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Research Article

The Impact of Coupling Function on Finite-Time Synchronization Dynamics of Multi-Weighted Complex Networks with Switching Topology

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This paper is not only concerned with the problem of finite-time synchronization control for a class of nonlinear coupling multi-weighted complex networks (NCMWCNs) with switching topology but also an attempt at using the derived results and Lyapunov stability theory to study the impact of nonlinear coupling function on finite-time synchronization dynamics of the raised network model. Firstly, different from the existing related results, based on the existing and new finite-time theories, two finite-time synchronization controllers are, respectively, designed to make the considered network achieve finite-time synchronization. Secondly, according to the obtained results, several finite-time synchronization dynamics criteria are established to show that nonlinear coupled function and the switching of outer-coupling matrix are how to impact finite-time synchronization dynamics. Finally, two illustrated examples are provided to verify the effectiveness of theoretical results proposed in this paper.

1. Introduction

During recent years, many researchers have paid close attention to synchronization dynamics problems of complex networks because synchronization dynamics is one of the most important collective behavior of complex networks [1–3] and many practical systems, including sensor network, communication network, neural networks, social network, and so on [4–6], can be modeled by complex networks. Therefore, many valuable and meaningful results for synchronization dynamics problems of complex networks have been obtained [7–13]. For example, based on passivity theory and Lyapunov stability theory, Kaviarasan et al. [7] investigated robust asymptotic synchronization of complex dynamical networks with uncertain inner coupling and successive delays via state feedback delayed control scheme. In [8], pinning synchronization problem is proposed for nonlinear coupling complex networks by Liu and Chen. Under the pinning control technique, the authors not only derived several global synchronization criteria for considered networks but also discussed the effect of nonlinear coupling function for synchronization dynamics from simulation aspect. By making use of pinning control strategies, Kaviarasan et al. [9] studied the problem of global synchronization on singular complex dynamical networks with Markovian switching and two additive time-varying delays. Moreover, in the real world, a lot of networks such as QQ networks, E-mail networks, and transportation networks, can be more properly modeled by multi-weighted complex networks [14, 15], in which the coupling forms among nodes are multiple. That is to say, the nodes in complex networks with multi-weights are connected by more than one weight. Thus, recently, synchronization and passivity dynamics problems on multi-weighted complex networks have aroused an increasing interest of some researchers [16–23]. For instance, in [16], Qiu et al. made a discussion on finite-time synchronization problem of linear coupling multi-weighted complex networks. Qin et al. [17], respectively, investigated global synchronization and H_∞

synchronization of linear coupling multi-weighted complex networks with fixed and switching topologies by making use of Lyapunov stability theory and inequality techniques. Besides these, in fact, due to some factors including external disturbance, limited communications, and so on [24, 25], there is inevitable switching in many dynamical systems, in which the switching may affect synchronization dynamics of systems. Therefore, it is interesting to study synchronization dynamics of multi-weighted complex networks with switching topology.

As a matter of fact, in many practical engineering areas, it is necessary and meaningful for a coupling dynamical system to achieve the desired dynamical behaviors in finite time interval [26–28]. Hence, a lot of results about finite-time synchronization dynamics problems for coupling systems have been obtained [29–34]. For example, based on finite-time control theory, Wang et al. [29] designed finite-time control rule to achieve global synchronization within convergence time for a class of linear coupling Markovian jump complex networks. In [30], the authors investigated finite-time synchronization problem of linear coupling complex networks and finite-time synchronization criteria were derived by exploring finite-time stability theory.

In reality, in some cases, nodes of practical coupling networks are entangled by nonlinear function method [35], such as the interactions between different neuron elements in brain dynamical networks and the interactions between different electrical elements in an electrical gird dynamical networks. Therefore, recently, the researches on dynamic behaviors for nonlinear coupling networks including consensus problems of nonlinear coupling multi-agent systems [35, 36], synchronization problems of nonlinear coupling neural networks and complex networks [8, 37–40], and so on [41] have witnessed an increasing interest. Although some valuable results about dynamic behaviors of nonlinear coupling networks have been developed (e.g., [8, 35–41]), in these existing works, the researchers mainly concentrated on how to derive sufficient condition criteria for considered nonlinear coupling systems. It is worth pointing out that nonlinear coupling function is one of important factors affecting synchronization dynamics. Regrettably, few researchers devoted themselves to exploring the impact of nonlinear coupling function on synchronization dynamics from theory aspect.

Motivated by the above analysis, the main purpose of this paper is to investigate the effect of nonlinear coupling function and outer-coupling matrix switching on finite-time synchronization dynamics for a class of nonlinear coupling multi-weighted complex networks (NCMWCNs) with switching topology based on stability and a novel finite-time theory. The contributions of our works include the following three aspects. First, a new class of NCMWCNs with switching topology is considered. Second, in order to address the novel finite-time control method proposed, two finite-time synchronization controllers built on the existing and a new finite-time synchronization theories are, respectively, designed to make the considered network model achieve global synchronization within finite time interval. Third, we not only derive sufficient condition for ensuring finite-time synchronization of the NCMWCNs with switching topology but also give synchronization dynamics criteria on the impact of nonlinear coupling function and outer-coupling matrix switching.

Notations. Some necessary notations that will be used throughout the article are introduced. $\|\cdot\|$ refers to the standard $L2$ norm in Euclidean space. The number N represents a positive integer. $R^{n\times m}$ is the set of real matrices and R^n denotes the n-dimensional Euclidean space. The superscript T denotes the matrix transposition. $I_n \in R^{n\times n}$ means an n-dimensional identity matrix. $X \geq Y > 0$ (respectively, $X > Y > 0$), where $X, Y \in R^{n\times n}$ are symmetric matrices, means that the matrix $X - Y$ is positive semidefinite (respectively, positive definite). If A is a matrix, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote its maximal eigenvalue and its minimum eigenvalue, respectively. The Kronecker product of matrices $A \in R^{m\times n}$ and $B \in R^{M\times N}$ is a matrix in $R^{mM\times nN}$ denoted as $A \otimes B$. The matrix diag() represents diagonal matrix. If the dimensions of matrices are not explicitly indicated that means they are suitable for any algebraic operations.

2. Model and Preliminaries

Firstly, consider a class of NCMWCNs with switching topology presented by

$$\begin{aligned}\dot{x}_i(t) = {} & f(x_i(t)) + \sum_{j=1}^{N} c_1 a_{ij}^{1,\sigma(t)} \Gamma_1 g(x_j(t)) \\ & + \sum_{j=1}^{N} c_2 a_{ij}^{2,\sigma(t)} \Gamma_2 g(x_j(t)) + \cdots \\ & + \sum_{j=1}^{N} c_\eta a_{ij}^{\eta,\sigma(t)} \Gamma_\eta g(x_j(t)) + u_i(t), \\ & \qquad i = 1, 2, \ldots, N,\end{aligned} \tag{1}$$

where $\sigma(t) : [0, \infty) \longrightarrow M = \{1, 2, \ldots, s\}$ is the topology switching signal, which is defined as the switching sequence

$$S = \{(m_0, t_0), \ldots, (m_i, t_i), \cdots \mid m_i \in M,\ i \in \mathbb{N}\}, \tag{2}$$

where t_0 is the initial time and m_i denotes the serial number of the activated subsystem at t_i. $c_l (l = 1, 2, \ldots, \eta)$ represents the coupling strength and $c_l > 0, \Gamma_l = \mathrm{diag}(\gamma_{l1}, \gamma_{l2}, \ldots, \gamma_{ln}) > 0$ is inner-coupling matrix and $\Gamma_l \in R^{n\times n}$ and $A^{l,m} = (a_{ij}^{l,m})_{N\times N}$ is outer-coupling matrix with the coupling weights in the lth coupling form and the mth topology. For each $m \in M$, there is $a_{ij}^{l,m} = a_{ji}^{l,m} > 0$ if node i and node j are connected. Otherwise, $a_{ij}^{l,m} = a_{ji}^{l,m} = 0$. Besides these, $f : R^n \longrightarrow R^n$ stands for the activity of ith node and is a vector-value function, $g : R^n \longrightarrow R^n$ is nonlinear coupling function, and $u_i(t)$ is the control input of ith node.

Remark 1. Recently, because multi-weighted complex networks can more accurately describe some practical engineering networks, e.g., public traffic networks and Mobile phone networks, some researchers began to pay more and more attention to dynamical behaviors of multi-weighted complex networks and have obtained some valuable and meaningful results [14–23]. However, it should be emphasized that, in these existing works [14–23], the authors concentrated on linear coupling multi-weighted complex networks. To the best of our knowledge, until now, there is still no discussion on synchronization dynamics problems for the NCMWCNs with switching topology. Hence, it is very significant to study finite-time synchronization for the NCMWCNs with switching topology. Besides this, in the above network model (1), there is no the restriction which is that outer-coupling matrix with coupling weights $A^{l,m} = (a_{ij}^{l,m})_{N\times N}$ must satisfy $a_{ii}^{l,m} = -\sum_{j=1,j\neq i}^{N} a_{ij}^{l,m}$. Thus, the considered network model (1) is more general. Actually, the similar scheme has been adopted in the literature [21].

Remark 2. From Remark 1, it is obtained that some practical engineering networks can be more accurately described by multi-weighted complex networks. For example, in the network (1), let $g(x_j(t)) = x_j(t)$, $\sigma(t) : [0,\infty) \longrightarrow M = \{1\}$ and $a_{ii}^{l,m} = -\sum_{j=1,j\neq i}^{N} a_{ij}^{l,m}$, and the network (1) becomes the addressed network model (1) in [14, 15]. It is clear that the proposed network model (1) in [14, 15] is one special case of the network (1) in this paper. According to [14, 15], it is seen that the effectiveness of the derived results is testified by the public traffic transfer networks. This reflects that the public traffic transfer networks in [14, 15] can be expressed by the network (1) of this paper. Furthermore, compared with the network model of [14, 15], it is not difficult to find that the network (1) of this paper can model more general public traffic transfer networks. Besides this, if the network (1) is composed of some Kuramoto oscillators [42], the network (1) becomes nonlinear coupling Kuramoto oscillator network with multi-weights and switching topology. These above show the significance of considering the multi-weighted coupling term and real physical meaning of the network (1).

