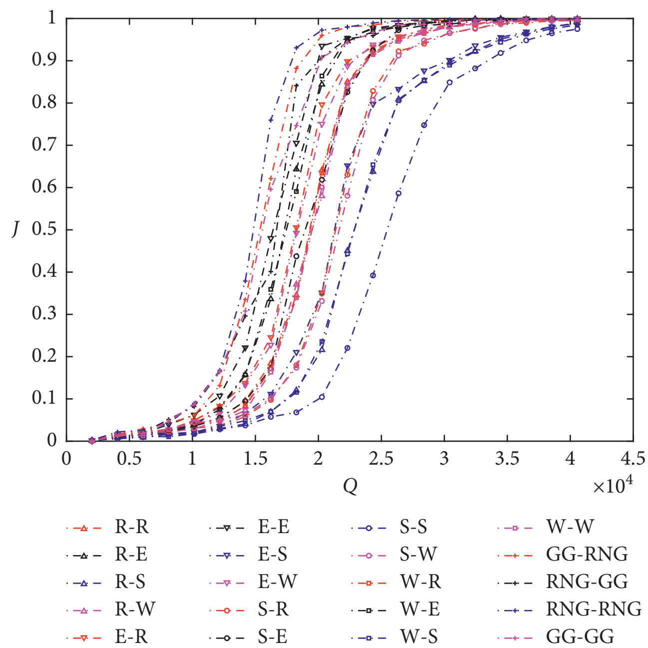


Figure 8: Completely coupled multilayer network performance as the total traffic flow increases.

If different layers are completely connected, as shown in Figure 8, when the traffic flow is lesser, the networks all run functionally. The figure is different from the randomly connected one, but the general trend does not change substantially. As the traffic flow increases, these scale-free multilayer networks have relatively strong ability to support more traffic flow and have higher anticongestion ability. The second group is the same, *R-S*, *E-S*, and *W-S*, with *S* as the upper-layer network. The next group includes networks with *S* as the lower-layer network. The characteristics of the other networks are similar and hold for less traffic flow.

4.2. Lower Layer and Upper Layer Have Different Numbers of Nodes. The number of nodes in the upper layer is normally smaller than that in the lower layer. The upper-layer nodes are randomly selected from the lower layer and $N^{\mathrm{U}} = 36$ and $N^{\mathrm{L}} = 400$. The total traffic flow Q is set to increase until approximately 4×10^4.

In the first condition, if different layers are randomly connected, as shown in Figure 9, multilayer networks with a scale-free network as the lower-layer network have relatively strong ability to support increased traffic flow, regardless of the type of upper-layer network. The remaining networks belong to the second group. The networks with random networks as the upper-layer networks have similar ability except *S-E*. Clearly, GG-RNG, RNG-GG, RNG-RNG, and GG-GG can support less traffic flow. When the total traffic flow Q reaches approximately 2×10^4, all the changing trends of these networks become stable and jammed.

Then, if the different layers are completely connected, as shown in Figure 10, the changing trends are similar as those of randomly coupled networks, but these networks can support more traffic flow. Additionally, a scale-free network as the lower-layer network maintains relatively strong support. The same, those networks with random networks as the upper-layer networks have similar ability except *S-E*. Correspondingly, GG-RNG, RNG-GG, RNG-RNG, and GG-GG can support less traffic flow.

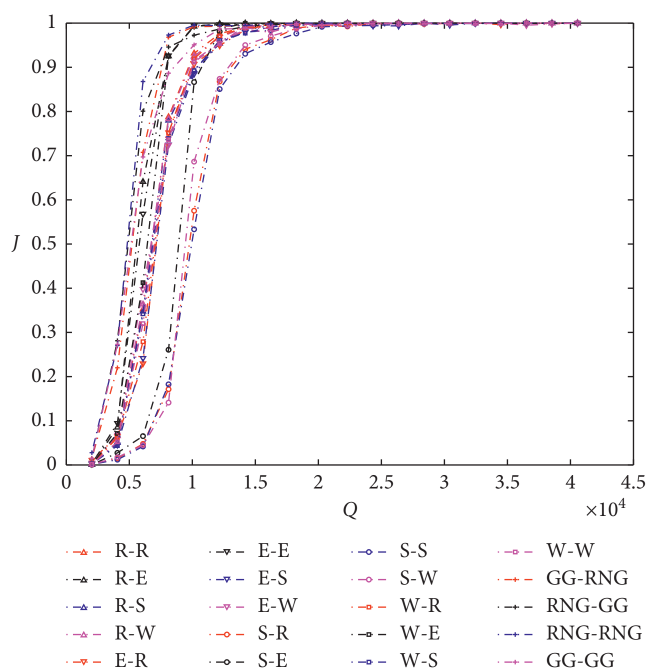


Figure 9: Randomly coupled multilayer network performance as the total traffic flow increases.

4.3. The Influence of Tunable Parameter τ. In the last section, we demonstrated the performance of network coupling

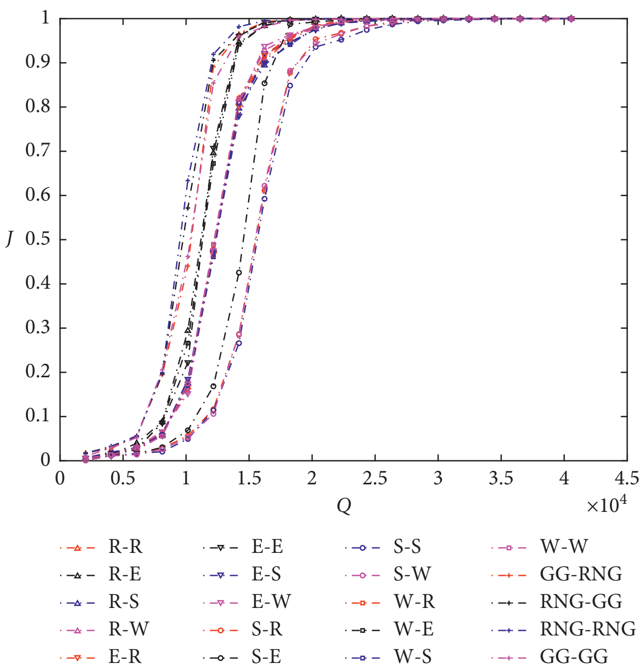


FIGURE 10: Completely coupled multilayer network performance as the total traffic flow increases.

methods in different situations with the same value of parameter τ. To further understand the influence of τ, we tested multilayer networks where the lower layer and upper layer have the same or different number of nodes, and we set $\tau \in [1, 2]$ to check the changing trend of the congestion factor J as Q increases.

