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Research Article

Nonautonomous Motion Study on Accelerated and Decelerated Lump Waves for a (3 + 1)-Dimensional Generalized Shallow Water Wave Equation with Variable Coefficients

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Under investigation in this paper is a $(3+1)$-dimensional variable-coefficient generalized shallow water wave equation. The exact lump solutions of this equation are presented by virtue of its bilinear form and symbolic computation. Compared with the solutions of the previous cases, these solutions contain two inhomogeneous coefficients, which can show some interesting nonautonomous characteristics. Three types of dispersion coefficients are considered, including the periodic, exponential, and linear modulations. The corresponding nonautonomous lump waves have different characteristics of trajectories and velocities. The periodic fission and fusion interaction between a lump wave and a kink soliton is discussed graphically.

1. Introduction

The $(3+1)$-dimensional Jimbo–Miwa (JM) equation [1]

$$2u_{yt} + u_{xxxy} + 3u_x u_{xy} + 3u_{xx}u_y - 3u_{xz} = 0, \qquad (1)$$

is the second equation in the well-known Kadomtsev–Petviashvili (KP) hierarchy [1–4]. This equation can characterize certain $(3+1)$-dimensional nonlinear wave phenomena in physics [1]. Very recently, lump solutions for all kinds of JM-like equations have aroused great interests [5–13]. This type of wave is localized in all directions in space [14–21], which is different from soliton and rogue wave [22–33]. For instance, lump and lump-kink solutions for equation (1) have been obtained in Reference [5]. Classes of lump-type solutions for equation (1) have been presented in References [6, 7]. Lump solutions of a reduced JM-like equation have been investigated in Reference [8]. Interaction solutions between lump-type and kink solutions for a $(3+1)$-dimensional JM-like equation have been studied in Reference [9]. Rogue wave and a pair of resonance stripe solutions of a reduced $(3+1)$-dimensional JM equation have been discussed in Reference [10]. New periodic wave, cross-kink wave, and the interaction phenomenon have been derived in Reference [11]. Interaction solutions for a reduced extended equation (1) equation have been analyzed in Reference [12].

With the inhomogeneities of the media and nonuniformities of the boundaries considered, the variable-coefficient models can often describe more realistic wave propagations in various physical scenes [34–36]. Note that the previous studies are mainly focused on the dynamics of lump waves in the constant-coefficient JM-like equations [5–13]. However, few studies examine the nonautonomous lump waves in variable-coefficient JM-like ones. In this paper, we will investigate a $(3+1)$-dimensional variable-coefficient generalized shallow water wave equation as follows [37]:

$$\alpha_1(t)u_{yt} + \alpha_2(t)u_{xxxy} + \alpha_3(t)u_x u_{xy} + \alpha_3(t)u_{xx}u_y + \alpha_4(t)u_{xz} = 0, \qquad (2)$$

where the parameters $\alpha_1(t), \alpha_2(t), \alpha_3(t)$, and $\alpha_4(t)$ are real functions of t. When $\alpha_1(t) = 2, \alpha_2(t) = 1, \alpha_3(t) = 3,$ and

$\alpha_4(t) = -3$, equation (2) is reduced to (1). Huang et al. [37] have given the bilinear Bäcklund transformation, soliton, and periodic wave solutions for equation (2). Liu and Zhu [38] have studied the breather wave solutions of equation (2). However, to our knowledge, the lump wave solutions and their nonautonomous characteristics (e.g., the accelerated and decelerated motions and trajectories) have not been reported yet. The present work aims at these aspects.

The paper is organized as follows. In Section 2, the nonautonomous lump solutions of equation (2) will be derived based on Hirota's bilinear form [39–42]. In Section 3, the accelerated and decelerated motions of lump waves will be investigated analytically. The characteristics of trajectories of waves will be also studied with different dispersion coefficients. In Section 4, the periodic fission and fusion interaction between a lump wave and a kink soliton will be discussed graphically. In Section 5, the conclusions will be given.

