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Research Article

Product Line Pricing under Marginal Moment Model with Network Effect

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The complex network effect of the product exhibits has a significant impact on the product line optimization design. The multinomial logit (MNL) model which is used to simulate consumer choice behavior is applied in most of product line optimization problems. However, its assumptions, independence of irrelevant alternatives (IIA) and the same Gumbel distribution of random error terms, are usually difficult to be met in practice. The marginal moment model (MMM) can be used when the mean and variance of consumer's utility error are known. The MMM not only has weak assumption conditions but also overcomes the IIA problem of MNL model. In this paper, we study the product pricing problem based on MMM with endogenous negative network effect. Firstly, we construct a variant of MMM considering network effect in product line optimization design. Secondly, we prove that the revenue function is concave in market share. We propose the solving methods of the model to obtain the optimal price, the corresponding market share, and the revenue under three different scenarios, i.e., developing single product, homogeneous products, and heterogeneous products. Finally, numerical experiments show that the proposed model can better simulate consumer choice behavior and potentially increase revenue.

1. Introduction

With the rapid development of Internet economy, the traditional consumption structure has been changed greatly, and the research on the influence of complex network effect for development and pricing of new products has become an important academic issue. Many products will exhibit network effect or network externality when the utility a consumer obtains from buying the products depends on the number of sales of the same or similar products [1]. In traditional process of product line pricing, it is assumed that utility is only determined by customer's personal features. However, if a product line has the characteristic of network effect, firms need to analyze how a customer's purchasing behavior is influenced by other customers and reassess the estimated market size. Since product price is directly related to utility and market size affects the cost structure of the product line, network effect also influences the process of pricing and quality design of a product line. For these firms that develop new products with network effect, a significant challenge is to simulate consumer choice behavior that incorporates network effect and use this model to design a product line to maximize revenue.

The network effect usually includes two aspects: positive network effect and negative network effect. For example, there are many products that display positive network effect, e.g., online games, social software (QQ, WeChat, Facebook, and microblog), online movies, and group purchase products. The more people that buy them or use them, the more utilities the consumers can get from buying these items. There are also some products that exhibit negative network effect, for example, the choice of traffic modes or the selection of personalized customers for products. If a firm produces a product line of women's bags or clothes, customers with personalized requirements may be reluctant to buy the products when more people buy the same or similar products in the product line. If there are too many people buying cars in a small city and resulting in traffic congestion, the utility of owning a car will decrease. If the customers of some luxury goods feel that these goods are possessed by too many consumers, the value or attractiveness of these goods will decline. The more people choose these products or services, the less utility the consumers who choose the same or similar items will obtain.

Many firms provide a product line with a variety of products that display network effect to meet different consumers' needs and obtain maximum profit or revenue. To design a product line, a firm needs to consider a lot of factors, such as pricing decision, quality of product, development cost, price sensitivity of consumer, and network effect.

In the process of product line design, consumer choice model [2] is vital to solve the product line optimization problem. There are two kinds of consumer choice models: one is deterministic choice model and the other is probabilistic choice model. Because probabilistic choice model can simulate consumer choice behavior more practically, it has been widely applied in marketing, operation management, transportation, and other fields.

Because of the simple formula and good simulation results, the multinomial logit (MNL) model [3] is one of the most widely used probabilistic choice models. However, its assumptions, independence of irrelevant alternatives (IIA) and the same Gumbel distribution of random error terms, are difficult to be met in practice. The IIA property shows that the ratio of choice probabilities of two alternatives does not depend on other alternatives, regardless of their similarities. This is usually exemplified by the famous "red bus/blue bus" paradox [1]. Some other probabilistic choice models, such as nested logit (NL) model, multinomial probit (MNP) model, or mixed logit (MIXL) model, are also limited in application due to the complex calculation process or pregiven specific distributions of measuring errors.

To better simulate consumer choice behavior, Natarajan et al. [4] propose a semiparametric choice model which is called as marginal moment model (MMM). The MMM can be applied when both the mean and the variance of utility error are known. The model does not need to know the specific distribution of error in advance and can also overcome the shortcoming of IIA in the MNL model; hence the MMM has better applicability in practice [5].

In the research of product pricing problem, many scholars study the influence of positive network effect on consumer choice behavior based on MNL model [6–9], and it has been found that the optimal prices may be different even for homogeneous products case. However, the issue of product optimization with considering negative network effect cannot be ignored and needs to be further solved. In many product line design optimization problems, there exist a lot of products with negative network effect [1] and the problem of IIA may not be met. In this case, consumer choice behavior simulation and product pricing would be very interesting practical issues. The network effect is usually endogenized in the utility of consumer choice model, and the MMM that overcomes IIA problem can better simulate consumer choice model; therefore, the MMM with network effect can solve the product line pricing problem better. This paper focuses on the influence of the consumer choice behavior based on MMM with endogenous negative network effect on product line design problem. The contributions of this research are given as follows:

(1) We apply the MMM with negative network effect to simulate consumer choice behavior in product line optimization problem. For the utility of consumer, we add a network effect term consisting of a coefficient of network effect and the sale quantity of the product.

(2) Under the scenarios of single product, homogeneous products, or heterogeneous products, we prove the concavity of revenue function in market share with considering the network effect and present the solving equations for the optimal price, the optimal market share, and the optimal revenue, respectively.

(3) By performing numerical experiments, we first demonstrate the variation of the optimal solutions with different parameters including network effect parameter, quality of product, price sensitivity parameter, and variance of utility error term. We then analyze the importance of considering network effect when a product line is designed. Finally, to illustrate the applicability of the proposed model, we test the robustness of solutions when the estimation of network effect parameters has some errors.

The remainder of this paper is organized as follows. In Section 2, we review the related literature papers about product line pricing and network effect. In Section 3, we propose a marginal moment model with negative network effect to simulate consumer choice behavior in product line design and present the solving methods of the optimal solutions. In Section 4, we present the results of some numerical experiments. In Section 5, we conclude the paper.

2. Literature Review

In this section, we review the literature papers related to this paper, including the product line pricing considering consumer choice behavior and the study of products with network effect.

2.1. Product Line Pricing. There are a lot of literature papers studying product pricing in the product line optimization design [10, 11]. The optimization objective is mainly maximizing profit [12, 13]. There are also some literatures which aim at obtaining maximin utility or minimax regret [14, 15]. And price is usually set as endogenous variable [16, 17].

Product line pricing problem is designed in different competitive market. For a monopoly company, Heese and Swaminathan [18] analyzed a stylized model of product line design with component commonality to reduce production costs. They showed the strategy could produce the products of higher quality and acquire higher revenue. In a bilateral monopoly market, Dong et al. [19] investigated the product line pricing with two-stage game. Different from the previous study, they extended to the complementary products and endogenized the price for both the retailer and the manufacturer. Under oligopoly, Yayla-Küllü et al. [20] analyzed how the capacity constraint and competition affected the product-mix decisions. They found that when the resources were limited, the firm should only offer the product that had the largest profit per unit of capacity.

Moreover, the simulation of consumer choice behavior is also vital for product line optimization design. Since McFadden [3] proposed multinomial logit (MNL) model, the model has been widely applied in many fields, such as marketing, economics, operation management, and transportation. But its restrictive assumptions, the independence of irrelevant alternatives (IIA) and the same Gumbel distribution of random error terms, are hard to be met in reality. Subsequently, some other probabilistic choice models are presented, for example, the nested logit (NL) model [21], multinomial probit (MNP) model [22], generalized extreme value (GEV) model [23], and mixed logit (MIXL) model [24].

Recently, some researchers propose new probabilistic choice models. Kim et al. [25] developed a probit choice model under sequential search, which avoided the computation of high-dimensional integrations for partial analytic model. To address the problem of assortment optimization, Blanchet et al. [26] introduced a Markov chain choice model, which was a good approximation for general choice model under mild assumptions. To alleviate the IIA property of MNL model, Natarajan et al. [4] proposed two kinds of semiparametric choice models—marginal distribution model (MDM) and marginal moment model (MMM). They could be used when only the distribution or mean and variance of the utility error were known. Mishra et al. [27] further studied the theoretical characteristic of MDM. They proved that MDM generalized the MNL, MMM, and GEV models.