The synchronization state $s(t)$ of the network (1) satisfies

$$\begin{aligned}\dot{s}(t) = {} & f(s(t)) + \sum_{j=1}^{N} c_1 a_{ij}^{1,\sigma(t)} \Gamma_1 g(s(t)) \\ & + \sum_{j=1}^{N} c_2 a_{ij}^{2,\sigma(t)} \Gamma_2 g(s(t)) + \cdots \\ & + \sum_{j=1}^{N} c_\eta a_{ij}^{\eta,\sigma(t)} \Gamma_\eta g(s(t)),\end{aligned} \tag{3}$$

where $s(t) = (s_1(t), s_2(t), \ldots, s_n(t))^T$.

Secondly, in order to derive the main results, we need to give the following Definition 3, some assumptions, and lemmas.

Definition 3. The network (1) is said to achieve global synchronization within finite time interval t^* if there exists a constant $t^* > 0$ such that $\lim_{t\longrightarrow t^*} \|x_i(t) - s(t)\| = 0$ and $\|x_i(t) - s(t)\| = 0$ for $t > t^*$, where $i \in \{1, 2, \ldots, N\}$ and $s(t) = (s_1(t), s_2(t), \ldots, s_n(t))^T \in R^n$ is the synchronization state of the network (1).

Assumption 4 (see [16]). There exist matrices $Q = \operatorname{diag}(q_1, q_2, \ldots, q_n)$ and $0 < H = \operatorname{diag}(h_1, h_2, \ldots, h_n)$, such that $f(\cdot)$ satisfies the following inequality:

$$\begin{aligned}& (\iota_1 - \iota_2)^T H [f(\iota_1) - f(\iota_2) - Q(\iota_1 - \iota_2)] \\ & \quad \leq -\kappa (\iota_1 - \iota_2)^T (\iota_1 - \iota_2),\end{aligned} \tag{4}$$

where $0 < \kappa \in R$ and ι_1 and $\iota_2 \in R^n$.

Assumption 5. The coupling function $g(\cdot)$ of the network (1) satisfies the Lipschitz condition and $g(0) = 0$. That means there exists constant $\nu > 0$ such that $\|g(x) - g(y)\| \leq \nu\|x - y\|$ and $\|g(x)\| \leq \nu\|x\|$, where $x, y \in R^n$.

Remark 6. Note that nonlinear functions $f(\cdot)$ and $g(\cdot)$ can be linearized by Assumptions 4 and 5, respectively. Assumption 4 is so-called the QUAD condition (or one-sided Lipschitz) [43] and Assumption 5 is the Lipschitz condition. In fact, Assumption 4 is more general than Assumption 5. Until now, in the research about synchronization problems of complex networks, the Lipschitz condition and the QUAD condition have been widely used to process nonlinear functions [1, 14–16, 18, 43, 44].

Lemma 7 (see [45]). *Suppose that a positive-definite and continuous $V(t)$ satisfies the following condition:*

$$\dot{V}(t) \leq -cV^{\eta}(t), \quad t \geq 0, \ V(0) \geq 0, \tag{5}$$

where $0 < \eta < 1$ and $c > 0$ are constants. Then, one has

$$V^{1-\eta}(t) \leq V^{1-\eta}(0) - c(1-\eta)t, \quad 0 \leq t \leq t^*, \tag{6}$$

and $V(t) = 0$, $t > t^*$, with t^* given by

$$t^* = \frac{V^{1-\eta}(0)}{c(1-\eta)}. \tag{7}$$

Lemma 8 (see [30]). *Assume that a continuous and positive-definite function $V(t)$ satisfies the following condition:*

$$\dot{V}(t) \leq l(t)V(t) - k(t)V^{\eta}(t), \quad t \geq t_0, \ V(t_0) \geq 0, \tag{8}$$

where $k(t) > 0$ and $l(t)$ are two functions, $0 < \eta < 1$. Then, for any given t_0, one has

$$\begin{aligned}V(t) \leq e^{\theta(t)} \Big[& V^{1-\eta}(t_0) \\ & - (1-\eta) \int_{t_0}^{t} k(s) e^{-(1-\eta)\theta(s)} ds \Big]^{1/(1-\eta)},\end{aligned} \tag{9}$$

where $\theta(t) = \int_{t_0}^{t} l(\tau)d\tau$, $\theta(s) = \int_{t_0}^{s} l(\tau)d\tau$, $t_0 \leq t \leq t_1$, and $V(t) = 0$, $t > t_1$, with t_1 given by

$$\int_{t_0}^{t_1} k(s) e^{-(1-\eta)\theta(s)} ds = \frac{1}{1-\eta} V^{1-\eta}(t_0). \tag{10}$$

Lemma 9. *Assume that a continuous and positive-definite function $V(t)$ satisfies the following condition:*

$$\dot{V}(t) \le -\left(c + cp(v)\right) V^{\eta}(t), \quad t \ge 0, \; V(0) \ge 0, \tag{11}$$

where $c > 0$, $p(v) > 0$ and $0 < \eta < 1$. Then, one has

$$V^{1-\eta}(t) \le V^{1-\eta}(0) - c(1-\eta)(1+p(v))t, \qquad 0 \le t \le t^*, \tag{12}$$

and $V(t) = 0$, $t > t^*$ with t^* given by

$$t^* = \frac{V^{1-\eta}(0)}{c(1-\eta)(1+p(v))}. \tag{13}$$

Proof. Let $l(t) = 0$, $k(t) = c + cp(v)$, and $t_0 = 0$; then combining Lemma 8, we can obtain Lemma 9. □

Lemma 10 (see [46]). *If $a_1, a_2, \ldots, a_n \ge 0$ and $0 < p \le 1$, then*

$$\left(\sum_{i=1}^{n} a_i\right)^{p} \le \sum_{i=1}^{n} a_i^{p}. \tag{14}$$

Lemma 11 (see [47]). *Assuming one positive definite matrix $Q > 0$, then*

$$2x^T y \le x^T Q^{-1} x + y^T Q y, \tag{15}$$

where $x, y \in R^n$.

3. Main Results

In this section, we, respectively, design two classes of finite-time synchronization control rules to realize global synchronization in finite time for the network (1). Furthermore, based on the obtained finite-time synchronization control rules, several finite-time synchronization dynamics criteria are established to show that nonlinear coupling function $g(\cdot)$ and the switching of outer-coupling matrix $A^{l,m}$ is how to impact finite-time synchronization dynamics of the network (1).

Letting (1) and (3), we have the following error system of the network (1):

$$\begin{aligned}\dot{e}_i(t) = {} & F(e_i(t)) + \sum_{j=1}^{N} c_1 a_{ij}^{1,m} \Gamma_1 G(e_j(t)) \\ & + \sum_{j=1}^{N} c_2 a_{ij}^{2,m} \Gamma_2 G(e_j(t)) + \cdots \\ & + \sum_{j=1}^{N} c_\eta a_{ij}^{\eta,m} \Gamma_\eta G(e_j(t)) + u_i(t), \\ & \qquad i = 1, 2, \ldots, N,\end{aligned} \tag{16}$$

where $e_i(t) = x_i(t) - s(t)$, $F(e_i(t)) = f(x_i(t)) - f(s(t))$, and $G(e_j(t)) = g(x_j(t))) - g(s(t)))$.

3.1. Based on Lemma 7, the Design of Finite-Time Synchronization Control Rule for the Network (1)

Theorem 12. *Under Assumptions 4 and 5, if there exists*

$$\begin{aligned}2I_N \otimes \Bigg[& HQ - \kappa I_n - \Xi \otimes H + \frac{1}{2}\sum_{l=1}^{\eta} c_l \Big(\nu^2 \|\varphi_{l,m}\| I_n \\ & + \left(A^{l,m} \otimes H\Gamma_l\right) \varphi_{l,m}^{-1} \left(A^{l,m} \otimes H\Gamma_l\right)^T \Big) \Bigg] \le 0,\end{aligned} \tag{17}$$

the network (1) can realize finite-time synchronization using the following controller

$$u_i(t) = -\varepsilon_i e_i(t) - cH^{(\beta-1)/2} \operatorname{sign}(e_i(t)) |e_i(t)|^{\beta}, \tag{18}$$

where $c > 0$ and $0 < \beta < 1$. Moreover, the settling time of synchronization t_{T1}^* satisfies

$$t_{T1}^* \le \frac{V^{1-\eta}(0)}{2c(1-\eta)}, \tag{19}$$

where $\eta = (1+\beta)/2$.