2. Nonautonomous Lump Solutions of Equation (2)

By using the transformation

$$u = 6\frac{\alpha_2(t)}{\alpha_3(t)}(\ln f)_x, \tag{3}$$

which is changed into $u = 2(\ln f)_x$ with the constraint $\alpha_3(t) = 3\times\alpha_2(t)$, the bilinear form for equation (2) is given as

$$\begin{aligned}\left[\alpha_2(t)D_x^3D_y + \alpha_4(t)D_xD_z + \alpha_1(t)D_yD_t\right]f\cdot f = \alpha_2(t)\\ \cdot\left(6f_{xx}f_{xy} - 6f_xf_{xxy} + 2ff_{xxxy} - 2f_{xxx}f_y\right)\\ + \alpha_4(t)\left(2ff_{xz} - 2f_xf_z\right) + \alpha_1(t)\left(2ff_{yt} - 2f_yf_t\right) = 0,\end{aligned} \tag{4}$$

where f is a real function of the spatial coordinates x, y, z and temporal coordinate t, and D_x, D_y, D_z, and D_t are the bilinear derivative operators, defined in Reference [43].

$$D_x^kD_y^lD_z^mD_t^n(f_1\cdot f_2) = \left(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}\right)^k\left(\frac{\partial}{\partial y_1} - \frac{\partial}{\partial y_2}\right)^l\left(\frac{\partial}{\partial z_1} - \frac{\partial}{\partial z_2}\right)^m\left(\frac{\partial}{\partial t_1} - \frac{\partial}{\partial t_2}\right)^n f_1(x_1,y_1,z_1,t_1)f_2(x_2,y_2,z_2,t_2)\Big|_{x_1=x_2,y_1=y_2,z_1=z_2,t_1=t_2}. \tag{5}$$

To search for the nonautonomous lump solutions of equation (2), we set

$$\begin{aligned}f &= w^2 + q^2 + a_9,\\ w &= a_1x + a_2y + a_3z + a_4(t),\\ q &= a_5x + a_6y + a_7z + a_8(t),\end{aligned} \tag{6}$$

where a_i $(i = 1,2,3,5,6,7,9)$ are the real constants, and $a_4(t)$ and $a_8(t)$ are unknown differentiable functions. In previous work, $a_4(t)$ and $a_8(t)$ were all considered as constants rather than real functions [44–48]. Hereby, we use an improved positive quadratic function to solve the JM equation with variable coefficients [37]. Substituting equation (6) into equation (4), we have

$$\begin{aligned}a_4(t) &= -\frac{(a_5(a_3a_6 - a_2a_7) + a_1(a_2a_3 + a_6a_7))\left(\int(\alpha_4(t)/\alpha_1(t))dt\right)}{a_2^2 + a_6^2},\\ a_8(t) &= -\frac{(a_2(a_3a_5 + a_1a_7) + a_6(a_5a_7 - a_1a_3))\left(\int(\alpha_4(t)/\alpha_1(t))dt\right)}{a_2^2 + a_6^2},\\ \alpha_2(t) &= \frac{(a_1a_6 - a_2a_5)(a_2a_7 - a_3a_6)a_9\alpha_4(t)}{3(a_1^2 + a_5^2)(a_1a_2 + a_5a_6)(a_2^2 + a_6^2)},\end{aligned} \tag{7}$$

where $(a_1a_2 + a_5a_6) \neq 0$. By using equation (7), the function f can be expressed as follows:

$$\begin{aligned}f = &\left[a_1x + a_2y + a_3z - \frac{(a_5(a_3a_6 - a_2a_7) + a_1(a_2a_3 + a_6a_7))\left(\int(\alpha_4(t)/\alpha_1(t))dt\right)}{a_2^2 + a_6^2}\right]^2\\ &+ \left[a_5x + a_6y + a_7z - \frac{(a_2(a_3a_5 + a_1a_7) + a_6(a_5a_7 - a_1a_3))\left(\int(\alpha_4(t)/\alpha_1(t))dt\right)}{a_2^2 + a_6^2}\right]^2 + a_9.\end{aligned} \tag{8}$$