First, multilayer networks with the same number of nodes and randomly connected layers are shown in Figure 11. Then, multilayer networks with the same number of nodes and completely connected layers are shown in Figure 12. Multilayer networks with different numbers of nodes and randomly connected layers are shown in Figure 13. Finally, multilayer networks with different numbers of nodes and completely connected layers are shown in Figure 14.

Clearly, as τ increases, the anticongestion ability of each multilayer network increases until reaching a stable plateau, and the figures become less compact, and the division of these groups became more clear.

4.4. The Influence of Tunable Parameter Θ. We set the link capacity of upper-layer links to Θ times that of the lower-layer links. To test the influence of the adjustable parameter Θ to the general system, we examined the anticongestion ability of the *S-S* network with different values of Θ, ranging from 1 to 30. Figure 15 shows that as Θ increases, the anticongestion ability of the *S-S* network generally increases. When $\Theta = 20$, the network has the highest anticongestion ability. As Θ increases further, the anticongestion ability decreases slightly and then continues to fluctuate.

5. Discussions and Conclusions

First, this paper reviews some recent studies on optimal network structure and analyzes research trends and hot spots. Although some studies on the optimal structure of multilayer networks have been conducted recently, most are based on very few types of networks, and the carrying capacity and anticongestion abilities of traffic networks are not considered [16, 31, 32]. Hence, this research is based on anticongestion ability and analyzes different traffic congestion conditions to test the network characteristics of various multilayer network structures to propose the optimal multilayer urban traffic network topology. Then, different basic network structures are described and introduced, namely, regular network (R), random graph (E), small-world network (W), scale-free network (S), relative neighbor graph (RNG), and Gabriel graph (GG), so that readers can gain a more comprehensive understanding of these traffic networks. The novelty of this study is that based on the coupling of these different network structures, the multilayer network properties are fully discussed, and the traffic impedance, a relatively novel research objective in the research of complex networks, is discussed. Although our study does not include all network models, it considers most of the combination of networks and is more comprehensive than previous studies. The representation of the results is more intuitive and convenient for traffic planners to use directly.

This study is based on simulation methods and using simulated data to test the properties of networks under different situations, which is in line with recent research trends. The results show that general network design is strongly related to the network topology of the different layers and their parameters and that the improved operation of existing networks also relies on these indicators. Large differences are observed when the upper-layer and lower-layer networks are randomly or completely connected. Additionally, for upper-layer and lower-layer networks with the same or different number of nodes, we have considered all these conditions and compared all the coupling methods for multilayer networks.

Scale-free multilayer networks have relatively strong ability to support more traffic flow. This means that the design of urban traffic networks should be based on scale-free multilayer networks. Four other network topologies, GG-RNG, RNG-GG, RNG-RNG, and GG-GG, are relatively weak, which means that we should avoid applying those designs. These measurement results have important guiding significance for urban traffic network planning and design. At present, the design of road routes is based mainly on the return on investment of a single line, so it is difficult to satisfy the needs of a comprehensive benefit analysis of traffic networks. Additionally, it is difficult to measure such factors such as the impact of new routes on the overall efficiency of the network, the change in users' travel habits, and the impact on the regional economy. When we have a deeper understanding of the structure of the network, we can design the road network more scientifically and rationally.

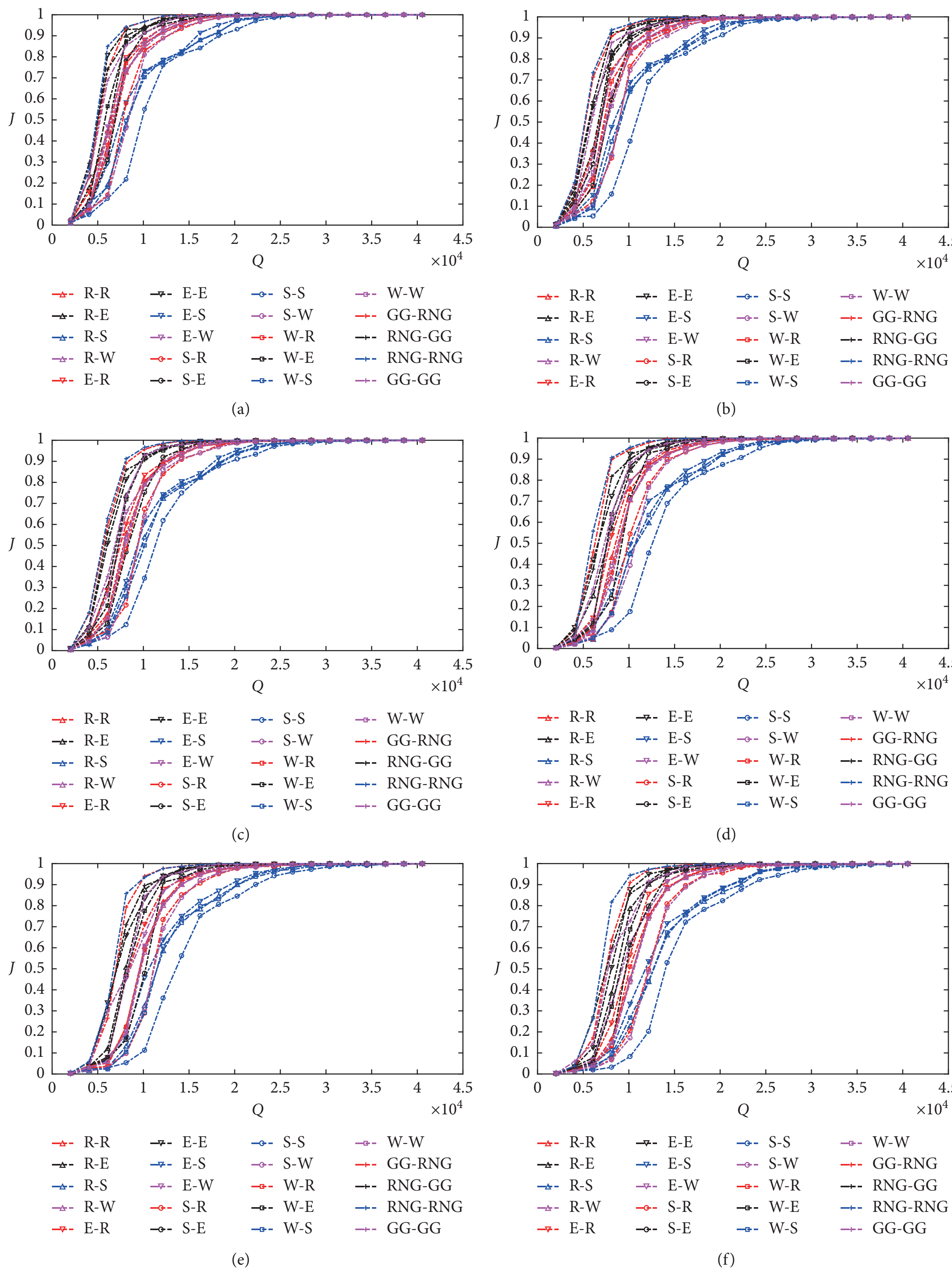


Figure 11: Continued.

