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Research Article

Robust Protocol-Based $\mathscr{H}_\infty$ Consensus Control of Time-Varying Uncertain Multiagent Systems Subject to Missing Measurements

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In this paper, we consider the design problem of a robust $\mathscr{H}_\infty$ consensus controller for discrete time-varying uncertain multiagent systems (DTVUMASs) with stochastic communication protocol (SCP) and missing measurements. The SCP is described by a set of random variables with a known probability to arrange signal transmission of addressed multiagent systems. Moreover, we depict the missing measurement phenomenon by a sequence of Bernoulli-distributed random variables having known probabilities. The controller parameters are designed to ensure that the closed-loop DTVUMASs satisfy the $\mathscr{H}_\infty$ performance with the satisfactory consensus criterion. Together with the completing squares approach and the stochastic analysis methodology, some sufficient conditions are proposed by solving coupled backward recursive Riccati difference equations (BRRDEs) to guarantee the $\mathscr{H}_\infty$ consensus performance. Finally, we present a numerical simulation example to illustrate the effectiveness of the designed controller design scheme.

1. Introduction

Over the past few decades, the consensus control problems for multiagent systems (MASs) were received special attention due to their extensive applications in a variety of fields, such as unmanned vehicle, robots formation, and target tracking. The major aim of the consensus control is to introduce a suitable controller for each agent, which can be designed by using its neighboring and its own local information, such that all agents reach some common features [1, 2]. The consensus control problems for first-order multiagent systems with switching topology and time delays were studied in [1], and the initiative works were conducted to solving the consensus control problems of first-order multiagent systems (FOMASs). Since then, some control problems under certain requirements were investigated for complex dynamical systems by taking the influences caused by nonlinearities, quantization, transmission losses, and communication noise into account, see, e.g., [3–5]. Besides, the consensus control problems of second-order multiagent systems (SOMASs) also received a lot of research attention. For example, the effects of fixed topology and switching topology were discussed in [6–9] and related methods were given. Subsequently, in [10], the leader-following consensus control problems of high-order multiagent systems (HOMASs) were solved by using a novel distributed event-triggered communication protocol based on state estimates of neighboring agents. In recent years, more and more researchers pay attention to the consensus control problems for general multiagent systems (GMASs) [2, 11–13], where some new consensus methods were presented by designing the controller parameters. For instance, a new non-fragile consensus control method based on the output feedback technique was provided to tackle the deception attacks in quantized GMASs.

Generally, we all expect the unlimited bandwidth during the communications in order to obtain satisfactory control performance, but it cannot be ensured in the engineering reality [14–16]. Accordingly, the limited bandwidth may lead to the conflicts between signal transmissions. Accompanying with the continuous studies, more and more researchers find that multiple transmission protocols are efficacious in networks for the limited bandwidth. In order to prevent the transmission conflicts between the signals, a suitable communication protocol can be utilized, for example, the round-robin protocol [17–19], the stochastic communication protocol (SCP) [20–22], and the try-once-discard protocol [23–25], and so on. Among these communication protocols, the SCP is widely used in the satellite network and wireless network. Under the scheduling of the SCP, only one agent can obtain and transmit information at each time, and the transport order among the agents is in a stochastic way. The distributed resilient filtering problem was constrained by SCP and RR protocols studied in [21], by applying recursive linear matrix inequality technique and stochastic analysis approach, and the state estimation error system achieved $\mathscr{H}_\infty$ consensus performance over a given finite-horizon. In [26], the distributed $\mathscr{H}_\infty$ consensus control problem was handled for discrete time-varying multiagent systems with the SCP, where a cooperative controller was designed for each agent such that the MASs achieved $\mathscr{H}_\infty$ consensus performance over a given finite-horizon by solving coupled BRRDEs. It should be noted that the MASs with time-varying parameters are common in network environments [27–30]. So far, the consensus control problem of time-varying MASs by considering the limited communication resources has not been fully investigated.

For the consensus control problems of MASs, the measurement information is commonly desired for the control synthesis. As we all know, the network-induced phenomena may occur, such as quantized transmissions [16, 23], input saturation [31], missing measurements [32, 33] and sensor saturations [34] due to the sensor aging, inherently limited bandwidth of the communication channel, network congestion, and so on. In order to improve the expressed authenticity to the original systems, it is vitally important to take the missing measurements into account with hope to reflect the practical engineering [33, 35]. In recent years, many researchers studied the robust control problems with missing measurements or packet dropouts [33, 35, 36]. As discussed in [33], the coupled BRRDEs were used to deal with the $\mathscr{H}_\infty$ consensus control problems subject to missing measurements and parameter uncertainties for discrete time-varying MASs. As such, we aim to study the consensus control issue for time-varying multiagent systems with SCP and missing measurements in depth, where a new control scheme will be proposed to ensure the design requirements.

To sum up, the motivation of this paper is to handle the $\mathscr{H}_\infty$ consensus problem for a class of DTVUMASs subject to the SCP, missing measurements, and parameter uncertainties. The main difficulties encountered when dealing with this problem are (1) how to better describe the $\mathscr{H}_\infty$ consensus performance of DTVUMASs with SCP, missing measurements, and parameter uncertainties? and (2) how to reduce the impact of missing measurements and parameter uncertainties on the $\mathscr{H}_\infty$ consensus performance of DTVUMASs? The main differences of the paper can be listed as follows: (1) the $\mathscr{H}_\infty$ consensus control problem is investigated, for the first time, for a class of DTVUMASs with SCP and missing measurements. (2) Some sufficient conditions are proposed to guarantee the corresponding $\mathscr{H}_\infty$ consensus condition over a finite-horizon. (3) A coupled BRRDE approach is presented to deal with the effects of SCP, parameter uncertainties, and missing measurements simultaneously. The rest of the paper consists of the following sections. The DTVUMASs with SCP and missing measurements are modeled in Section 2, and the preliminary work is introduced. In Section 3, a consensus controller is designed and the related controller parameters are obtained in terms of the solutions to two coupled BRRDEs. Next, in Section 4, some simulations are provided to further demonstrate the effectiveness of the new controller design scheme. Finally, the summarization of this paper is given in Section 5.

Notations: $\mathbb{R}^n$ stands for the n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ stands for a set of all $n \times m$ real matrices. 1_n depicts an n-dimensional column vector with all elements being 1. diag$\{\cdots\}$ means a block diagonal matrix. The space of square-summable n-dimensional vector function over $[0, T]$ is denoted by $l_2([0,T];\mathbb{R}^n)$. The notation $U \geq V$ ($U > V$) with U and V being symmetric matrices means that $U - V$ is a positive semidefinite (positive definite) matrix. The superscripts T and $\dagger$ denote, respectively, the transpose and the Moore–Penrose pseudoinverse. $\mathbb{E}\{x\}$ represents the expectation of random variable x. Prob$\{\cdot\}$ means the occurrence probability of the event "$\cdot$". $\|x\|$ is the norm of x in Euclidean space. $\otimes$ is the Kronecker product of matrices. $\circ$ denotes the Hadamard product of matrices. $\delta(a)$ is the Kronecker delta function, where $\delta(0) = 1$ and $\delta(a) = 0$ if $a \neq 0$.

2. Problem Formulation

In this paper, we consider the multiagent systems with N agents. The directed connected graph is $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{H})$, where $\mathscr{V} = \{1, 2, \ldots, N\}$ stands for N vertices, $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is the edge set, and $\mathscr{H} = [h_{ij}]$ is the weighting adjacency matrix with nonnegative adjacency element. If $(i, j) \in \mathscr{E}$, then $h_{ij} > 0$, it means that agent i can obtain information from agent j, otherwise $h_{ij} = 0$, which means that agent i cannot obtain information from agent j. Furthermore, self-edges (i, i) are not allowed. $\overline{\mathscr{H}}$ represents the Laplacian of the directed graph, which can be expressed as $\overline{\mathscr{H}} = \mathscr{H} - \overline{\mathscr{D}}$, thereinto $\overline{\mathscr{D}} = \mathrm{diag}\{\mathrm{deg}_{\mathrm{in}}^{(1)}, \mathrm{deg}_{\mathrm{in}}^{(2)}, \ldots, \mathrm{deg}_{\mathrm{in}}^{(N)}\}$. The neighborhood of agent i is denoted by $\mathscr{N}_i = \{j \in \mathscr{V} : (i, j) \in \mathscr{E}\}$. The in-degree of agent i is defined as $\mathrm{deg}_{\mathrm{in}}^{(i)} = \sum_{j\in\mathscr{N}_i} h_{ij}$.

Consider a class of DTVUMASs with the following dynamics equation:

$$\begin{cases} x_{i,k+1} = \left(A_k + \Delta A_{i,k}\right)x_{i,k} + B_k u_{i,k} + E_k \omega_{i,k}, \\ y_{i,k} = \alpha_{i,k} C_k x_{i,k} + D_k v_{i,k}, \\ z_{i,k} = L_k x_{i,k}, \end{cases} \tag{1}$$

where $x_{i,k} \in \mathbb{R}^{n_x}$ is the system state, $y_{i,k} \in \mathbb{R}^{n_y}$ denotes the measurement output, $u_{i,k} \in \mathbb{R}^{n_u}$ represents the control input, and $z_{i,k} \in \mathbb{R}^{n_z}$ is the controlled output. A_k, B_k, C_k, D_k, E_k, and L_k are known time-varying matrices of appropriate dimensions, and $\omega_{i,k} \in \mathbb{R}^{n_\omega}$ and $v_{i,k} \in \mathbb{R}^{n_v}$ are the external disturbances belonging to $l_2([0,T];\mathbb{R}^n)$. The random variables $\alpha_{i,k}$ taking values 1 or 0 are Bernoulli-distributed sequences and obey the following statistical property:

$$\begin{aligned} \text{Prob}\left\{\alpha_{i,k} = 1\right\} &= \overline{\alpha}, \\ \text{Prob}\left\{\alpha_{i,k} = 0\right\} &= 1 - \overline{\alpha}, \end{aligned} \tag{2}$$

where $\overline{\alpha} \in [0,1]$ is a known constant. $\Delta A_{i,k}\,(i \in \mathscr{V})$ is time-varying parameter uncertainty, which satisfies

$$\Delta A_{i,k} = M_{i,k} F_{i,k} G_k, \tag{3}$$

where $M_{i,k}$ and G_k are known time-varying real-valued matrices, and the unknown time-varying matrix $F_{i,k}$ satisfies

$$F_{i,k}^T F_{i,k} \le I, \quad \forall k \in [0,T]. \tag{4}$$

In this paper, if $\zeta_{i,k} \in \mathscr{N}_i$ is the neighboring agent of the agent i that means agent i can obtain the information from $\zeta_{i,k}$ at time instant k. Due to the scheduling behavior of the SCP, $\zeta_{i,k}$ can be regarded as a sequence of random variables, and all of the random variables are independent of each other. The probability of $\zeta_{i,k} = j$, which reflects the opportunity that agent i transmits the information to agent j at time instant k, is defined as

$$\text{Prob}\left\{\zeta_{i,k} = j\right\} = p_{i,k}^j, \quad \forall i \in 1,2,\ldots,N, \tag{5}$$

where $p_{i,k}^j \ge 0 \;\; (j \in \mathscr{N}_i)$ is the occurrence probability and $\sum_{j\in\mathscr{N}_i} p_{i,k}^j = 1$. In general, suppose that $p_{i,k}^j = 0$ for $j \notin \mathscr{N}_i$.