Thus, we can present a class of lump solutions of the $(3+1)$-dimensional variable-coefficient JM equation:

$$u = \frac{4a_1w + 4a_5q}{f}, \tag{9}$$

and the expressions of functions w and q are given as, respectively,

$$w = a_1x + a_2y + a_3z - \frac{\left(a_5\left(a_3a_6 - a_2a_7\right) + a_1\left(a_2a_3 + a_6a_7\right)\right)\left(\int \left(\alpha_4(t)/\alpha_1(t)\right)dt\right)}{a_2^2 + a_6^2},$$
$$q = a_5x + a_6y + a_7z - \frac{\left(a_2\left(a_3a_5 + a_1a_7\right) + a_6\left(a_5a_7 - a_1a_3\right)\right)\left(\int \left(\alpha_4(t)/\alpha_1(t)\right)dt\right)}{a_2^2 + a_6^2}. \quad (10)$$

If the solutions $u(x, y, z, t)$ are lumps, then they need to satisfy the following condition:

$$\lim_{x^2+y^2 \longrightarrow \infty} u(x, y, z, t) = 0, \quad \forall (z, t) \in R^2. \quad (11)$$

By selecting the parameter values $a_1 = 2, a_2 = 1, a_3 = 2, a_5 = -1, a_6 = 1, a_7 = 1, a_9 = 1, t = 0, z = 0$, the lump wave described by solution (9) is depicted in Figures 1(a) and 1(b). One can observe a bright-dark lump; the values of the width (distance between the peak and valley) and amplitude of which are $2\sqrt{5}/5$ and $2\sqrt{5}$, respectively. The analytic expression for trajectory of the peak of the lump is $\sqrt{5}x + 2\sqrt{5}y - 1 = 0$, while for the valley is $\sqrt{5}x + 2\sqrt{5}y + 1 = 0$. Thus, these two trajectories are parallel to each other. As shown in Figure 1(c), the lump wave propagates along a straight line. Obviously, as time increases, the lump wave moves from $-\infty$ to $+\infty$ on the $x - y$ plane with the constant velocity $5\sqrt{5}/4$. Moreover, due to the existence of the variable coefficients $\alpha_1(t)$ and $\alpha_2(t)$ in the solution (9), the lump wave may show more characteristics which are absent in constant-coefficient JM equations. We will discuss this in detail in the following section.

3. Accelerated and Decelerated Motions and Characteristics of Trajectory

In this section, through symbolic computation, we investigate the nonautonomous characteristics of the lump solution of equation (2). We find that the variable coefficients $\alpha_1(t)$ and $\alpha_2(t)$ do not affect the width and amplitude of the lump wave. For simplicity, we hereby focus on the effects of the dispersion coefficient $\alpha_2(t)$ on the wave. Therefore, we hereby suppose $\alpha_1(t)$ is constant and analyze the dynamic characteristics of the nonautonomous lump wave, including the trajectory and velocity.

We first consider the periodic dispersion modulation. Figure 2(a) shows a segment-typed trajectory for a nonautonomous lump wave with $\alpha_2(t) = -(12/5)\cos(t)$. In order to analyze the trajectory and velocity clearly, we give the coordinates of the peak and valley as $((\sqrt{5}/5) + 20\sin(t), -10\sin(t))$ and $(-(\sqrt{5}/5) + 20\sin(t), -10\sin(t))$, respectively. By using the computation, we can find that both the trajectories of peak and valley are similar to the constant-coefficient case mentioned above. However, because of the periodic modulation, the range of motion of the lump wave along the x-axis (y-axis) is confined to $(-20, 20)$ $[(-10, 10)]$. And the coordinates of two endpoints are A (20, −10) and B (−20, 10). The lump wave propagates between these two points. In addition, the velocity of the lump is variable as a result of periodic modulation, the expression of which is $v = 10\sqrt{5}|\cos t|$. As demonstrated in Figure 2(b), the velocity of the lump wave periodically varies with time. The period T is equal to π. When $t = n \times \pi$ (n is an integer), the velocity reaches the maximum $10\sqrt{5}$ (O). Similarly, when $t = (\pi/2) + n \times \pi$, the velocity reaches the minimum 0 (A or B). We can conclude that the periodic dispersion does lead to accelerated and decelerated motions of the lump wave.