2.2. Network Effect. There are two streams of researches on the products with network effects. One is global network effect, where the utility consumers get from buying a product depends on the total sales of that product; the other is local network effect, where the consumer's utility only depends on the number of purchases made by his "neighbors". Our work only considers global network effect. For local network effect, please refer to Candogan et al. [8], Bloch and Quérou [28], and references therein for recent researches.

The research on network effect mainly focuses on the global network effect [29, 30]. For new product pricing problem, Du et al. [9] established the MNL model with network effect. The authors analyzed the properties of the optimal market share and the optimal price for homogeneous coefficients and heterogeneous coefficients case. They found that the optimal prices might be different when network effect existed even for homogeneous case. Different from the literature Du et al. [9], Wang and Wang [1] investigated the assortment optimization problem under MNL with endogenous network effects. They found that the quasi-revenue-ordered assortment was optimal under certain conditions. In addition, they showed that it was very necessary to consider the network effect if such effects did exist.

3. Product Line Design Based on MMM with Network Effect

Natarajan et al. [4] proposed a marginal moment model (MMM). Assume that there are n $(i = 1, \ldots, n)$ products to be developed to maximize the revenue. We assume that the competitors, in the short run, do not respond to the firm's new products [31, 32]. The market has m $(j = 1, \ldots, m)$ potential consumers. If consumer j purchases product i, the consumer has a utility u_{ij} and

$$u_{ij} = v_{ij} + \varepsilon_{ij}, \tag{1}$$

where v_{ij} is the deterministic component of the utility from the observed product and consumer attributes and ε_{ij} is a random variable that represents consumer-specific idiosyncrasies. Assume that we know the mean 0 and variance σ_{ij}^2 of the random error; that is, the utility u_{ij} has the mean v_{ij} and variance σ_{ij}^2 and the choice probability (market share) is given as [4, 33]

$$q_{ij} = \frac{1}{2}\left(1 + \frac{v_{ij} - \lambda_j}{\sqrt{\left(v_{ij} - \lambda_j\right)^2 + \sigma_{ij}^2}}\right), \tag{2}$$

where the Lagrange multiple λ_j is found by solving

$$\sum_{i=1}^{n} \frac{1}{2}\left(1 + \frac{v_{ij} - \lambda_j}{\sqrt{\left(v_{ij} - \lambda_j\right)^2 + \sigma_{ij}^2}}\right) = 1. \tag{3}$$

In our model, we consider a variant of MMM that incorporates network effects in consumer's utilities. The consumer's utility that is similar to the literature of Du et al. [9] is decided by the quality of product y_i, its price p_i, its overall consumption x_i, price sensitivity parameter b_{ij} $(b_{ij} \geq 0)$, and the network effect sensitivity parameter α_{ij}, and

$$v_{ij} = y_i - b_{ij} p_i + \alpha_{ij} x_i. \tag{4}$$

The parameter α_{ij} represents the strength that consumer j's utility is affected by the overall consumption of product i. The larger the parameter α_{ij}, the more sensitive the network effect of consumer purchasing the product. The network effect on v_{ij} only depends on the total consumption level of product i. Throughout, we assume that $\alpha_{ij} \leq 0$, and we normalize the total market size to 1 $(m = 1)$. Thus the choice probability q_i is equal to the total market consumption of product i, i.e., $x_i = q_i$. (For a general m, $x_i = mq_i$, we can redefine $\widetilde{\alpha}_i = m\alpha_i$ and the two problems are equivalent [9].) And we have $v_i = y_i - b_i p_i + \alpha_i q_i$ and then the choice probability is defined as

$$q_i = \frac{1}{2}\left(1 + \frac{y_i - b_i p_i + \alpha_i q_i - \lambda}{\sqrt{\left(y_i - b_i p_i + \alpha_i q_i - \lambda\right)^2 + \sigma_i^2}}\right). \tag{5}$$

In model (5), network effect is endogenized in the utility of MMM in the form of network strength parameter. Each consumer buys at most one product. Product 0 indicates a no-purchase option in the product line or an external option. For the no-purchase option, the utility is v_0 and the variance of error term is σ_0^2. Meanwhile, both parameters v_0 and σ_0 are known. The choice probability of no-purchase option is

$$q_0 = \frac{1}{2}\left(1 + \frac{v_0 - \lambda}{\sqrt{(v_0 - \lambda)^2 + \sigma_0^2}}\right). \quad (6)$$

By (6), we have $\lambda = v_0 - (1/2)\sigma_0(2q_0 - 1)\cdot(q_0 - q_0^2)^{-1/2}$. And by (5), the price of product i is defined by

$$\begin{aligned} p_i = \frac{1}{b_i}\Big[& y_i + \alpha_i q_i - v_0 + \frac{1}{2}\sigma_0(2q_0 - 1)\cdot(q_0 - q_0^2)^{-1/2} \\ & - \frac{1}{2}\sigma_i(2q_i - 1)(q_i - q_i^2)^{-1/2}\Big], \end{aligned} \quad (7)$$

where $\sigma_i > 0$ and $\sigma_0 > 0$. The development costs of all products are assumed to be 0 [9]. Defining $\mathbf{q} = (q_0, q_1, \ldots, q_n)$, the firm's total revenue is

$$\begin{aligned} \pi(\mathbf{q}) &= \sum_{i=1}^{n} q_i \cdot p_i(\mathbf{q}) \\ &= \sum_{i=1}^{n}\frac{\alpha_i}{b_i}q_i^2 + \sum_{i=1}^{n}\frac{q_i}{b_i}\cdot\Big[y_i - v_0 + \frac{\sigma_0}{2}(2q_0 - 1) \\ &\quad \cdot (q_0 - q_0^2)^{-1/2}\Big] \\ &\quad - \sum_{i=1}^{n}\frac{\sigma_i}{2b_i}(2q_i^2 - q_i)(q_i - q_i^2)^{-1/2}. \end{aligned} \quad (8)$$

The product line optimization problem is stated as

$$\begin{aligned} \max \quad & \pi(q_0, q_1, q_2, \ldots, q_n) \\ s.t. \quad & \sum_{i=1}^{n} q_i + q_0 = 1 \\ & q_i, q_0 \geq 0 \quad (i = 1, 2, \ldots, n). \end{aligned} \quad (9)$$

In the following sections, we study that how network effects impact the optimal prices, the corresponding market shares, and the optimal revenues in different scenarios, including developing one new product, homogeneous scenarios of multiple products with the same coefficients, and heterogeneous cases of multiple products with different parameters.

3.1. Pricing for One Product. In this section, we will investigate the product pricing problem when one product is developed. The quality of the product is y, the price sensitivity parameter is b, the network effect parameter is α, the variance of utility error term is σ^2, and the purchasing probability of the product is q. By (7), the price of the product is

$$\begin{aligned} p = \frac{1}{b}\Big[& y + \alpha q - v_0 + \frac{1}{2}\sigma_0(2q_0 - 1)(q_0 - q_0^2)^{-1/2} \\ & - \frac{1}{2}\sigma(2q - 1)(q - q^2)^{-1/2}\Big]. \end{aligned} \quad (10)$$

We assume $\sigma = \sigma_0 > 0$ and $q + q_0 = 1$, so the price is changed as

$$\begin{aligned} p = \frac{1}{b}\Big[& y + \alpha q - v_0 + \frac{1}{2}\sigma_0(1 - 2q)(q - q^2)^{-1/2} \\ & - \frac{1}{2}\sigma(2q - 1)(q - q^2)^{-1/2}\Big] = \frac{1}{b}(y - v_0 + \alpha q) \\ & + \frac{\sigma}{b}(1 - 2q)(q - q^2)^{-1/2}. \end{aligned} \quad (11)$$

The firm's revenue is

$$\begin{aligned} \pi_1(q) &= p \cdot q \\ &= \frac{\alpha}{b}q^2 + \frac{y - v_0}{b}q + \frac{\sigma}{b}q(1 - 2q)(q - q^2)^{-1/2}. \end{aligned} \quad (12)$$

The optimal product pricing optimization problem is

$$\begin{aligned} \max \quad & \pi_1(q) \\ s.t. \quad & q \geq 0. \end{aligned} \quad (13)$$

The following proposition proves that $\pi_1(q)$ is concave in q when only one product is developed. When network effect is considered, the optimal price and the optimal market share can be derived by the implicit equations; then the maximum revenue can be obtained by formula (12) according to the corresponding optimal price and the optimal market share.

