Hindawi
Complexity
Volume 2019, Article ID 6927528, 16 pages
https://doi.org/10.1155/2019/6927528

Research Article

Event-Triggered Stability Analysis of Semi-Markovian Jump Networked Control System with Actuator Faults and Time-Varying Delay via Bessel–Legendre Inequalities

Hongqian Lu ,[1] Chaoqun Guo,[1] Yue Hu,[1] and Wuneng Zhou[2]

[1]*School of Electrical Engineering and Automation, Qilu University of Technology (Shandong Academy of Sciences), Jinan 250353, China*
[2]*College of Information Science and Technology, Donghua University, Shanghai 201620, China*

Correspondence should be addressed to Hongqian Lu; hqlu@163.com

Received 1 June 2019; Revised 6 October 2019; Accepted 14 October 2019; Published 31 October 2019

Academic Editor: Hassan Zargarzadeh

This paper discusses the stability of semi-Markovian jump networked control system containing time-varying delay and actuator faults. The system dynamic is optimized while the network resource is saved by introducing an improved static event-triggered mechanism. For deriving a less conservative stability criterion, the Bessel–Legendre inequalities approach is employed to the stability analysis and plays a major role. By constructing the enhanced Lyapunov–Krasovskii functional (LKF) relevant to the Legendre polynomials, a stability criterion with lower conservativeness indexed by N is derived, and the conservativeness will decrease as N increases. In addition, a controller is designed. To prove the validity of this paper, numerical examples are provided at the last.

1. Introduction

Networked control system is widely used and researched due to its many advantages such as sharing network resources, cutting down system cost, and improving system stability. The networked control system is a feedback control system consisting of the controller, actuator, controlled object, and sensor [1]. There are many unavoidable problems in the networked control system. Among them, the network time delay has a large impact on stability of the networked control system [2, 3]. The networked control time delay occurs in the transmission process of information. This paper ignores the process delay caused by the calculation of sensors, controllers, and actuators themselves and only considers the network time delay which is called the network time-varying delay. In addition, an actual networked control system may suffer from actuator faults resulting in losing partial or total effectiveness of executing control actions, which can also cause instability of the networked control system. Thus, researching stability of the networked control system containing actuator faults is of vital importance [4]. Observer-based fault estimation for a nonlinear system was studied in [5]. A reliable controller for system with actuator faults was researched in [6]. Chance constrained control was proposed in [7] for stochastic nonlinear systems.

As a result of the inevitable external interferences, parameters and structures of the model may suddenly change. The jumping of parameters is usually subject to Markovian jump process. Thus, the Markovian jump has received wide attention in control systems. It is a useful tool in constructing the model structures which could abrupt change in their parameters [8]. Many research results about the Markovian jump systems are proposed. The stability of the Markovian jump networked control system was analyzed in [9] and the filter problem of the networked control system using the Markovian jump method was researched in [10]. It is worth mentioning that the actuator faults usually have the random property like Markovian jump. Literatures [11–13] all studied the control problem of the system with Markovian jump actuator faults. However, the application of the Markovian jump structure is limited because of the assumption that sojourn-time l obeys exponential distribution, and the transition rate is a constant. Nevertheless, in the practical applications, such an assumption is not always satisfied. To overcome this defect, we slack probability distribution of sojourn-time as a general probability distribution. This will make the transition rate become time-varying [14–16]. Correspondingly, this stochastic process is often called as semi-Markovian jump process. Nowadays, there are many research results about semi-Markovian jump. For instance, robust sliding mode control of the uncertain semi-Markovian jump system was researched in [17]. An observer was established for the fuzzy system containing stochastic actuator faults in [18]. This paper will further study the stability of the semi-Markovian jump networked control system with actuator faults and delays.

For cutting down the conservatism of the delay upper bound, in terms of technology, a series of approaches are developed aiming to decrease the approximation error of term $\int_{t-d_2}^{t} \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s$ in the derivative of LKF. At first, a model transformation method is employed to deal with that integral term, which has large conservatism [19]. Afterwards, a free weighting matrix method was studied in [20] to reduce the conservatism of stability criterion. Nevertheless, this approach will raise decision variables. Then, Jensen inequality was widely employed to overcome this defect [21]. Besides, Wirtinger-based inequality, as a more advanced technology than Jensen inequality, was introduced in various control systems [22]. Moreover, free-matrix-based integral inequality was applied in [23] for researching filtering problem for the neural network system. A kind of integral inequality method based on auxiliary function was introduced in [24]. These techniques are used to approximate the derivative of the LKF more accurately so as to obtain less conservative linear matrix inequalities in the stability criterion. But, they all have some manipulations permitting to use a finite linear matrix inequality to test the infinite dimensional stability problem [25–28]. In this paper, we use the Bessel–Legendre inequalities method to tightly approximate the integral term of quadratic functional to the quadratic term of matrix functional about Legendre polynomials. For example, term "$-\int_{t-\tau_M}^{t} \dot{x}(t)W\dot{x}(t)\mathrm{d}s$" in Lyapunov–Krasovskii functional derivative is less than or equal to the "$-\tau_M \Psi_N^T W_N \Psi_N$", where $W_N = \mathrm{diag}\{W, 3W, \ldots, (2N+1)W\}$, $\Psi_N = (1/\tau_M)\int_{t-\tau_M}^{t} \mathscr{L}_N((s-\tau_M)/\tau_M)x(s)\mathrm{d}s$, $N \geq 0$, and the $\mathscr{L}_N$ is the "shifted" Legendre polynomial matrix. Moreover, we construct an improved LKF which is relevant to Legendre polynomials and make use of the reciprocally convex combination method [29–31] to obtain the stability criterion. The derived stability criterion with lower conservativeness is governed by the order N. The conservatism of the criterion decreases as N increases. To the author's best knowledge, for the semi-Markovian jump networked control system containing actuator faults and time-varying delay, employing Bessel–Legendre inequalities to investigate stability has not been fully researched and is of vital significance.

In fact, network channel resources are limited. In the past, most articles applied a periodic triggered mechanism to study networked control systems. The traditional periodic triggered mechanism is fixed to deal with various disturbances. This will cause an increase in the amount of computation of controller because the system is not always under disturbance [32–34]. The event-triggered scheme can decrease the waste of channel resources effectively compared with the traditional periodic triggering scheme. Thus, event-triggered stability analysis not only guarantees the stability of system but also reduces the burden of channel. The distribute control under the event-triggered scheme was researched in [35]. An event-triggered controller for the discrete networked control systems was designed in [36]. The event-triggered real-time scheduling strategy was introduced in [37] for the T-S fuzzy system. Recently, the researchers have developed some improved ETSs for different control systems and different property requirements. Decentralized ETS was researched for the networked fuzzy system in [38]. Distributed ETS for a multiagent system was dealt in [39]. In terms of accelerating system dynamics, we usually expect the high frequency of release at the beginning process. A new static ETS was introduced for a discrete nonlinear system in [40]. Correspondingly, the dynamic ETS which contains the dynamic variables was studied for the networked multiagent system in [41]. Now, in this paper, for the purpose of improving system dynamics while reducing transmission burden, we will introduce an improved static ETS for this semi-Markovian jump networked control system containing actuator faults and time-varying delay.

Thus, there exist several questions to further investigate the semi-Markovian jump networked control system containing actuator faults and delay based on the Bessel–Legendre inequalities approach as follows:

(1) Is it feasible to complete the analysis of this semi-Markovian jump networked control system containing actuator faults and network delay by employing the Bessel–Legendre inequalities approach?

(2) How to establish an improved ETS for this comprehensive continuous system with time delay to raise the release frequency in beginning stage and then gradually reduce the release rate? Such an ETS can not only reduce the waste of channel resources but also accelerate the system dynamics.

(3) Based on the approach of Bessel–Legendre inequalities, how can we design an effective event-triggered controller to control the semi-Markovian jump networked control system containing actuator faults and time-varying delay?

Despite all questions mentioned above are necessary to solve, no one has ever carried out. Now, this paper is committed to dealing with these problems.

In general, the major contributions are (1) A comprehensive model such as the semi-Markovian jump networked control system with actuator faults and time-varying delay is researched. Different from previous research studies, this paper applies Bessel–Legendre inequalities approach to analyze the stability of the system and constructs an appropriate Lyapunov–Krasovskii functional with respect to

Legendre polynomials. Finally, an affine linear matrix inequality is obtained. The acquired stability criterion which is indexed by N has lower conservatism. And the larger N is, the lower conservatism is. (2) This paper establishes an improved static ETS for this comprehensive continuous system to effectively shorten system dynamic process and decrease the burden of transmission. (3) A valid event-triggered controller is ingeniously designed.

The remainder of this paper can be summarized as below. Section 2 shows definitions and problem formulation. In addition, an improved static ETS is established. In Section 3, the result of stability analysis is presented. An effective controller is designed in Section 4. Section 5 provides the numerical examples to verify the effectiveness of the research results. Finally, Section 6 shows the conclusions.

Notations. $\mathscr{E}\{\cdot\}$ denotes the mathematical expectation. $\mathbb{N}$ denotes the natural number. $\mathbb{S}_+^N$ is the set of symmetric positive definite matrices of $\mathbb{R}^{n\times n}$, and $\mathbb{R}^{n\times n}$ represents the set of all $n\times n$ matrices. For any matrix A, $He(A)=A+A^T$.

2. Definitions and Problem Formulation

The general state space model of the semi-Markovian jump networked control system can be expressed as follows:

$$\dot{x}(t)=A(r(t))x(t)+B(r(t))u(t), \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the system state and $u(t)\in\mathbb{R}^p$ is the control input. When $t\geq 0$, $\{r(t)\}$ is the semi-Markovian jump process and $r(t)=i\in\mathscr{S}$, $\mathscr{S}=\{1,2,\ldots,M\}$. At $t=0$, the initial value $x(0)=x_0$, and the initial mode of the semi-Markovian jump process is $r(0)=r_0$. $A(r(t))$ is system matrix with appropriate dimensions. Due to $r(t)=i$, then $A(r(t))$ is expressed by A_i.

The sojourn time of the semi-Markovian jump process does not follow the exponential distribution. Thus, the semi-Markovian jump process has the following transition probabilities:

$$P_r\{r(t+l)=j\mid r(t)=i\}=\begin{cases}\lambda_{ij}(l)l+o(l), & i\neq j,\\ 1+\lambda_{ii}(l)l+o(l), & i=j,\end{cases} \tag{2}$$

and l is the sojourn time. $\lambda_{ij}(l)$ denotes the transition rate from mode i at time t to mode j at time $t+l$ for $i\neq j$. $\lambda_{ii}(l)=-\sum_{j=1,j\neq i}^{M}\lambda_{ij}(l)$. $o(l)$ is the high order infinitesimal of l and $\lim_{l\longrightarrow 0}(o(l)/l)=0$. We define that $\lambda_i(l)$ is the system transition rate of mode i, and then the $\lambda_i(l)$ satisfies

$$\lambda_i(l)=\frac{f_i(l)}{1-F_i(l)}. \tag{3}$$

At mode i, function $f_i(l)$ denotes the probability distribution of sojourn time and function $F_i(l)$ denotes cumulative distribution of sojourn time.

Due to that the bandwidth is limited and the capability of the controller, sensor, and actuator is finite, the traditional periodic-triggered mechanism which will send some unnecessary signals under the most cases without disturbances can cause waste of network resources. In order to shorten the system dynamics while decreasing the load of network transmission, this paper introduces an improved event-triggered scheme.