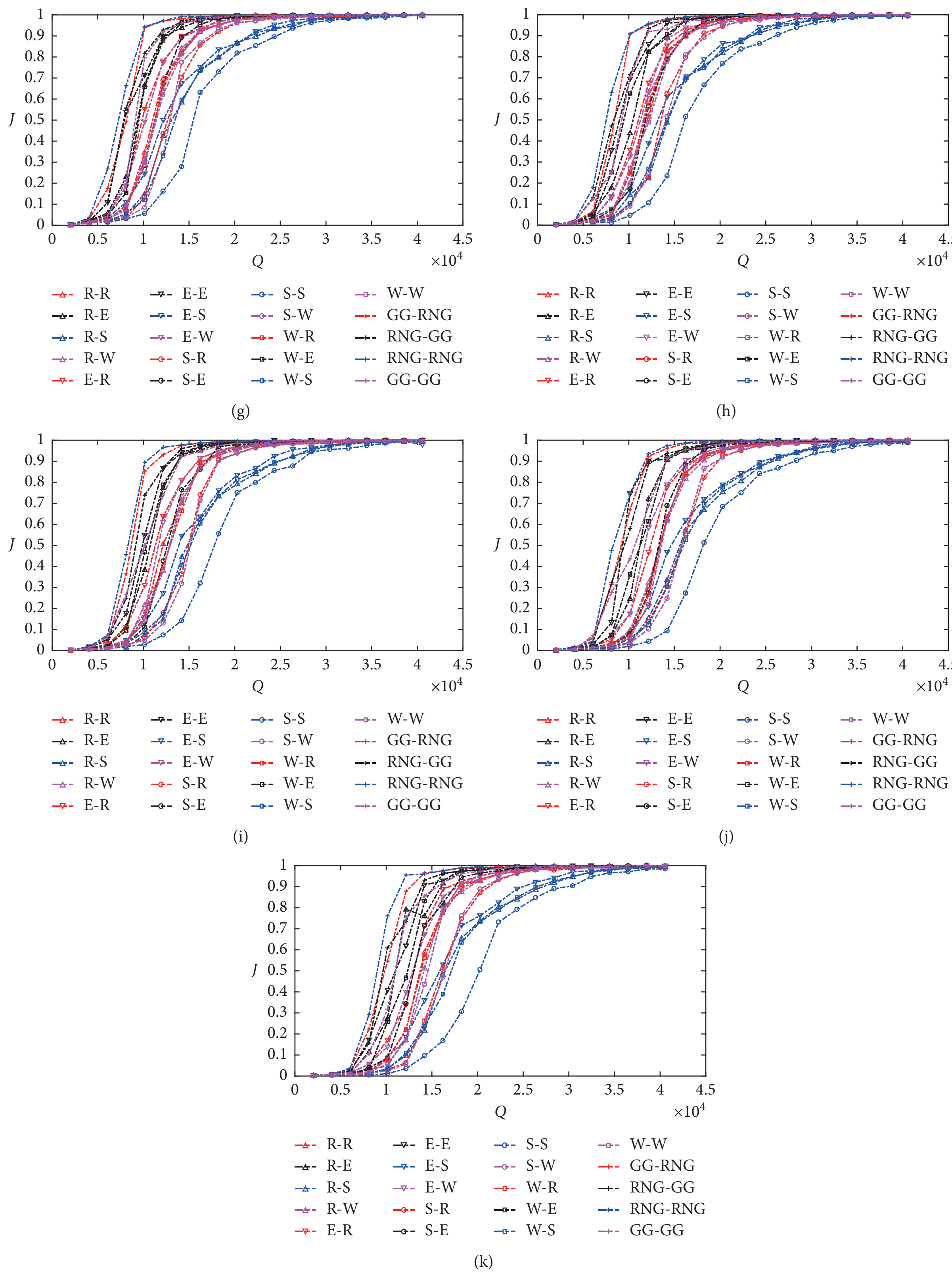
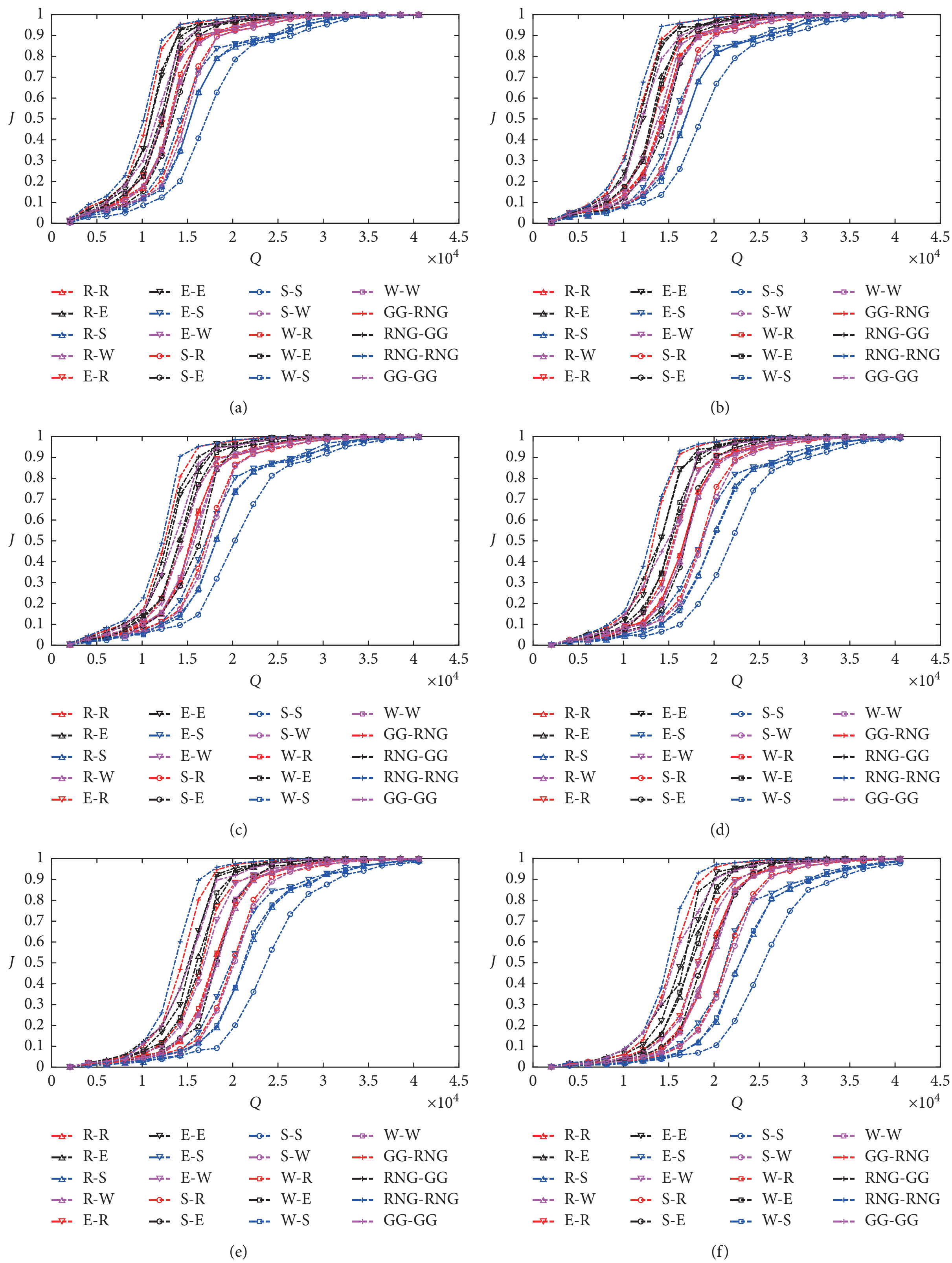