Based on the SCP scheduling, a cooperative controller of the agent i can be designed in the following form:

$$u_{i,k} = K_k h_{i\zeta_{i,k}}\left(y_{\zeta_i,k} - y_{i,k}\right) = K_k \sum_{j\in\mathscr{N}_i} h_{ij}\lambda_{j,k}^i\left(y_{j,k} - y_{i,k}\right), \tag{6}$$

where $K_k \in \mathbb{R}^{n_u \times n_y}\,(k \in [0,T])$ are feedback gain matrices to be determined and $\lambda_{j,k}^i = \delta(j - \zeta_{i,k}) \;\; (j \in \mathscr{N}_i)$.

Defining $u_k \triangleq \text{col}_N\{u_{i,k}\}$ and $y_k \triangleq \text{col}_N\{y_{i,k}\}$, it can be concluded from (6) that

$$u_k = \mathscr{K}_k\left(\overrightarrow{\mathscr{H}}_{\zeta_k} \otimes I_{n_y}\right) y_k, \tag{7}$$

where

$$\begin{aligned} \mathscr{K}_k &= I_N \otimes K_k, \\ \overrightarrow{\mathscr{H}}_{\zeta_k} &= \mathscr{H} \circ \Lambda_k - \overrightarrow{\mathscr{D}}_k, \\ \Lambda_k &= \left[\lambda_{j,k}^i\right]_{N\times N}, \\ \deg_{\text{in},k}^{(i)} &= \sum_{j\in\mathscr{N}_i} h_{ij}\lambda_{j,k}^i, \\ \overrightarrow{\mathscr{D}}_k &= \text{diag}\left\{\deg_{\text{in},k}^{(1)}, \deg_{\text{in},k}^{(2)}, \ldots, \deg_{\text{in},k}^{(N)}\right\}, \\ \mathscr{H} &= \begin{bmatrix} 0 & h_{12} & \cdots & h_{1N} \\ h_{21} & 0 & \cdots & h_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1} & h_{N2} & \ldots & 0 \end{bmatrix}. \end{aligned} \tag{8}$$

Remark 1. The Laplacian matrix of the multiagent systems is defined as $\overline{\mathscr{H}} = \mathscr{H} - \overline{\mathscr{D}}$ in this paper. In (7), the Laplacian of the time-varying directed graph is redefined as $\overrightarrow{\mathscr{H}}_{\zeta_k} = \mathscr{H} \circ \Lambda_k - \overrightarrow{\mathscr{D}}_k$ due to introduction of SCP, and the in-degree of the agent i is $\deg_{\text{in},k}^{(i)} = \sum_{j\in\mathscr{N}_i} h_{ij}\lambda_{j,k}^i$. The self information of the agent i in (6) is the measurement information $y_{i,k}$. That is, the controller (6) is designed based on the self-measurement information $y_{i,k}$ of the agent i and its interaction information $y_{j,k} \;\; (j \in \mathscr{N}_i)$ of the neighborhood agent, so that all the agents eventually tend to be consensus under this controller.

The DTVUMASs are interfered not only by parameter uncertainties but also by external disturbance in this paper. Next, the following definition with respect to $\mathscr{H}_\infty$ performance is introduced.

Definition 1 (see [26]). The time-varying UMASs (1) have the $\mathscr{H}_\infty$ consensus performance if

$$\mathbb{E}\left\{\sum_{i=1}^N \sum_{k=0}^T \left(\left\|\overline{z}_{i,k}\right\|^2 - \gamma^2\left(\left\|\omega_{i,k}\right\|^2 + \left\|v_{i,k}\right\|^2\right)\right) - \gamma^2 \sum_{i=1}^N \overline{x}_{i,0}^T W \overline{x}_{i,0}\right\} < 0 \tag{9}$$

holds over the finite horizon $[0,T]$, where $\gamma > 0$ is a specified disturbance attenuation level, and $W = W^T > 0$ is a weighted matrix, $\overline{z}_{i,k} = z_{i,k} - (1/N)\sum_{i=1}^N z_{i,k}$, and $\overline{x}_{i,0} = x_{i,0} - (1/N)\sum_{i=1}^N x_{i,0}$.

In this paper, the purpose is to design the controller parameter matrices $K_k\,(k \in [0,T])$ such that, for all parameter uncertainties, missing measurements, and SCP, the DTVUMASs have the $\mathscr{H}_\infty$ consensus performance criterion. Accordingly, we need to propose the related sufficient and necessary condition to ensure the above objective.

3. Control Scheme Design

In this section, the main method is given to fulfill the mentioned control requirement. To begin, we introduce the following useful Lemmas in order to help further derivations.

Lemma 1 (see [26]). *Through the process of mapping $\kappa(\cdot)$, $\zeta_{i,k}$ $(i = 1,2,\ldots,N)$ can be mapped to $\varrho_k \in \kappa \triangleq \{1,2,\ldots,N^N\}$, that is,*

$$\varrho_k = \kappa(\zeta_k) \triangleq \sum_{i=1}^N N^{i-1}\left(\zeta_{i,k} - 1\right) + 1. \tag{10}$$

Furthermore, as shown below, if the value of ϱ_k is given, then the following from of $\phi_i(\varrho_k)$ $(i = 1,2,\ldots,N)$ and $\zeta_{i,k}$ can be obtained:

$$\zeta_{i,k} = \phi_{i,\varrho_k} \triangleq \text{mod}\left(\frac{\varrho_k - 1}{N^{i-1}}, N\right) + 1. \tag{11}$$

Lemma 2 (see [26]). *The probability of $\varrho_k = i \in \kappa$ can be expressed as follows:*

$$\overline{p}_{i,k} \triangleq \text{Prob}\{\varrho_k = i\} = \prod_{j=1}^{N} \text{Prob}\left(\zeta_{j,k} = \phi_{j,i}\right) = \prod_{j=1}^{N} p_{j,k}^{\phi_{j,i}}, \quad (12)$$

where $k \in [0, T]$.

In order to make the purpose clear and concise, we introduce the following symbols:

$$\begin{aligned}
\Xi_k^{(ij)} &= \begin{cases} -\left(\alpha_{i,k} - \overline{\alpha}\right)\deg_{\text{in}}^{(i)}\lambda_{j,k}^{i}C_k, & i = j, \\ \left(\alpha_{j,k} - \overline{\alpha}\right)h_{ij}\lambda_{j,k}^{i}C_k, & i \neq j, \end{cases} \\
\Xi_k &= \left[\Xi_k^{(ij)}\right]_{N\times N}, \\
\alpha_k &= \text{diag}\{\alpha_{1,k} \;\; \alpha_{2,k} \;\; \cdots \;\; \alpha_{N,k}\}, \\
x_k &= \left[\, x_{1,k}^T \;\; x_{2,k}^T \;\; \cdots \;\; x_{N,k}^T \,\right]^T, \\
z_k &= \left[\, z_{1,k}^T \;\; z_{2,k}^T \;\; \cdots \;\; z_{N,k}^T \,\right]^T, \\
w_k &= \left[\, w_{1,k}^T \;\; w_{2,k}^T \;\; \cdots \;\; w_{N,k}^T \,\right]^T, \\
v_k &= \left[\, v_{1,k}^T \;\; v_{2,k}^T \;\; \cdots \;\; v_{N,k}^T \,\right]^T, \\
\mathscr{B}_k &= I_N \otimes B_k, \\
\mathscr{C}_k &= I_N \otimes C_k, \\
\mathscr{D}_k &= I_N \otimes D_k, \\
\mathscr{G}_k &= I_N \otimes G_k, \\
\mathscr{M}_k &= \text{diag}\{M_{1,k} \;\; M_{2,k} \;\; \cdots \;\; M_{N,k}\}, \\
\mathscr{F}_k &= \text{diag}\{F_{1,k} \;\; F_{2,k} \;\; \cdots \;\; F_{N,k}\}.
\end{aligned} \quad (13)$$

In the sequel, the closed-loop system of dynamics system (1) under controller (6) can be written as

$$\begin{cases} x_{k+1} = \left(I_N \otimes A_k + \mathscr{M}_k\mathscr{F}_k\mathscr{G}_k + \left(I_N \otimes B_kK_k\right)\Xi_k\right)x_k \\ \quad +\left(\check{\Lambda}_{\varrho_k} \otimes \overline{\alpha}B_kK_kC_k\right)x_k + \left(I_N \otimes E_k\right)\omega_k + \left(\check{\Lambda}_{\varrho_k} \otimes B_kK_kD_k\right)v_k, \\ z_k = \left(I_N \otimes L_k\right)x_k, \end{cases} \quad (14)$$

where $\check{\Lambda}_{\varrho_k} = \overrightarrow{\mathscr{H}}_{\zeta_k}$.

Remark 2. Recently, the SCP is employed to depict some random access protocols and multiple access protocols used in the industrial network. Under the influence of SCP behavior, all agents have the same status in the communication network, and the opportunities for each agent utilizing the communication network are randomly assigned according to a certain probability. When the system is not affected by SCP, agents transmit information according to a fixed topology; on the contrary, under the influence of SCP, the higher the probability of agents obtaining information, the more likely they are to transmit information. In this way, the signal conflicts could be effectively avoided.

Letting

$$\begin{aligned}
\overline{x}_{i,k} &= x_{i,k} - \frac{1}{N}\sum_{i=1}^{N} x_{i,k}, \\
\overline{x}_k &= \left[\, \overline{x}_{1,k}^T \;\; \overline{x}_{2,k}^T \;\; \cdots \;\; \overline{x}_{N,k}^T \,\right]^T, \\
\overline{z}_k &= \left[\, \overline{z}_{1,k}^T \;\; \overline{z}_{2,k}^T \;\; \cdots \;\; \overline{z}_{N,k}^T \,\right]^T,
\end{aligned} \quad (15)$$

then we have

$$\begin{aligned}
\overline{x}_k &= x_k - \frac{1}{N}\left(1_N \otimes I_{n_x}\right)\left(1_N^T \otimes I_{n_x}\right)x_k \\
&= \left[\left(I_N - \frac{1}{N}1_N1_N^T\right) \otimes I_{n_x}\right]x_k.
\end{aligned} \quad (16)$$

Similarly, $\overline{z}_k = [(I_N - (1/N)1_N1_N^T) \otimes I_{n_z}]z_k$.