Assume that the sampled states at $t_0h, t_1h, t_2h, \ldots$, satisfy event-triggering condition, then we call the $t_0h, t_1h, t_2h, \ldots$, as release times. The signals at $t_0h, t_1h, t_2h, \ldots$, can be sent from the sensor to controller. However, there exists a time delay when signal arrives at the actuator from controller. Suppose that $\tau_k\in(0,\overline{\tau})$ is the network time delay and $\overline{\tau}$ is a positive scalar. Thus, $x(t_0h), x(t_1h), x(t_2h), \ldots$ are transmitted to the actuator at the time $t_0h+\tau_0, t_1h+\tau_1, t_2h+\tau_2, \ldots$, respectively. We redescribe system (1) as

$$\dot{x}(t)=A_ix(t)+B_iu(t_kh). \tag{4}$$

Due to $u(t)=K(r(t))x(t)$, thus,

$$\dot{x}(t)=A_ix(t)+B_iK_ix(t_kh). \tag{5}$$

Considering the time delay in the network transmission process, we need to set up the model of the network time delay. Define that

$$\phi_k=\min\{\alpha\mid t_kh+\tau_k+\alpha h\geq t_{k+1}h+\tau_{k+1},\quad \alpha=0,1,2,\ldots\}. \tag{6}$$

Let

$$\begin{aligned}\mathbb{I}_\alpha&=[t_kh+\tau_k+(\alpha-1)h, t_kh+\tau_k+\alpha h),\\ \mathbb{I}_{\phi k}&=[t_kh+\tau_k+(\phi_k-1)h, t_{k+1}h+\tau_{k+1}),\\ &\qquad\alpha=1,2,\ldots,\phi_k-1.\end{aligned} \tag{7}$$

Then,

$$[t_kh+\tau_k, t_{k+1}h+\tau_{k+1})=\bigcup_{\alpha=1}^{\phi_k}\mathbb{I}_\alpha. \tag{8}$$

For $t\in[t_kh+\tau_k, t_{k+1}h+\tau_{k+1})$, define that

$$\begin{aligned}\tau(t)&=\begin{cases}t-t_kh, & t\in\mathbb{I}_1,\\ t-t_kh-h, & t\in\mathbb{I}_2,\\ \quad\vdots & \quad\vdots\\ t-t_kh-(\phi_k-1)h, & t\in\mathbb{I}_{\phi_k},\end{cases}\\ e_k(t)&=\begin{cases}0, & t\in\mathbb{I}_1,\\ x(t_kh)-x(t_kh+h), & t\in\mathbb{I}_2,\\ \quad\vdots & \quad\vdots\\ x(t_kh)-x(t_kh+(\phi_k-1)h), & t\in\mathbb{I}_{\phi_k}.\end{cases}\end{aligned} \tag{9}$$

Apparently, $0<\tau_k\leq\tau(t)\leq\overline{\tau}+h$. Let $\tau_M=\overline{\tau}+h$, then $0<\tau_k\leq\tau(t)\leq\tau_M$.

We expect the proposed ETS to achieve the situation that more packets are transmitted during transient response to make the system approach stability faster. And when the system nears the steady state, the triggering should be limited. Next, we introduce the following improved static event-triggered mechanism:

$$e_k^T(t)\Lambda e_k(t)\leq\sigma_k(t)x^T(t-\tau(t))\Lambda x(t-\tau(t)), \tag{10}$$

where Λ is symmetric positive definite matrix, and time-varying function $\sigma_k(t)$ meets

$$\sigma_k(t) = \sigma_{k\alpha}, \quad t \in \mathbb{I}_\alpha,\ \alpha = 1, 2, \ldots, \phi_k,$$
$$\sigma_{k(\alpha+1)} = \overline{\sigma} + \frac{e_{k\alpha}^T e_{k\alpha}}{\varepsilon + e_{k\alpha}^T e_{k\alpha}} (\sigma_{k\alpha} - \overline{\sigma}),$$
$$e_{k\alpha} = x(t_k h) - x(t_k h + (\alpha - 1)h), \quad \alpha = 1, 2, \ldots, \phi_k, \tag{11}$$

where $\overline{\sigma}$ is the upper bound of $\sigma_{k\alpha}$ and $\overline{\sigma} \in [0, 1]$, $0 < \sigma_{k1} < \overline{\sigma}$, known constant $\varepsilon > 0$.

Using $e_k(t)$ and $\tau(t)$, for $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$, we rewrite (5) as

$$\dot{x}(t) = A_i x(t) + B_i K_i x(t - \tau(t)) + B_i K_i e_k(t). \tag{12}$$

In practical systems, the actuators may experience failures, which may cause the instability of real system. Therefore, considering the actuator faults is meaningful, under the even-triggered mechanism, we define $u_f(t_k h)$ as the control signal sent from the actuator and

$$u_f(t_k h) = \beta(r(t)) u(t_k h), \tag{13}$$

where $\beta(r(t)) = \operatorname{diag}(\beta_{1r(t)}, \beta_{2r(t)}, \ldots, \beta_{pr(t)})$ and $0 \le \underline{\beta}_{wr(t)} \le \beta_{wr(t)} \le \overline{\beta}_{wr(t)} \le 1$, $w = 1, 2, \ldots, p$. The $\underline{\beta}_{wr(t)}$ and $\overline{\beta}_{wr(t)}$ denote the failure bounds of the pth actuator under the fault mode i. We use β_i denoting $\beta(r(t))$, then $\beta_i = \operatorname{diag}(\beta_{1i}, \beta_{2i}, \ldots, \beta_{pi})$ with $0 \le \underline{\beta}_{wi} \le \beta_{wi} \le \overline{\beta}_{wi} \le 1$. It is easy to see that if $\underline{\beta}_{wi} = \overline{\beta}_{wi} = 0$, then the w th actuator completely outage in the ith fault mode; if $\underline{\beta}_{wi} = \overline{\beta}_{wi} = 1$, then the w th actuator under fault mode i has no failure. Define

$$\widehat{\beta}_i = \operatorname{diag}\left(\frac{\underline{\beta}_{1i} + \overline{\beta}_{1i}}{2}, \frac{\underline{\beta}_{2i} + \overline{\beta}_{2i}}{2}, \ldots, \frac{\underline{\beta}_{pi} + \overline{\beta}_{pi}}{2}\right),$$
$$\breve{\beta} = \operatorname{diag}\left(\frac{\overline{\beta}_{1i} - \underline{\beta}_{1i}}{2}, \frac{\overline{\beta}_{2i} - \underline{\beta}_{2i}}{2}, \ldots, \frac{\overline{\beta}_{pi} - \underline{\beta}_{pi}}{2}\right). \tag{14}$$

Rewrite β_i as

$$\beta_i = \widehat{\beta}_i + \Delta_i, \tag{15}$$

where $\Delta_i = \operatorname{diag}(\delta_{1i}, \delta_{2i}, \ldots, \delta_{pi})$, and $|\delta_{wi}| \le ((\overline{\beta}_{wi} - \underline{\beta}_{wi})/2)$, $w = 1, 2, \ldots, p$.

Next, the complete model is shown as

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i \beta_i K_i x(t - \tau(t)) + B_i \beta_i K_i e_k(t), \\ x(t) &= \phi(t), \quad t \in [-\tau_M, 0], \end{aligned} \tag{16}$$

where the function $\Phi(t)$ is continuous on $[-\tau_M, 0]$. Note that

$$\begin{aligned} 0 < \tau_k \le \tau(t) \le \tau_M, \\ d_1 \le \dot{\tau}(t) \le d_2, \end{aligned} \tag{17}$$

where scalars $d_1 < 0$ and $d_2 > 0$.

This paper analyzes the stability of system (16) via the Bessel–Legendre inequalities. Before the stability research, we first give some definitions and lemmas to facilitate analysis.

Lemma 1 (see [42]). *Assume that $W_1, W_2 \in \mathbb{R}^n$ are symmetric positive matrices. If there exist the symmetric matrices $X_1, X_2 \in \mathbb{R}^n$ and the matrices $Y_1, Y_2 \in \mathbb{R}^n$ such that for $\varepsilon = 0, 1$, the*

$$\begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} - \varepsilon \begin{bmatrix} X_1 & Y_1 \\ Y_1^T & 0 \end{bmatrix} - (1 - \varepsilon) \begin{bmatrix} 0 & Y_2 \\ Y_2^T & 0 \end{bmatrix} \ge 0, \tag{18}$$

holds, then the following inequality is true for all $\varepsilon \in (0, 1)$:

$$\begin{bmatrix} \frac{1}{\varepsilon} W_1 & 0 \\ 0 & \frac{1}{1-\varepsilon} W_2 \end{bmatrix} \ge \begin{bmatrix} W_1 & 0 \\ 0 & W_2 \end{bmatrix} + \varepsilon \begin{bmatrix} 0 & Y_1 \\ Y_1^T & X_2 \end{bmatrix} + (1 - \varepsilon) \begin{bmatrix} X_1 & Y_2 \\ Y_2^T & 0 \end{bmatrix}. \tag{19}$$

Next, we give the definition of the "shifted" Legendre polynomials which is defined over the interval $[0, 1]$.

Definition 1. For all $u \in [0, 1]$, $f, q \in \mathbb{N}$, the "shifted" Legendre polynomials is the following formula:

$$L_f(u) = (-1)^f \sum_{q=0}^{f} \rho_q^f u^q, \tag{20}$$

where $\rho_q^f = (-1)^q \binom{f}{q} \binom{f+q}{q}$, $\binom{f}{q}$ means the binomial coefficients, and $\binom{f}{q} = f!/((f - q)!q!)$.

According to the Legendre polynomials, we define polynomial matrix $\mathbb{L}_N$:

$$\mathbb{L}_N(u) := \left[L_0(u) I_n \ \ L_1(u) I_n \ \ \ldots \ \ L_N(u) I_n \right]^T, \tag{21}$$

where $n \in \mathbb{N}$ and $N \in \mathbb{N}$. The Legendre polynomials have the orthogonality property which results in the application of the Legendre polynomials. For any symmetric positive definite matrix W

$$\int_0^1 L_N(u) W^{-1} \mathbb{L}_N^T(u) \mathrm{d}u = \overline{W}_N^{-1}, \tag{22}$$

is true, where $\overline{W}_N = \operatorname{diag}\{W, 3W, \ldots, (2N + 1)W\}$.