Figure 11: The influence of tunable parameter τ on multilayer networks (with the same number of nodes) when the networks are randomly connected. (a) $\tau = 1$. (b) $\tau = 1.1$. (c) $\tau = 1.2$. (d) $\tau = 1.3$. (e) $\tau = 1.4$. (f) $\tau = 1.5$. (g) $\tau = 1.6$. (h) $\tau = 1.7$. (i) $\tau = 1.8$. (j) $\tau = 1.9$. (k) $\tau = 2$.



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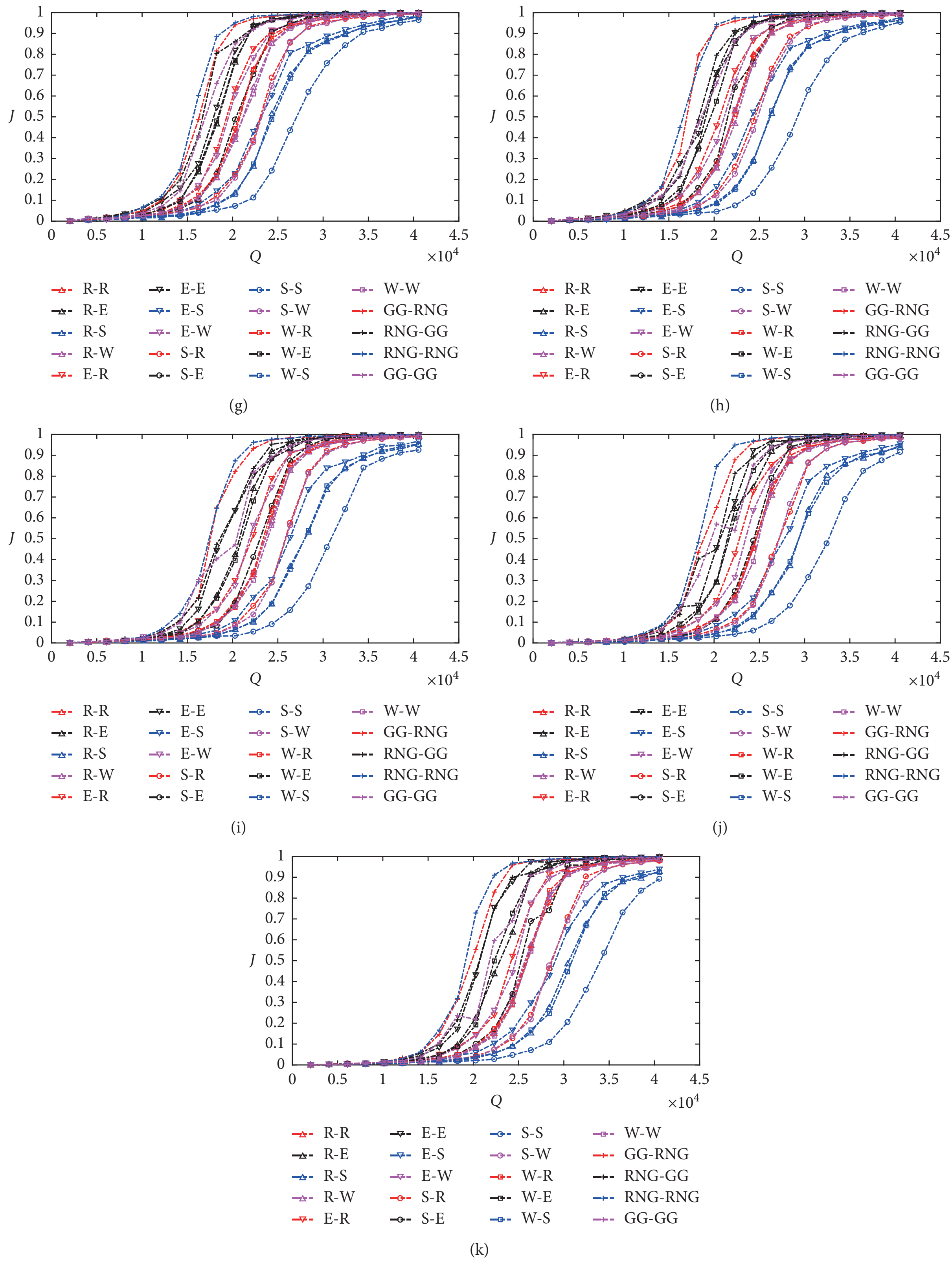
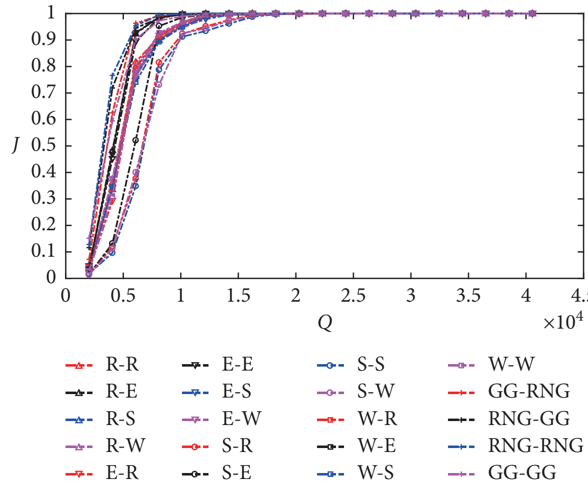
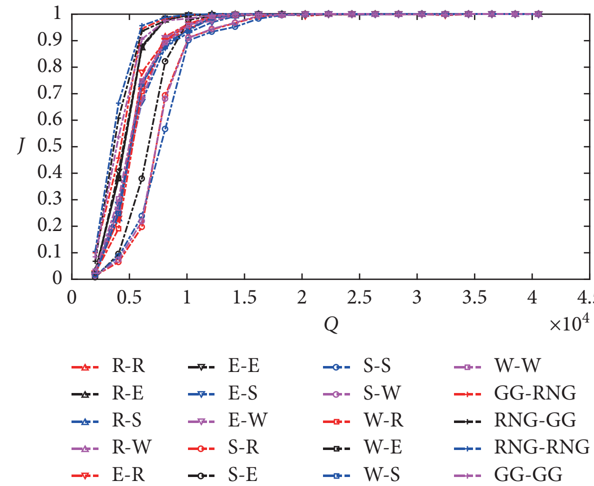