Proof. Consider the following Lyapunov-Krasovskii functional for the network (1) as

$$V_{T1}(e(t), t) = \sum_{i=1}^{N} e_i^T(t) H e_i(t). \tag{20}$$

Computing $V_{T1}^{+}(e(t), t)$ along the trajectory of error system (16), we can obtain

$$\begin{aligned}V_{T1}^{+}(e(t), t) &= 2\sum_{i=1}^{N} e_i^T(t) H \dot{e}_i(t) \\ &= 2\sum_{i=1}^{N} e_i^T(t) H \Bigg[F(e_i(t)) + \sum_{l=1}^{\eta}\sum_{j=1}^{N} c_l a_{ij}^{l,m} \Gamma_l G(e_j(t)) - \varepsilon_i e_i(t) - cH^{(\beta-1)/2} \operatorname{sign}(e_i(t)) \\ &\quad \cdot |e_i(t)|^{\beta} \Bigg].\end{aligned} \tag{21}$$

From Assumption 4, we can get

$$e_i^T(t) HF(e_i(t)) \le e_i^T(t) HQe_i(t) - \kappa e_i^T(t) e_i(t). \tag{22}$$

Applying Lemma 11 and Assumption 5, we have

$$\begin{aligned}2\sum_{i=1}^{N} e_i^T(t) H \sum_{l=1}^{\eta}\sum_{j=1}^{N} c_l a_{ij}^{l,m} \Gamma_l G(e_j(t)) &= 2\sum_{l=1}^{\eta} c_l e^T(t) \big(A^{l,m} \\ & \otimes H\Gamma_l\big) G(e(t)) \le \sum_{l=1}^{\eta} c_l \Big\{ G^T(e(t)) \varphi_{l,m} G(e(t)) \\ & + e^T(t) \left[\left(A^{l,m} \otimes H\Gamma_l\right) \varphi_{l,m}^{-1} \left(A^{l,m} \otimes H\Gamma_l\right)^T \right] e(t) \Big\} \\ & \le \sum_{l=1}^{\eta} c_l \Big\{ \nu^2 \|\varphi_{l,m}\| e^T(t) e(t) \\ & + e^T(t) \left[\left(A^{l} \otimes H\Gamma_l\right) \varphi_{l,m}^{-1} \left(A^{l} \otimes H\Gamma_l\right)^T \right] e(t) \Big\},\end{aligned} \tag{23}$$

where $e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$, $e_i(t) = (e_{i1}(t), e_{i2}(t), \dots, e_{in}(t))^T$, $G(e(t)) = (G^T(e_1(t)), G^T(e_2(t)), \dots, G^T(e_N(t)))^T$, and $G(e_i(t)) = (G(e_{i1}(t)), G(e_{i2}(t)), \dots, G(e_{in}(t)))^T$.

Substituting (22)-(23) into (21) and using the inequality (17) in Theorem 12, we have

$$\begin{aligned}
V_{T1}^{+}(e(t),t) &\le 2e^T(t)(I_N \otimes HQ)e(t) - 2\kappa e^T(t)e(t) \\
&+ \sum_{l=1}^{\eta} c_l \Big\{ \nu^2 \|\varphi_{l,m}\| \\
&\cdot e^T(t)e(t) + e^T(t)\Big[(A^{l,m} \otimes H\Gamma_l)\varphi_{l,m}^{-1}(A^{l,m} \\
&\otimes H\Gamma_l)^T \Big] e(t) \Big\} - 2e^T(t)(\Xi \otimes H)e(t) \\
&- 2c\sum_{i=1}^{N} e_i^T(t) H^{(\beta+1)/2}\operatorname{sign}(e_i(t))|e_i(t)|^{\beta} \\
&\le e^T(t) \\
&\cdot \Big\{ 2I_N \otimes \Big[HQ - \kappa I_n - \Xi \otimes H + \frac{1}{2}\sum_{l=1}^{\eta} c_l \Big(\nu^2 \|\varphi_{l,m}\| \\
&\cdot I_n + (A^{l,m} \otimes H\Gamma_l)\varphi_{l,m}^{-1}(A^{l,m} \otimes H\Gamma_l)^T \Big) \Big] \Big\} e(t) \\
&- 2c\sum_{i=1}^{N} e_i^T(t) H^{(\beta+1)/2}\operatorname{sign}(e_i(t))|e_i(t)|^{\beta} \\
&\le -2c\sum_{i=1}^{N} e_i^T(t) H^{(\beta+1)/2}\operatorname{sign}(e_i(t))|e_i(t)|^{\beta},
\end{aligned} \tag{24}$$

where $\Xi = \operatorname{diag}\{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N\}$.

By Lemma 10, we can obtain

$$\begin{aligned}
&-\sum_{i=1}^{N} e_i^T(t) H^{(\beta+1)/2}\operatorname{sign}(e_i(t))|e_i(t)|^{\beta} \\
&= -\sum_{i=1}^{N}\sum_{j=1}^{n} h_j^{(\beta+1)/2} |e_{ij}(t)|^{1+\beta} \\
&= -\sum_{i=1}^{N} \Big(\sum_{j=1}^{n} h_j e_{ij}^2(t) \Big)^{(1+\beta)/2} \\
&= -\sum_{i=1}^{N} \big(e_i^T(t) He_i(t) \big)^{(1+\beta)/2} \\
&\le -\Big(\sum_{i=1}^{N} e_i^T(t) He_i(t) \Big)^{(1+\beta)/2}.
\end{aligned} \tag{25}$$

Combining (24) and (25), we have

$$V_{T1}^{+}(e(t),t) \le -2c(V(e(t),t))^{(1+\beta)/2}. \tag{26}$$

By Lemma 7, $V(e(t),t)$ converges to zero in finite-time t_{T1}^* and t_{T1}^* is gotten by

$$t_{T1}^* \le t^*, \tag{27}$$

where $t^* = V^{1-\eta}(0)/2c(1-\eta)$, $\eta = (1+\beta)/2$. That means the error vector $e_i(t) \in R^n (i = 1, 2, \dots, N)$ will converge to zero within t_{T1}^*. Thus, we can obtain $e_i(t) = 0$ if $t \ge t_{T1}^*$. According to Definition 3, we have $\|x_i(t) - s(t)\| = 0$ if $t \ge t_{T1}^*$. Hence, global synchronization of the network (1) will be achieved in finite-time t_{T1}^*. This completes the proof. □

3.2. Based on Lemma 9, the Design of Finite-Time Synchronization Control Rule for the Network (1)

Theorem 13. *Under Assumptions 4 and 5, if there exists*

$$\begin{aligned}
&2I_N \otimes \Big[HQ - \kappa I_n - \Xi \otimes H + \frac{1}{2}\sum_{l=1}^{\eta} c_l \Big(\nu^2 \|\varphi_{l,m}\| I_n \\
&+ (A^{l,m} \otimes H\Gamma_l)\varphi_{l,m}^{-1}(A^{l,m} \otimes H\Gamma_l)^T \Big) \Big] \le 0,
\end{aligned} \tag{28}$$

the network (1) can realize finite-time synchronization using the following controller:

$$\begin{aligned}
u_i(t) &= -\varepsilon_i e_i(t) \\
&- c(1 + p(\nu)) H^{(\beta-1)/2}\operatorname{sign}(e_i(t))|e_i(t)|^{\beta},
\end{aligned} \tag{29}$$

where $c > 0$, $p(\nu) > 0$, and $0 < \beta < 1$. Moreover, the settling time of synchronization t_{T2}^* satisfies

$$t_{T2}^* \le \frac{V^{1-\eta}(0)}{2c(1-\eta)(1+p(\nu))}, \tag{30}$$

where $\eta = (1+\beta)/2$.

Proof. Consider the Lyapunov-Krasovskii functional for the network (1) as

$$V_{T2}(e(t),t) = \sum_{i=1}^{N} e_i^T(t) He_i(t). \tag{31}$$

□

The next proof is similar to that of Theorem 12.

Remark 14. According to Theorems 12 and 13, although the controllers (18) and (29) can be, respectively, designed to realize finite-time synchronization of the network (1), it is not difficult to find that the synchronization finite-time estimation approach in Theorem 13 is more practical than that of Theorem 12. This can be testified by t_{T1}^* and t_{T2}^*. According to inequalities (19) and (30), it is seen that t_{T2}^* is closely related to $p(\nu)$; otherwise, t_{T1}^* is not. By using Assumption 5, we can obtain that there is $\|g(x(t))\|/\|x(t)\| \le \nu$. This shows that nonlinearity of $g(x(t))$ can be reflected by ν. For example, assuming that there are $g^{(1)}(x(t))$ and $g^{(2)}(x(t))$ and nonlinearity of $g^{(1)}(x(t))$ is more serious than that of $g^{(2)}(x(t))$, then combining Assumption 5, $\|g^{(2)}(x(t))\|/\|x(t)\| \le \nu^{(2)} < \|g^{(1)}(x(t))\|/\|x(t)\| \le \nu^{(1)}$ must exist, where $\nu^{(1)} > \nu^{(2)} > 0$. Thus, more feasible $p(\nu)$ can be chosen by using ν. In order to further investigate the effectiveness of t^*_{T1} and t^*_{T2}, the following several interesting and useful Corollaries are derived.

Remark 15. It can be seen from Theorems 12 and 13 that nonlinear coupling function $g(\cdot)$ of the network (1) is linearized by the Lipschitz condition in Assumption 5. Actually, from the process of proving Theorems 12 and 13, it is observed that some nonlinearity bound conditions such as the sector-bound nonlinearity condition and the QUAD condition can replace Assumption 5 to process nonlinear coupling function $g(\cdot)$ of the network (1). It should be pointed out that the sector-bound nonlinearity condition and the QUAD condition [7, 10, 43, 44] are more general that the Lipschitz condition. Therefore, based on the above two techniques, the derived results have lower conservatism than Theorems 12 and 13 built on the Lipschitz condition. How do we get the related results? This is one of interesting topics in the future.