In the process of stability analysis, we need to use the following techniques with respect to the differential about the Legendre polynomials matrix:

$$\frac{\mathrm{d}}{\mathrm{d}u} \mathbb{L}_N(u) = \overline{\Upsilon}_N \mathbb{L}_N(u) = \Upsilon_N \mathbb{L}_{N-1}(u), \tag{23}$$

$$\frac{\mathrm{d}}{\mathrm{d}u} (u \mathbb{L}_N(u)) = \mathbb{L}_N(u) + \Xi_N \mathbb{L}_N(u), \tag{24}$$

where $\overline{\Upsilon}_N = \left[\Upsilon_N \ \ 0_{n(N+1),n}\right]$, $\Upsilon_N = \upsilon_N \otimes I_n$, and $\Xi_N = \widehat{e}_N \otimes I_n$. The matrix $\upsilon_N \in \mathbb{R}^{(N+1)\times N}$ is defined as

$$\upsilon_N(f,q) = \begin{cases} 0, & \text{if } f \geq q, \\ (2f-1)\left(1-(-1)^{f+q}\right), & \text{if } f < q. \end{cases} \tag{25}$$

The matrix $\widehat{e}_N \in \mathbb{R}^{(N+1)\times(N+1)}$ is defined as

$$\widehat{e}_N(f,q) = \begin{cases} 0, & \text{if } f > q, \\ f, & \text{if } f = q, \\ 2f-1, & \text{if } k < q. \end{cases} \tag{26}$$

Noting that the "shifted" Legendre polynomials are defined on the interval $[0,1]$, we give the evaluation values of the polynomial matrix at $u = 0$ and $u = 1$:

$$\mathbb{L}_N(0) = \begin{bmatrix} I_n \\ -I_n \\ \vdots \\ (-1)^N I_n \end{bmatrix} := \overline{\chi}_N, \tag{27}$$

$$\mathbb{L}_N(1) = \begin{bmatrix} I_n \\ I_n \\ \vdots \\ I_n \end{bmatrix} := \chi_N. \tag{28}$$

The next lemma expresses the Bessel–Legendre inequality.

Lemma 2 (see [43]). *For $x \in \mathscr{L}_2([a,b] \longrightarrow \mathbb{R}^n)$, any appropriate dimensional matrix W which is a symmetric positive definite, and any $N \in \mathbb{N}$, the inequality*

$$\int_a^b x^T(s) W x(s) ds \geq (b-a) \Psi_N^T \overline{W}_N \Psi_N, \tag{29}$$

holds, where

$$\begin{aligned} \Psi_N &= \frac{1}{b-a}\int_a^b \mathbb{L}_N\left(\frac{s-a}{b-a}\right) x(s) ds, \\ \overline{W}_N &= \operatorname{diag}\{W, 3W, \ldots, (2N+1)W\}. \end{aligned} \tag{30}$$

Definition 2. For any $r_0 \in \mathcal{S}$ and any initial state $\Phi(t) \in [-\tau_M, 0]$, if the following inequality

$$\lim_{t\longrightarrow\infty} \mathscr{E}\left\{\int_0^t \|x(s)\|^2 \mathrm{d}s \mid (\Phi(t), r_0)\right\} \leq \infty, \tag{31}$$

is true, then system (16) is stochastically stable.

3. Stability Analysis

In this part, we research the stochastic stability of system (16). The less conservative stability criterion is given in Theorem 1.

Theorem 1. *Given $N \in \mathbb{N}$, scalar $\varepsilon > 0$, if there exist matrix $P_N(j) \in \mathbb{S}_+^{(2N+3)n}$, matrices $Q_1, Q_2, W \in \mathbb{S}_+^n$, and matrices $Y_1, Y_2 \in \mathbb{R}^{(N+1)n\times(N+1)n}$ such that for all $(\tau(t), \dot{\tau}(t)) \in \mathscr{H}$ the following inequality*

$$\Theta_N' = \begin{bmatrix} \overline{\Theta}_N & \Pi_N^T \begin{bmatrix} \frac{\tau_M - \tau(t)}{\tau_M} Y_1 \\ 0 \end{bmatrix} & \Pi_N^T \begin{bmatrix} 0 \\ \frac{\tau(t)}{\tau_M} Y_2^T \end{bmatrix} & \tau_M H_N^T \\ * & -\frac{\tau_M - \tau(t)}{\tau_M}\overline{W}_N & 0 & 0 \\ * & * & -\frac{\tau(t)}{\tau_M}\overline{W}_N & 0 \\ * & * & * & W - 2I_n \end{bmatrix} \leq 0, \tag{32}$$

holds, then system (16) is stochastically stable, where

$$\begin{aligned}
\mathscr{H} &= \mathscr{C}o\{(0,0), (0,d_2), (\tau_M,0), (\tau_M,d_1)\}, \\
\overline{\Theta}_N &= He\left(\left(E_{1,N} + \dot{\tau}(t) E_{2,N}\right)^T P_N(i) \Phi_N\right) \\
&\quad + \Phi_N^T \left(\sum_{j=1}^M \overline{\lambda}_{i,j} P_N(j)\right) \Phi_N + E_{3,N} - \prod_N^T \Lambda_N \Pi_N, \\
\Pi_N &= \begin{bmatrix} \chi_N & -\overline{\Upsilon}_N & 0 & -\overline{\chi}_N & 0 & 0 \\ 0 & 0 & -\overline{\Upsilon}_N & \chi_N & -\overline{\chi}_N & 0 \end{bmatrix}, \\
\overline{W}_N &= \operatorname{diag}\{W, 3W, \ldots, (2N+1)W\}, \\
H_N &= \begin{bmatrix} A_i & 0 & 0 & B_i\beta_i K_i & 0 & B_i\beta_i K_i \end{bmatrix}, \\
E_{1,N} &= \begin{bmatrix} A_i & 0 & 0 & B_i\beta_i K_i & 0 & B_i\beta_i K_i \\ \chi_N & -\overline{\Upsilon}_N & 0 & -\overline{\chi}_N & 0 & 0 \\ 0 & 0 & -\overline{\Upsilon}_N & \chi_N & -\overline{\chi}_N & 0 \end{bmatrix}, \\
E_{2,N} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\Xi_N + \overline{\Upsilon}_N & 0 & \overline{\chi}_N & 0 & 0 \\ 0 & 0 & \Xi_N & -\overline{\chi}_N & 0 & 0 \end{bmatrix}, \\
\Phi_N &= \begin{bmatrix} I_n & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau(t) I_{nN} & 0 & 0 & 0 & 0 \\ 0 & 0 & (\tau_M - \tau(t)) I_{nN} & 0 & 0 & 0 \end{bmatrix}, \\
E_{3,N} &= \operatorname{diag}\{Q_1, 0, 0, (1-\dot{\tau}(t))(Q_2 - Q_1) + \sigma_k(t)\Lambda, -Q_2, -\Lambda\}, \\
\Lambda_N &= \begin{bmatrix} \overline{W}_N & 0 \\ 0 & \overline{W}_N \end{bmatrix} + \frac{\tau(t)}{\tau_M}\begin{bmatrix} 0 & Y_1 \\ Y_1^T \overline{W}_N \end{bmatrix} + \frac{\tau_M - \tau(t)}{\tau_M}\begin{bmatrix} \overline{W}_N & Y_2 \\ Y_2^T & 0 \end{bmatrix},
\end{aligned} \tag{33}$$

and the unit matrix $I_{nN} \in \mathbb{R}^{(N+1)n\times(N+1)n}$ and $I_n \in \mathbb{R}^{n\times n}$.

Proof. Firstly, we construct the LKF as

$$V_N(x(t),\dot{x}(t),r(t),t) = V_{1N}(x(t),r(t),t) + V_{2N}(x(t),t) + V_{3N}(x(t),\dot{x}(t),t), \tag{34}$$

where

$$\begin{aligned} V_{1N}(x(t),r(t),t) &= \xi_N^T(t)P_N(r(t))\xi_N(t),\\ V_{2N}(x(t),t) &= \int_{t-\tau(t)}^{t} x^T(s)Q_1x(s)\mathrm{d}s + \int_{t-\tau_M}^{t-\tau(t)} x^T(s)Q_2x(s)\mathrm{d}s,\\ V_{3N}(x(t),\dot{x}(t),t) &= \tau_M\int_{t-\tau_M}^{t}\int_{\theta}^{t}\dot{x}^T(s)W\dot{x}(s)\mathrm{d}s\,\mathrm{d}\theta, \end{aligned} \tag{35}$$

where

$$\begin{aligned} \xi_N^T(t) &= \left[\, x^T(t) \quad \tau(t)\Psi_{1,N}^T(t) \quad (\tau_M-\tau(t))\Psi_{2,N}^T(t)\,\right],\\ \Psi_{1,N}(t) &= \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}\mathbb{L}_N\left(\frac{s-t+\tau(t)}{\tau(t)}\right)x(s)\mathrm{d}s,\\ \Psi_{2,N}(t) &= \frac{1}{\tau_M-\tau(t)}\int_{t-\tau_M}^{t-\tau(t)}\mathbb{L}_N\left(\frac{s-t+\tau_M}{\tau_M-\tau(t)}\right)x(s)\mathrm{d}s. \end{aligned} \tag{36}$$

For deriving the stability criterion, we fist calculate the weak infinitesimal generator $\mathscr{L}V_N$ of LKF. The main task is to obtain an upper bound of the weak infinitesimal generator $\mathscr{L}V_N$ by employing the Bessel–Legendre polynomials method. Through using Lemma 1 and Lemma 2, we will carry out a series of relaxations and simplifications on $\mathscr{L}V_N$ to get a tight upper bound. Next, we calculate the $\mathscr{L}V_N$:

$$\mathscr{L}V_N = \mathscr{L}V_{1N} + \sum_{k=2}^{3}\mathscr{L}V_{kN}. \tag{37}$$

Among them,

$$\mathscr{L}V_{1N} = \lim_{\Delta\longrightarrow 0}\frac{\mathscr{E}[V_{1N}(t+\Delta),r(t+\Delta)\,|\,x(t),r(t)] - V_{1N}(x(t),r(t))}{\Delta}. \tag{38}$$

Due to $r(t) = i \in \mathscr{S}$, making use of transition probability of the semi-Markovian jump process, we have

$$\begin{aligned} \mathscr{L}V_{1N} &= \lim_{\Delta\longrightarrow 0}\frac{1}{\Delta}\left\{\mathscr{E}\left[\sum_{j=1,j\neq i}^{M} Pr\{r(t+\Delta)=j\,|\,r(t)=i\}\xi_N^T(t+\Delta)P_N(j)\xi_N(t+\Delta)\right.\right.\\ &\quad\left.\left. + Pr\{r(t+\Delta)=i\,|\,r(t)=i\}\xi_N^T(t+\Delta)P_N(i)\xi_N(t+\Delta)\right] - \xi_N^T(t)P_N(i)\xi_N(t)\right\}\\ &= \lim_{\Delta\longrightarrow 0}\frac{1}{\Delta}\left\{\mathscr{E}\left[\sum_{j=1,j\neq i}^{M}\frac{q_{ij}(F_i(l+\Delta)-F_i(l))}{1-F_i(l)}\xi_N^T(t+\Delta)P_N(j)\xi_N(t+\Delta)\right.\right.\\ &\quad\left.\left. + \frac{1-F_i(l+\Delta)}{1-F_i(l)}\xi_N^T(t+\Delta)P_N(i)\xi_N(t+\Delta)\right] - \xi_N^T(t)P_N(i)\xi_N(t)\right\}. \end{aligned} \tag{39}$$

The derivation process of equation (39) is aided by (3), and the following definitions that $\lambda_{ij}(l) \triangleq q_{ij}\lambda_i(l)$, $i \neq j$, and $\lambda_{ii}(l) \triangleq -\sum_{j=1,j\neq i}^{M}\lambda_{ij}(l)$, where q_{ij} is the transition probability intensity, and $q_{ii} = -\sum_{j=1,j\neq i}^{M}q_{ij} = -1$ [44]. In equation (39), $Pr\{r(t+\Delta)=j\,|\,r(t)=i\} = \lambda_{ij}(l)\Delta = q_{ij}\lambda_i(l)\Delta = q_{ij}(f_i(l)\Delta)/(1-F_i(l)) = q_{ij}((F_i(l+\Delta)-F_i(l))\Delta)/((1-F_i(l))\Delta) = (q_{ij}(F_i(l+\Delta)-F_i(l)))/(1-F_i(l))$, $Pr\{r(t+\Delta)=i\,|\,r(t)=i\} = 1+\lambda_{ii}\Delta = 1-\sum_{j=1,j\neq i}^{M}\lambda_{ij}(l)\Delta = 1-\sum_{j=1,j\neq i}^{M}(q_{ij}(F_i(l+\Delta)-F_i(l)))/(1-F_i(l)) = (1-F_i(l+\Delta))/(1-F_i(l))$.