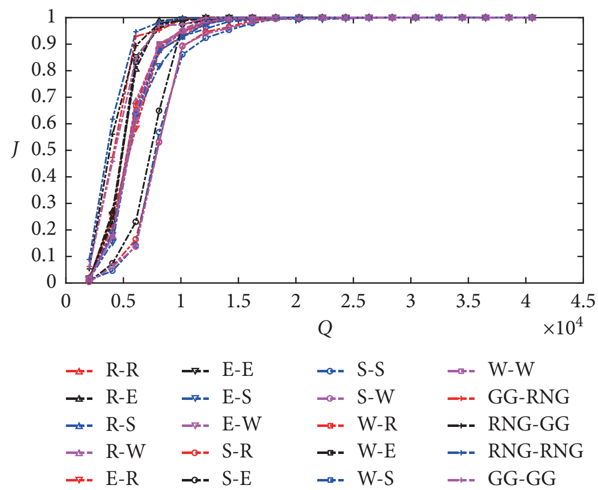
Figure 12: The influence of tunable parameter τ on multilayer networks (with the same number of nodes) when the networks are completely connected. (a) $\tau = 1$. (b) $\tau = 1.1$. (c) $\tau = 1.2$. (d) $\tau = 1.3$. (e) $\tau = 1.4$. (f) $\tau = 1.5$. (g) $\tau = 1.6$. (h) $\tau = 1.7$. (i) $\tau = 1.8$. (j) $\tau = 1.9$. (k) $\tau = 2$.



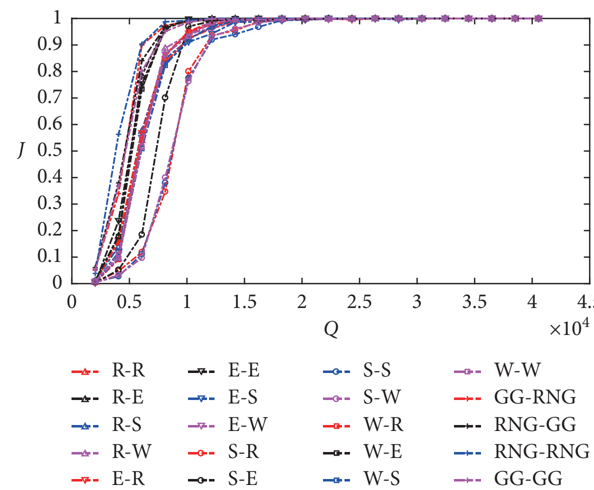
(a)



(b)



(c)



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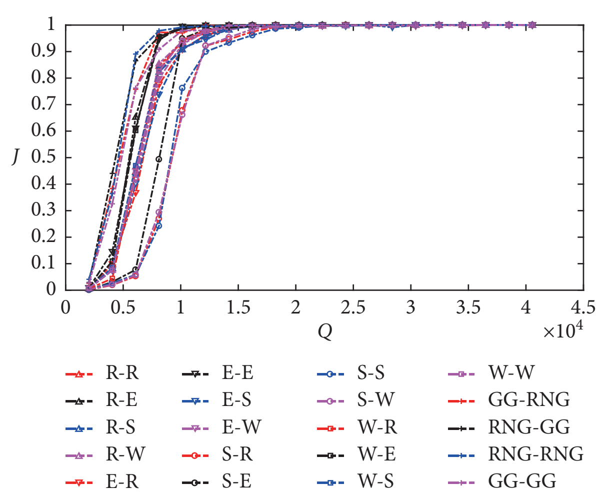

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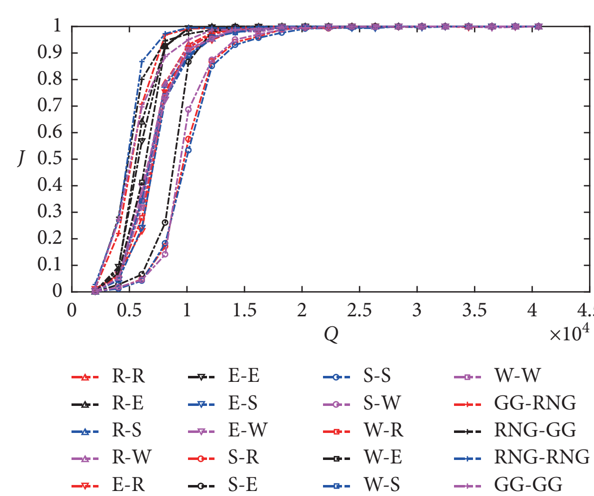

(f)