Remark 16. In fact, it can be further obtained that the conservatism of the proposed method based on Theorems 12 and 13 is closely related to Assumption 4 and Lemmas 7 and 9, the considered Lyapunov-Krasovskii functional, and the controllers (18) and (29). For example, assume the Lyapunov-Krasovskii functional for the network (1) $V_{T1}(e(t),t) = \sum_{i=1}^{N} e_i^T(t)e_i(t)$. Thus, in Assumption 4 and Theorems 12 and 13, let $H = I$; then sufficient conditions for finite-time synchronization of the network (1) can be obtained. It is clear that the new derived results have higher conservatism than Theorems 12 and 13. From the above, it is seen that, in order to get the results, besides choosing a new $V_{T1}(e(t),t)$, in Assumption 4, H is replaced by I. Although Assumption 4 with $H = I$ still holds, its conservatism becomes high. Furthermore, comparing Theorems 12 and 13, it can also be found that $V^+_{T1}(e(t),t) \le -2c(V(e(t),t))^{(1+\beta)/2}$ and $V^+_{T2}(e(t),t) \le -2c\widetilde{p}(\nu)(V(e(t),t))^{(1+\beta)/2}$, where $\widetilde{p}(\nu) = 1 + p(\nu)$, $p(\nu) > 0$, $c > 0$, and $0 < \beta < 1$. Because of $V^+_{T2}(e(t),t) < V^+_{T1}(e(t),t) \le -2c(V(e(t),t))^{(1+\beta)/2}$, Theorem 12 must hold if Theorem 13 holds. Otherwise, the conclusion does not hold. That means the conservatism of Theorem 12 is higher than that of Theorem 13. This is caused by the finite-time control scheme of controllers (18) and (29) built on Lemmas 7 and 9, respectively. This validates that more feasible control approach can also reduce the conservatism of the derived results. How to further explore more effective finite-time control method is another valuable problem.

Remark 17. Notice that if the network (1) is large-scaled, the dimension of LMIs (17) and (28) in Theorems 12 and 13 becomes high. This causes that it might not be available to easily realize LMIs (17) and (28) in practice. How do we establish low-dimensional LMIs conditions? Because of $A^{l,m} \otimes H\Gamma_l = (A^{l,m} \otimes H\Gamma_l)^T$, according to LMIs (17) and (28), it is derived that $\Psi^m = \lambda_{\max}(HQ) - \kappa - \lambda_{\min}(\Xi \otimes H) + (1/2)\sum_{l=1}^{\eta} c_l(\nu^2\|\varphi_{l,m}\| + \lambda^{l,m}_{\max}(\varphi^{-1}_{l,m})(\lambda^{l,m}_{\max}(\mathbb{A}^{l,m}))^2$, where $\mathbb{A}^{l,m} = A^{l,m} \otimes H\Gamma_l$. Letting $\Psi^m \le 0$ hold, then it is clear that $\Psi^m \le 0$ is larger than LMIs (17) and (28). Thus, low-dimensional condition in Theorems 12 and 13 is obtained. It needs to be emphasized that although the condition $\Psi^m \le 0$ is more practical than the conditions (17) and (28), its conservatism is higher than LMIs (17) and (28). How do we balance its conservatism and feasibility? This is a more attractive and open question.

3.3. Based on Theorems 12 and 13, the Impact Analysis of Nonlinear Coupling Function $g(\cdot)$ and the Switching of Outer-Coupling Matrix $A^{l,m}$ on Finite-Time Synchronization Dynamics of the Network (1)

Corollary 18. *Under Theorem 12 and the controller (18), if $A^{l,m} > 0$, $e(t) > 0$, and $G(e(t)) > 0$, or if $A^{l,m} > 0$, $e(t) < 0$, and $G(e(t)) < 0$, or if $A^{l,m} < 0$, $e(t) > 0$, and $G(e(t)) < 0$, or if $A^{l,m} < 0$, $e(t) < 0$, and $G(e(t)) > 0$, where $e(t) = x(t) - s(t)$ and $G(e(t)) = g(x(t)) - g(s(t))$, with increasing nonlinearity of nonlinear coupling function $g(x(t))$, global synchronization dynamics of the network (1) within finite-time t^*_{C1} will develop poorer.*

Proof. From Theorem 12, there are

$$V_{C1}(e(t),t) = \sum_{i=1}^{N} e_i^T(t) He_i(t), \tag{32}$$

$$\begin{aligned} 2\sum_{i=1}^{N} e_i^T(t) H \sum_{l=1}^{\eta}\sum_{j=1}^{N} c_l a_{ij}^{l,m} \Gamma_l G(e_j(t)) \\ = 2\sum_{l=1}^{\eta} c_l e^T(t)\left(A^{l,m} \otimes H\Gamma_l\right) G(e(t)). \end{aligned} \tag{33}$$

Similar to the proof of Theorem 12, we have

$$\begin{aligned} V^+_{C1}(e(t),t) &\le 2e^T(t)\left(I_N \otimes HQ\right)e(t) - 2\kappa e^T(t)e(t) \\ &+ 2\sum_{l=1}^{\eta} c_l e^T(t)\left(A^{l,m} \otimes H\Gamma_l\right) G(e(t)) \\ &- 2e^T(t)(\Xi \otimes H)e(t) - 2c(V(e(t),t))^{(1+\beta)/2}. \end{aligned} \tag{34}$$

Combining inequalities (23), (24), and (34), there is $V^+_{C1}(e(t),t) \le V^+_{T1}(e(t),t) \le -2c(V(e(t),t))^{(1+\beta)/2} \le 0$ under Theorem 12.

According to equality (33), it is derived that if $A^{l,m} > 0$, $e(t) > 0$, and $G(e(t)) > 0$, or if $A^{l,m} > 0$, $e(t) < 0$, and $G(e(t)) < 0$, or if $A^{l,m} < 0$, $e(t) > 0$, and $G(e(t)) < 0$, or if $A^{l,m} < 0$, $e(t) < 0$, and $G(e(t)) > 0$, we have

$$2\sum_{l=1}^{\eta} c_l e^T(t)\left(A^{l,m} \otimes H\Gamma_l\right) G(e(t)) > 0. \tag{35}$$

Next, we prove the relationship between nonlinearity of nonlinear coupling function $g(x(t))$ and the equality (35). Letting $x(t)$ be fixed, then if nonlinearity of nonlinear coupling function $g(x(t))$ is more serious, $\|g(x(t))\|$ must become larger. This leads to $\|G(e(t))\|$ increasing. Therefore, with increasing nonlinearity of $g(x(t))$, $G(e(t)) > 0$ and $G(e(t)) < 0$ will increase and decrease, respectively. This causes that, under $A^{l,m} > 0$, $e(t) > 0$, and $G(e(t)) > 0$, or $A^{l,m} > 0$, $e(t) < 0$, and $G(e(t)) < 0$, or $A^{l,m} < 0$, $e(t) > 0$, and $G(e(t)) < 0$, or $A^{l,m} < 0$, $e(t) < 0$, and $G(e(t)) > 0$, the above equality (35) must be larger if nonlinearity of $g(x(t))$ develops more serious. This makes $V^{+}_{C1}(e(t), t) \le 0$ increase. This completes the proof. □

Similar to the proof of Corollary 18, we can obtain Corollaries 19–21.

Corollary 19. *Under Theorem 12 and the controller (18), if* $A^{l,m} > 0$, $e(t) < 0$, *and* $G(e(t)) > 0$, *or if* $A^{l,m} > 0$, $e(t) > 0$, *and* $G(e(t)) < 0$, *or if* $A^{l,m} < 0$, $e(t) > 0$, *and* $G(e(t)) > 0$, *or if* $A^{l,m} < 0$, $e(t) < 0$, *and* $G(e(t)) < 0$, *where* $e(t) = x(t) - s(t)$ *and* $G(e(t)) = g(x(t)) - g(s(t))$, *with increasing nonlinearity of nonlinear coupling function* $g(x(t))$, *global synchronization dynamics of the network (1) within finite-time* t^{*}_{C2} *will become better.*

Corollary 20. *Suppose that function* $p(\nu)$ *is decreasing function and* $p(\nu) > 0$. *Then, under Theorem 13 and the controller (29), if* $A^{l,m} > 0$, $e(t) > 0$, *and* $G(e(t)) > 0$, *or if* $A^{l,m} > 0$, $e(t) < 0$, *and* $G(e(t)) < 0$, *or if* $A^{l,m} < 0$, $e(t) > 0$, *and* $G(e(t)) < 0$, *or if* $A^{l,m} < 0$, $e(t) < 0$, *and* $G(e(t)) > 0$, *where* $e(t) = x(t) - s(t)$ *and* $G(e(t)) = g(x(t)) - g(s(t))$, *with increasing nonlinearity of nonlinear coupling function* $g(x(t))$, *global synchronization dynamics of the network (1) within finite-time* t^{*}_{C3} *will be poorer and synchronization convergence time* t^{*}_{C3} *of the network (1) becomes larger.*

Corollary 21. *Suppose that function* $p(\nu)$ *is increasing function and* $p(\nu) > 0$. *Then, under Theorem 13 and the controller (29), if* $A^{l,m} < 0$, $e(t) > 0$, *and* $G(e(t)) > 0$, *or if* $A^{l,m} < 0$, $e(t) < 0$, *and* $G(e(t)) < 0$, *or if* $A^{l,m} > 0$, $e(t) > 0$, *and* $G(e(t)) < 0$, *or if* $A^{l,m} > 0$, $e(t) < 0$, *and* $G(e(t)) > 0$, *where* $e(t) = x(t) - s(t)$ *and* $G(e(t)) = g(x(t)) - g(s(t))$, *with increasing nonlinearity of nonlinear coupling function* $g(x(t))$, *global synchronization dynamics of the network (1) within finite-time* t^{*}_{C4} *will become better and synchronization convergence time* t^{*}_{C4} *of the network (1) will be smaller.*

Remark 22. In Corollaries 20 and 21, it is seen that function $p(\nu) > 0$ is decreasing function and increasing function, respectively. Why? Similar to the proof of Corollary 18, it is obtained that, in Corollary 21, if $e(t) \neq 0$, there is $\sum_{l=1}^{\eta} c_l e^{T}(t)\mathbb{A}^{l,m} * G(e(t)) < 0$, where $\mathbb{A}^{l,m} = A^{l,m} \otimes H\Gamma_l$. Moreover, with increasing nonlinearity of nonlinear coupling function $g(\cdot)$, $\sum_{l=1}^{\eta} c_l e^{T}(t)\mathbb{A}^{l,m}G(e(t)) < 0$ becomes smaller. This causes $V^{+}_{C4}(e(t), t) \le 0$ to decrease. Thus, synchronization time t^{*}_{C4} will be smaller. Therefore, in inequality (30) of Theorem 13, let $p(\nu) > 0$ be increasing function and then it can be realized. In Corollary 20, by using the similar analysis, it is needed to make $p(\nu) > 0$ be decreasing function.

