Meteorological data-based optimal control strategy for microalgae cultivation in open pond systems

Supplementary material

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S1 Model presentation

S1.1 Temperature model

The formal expression of each heat flow in Equation 8 was extracted from [1]. Assuming the water surface to be gray-diffuse, the radiation $Q_{ra,p}$ from the pond surface to the atmosphere was given by Stefan-Boltzmann's fourth power law:

$$Q_{ra,p} = -\varepsilon_w \sigma_{SB} T_p^4 S, \tag{S1-1}$$

where ε_w is the emissivity of water and σ_{SB} is the Stefan-Boltzmann constant (W m^{-2} K^{-4}). The heat flow associated with solar irradiance $Q_{ra,s}$ was expressed as follows:

$$Q_{ra,s} = (1 - f_a) H_s S, \tag{S1-2}$$

where f_a is the theoretical photosynthetic efficiency (the fraction of PAR that is effectively used during the photosynthetic process). The heat flow generated by air radiation was described as:

$$Q_{ra,a} = \varepsilon_a \varepsilon_w \sigma_{SB} T_a^4 S, \tag{S1-3}$$

where ε_a is the emissivity of the air and T_a is the air temperature (K). The evaporation heat flow Q_{ev} was given by the equation:

$$Q_{ev} = -m_e L_w S, \tag{S1-4}$$

where L_w is the water latent heat (J kg^{-1}), whereas m_e is the evaporation rate (kg m^{-2} s^{-1}). The following expression was used to calculate m_e:

$$m_e = K \left(\frac{P_w}{T_p} - \frac{RH \cdot P_a}{T_a} \right) \frac{M_w}{R}, \tag{S1-5}$$

where K is the mass transfer coefficient (m s^{-1}), P_w and P_a are, respectively, the saturated vapor pressure (Pa) at T_p and T_a, evaluated through the following empirical correlation:

$$P_i = 3385.5 \cdot e^{\left(-8.0929 + 0.97608 (T_i + 42.607 - 273.15)^{0.5}\right)}, \tag{S1-6}$$

where the i index represents air or water, RH is the relative air humidity above the pond surface, M_w is the molecular weight of water (kg mol^{-1}) and R is the universal ideal gas constant (Pa m^3 mol^{-1} K^{-1}). The mass transfer coefficient K in (S1-5) was calculated through the following two correlations:

$$Sh = 0.035 Re^{0.8} Sc^{1/3} \qquad \text{for turbulent flows,} \tag{S1-7a}$$

$$Sh = 0.628 Re^{0.5} Sc^{1/3} \qquad \text{for laminar flows,} \tag{S1-7b}$$

where $Sh = KL/D_{w,a}$, $Re = Lv_w/\nu_a$ and $Sc = \nu_a/D_{w,a}$. The dimensionless variables Sh, Re and Sc are, respectively, the Sherwood, Reynolds and Schmidt numbers, L is the characteristic pond length (m), $D_{w,a}$ is the mass diffusion coefficient of water vapor in air (m^2 s^{-1}), v_w is the wind velocity (m s^{-1}) and ν_a is the air kinematic viscosity (m^2 s^{-1}). The convective flow Q_{conv}, defined as:

$$Q_{conv} = h_{conv}(T_a - T_p)S \tag{S1-8}$$

was calculated by evaluating the heat transfer coefficient h_{conv} (W m^{-2} K^{-1}) value through the following set of correlations:

$$Nu = 0.035 Re^{0.8} Pr^{1/3} \qquad \text{for turbulent flows,} \tag{S1-9a}$$

$$Nu = 0.628 Re^{0.5} Pr^{1/3} \qquad \text{for laminar flows,} \tag{S1-9b}$$

where Nu=$h_{conv}L/\lambda_a$ and Pr=ν_a/α_a, λ_a is the air thermal conductivity (W m^{-1} K^{-1}) and α_a the air thermal diffusivity (m^2 s^{-1}). Meteorological stations measure the wind velocity at a certain height z_0 (m), which usually differs from the height at which v_w is needed in the previous correlations (0.5 m). The conversion of the wind velocity from the height z_0 to 0.5 m was performed by using the following expression:

$$v_w = v_0 \left(\frac{z}{z_0}\right)^{\alpha}, \tag{S1-10}$$

where v_0 is the wind velocity (m s^{-1}) measured at height z_0 and α is a power law exponent. The equation that describes the conductive heat flow between the pond and the soil was based on Fourier's law:

$$Q_{cond} = k_s S \frac{dT_s}{dz}(z = 0), \tag{S1-11}$$

where k_s is the soil conductivity (W m^{-1} K^{-1}) and T_s the soil temperature (K). The value of T_s was obtained from the following equation and initial/boundary conditions:

$$c_{p_s} \rho_s \frac{dT_s}{dt}(z, t) = k_s \frac{d^2T_s}{dz^2}(z, t) \tag{S1-12}$$

$$\begin{cases} T_s(t, z = 0) = T_p(t) & \text{b.c.(1)} \\ T_s(t, z = l_{s_{ref}}) = T_{s_{ref}} & \text{b.c.(2)} \\ \frac{d^2T_s}{dz^2}(t = 0) = 0 & \text{i.c.} \end{cases} \tag{S1-13}$$

where c_{p_s} is the soil specific heat capacity (J kg^{-1} K^{-1}), ρ_s is the soil density (kg m^{-3}), and $T_{s_{ref}}$ is the soil temperature (K) at the reference depth $l_{s_{ref}}$ (m). The heat flow associated with fresh medium inflow Q_{in} was computed from the equation:

$$Q_i = \rho_w c_{p_w} q^{in}(T^{in} - T_p) \tag{S1-14}$$

where T^{in} is the water inflow temperature (K). Finally, the rain heat flow Q_r was expressed as:

$$Q_r = \rho_w c_{p_w} v_r (T_a - T_p) S. \tag{S1-15}$$

All the parameters values used in the energy balance are tabulated in the study of [1] and reported in table S1-1.

Table S1-1: Parameter values of the energy balance

	Water Parameters	
ρ_w	water density	998 (kg·m^{-3})
c_{p_w}	water heat capacity	$4.18 \cdot 10^3$ (J·kg^{-1}·K^{-1})
L_w	water latent heat	$2.45 \cdot 10^6$ (J·kg^{-1})
ε_w	water emissivity	0.97 (-)
M_w	water molecular weight	0.018 (kg·mol^{-1})
	Soil Parameters	
k_s	soil thermal conductivity	1.7 (W·m^{-1}·K^{-1})
c_{p_s}	soil heat capacity	$1.25 \cdot 10^3$ (J·kg^{-1}·K^{-1})
ρ_s	soil density	$1.9 \cdot 10^3$ (kg·m^{-3})
$T_{s_{ref}}$	soil temperature at $l_{s_{ref}} = 4.5$m	286.75 (K)
	Air Parameters	
ε_a	air emissivity	0.8 (-)
ν_a	air kinematics viscosity	$1.5 \cdot 10^{-5}$ (m·s^{-1})
λ_a	air thermal conductivity	$2.6 \cdot 10^{-2}$ (W·m^{-1}·K^{-1})
α_a	air thermal diffusivity	$2.2 \cdot 10^{-5}$(m·s^{-1})
$D_{w,a}$	mass diffusion coefficient of water vapor in air	$2.4 \cdot 10^{-5}$ (m·s^{-2})
	Pond parameters	
V_0	initial pond volume	30 (m^3)
S	pond surface	100 (m^2)
L	pond characteristic length	10 (m)
α	power law exponent	0.29 (-)
z	wind velocity height	0.5 (m)
z_0	wind sensor height	10 (m)
f_a	algal absorption fraction	2.5 (%)
	Universal constants	
σ_{SB}	Stephan-Boltzmann constant	$5.67 \cdot 10^{-8}$ (W·m^{-2}·K^{-4})
R	ideal gas constant	8.314 (Pa·m^3·mol^{-1}·K^{-1})

S1.2 Determining the solar irradiance from cloudiness data

The solar irradiance H_s was calculated from cloudiness data CC, by using the Kasten and Czeplak correlation, recommended by [2] due to its high accuracy (the average error on solar irradiance estimates is generally within 20 W m^{-2}):

$$H_s = \begin{cases} 0 & \text{if } \omega < -\omega_s \text{ or } \omega > \omega_s \\ \frac{H_c(4-3(CC/8)^{3.4})}{4} & \text{if } -\omega_s \leq \omega \leq \omega_s, \end{cases} \tag{S1-16}$$

where CC is the cloudiness expressed in OKTAS (range 0÷8), H_c is the clear-sky total irradiance (W m^{-2}), ω is the hour angle which varies from -π to π over a day, $-\omega_s$ and ω_s are, respectively, the hour angle values at sunrise and sunset. The variable ω_s is calculated from the following equation:

$$\cos\omega_s = -\tan\delta\tan\phi, \tag{S1-17}$$

where ϕ is the local latitude and δ is the solar declination angle. The term δ was evaluated as follows ([3]):

$$\delta = 23.45\frac{\pi}{180}\sin\left(2\pi\frac{284+N}{365}\right), \tag{S1-18}$$

where N is the Julian day of the year ($N = 1$ for the first day of January). The clear-sky radiation H_c was given by ([3]):