When Δ is small, we can get that

$$\xi_N(t+\Delta) = \Delta\dot{\xi}_N(t) + \xi_N(t) + o(\Delta). \tag{40}$$

Then, (39) becomes

$$\mathscr{L}V_{1N} = \lim_{\Delta\longrightarrow 0}\frac{1}{\Delta}\left\{\mathscr{E}\left[\sum_{j=1,j\neq i}^{M}\frac{q_{ij}\left(F_i(l+\Delta)-F_i(l)\right)}{1-F_i(l)}\left[\Delta\dot{\xi}_N(t)+\xi_N(t)+o(\Delta)\right]^T P_N(j)\right.\right.$$
$$\cdot\left[\Delta\dot{\xi}_N(t)+\xi_N(t)+o(\Delta)\right]+\frac{1-F_i(l+\Delta)}{1-F_i(l)}\left[\Delta\dot{\xi}_N(t)+\xi_N(t)+o(\Delta)\right]^T \tag{41}$$
$$\left.\left.\cdot P_N(i)\left[\Delta\dot{\xi}_N(t)+\xi_N(t)+o(\Delta)\right]\right]-\xi_N(t)^T P_N(i)\xi_N(t)\right\}.$$

Due to

$$\lim_{\Delta\longrightarrow 0}\frac{\Delta^2}{\Delta}=0,$$
$$\lim_{\Delta\longrightarrow 0}\frac{q_{ij}\left(F_i(l+\Delta)-F_i(l)\right)}{1-F_i(l)}=0, \tag{42}$$

then

$$\mathscr{L}V_{1N} = \lim_{\Delta\longrightarrow 0}\left\{\frac{1}{\Delta}\mathscr{E}\left[\sum_{j=1,j\neq i}^{M}\frac{q_{ij}\left(F_i(l+\Delta)-F_i(l)\right)}{1-F_i(l)}\left[\xi_N^T(t)P_N(j)\xi_N(t)\right]\right]\right.$$
$$+\mathscr{E}\left[\frac{1-F_i(l+\Delta)}{1-F_i(l)}\left[\dot{\xi}_N^T(t)P_N(i)\xi_N(t)+\xi_N^T(t)P_N(i)\dot{\xi}_N(t)\right]\right.$$
$$\left.\left.+\frac{1}{\Delta}\frac{1-F_i(l+\Delta)}{1-F_i(l)}\xi_N^T(t)P_N(i)\xi_N(t)\right]-\frac{1}{\Delta}\xi_N^T(t)P_N(i)\xi_N(t)\right\},$$
$$=\lim_{\Delta\longrightarrow 0}\left\{\frac{1}{\Delta}\mathscr{E}\left[\sum_{j=1,j\neq i}^{M}\frac{q_{ij}\left(F_i(l+\Delta)-F_i(l)\right)}{1-F_i(l)}\left[\xi_N^T(t)P_N(j)\xi_N(t)\right]\right]\right. \tag{43}$$
$$+\mathscr{E}\left[\frac{1-F_i(l+\Delta)}{1-F_i(l)}\left[\dot{\xi}_N^T(t)P_N(i)\xi_N(t)+\xi_N^T(t)P_N(i)\dot{\xi}_N(t)\right]\right.$$
$$\left.\left.+\frac{1}{\Delta}\frac{F_i(l)-F_i(l+\Delta)}{1-F_i(l)}\xi_N^T(t)P_N(i)\xi_N(t)\right]\right\}.$$

According to

$$\lim_{\Delta\longrightarrow 0}\frac{1-F_i(l+\Delta)}{1-F_i(l)}=1, \tag{44}$$

we can get

$$\mathscr{L}V_{1N}=\mathscr{E}\left\{\dot{\xi}_N^T(t)P_N(i)\xi_N(t)+\xi_N^T(t)P_N(i)\dot{\xi}_N(t)+\lim_{\Delta\longrightarrow 0}\frac{1}{\Delta}\frac{F_i(l+\Delta)-F_i(l)}{1-F_i(l)}\left[\sum_{j=1,j\neq i}^{M}q_{ij}\xi_N^T(t)P_N(j)\xi_N(t)-\xi_N^T(t)P_N(i)\xi_N(t)\right]\right\}. \tag{45}$$

Notice that

$$\lim_{\Delta\longrightarrow 0}\frac{1}{\Delta}\frac{F_i(l+\Delta)-F_i(l)}{1-F_i(l)}=\frac{1}{1-F_i(l)}\lim_{\Delta\longrightarrow 0}\frac{F_i(l+\Delta)-F_i(l)}{\Delta}, \tag{46}$$

according to (3) and (46) becomes $\lambda_i(l)$. Then, $\mathscr{L}V_{1N}$ becomes

$$\mathscr{L}V_{1N} = \mathscr{E}\left\{\dot{\xi}_N^T(t)P_N(i)\xi_N(t) + \xi_N^T(t)P_N(i)\dot{\xi}_N(t) + \xi_N^T(t)\left(\sum_{j=1}^{M}\lambda_{ij}(l)P_N(j)\right)\xi_N(t)\right\}. \quad (47)$$

By the same approach in [44],

$$\mathscr{L}V_{1N} = \dot{\xi}_N^T(t)P_N(i)\xi_N(t) + \xi_N^T(t)P_N(i)\dot{\xi}_N(t) + \xi_N^T(t)\left(\sum_{j=1}^{M}\overline{\lambda}_{ij}P_N(j)\right)\xi_N(t), \quad (48)$$

where $\overline{\lambda}_{ij} \coloneqq \mathscr{E}\{\lambda_{ij}(l)\} = \int_0^\infty \lambda_{ij}(l)f_i(l)\mathrm{d}l$.

Before solving $\mathscr{L}V_{2N}$ and $\mathscr{L}V_{3N}$, we do some processes on (48) in order to conveniently derive inequality (32) in Theorem 1. We need to define that

$$\eta_N^T(t) = \left[x^T(t)\ \ \Psi_{1,N}^T(t)\ \ \Psi_{2,N}^T(t)\ \ x^T(t-\tau(t))\ \ x^T(t-\tau_M)\ \ e_k^T(t)\right]. \quad (49)$$

Then,

$$\xi_N^T(t) = \eta_N^T(t)\Phi_N^T, \quad (50)$$

where Φ_N is defined in Theorem 1. In addition,

$$\dot{\xi}_N^T(t) = \left[\dot{x}^T(t)\quad \frac{\mathrm{d}}{\mathrm{d}t}\left[\tau(t)\Psi_{1,N}^T(t)\right]\quad \frac{\mathrm{d}}{\mathrm{d}t}\left[(\tau_M-\tau(t))\Psi_{2,N}^T(t)\right]\right]. \quad (51)$$

Among them,

$$\begin{aligned}
\dot{x}(t) &= A_i x(t) + B_i\beta_i K_i x(t-\tau(t)) + B_i\beta_i K_i e_k(t),\\
\Psi_{1,N}(t) &= \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}\mathbb{L}_N\left(\frac{s-t+\tau(t)}{\tau(t)}\right)x(s)\mathrm{d}s,\\
\Psi_{2,N}(t) &= \frac{1}{\tau_M-\tau(t)}\int_{t-\tau_M}^{t-\tau(t)}\mathbb{L}_N\left(\frac{s-t+\tau_M}{\tau_M-\tau(t)}\right)x(s)\mathrm{d}s.
\end{aligned} \quad (52)$$

Set $\lambda = (s-t+\tau(t))/(\tau(t))$, then $s(\lambda) = \lambda\tau(t)+t-\tau(t)$, and rewrite $\Psi_{1,N}(t)$ as

$$\Psi_{1,N}(t) = \int_0^1 \mathbb{L}_N(\lambda)x(s(\lambda))\mathrm{d}\lambda. \quad (53)$$

Then,

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left[\tau(t)\Psi_{1,N}(t)\right] &= \dot{\tau}(t)\Psi_{1,N}(t) + \tau(t)\int_0^1 \mathbb{L}_N(\lambda)\dot{x}(s(\lambda)) \cdot [\lambda\dot{\tau}(t) + 1 - \dot{\tau}(t)]\mathrm{d}\lambda\\
&= \dot{\tau}(t)\Psi_{1,N}(t) + \dot{\tau}(t)\tau(t)\int_0^1 \lambda\mathbb{L}_N(\lambda)\dot{x}(s(\lambda))\mathrm{d}\lambda\\
&\quad + (1-\dot{\tau}(t))\tau(t)\int_0^1 \mathbb{L}_N(\lambda)\dot{x}(s(\lambda))\mathrm{d}\lambda.
\end{aligned} \quad (54)$$

Using the subsection integration method and employing (24) and (28), we can get

$$\begin{aligned}
\tau(t)\int_0^1 \lambda\mathbb{L}_N(\lambda)\dot{x}(s(\lambda))\mathrm{d}\lambda &= \int_0^1 \lambda\mathbb{L}_N(\lambda)dx(s(\lambda))\\
&= \mathbb{L}_N(1)x(t) - \int_0^1 x(s(\lambda))\frac{d}{d\lambda}(\lambda\mathbb{L}_N(\lambda))\mathrm{d}\lambda\\
&= \mathbb{L}_N(1)x(t) - \Psi_{1,N}(t) - \Xi_N\Psi_{1,N}(t)\\
&= \chi_N x(t) - \Psi_{1,N}(t) - \Xi_N\Psi_{1,N}(t).
\end{aligned} \quad (55)$$

In addition, we deal with $\tau(t)\int_0^1 \mathbb{L}_N(\lambda)\dot{x}(s(\lambda))\mathrm{d}\lambda$. Similarly, using the subsection integration and employing (23) and (27), we obtain

$$\begin{aligned}
\tau(t)\int_0^1 \mathbb{L}_N(\lambda)\dot{x}(s(\lambda))\mathrm{d}\lambda &= \int_0^1 \mathbb{L}_N(\lambda)\mathrm{d}x(s(\lambda))\\
&= \mathbb{L}_N(1)x(t) - \mathbb{L}_N(0)x(t-\tau(t))\\
&\quad - \int_0^1 x(s(\lambda))\frac{\mathrm{d}}{\mathrm{d}\lambda}(\mathbb{L}_N(\lambda))\mathrm{d}\lambda\\
&= \chi_N x(t) - \overline{\chi}_N x(t-\tau(t)) - \overline{\Upsilon}_N\Psi_{1,N}(t).
\end{aligned} \quad (56)$$

Thus,

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left[\tau(t)\Psi_{1,N}(t)\right] &= \dot{\tau}(t)\Psi_{1,N}(t) + \dot{\tau}(t)\left[\chi_N x(t) - \Psi_{1,N}(t)\right.\\
&\quad \left. - \Xi_N\Psi_{1?N}(t)\right] + (1-\dot{\tau}(t))\left[\chi_N x(t)\right.\\
&\quad \left. - \overline{\chi}_N x(t-\tau(t)) - \overline{\Upsilon}\Psi_{1,N}(t)\right].
\end{aligned} \quad (57)$$