Remark 23. By Theorem 13 and Corollaries 20 and 21, it is obtained that the proposed finite-time computing approach can not only estimate synchronization time of the network (1) but also reflect that nonlinear coupling function $g(\cdot)$, coupling matrix $A^{l,m}$, the initial conditions, and synchronization states are how to impact synchronization dynamics of the network (1). This reveals the relationship between the multi-weighted coupling term and the finite-time synchronization dynamics in the network (1). Recently, although some wonderful works about finite-time synchronization problems of nonlinear coupling systems such as stochastic chaotic neural networks [2], Lur'e networks [41], and so on [27] have been developed, synchronization time of the addressed nonlinear coupling systems can be estimated by the derived finite-time approaches. Unfortunately, it is pity that the obtained finite-time results cannot reflect the effect of coupling term on finite-time synchronization dynamics of the considered coupling systems. This testifies that compared with the existing results [2, 27, 41], the main advantage of the proposed finite-time approach is more feasible.

Corollary 24. *Under Corollaries 18 and 20, global synchronization dynamics of the network (1) within finite time* t^{*}_{T1} *is poorer than that of the network (1) within finite time* t^{*}_{T2} *and synchronization convergence time* t^{*}_{T1} *of the network (1) is larger than synchronization convergence time* t^{*}_{T2} *of the network (1).*

Proof. From the proof of Corollaries 18 and 20, there must be $V^{+}_{C3}(e(t), t) < V^{+}_{C1}(e(t), t) \le 0$. This shows that, under Corollaries 18 and 20, finite-time synchronization dynamics of the network (1) with the controller (29) is better than that of the network (1) with the controller (18). Thus, there must be $t^{*}_{T1} > t^{*}_{T2} > 0$. Because of $t^{*}_{T1} \le V^{1-\eta}(0)/2c(1-\eta)$ and $t^{*}_{T2} \le V^{1-\eta}(0)/2c(1-\eta)(1+p(\nu))$, where $c > 0$ and $p(\nu) > 0$, we can make $t^{*}_{T1} > t^{*}_{T2} > 0$ hold. The proof is completed. □

Corollary 25. *Under Corollaries 19 and 21, global synchronization dynamics of the network (1) within finite-time* t^{*}_{T1} *is poorer than that of the network (1) within finite-time* t^{*}_{T2} *and synchronization convergence time* t^{*}_{T1} *of the network (1) is larger than synchronization convergence time* t^{*}_{T2} *of the network (1).*

Proof. Similar to the proof of Corollary 24, we can derive Corollary 25. □

Remark 26. By Corollaries 18–21, it is difficult to obtain nonlinearity of nonlinear coupling function being how to impact global synchronization in finite time for the network (1). The reason is that the impact of nonlinear coupling function on finite time synchronization dynamics of the network (1) is not only related to $G(e(t)) > 0$ but also closely connected with $A^{l,m} > 0$, $e(t) < 0$, and $G(e(t)) > 0$, where $e(t) = x(t) - s(t)$ and $G(e(t)) = g(x(t))) - g(s(t)))$. All these show that the impact of nonlinear coupling function on finite time synchronization dynamics of the network (1) is decided by the initial state $x(0)$, nonlinear coupling function $g(x(t))$, synchronization state $s(t)$, and coupling matrix $A^{l,m}$. Furthermore, Corollaries 24 and 25 further testify that synchronization time estimation scheme of Theorem 13 is more feasible and reasonable than that of Theorem 12. Therefore, for nonlinear coupling systems, how to design more scientific and practical controller is very meaningful and valuable.

Similar to the proof of Corollary 18 and letting $s_1 \in M$, $(s_1 + 1) \in M$, then we can also obtain the switching of $A^{l,m}$ being how to affect finite-time synchronization dynamics of the network (1).

Corollary 27. *Under Theorem 13 and the controller (29), if $e(t) > 0$ and $G(e(t)) > 0$, or if $e(t) < 0$ and $G(e(t)) < 0$, where $e(t) = x(t) - s(t)$ and $G(e(t)) = g(x(t)) - g(s(t))$, finite-time synchronization dynamics of the network (1) based on $A^{l,s_1} > 0$ and $A^{l,s_1+1} > 0$ is poorer than that of the network (1) built on $A^{l,s_1} > 0$ and $A^{l,s_1+1} < 0$ or $A^{l,s_1} < 0$ and $A^{l,s_1+1} > 0$.*

Corollary 28. *Under Theorem 13 and the controller (29), if $e(t) > 0$ and $G(e(t)) < 0$, or if $e(t) < 0$ and $G(e(t)) > 0$, where $e(t) = x(t) - s(t)$ and $G(e(t)) = g(x(t)) - g(s(t))$, finite-time synchronization dynamics of the network (1) based on $A^{l,s_1} < 0$ and $A^{l,s_1+1} < 0$ is poorer than that of the network (1) built on $A^{l,s_1} > 0$ and $A^{l,s_1+1} < 0$ or $A^{l,s_1} < 0$ and $A^{l,s_1+1} > 0$.*

Corollary 29. *Under Theorem 13 and the controller (29), if $e(t) > 0$ and $G(e(t)) > 0$, or if $e(t) < 0$ and $G(e(t)) < 0$, where $e(t) = x(t) - s(t)$ and $G(e(t)) = g(x(t)) - g(s(t))$, finite-time synchronization dynamics of the network (1) based on $A^{l,s_1} < 0$ and $A^{l,s_1+1} < 0$ is better than that of the network (1) built on $A^{l,s_1} > 0$ and $A^{l,s_1+1} < 0$ or $A^{l,s_1} < 0$ and $A^{l,s_1+1} > 0$.*

Corollary 30. *Under Theorem 13 and the controller (29), if $e(t) > 0$ and $G(e(t)) < 0$, or if $e(t) < 0$ and $G(e(t)) > 0$, where $e(t) = x(t) - s(t)$ and $G(e(t)) = g(x(t)) - g(s(t))$, finite-time synchronization dynamics of the network (1) based on $A^{l,s_1} > 0$ and $A^{l,s_1+1} > 0$ is better than that of the network (1) built on $A^{l,s_1} > 0$ and $A^{l,s_1+1} < 0$ or $A^{l,s_1} < 0$ and $A^{l,s_1+1} > 0$.*

Remark 31. Note that there is no function relationship between t^*_{T2} and switching of $A^{l,m}$. Therefore, the weakness of the application of the proposed method is that the impact of the switching on finite-time synchronization dynamics of the network (1) cannot be reflected by t^*_{T2}. This shows that synchronization finite time estimation approach located in Theorem 13 still exists in some conservatism. In the future, it would be very interesting to further investigate the issue. Besides this, from Corollaries 18–28, there are $A^{l,m} > 0$ or $A^{l,m} < 0$. Therefore, one has $\lambda(A^{l,m}) > 0$ or $\lambda(A^{l,m}) < 0$. Letting $A^{l,m} = [-2, 1, 1; 1, -2, 1; 1, 1, -2]$, then one gets that $\lambda(A^{l,m})$ is -3, -3, and 0, respectively. This shows that if $a^{l,m}_{ii} = -\sum_{j=1,j\neq i}^{N} a^{l,m}_{ij}$, $A^{l,m} > 0$ and $A^{l,m} < 0$ may not hold. Therefore, if coupling matrix $A^{l,m}$ satisfies diffusive coupled condition, under Theorems 12 and 13, it is difficult to obtain the impact of nonlinear coupling function $g(\cdot)$ on finite-time synchronization dynamics of the network (1) from theory aspect.

Remark 32. Compared with the nonfinite-time control, finite-time control can improve robust performance and antidisturbance performance of systems [2]. Therefore, recently, besides finite-time synchronization control of complex networks, finite-time scaled consensus control of multi-agent systems has been paid close attention [48, 49]. In [48, 49], based on linear iterations and graph theory, Shang investigated finite-time scaled consensus control about discrete-time multi-agent system. It is seen that [48, 49] and this paper consider finite-time control problems about coupling systems. Moreover, it can also be found that [48, 49] addressed scaled consensus of linear coupling discrete-time multi-agent system within finite steps and the derived finite time in [48, 49] is a positive integer. In this paper, synchronization of NCMWCNs within finite time t^* is proposed and the obtained finite time t^* is a real number and greater than zero.

Remark 33. It is worth noting that because finite time control is mainly dependent on the initial conditions and fixed-time control does not rely on the initial conditions [50, 51]; in a few years recently, fixed-time control problems such as fixed-time synchronization [52], fixed-time group consensus [50], and fixed-time group tracking [51] began to be widely studied. For example, in [50, 51], according to graph theory and Lyapunov stability theory, fixed-time group consensus and fixed-time group tracking for multi-agent systems were investigated, respectively. Compared with coupling function $g(\cdot)$ of the network (1), nonlinear function $f(\cdot)$ of the considered multi-agent systems in [50, 51] is more general. Besides this, in [50, 51], multi-agent systems were coupled by linear coupling ways. If there is nonlinear coupling relationship in each agent of multi-agent systems, how to further explore fix-time consensus is a challenging and attractive question. Furthermore, from this paper and [50–52], it is seen that the similarity and difference between finite-time synchronization and fixed-time synchronization are as follows: (I) the similarity is how to get synchronization time for the addressed systems and (II) the difference is that finite time synchronization is closely related to the initial conditions; otherwise, fixed-time synchronization is not.