Next, with the dispersion coefficient $\alpha_2(t)$ being in the form of $\alpha_2(t) = (1/5)e^{-t}$, we show another type of trajectory for the lump wave in Figure 3(a). Similarly, the coordinates of the peak and valley appear as $((\sqrt{5}/5) + (5/3)e^{-t}, -(5/6)e^{-t})$ and $(-(\sqrt{5}/5) + (5/3)e^{-t}, -(5/6)e^{-t})$. It is obvious that the trajectory of the lump wave is a half-line, which is different from the case of periodic modulation. The range of motion of the wave is confined to in the second quadrant on the $x - y$ plane. As the time t increases, it moves from infinity to the origin O. Figure 3(b) depicts the velocity of the lump wave whose expression is $v = (5\sqrt{5}/6)\sqrt{e^{-2t}}$. The velocity decreases gradually with the time, and when $t \longrightarrow +\infty$, it is close to zero. That means the lump wave will eventually stop at the origin O.

Finally, we suppose that the dispersion coefficient $\alpha_2(t)$ is the linear function of t, which is taken as $\alpha_2(t) = -(2/5)t$. As above, we can easily calculate the coordinates of the peak and valley which appear as $((\sqrt{5}/5) + (5/3)t^2, -(5/6)t^2)$ and $(-(\sqrt{5}/5) + (5/3)t^2, -(5/6)t^2)$. As shown in Figure 4(a), the trajectory is similar to the case of exponential modulation. Nevertheless, the difference is that the lump wave is not localized at a fixed point eventually; on the contrary, it walks back. We then obtain the expression of the velocity for the lump $v = (5\sqrt{5}/3)|t|$. Figure 4(b) shows the velocity curve of the wave. As time increases ($t < 0$), the velocity of the wave decreases gradually. At $t = 0$, the velocity reaches the minimum 0. Then, the lump wave starts to accelerate with time ($t > 0$). Compared with the previous cases, we discover that the velocity of the lump wave varies with time linearly.

4. Interaction between Single-Lump Wave and One-Kink Soliton

In this section, we study the interaction between single-lump wave and one-kink soliton. We assume that the function f in equation (4) has the following forms (here, we use f_1, w_1, q_1, and l_1 to distinguish from the single-lump case):

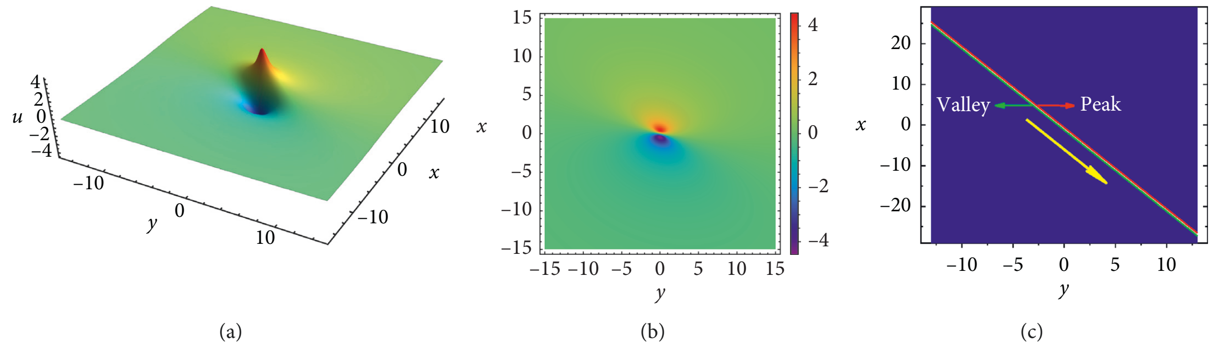


Figure 1: (a) The three-dimensional plot of the nonautonomous lump wave via the solution (9). (b) The density plot of (a). (c) The line-typed trajectory of the nonautonomous lump wave via the solution (9). The relevant parameters are set to $a_1 = 2, a_2 = 1, a_3 = 2, a_5 = -1, a_6 = 1, a_7 = 1, a_9 = 1, z = 0, t = 0$.