Defining $\mathscr{N} \triangleq I_N - (1/N)1_N1_N^T$, formula (14) can be rewritten as

$$\begin{cases} \overline{x}_{k+1} = \left[\left(\mathscr{N} \otimes A_k\right) + \left(\mathscr{N} \otimes M_{i,k}F_{i,k}G_k\right) + \left(\mathscr{N} \otimes B_kK_k\Xi_k^{(ij)}\right)\right]x_k \\ \quad +\left(\mathscr{N}\check{\Lambda}_{\rangle\#x2009;)_k} \otimes \overline{\alpha}B_kK_kC_k\right)x_k + \left(\mathscr{N} \otimes E_k\right)\omega_k \\ \quad +\left(\mathscr{N}\check{\Lambda}_{\rangle\#x2009;)_k} \otimes B_kK_kD_k\right)v_k, \\ \overline{z}_k = \left(\mathscr{N} \otimes L_k\right)x_k. \end{cases} \quad (17)$$

In addition, we can get the following formula:

$$\begin{aligned}
\sum_{j=1}^{N}\left(\check{\Lambda}_{\varrho_k}\right)_{i,j} &= \sum_{j=1}^{N}\left(\mathscr{H} \circ \Lambda_k\right)_{i,j} - \deg_{\text{in},k}^{(i)} \\
&= \left(\mathscr{H}\Lambda_k^T\right)_{i,i} - \sum_{j\in\mathscr{N}_i} h_{ij}\lambda_{j,k}^{i} = 0.
\end{aligned} \quad (18)$$

Afterwards, (17) can be rewritten as follows via $\check{\Lambda}_{\varrho_k}\mathscr{N} = \check{\Lambda}_{\varrho_k}$:

$$\begin{cases} \overline{x}_{k+1} = \left(\mathscr{A}_k + \mathscr{B}_{\varrho_k,k}\right)\overline{x}_k + \mathscr{D}_{\varrho_k,k}\overline{v}_k + \mathscr{T}_{\varrho_k,k}, \\ \overline{z}_k = \overline{\mathscr{L}}_k\overline{x}_k, \end{cases} \quad (19)$$

where

$$\begin{aligned}
\overline{v}_k &= \left[\left(I_N \otimes \left(\varepsilon_kF_{i,k}G_k\right)x_k\right)^T \omega_k^T v_k^T\right]^T, \\
\overline{\mathscr{L}}_k &= \mathscr{N} \otimes L_k, \\
\mathscr{A}_k &= \mathscr{N} \otimes A_k, \\
\mathscr{B}_{\varrho_k,k} &= \mathscr{N} \otimes \left(B_kK_k\Xi_k^{(ij)}\right), \\
T_k &= \overline{\alpha}K_kC_k\overline{x}_k, \\
\mathscr{T}_{\varrho_k,k} &= \mathscr{N}\check{\Lambda}_{\varrho_k} \otimes \left(B_kT_k\right), \\
\mathscr{D}_{\varrho_k,k} &= \left[\mathscr{N} \otimes \left(\varepsilon_k^{-1}M_k\right)\mathscr{N} \otimes E_k\mathscr{N}\check{\Lambda}_{\varrho_k} \otimes \left(B_kK_kD_k\right)\right],
\end{aligned} \quad (20)$$

and $\varepsilon_k > 0$.

Note that the multiagent system is affected by parameter uncertainties. We reconstruct the $\mathscr{H}_\infty$ consensus performance as follows:

$$\mathbb{E}\Big\{\Big(\|\overline{z}_k\|^2_{[0,T]} - \gamma^2\|\overline{v}_k\|^2_{[0,T]} + \gamma^2\|\varepsilon_k G_k \overline{x}_k\|^2_{[0,T]}\Big) - \gamma^2\overline{x}_0^T(I_N \otimes W)\overline{x}_0\Big\} < 0. \tag{21}$$

If (21) is satisfied, then (8) is satisfied too.

According to the probability of ϱ_k given by Lemma 2, we have

$$\overline{\Lambda}_k \triangleq \mathbb{E}\{\check{\Lambda}_{\rangle\#x2009;)_k}\} = \sum_{i\in\kappa}\overline{p}_{i,k}\check{\Lambda}_i. \tag{22}$$

Defining $\widetilde{\Lambda}_{\varrho_k} \triangleq \check{\Lambda}_{r_k} - \overline{\Lambda}_k$, then the time-varying multiagent system (19) can be constructed as follows:

$$\begin{cases} \overline{x}_{k+1} = \big(\mathscr{A}_k + \overline{\mathscr{B}}_k + \widetilde{\mathscr{B}}_{\varrho_k,k}\big)\overline{x}_k + \big(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\big)\overline{v}_k + \overline{\mathscr{T}}_k + \widetilde{\mathscr{T}}_{\varrho_k,k}, \\ \overline{z}_k = \overline{\mathscr{L}}_k\overline{x}_k, \end{cases} \tag{23}$$

where

$$\begin{aligned}
\Xi^{(ij)}_{k,\overline{\Lambda}_k} &= \begin{cases} -\big(\alpha_{i,k}-\overline{\alpha}\big)\deg^{(i)}_{\text{in}}\overline{p}_{j,k}\lambda^i_{j,k}C_k, & i=j, \\ \big(\alpha_{i,k}-\overline{\alpha}\big)h_{i,j}\overline{p}_{j,k}\lambda^i_{j,k}C_k, & i\neq j, \end{cases} \\
\Xi^{(ij)}_{k,\widetilde{\Lambda}_k} &= \begin{cases} -\big(\alpha_{i,k}-\overline{\alpha}\big)\deg^{(i)}_{\text{in}}\big(1-\overline{p}_{j,k}\big)\lambda^i_{j,k}C_k, & i=j, \\ \big(\alpha_{i,k}-\overline{\alpha}\big)h_{i,j}\big(1-\overline{p}_{j,k}\big)\lambda^i_{j,k}C_k, & i\neq j, \end{cases} \\
\overline{\mathscr{B}}_k &= \mathscr{N}\otimes\Big(B_kK_k\Xi^{(ij)}_{k,\overline{\Lambda}_k}\Big), \\
\widetilde{\mathscr{B}}_{\varrho_k,k} &= \mathscr{N}\otimes\Big(B_kK_k\Xi^{(ij)}_{k,\widetilde{\Lambda}_k}\Big), \\
\overline{\mathscr{T}}_k &= \mathscr{N}\otimes(\overline{\alpha}B_kK_kC_k)\overline{x}_k, \\
\widetilde{\mathscr{T}}_{\varrho_k,k} &= \mathscr{N}\widetilde{\Lambda}_{\varrho_k,k}\otimes(\overline{\alpha}B_kK_kC_k)\overline{x}_k, \\
\widetilde{\mathscr{D}}_{\varrho_k,k} &= \big[0\;0\;\mathscr{N}\widetilde{\Lambda}_{\varrho_k}\otimes(B_kK_kD_k)\big], \\
\overline{\mathscr{D}}_k &= \big[\mathscr{N}\otimes\big(\varepsilon_k^{-1}M_{i,k}\big)\;\mathscr{N}\otimes E_k\;\mathscr{N}\overline{\Lambda}_k\otimes(B_kK_kD_k)\big].
\end{aligned} \tag{24}$$

Now, the following sufficient and necessary condition regarding the $\mathscr{H}_\infty$ performance constraint under SCP can be obtained.

Lemma 3. *Consider DTVUMASs (23) under the SCP scheduling. For any nonzero* $\{\overline{v}_k\}_{k\in[0,T]} \in l_2([0,T];\mathbb{R}^{n_{\overline{v}}})$, *given the disturbance attention level* $\gamma > 0$ *and the positive definite matrix W, if and only if there exist a set of real-valued matrices* $\{K_k\}_{k\in[0,T]}$, *a set of positive scalars* $\{\varepsilon_k\}_{k\in[0,T]}$, *and a set of nonnegative definite matrices* $\{P_k\}_{k\in[0,T]}$ *with the terminal condition* $P_{T+1} = 0$ *satisfying the following BRRDE:*

$$\begin{aligned}
P_k &= \mathscr{A}_k^TP_{k+1}\mathscr{A}_k + \overline{\alpha}\mathscr{A}_k^TP_{k+1}\mathscr{D}_k + \overline{\alpha}\mathscr{D}_k^TP_{k+1}\mathscr{A}_k + \overline{\alpha}\mathscr{A}_k^TP_{k+1}\mathscr{L}_k \\
&\quad + \overline{\alpha}\mathscr{L}_k^TP_{k+1}\mathscr{A}_k + \overline{\alpha}^2\mathscr{D}_k^TP_{k+1}\mathscr{D}_k + \overline{\alpha}^2\mathscr{D}_k^TP_{k+1}\mathscr{L}_k \\
&\quad + \overline{\alpha}^2\mathscr{L}_k^TP_{k+1}\mathscr{D}_k + \overline{\alpha}^2\mathscr{L}_k^TP_{k+1}\mathscr{L}_k + \Pi^{\mathscr{P}}_{k,\overline{\Lambda}_k} + \Pi^{\mathscr{P}}_{k,\widetilde{\Lambda}_k} \\
&\quad + \overline{\mathscr{L}}_k^T\overline{\mathscr{L}}_k + \gamma^2\varepsilon_k^2\mathscr{G}_k^T\mathscr{G}_k \\
&\quad + \Big(\overline{\mathscr{D}}_k^TP_{k+1}\mathscr{A}_k + \widetilde{\mathscr{D}}^T_{\varrho_k,k}P_{k+1}\mathscr{A}_k + \overline{\alpha}\overline{\mathscr{D}}_k^TP_{k+1}\mathscr{D}_k \\
&\quad + \overline{\alpha}\overline{\mathscr{D}}_k^TP_{k+1}\mathscr{L}_k + \overline{\alpha}\widetilde{\mathscr{D}}^T_{\varrho_k,k}P_{k+1}\mathscr{D}_k + \overline{\alpha}\widetilde{\mathscr{D}}^T_{\varrho_k,k}P_{k+1}\mathscr{L}_k\Big)^T \\
&\quad \phi_k^{-1}\Big(\overline{\mathscr{D}}_k^TP_{k+1}\mathscr{A}_k + \widetilde{\mathscr{D}}^T_{\varrho_k,k}P_{k+1}\mathscr{A}_k + \overline{\alpha}\overline{\mathscr{D}}_k^TP_{k+1}\mathscr{D}_k \\
&\quad + \overline{\alpha}\overline{\mathscr{D}}_k^TP_{k+1}\mathscr{L}_k + \overline{\alpha}\widetilde{\mathscr{D}}^T_{\varrho_k,k}P_{k+1}\mathscr{D}_k + \overline{\alpha}\widetilde{\mathscr{D}}^T_{\varrho_k,k}P_{k+1}\mathscr{L}_k\Big),
\end{aligned} \tag{25}$$