Proposition 1. *When only one product is developed, $\pi_1(q)$ is concave in q, and the optimal price and the optimal market share can be obtained by the following implicit equations:*

$$\begin{aligned} & 4\alpha q + 2(y - v_0) + \sigma(q - q^2)^{-3/2}(4q^3 - 6q^2 + q) = 0 \\ & p = \frac{1}{b}(y - v_0 + \alpha q) + \frac{\sigma}{b}(1 - 2q)(q - q^2)^{-1/2}. \end{aligned} \quad (14)$$

Proof. Because

$$\begin{aligned}
\frac{\partial \pi_1(q)}{\partial q} &= 2\frac{\alpha}{b}q + \frac{y-v_0}{b} + \frac{\sigma}{b}\Big[(1-4q)\left(q-q^2\right)^{-1/2} \\
&\quad + \left(q-2q^2\right)\left(-\frac{1}{2}\right)\left(q-q^2\right)^{-3/2}(1-2q)\Big] \\
&= \frac{2\alpha}{b}q + \frac{y-v_0}{b} \\
&\quad + \frac{\sigma}{2b}\left(q-q^2\right)^{-3/2}\left(4q^3-6q^2+q\right), \\
\frac{\partial^2 \pi_1(q)}{\partial q^2} &= \frac{2\alpha}{b} + \frac{\sigma}{2b}\Big[\left(-\frac{3}{2}\right)\left(q-q^2\right)^{-5/2}(1-2q) \\
&\quad \cdot \left(4q^3-6q^2+q\right) \\
&\quad + \left(q-q^2\right)^{-3/2}\left(12q^2-12q+1\right)\Big] \\
&= \frac{2\alpha}{b} + \frac{\sigma}{2b}\left(q-q^2\right)^{-5/2}\left(-q^2-\frac{1}{2}q\right),
\end{aligned} \tag{15}$$

and $b \geq 0, \sigma > 0, \alpha \leq 0$, so $\partial^2\pi_1(q)/\partial q^2 \leq 0$, $\pi_1(q)$ is concave in q.

By $\partial\pi_1(q)/\partial q = 0$, $(2\alpha/b)q + (y-v_0)/b + (\sigma/2b)(q-q^2)^{-3/2}(4q^3-6q^2+q) = 0$. Then the optimal q^*, p^* can be obtained by (14): □

Even if the network effect is taken into account, Proposition 1 proves that the revenue function is concave in choice probability when only one product is developed. Meanwhile, the optimal price and the corresponding choice probability can be solved, and the optimal revenue can also be obtained. Therefore, when the firm plans to develop one new product to obtain maximum revenue, we find that the optimal choice probability (market share) is related to the network effect, the quality, the utility of no-purchase option, and the variance of utility error term; in addition to the above factors, the optimal price is also related to the price sensitivity parameter.

3.2. Pricing for the Homogeneous Products. In this section, we consider a special case that all n products have the same coefficients; i.e., all products have the same quality, price sensitivity parameter, network effect parameter, and the variance of utility error term, i.e., $y_i = y$, $\alpha_i = \alpha$, $b_i = b$, and $\sigma_i = \sigma$. Without loss of generality, we assume that the price sensitivity parameter is $b = 1$ in the remainder of this section.

By (7), the price can be rewritten as

$$\begin{aligned}
p_i &= y + \alpha q_i - v_0 + \frac{1}{2}\sigma_0(2q_0-1)\left(q_0-q_0^2\right)^{-1/2} \\
&\quad - \frac{1}{2}\sigma(2q_i-1)\left(q_i-q_i^2\right)^{-1/2},
\end{aligned} \tag{16}$$

and the firm's total revenue (8) is expressed as

$$\begin{aligned}
\pi_2(\mathbf{q}) &= \sum_{i=1}^{n}(y-v_0+\alpha q_i)q_i + \sum_{i=1}^{n}q_i \cdot \frac{\sigma_0}{2}(2q_0-1) \\
&\quad \cdot \left(q_0-q_0^2\right)^{-1/2} \\
&\quad - \sum_{i=1}^{n}q_i\frac{\sigma}{2}(2q_i-1)\left(q_i-q_i^2\right)^{-1/2}.
\end{aligned} \tag{17}$$

Thus the product line optimization problem can be formulated as follows:

$$\begin{aligned}
\max \quad & \pi_2(q_0, q_1, q_2, \ldots, q_n) \\
s.t. \quad & \sum_{i=1}^{n} q_i + q_0 = 1 \\
& q_i, q_0 \geq 0 \quad (i = 1, 2, \ldots, n).
\end{aligned} \tag{18}$$

The following proposition shows that when the network effect exists, the revenue function $\pi_2(q_0, q_1, q_2, \ldots, q_n)$ is still concave in choice probability q_i and the optimal price and the optimal market share can still be obtained by implicit equations. Meanwhile, the optimal revenue can be obtained by formula (17).

Proposition 2. *The revenue function $\pi_2(q_0, q_1, q_2, \ldots, q_n)$ is concave in q_i, and the optimal price and the optimal market share can be obtained by the following implicit equations:*

$$\begin{aligned}
& 2\alpha q_i + y - v_0 \\
& \quad + \frac{1}{2}\sigma_0\left(q_0-q_0^2\right)^{-3/2}\left(-2q_0^3+3q_0^2-\frac{1}{2}q_0-\frac{1}{2}\right) \\
& \quad - \frac{\sigma}{2}\left(q_i-q_i^2\right)^{-3/2}\left(-2q_i^3+3q_i^2-\frac{1}{2}q_i\right) = 0 \\
& p_i = y - v_0 + \alpha q_i + \frac{1}{2}\sigma_0(2q_0-1)\left(q_0-q_0^2\right)^{-1/2} \\
& \quad - \frac{1}{2}\sigma(2q_i-1)\left(q_i-q_i^2\right)^{-1/2} \\
& \qquad\qquad q_0 = 1 - \sum_{i=1}^{n} q_i.
\end{aligned} \tag{19}$$

Proof. For $\pi_2(\mathbf{q})$, let us divide it into three sections. Let $t_1 = \sum_{i=1}^{n}(y-v_0+\alpha q_i)q_i$, and then

$$\begin{aligned}
\frac{\partial t_1}{\partial q_i} &= \alpha q_i + \alpha q_i + y - v_0 = 2\alpha q_i + y - v_0, \\
\frac{\partial^2 t_1}{\partial q_i^2} &= 2\alpha \leq 0.
\end{aligned} \tag{20}$$