Next, we deal with $(\mathrm{d}/\mathrm{d}t)[(\tau_M-\tau(t))\Psi_{2,N}(t)]$. Set $u = (s-t+\tau_M)/(\tau_M-\tau(t))$ and $s(u) = u\tau_M - u\tau(t) + t - \tau_M$, and then we rewrite $\Psi_{2,N}(t)$ as

$$\Psi_{2,N}(t) = \int_0^1 \mathbb{L}_N(u)x(s(u))\mathrm{d}u. \quad (58)$$

Then,

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left[(\tau_M-\tau(t))\Psi_{2,N}(t)\right] &= -\dot{\tau}(t)\Psi_{2,N}(t) + (\tau_M-\tau(t)) \cdot \int_0^1 \mathbb{L}_N(u)\dot{x}(s(u))[-u\dot{\tau}(t)+1]\mathrm{d}u\\
&= -\dot{\tau}(t)\Psi_{2,N}(t) - \dot{\tau}(t)(\tau_M-\tau(t)) \cdot \int_0^1 u\mathbb{L}_N(u)\dot{x}(s(u))\mathrm{d}u\\
&\quad + (\tau_M-\tau(t))\int_0^1 \mathbb{L}_N(u)\dot{x}(s(u))\mathrm{d}u,
\end{aligned} \quad (59)$$

where

$$\begin{aligned}(\tau_M-\tau(t))\int_0^1 u\mathbb{L}_N(u)\dot{x}(s(u))\mathrm{d}u &= \int_0^1 u\mathbb{L}_N(u)\mathrm{d}x(s(u))\\ &= \mathbb{L}_N(1)x(t-\tau(t))-\Psi_{2,N}(t)-\Xi_N\Psi_{2,N}(t)\\ &= \chi_N x(t-\tau(t))-\Psi_{2,N}(t)-\Xi_N\Psi_{2,N}(t),\\ (\tau_M-\tau(t))\int_0^1 \mathbb{L}_N(u)\dot{x}(s(u))\mathrm{d}u &= \int_0^1 \mathbb{L}_N(u)\mathrm{d}x(s(u))\\ &= \mathbb{L}_N(1)x(t-\tau(t))-\mathbb{L}_N(0)x(t-\tau_M)-\overline{\Upsilon}_N\Psi_{2,N(t)}\\ &= \chi_N x(t-\tau(t))-\overline{\chi}_N x(t-\tau_M)-\overline{\Upsilon}_N\Psi_{2,N}(t).\end{aligned} \tag{60}$$

Thus, $(\mathrm{d}/\mathrm{d}t)[(\tau_M-\tau(t))\Psi_{2,N}(t)]$ becomes

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\left[(\tau_M-\tau(t))\Psi_{2,N}(t)\right] &= -\dot{\tau}(t)\Psi_{2,N}(t)-\dot{\tau}(t)\left[\chi_N x(t-\tau(t))-\Psi_{2,N}(t)-\Xi_N\Psi_{2,N}(t)\right]\\ &\quad+\chi_N x(t-\tau(t))-\overline{\chi}_N x(t-\tau_M)-\overline{\Upsilon}_N\Psi_{2,N}(t).\end{aligned} \tag{61}$$

Combining $\dot{x}(t)$ and (57) and (61), we can obtain

$$\dot{\xi}_N(t)=\left(E_{1,N}+\dot{\tau}(t)E_{2,N}\right)\eta_N(t), \tag{62}$$

where $E_{1,N}$ and $E_{2,N}$ are defined in Theorem 1.

Now, we use the obtained (50) and (62) to rewrite the $\mathscr{L}V_{1N}$ in (48),

$$\begin{aligned}\mathscr{L}V_{1N} &= \eta_N^T(t)He\left(\left(E_{1,N}+\dot{\tau}(t)E_{2,N}\right)^T P_N(i)\Phi_N\right)\eta_N(t)\\ &\quad+\eta_N^T(t)\Phi_N^T\left(\sum_{j=1}^{M}\overline{\lambda}_{i,j}P_N(j)\right)\Phi_N\eta_N(t).\end{aligned} \tag{63}$$

Next, we will give $\mathscr{L}V_{2N}$ and $\mathscr{L}V_{3N}$. Add term $e_k^T(t)\Lambda e_k(t)-e_k^T(t)\Lambda e_k(t)$ on $\mathscr{L}V_{2N}$, then

$$\begin{aligned}\mathscr{L}V_{2N} &= x^T(t)Q_1x(t)+(1-\dot{\tau}(t))x^T(t-\tau(t))(Q_2-Q_1)x(t-\tau(t))\\ &\quad-x^T(t-\tau_M)Q_2x(t-\tau_M)+e_k^T(t)\Lambda e_k(t)-e_k^T(t)\Lambda e_k(t)\\ &\leq \eta_N^T(t)E_{3,N}\eta_N(t),\end{aligned} \tag{64}$$

where the matrix $E_{3,N}$ is defined in Theorem 1.

$$\begin{aligned}\mathscr{L}V_{3N} &= \tau_M^2\dot{x}^T(t)W\dot{x}(t)-\tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)W\dot{x}(s)\mathrm{d}s\\ &= \tau_M^2\eta_N^T(t)H_N^TWH_N\eta_N(t)-\tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)W\dot{x}(s)\mathrm{d}s,\end{aligned} \tag{65}$$

where

$$H_N=\left[\,A_i\ \ 0\ \ 0\ \ B_i\beta_iK_i\ \ 0\ \ B_i\beta_iK_i\,\right]. \tag{66}$$

Based on Lemma 2, we have

$$\tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)W\dot{x}(s)\mathrm{d}s\geq\begin{bmatrix}\overline{\Psi}_{1,N}\\ \overline{\Psi}_{2,N}\end{bmatrix}^T\begin{bmatrix}\tau_M\tau(t)\overline{W}_N & 0\\ 0 & \tau_M(\tau_M-\tau(t))\overline{W}_N\end{bmatrix}\begin{bmatrix}\overline{\Psi}_{1,N}\\ \overline{\Psi}_{2,N}\end{bmatrix}, \tag{67}$$

where

$$\overline{\Psi}_{1,N} = \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}\mathbb{L}_N\left(\frac{s-t+\tau(t)}{\tau(t)}\right)\dot{x}(s)\mathrm{d}s,$$
$$\overline{\Psi}_{2,N} = \frac{1}{\tau_M-\tau(t)}\int_{t-\tau_M}^{t-\tau(t)}\mathbb{L}_N\left(\frac{s-t+\tau_M}{\tau_M-\tau(t)}\right)\dot{x}(s)\mathrm{d}s. \quad (68)$$

Employing the subsection integration method, we derive

$$\begin{bmatrix}\tau(t)\overline{\Psi}_{1,N}\\(\tau_M-\tau(t))\overline{\Psi}_{2,N}\end{bmatrix} = \begin{bmatrix}\chi_N x(t)-\overline{\chi}_N x(t-\tau(t))-\overline{\Upsilon}_N\Psi_{1,N}\\ \chi_N x(t-\tau(t))-\overline{\chi}_N x(t-\tau_M)-\overline{\Upsilon}_N\Psi_{2,N}\end{bmatrix} = \Pi_N\eta_N(t), \quad (69)$$

where

$$\Pi_N = \begin{bmatrix}\chi_N & -\overline{\Upsilon}_N & 0 & -\overline{\chi}_N & 0 & 0\\ 0 & 0 & \overline{\Upsilon}_N & \chi_N & -\overline{\chi}_N & 0\end{bmatrix}. \quad (70)$$

Thus,

$$-\tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)W\dot{x}(s)\mathrm{d}s \le -\eta_N^T(t)\prod_N^T\begin{bmatrix}\frac{\tau_M}{\tau(t)}\overline{W}_N & 0\\ 0 & \frac{\tau_M}{\tau_M-\tau(t)}\overline{W}_N\end{bmatrix}\Pi_N\eta_N(t). \quad (71)$$

Next, using Lemma 1 and 2 and choosing $X_1 = W_N - Y_1W_N^{-1}Y_1^T$, $X_2 = W_N - Y_2^TW_N^{-1}Y_2$ [43], we get the following inequality:

$$-\tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)W\dot{x}(s)\mathrm{d}s \le -\eta_N^T(t)\prod_N^T\Lambda_N\Pi_N\eta_N(t) + \eta_N^T(t)\prod_N^T\Lambda_N'\Pi_N\eta_N(t), \quad (72)$$

where

$$\Lambda_N = \begin{bmatrix}\overline{W}_N & 0\\ 0 & \overline{W}_N\end{bmatrix} + \frac{\tau(t)}{\tau_M}\begin{bmatrix}0 & Y_1\\ Y_1^T & \overline{W}_N\end{bmatrix} + \frac{\tau_M-\tau(t)}{\tau_M}\begin{bmatrix}\overline{W}_N & Y_2\\ Y_2^T & 0\end{bmatrix},$$
$$\Lambda_N' = \begin{bmatrix}\frac{\tau_M-\tau(t)}{\tau_M}Y_1\overline{W}_N^{-1}Y_1^T & 0\\ 0 & \frac{\tau(t)}{\tau_M}Y_2^T\overline{W}_N^{-1}Y_2\end{bmatrix}. \quad (73)$$

Then, we rewrite $\mathscr{L}V_{3N}$ as follows:

$$\mathscr{L}V_{3N} = \eta_N^T(t)\left[\tau_M^2H_N^TWH_N + \prod_N^T\Lambda_N'\Pi_N - \prod_N^T\Lambda_N\Pi_N\right]\eta_N(t). \quad (74)$$

Combining (63), (64), and (74), $\mathscr{L}V_N$ satisfies that

$$\mathscr{L}V_N \le \eta_N^T(t)\Bigg[He\left(\left(E_{1,N}+\dot{\tau}(t)E_{2,N}\right)^TP_N(i)\Phi_N\right) + \Phi_N^T\left(\sum_{j=1}^{M}\overline{\lambda}_{i,j}P_N(j)\right)\Phi_N + E_{3,N} + \tau_M^2H_N^TWH_N + \prod_N^T\Lambda_N'\Pi_N - \prod_N^T\Lambda_N\Pi_N\Bigg]\eta_N(t) = \eta_N^T(t)\Theta_N\eta_N(t), \quad (75)$$

where

$$\Theta_N = \overline{\Theta}_N + \prod_N^T\Lambda_N'\Pi_N + \tau_M^2H_N^TWH_N,$$
$$\overline{\Theta}_N = He\left(\left(E_{1,N}+\dot{\tau}(t)E_{2,N}\right)^TP_N(i)\Phi_N\right) + \Phi_N^T\left(\sum_{j=1}^{M}\overline{\lambda}_{i,j}P_N(j)\right)\Phi_N + E_{3,N} - \prod_N^T\Lambda_N\Pi_N. \quad (76)$$

According to

$$(I_n-W)W^{-1}(I_n-W) > 0, \quad (77)$$

thus

$$-W^{-1} \leq -(2I_n - W). \tag{78}$$

By Schur complement, if $\Theta_N \leq 0$, then $\Theta'_N \leq 0$ which is defined in (32) in Theorem 1. As we can see, Θ'_N is multiaffine on $\tau(t)$ and $\dot{\tau}(t)$, where $(\tau(t), \dot{\tau}(t)) \in \mathscr{H} = \mathscr{C}o\{(0,0), (0,d_2), (\tau_M, 0), (\tau_M, d_1)\}$. There exists $\varepsilon > 0$ such that $\mathscr{L}V_N \leq -\varepsilon x(t)^2$. Thus, system (16) is stochastically stable for any delay satisfying the set $\mathscr{H}$. The proof is complete.