subject to

$$\begin{cases} \phi_k = \gamma^2 I - \overline{\mathscr{D}}_k^TP_{k+1}\overline{\mathscr{D}}_k - \overline{\mathscr{D}}_k^TP_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k} - \widetilde{\mathscr{D}}^T_{\varrho_k,k}P_{k+1}\overline{\mathscr{D}}_k - \Omega_{1,k+1} > 0, \\ P_0 < \gamma^2(I_N\otimes W), \end{cases} \tag{26}$$

where

$$\begin{aligned}
\Pi^P_{k,\overline{\Lambda}_k} &\triangleq \text{diag}\Big\{\Pi^{P(1)}_{k,\overline{\Lambda}_k}\;\Pi^{P(2)}_{k,\overline{\Lambda}_k}\;\cdots\;\Pi^{P(N)}_{k,\overline{\Lambda}_k}\Big\}, \\
\widetilde{\alpha}^* &= \overline{\alpha}(1-\overline{\alpha}), \\
\Pi^{P(s)}_{k,\overline{\Lambda}_k} &= \widetilde{\alpha}^*\big(\deg^{(s)}_{in,k}\big)^2C_k^TK_k^TB_k^TP^{k+1}_{ss}B_kK_kC_k \\
&\quad - \sum_{i=1,i\neq s}^{N}\widetilde{\alpha}^*h_{is}\lambda^i_{s,k}\deg^{(s)}_{in,k}C_k^TK_k^TB_k^T\big(P^{k+1}_{is}+P^{k+1}_{si}\big)B_kK_kC_k \\
&\quad + \sum_{i,j=1,i,j\neq s}^{N}\widetilde{\alpha}^*h_{is}\lambda^i_{s,k}h_{js}\lambda^j_{s,k}C_k^TK_k^TB_k^TP^{k+1}_{ij}B_kK_kC_k, \\
&\qquad (s=1,2,\ldots,N), \\
\Pi^{\mathscr{P}}_{k,\overline{\Lambda}_k} &= \mathscr{N}\otimes\Pi^{P(s)}_{k,\overline{\Lambda}_k}, \\
\Pi^{\mathscr{P}}_{k,\widetilde{\Lambda}_k} &= \mathscr{N}\otimes\Pi^{P(s)}_{k,\widetilde{\Lambda}_k}, \\
\mathscr{L}_k &= \mathscr{N}\widetilde{\Lambda}_\varrho\otimes(B_kK_kC_k), \\
\mathscr{D}_k &= \mathscr{N}\overline{\Lambda}_k\otimes(B_kK_kC_k), \\
\Omega_{1,k+1} &= \sum_{i\in\kappa}\overline{p}_{i,k}\widetilde{\mathscr{D}}^T_{i,k}P_{k+1}\widetilde{\mathscr{D}}_{i,k}, \\
\Omega_{2,k+1} &= \sum_{i\in\kappa}\overline{p}_{i,k}\widetilde{\mathscr{D}}^T_{i,k}P_{k+1}\widetilde{\mathscr{T}}_{i,k}, \\
\Omega_{3,k+1} &= \sum_{i\in\kappa}\overline{p}_{i,k}\widetilde{\mathscr{T}}^T_{i,k}P_{k+1}\widetilde{\mathscr{T}}_{i,k}.
\end{aligned} \tag{27}$$

Then, the time-varying system (23) with the SCP scheduling satisfies the $\mathscr{H}_\infty$ performance constraint (21).

Proof (sufficiency). Choosing the proper Lyapunov-like function with respect to $\{P_k\}_{k\in[0,T]}$, it can be obtained that

$$
\begin{aligned}
&\mathbb{E}\{V_{k+1}-V_k\}\\
&\quad=\mathbb{E}\left\{\overline{x}_{k+1}^T P_{k+1}\overline{x}_{k+1}-\overline{x}_k^T P_k\overline{x}_k\right\}\\
&\quad=\mathbb{E}\Big\{\Big(\big(\mathscr{A}_k+\overline{\mathscr{B}}_k+\widetilde{\mathscr{B}}_{\varrho_k,k}\big)\overline{x}_k+\big(\overline{\mathscr{D}}_k+\widetilde{\mathscr{D}}_{\varrho_k,k}\big)\overline{v}_k+\overline{\mathscr{T}}_k+\widetilde{\mathscr{T}}_{\varrho_k,k}\Big)^T\\
&\qquad P_{k+1}\Big(\big(\mathscr{A}_k+\overline{\mathscr{B}}_k+\widetilde{\mathscr{B}}_{\varrho_k,k}\big)\overline{x}_k+\big(\overline{\mathscr{D}}_k+\widetilde{\mathscr{D}}_{\varrho_k,k}\big)\overline{v}_k+\overline{\mathscr{T}}_k+\widetilde{\mathscr{T}}_{\varrho_k,k}\Big)-\overline{x}_k^T P_k\overline{x}_k\Big\}\\
&\quad=\mathbb{E}\Big\{\overline{x}_k^T\mathscr{A}_k^T P_{k+1}\mathscr{A}_k\overline{x}_k+2\overline{x}_k^T\mathscr{A}_k^T P_{k+1}\overline{\mathscr{D}}_k\overline{v}_k+2\overline{x}_k^T\mathscr{A}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}\overline{v}_k\\
&\qquad+2\overline{x}_k^T\mathscr{A}_k^T P_{k+1}\overline{\mathscr{T}}_k+2\overline{x}_k^T\mathscr{A}_k^T P_{k+1}\widetilde{\mathscr{T}}_{\varrho_k,k}+\overline{x}_k^T\overline{\mathscr{B}}_k^T P_{k+1}\overline{\mathscr{B}}_k\overline{x}_k+\overline{x}_k^T\widetilde{\mathscr{B}}_{\varrho_k,k}^T P_{k+1}\widetilde{\mathscr{B}}_{\varrho_k,k}\overline{x}_k+\overline{v}_k^T\overline{\mathscr{D}}_k^T P_{k+1}\overline{\mathscr{D}}_k\overline{v}_k\\
&\qquad+2\overline{v}_k^T\overline{\mathscr{D}}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}\overline{v}_k+2\overline{v}_k^T\overline{\mathscr{D}}_k^T P_{k+1}\overline{\mathscr{T}}_k+2\overline{v}_k^T\overline{\mathscr{D}}_k^T P_{k+1}\widetilde{\mathscr{T}}_{\varrho_k,k}+\overline{v}_k^T\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}\overline{v}_k\\
&\qquad+2\overline{v}_k^T\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\overline{\mathscr{T}}_k+2\overline{v}_k^T\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\widetilde{\mathscr{T}}_{\varrho_k,k}+\overline{\mathscr{T}}_k^T P_{k+1}\overline{\mathscr{T}}_k+2\overline{\mathscr{T}}_k^T P_{k+1}\widetilde{\mathscr{T}}_{\varrho_k,k}+\widetilde{\mathscr{T}}_{\varrho_k,k}^T P_{k+1}\widetilde{\mathscr{T}}_{\varrho_k,k}-\overline{x}_k^T P_k\overline{x}_k\Big\}
\end{aligned}
\tag{28}
$$

Notice that the statistical properties of $\widetilde{\Lambda}_{\varrho_k}$ satisfy

$$
\begin{cases}
\mathbb{E}\left\{\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}\right\}=\Omega_{1,k+1},\\
\mathbb{E}\left\{\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\widetilde{\mathscr{T}}_{\varrho_k,k}\right\}=\Omega_{2,k+1},\\
\mathbb{E}\left\{\widetilde{\mathscr{T}}_{\varrho_k,k}^T P_{k+1}\widetilde{\mathscr{T}}_{\varrho_k,k}\right\}=\Omega_{3,k+1}.
\end{cases}
\tag{29}
$$

Hence, from (28) and (29), one has

$$
\begin{aligned}
&\mathbb{E}\{V_{k+1}-V_k\}\\
&=\mathbb{E}\Big\{\overline{x}_k^T\mathscr{A}_k^T P_{k+1}\mathscr{A}_k\overline{x}_k+2\overline{x}_k^T\mathscr{A}_k^T P_{k+1}\overline{\mathscr{D}}_k\overline{v}_k+2\overline{x}_k^T\mathscr{A}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}\bar{\Lambda}_k+2\overline{\alpha}\overline{x}_k^T\mathscr{A}_k^T P_{k+1}\mathscr{D}_k\overline{x}_k+2\overline{\alpha}\overline{x}_k^T\mathscr{A}_k^T P_{k+1}\mathscr{L}_k\overline{x}_k\\
&\quad+\overline{v}_k^T\overline{\mathscr{D}}_k^T P_{k+1}\overline{\mathscr{D}}_k\overline{v}_k+2\overline{v}_k^T\overline{\mathscr{D}}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}\overline{v}_k+2\overline{\alpha}\overline{v}_k^T\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{D}_k\overline{x}_k+2\overline{\alpha}\overline{v}_k^T\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{L}_k\overline{x}_k+\overline{v}_k^T\Omega_{1,k+1}\overline{v}_k+2\overline{\alpha}\overline{v}_k^T\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\mathscr{D}_k\overline{x}_k\\
&\quad+2\overline{v}_k^T\Omega_{2,k+1}+\overline{\alpha}^2\overline{x}_k^T\mathscr{D}_k^T P_{k+1}\mathscr{D}_k\overline{x}_k+2\overline{\alpha}^2\overline{x}_k^T\mathscr{D}_k^T P_{k+1}\mathscr{L}_k\overline{x}_k+\Omega_{3,k+1}+\overline{x}_k^T\Pi_{k,\bar{\Lambda}_k}^{\mathscr{P}}\overline{x}_k+\overline{x}_k^T\Pi_{k,\widetilde{\Lambda}_k}^{\mathscr{P}}\overline{x}_k-\overline{x}_k^T P_k\overline{x}_k\Big\}\\
&=\mathbb{E}\Big\{\overline{x}_k^T\Big(\mathscr{A}_k^T P_{k+1}\mathscr{A}_k+\overline{\alpha}\mathscr{A}_k^T P_{k+1}\mathscr{D}_k+\overline{\alpha}\mathscr{D}_k^T P_{k+1}\mathscr{A}_k+\overline{\alpha}\mathscr{A}_k^T P_{k+1}\mathscr{L}_k+\overline{\alpha}\mathscr{L}_k^T P_{k+1}\mathscr{A}_k+\overline{\alpha}^2\mathscr{D}_k^T P_{k+1}\mathscr{D}_k+\overline{\alpha}^2\mathscr{D}_k^T P_{k+1}\mathscr{L}_k\\
&\quad+\overline{\alpha}^2\mathscr{L}_k^T P_{k+1}\mathscr{D}_k+\overline{\alpha}^2\mathscr{L}_k^T P_{k+1}\mathscr{L}_k+\Pi_{k,\bar{\Lambda}_k}^{\mathscr{P}}+\Pi_{k,\widetilde{\Lambda}_k}^{\mathscr{P}}-P_k\overline{x}_k\Big)+\Big(\overline{v}_k^T\overline{\mathscr{D}}_k^T P_{k+1}\widetilde{\mathscr{D}}_k+\overline{\mathscr{D}}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}+\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\overline{\mathscr{D}}_k+\Omega_{1,k+1}\Big)\overline{v}_k\\
&\quad+2\overline{x}_k^T\Big(A_k^T P_{k+1}\overline{\mathscr{D}}_k+A_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}+\overline{\alpha}\mathscr{D}_k^T P_{k+1}\widetilde{\mathscr{D}}_k+\overline{\alpha}\mathscr{L}_k^T P_{k+1}\overline{\mathscr{D}}_k+\overline{\alpha}\mathscr{D}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}+\overline{\alpha}\mathscr{L}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}\Big)\overline{v}_k\Big\}.
\end{aligned}
\tag{30}
$$