Let $t_2 = \sum_{i=1}^{n} q_i \cdot (\sigma_0/2)(2q_0-1)(q_0-q_0^2)^{-1/2}$, and then

$$\begin{aligned}
\frac{\partial t_2}{\partial q_i} &= \frac{1}{2}\sigma_0(2q_0-1)\left(q_0-q_0^2\right)^{-1/2} + \sum_{i=1}^{n}q_i \cdot \frac{1}{2}\sigma_0\Big[(-2) \\
&\quad \cdot \left(q_0-q_0^2\right)^{-1/2} + (2q_0-1)\left(-\frac{1}{2}\right)\left(q_0-q_0^2\right)^{-3/2} \\
&\quad \cdot (-1+2q_0)\Big] \\
&= \frac{1}{2}\sigma_0\left(q_0-q_0^2\right)^{-3/2}\left(-2q_0^3+3q_0^2-q_0-\frac{1}{2}\sum_{i=1}^{n}q_i\right),
\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 t_2}{\partial q_i^2} &= \frac{1}{2}\sigma_0 \left[\left(-\frac{3}{2}\right)\left(q_0 - q_0^2\right)^{-5/2}(-1+2q_0) \right.\\ &\quad \cdot \left(-2q_0^3 + 3q_0^2 - q_0 - \frac{1}{2}\sum_{i=1}^{n} q_i\right) + \left(q_0 - q_0^2\right)^{-3/2} \\ &\quad \left. \cdot \left(6q_0^2 - 6q_0 + \frac{1}{2}\right)\right] = \frac{1}{2}\sigma_0 \left(q_0 - q_0^2\right)^{-5/2} \\ &\quad \cdot \left[\left(-\frac{1}{2}\right)\left(q_0 - \frac{5}{4}\right)^2 + \frac{1}{32}\right] \le 0.\end{aligned} \tag{21}$$

Let $t_3 = -\sum_{i=1}^{n} q_i(\sigma/2)(2q_i - 1)(q_i - q_i^2)^{-1/2} = -(\sigma/2)\sum_{i=1}^{n}(2q_i^2 - q_i)(q_i - q_i^2)^{-1/2}$, and then

$$\begin{aligned}\frac{\partial t_3}{\partial q_i} &= -\frac{\sigma}{2}\left(q_i - q_i^2\right)^{-3/2}\left(-2q_i^3 + 3q_i^2 - \frac{1}{2}q_i\right), \\ \frac{\partial^2 t_3}{\partial q_i^2} &= -\frac{\sigma}{2}\left(q_i - q_i^2\right)^{-5/2}\left(\frac{1}{2}q_i^2 + \frac{1}{4}q_i\right) \le 0.\end{aligned} \tag{22}$$

So $\partial^2\pi_2/\partial q_i^2 = \partial^2 t_1/\partial q_i^2 + \partial^2 t_2/\partial q_i^2 + \partial^2 t_3/\partial q_i^2 \le 0$. And $\partial^2\pi_2/\partial q_i\partial q_j = \partial^2 t_2/\partial q_i^2 \le 0$ $(i \ne j)$. For the Hessian matrix of revenue function $\pi_2(\mathbf{q})$, let $\partial^2\pi_2/\partial q_i^2 = B_i \le 0$, $\partial^2\pi_2/\partial q_i\partial q_j = A \le 0$, and $|B| = \min_i |B_i|$, $|B| \ge |A|$. Because

$$\begin{aligned}&\begin{vmatrix} B & A & \cdots & A \\ A & B & \cdots & A \\ \vdots & \vdots & \ddots & \vdots \\ A & A & \cdots & B \end{vmatrix} = \begin{vmatrix} B+(\mathrm{n}-1)A & A & \cdots & A \\ B+(\mathrm{n}-1)A & B & \cdots & A \\ \vdots & \vdots & \ddots & \vdots \\ B+(\mathrm{n}-1)A & A & \cdots & B \end{vmatrix} \\ &= [B+(\mathrm{n}-1)A]\begin{vmatrix} 1 & A & \cdots & A \\ 1 & B & \cdots & A \\ \vdots & \vdots & \ddots & \vdots \\ 1 & A & \cdots & B \end{vmatrix} = [B+(\mathrm{n}-1)A] \\ &\quad \cdot (B-A)^{n-1} \\ &\begin{cases} \le 0, & if\ n\ is\ an\ odd\ number; \\ > 0, & if\ n\ is\ an\ even\ number. \end{cases}\end{aligned} \tag{23}$$

Assuming that $B_i = B + \Delta_i (\Delta_i \le 0, B_i \le 0)$, then

$$\begin{aligned}&\begin{vmatrix} B & A & \cdots & A & \cdots & A \\ A & B & \cdots & A & \cdots & A \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ A & A & \cdots & B_i & \cdots & A \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A & A & \cdots & A & \cdots & B \end{vmatrix} = \begin{vmatrix} B & A & \cdots & A & \cdots & A \\ A & B & \cdots & A & \cdots & A \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ A & A & \cdots & B & \cdots & A \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A & A & \cdots & A & \cdots & B \end{vmatrix} \\ &+ \begin{vmatrix} B & A & \cdots & A & \cdots & A \\ A & B & \cdots & A & \cdots & A \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \Delta_i & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ A & A & \cdots & A & \cdots & B \end{vmatrix} = [B+(\mathrm{n}-1)A] \\ &\cdot (B-A)^{n-1} + \Delta_i \cdot (-1)^{i+i} \\ &\cdot \begin{vmatrix} B & A & \cdots & A \\ A & B & \cdots & A \\ \vdots & \vdots & \ddots & \vdots \\ A & A & \cdots & B \end{vmatrix}_{(\mathrm{n}-1)\times(\mathrm{n}-1)} = [B+(\mathrm{n}-1)A] \\ &\cdot (B-A)^{n-1} + \Delta_i \cdot [B+(\mathrm{n}-2)A] \cdot (B-A)^{n-2} \\ &\begin{cases} \le 0, & if\ n\ is\ an\ odd\ number; \\ > 0, & if\ n\ is\ an\ even\ number. \end{cases}\end{aligned} \tag{24}$$

We can obtain the similar conclusion when there are multiple B_i in the above matrix. So $\pi_2(q_0, q_1, q_2, \ldots, q_n)$ is concave in q_i. By $\partial\pi_2(\mathbf{q})/\partial q_i = 0$, we have

$$\begin{aligned}&2\alpha q_i + y - v_0 \\ &\quad + \frac{1}{2}\sigma_0\left(q_0 - q_0^2\right)^{-3/2}\left(-2q_0^3 + 3q_0^2 - \frac{1}{2}q_0 - \frac{1}{2}\right) \\ &\quad - \frac{\sigma}{2}\left(q_i - q_i^2\right)^{-3/2}\left(-2q_i^3 + 3q_i^2 - \frac{1}{2}q_i\right) = 0.\end{aligned} \tag{25}$$

Therefore, the optimal price and the market share can be solved by the implicit equations (19). □

According to Proposition 2, we find that when multiple homogeneous products are developed, the revenue function is still concave and the optimal price and the optimal choice probability are also obtained. Note that the optimal prices with network effect are different from the optimal prices without network effect. In literature [9], when network effect is positive in homogeneous scenario, the optimal solution takes one of two different forms. However, when network effect is negative, the optimal solution is unique.

3.3. Pricing for the Heterogeneous Products. Different from the homogeneous case, in this section, we consider the general case (9) when the parameters of all products are distinct; i.e., the quality levels of products (y_i), the price sensitivity parameters (b_i), network effect parameters (α_i), or the variances of utility error terms (σ_i^2) are different for various products.

For the revenue function (8), the product line pricing problem is described by (9). Through the following proposition, we find that the revenue function $\pi(q_0, q_1, q_2, \ldots, q_n)$

is concave in choice probability q_i when $b_i = b$ $(i = 1, \ldots, n)$ even though the parameters of all products are heterogeneous and network effect exists. Meanwhile, the optimal price and the optimal market share can also be derived by the implicit equations; and according to the corresponding optimal price and the optimal market share, the maximum revenue can be obtained by formula (8).