It is should be mentioned that the solution of LMI in Theorem 1 with allowable delay set $\mathscr{H}$ has lower conservatism than that with allowable delay set $[0, \tau_M] \times [d_1, d_2]$. In fact, in the instance of set $[0, \tau_M] \times [d_1, d_2]$, the vertices $(0, d_1)$ and (τ_M, d_2) cannot be reached at any time for the impossible circumstances that $\dot{\tau}(t)$ is negative when $\tau(t) = 0$ and $\dot{\tau}(t)$ is positive when $\tau(t) = \tau_M$. □

4. Stabilization Analysis

This section studies the stabilization for system (16). We will design a controller based on Theorem 1. The next theorem shows the main result.

Theorem 2. *Given $N \in \mathbb{N}$, scalar $\varepsilon > 0$, if there exist $\widehat{P}_{1,N}(i) \in \mathbb{S}_+^n$, $\widehat{P}_{2,N}(i) \in \mathbb{S}_+^{(N+1)n}$, $\widehat{P}_{3,N}(i) \in \mathbb{S}_+^{(N+1)n}$, matrices $Q_1, Q_2, W \in \mathbb{S}_+^n$, and matrices $Y_1, Y_2 \in \mathbb{R}^{(N+1)n \times (N+1)n}$ such that the following inequality*

$$\widehat{\Theta}'_N = \begin{bmatrix} \overline{\Theta}'_N & \prod_N^T \begin{bmatrix} \dfrac{\tau_M - \tau(t)}{\tau_M} Y_1 \\ 0 \end{bmatrix} & \prod_N^T \begin{bmatrix} 0 \\ \dfrac{\tau(t)}{\tau_M} Y_2^T \end{bmatrix} & \tau_M \widehat{H}_N^T \\ * & -\dfrac{\tau_M - \tau(t)}{\tau_M} \overline{W}_N & 0 & 0 \\ * & * & -\dfrac{\tau(t)}{\tau_M} \overline{W}_N & 0 \\ * & * & * & W - 2I \end{bmatrix} \leq 0, \tag{79}$$

is true for any $(\tau(t), \dot{\tau}(t)) \in \mathscr{H}$, then system (16) is stochastically stable, where

$$\begin{aligned}
\mathscr{H} &= \mathscr{C}o\{(0,0), (0,d_2), (\tau_M, 0), (\tau_M, d_1)\}, \\
\overline{\Theta}'_N &= He\left(\left(E'_{1,N} + \dot{\tau}(t) E_{2,N}\right)^T P_N(i) \Phi_N\right) + \Phi_N^T \left(\sum_{j=1}^M \overline{\lambda}_{i,j} P_N(j)\right) \Phi_N + E_{3,N} - \prod_N^T \Lambda_N \Pi_N, \\
E'_{1,N} &= \begin{bmatrix} A_i & 0 & 0 & \widehat{P}_{1,N}(i) - 2I_n & 0 & \widehat{P}_{1,N}(i) - 2I_n \\ \chi_N & -\overline{\Upsilon}_N & 0 & -\overline{\chi}_N & 0 & 0 \\ 0 & 0 & -\overline{\Upsilon}_N & \chi_N & -\overline{\chi}_N & 0 \end{bmatrix}, \\
P_N(i) &= \begin{bmatrix} \widehat{P}_{1,N}(i) & 0 & 0 \\ 0 & \widehat{P}_{2,N}(i) & 0 \\ 0 & 0 & \widehat{P}_{3,N}(i) \end{bmatrix}, \\
\widehat{H}_N &= \begin{bmatrix} A_i & 0 & 0 & \widehat{P}_{1,N}(i) - 2I_n & 0 & \widehat{P}_{1,N}(i) - 2I_n \end{bmatrix},
\end{aligned} \tag{80}$$

and other terms are defined in (33).

Proof. If (33) holds, system (16) is called stochastically stable. In Theorem 1, there exist some nonlinear terms like $E_{1,N}^T P_N(i) \Phi_N$ because of the existence of K_i. Thus, we need to eliminate the nonlinear terms in inequality (32).

Due to the $B_i \beta_i K_i \in \mathbb{R}^{n \times n}$ is just one term in $E_{1,N}$, and $P_N(i)$ is in $\mathbb{N}^{(2N+3)n \times (2N+3)n}$, thus we divide matrix $P_N(i)$ into blocks. Then, the $P_N(i)$ becomes that

$$P_N(i) = \begin{bmatrix} \widehat{P}_{1,N}(i) & 0 & 0 \\ 0 & \widehat{P}_{2,N}(i) & 0 \\ 0 & 0 & \widehat{P}_{3,N}(i) \end{bmatrix}, \tag{81}$$

where $\widehat{P}_{1,N}(i) \in \mathbb{S}_+^n$, $\widehat{P}_{2,N}(i) \in \mathbb{S}_+^{(N+1)n}$, and $\widehat{P}_{3,N}(i) \in \mathbb{S}_+^{(N+1)n}$.

Obviously, $K_i^T \beta_i^T B_i^T \widehat{P}_{1,N}(i)$ and $\widehat{P}_{1,N}(i) B_i \beta_i K_i$ are nonlinear terms in $E_{1,N} P_N(i)$. To eliminate the nonlinear terms, we define $K_i = -\beta_i^{-1} B_i^{-1} \widehat{P}_{1,N}^{-1}(i)$. Then, the $K_i^T \beta_i^T B_i^T \widehat{P}_{1,N}(i)$ and $\widehat{P}_{1,N}(i) B_i \beta_i K_i$ become $-I_n$.

In addition, the term $B_i \beta_i K_i$ which is in H_N in (32) will become $-\widehat{P}_{1,N}^{-1}(i)$. Due to

$$\left(I_n - \widehat{P}_{1,N}(i)\right) \widehat{P}_{1,N}^{-1}(i) \left(I_n - \widehat{P}_{1,N}(i)\right) \geq 0, \tag{82}$$

thus

$$-\widehat{P}_{1,N}^{-1}(i) \leq \widehat{P}_{1,N}(i) - 2I_n. \tag{83}$$

We replace $B_i \beta_i K_i$ in (32) with $\widehat{P}_{1,N}(i) - 2I_n$, then we obtain

$$\widehat{H}_N = \begin{bmatrix} A_i & 0 & 0 & \widehat{P}_{1,N}(i) - 2I_n & 0 & \widehat{P}_{1,N}(i) - 2I_n \end{bmatrix}. \tag{84}$$

Table 1: The controller gains K_1 and K_2 and trigger matrices Λ_1 and Λ_2.

t	0	1	...	20
Λ_1	$\begin{bmatrix} 1.88 & -0.02 \\ -0.12 & 1.90 \end{bmatrix}$	$\begin{bmatrix} 1.33 & 0.95 \\ 0.40 & 7.23 \end{bmatrix}$	...	$\begin{bmatrix} 1.77 & 0.13 \\ 0.13 & 6.14 \end{bmatrix}$
Λ_2	$\begin{bmatrix} 2.50 & 0.07 \\ 0.07 & 2.40 \end{bmatrix}$	$\begin{bmatrix} 2.13 & 0.95 \\ 0.40 & 1.85 \end{bmatrix}$	...	$\begin{bmatrix} 1.97 & -0.22 \\ -0.03 & 5.14 \end{bmatrix}$
K_1	$\begin{bmatrix} -0.9129 & 0.4357 \end{bmatrix}$	$\begin{bmatrix} 1.1887 & -2.0034 \end{bmatrix}$	...	$\begin{bmatrix} -1.4731 & 0.7982 \end{bmatrix}$
K_2	$\begin{bmatrix} -0.9029 & -0.3271 \end{bmatrix}$	$\begin{bmatrix} 0.1735 & -1.1657 \end{bmatrix}$	...	$\begin{bmatrix} 1.1003 & 2.1372 \end{bmatrix}$

Thus, we get

$$\widehat{\Theta}_N = \overline{\Theta}'_N + \prod_N^T \Lambda'_N \Pi_N + \tau_M^2 \widehat{H}_N^T W \widehat{H}_N, \tag{85}$$

where

$$\overline{\Theta}'_N = He\left(\left(E'_{1,N} + \dot{\tau}(t) E_{2,N}\right)^T P_N(i) \Phi_N\right) + \Phi_N^T \left(\sum_{j=1}^M \overline{\lambda}_{i,j} P_N(j)\right) \Phi_N + E_{3,N} - \prod_N^T \Lambda_N \Pi_N. \tag{86}$$

Π_N and Λ_N are defined in (33) and $P_N(i)$ is defined in (81). By Schur complement, if $\widehat{\Theta}_N \leq 0$, then $\widehat{\Theta}'_N \leq 0$ which is defined in Theorem 2. Obviously, $\widehat{\Theta}'_N \geq \Theta'_N$ which is defined in Theorem 1, thus if $\widehat{\Theta}'_N \leq 0$, then $\Theta'_N \leq 0$. According to Theorem 1, system (16) is stochastically stable for any delay satisfying $\mathscr{H}$. The proof is complete. □

5. Numerical Examples

Example 1. Consider the following event-triggered semi-Markovian jump networked control system with actuator faults and time-varying delay:

$$A_1 = \begin{bmatrix} -2.1 & 1.6 \\ 1.5 & -1.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -2 & 1.7 \\ -1.2 & -2.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.9 \\ 0.9 \end{bmatrix}, \quad \overline{\lambda}_{i,j} = \begin{bmatrix} 0.8 & -0.8 \\ -0.5 & 0.5 \end{bmatrix}. \tag{87}$$

Take $\overline{\sigma} = 0.3$, $\sigma_{k1} = 0.01$, $\varepsilon = 0.01$, $h = 0.18$, $\beta_1 = 0.3$, $\beta_2 = 0.2$, $N = 0$. The designed controller gains K_1 and K_2 and triggering parameters Λ_1 and Λ_2 at different times are given in Table 1. The state trajectories $x(t)$ and semi-Markovian chain $r(t)$ of the system are shown in Figure 1.

Under the improved static ETS, the time-varying parameter $\sigma_k(t)$ and the situation of release are shown in Figure 2. From Figure 2, $\sigma_k(t)$ changes from 0.01 to 0.3 and the release frequency at the initial stage is larger than other times. In addition, only 25.2% sampled signals are released. Consequently, this proposed ETS cuts down the dynamic process and reduces the transmission burden.

Next, set $h = 0.35$. Under the improved static ETS with time-varying parameter $\sigma_k(t)$, the release instants and release interval are given in Figure 3. If we replace $\sigma_k(t)$ with the constant σ, the corresponding release instants and release intervals are described in Figure 4. It is distinct that the transmission frequency at the beginning times in Figure 3 is higher than that in Figure 4.