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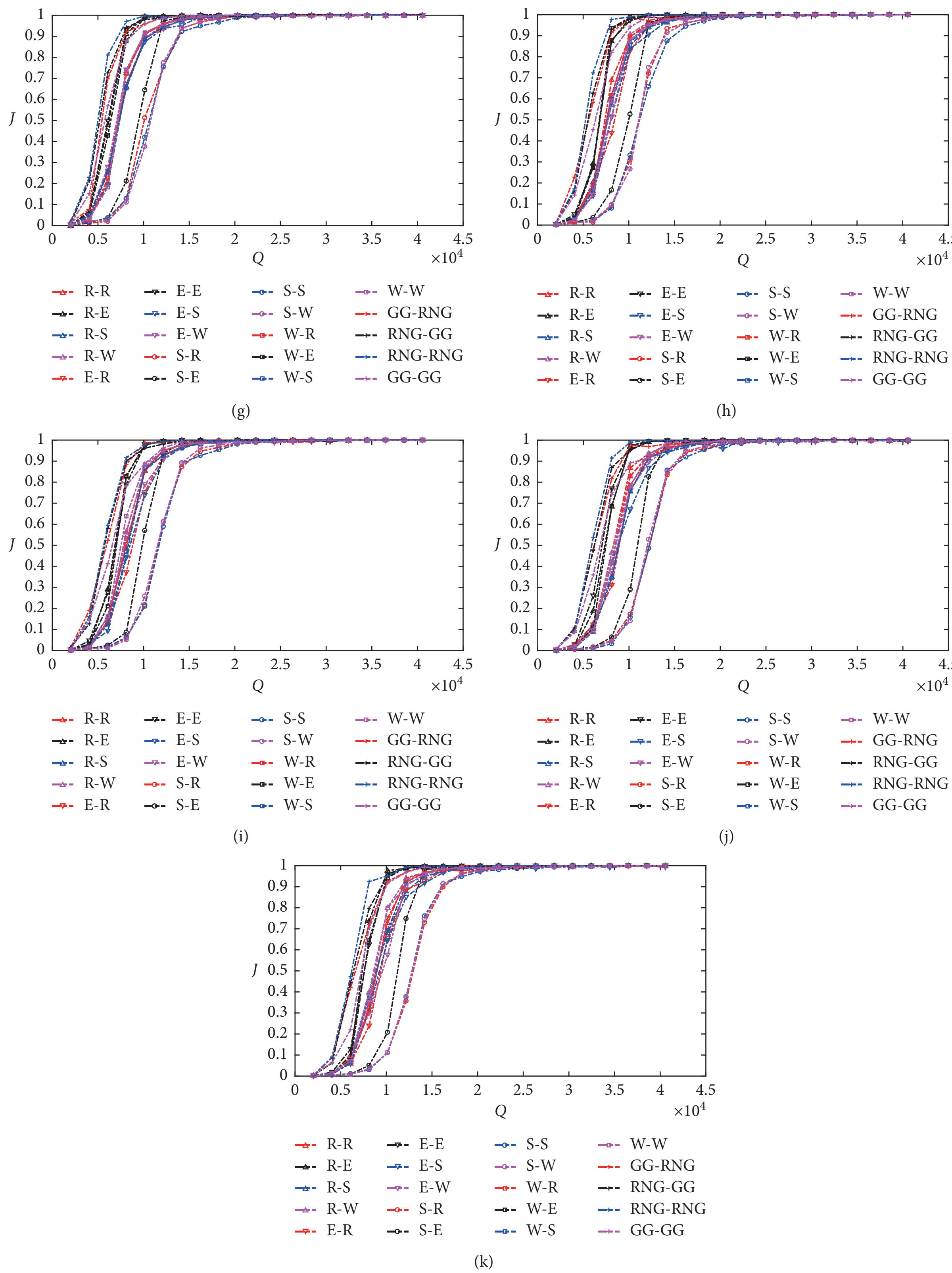
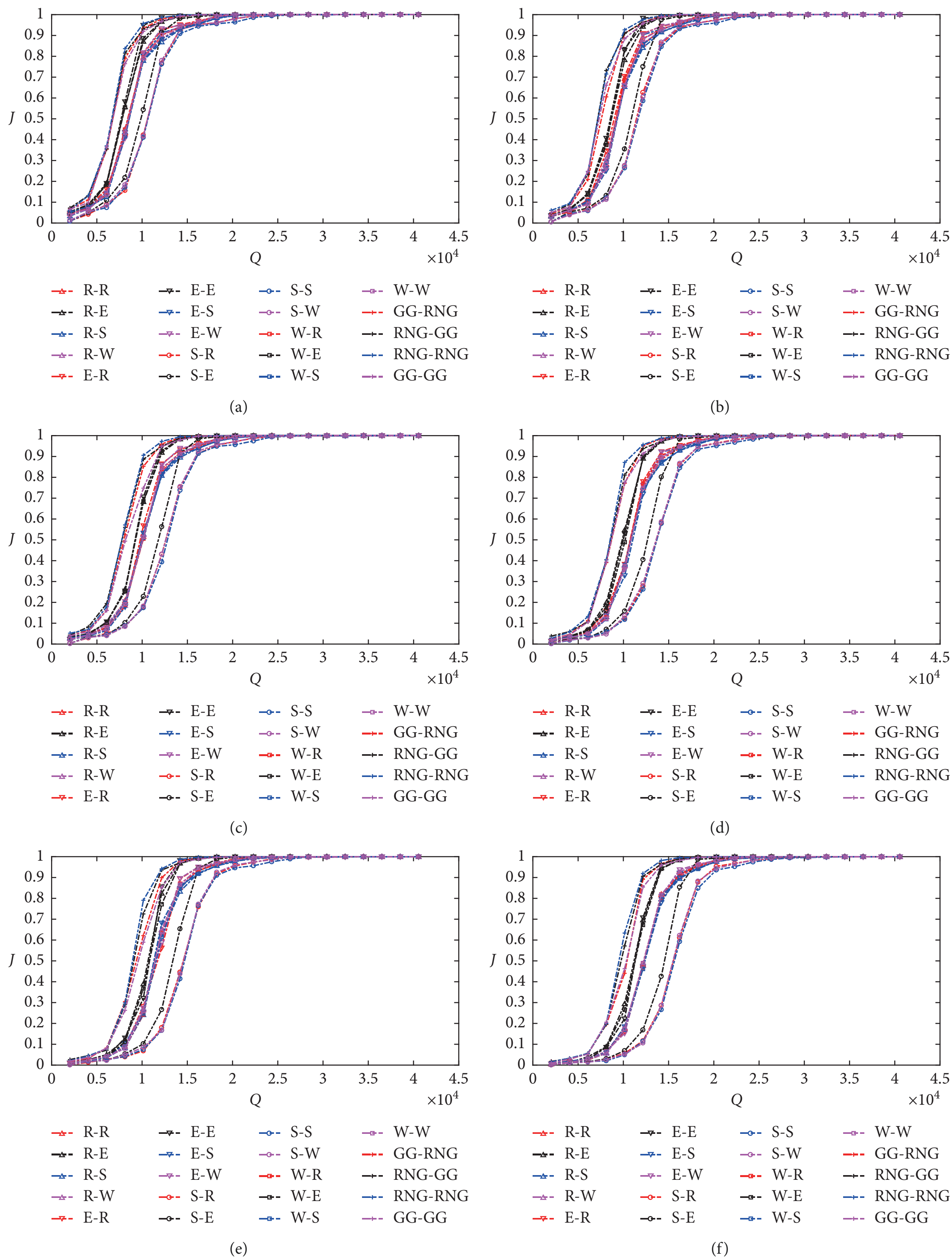