Now, it is in the right position to deal with the $\mathscr{H}_\infty$ performance for disturbance $\{\overline{v}_k\}_{k\in[0,T]}\in l_2([0,T];\mathbb{R}^{n_{\overline{v}}})$. From (30), we know that

$$
\begin{aligned}
J_1 \triangleq\, & \mathbb{E}\left\{\|\overline{z}_k\|^2_{[0,T]} - \gamma^2\|\overline{\nu}_k\|^2_{[0,T]} + \|\gamma^2\varepsilon_k G_k\overline{x}_k\|^2_{[0,T]}\right\} \\
=\, & \mathbb{E}\left\{\overline{x}_0^T P_0\overline{x}_0 - \overline{x}_{T+1}^T P_{T+1}\overline{x}_{T+1}\right\} + \sum_{k=0}^{T}\mathbb{E}\left\{\|\overline{z}_k\|^2 - \gamma^2\|\overline{\nu}_k\|^2 + \gamma^2\|\varepsilon_k G_k\overline{x}_k\|^2 + \overline{x}_k^T\left(\mathscr{A}_k^T P_{k+1}\mathscr{A}_k + \overline{\alpha}\mathscr{A}_k^T P_{k+1}\mathscr{D}_k + \overline{\alpha}\mathscr{D}_k^T P_{k+1}\mathscr{A}_k\right.\right. \\
& + \overline{\alpha}\mathscr{A}_k^T P_{k+1}\mathscr{L}_k + \overline{\alpha}\mathscr{L}_k^T P_{k+1}\mathscr{A}_k + \overline{\alpha}^2\mathscr{D}_k^T P_{k+1}\mathscr{D}_k + \overline{\alpha}^2\mathscr{D}_k^T P_{k+1}\mathscr{L}_k + \overline{\alpha}^2\mathscr{L}_k^T P_{k+1}\mathscr{D}_k + \overline{\alpha}^2\mathscr{L}_k^T P_{k+1}\mathscr{L}_k + \Pi^{\mathscr{P}}_{k,\overline{\Lambda}_k} + \Pi^{\mathscr{P}}_{k,\widetilde{\Lambda}_k} - P_k\Big)\overline{x}_k \\
& + \overline{\nu}_k^T\left(\overline{\mathscr{D}}_k^T P_{k+1}\overline{\mathscr{D}}_k + \overline{\mathscr{D}}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k} + \widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\overline{\mathscr{D}}_k + \Omega_{1,k+1}\right)\overline{\nu}_k + 2\overline{x}_k^T\Big(A_k^T P_{k+1}\overline{\mathscr{D}}_k + A_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k} \\
& \left.+ \overline{\alpha}\mathscr{D}_k^T P_{k+1}\widetilde{\mathscr{D}}_k + \overline{\alpha}\mathscr{D}_k^T P_{k+1}\widetilde{\mathscr{D}}_k + \overline{\alpha}\mathscr{D}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k} + \overline{\alpha}\mathscr{L}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}\Big)\overline{\nu}_k\right\} = \mathbb{E}\left\{\overline{x}_0^T P_0\overline{x}_0 - \overline{x}_{T+1}^T P_{T+1}\overline{x}_{T+1}\right\} \\
& + \sum_{k=0}^{T}\mathbb{E}\Big\{\overline{x}_k^T\Big(\mathscr{A}_k^T P_{k+1}\mathscr{A}_k + \overline{\alpha}\mathscr{A}_k^T P_{k+1}\mathscr{D}_k + \overline{\alpha}\mathscr{D}_k^T P_{k+1}\mathscr{A}_k + \overline{\alpha}\mathscr{A}_k^T P_{k+1}\mathscr{L}_k + \overline{\alpha}\mathscr{L}_k^T P_{k+1}\mathscr{A}_k + \overline{\alpha}^2\mathscr{D}_k^T P_{k+1}\mathscr{D}_k \\
& + \overline{\alpha}^2\mathscr{D}_k^T P_{k+1}\mathscr{L}_k + \overline{\alpha}^2\mathscr{L}_k^T P_{k+1}\mathscr{D}_k + \overline{\alpha}^2\mathscr{L}_k^T P_{k+1}\mathscr{L}_k + \Pi^{\mathscr{P}}_{k,\overline{\Lambda}_k} + \Pi^{\mathscr{P}}_{k,\widetilde{\Lambda}_k}\overline{\mathscr{L}}_k^T + \overline{\mathscr{L}}_k + \gamma^2\varepsilon_k^2 G_k^T G_k - P_k\Big)\overline{x}_k \\
& - \overline{\nu}_k^T\left(\gamma^2 I - \overline{\mathscr{D}}_k^T P_{k+1}\overline{\mathscr{D}}_k - \overline{\mathscr{D}}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k} - \widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\overline{\mathscr{D}}_k - \Omega_{1,k+1}\right)\overline{\nu}_k + 2\overline{x}_k^T\Big(A_k^T P_{k+1}\overline{\mathscr{D}}_k + A_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k} \\
& + \overline{\alpha}\mathscr{D}_k^T P_{k+1}\widetilde{\mathscr{D}}_k + \overline{\alpha}\mathscr{D}_k^T P_{k+1}\widetilde{\mathscr{D}}_k + \overline{\alpha}\mathscr{D}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k} + \overline{\alpha}\mathscr{L}_k^T P_{k+1}\widetilde{\mathscr{D}}_{\varrho_k,k}\Big)\overline{\nu}_k\Big\}.
\end{aligned}
\tag{31}
$$

By applying the completing squares method, it can be observed that

$$
\begin{aligned}
J_1 =\, & \mathbb{E}\left\{\overline{x}_0^T P_0\overline{x}_0 - \overline{x}_{T+1}^T P_{T+1}\overline{x}_{T+1}\right\} + \sum_{k=0}^{M}\mathbb{E}\left\{\overline{x}_0^T P_0\overline{x}_0 - \overline{x}_{T+1}^T P_{T+1}\overline{x}_{T+1}\right\} \\
& + \sum_{k=0}^{T}\mathbb{E}\Big\{\overline{x}_k^T\Big(\mathscr{A}_k^T P_{k+1}\mathscr{A}_k + \overline{\alpha}\mathscr{A}_k^T P_{k+1}\mathscr{D}_k + \overline{\alpha}\mathscr{D}_k^T P_{k+1}\mathscr{A}_k + \overline{\alpha}\mathscr{A}_k^T P_{k+1}\mathscr{L}_k + \overline{\alpha}\mathscr{L}_k^T P_{k+1}\mathscr{A}_k + \overline{\alpha}^2\mathscr{D}_k^T P_{k+1}\mathscr{D}_k \\
& + \overline{\alpha}^2\mathscr{D}_k^T P_{k+1}\mathscr{L}_k + \overline{\alpha}^2\mathscr{L}_k^T P_{k+1}\mathscr{D}_k + \overline{\alpha}^2\mathscr{L}_k^T P_{k+1}\mathscr{L}_k + \Pi^{\mathscr{P}}_{k,\overline{\Lambda}_k} + \Pi^{\mathscr{P}}_{k,\widetilde{\Lambda}_k} + \overline{\mathscr{L}}_k^T\overline{\mathscr{L}}_k + \gamma^2\varepsilon_k^2 G_k^T G_k - P_k \\
& + \left(\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{A}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\mathscr{A}_k + \overline{\alpha}\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{D}_k + \overline{\alpha}\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{L}_k + \overline{\alpha}\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\mathscr{D}_k + \overline{\alpha}\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\mathscr{L}_k\right)^T\phi_k^{-1} \\
& + \left(\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{A}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\mathscr{A}_k + \overline{\alpha}\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{D}_k + \overline{\alpha}\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{L}_k + \overline{\alpha}\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\mathscr{D}_k + \overline{\alpha}\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\mathscr{L}_k\right)\Big)\overline{x}_k \\
& + \left(\overline{\nu}_k - \overline{\nu}_k^*\right)^T\phi_k\left(\overline{\nu}_k - \overline{\nu}_k^*\right)\Big\},
\end{aligned}
\tag{32}
$$

where

$$
\begin{aligned}
\overline{\nu}_k^* =\, & \phi_k^{-1}\Big(\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{A}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\mathscr{A}_k + \overline{\alpha}\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{D}_k \\
& + \overline{\alpha}\overline{\mathscr{D}}_k^T P_{k+1}\mathscr{L}_k + \overline{\alpha}\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\mathscr{D}_k + \overline{\alpha}\widetilde{\mathscr{D}}_{\varrho_k,k}^T P_{k+1}\mathscr{L}_k\Big)^T\overline{x}_k.
\end{aligned}
\tag{33}
$$

Under the terminal condition $P_{T+1} = 0$, the following result can be obtained because of $\phi_k > 0$ and $P_0 < \gamma^2(I_N \otimes W)$:

$$
\begin{aligned}
\widetilde{J}_1 =\, & \mathbb{E}\left\{\|\overline{z}_k\|^2_{[0,T]} - \gamma^2\|\overline{\nu}_k\|^2_{[0,T]} + \gamma^2\|\varepsilon_k G_k\overline{x}_k\|^2_{[0,T]}\right\} \\
& - \gamma^2\mathbb{E}\left\{\overline{x}_0^T(I_N \otimes W)\overline{x}_0\right\} \\
\le\, & \mathbb{E}\left\{\|\overline{z}_k\|^2_{[0,T]} - \gamma^2\|\overline{\nu}_k\|^2_{[0,T]} + \gamma^2\|\varepsilon_k G_k\overline{x}_k\|^2_{[0,T]}\right\} \\
& - \mathbb{E}\left\{\overline{x}_0^T P_0\overline{x}_0\right\} \\
=\, & J_1 - \mathbb{E}\left\{\overline{x}_0^T P_0\overline{x}_0\right\} \\
\le\, & -\mathbb{E}\left\{\sum_{k=0}^{T}\left(\overline{\nu}_k - \overline{\nu}_k^*\right)^T\phi_k\left(\overline{\nu}_k - \overline{\nu}_k^*\right)\right\} \le 0.
\end{aligned}
\tag{34}
$$

At this point, the $\mathscr{H}_\infty$ performance (21) is satisfied and this is the end of the proof of sufficiency. □

Necessity. This proof has been given by Lemma 3 in [37], so it will not be repeated here.