Remark 34. In the last few years, some valuable and meaningful results about synchronization dynamics problems of linear coupling complex or nonlinear coupling complex networks have been obtained [1–25, 29–34, 37–40]. However, these existing works mainly concentrated on how to derive sufficient conditions for synchronization problems of the considered complex networks. In this paper, the research about sufficient conditions of finite-time synchronization and synchronization dynamics criteria on the impact of nonlinear coupling function and outer-coupling matrix switching for the addressed complex networks is explored. All these show that our derived results enrich and complement the earlier works.

4. Numerical Examples

In order to test the effectiveness of the derived results, two examples are given. Define the synchronization total error being $e(t) = \sum_{i=1}^{3}\sum_{j=1}^{2} e_{ij}(t)$, and assume $N = 3$, $c_1 = c_2 = 1$, $m = 1, 2$, and $l = 1, 2$; then the network (1) with the controller (29) is as follows:

$$\begin{aligned}\dot{x}_i(t) &= f(x_i(t)) + \sum_{j=1}^{3} a_{ij}^{1,m}\Gamma_1 g(x_j(t)) \\ &\quad + \sum_{j=1}^{3} a_{ij}^{2,m}\Gamma_2 g(x_j(t)) - \varepsilon_i e_i(t) \\ &\quad - c(1+p(\nu)) H^{(\beta-1)/2}\operatorname{sign}(e_i(t))|e_i(t)|^{\beta},\end{aligned} \quad (36)$$

where

$$f(x_i(t)) = \begin{bmatrix} -3\tanh(3x_{i1}(t)) \\ -3\tanh(3x_{i2}(t)) \end{bmatrix}, \qquad \Gamma_1 = \Gamma_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (37)$$

Example 1. According to $A^{l,m}$, $e(t)$, and $G(e(t))$ of Corollaries 20 and 21, it is observed that there exist four cases in Corollaries 20 and 21. Because the simulation process of the other cases including cases III and IV of Corollaries 20 and 21, cases I-IV of Corollaries 18 and 19 are similar to that of cases I and II in Corollaries 20 and 21; this example gives the simulation results of cases I and II in Corollaries 20 and 21.

Case I of Corollary 20. $A^{l,m} > 0$, $e(t) > 0$, and $G(e(t)) > 0$, where $e(t) = x(t) - s(t)$ and $G(e(t)) = g(x(t)) - g(s(t))$. Let $x_1(0) = (1, 1.5)^T$, $x_2(0) = (2, 2.5)^T$, $x_3(0) = (3, 3.5)^T$, and

$$A_{C3-CI}^{l,m} = \begin{bmatrix} 1.4 & 0.3 & 0.6 \\ 0.3 & 1.5 & 0.5 \\ 0.6 & 0.5 & 1.6 \end{bmatrix}, \qquad g_{(C3-CI)}^{(1)}(x_i(t)) = \begin{bmatrix} \tanh(x_{i1}(t)) \\ \tanh(x_{i2}(t)) \end{bmatrix}. \quad (38)$$

From Assumptions 4–5, we have $\kappa = 1$, $H = I_2$, $Q = 0$, $\nu^{(1)} = 1.2$, and $\varphi_{l,m} = I_6$. Choosing $\Xi = 6I_3$, it is easily testified that inequality (28) in Theorem 13 holds. Thus, we have $t_{(C3-CI)}^{*(1)} < 0.81$, where $\beta = 0.4$, $c = 1$, $p^{(1)}(\nu) = 12$, and $V(0) = 34.75$. Next, making $g_{(C3-CI)}^{(2)}(x_i(t)) = [0.7\tanh(0.7x_{i1}(t)), 0.7\tanh(0.7x_{i2}(t))]^T$ and $g_{(C3-CI)}^{(3)}(x_i(t)) = [0.3\tanh(0.3x_{i1}(t)), 0.3\tanh(0.3x_{i2}(t))]^T$, we derive that inequality (28) in Theorem 13 still holds, where $\Xi = 6I_3$, $\nu^{(2)} = 0.9$, and $\nu^{(3)} = 0.5$. Taking $p^{(2)}(\nu) = 12.3$, $p^{(3)}(\nu) = 12.5$, and $c = 1$, we get $t_{(C3-CI)}^{*(2)} < 0.79$ and $t_{(C3-CI)}^{*(3)} < 0.78$.

Case II of Corollary 20. $A^{l,m} > 0$, $e(t) < 0$, and $G(e(t)) < 0$. Let $A_{C3-CII}^{l,m} = A_{C3-CI}^{l,m}$, $g_{(C3-CII)}^{(\varrho)}(x_i(t)) = g_{(C3-CI)}^{(\varrho)}(x_i(t))$, $x_1(0) = (-1, -1.5)^T$, $x_2(0) = (-2, -2.5)^T$, and $x_3(0) = (-3, -3.5)^T$, $\varrho = 1, 2, 3$. From the above case I and $g_{(C3-CII)}^{(\varrho)}(x_i(t))$, there are $\Xi = 6I_3$, $\kappa = 1$, $H = I_2$, $Q = 0$, $\nu^{(1)} = 1.2$, $\nu^{(2)} = 0.9$, and $\nu^{(3)} = 0.5$. Choosing $p^{(1)}(\nu) = 12$, $p^{(2)}(\nu) = 12.3$, $p^{(3)}(\nu) = 12.5$, and $c = 1$, we get $t_{(C3-CII)}^{*(1)} < 0.81$, $t_{(C3-CII)}^{*(2)} < 0.79$, and $t_{(C3-CII)}^{*(3)} < 0.78$.

Case I of Corollary 21. $A^{l,m} < 0$, $e(t) > 0$, and $G(e(t)) > 0$. Taking $A_{C4-CI}^{l,m} = -A_{C3-CI}^{l,m}$, $g_{(C4-CI)}^{(\varrho)}(x_i(t)) = g_{(C3-CI)}^{(\varrho)}(x_i(t))$, $x_1(0) = (1, 1.5)^T$, $x_2(0) = (2, 2.5)^T$ and $x_3(0) = (3, 3.5)^T$, $\Xi = 6I_3$, $\kappa = 1$, $H = I_2$, $Q = 0$, $\nu^{(1)} = 1.2$, $\nu^{(2)} = 0.9$, $\nu^{(3)} = 0.5$, $p^{(1)}(\nu) = 12.5$, $p^{(2)}(\nu) = 12.3$, $p^{(3)}(\nu) = 12$, and $c = 1$, we obtain $t_{(C4-CI)}^{*(1)} < 0.78$, $t_{(C4-CI)}^{*(2)} < 0.79$, and $t_{(C4-CI)}^{*(3)} < 0.81$.

Case II of Corollary 21. $A^{l,m} < 0$, $e(t) < 0$, and $G(e(t)) < 0$. We choose $A_{C4-CI}^{l,m} = -A_{C3-CI}^{l,m}$, $g_{(C4-CI)}^{(\varrho)}(x_i(t)) = g_{(C3-CI)}^{(\varrho)}(x_i(t))$, $x_1(0) = (-1, -1.5)^T$, $x_2(0) = (-2, -2.5)^T$, and $x_3(0) = (-3, -3.5)^T$. Similar to case I, it is obtained that $t_{(C4-CII)}^{*(1)} < 0.78$, $t_{(C4-CII)}^{*(2)} < 0.79$, $t_{(C4-CII)}^{*(3)} < 0.81$, $\Xi = 6I_3$, $\kappa = 1$, $H = I_2$, $Q = 0$, $\nu^{(1)} = 1.2$, $nu^{(2)} = 0.9$, $\nu^{(3)} = 0.5$, $p^{(1)}(\nu) = 12.5$, $p^{(2)}(\nu) = 12.3$, $p^{(3)}(\nu) = 12$, and $c = 1$.

Example 2. From Corollaries 27–30, it is seen that the other derived results in Corollaries 27–30 are similar to that of cases I-1 and I-2 in Corollaries 27 and 29, respectively. Therefore, in this example, we only give the simulation results of cases I-1 and I-2 Corollaries 27 and 29. According to Example 1 and Theorem 13, there are $g(x_i(t)) = [\tanh(x_{i1}(t)), \tanh(x_{i2}(t))]^T$, $\kappa = 1$, $H = I_2$, $Q = 0$, $\nu^{(1)} = 1.2$, and $\varphi_{l,m} = I_6$.

Case I-1 of Corollary 27. $e(t) > 0$, $G(e(t)) > 0$, $A^{l,s_1} > 0$, and $A^{l,s_1+1} > 0$. Letting $x_1(0) = (1, 1.5)^T$, $x_2(0) = (2, 2.5)^T$, $x_3(0) = (3, 3.5)^T$, and

$$A_{C7-CI-1}^{l,1} = A_{C7-CI-1}^{l,2} = \begin{bmatrix} 1.4 & 0.3 & 0.6 \\ 0.3 & 1.5 & 0.5 \\ 0.6 & 0.5 & 1.6 \end{bmatrix}, \quad (39)$$

then we have $\Xi = 6I_3$, $\beta = 0.4$, $c = 1$, $p(\nu) = 12$, $V(0) = 34.75$, and $t_{C7-CI-1}^{*} < 0.81$.