Proposition 3. *For each product i $(i = 1, \ldots, m)$, when $b_i = b$, the revenue function $\pi(q_0, q_1, q_2, \ldots, q_n)$ is concave in q_i, the optimal price and the optimal market share can be obtained by the following implicit equations:*

$$\begin{aligned}
&4\left(2\alpha_i q_i + y_i - v_0\right) \\
&\quad + \sigma_0\left(q_0 - q_0^2\right)^{-3/2}\left(-4q_0^3 + 6q_0^2 - q_0 - 1\right) \\
&\quad + \sigma_i\left(q_i - q_i^2\right)^{-3/2}\left(4q_i^3 - 6q_i^2 + q_i\right) = 0 \\
&p_i = \frac{1}{b_i}\left[y_i + \alpha_i q_i - v_0 + \frac{1}{2}\sigma_0\left(2q_0 - 1\right)\left(q_0 - q_0^2\right)^{-1/2}\right. \\
&\quad \left. - \frac{1}{2}\sigma_i\left(2q_i - 1\right)\left(q_i - q_i^2\right)^{-1/2}\right] \\
&q_0 = 1 - \sum_{i=1}^{n} q_i.
\end{aligned} \tag{26}$$

Proof. Let us, respectively, consider each part of the revenue function (8).

Let $w_1 = \sum_{i=1}^{n}(\alpha_i/b_i)q_i^2$, $\partial w_1/\partial q_i = (2\alpha_i/b_i)q_i$, and $\partial^2 w_1/\partial q_i^2 = 2\alpha_i/b_i \le 0$. And $\sum_{i=1}^{n}(q_i/b_i)\cdot(y_i - v_0)$ is linear in q_i.

Let $w_2 = \sum_{i=1}^{n}(q_i/b_i)\cdot(\sigma_0/2)(2q_0-1)(q_0-q_0^2)^{-1/2}$, and then

$$\begin{aligned}
\frac{\partial w_2}{\partial q_i} &= \left(q_0 - q_0^2\right)^{-3/2} \\
&\quad \cdot \left[-\frac{\sigma_0}{4}\sum_{i=1}^{n}\frac{q_i}{b_i} + \frac{\sigma_0}{2b_i}\left(-q_0 + 3q_0^2 - 2q_0^3\right)\right], \\
\frac{\partial^2 w_2}{\partial q_i^2} &= \left(q_0 - q_0^2\right)^{-5/2} \\
&\quad \cdot \left[\frac{3\sigma_0}{8}\left(2q_0 - 1\right)\sum_{i=1}^{n}\frac{q_i}{b_i} - \frac{\sigma_0}{2b_i}\left(q_0 - q_0^2\right)\right].
\end{aligned} \tag{27}$$

When $b_i = b$,

$$\begin{aligned}
\frac{\partial^2 w_2}{\partial q_i^2} &= \left(q_0 - q_0^2\right)^{-5/2}\cdot\frac{\sigma_0}{8b}\left(-2q_0^2 + 5q_0 - 3\right) \\
&= \left(q_0 - q_0^2\right)^{-5/2}\cdot\frac{\sigma_0}{8b}\left[-2\left(q_0 - \frac{5}{4}\right)^2 + \frac{1}{8}\right] \le 0.
\end{aligned} \tag{28}$$

Let $w_3 = -\sum_{i=1}^{n}(\sigma_i/2b_i)(2q_i^2 - q_i)(q_i - q_i^2)^{-1/2}$, and then

$$\begin{aligned}
\frac{\partial w_3}{\partial q_i} &= \frac{\sigma_i}{2b_i}\left(1 - 4q_i\right)\left(q_i - q_i^2\right)^{-1/2} \\
&\quad - \frac{\sigma_i}{4b_i}\left(q_i - 4q_i^2 + 4q_i^3\right)\left(q_i - q_i^2\right)^{-3/2} \\
&= -\frac{\sigma_i}{2b_i}\left(q_i - q_i^2\right)^{-3/2}\left(-2q_i^3 + 3q_i^2 - \frac{1}{2}q_i\right), \\
\frac{\partial^2 w_3}{\partial q_i^2} &= \frac{\sigma_i}{8b_i}\left(q_i - q_i^2\right)^{-5/2}\left(-2q_i^2 - q_i\right) \le 0.
\end{aligned} \tag{29}$$

So when $b_i = b$, $\partial^2\pi(\mathbf{q})/\partial q_i^2 \le 0$. And $\partial^2\pi(\mathbf{q})/\partial q_i\partial q_j = \partial^2 w_2/\partial q_i^2 \le 0$ $(i \ne j)$. The rest of the proof is similar to the proof of Proposition 2, so $\pi(q_0, q_1, q_2, \ldots, q_n)$ is concave in q_i. By $\partial\pi(\mathbf{q})/\partial q_i = 0$, we have

$$\begin{aligned}
\frac{\partial \pi}{\partial q_i} &= \frac{1}{b}\cdot\left(2\alpha_i q_i + y_i - v_0\right) \\
&\quad + \frac{\sigma_0}{4b}\left(q_0 - q_0^2\right)^{-3/2}\left(-4q_0^3 + 6q_0^2 - q_0 - 1\right) \\
&\quad + \frac{\sigma_i}{4b}\left(q_i - q_i^2\right)^{-3/2}\left(4q_i^3 - 6q_i^2 + q_i\right) = 0.
\end{aligned} \tag{30}$$

The preceding equation is reduced to

$$\begin{aligned}
&4\left(2\alpha_i q_i + y_i - v_0\right) \\
&\quad + \sigma_0\left(q_0 - q_0^2\right)^{-3/2}\left(-4q_0^3 + 6q_0^2 - q_0 - 1\right) \\
&\quad + \sigma_i\left(q_i - q_i^2\right)^{-3/2}\left(4q_i^3 - 6q_i^2 + q_i\right) = 0.
\end{aligned} \tag{31}$$

Then the optimal market share and the optimal price can be obtained by the implicit equations (26). □

In heterogeneous scenario, when the price sensitivity parameters of different products are the same, the revenue function is also concave in choice probability. Meanwhile, we find that the optimal prices of all products are generally distinct. In addition, in literature [9], the optimal choice probability may be one of two different structures; however, when the products exhibit negative network effect, the optimal choice probability is unique.

4. Numerical Experiments

In this section, we present some numerical experiments to show the influence of network effect parameters on the results. First, we study the variation of optimal solutions with different parameters for developing one product, homogeneous coefficients, and heterogeneous coefficients case. Second, we analyze the importance of considering network effect if it does exist. Finally, we test the robustness of solutions when the estimation of network effect parameter has some error.

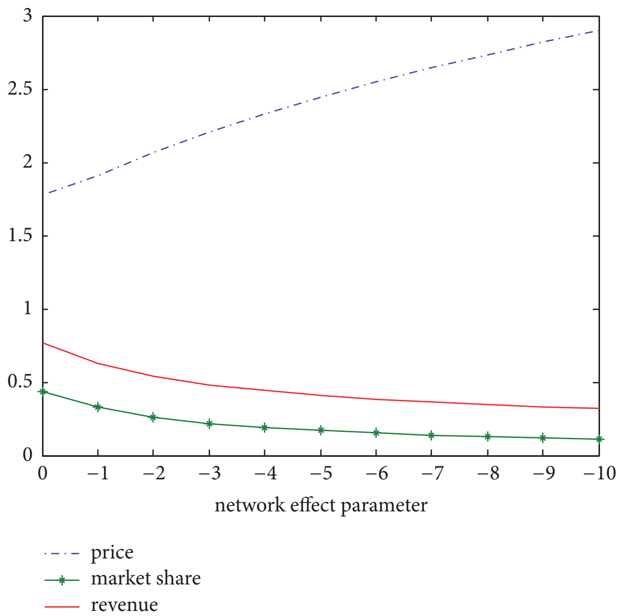

Figure 1: The variation of optimal solutions with network effect parameter.