Figure 13: The influence of tunable parameter τ on multilayer networks (with different numbers of nodes) when the networks are randomly connected. (a) $\tau = 1$. (b) $\tau = 1.1$. (c) $\tau = 1.2$. (d) $\tau = 1.3$. (e) $\tau = 1.4$. (f) $\tau = 1.5$. (g) $\tau = 1.6$. (h) $\tau = 1.7$. (i) $\tau = 1.8$. (j) $\tau = 1.9$. (k) $\tau = 2$.



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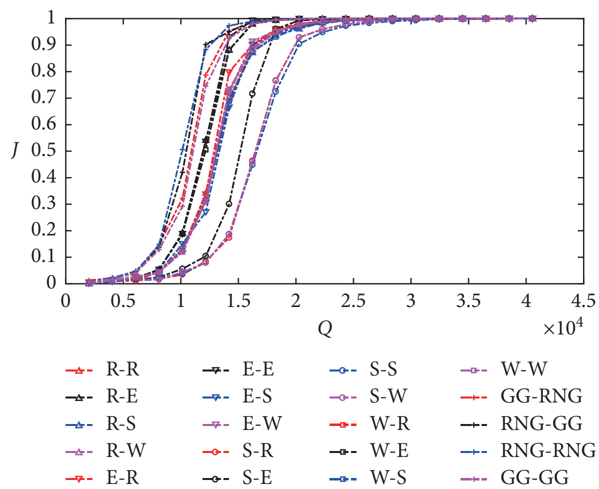
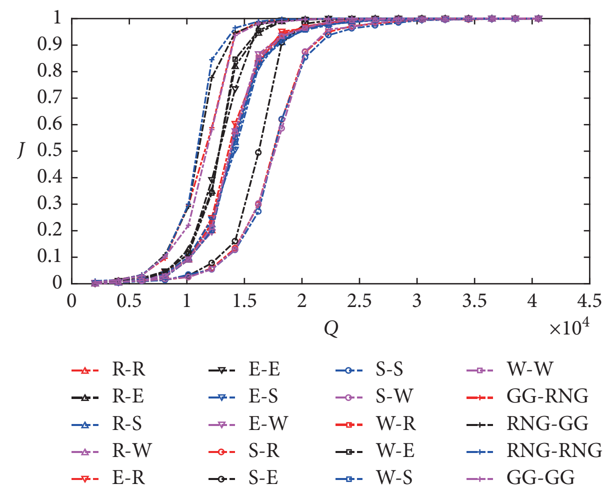
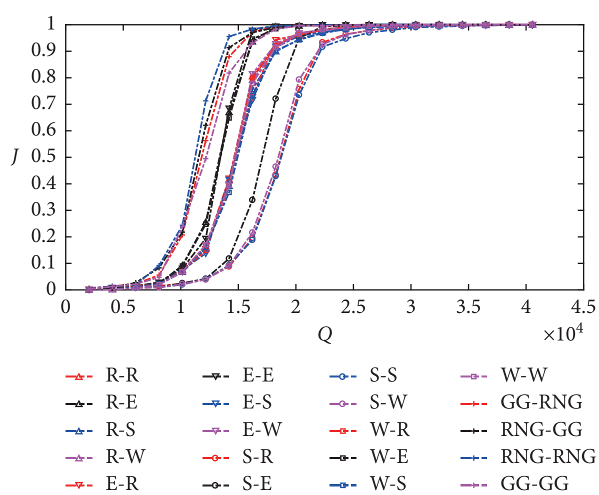


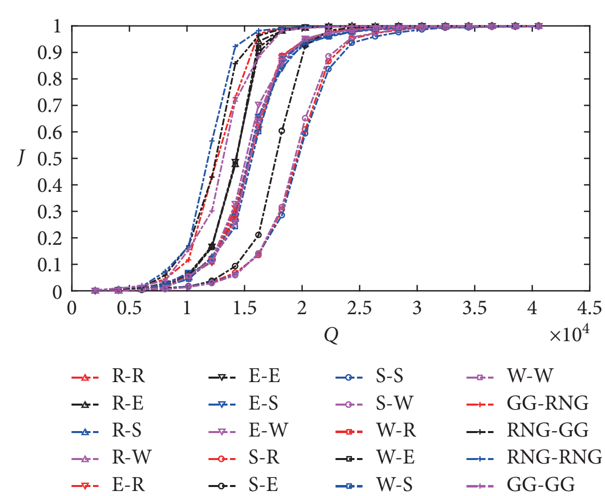
(g)



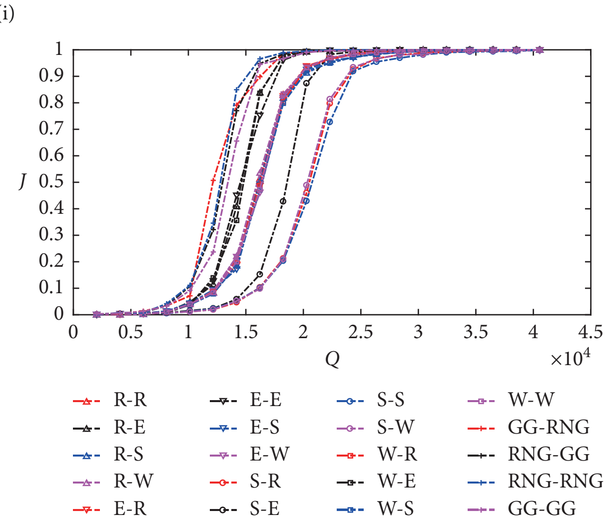
(h)



(i)



(j)



(k)

FIGURE 14: The influence of tunable parameter τ on multilayer networks (with different numbers of nodes) when the networks are completely connected. (a) $\tau = 1$. (b) $\tau = 1.1$. (c) $\tau = 1.2$. (d) $\tau = 1.3$. (e) $\tau = 1.4$. (f) $\tau = 1.5$. (g) $\tau = 1.6$. (h) $\tau = 1.7$. (i) $\tau = 1.8$. (j) $\tau = 1.9$. (k) $\tau = 2$.