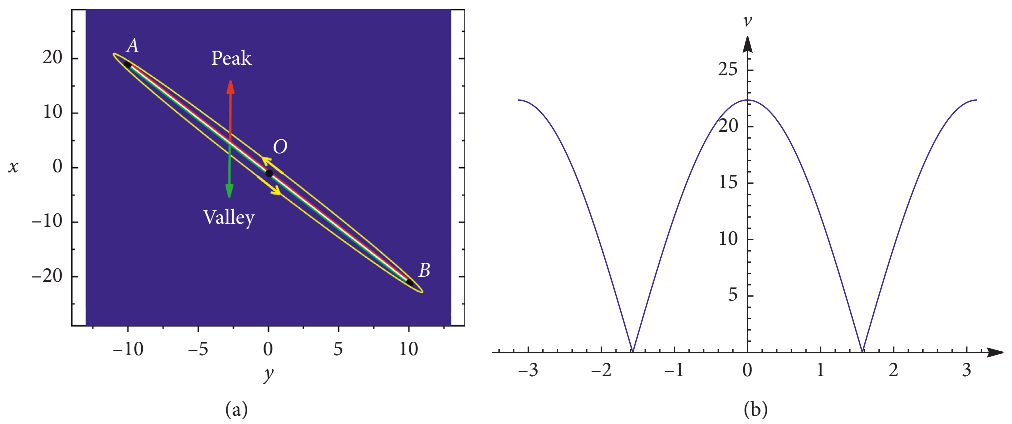


Figure 2: (a) The segment-typed trajectory of the nonautonomous lump wave via the solution (9). (b) The velocity curve of the nonautonomous lump wave with time. The variable coefficients are taken as $\alpha_1(t) = 2$ and $\alpha_2(t) = -(12/5)\cos(t)$. The other parameters are the same as Figure 1.

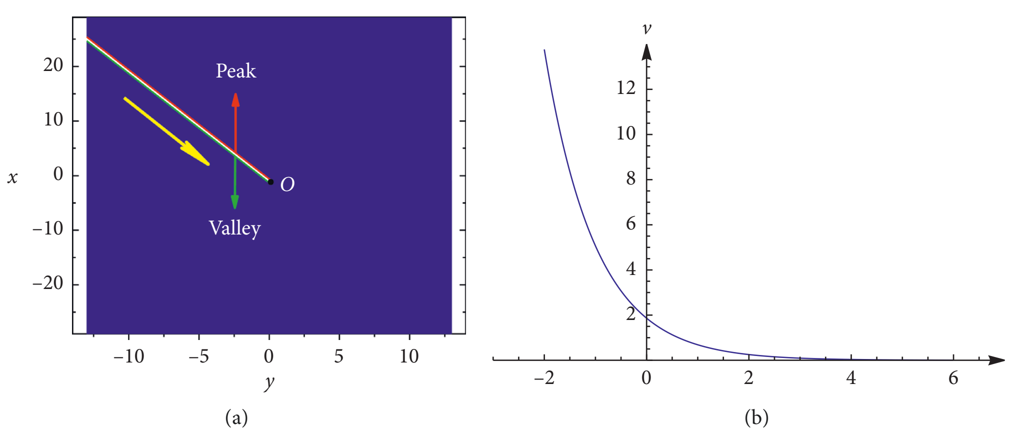
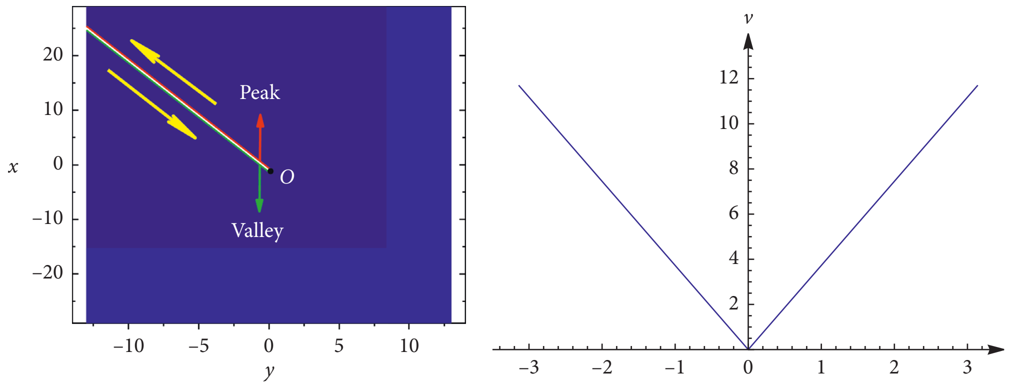