Example 2. Consider the following parameters of system (16):

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2.2 & 1.8 \\ -1.2 & -0.5 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.1 & 1.5 \\ -1 & 1.4 \end{bmatrix}, \quad K_1 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \overline{\lambda}_{i,j} = \begin{bmatrix} 0.8 & -0.8 \\ -0.5 & 0.5 \end{bmatrix}, \quad \beta_1 = \beta_2 = 0.2, \ \sigma = 0.1. \tag{88}$$

Take $\overline{\sigma} = 0.3$, $\sigma_{k1} = 0.01$, $\varepsilon = 0.01$, $h = 0.18$, $\beta_1 = 0.2$, and $\beta_2 = 0.2$. Under different N, we obtain corresponding upper bounds of network time delay with $(\tau(t), \dot{\tau}(t)) \in \mathscr{H}$. If $N = 0$, then $\tau_M = 1.2161$. If $N = 1$, then $\tau_M = 1.7572$. If $N = 2$, then $\tau_M = 2.2546$. Table 2 shows the comparison of the upper bound of network time delay with other papers.

From Table 2, we can see that when $N = 0$, $\tau_M = 1.2161$ is bigger than other papers. The upper bound of τ_M increases

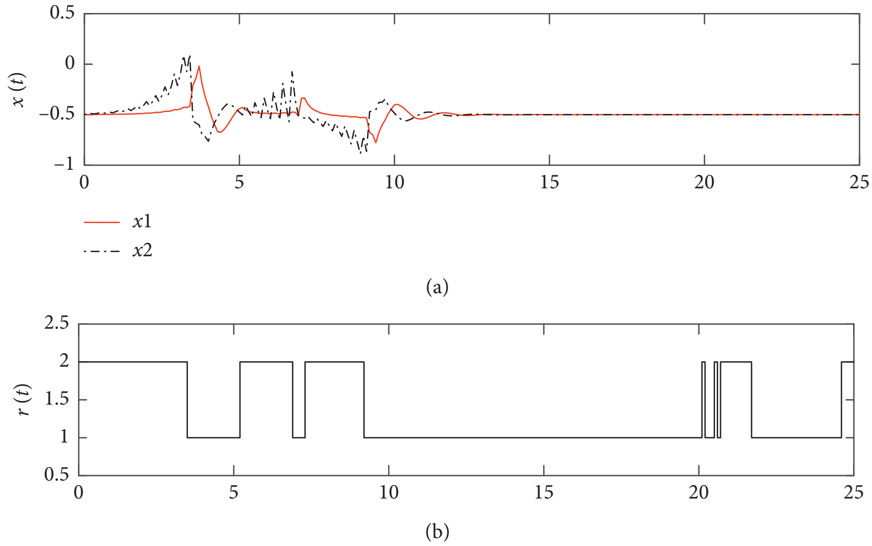


Figure 1: State trajectory $x(t)$ and semi-Markovian chain $r(t)$ of the networked control system containing actuator faults and time-varying delay.

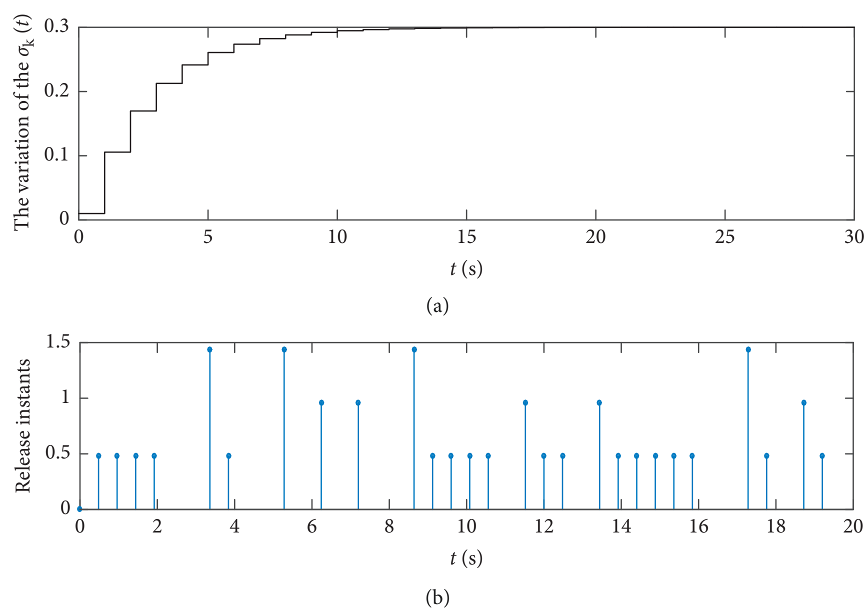


Figure 2: The parameter $\sigma_k(t)$ and the release instants.

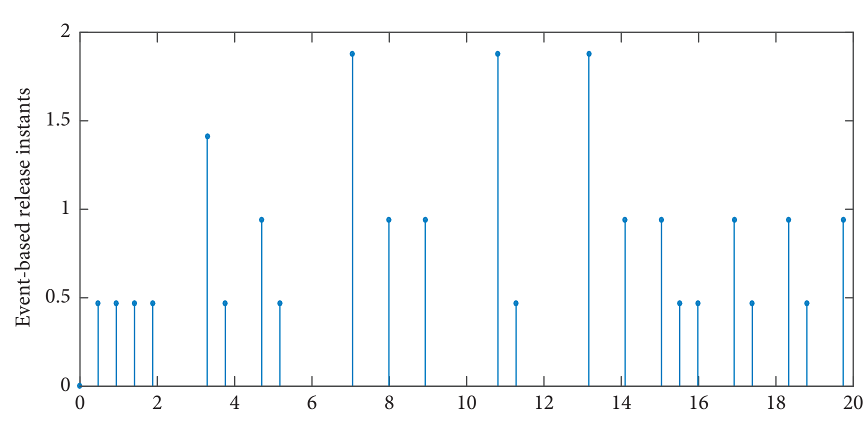


Figure 3: The release instants and release intervals of the improved static ETS.

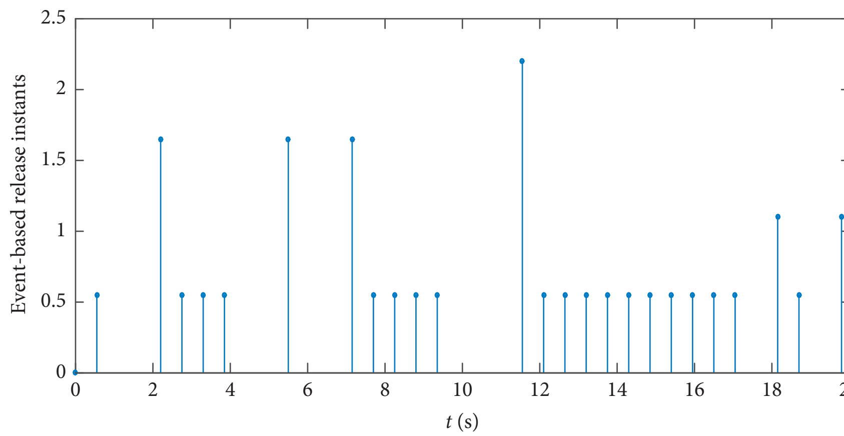


Figure 4: The release instants and release intervals of the ETS with constant parameter σ.

Table 2: The value of τ_M corresponding to different $\dot{\tau}(t)\,(d_2 = -d_1 = \dot{d}(t))$.

Method\$\dot{d}(t)$	0.1	0.2	0.3	0.8
$N = 2$	2.2546	2.1362	2.1041	1.6771
$N = 1$	1.7572	1.6035	1.4722	1.1095
$N = 0$	1.2161	0.9773	0.8554	0.6161
[45]	0.4945	0.4703	0.3634	0.3544
[10]	0.744	0.66	0.51	0.43

as N increases. Thus, the conservativeness of the stability criterion decreases as N increases.

6. Conclusions

To sum up, event-triggered control problem for the networked control system with network delay and stochastic jumping parameters is investigated. The jumping among the parameters is subject to semi-Markovian jump process. More in combination with the actual situation, the actuator faults which also have the semi-Markovian jump property are considered. The feature of this paper is that an improved static ETS is proposed to change the trigger frequency at different stages. Consequently, the burden of transmission is reduced and the system dynamic process is shortened. A stability criterion with lower conservativeness is obtained with the help of applying the Bessel–Legendre inequalities approach and constructing an appropriate LKF. The criterion is indexed by N. The conservatism will decrease when N increases. Moreover, for this comprehensive system model, this paper designs an effective event-triggered controller. Finally, for verifying the validity of the results, numerical examples are presented.

The further topics of the research can be that the investigation of the improved event-triggered filtering problem of the semi-Markovian jump networked control system containing actuator faults. The further improvement of ETS for some requirements of this comprehensive system.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grant No. 61573095 and Natural Science Foundation of Shandong Province under Grant No. ZR2014FM019.

References

[1] H. Su, Y. Qu, S. Gao, H. Song, and K. Wang, "A model of feedback control system on network and its stability analysis," *Communications in Nonlinear Science and Numerical Simulation*, vol. 18, no. 7, pp. 1822–1831, 2013.

[2] J. Chen, S. Meng, and J. Sun, "Stability analysis of networked control systems with aperiodic sampling and time-varying delay," *IEEE Transactions on Cybernetics*, vol. 47, no. 99, pp. 1–9, 2016.

[3] J. Zhang, J. Lam, and Y. Xia, "Output feedback delay compensation control for networked control systems with random delays," *Information Sciences*, vol. 265, pp. 154–166, 2014.

[4] J. Zhang, Z. Pang, Y. Zhou et al., "Active fault tolerant control for networked control systems with actuator fault," in *Proceedings of the Chinese Control Conference*, Dalian, China, July 2016.

[5] X. Xie, D. Yue, and J. H. Park, "Observer-based fault estimation for discrete-time nonlinear systems and its application: a weighted switching approach," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 66, no. 11, pp. 4377–4387, 2019.

[6] E. Tian, D. Yue, and C. Peng, "Reliable control for networked control systems with probabilistic actuator fault and random delays," *Journal of the Franklin Institute*, vol. 347, no. 10, pp. 1907–1926, 2010.

[7] E. Tian, Z. Wang, L. Zou, and D. Yue, "Chance-constrained H∞ control for a class of time-varying systems with stochastic

nonlinearities: the finite-horizon case," *Automatica*, vol. 107, pp. 296–305, 2019.

[8] Z. Chen and Q. Huang, "Globally exponential stability and stabilization of interconnected Markovian jump system with mode-dependent delays," *International Journal of Systems Science*, vol. 47, no. 1, pp. 14–31, 2016.

[9] A. Khalil and J. Wang, "Robust stabilization of networked control systems using the Markovian jump system approach," in *Proceedings of the UKACC International Conference on Control*, Cardiff, UK, September 2012.

[10] H. Wang, P. Shi, and R. K. Agarwal, "Network-based event-triggered filtering for Markovian jump systems," *International Journal of Control*, vol. 89, no. 6, pp. 1096–1110, 2015.

[11] M. Maki, J. Jiang, and K. Hagino, "A stability guaranteed active fault-tolerant control system against actuator failures," *International Journal of Robust and Nonlinear Control*, vol. 14, no. 12, pp. 1061–1077, 2004.

[12] H. Fan, B. Liu, W. Wang, and C. Wen, "Adaptive fault-tolerant stabilization for nonlinear systems with Markovian jumping actuator failures and stochastic noises," *Automatica*, vol. 51, pp. 200–209, 2015.

[13] J.-W. Zhu and G.-H. Yang, "Adaptive sliding-mode control for stochastic Markovian jumping systems with actuator faults," *IET Control Theory & Applications*, vol. 10, no. 6, pp. 664–673, 2016.