Case I-2 of Corollary 27. $e(t) > 0$, $G(e(t)) > 0$, $A^{l,s_1} > 0$, and $A^{l,s_1+1} < 0$. Making $x_1(0) = (1, 1.5)^T$, $x_2(0) = (2, 2.5)^T$, $x_3(0) = (3, 3.5)^T$, $A_{C7-CI-2}^{l,1} = A_{C7-CI-1}^{l,1}$, and $A_{C7-CI-2}^{l,2} = -A_{C7-CI-1}^{l,1}$, we get $\Xi = 6I_3$ and $t_{C7-CI-2}^{*} < 0.81$, where $\beta = 0.4$, $c = 1$, $p(\nu) = 12$, and $V(0) = 34.75$.

Case I-1 of Corollary 29. $e(t) > 0$, $G(e(t)) > 0$, $A^{l,s_1} < 0$, and $A^{l,s_1+1} < 0$. Choosing $A_{C7-CI-2}^{l,m} = -1.1A_{C7-CI-1}^{l,1}$, $x_1(0) = (1, 1.5)^T$, $x_2(0) = (2, 2.5)^T$, and $x_3(0) = (3, 3.5)^T$, there are $\Xi = 6I_3$ and $t_{C9-CI-1}^{*} < 0.81$, where $\beta = 0.4$, $c = 1$, $p(\nu) = 12$, and $V(0) = 34.75$.

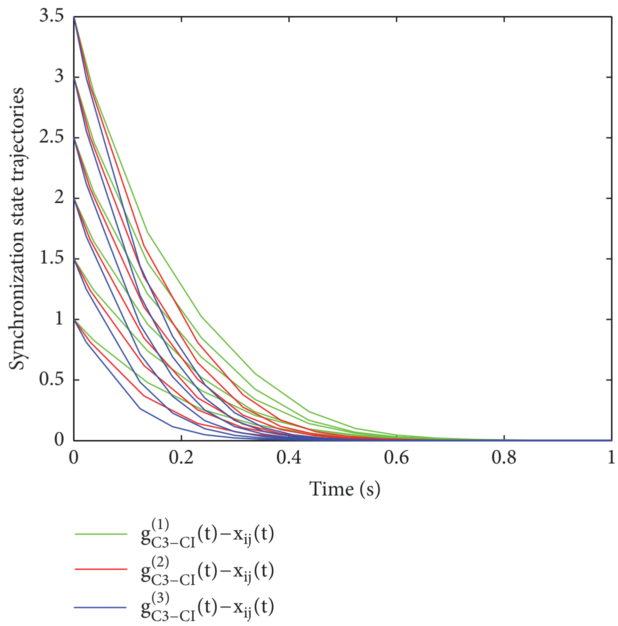

Figure 1: Synchronization state trajectories of the network (36) for the case I of Corollary 20.

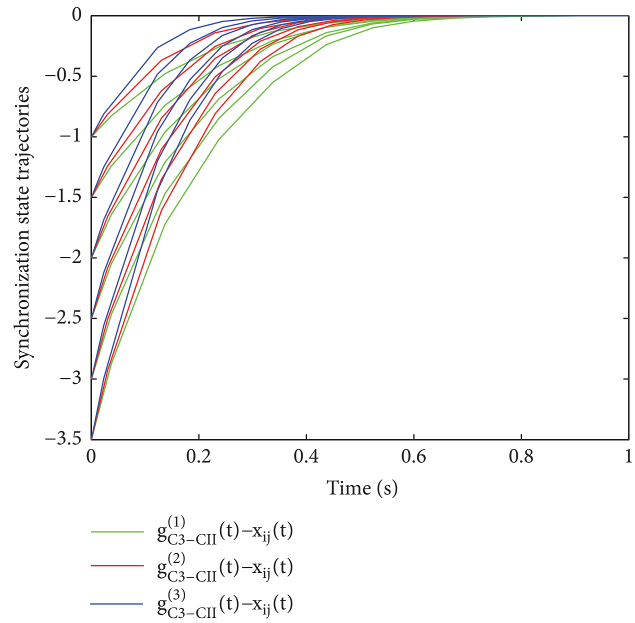

Figure 3: Synchronization state trajectories of the network (36) for the case II of Corollary 20.

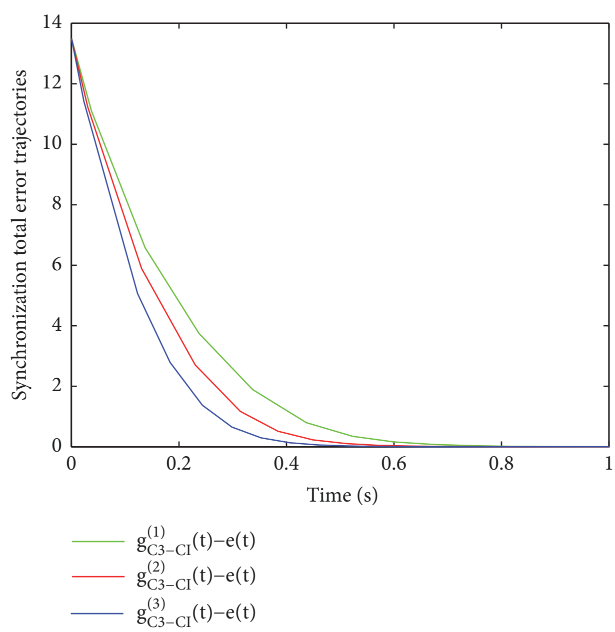

Figure 2: Synchronization total error trajectories of the network (36) for the case I of Corollary 20.

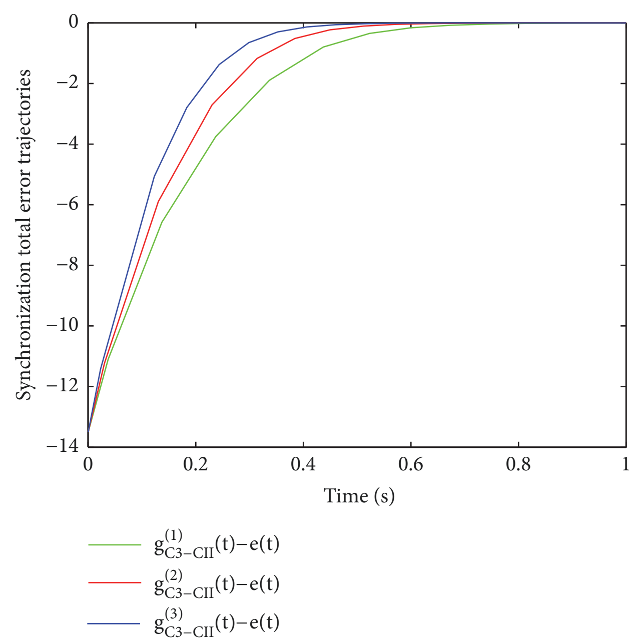

Figure 4: Synchronization total error trajectories of the network (36) for the case II of Corollary 20.

Case I-2 of Corollary 29. $e(t) > 0$, $G(e(t)) > 0$, $A^{l,s_1} > 0$, and $A^{l,s_1+1} < 0$. Taking $A^{l,1}_{C7-CI-2} = 1.1A^{l,1}_{C7-CI-1}$, $x_1(0) = (1,1.5)^T$, $x_2(0) = (2,2.5)^T$, $x_3(0) = (3,3.5)^T$, and $A^{l,2}_{C7-CI-2} = -1.1A^{l,1}_{C7-CI-1}$, we obtain $\Xi = 6I_3$ and $t^*_{C9-CI-2} < 0.81$, where $\beta = 0.4$, $c = 1$, $p(\nu) = 12$, and $V(0) = 34.75$.

Remark 35. In Figures 1–8, trajectories marked with green, red, and blue, respectively, represent synchronization state and total error trajectories of the network (36) with nonlinear coupling functions $g^{(1)}_{(C\widehat{k}-C\widehat{I})}(x_i(t))$, $g^{(2)}_{(C\widehat{k}-C\widehat{I})}(x_i(t))$, and $g^{(3)}_{(C\widehat{k}-C\widehat{I})}(x_i(t))$, where $\widehat{k} = 3,4$ and $\widehat{I} = I,II$. From the simulation results, it is observed that the impact of nonlinear coupling function on finite-time synchronization dynamics of the network (36) is that, in cases I-II of Corollaries 20 and 21, with increasing nonlinearity of nonlinear coupling function $g(\cdot)$, finite-time synchronization dynamics of the

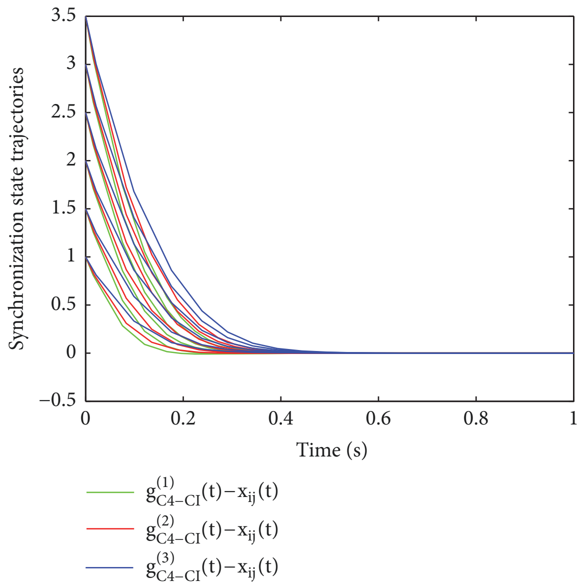

Figure 5: Synchronization state trajectories of the network (36) for the case I of Corollary 21.