$$H_c = H_{d,c} + H_{D,c} = (\tau_{d,c} + \tau_{D,c})H_0, \tag{S1-19}$$

where $\tau_{d,c} = H_{d,c}/H_0$ and $\tau_{D,c}$= $H_{D,c}/H_0$. $H_{d,c}$ and $H_{D,c}$ are, respectively, the diffuse and the direct components of the clear-sky total irradiance (W m^{-2}). The variable H_0, defined as the amount of solar radiation reaching the external surface of the atmosphere, was given by ([3]):

$$H_0 = I_{sc}\left(1 + 0.033\cos\frac{360N}{365}\right)\cos\theta_z, \tag{S1-20}$$

where I_{sc} is the solar constant (1367 W m^{-2}) and θ_z is the zenith angle (the angle between the vertical axis and the sun direction), evaluated through the expression:

$$\cos\theta_z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega. \tag{S1-21}$$

The variable $\tau_{D,c}$ was obtained through the following expression ([2]):

$$\tau_{D,c} = \exp(-\gamma T_{LK} A), \tag{S1-22}$$

where T_{LK} is an empirical coefficient (set to 2.74). The term γ was given by the equation:

$$\gamma = \frac{1}{9.4 + 0.9A} \tag{S1-23}$$

and A is the pressure-corrected air-mass, obtained from the following correlation ([2]):

$$A = \frac{1}{\cos\theta_z + 0.50572\left(96.07995 - \theta_z \frac{180}{\pi}\right)^{-1.6364}}. \tag{S1-24}$$

Finally, the variable $\tau_{d,c}$ is determined through the Erbs correlation (recommended by [3]):

$$K_d = \begin{cases} 1.0 - 0.09k_T & \text{if } k_T \leq 0.22 \\ 0.9511 - 0.1604k_T + 4.388{k_T}^2 + \\ -16.638{k_T}^3 + 12.336{k_T}^4 & \text{if } 0.22 < k_T \leq 0.8 \\ 0.16527 & \text{if } k_T > 0.8 \end{cases} \tag{S1-25}$$

where $K_d = \tau_{d,c}/(\tau_{d,c} + \tau_{D,c})$ and $k_T = \tau_{d,c} + \tau_{D,c}$.

S2 Meteorological data

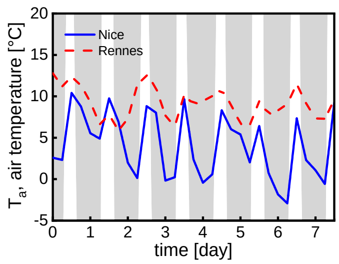
The weather data extracted from the European Centre for Medium-Range Weather Forecast (ECMWF) website were: the air temperature T_a, the sky cloudiness CC, the relative humidity RH, the wind velocity v_w and the rain volumetric flux v_r during the simulated cultivation period. Figures S2-1, S2-2 and S2-3 represent, respectively, the implemented meteorological data for the Winter, Spring and Summer case studies.

Bibliography

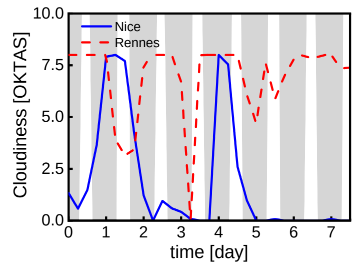
[1] Q. Béchet, A. Shilton, J. B. K. Park, R. J. Craggs, B. Guieysse, Universal temperature model for shallow algal ponds provides improved accuracy, Environ. Sci. Technol. 45 (2011) 3702–3709. doi:10.1021/es1040706.

[2] T. R. Marthews, Y. Malhi, H. Iwata, Calculating downward longwave radiation under clear and cloudy conditions over a tropical lowland forest site: an evaluation of model schemes for hourly data, Theor. Appl. Climatol. 107(3-4) (2012) 461–477. doi:10.1007/s00704-011-0486-9.

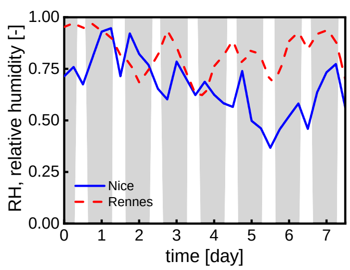
[3] J. A. Duffie, W. A. Beckman, Solar Engineering of Thermal Processes, Little, Brown & Co., Boston, 1958.