[14] F. Li, P. Shi, and L. Wu, "Stochastic stability of semi-Markovian jump systems," *International Journal of Robust and Nonlinear Control*, vol. 24, no. 18, pp. 3317–3310, 2015.

[15] J. Huang and Y. Shi, "Stochastic stability and robust stabilization of semi-Markov jump linear systems," *International Journal of Robust and Nonlinear Control*, vol. 23, no. 18, pp. 2028–2043, 2013.

[16] H. Shen, J. H. Park, Z. G. Wu, and Z. Zhang, "Finite-time H∞ synchronization for complex networks with semi-Markov jump topology," *Communications in Nonlinear Science and Numerical Simulation*, vol. 24, pp. 40–51, 2015.

[17] Q. Zhou, D. Yao, J. Wang, and C. Wu, "Robust control of uncertain semi-Markovian jump systems using sliding mode control method," *Applied Mathematics and Computation*, vol. 286, no. C, pp. 72–87, 2016.

[18] S. Wang, Y. Jiang, Y. Li, and D. Liu, "Reliable observer-based H∞ control for discrete-time fuzzy systems with time-varying delays and stochastic actuator faults via scaled small gain theorem," *Neurocomputing*, vol. 147, no. 1, pp. 251–259, 2015.

[19] K. Q. Gu and S. I. Niculescu, "Further remarks on additional dynamics in various model transformations of linear delay systems," *IEEE Transactions on Automatic Control*, vol. 46, no. 3, pp. 497–500, 2001.

[20] Y. He, M. Wu, and J. She, "Delay-dependent exponential stability of delayed neural networks with time-varying delay," *IEEE Transactions on Circuits and Systems II Express Briefs*, vol. 53, no. 7, pp. 553–557, 2006.

[21] J.-H. Kim, "Further improvement of Jensen inequality and application to stability of time-delayed systems," *Automatica*, vol. 64, no. C, pp. 121–125, 2016.

[22] O. M. Kwon, M. J. Park, J. H. Park, S. M. Lee, and E. J. Cha, "Improved results on stability of linear systems with time-varying delays via Wirtinger-based integral inequality," *Journal of the Franklin Institute*, vol. 351, no. 12, pp. 5386–5398, 2014.

[23] H. J. Yu, Y. He, and M. Wu, "Improved generalized H_2 filtering for static neural networks with time-varying delay via free-matrix-based integral inequality," *Mathematical Problems in Engineering*, vol. 2018, Article ID 5147565, 9 pages, 2018.

[24] P. Park, W. I. Lee, and S. Y. Lee, "Auxiliary function-based integral/summation inequalities: application to continuous/discrete time-delay systems," *International Journal of Control, Automation and Systems*, vol. 14, no. 1, pp. 3–11, 2016.

[25] P. Park, W. I. Lee, and S. Y. Lee, "Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems," *Journal of the Franklin Institute*, vol. 352, no. 4, pp. 1378–1396, 2015.

[26] A. Seuret and F. Gouaisbaut, "Hierarchy of LMI conditions for the stability analysis of time-delay systems," *Systems & Control Letters*, vol. 81, pp. 1–7, 2015.

[27] A. Seuret and F. Gouaisbaut, "Complete quadratic lyapunov functionals using bessel-legendre inequality," in *Proceedings of the European Control Conference*, Linz, Austria, June 2014.

[28] A. Seuret, F. Gouaisbaut, and Y. Ariba, "Complete quadratic Lyapunov functionals for distributed delay systems," *Automatica*, vol. 62, no. C, pp. 168–176, 2015.

[29] A. Seuret, K. Liu, and F. Gouaisbaut, "Generalized reciprocally convex combination lemmas and its application to time-delay systems," *Automatica*, vol. 95, pp. 488–493, 2018.

[30] C.-K. Zhang, Y. He, L. Jiang, M. Wu, and Q.-G. Wang, "An extended reciprocally convex matrix inequality for stability analysis of systems with time-varying delay," *Automatica*, vol. 85, pp. 481–485, 2017.

[31] C.-K. Zhang, Y. He, L. Jiang, Q.-G. Wang, and M. Wu, "Stability analysis of discrete-time neural networks with time-varying delay via an extended reciprocally convex matrix inequality," *IEEE Transactions on Cybernetics*, vol. 47, no. 10, pp. 3040–3049, 2017.

[32] P. Pan, S. Jiang, and F. Pan, "Event-triggered robust control for networked control systems with actuator failures and time-varying transmission delays," *International Journal of Intelligent Computing and Cybernetics*, vol. 8, no. 2, pp. 172–186, 2015.

[33] J. Liu and D. Yue, "Event-triggering in networked systems with probabilistic sensor and actuator faults," *Information Sciences*, vol. 240, 2013.

[34] D. Yue, E. Tian, and Q.-L. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 475–481, 2013.

[35] M. Guinaldo, D. Lehmann, J. Sanchez et al., "Distributed event-triggered control with network delays and packet losses," in *Proceedings of the IEEE Conference on Decision and Control*, Maui, HI, USA, February 2013.

[36] S. Hu, D. Yue, C. Peng, X. Xie, and X. Yin, "Event-triggered controller design of nonlinear discrete-time networked control systems in T-S fuzzy model," *Applied Soft Computing*, vol. 30, no. C, pp. 400–411, 2015.

[37] X. Xie, Q. Zhou, D. Yue, and H. Li, "Relaxed control design of discrete-time Takagi? Sugeno fuzzy systems: an event-triggered real-time scheduling approach," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 12, pp. 2251–2262, 2017.

[38] Z. X. Zhong, C. M. Lin, Z. Shao et al., "Decentralized event-triggered control for large-scale networked fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 26, 2016.

[39] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 5, pp. 1291–1297, 2012.

[40] E. Tian, Z. Wang, L. Zou, and D. Yue, "Probabilistic-constrained filtering for a class of nonlinear systems with improved static event-triggered communication," *International Journal of Robust and Nonlinear Control*, vol. 29, no. 5, pp. 1484–1498, 2019.

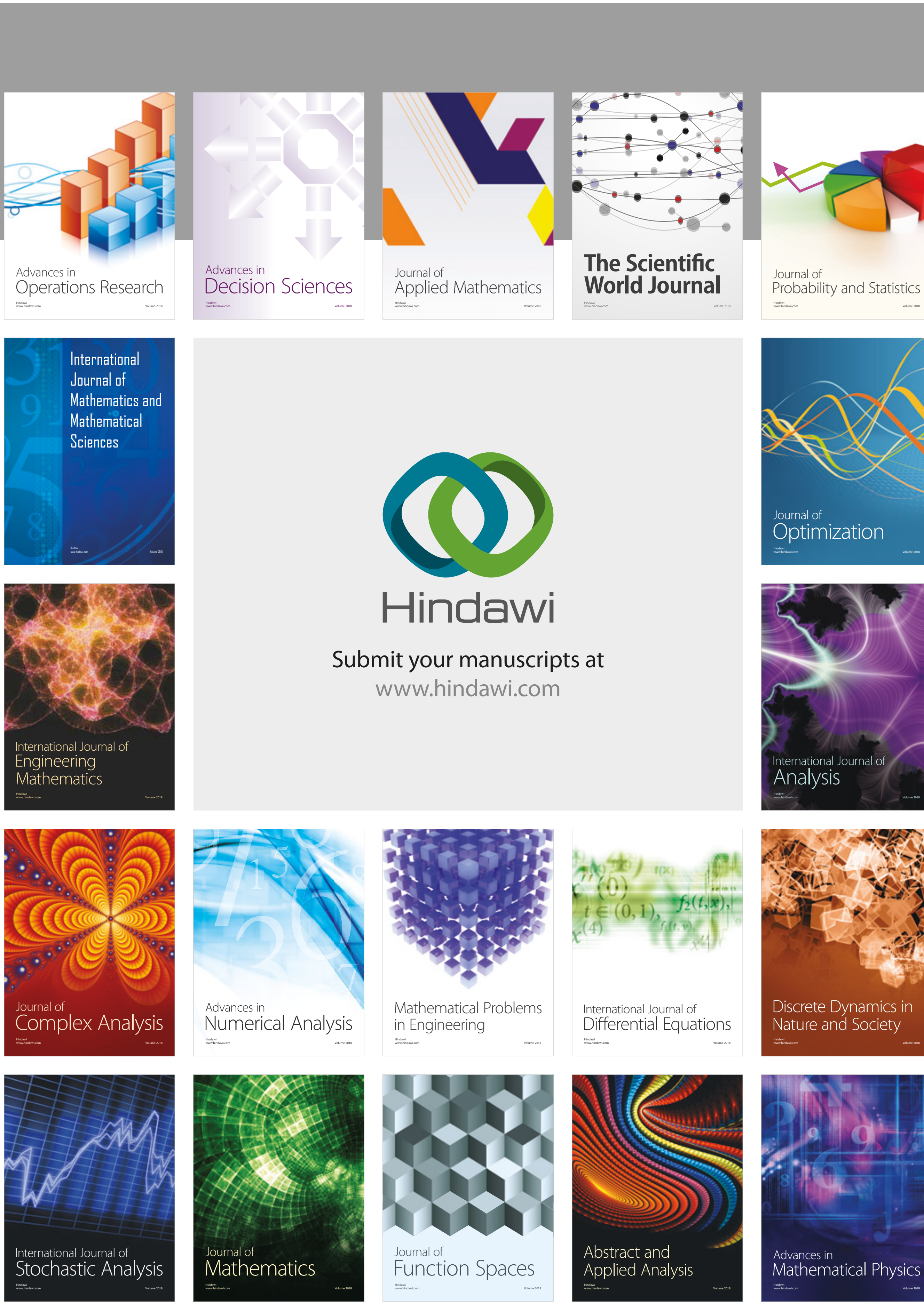
[41] X. Ge and Q.-L. Han, "Distributed formation control of networked multi-agent systems using a dynamic event-triggered communication mechanism," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 10, pp. 8118–8127, 2017.

[42] P. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[43] X. M. Zhang, Q. L. Han, A. Seuret, and F. Gouaisbaut, "An improved reciprocally convex inequality and an augmented lyapunov-krasovskii functional for stability of linear systems with time-varying delay," *Automatica*, vol. 84, pp. 221–226, 2017.

[44] Y. Wei, J. H. Park, H. R. Karimi, Y.-C. Tian, and H. Jung, "Improved stability and stabilization results for stochastic synchronization of continuous-time semi-Markovian jump neural networks with time-varying delay," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 6, pp. 2488–2501, 2018.

[45] H. Wang, P. Shi, C.-C. Lim, and Q. Xue, "Event-triggered control for networked Markovian jump systems," *International Journal of Robust and Nonlinear Control*, vol. 25, no. 17, pp. 3422–3438, 2015.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
The Scientific World Journal
Journal of Probability and Statistics
International Journal of Mathematics and Mathematical Sciences
Journal of Optimization
Hindawi
Submit your manuscripts at
www.hindawi.com
International Journal of Engineering Mathematics
International Journal of Analysis
Journal of Complex Analysis
Advances in Numerical Analysis
Mathematical Problems in Engineering
International Journal of Differential Equations
Discrete Dynamics in Nature and Society
International Journal of Stochastic Analysis
Journal of Mathematics
Journal of Function Spaces
Abstract and Applied Analysis
Advances in Mathematical Physics