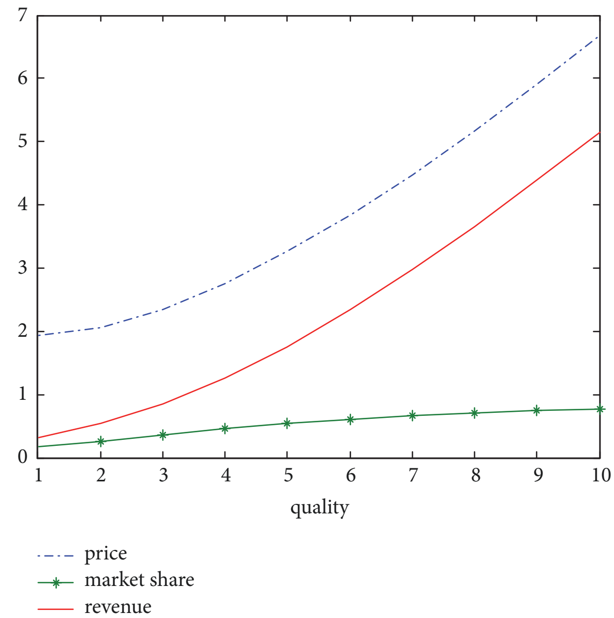

Figure 2: The variation of optimal solutions with the quality.

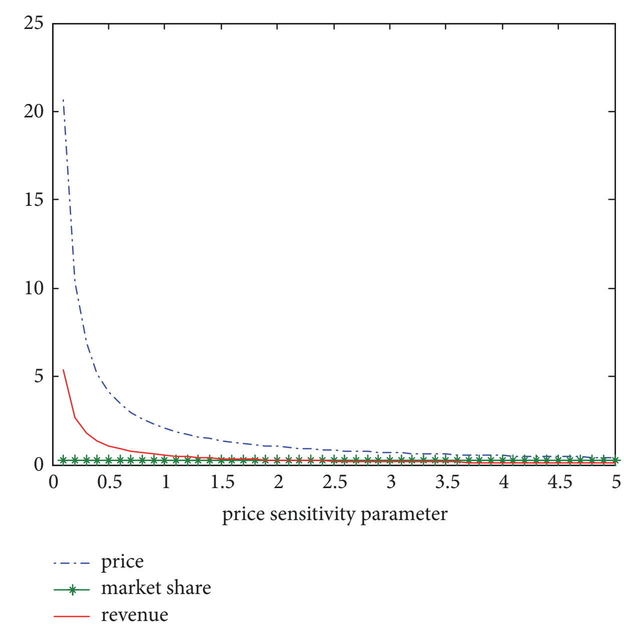

Figure 3: The variation of optimal solutions with the price sensitivity parameter.

4.1. Variation of the Optimal Solutions with Different Parameters. We consider the influence of different parameters on the optimal price, the optimal market share, and the optimal revenue when different products are developed.

4.1.1. Develop One Product. We first present the variation of the optimal results with different parameters when only one product is developed. For ease of analysis, we consider the case where only one parameter changes and the other parameters remain the same.

To analyze the influence of network effect parameter on the optimal solutions, let the variation range of α be set as $[-10, 0]$ and the other parameters be set as $y = 2$, $b = 1$, $\sigma = 1$, and $v_0 = 0.5$. The variation of optimal solutions with network effect parameter α is showed in Figure 1.

As shown in Figure 1, with the network effect parameter reducing, the optimal price gradually is enhanced and the optimal market share or the revenue is going down. That means the network effect is a key factor for the product development if it exists. If the network effect is almost negligible, the optimal price is relatively low, the corresponding market share is large, and the earning revenue is high. Because we only consider the negative network effect, when the absolute value of network effect goes up, the optimal price may be set higher, but both the corresponding market share and the revenue reduce gradually.

We next analyze the influence of the product's quality on the optimal solutions when the network effect exists. We assume the product's quality y lies in $[1, 10]$; let $\alpha = -2$, $b = 1$, $\sigma = 1$, and $v_0 = 0.5$. Figure 2 shows the variation of optimal solutions with the quality.

From Figure 2 we find that the optimal price, the corresponding market share, and the revenue all go up with the quality of product increasing. That is to say, even when network effect exists, improving quality is good for the seller when development costs are not taken into account. Because of the better quality, the price of product may be set higher, the corresponding market share is large, and the earning revenue also increases.

We then show the influence of price sensitivity parameter on the optimal solutions when the network effect exists. We assume $b \in [0.1, 5]$, $\alpha = -2$, $y = 2$, $\sigma = 1$, and $v_0 = 0.5$. Figure 3 shows the variation of optimal solutions with the price sensitivity parameter b.

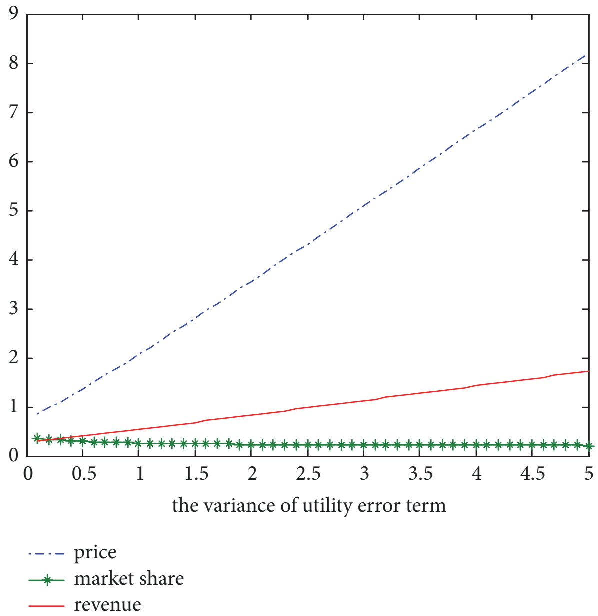

Figure 4: The variation of the optimal solutions with the variance of utility error term.

As shown in Figure 3, when the network effect is considered, the optimal price, the corresponding market share, and the optimal revenue all go down with the price sensitivity parameter becoming large. When the sensitivity of consumer to the price is relatively low, the price could be high and the earning revenue is also large. However, when the price sensitivity parameter of consumer to the product is large, the optimal price should be set lower and the corresponding revenue also decreases. When the price sensitivity parameter is large enough, the optimal price, the corresponding market share, and the optimal revenue are almost 0.

To analyze the influence of the variance of the utility error term on the optimal solutions when the network effect is considered, we set $\sigma \in [0.1, 5]$, $\alpha = -2$, $y = 2$, $b = 1$, and $v_0 = 0.5$. Figure 4 shows the variation of optimal solutions with the variance of utility error term σ.

From Figure 4 we find that as the variance of utility error term increases, the optimal price and the revenue go up and the market share decreases. That means that if the estimated utility has a large error, the price of product may be increased and the earning revenue also goes up, but the corresponding market share reduces.

When different products have the homogeneous coefficients, even though the network effect exists, the influences of different parameters on the optimal solutions (the optimal price, the optimal market share, or the optimal revenue) are similar to that when one product is developed. Next, we will study the variation of the optimal solutions with different parameters when all products have the heterogeneous coefficients and the network effect exists.

4.1.2. Heterogeneous Case. We consider the influence of different parameters on the optimal solutions when all products are heterogeneous and the network effect exists. Without loss of generality, we assume only two products are developed in the market.

First, we consider the influence of the network effect parameter on the optimal solutions. For ease of exposition, we assume the network effect parameters of two products are the same and $\alpha_1 = \alpha_2 \in [-10, 0]$. Let $y_1 = 2$, $y_2 = 3$, $b = 1$, $v_0 = 0.5$, $\sigma_0 = 0.5$, $\sigma_1 = 1$, and $\sigma_2 = 0.8$. Figure 5 shows the variation of the optimal solutions with the network effect parameter.

As shown in Figure 5, the variations of the optimal prices, the optimal market shares, and the optimal revenues are distinct. With the decrease of the network effect, the optimal prices of two products first reduce and then increase, the market shares of two products and the total market share decrease generally, and the optimal revenues of two products and the total revenue also go down gradually. When the network effect decreases, the optimal prices should be set higher, but the corresponding optimal revenues decrease, which is similar to that one product developed.

Second, we study the variation of the optimal solutions with the quality of product when the network effect exists. We assume the qualities of two products are the same and $y_1 = y_2 \in [1, 10]$. Let $\alpha_1 = -2$, $\alpha_2 = -1$, $b = 1$, $v_0 = 0.5$, $\sigma_0 = 0.5$, $\sigma_1 = 1$, and $\sigma_2 = 0.8$. Figure 6 shows the variation of the optimal solutions with the quality of product.