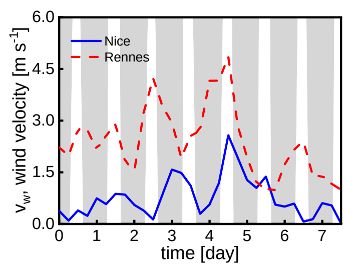
(a) Winter: Air temperature T_a



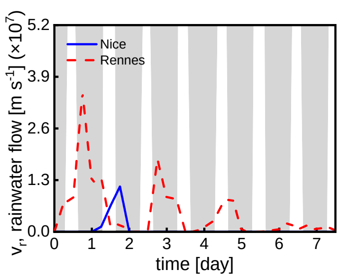
(b) Winter: Cloudiness



(c) Winter: Relative humidity RH

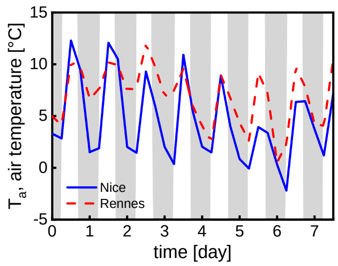


(d) Winter: Wind velocity

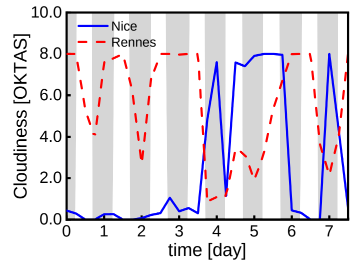


(e) Winter: Rain

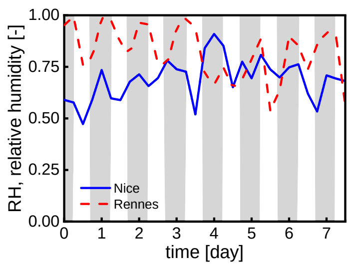
Figure S2-1: Meteorological data: Winter case (Plain line: Nice; dashed line: Rennes. The background is colored in white at daytime and in grey at night-time.)



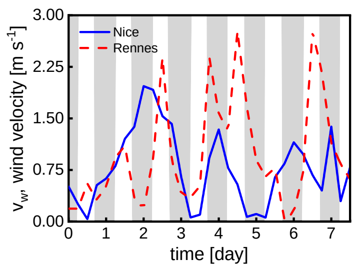
(a) Spring: Air temperature T_a



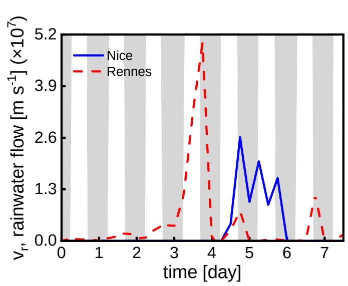
(b) Spring: Cloudiness



(c) Spring: Relative humidity RH

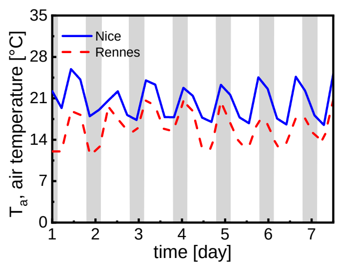


(d) Spring: Wind velocity

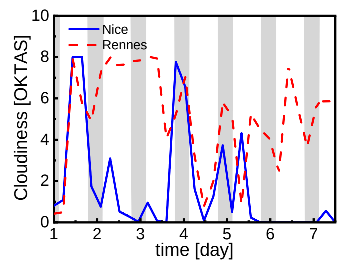


(e) Spring: Rain

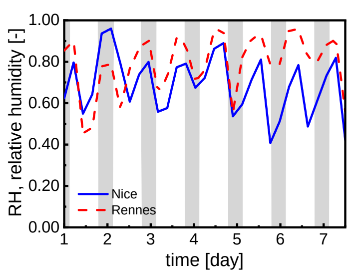
Figure S2-2: Meteorological data: Spring case (Plain line: Nice; dashed line: Rennes. The background is colored in white at daytime and in grey at night-time.)



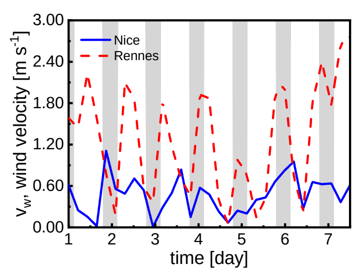
(a) Summer: Air temperature T_a



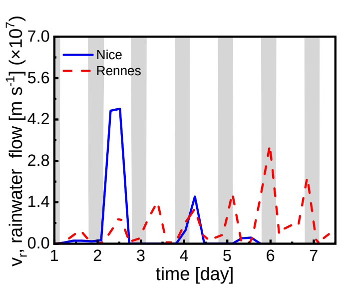
(b) Summer: Cloudiness



(c) Summer: Relative humidity RH



(d) Summer: Wind velocity



(e) Summer: Rain

Figure S2-3: Meteorological data: Summer case (Plain line: Nice; dashed line: Rennes. The background is colored in white at daytime and in grey at night-time.)