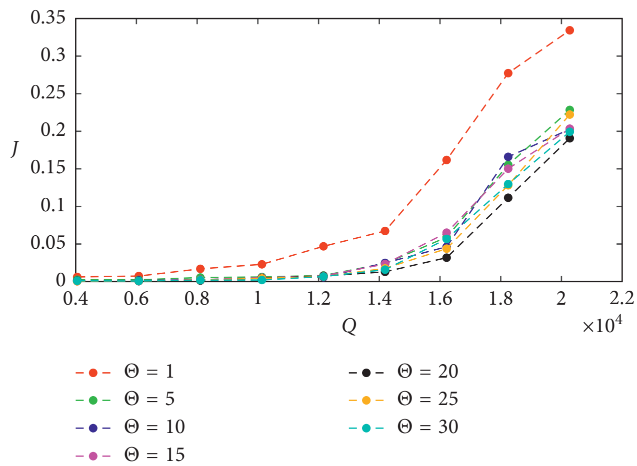


Figure 15: The influence of tunable parameter Θ.

We also considered the impact of tunable parameters τ and Θ. Regardless of the condition, as τ increases, the anticongestion ability of each multilayer network increases. For Θ, there exists a critical value, where before this value, the anticongestion ability increases but beyond this value, the anticongestion ability decreases slightly and then continues to fluctuate.

With a deeper consideration of the network topology, we can optimize multilayer networks. Additionally, this research deepens the understanding of the coupling relationship. Meanwhile, limited by the finite computational ability, we only calculated hundreds of traffic network nodes, which can partially represent the general trends and properties of multilayer networks. However, we still need more data to apply our analysis to a real project and further the optimization process. Furthermore, the assignment of traffic and link capacity should further consider the real location and distribution of urban populations.

Data Availability

The simulated data used to support the findings of this study are included within the article, and related codes are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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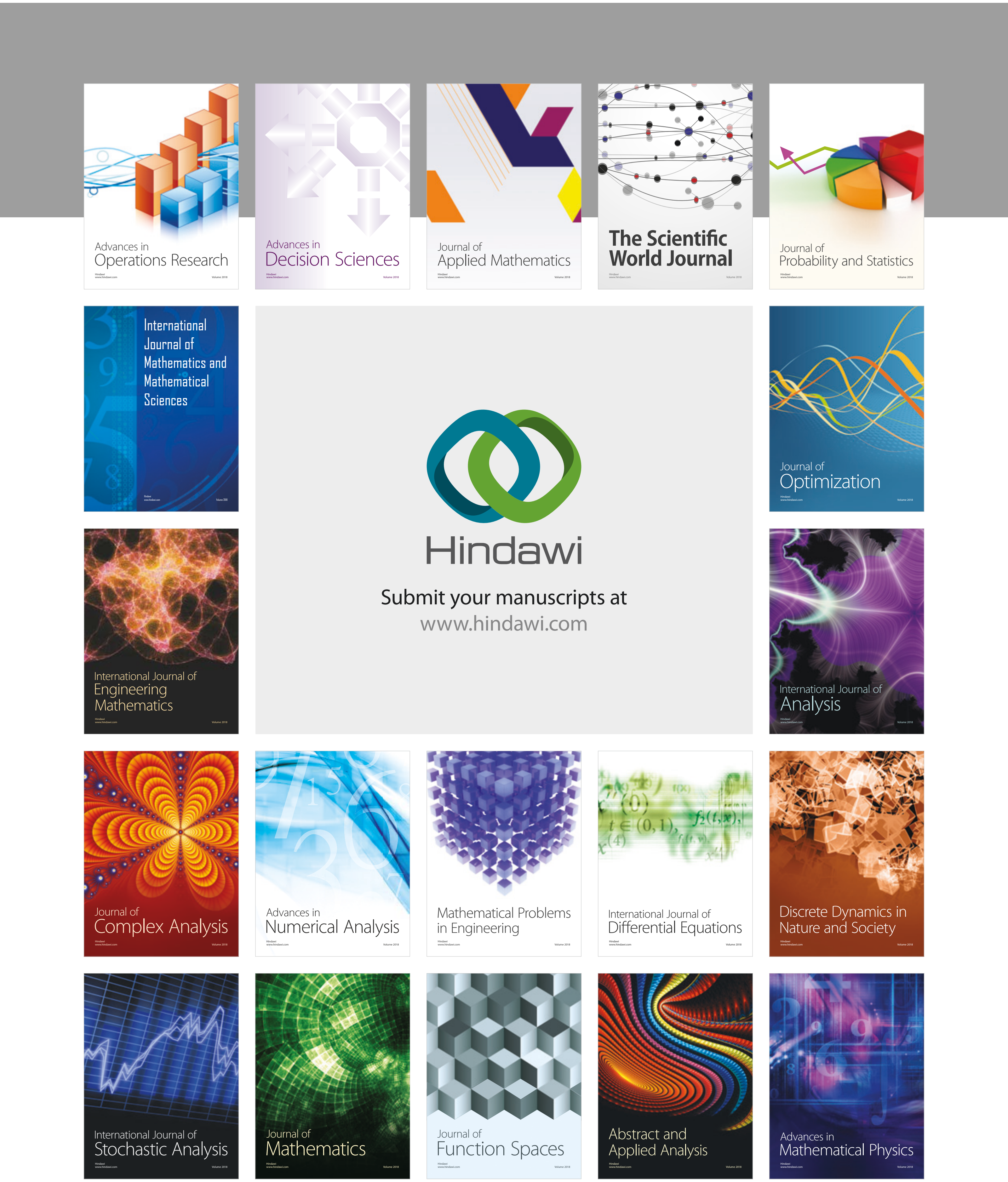
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