In order to make clear how $\overline{\alpha}$ and γ affect the feasible solution, denote $\mathscr{T}^*_{\varrho_k.k} \triangleq \mathscr{N}\check{\Lambda}_{r_k} \otimes (\overline{\alpha} B_k K_k C_k)$, then (13) can be recomposed as

$$P_k = \left(\mathscr{A}_k + \mathscr{T}^*_{\varrho_k,k}\right)^T P_{k+1}\left[P_{k+1}^{-1} + \left(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\right)\phi_k^{-1}\left(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\right)^T\right] P_{k+1}\left(\mathscr{A}_k + \mathscr{T}^*_{\varrho_k,k}\right) + \frac{1-\overline{\alpha}}{\overline{\alpha}}\left(\Pi^{\mathscr{P}}_{k,\overline{\Lambda}_k} + \Pi^{\mathscr{P}}_{k,\widetilde{\Lambda}_k}\right) + \gamma^2\varepsilon_k^2 G_k^T G_k + \overline{\mathscr{L}}_k^T\overline{\mathscr{L}}_k. \tag{35}$$

Until now, Lemma 3 reveals that $\mathscr{H}_\infty$ performance can be satisfied under the SCP scheduling as long as the BRRDE is solvable. Then, we are ready to design the controller parameter matrices K_k under the worst situation $\overline{\nu}_k = \overline{\nu}_k^* \triangleq \phi_k^{-1}(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\rangle\#x2009;)_k,k})^T P_{k+1}(\mathscr{A}_k + \overline{\alpha}\mathscr{D}_k + \overline{\alpha}\mathscr{L}_k)\overline{x}_k$. Therefore, the closed-loop multiagent system (12) can be recomposed as follows:

$$\overline{x}_{k+1} = \left(\mathscr{A}_k + \overline{\mathscr{B}}_k + \widetilde{\mathscr{B}}_{\varrho_k,k}\right)\overline{x}_k + \left(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\right)\phi_k^{-1}\left(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\right)^T P_{k+1} \times (\mathscr{A}_k + \overline{\alpha}\mathscr{D}_k + \overline{\alpha}\mathscr{L}_k)\overline{x}_k + \overline{\mathscr{T}}_k + \widetilde{\mathscr{T}}_{\varrho_k,k}. \tag{36}$$

Theorem 1. *Consider the DTVUMASs (1) under the SCP behavior scheduling governed by (5). Given the disturbance attenuation level $\gamma > 0$ and the positive definite matrix W, there exist controller parameters $\{K_k\}_{k\in[0,T]}$ and the positive scalar $\varepsilon_k > 0$ for the cooperative controllers (6) if the solutions $\{\varepsilon_k, P_k, \mathscr{K}_k\}_{k\in[0,T]}$ satisfy the BRRDE (25) as well as the following recursive RDE:*

$$\begin{cases} Q_k = \Delta_k^T Q_{k+1}\Delta_k + \Pi_k^Q + \overline{\mathscr{L}}_k^T\overline{\mathscr{L}}_k - \Delta_k^T Q_{k+1}\mathscr{B}_k\Gamma_k^{-1}\mathscr{B}_k^T Q_{k+1}^T\Delta_k, \\ Q_{T+1} = 0, \end{cases} \tag{37}$$

with a solution $\{Q_k, \mathscr{K}_k\}_{k\in[0,T]}$ satisfying

$$\begin{cases} Q_{k+1} \geq 0, \\ \Gamma_k = \mathscr{B}_k^T Q_{k+1}\mathscr{B}_k + I > 0, \\ \Delta_k = \mathscr{A}_k + \left(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\rangle\varrho)_k,k}\right)\phi_k^{-1}\left(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\rangle\varrho)_k,k}\right)^T \\ \quad P_{k+1}(\mathscr{A}_k + \overline{\alpha}\mathscr{D}_k + \overline{\alpha}\mathscr{L}_k), \\ \mathscr{K}_k = \arg\min_{\mathscr{K}_k} \operatorname{norm}\left(\overline{\alpha}\mathscr{K}_k(\mathscr{N}\overline{\Lambda}_k \otimes C_k) + \Gamma_k^{-1}\mathscr{B}_k^T Q_{k+1}^T\Delta_k\right), \end{cases} \tag{38}$$

and Π_k^Q is defined by

$$\Pi_k^Q \triangleq \operatorname{diag}\left\{\Pi_k^{Q(1)}, \Pi_k^{Q(2)}, \ldots, \Pi_k^{Q(N)}\right\}, \tag{39}$$

with

$$\begin{aligned}\Pi^{Q(s)}_{k,\overline{\Lambda}_k} &= \widetilde{\alpha}^*\left(\deg_{in}^{(s)}\right)^2 C_k^T\overline{\Lambda}_k^T K_k^T B_k^T Q_{ss}^{k+1} B_k K_k \overline{\Lambda}_k C_k \\ &- \sum_{i=1,i\neq s}^{N}\widetilde{\alpha}^* h_{is}\deg_{in}^{(s)} C_k^T\overline{\Lambda}_k^T K_k^T B_k^T\left(Q_{is}^{k+1} + Q_{si}^{k+1}\right)B_k K_k\overline{\Lambda}_k C_k \\ &+ \sum_{i,j=1,i,j\neq s}^{N}\widetilde{\alpha}^* h_{is}h_{js} C_k^T\overline{\Lambda}_k^T K_k^T B_k^T Q_{ij}^{k+1} B_k K_k \overline{\Lambda}_k C_k, \quad s = 1, 2, \cdots, N.\end{aligned} \tag{40}$$

Then, we can conclude that the closed-loop multiagent system (12) achieves the $\mathscr{H}_\infty$ performance requirement (8).

Proof. It can be seen from Lemma 3 that the system (12) satisfies the prespecified $\mathscr{H}_\infty$ performance (21) if the solution P_k to (25) can be calculated under the condition $\phi_k > 0$ and $P_0 < \gamma^2(I_N \otimes W)$. In this case, we can express the disturbance in the worst case as $\overline{\nu} = \overline{\nu}_k^* \triangleq \phi_k^{-1}(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k})^T P_{k+1}(\mathscr{A}_k + \overline{\alpha}\mathscr{D}_k + \overline{\alpha}\mathscr{L}_k)\overline{x}_k$, and it can be known that the prespecified $\mathscr{H}_\infty$ performance (21) is satisfied. We will design the controller parameter matrices K_k under the situation of worst-case disturbance later. Now, define the following cost function

$$J_2 = \mathbb{E}\left\{\sum_{i=1}^{N}\left\|\overline{z}_{i,k}\right\|^2_{[0,T]} + \sum_{i=1}^{N}\left\|\overline{u}_{i,k}\right\|^2_{[0,T]}\right\}, \tag{41}$$

or equivalently

$$J_2 = \mathbb{E}\left\{\left\|\overline{z}_k\right\|^2_{[0,T]} + \left\|\overline{u}_k\right\|^2_{[0,T]}\right\}. \tag{42}$$

For the solution $\{Q_k\}_{k\in[0,T]}$ of the BRRDE (37), it follows from (12) that

$$\begin{aligned}
&\mathbb{E}\left\{\overline{x}_{k+1}^T Q_{k+1}\overline{x}_{k+1} - \overline{x}_k^T Q_k \overline{x}_k\right\} \\
&= \mathbb{E}\Bigg\{\overline{x}_k^T\Big(\mathscr{A}_k + \overline{\mathscr{B}}_k + \widetilde{\mathscr{B}}_{\varrho_k,k} + \big(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\big)\phi_k^{-1}\big(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\big)^T P_{k+1}\left(\mathscr{A}_k + \overline{\alpha}\mathscr{D}_k + \overline{\alpha}\mathscr{L}_k\right)\Big)^T \\
&\quad\times Q_{k+1}\Big(\mathscr{A}_k + \overline{\mathscr{B}}_k + \widetilde{\mathscr{B}}_{\varrho_k,k} + \big(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\big)\phi_k^{-1}\big(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\big)^T P_{k+1}\left(\mathscr{A}_k + \overline{\alpha}\mathscr{D}_k + \overline{\alpha}\mathscr{L}_k\right)\Big)\overline{x}_k - \overline{x}_k^T Q_k \overline{x}_k \\
&\quad + 2\overline{x}_k^T\Big(\mathscr{A}_k + \big(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\big)\phi_k^{-1}\big(\overline{\mathscr{D}}_k + \widetilde{\mathscr{D}}_{\varrho_k,k}\big)^T P_{k+1}\left(\mathscr{A}_k + \overline{\alpha}\mathscr{D}_k + \overline{\alpha}\mathscr{L}_k\right)\Big)Q_{k+1}\big(\overline{\mathscr{T}}_k + \widetilde{\mathscr{T}}_{\varrho_k,k}\big) \\
&\quad + \Big(\overline{\mathscr{T}}_k^T Q_{k+1}\overline{\mathscr{T}}_k + 2\overline{\mathscr{T}}_k^T Q_{k+1}\widetilde{\mathscr{T}}_{\varrho_k,k} + \epsilon_{k+1}\Big)\Bigg\} = \mathbb{E}\Big\{\overline{x}_k^T\Delta_k^T Q_{k+1}\Delta_k\overline{x}_k + \overline{x}_k^T\big(\Pi_k^Q - Q_k\big)\overline{x}_k + 2\overline{x}_k^T\Delta_k^T Q_{k+1}\mathscr{B}_k\big(\overline{T}_k + \widetilde{T}_{\varrho_k,k}\big) \\
&\quad + \big(\overline{T}_k + \widetilde{T}_{\varrho_k,k}\big)^T\mathscr{B}_k^T Q_{k+1}\times\mathscr{B}_k\big(\overline{T}_k + \widetilde{T}_{\varrho_k,k}\big)\Big\},
\end{aligned} \tag{43}$$