We find from Figure 6 that the optimal prices, the corresponding market shares, and the optimal revenues gradually go up with the quality increasing even if the network effect exists. This tendency is the same as the case that one product is developed. For the sellers, if the quality of product can be improved, the sale price can be set higher and the market share and the earning revenue are also increased.

Next, we analyze the influence of the price sensitivity parameter on the optimal solutions. We assume $b \in [0.1, 5]$, $\alpha_1 = -2$, $\alpha_2 = -1$, $y_1 = 2$, $y_2 = 3$, $v_0 = 0.5$, $\sigma_0 = 0.5$, $\sigma_1 = 1$, and $\sigma_2 = 0.8$. Figure 7 shows the variation of the optimal solutions with the price sensitivity parameter when the network effect exists.

From Figure 7, we can see that the optimal prices and the optimal revenues both become small when the price sensitivity parameter gets big, but the corresponding market share is constant even if the price sensitivity parameter changes. For the market share, we can find from Proposition 3 that the market share has nothing to do with the price sensitivity parameter. In addition, when the price sensitivity of consumer is large, the seller should cut price to obtain the optimal revenue.

Finally, we investigate the influence of the variance of utility error term on the optimal solutions when the network effect exists. We assume that the variances of utility error terms are all uniform and $\sigma_0 = \sigma_1 = \sigma_2 \in [0.1, 10]$. Let $\alpha_1 = -2$, $\alpha_2 = -1$, $y_1 = 2$, $y_2 = 3$, $v_0 = 0.5$, and $b = 1$. Figure 8 shows the variation of the optimal solutions with the variance of utility error term.

From Figure 8 we can find that the variations are distinct among the optimal prices, the optimal market shares, and the optimal revenues. As the variance of utility error term becomes large, the optimal prices of two products increase, the optimal market shares of two products and the total

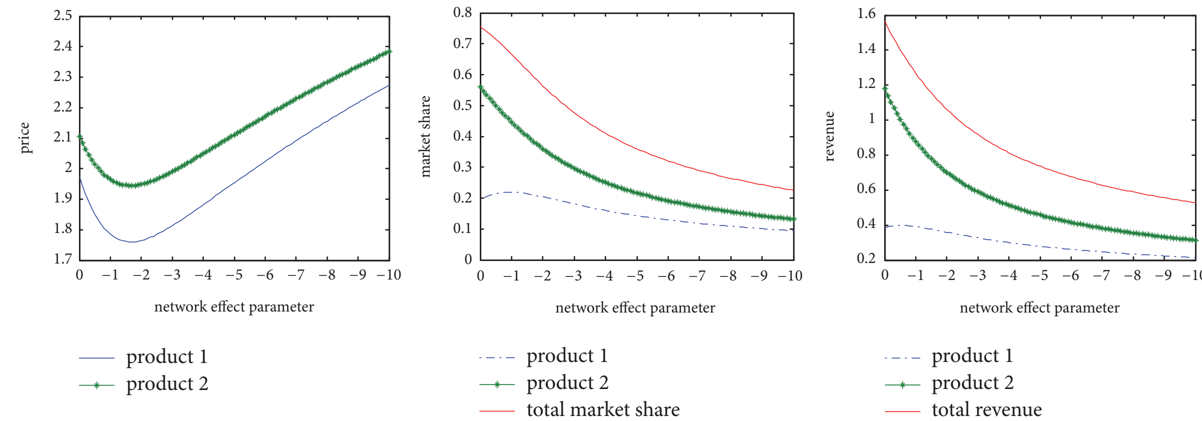

FIGURE 5: The influence of the network effect parameter on the optimal solutions.

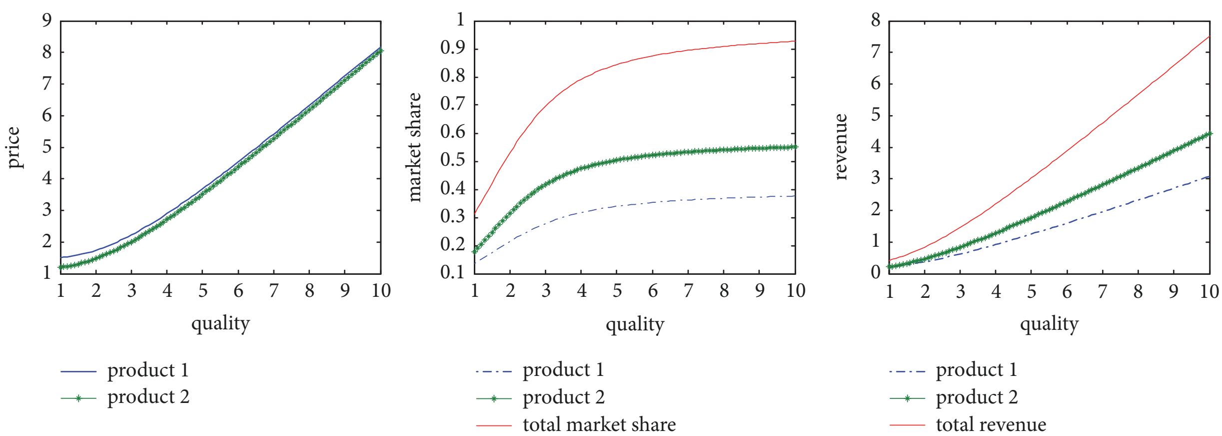

FIGURE 6: The influence of the quality of product on the optimal solutions.

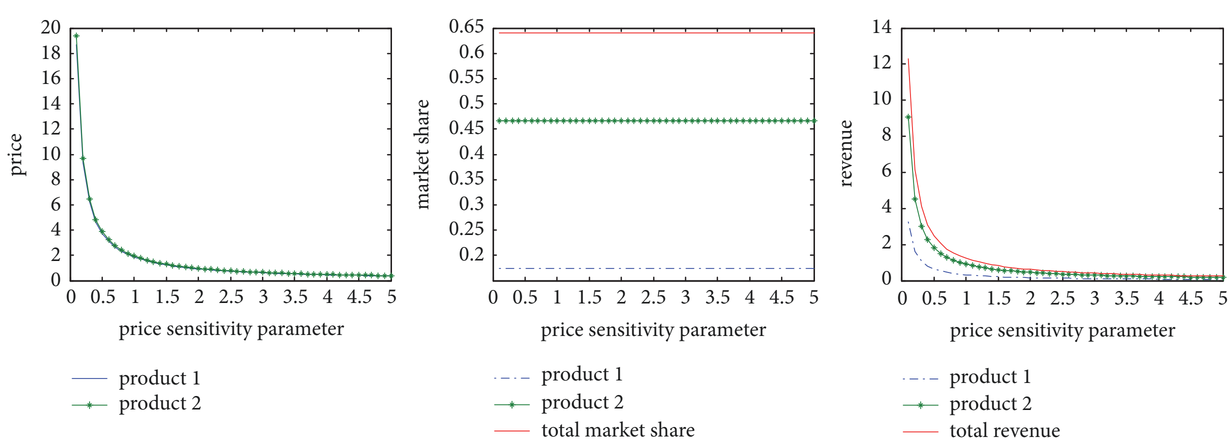

FIGURE 7: The influence of the price sensitivity parameter on the optimal solutions.

market share reduce, and the revenue of product 1 gradually increases, but the revenue of product 2 and the total revenue first decrease and then increase. It can be seen that the variance of the estimated utility can affect the price of product, the market share, and the earning revenue.