Figure 3: (a) The half-line-typed trajectory of the nonautonomous lump wave via the solution (9). (b) The velocity curve of the nonautonomous lump wave with time. The variable coefficients are taken as $\alpha_1(t) = 2$ and $\alpha_2(t) = (1/5)e^{-t}$. The other parameters are the same as Figure 1.



(a) (b)

Figure 4: (a) The half-line-typed trajectory of the nonautonomous lump wave via the solution (9). (b) The velocity curve of the nonautonomous lump wave with time. The variable coefficients are taken as $\alpha_1(t) = 2$ and $\alpha_2(t) = -(2/5)t$. The other parameters are the same as Figure 1.

$$f_1 = w_1^2 + q_1^2 + l_1 + a_9, \quad (12)$$

with

$$\begin{aligned} w_1 &= a_1 x + a_2 y + a_3 z + a_4(t), \\ q_1 &= a_5 x + a_6 y + a_7 z + a_8(t), \\ l_1 &= m \times e^{k_1 x + k_2 y + k_3 z + k_4(t)}, \end{aligned} \quad (13)$$

where m, a_i $(i = 1, 2, 3, 5, 6, 7, 9)$, and k_j $(j = 1, 2, 3)$ are real coefficients and $a_4(t), a_8(t), k_4(t)$ are unknown differentiable functions, which are to be determined later. After substituting equation (12) into equation (4), taking

$$\Lambda = \frac{\left(a_1^2 + a_5^2\right)^2 \left(a_2^2 + a_6^2\right)^2}{\left(a_2 a_5 - a_1 a_6\right)\left(a_1 a_2 + a_5 a_6\right)\left(a_2 a_7 - a_3 a_6\right)\left(a_2 a_3 + a_6 a_7\right)^2}, \quad (14)$$

we can get the constraining equations for the parameters

$$\alpha_4(t) = -3\Lambda\left(a_2^2 + a_6^2\right)k_3^2 \alpha_2(t), \quad (15)$$

$$\begin{aligned} a_4(t) &= 3\Lambda\left(a_5\left(a_3 a_6 - a_2 a_7\right) + a_1\left(a_2 a_3 + a_6 a_7\right)\right)k_3^2\left(\int \frac{\alpha_2(t)}{\alpha_1(t)} dt\right), \\ a_8(t) &= 3\Lambda\left(a_2\left(a_3 a_5 + a_1 a_7\right) + a_6\left(a_5 a_7 - a_1 a_3\right)\right)k_3^2\left(\int \frac{\alpha_2(t)}{\alpha_1(t)} dt\right), \\ k_4(t) &= \frac{\Lambda\left(a_1^2 + a_5^2\right)\left(a_2^2 + a_6^2\right)\left(a_1\left(3a_3 a_2^2 + 4a_6 a_7 a_2 - a_3 a_6^2\right) + a_5\left(-a_7 a_2^2 + 4a_3 a_6 a_2 + 3a_6^2 a_7\right)\right)k_3^3\left(\int \left(\alpha_2(t)/\alpha_1(t)\right) dt\right)}{\left(a_1 a_2 + a_5 a_6\right)^2\left(a_2 a_3 + a_6 a_7\right)}, \\ a_9 &= \frac{\left(a_1 a_2 + a_5 a_6\right)^2\left(a_2 a_3 + a_6 a_7\right)^2}{\left(a_1^2 + a_5^2\right)\left(a_2^2 + a_6^2\right)^2 k_3^2}, \\ k_1 &= \frac{\left(a_1^2 + a_5^2\right)\left(a_2^2 + a_6^2\right)k_3}{\left(a_1 a_2 + a_5 a_6\right)\left(a_2 a_3 + a_6 a_7\right)}, \\ k_2 &= \frac{\left(a_2^2 + a_6^2\right)k_3}{a_2 a_3 + a_6 a_7}, \end{aligned} \quad (16)$$