where $\overline{T}_k = \mathscr{N}\overline{\Lambda}_k \otimes (\overline{\alpha}K_kC_k)\overline{x}_k$ and $\widetilde{T}_k = \mathscr{N}\widetilde{\Lambda}_{\varrho_k,k}\otimes(\overline{\alpha}K_kC_k)\overline{x}_k$. Similar to Lemma 3, it has

$$\begin{aligned}
J_2 &= \sum_{k=0}^{T}\mathbb{E}\Big\{\left\|\overline{z}_k\right\|_{[0,T]}^2 + \left\|\overline{u}_k\right\|_{[0,T]}^2 + \overline{x}_k^T\Delta_k^T Q_{k+1}\Delta_k\overline{x}_k + \overline{x}_k^T\big(\Pi_k^Q - Q_k\big)\overline{x}_k + 2\overline{x}_k^T\Delta_k^T Q_{k+1}\times\big(\overline{\mathscr{T}}_k + \widetilde{\mathscr{T}}_{\varrho_k,k}\big) \\
&\quad + \big(\overline{\mathscr{T}}_k + \widetilde{\mathscr{T}}_{\varrho_k,k}\big)^T Q_{k+1}\big(\overline{\mathscr{T}}_k + \widetilde{\mathscr{T}}_{\varrho_k,k}\big) + \mathbb{E}\left\{\overline{x}_0^T Q_0\overline{x}_0 - \overline{x}_{T+1}^T Q_{T+1}\overline{x}_{T+1}\right\} \\
&= \sum_{k=0}^{T}\mathbb{E}\Big\{\overline{x}_k^T\Big(\Delta_k^T Q_{k+1}\Delta_k + \Pi_k^Q - Q_k + \overline{\mathscr{L}}_k^T\overline{\mathscr{L}}_k\Big)\overline{x}_k + 2\overline{x}_k^T\Delta_k^T Q_{k+1}\mathscr{B}_k\big(\overline{T}_k + \widetilde{T}_{\varrho_k,k}\big) + \big(\overline{T}_k + \widetilde{T}_{\varrho_k,k}\big)^T\times\mathscr{B}_k^T Q_{k+1}\mathscr{B}_k\big(\overline{T}_k + \widetilde{T}_{\varrho_k,k}\big)\Big\} \\
&\quad + \mathbb{E}\left\{\overline{x}_0^T Q_0\overline{x}_0 - \overline{x}_{T+1}^T Q_{T+1}\overline{x}_{T+1}\right\}.
\end{aligned} \tag{44}$$

By using the completing squares method again, it follows that

$$\begin{aligned}
J_2 &= \sum_{k=0}^{T}\mathbb{E}\Big\{\overline{x}_k^T\Big(\Delta_k^T Q_{k+1}\Delta_k + \Pi_k^Q - Q_k + \overline{\mathscr{L}}_k^T\overline{\mathscr{L}}_k - \Delta_k^T Q_{k+1}\mathscr{B}_k\big(\mathscr{B}_k^T Q_{k+1}\mathscr{B}_k + I\big)^{-1}\mathscr{B}_k^T Q_{k+1}^T\Delta_k\Big)\overline{x}_k \\
&\quad + \big(\overline{T}_k + \widetilde{T}_{\varrho_k,k} - T_k^*\big)^T\big(\mathscr{B}_k^T Q_{k+1}\mathscr{B}_k + I\big)\big(\overline{T}_k + \widetilde{T}_{\varrho_k,k} - T_k^*\big) + \mathbb{E}\left\{\overline{x}_0^T Q_0\overline{x}_0 - \overline{x}_{T+1}^T Q_{T+1}\overline{x}_{T+1}\right\},
\end{aligned} \tag{45}$$

where $T_k^* = -(\mathscr{B}_k^T Q_{k+1}\mathscr{B}_k + I)^{-1}\mathscr{B}_k^T Q_{k+1}^T\Delta_k\overline{x}_k$.

We can obtain the best choice of $\mathscr{K}_k$ shown on (37) and (38), and the proof is complete. □

Theorem 2. *Consider the DTVUMASs (1) under the SCP scheduling governed by (5). For given the disturbance attenuation level $\gamma > 0$ and the positive definite matrix W, there exist controller parameters $\{K_k\}_{k\in[0,T]}$ for the cooperative controllers (6), if there exist solutions $\{\varepsilon_k, P_k, \mathscr{K}_k\}_{k\in[0,T]}$ and $\{Q_k, \mathscr{K}_k\}_{k\in[0,T]}$ satisfying the coupled BRRDEs (25) and (37) subject to (38) with the controller parameters given by*

$$K_k = -\begin{bmatrix} \nu_{1,k} & \nu_{2,k} & \cdots & \nu_{N,k}\end{bmatrix}\begin{bmatrix} \psi_{1,k} & \psi_{2,k} & \cdots & \psi_{N,k}\end{bmatrix}^{\dagger}, \tag{46}$$

where

$$\begin{aligned}
\mathscr{N}\overline{\Lambda}_k\otimes C_k &\triangleq \begin{bmatrix} \left(\psi_{1,k}\right)^T & \left(\psi_{2,k}\right)^T & \cdots & \left(\psi_{N,k}\right)^T \end{bmatrix}^T \Gamma_k^{-1}\mathscr{B}_k^T Q_{k+1}^T\Delta_k \\
&\triangleq \begin{bmatrix} \left(\nu_{1,k}\right)^T & \left(\nu_{2,k}\right)^T & \cdots & \left(\nu_{N,k}\right)^T \end{bmatrix}^T.
\end{aligned} \tag{47}$$

Then, the closed-loop system (12) satisfies the $\mathscr{H}_\infty$ performance requirement (8).

Step 1. Set $k = T$ and the terminal condition $P_{T+1} = Q_{T+1} = 0$.
Step 2. Calculate Γ_k by (38) firstly, then solve (46), after that the controller parameter K_k can be obtained. Proceed to the next step, otherwise skip to the last step.
Step 3. Calculate ϕ_k in equation (26). If $\phi_k > 0$, move on to the next step, else jump to the last step.
Step 4. Calculate P_k and Q_k in equations (25) and (37), respectively. Let $k = k - 1$ until $k = 0$ stop, set back to the second step, else move on to the next step.
Step 5. If $\{\Gamma_k > 0, \phi_k > 0, P_0 < \gamma^2 \otimes W\}$ cannot be satisfied, then this algorithm is infeasible. Stop.

Algorithm 1: Algorithm CC.

Proof. The form of controller parameters $\mathscr{K}_k$ has been given in Theorem 1, which is shown in (38), and it can be rewritten as

$$\begin{aligned}\mathscr{K}_k &= \arg\min_{\mathscr{K}_k} \operatorname{norm}\left(\bar{\alpha}\mathscr{K}_k\left(\mathscr{N}\bar{\Lambda}_k \otimes C_k\right) + \Gamma_k^{-1}\mathscr{B}_k^T Q_{k+1}^T \Delta_k\right),\\ K_k &= \arg\min_{K_k} \operatorname{norm}\left(\bar{\alpha}K_k\left[\begin{array}{cccc}\psi_{1,k} & \psi_{2,k} & \cdots & \psi_{N,k}\end{array}\right]\right.\\ &\quad \left. + \left[\begin{array}{cccc}\nu_{1,k} & \nu_{2,k} & \cdots & \nu_{N,k}\end{array}\right]\right).\end{aligned} \tag{48}$$

Apparently, the controller parameter K_k can be obtained by applying the Moore–Penrose pseudoinverse. □

Remark 3. Up to now, the controller parameters K_k have been obtained under the missing measurements, parameter uncertainties, and SCP. If $\alpha_{i,k} = 1, M_{i,k} = G_k = 0$ in this paper, it means the missing measurements and parameter uncertainties are not considered, and the system is only affected by SCP, where the method has been given in [26]. Compared with [26], the multiagent systems considered in this paper are more general, and the results of this paper can be applied to more complex systems.

By observing the main results in the aforementioned theorems, the following consensus control (CC) (Algorithm 1) algorithm under the SCP can be given.

Remark 4. The DTVUMASs have been considered in this paper. According to (46), the controller parameter matrices K_k have been obtained. Since the BRRDEs need to be calculated at each step, the computing burden is increased. But, with the continuous development of computer science and technology, this concern may not cause too much impact. In addition, the RDE has a very important application in many fields. Thus, its solvability has gradually become a hot issue that needs to be studied, which is also the direction that the author needs to investigate in the future.

Remark 5. In fact, the influences from RR protocol and missing measurements have been discussed in [38] for multiagent systems with $\mathscr{H}_\infty$ consensus constraint. Compared with the method in [38], it can be seen two major differences: (i) a new protocol (i.e., SCP) and the time-varying parameter uncertainty have been considered simultaneously and (ii) a new consensus condition is given to fulfill the desired performance requirements. It is worth mentioning that the related $\mathscr{H}_\infty$ consensus performance is affected by the average state of controlled output $\bar{z}_k$, the average state of initial system state $\bar{x}_0$, the external disturbance $\bar{v}_k$, and the time-varying parameter uncertainty $\Delta A_{i,k}$. Hence, special effort should be made to handle the induced impacts. On the contrary, it needs to point out that the SCP is a dynamical protocol compared with the RR protocol as a static protocol; therefore, the proposed results have more wider application areas especially for the complex network environments. Overall, the DTVUMASs considered in this paper are more general, and the newly proposed control method possesses potential application domains.

4. A Simulation Example

In this section, the effectiveness of the proposed controller is shown by a simulation example.