4.2. The Importance of Considering Network Effect. In this section, we demonstrate the importance of considering network effect when making pricing decision if network effect does exist. Otherwise, the seller will suffer a certain loss. The test is presented by three parts of numerical experiments,

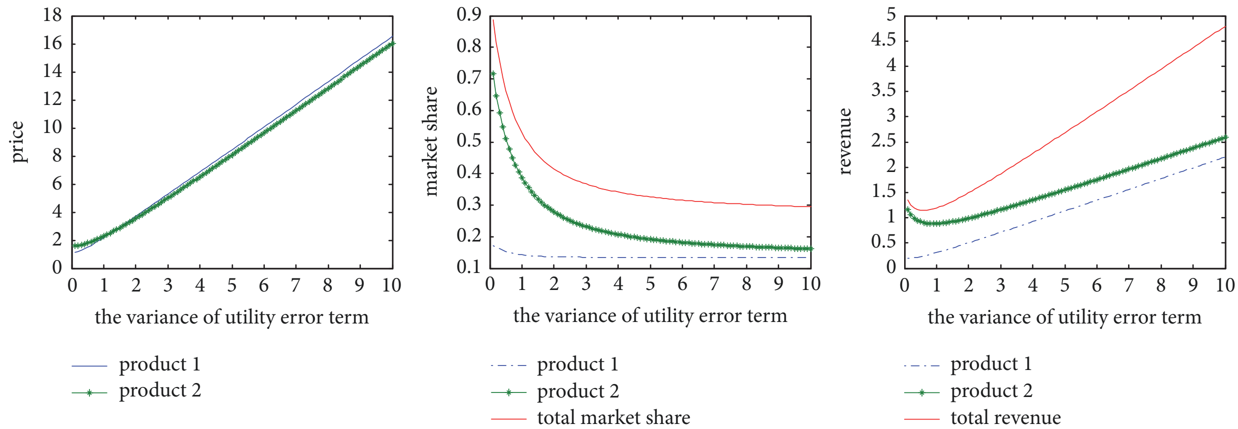

Figure 8: The influence of the variance of utility error term on the optimal solutions.

including developing one product, homogeneous products, and heterogeneous case.

For one product, we set $y = 2, b = 1, \sigma = 1$, and $v_0 = 0.5$. For the homogeneous case, we assume that only two products are developed and fix $y = 2, \sigma = 1$, and $v_0 = 0.5$. For the heterogeneous case, there are also two products in the market, and $y_1 = 2, y_2 = 3, b = 1, v_0 = 0.5, \sigma_0 = 0.5, \sigma_1 = 1$, and $\sigma_2 = 0.8$. We assume that the network effect parameter $\alpha \in [-10, 0]$.

In Table 1, the values of the optimal revenues π_1, π_2, and π are obtained by Propositions 1, 2, and 3. The values of π_1^0, π_2^0, and π^0 indicate the revenue without considering the network effect. Where the prices $\mathbf{p}^0$ are solved by Propositions 1, 2, and 3 with $\alpha = 0$, the corresponding market shares $\mathbf{q}^0$ are derived by (5) with the actual α. In general, $\mathbf{q}^0 \neq \mathbf{q}$ unless the actual network effect parameter is 0. The revenue without considering the network effect is earned by $\mathbf{p}^0 \cdot \mathbf{q}^0$. The values of l_1, l_2, and l indicate the loss of revenue if the pricing decision does not consider the network effect.

We can see from Table 1 that the optimal revenues outweigh the revenues which are obtained when the network effects are not considered but those do exist, whenever there are products, homogeneous products, or heterogeneous cases in the market. From Table 1, when $\alpha = -10$, the loss is as high as 11%. With the network effect parameter becoming small, the loss could be even higher. Therefore, if there exist network effects in the market, the network effect needs to be considered when making pricing decision; otherwise, the seller may suffer some losses.

4.3. Robustness of the Solutions. In this section, we investigate the robustness of the revenue if the network effect parameter has some estimation error. Without loss of generality, we only consider the homogeneous case. Assume that the seller has an estimation $\tilde{\alpha}$ to the network effect parameter and the pricing decision $\tilde{\mathbf{p}}(\tilde{\mathbf{q}})$ is made with $\tilde{\alpha}$. But the network effect parameter is actually α. According to $\tilde{\mathbf{p}}(\tilde{\mathbf{q}})$, the consumers have the corresponding market share $\overline{\mathbf{q}}$ based on α. If $\alpha = \tilde{\alpha}$, $\overline{\mathbf{q}} = \tilde{\mathbf{q}}$. Otherwise, $\overline{\mathbf{q}}$ and $\tilde{\mathbf{q}}$ are usually different. And the revenue is $\tilde{\pi} = \tilde{\mathbf{p}}(\tilde{\mathbf{q}}) \cdot \overline{\mathbf{q}}$.

For the homogeneous case, we assume that there exist two products to be developed. Let $y = 2$, $\sigma = 1$, and $v_0 = 0.5$. Define $l = 100 \times ((\pi^* - \tilde{\pi})/\pi^*)$ as the revenue loss, where π^* defines the optimal revenue when the network effect parameter does not have any estimation error. Table 2 shows the results of revenues and the loss.

As shown in Table 2, when the network effect parameter has some estimation error, the revenue shows good robustness. From Table 2, we can see that the highest revenue loss percentage is 0.12% and most percentages are below 0.1%. We also can find from Figure 1 that the revenue curve is relatively flat with the network effect changing. When the estimation error of network effect parameter is within a reasonable range, we may obtain the optimal robust revenue. Therefore, our model shows well robustness.

5. Conclusions

Nowadays, with the rapid development of e-commerce, the influence of complex network effect on the development and pricing of new products cannot be ignored. Consumer choice behavior not only has positive network effect but also has negative network effect, which is also common in practice. In addition, the assumptions of MNL model, which is one of the most widely used consumer choice models, may not be met in some practical cases. Therefore, in this paper, we study the product line pricing problem considering negative network effect based on the MMM. We establish an improved MMM with endogenous network effect. By proving the concavity of the profit function, we obtain the solving equations of the optimal price, the corresponding market share, and the optimal revenue in three different markets, including developing one product, homogeneous products, and heterogeneous case. Through numerical experiments, we first show the variation of the optimal solutions with different parameters, then analyze the importance of considering network effect if it does exist, and finally test the robustness of solution when the estimation of network effect parameter has a certain error.

Our paper also has some limitations that are showed as follows. First, we only study the product pricing with

Table 1: The comparison of revenues whether considering network effect.

α	0	-1	-2	-3	-4	-5	-10
π_1	0.7666	0.6248	0.5393	0.4822	0.4409	0.4093	0.3182
π_1^0	0.7666	0.6225	0.5320	0.4694	0.4231	0.3872	0.2823
l_1 (%)	0	0.37	1.35	2.65	4.04	5.40	11.28
π_2	1.0469	0.9166	0.8217	0.7507	0.6957	0.6517	0.5171
π_2^0	1.0469	0.9160	0.8188	0.7442	0.6850	0.6368	0.4848
l_2 (%)	0	0.07	0.35	0.87	1.54	2.29	6.25
π	1.5646	1.2679	1.0610	0.9171	0.8140	0.7368	0.5280
π^0	1.5646	1.2583	1.0531	0.9138	0.8132	0.7368	0.5222
l (%)	0	0.76	0.74	0.36	0.10	0	1.10

Table 2: The robustness of revenue for estimation of network effect.

$\tilde{\alpha}$	-2.5	-2.8	-3	-3.2	-3.5	-3.8
$\alpha = -2.8$						
$\tilde{\pi}$	0.7633	0.7634	0.7634	0.7633	0.7629	0.7625
l (%)	0.01	0	<0.01	0.01	0.07	0.12
$\alpha = -3.5$						
$\tilde{\pi}$	0.7207	0.7211	0.7214	0.7215	0.7216	0.7215
l (%)	0.12	0.07	0.03	0.01	0	0.01

negative network effect. We may also explore the product line optimization problem considering positive network effect. Second, we simulate the consumer choice behavior based on MMM with network effect. We could further introduce another probabilistic choice model to describe consumer choice behavior when the network effect is considered. Finally, we only obtain the solving equations of the optimal solutions. We would continue to explore the analytical expression of the optimal price or the optimal revenue. These issues would be the topics of our future work.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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