with the constraint

$$\begin{aligned} &m > 0, \\ &\left(a_2 a_5 - a_1 a_6\right)\left(a_1 a_2 + a_5 a_6\right)\left(a_2 a_7 - a_3 a_6\right)\left(a_2 a_3 + a_6 a_7\right)\alpha_1(t) \neq 0. \end{aligned} \quad (17)$$

In consideration of the transformation $u = 2(\ln f)_x$, we can get the single-lump and one-kink soliton waves to the $(3+1)$-dimensional variable-coefficient JM equation:

$$u = \frac{2\left(2a_1 w_1 + 2a_5 q_1 + k_1 l_1\right)}{w_1^2 + q_1^2 + l_1 + a_9}. \quad (18)$$

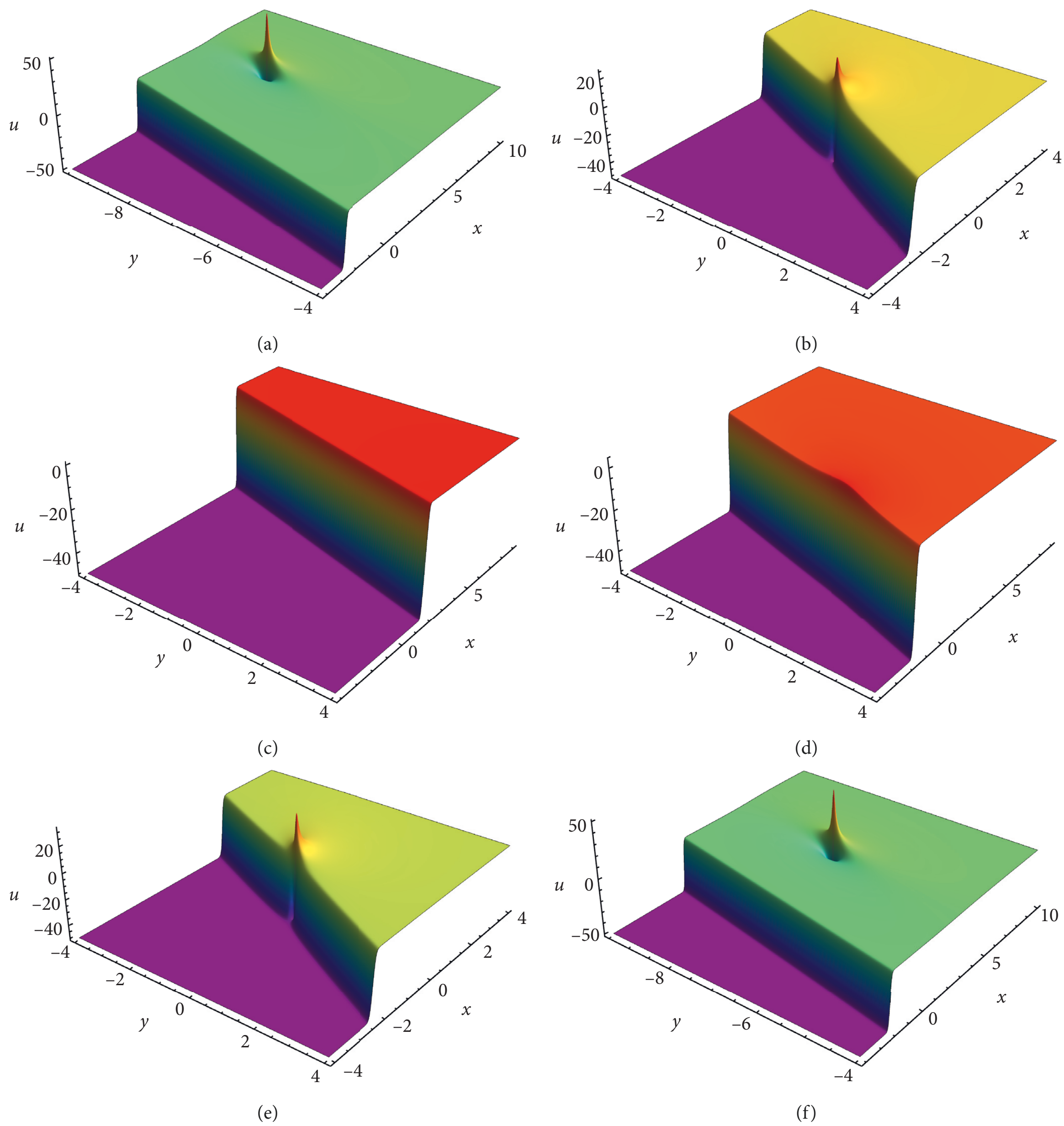


Figure 5: The three-dimensional plot of periodic interaction between single-lump wave and one-kink soliton via the solution (18). The relevant parameters are $a_1 = 2, a_2 = 2, a_3 = -1, a_5 = -1, a_6 = 1.5, a_7 = 1, k_3 = 1, m = 0.1, \alpha_1(t) = 2, \alpha_2(t) = 0.02\cos(t), z = 0$. (a) $t = -1$; (b) $t = 0$; (c) $t = 1$; (d) $t = 3$; (e) $t = 3.15$; (f) $t = 4$.

We can also find out that the solution (18) is composed of the rational and exponential functions, which describe the propagation behaviors of a lump wave and a soliton, respectively. Besides, it is worth noticing that the variable coefficients $\alpha_1(t)$ and $\alpha_2(t)$ appear in both two functions. It means that we can choose different dispersion coefficients $[\alpha_2(t)]$ to control the trajectories and velocities of the lump wave and soliton, even for their interaction. This is different from the case in constant-coefficient JM equations. In Figure 5, we can observe the interaction between single-lump wave and one-kink soliton with the parameters selected as $a_1 = 2, a_2 = 2, a_3 = -1, a_5 = -1, a_6 = 1.5, a_7 = 1, k_3 = 1, m = 0.1, \alpha_1(t) = 2, \alpha_2(t) = 0.02\cos(t)$. From $t = -1$ to $t = 0$, the lump and soliton are moving towards each other at the same time and they collide when $t = 0$. After that, the lump vanishes and the process could be seen as the fusion behavior. When $t = 3$, we can see that the lump wave appears once again. Then, they are moving away from each other which may be called as the fission behavior. When $t = 4$, the lump wave goes back to its initial location and so does the kink soliton. They repeat the same behavior in the next period. It is pointed out that the velocities of the lump wave and kink soliton vary with time. The lump wave propagates on a segment while the soliton moves between two parallel lines.

5. Conclusion

In conclusion, we have studied the $(3+1)$-dimensional variable-coefficient generalized shallow water wave equation, which characterizes the flow below a pressure surface in a fluid. Through the Hirota method, we have obtained nonautonomous lump solutions for equation (2). We have found that the variable coefficient affects the velocity and trajectory of the single-lump wave. Besides, we have observed that the dispersion coefficient has influence on the interaction between the lump wave and kink soliton.

Data Availability

Codes that related on the simulations can be made available on request (china907a@163.com).

Disclosure

Weiqin Chen, Qingfeng Guan, and Chaofan Jiang are co-first authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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