Consider the DTVUMASs (1) with the following parameters:

$$\begin{aligned}
A_k &= \begin{bmatrix} 0.7 + 0.05\sin(0.2k) & 0.7 + 0.4\sin(k) \\ 0.41 & -0.69 + 0.25\sin(0.5k) \end{bmatrix}\\
L_k &= \begin{bmatrix} 0.4 & -0.5 \end{bmatrix}\\
B_k &= -0.6em \begin{bmatrix} 0.25 + 0.09\cos(0.2k) & 0.06 & -0.07 \\ 0.09 & 0.08 & -0.004\cos(k) \end{bmatrix},\\
G_k &= \begin{bmatrix} 0.5 & -0.65 \end{bmatrix},\\
C_k &= \begin{bmatrix} 0.9 + 0.06\sin(k) & 0.3 + 0.01\sin(4k) \\ 0.5 & -0.6 \\ 0.8 & 0.6\sin(0.2k) \end{bmatrix},\\
E_k &= \begin{bmatrix} 0.04 \\ 0.024 \end{bmatrix},\\
F_{i,k} &= \begin{cases} 0.82\sin(k), & i = 1, \\ 0.82\cos(2k), & i = 2, \\ 0.51\cos(-2k)\sin(k), & i = 3, \end{cases}\\
D_k &= \begin{bmatrix} -0.3 \\ 0.06 \\ 0.02 \end{bmatrix}\\
M_{i,k} &= \begin{matrix} \begin{bmatrix} 0.4 & 0.1 \end{bmatrix}^T, & i = 1, \\ \begin{bmatrix} 0.5 & -0.2 \end{bmatrix}^T, & i = 2, \\ \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}^T, & i = 3, \end{matrix}\\
\mathscr{H} &= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.
\end{aligned} \tag{49}$$

Let the $\mathscr{H}_\infty$ consensus performance level γ and the time horizon T be 0.89 and 35, respectively. The matrix W and ε_k

Table 1: The occurrence probabilities of $p_{i,k}^{j}$.

$p_{i,k}^{j}$	$j=1$	$j=2$	$j=3$
$i=1$	0	0.7	0.3
$i=2$	0.5	0	0.5
$i=3$	0.4	0.6	0

Table 2: The output-feedback controller gains at each time step.

k	1	2	3	...
K_k	$\begin{bmatrix} 0.0267 & -0.1025 & 0.0035 \\ -0.0362 & 0.1299 & -0.0054 \\ -0.0163 & 0.0593 & -0.0023 \end{bmatrix}$	$\begin{bmatrix} 0.0169 & -0.1052 & 0.0103 \\ -0.0180 & 0.1056 & -0.0111 \\ -0.0104 & 0.0623 & -0.0064 \end{bmatrix}$	$\begin{bmatrix} -0.0188 & 0.0606 & -0.0240 \\ -0.0248 & 0.2069 & -0.0482 \\ -0.0051 & 0.0592 & -0.0110 \end{bmatrix}$	...

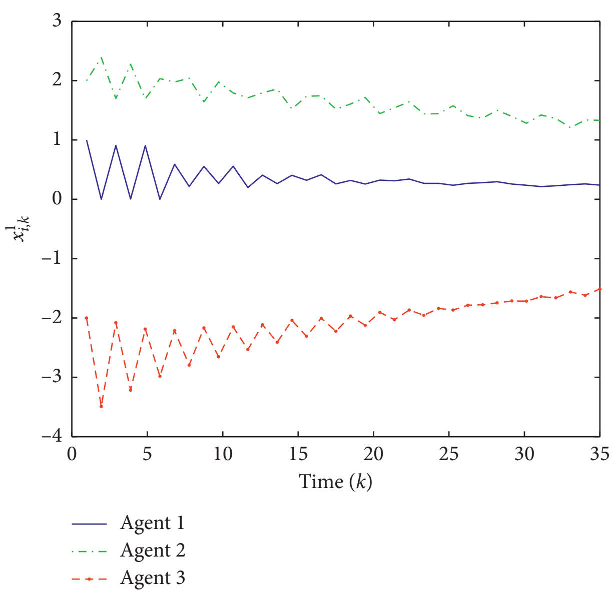

Figure 1: The state trajectories of $x_{i,k}^{1}\,(i=1,2,3)$ without the controller.

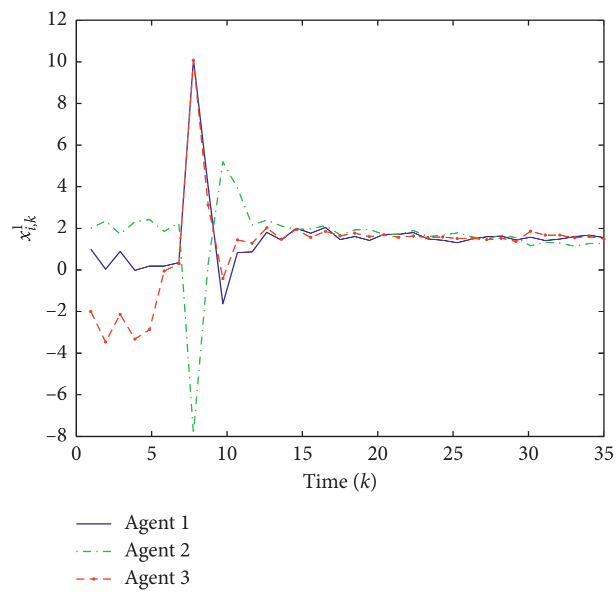

Figure 3: The state trajectories of $x_{i,k}^{1}\,(i=1,2,3)$.

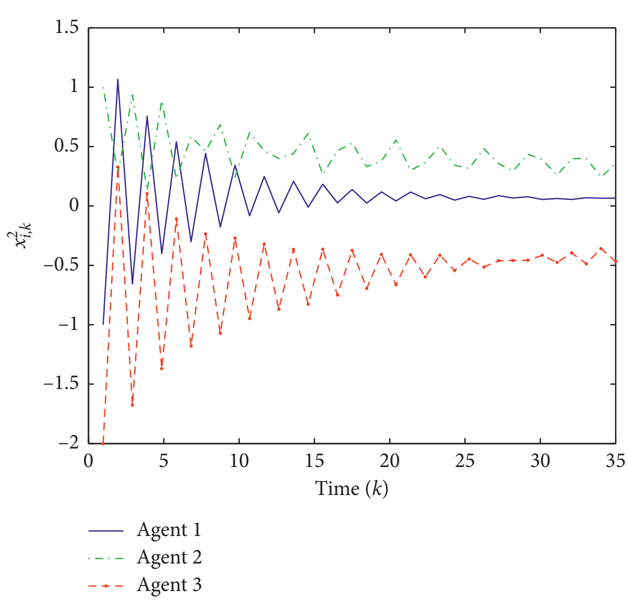

Figure 2: The state trajectories of $x_{i,k}^{2}\,(i=1,2,3)$ without the controller.

could be selected as $10I$ and 1.2, respectively. Also, the missing measurement probability $\overline{\alpha}$ is given by 0.80. The initial $x_{i,0}$ $(i=1,2,3)$ are selected as $x_{1,0}=[1\ -1]^T$, $x_{2,0}=[2\ 1]^T$, and $x_{3,0}=[-2\ -2]^T$. The occurrence probabilities $p_{i,k}^{j}$ are selected as shown in Table 1.

By implementing Algorithm CC, the controller parameters and the simulation results can be obtained. The desired output feedback controller parameters are listed in Table 2. Figures 1 and 2 plot the trajectories of $x_{i,k}^{1}$ and $x_{i,k}^{2}$ without the controller. Figures 3 and 4 plot the trajectories of $x_{i,k}^{1}$ and $x_{i,k}^{2}$. Figure 5 plots the trajectory of output consensus errors $\overline{z}_{i,k}$ $(i=1,2,3)$. Comparing Figure 2 with Figure 4 , if the agents are without the controller, the states of the agents are divergent. Therefore, the effectiveness of the proposed controller can be further verified.

5. Conclusion

In this paper, the $\mathscr{H}_\infty$ consensus control problem has been tackled for DTVUMASs with missing measurements and parameter uncertainties under the SCP. To avoid signals

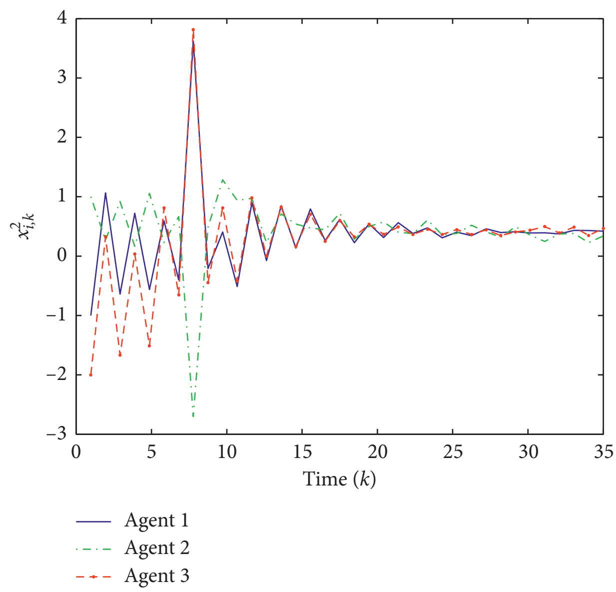

FIGURE 4: The state trajectories of $x_{i,k}^2$ $(i = 1, 2, 3)$.

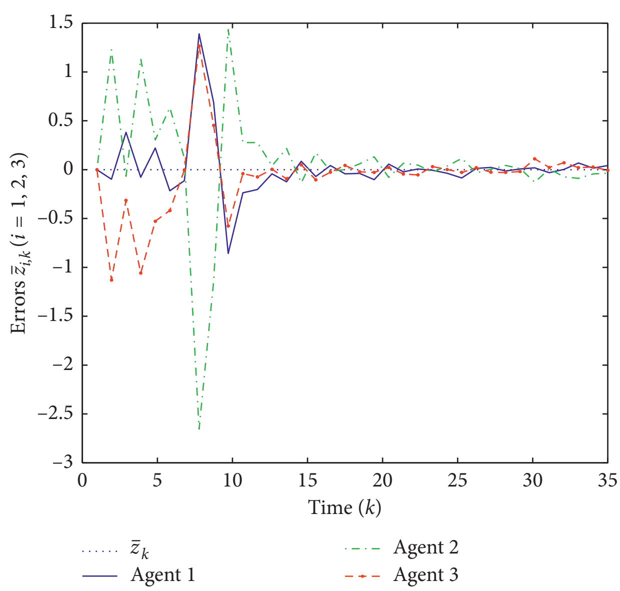

FIGURE 5: Output consensus error $\bar{z}_{i,k}$ $(i = 1, 2, 3)$.

conflict as much as possible, SCP has been adopted in this paper. By employing the SCP scheduling and completing squares method, the consensus performance with a given $\mathscr{H}_\infty$ disturbance attenuation level has been ensured, where some sufficient conditions over the finite horizon $[0, T]$ have been obtained. In particular, the controller gains have been characterized by the solutions to two BRRDEs. Finally, a simulation has been adopted to show the validity of the proposed consensus control approach. Based on the main results, a new method can be given to handle the uncertain occurrence probabilities with hope to further characterize the complicated cases.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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