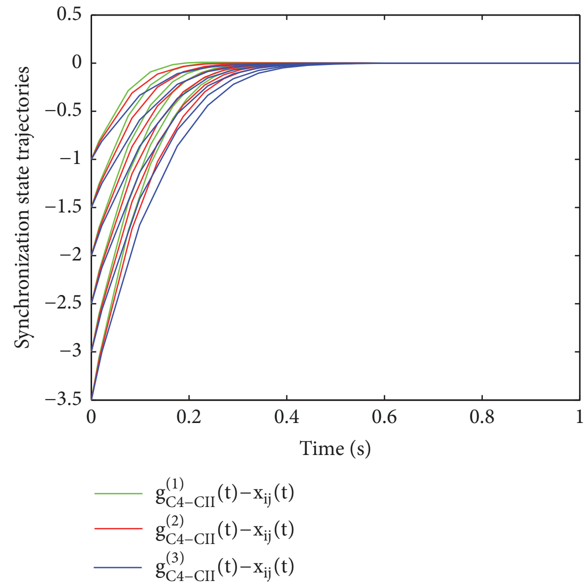

Figure 7: Synchronization state trajectories of the network (36) for the case II of Corollary 21.

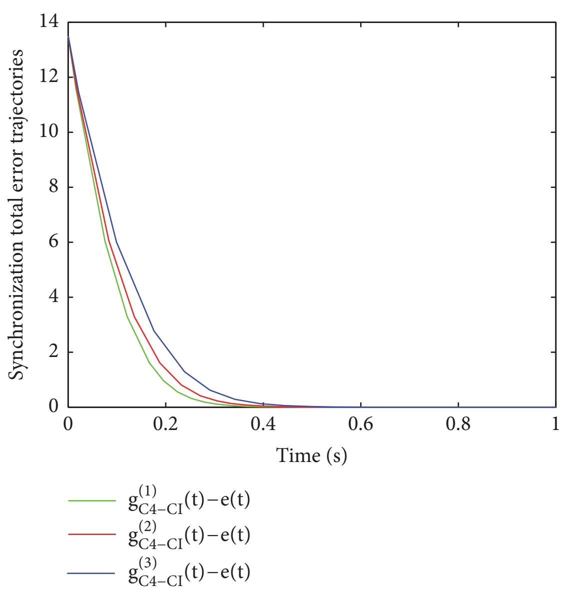

Figure 6: Synchronization total error trajectories of the network (36) for the case I of Corollary 21.

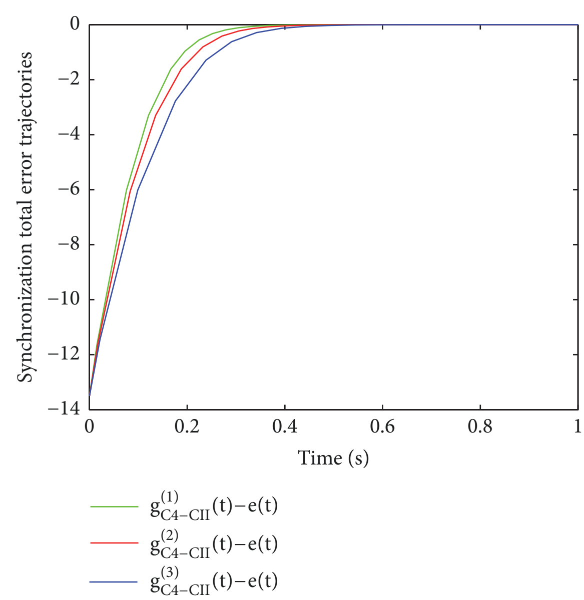

Figure 8: Synchronization total error trajectories of the network (36) for the case II of Corollary 21.

network (36) becomes poorer or better, respectively. Furthermore, it can also seen that, in cases I-II of Corollaries 20 and 21, there are $t^{*(1)}_{(C3-C\widetilde{I})} < 0.81$, $t^{*(2)}_{(C3-C\widetilde{I})} < 0.79$, $t^{*(3)}_{(C3-C\widetilde{I})} < 0.78$ and $t^{*(1)}_{(C4-C\widetilde{I})} < 0.78$, $t^{*(2)}_{(C4-C\widetilde{I})} < 0.79$, and $t^{*(3)}_{(C4-C\widetilde{I})} < 0.81$, where $\widetilde{I} = I$ and II. Thus, more feasible $p(\nu)$ is chosen to make $t^{*(1)}_{(C3-C\widetilde{I})} > t^{*(2)}_{(C3-C\widetilde{I})} > t^{*(3)}_{(C3-C\widetilde{I})} > 0$ and $0 < t^{*(1)}_{(C4-C\widetilde{I})} < t^{*(2)}_{(C4-C\widetilde{I})} < t^{*(3)}_{(C4-C\widetilde{I})}$ hold, respectively. That means the proposed finite-time estimation approach of Theorem 13 can accurately describe that nonlinearity of $g(\cdot)$ is how to impact global synchronization dynamic of the network (1) within finite time interval.

Remark 36. According to Figures 9–12, one can see that, in cases I-1 and I-2 of Corollaries 27 and 29, the effect of the switching of coupling matrix $A^{l,m}$ on finite-time synchronization dynamic of the network (1) is not only related to the switching of $A^{l,m}$ but also connected with the initial condition and nonlinear coupling function $g(\cdot)$. Moreover, from Example 2, one can obtain $t^*_{C7-CI-1} < 0.81$, $t^*_{C7-CI-2} < 0.81$, $t^*_{C9-CI-1} < 0.81$, and $t^*_{C9-CI-2} < 0.81$, respectively.

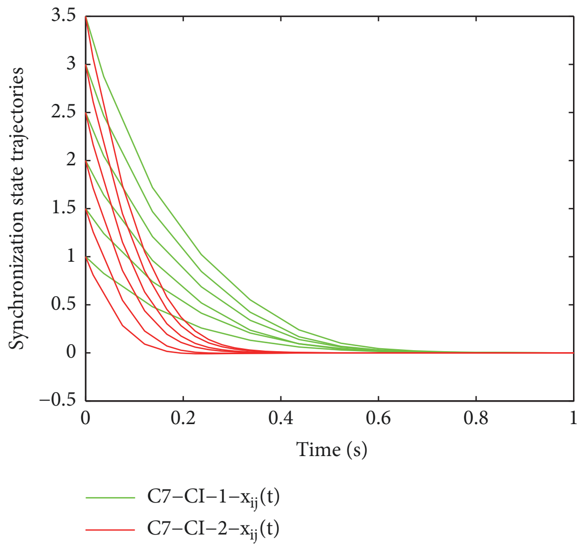

Figure 9: Synchronization state trajectories of the network (36) for the case I-1 and I-2 of Corollary 27.

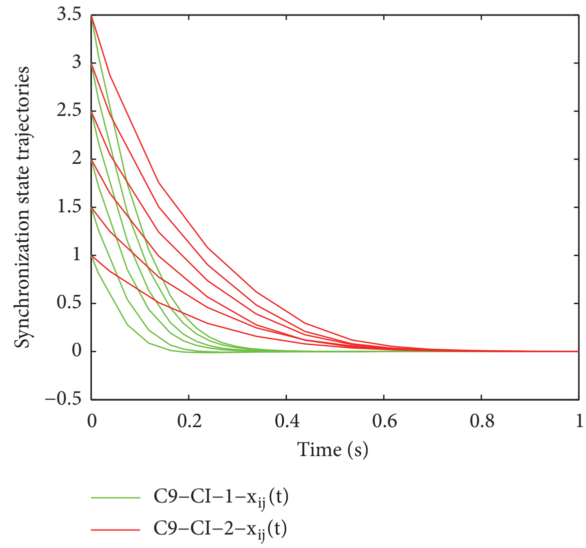

Figure 11: Synchronization state trajectories of the network (36) for the case I-1 and I-2 of Corollary 29.

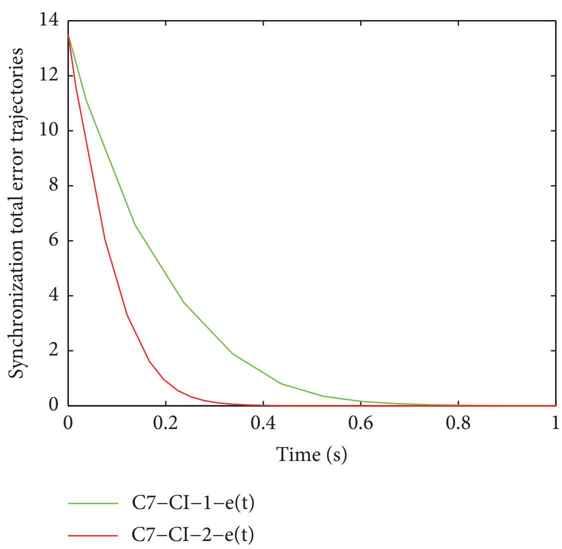

Figure 10: Synchronization total error trajectories of the network (36) for the case I-1 and I-2 of Corollary 27.

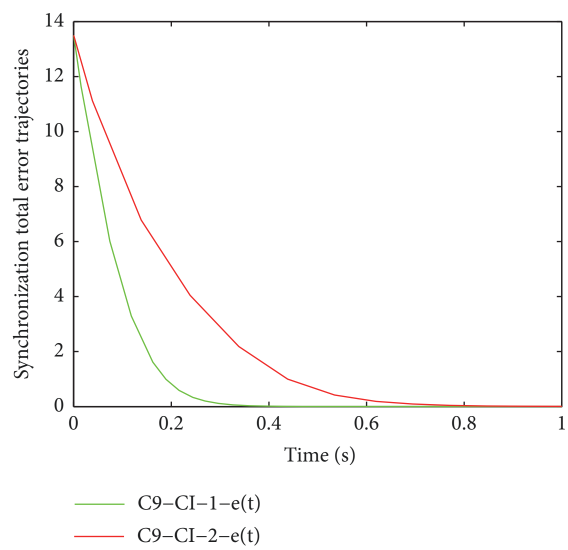

Figure 12: Synchronization total error trajectories of the network (36) for the case I-1 and I-2 of Corollary 29.

This shows that the proposed finite-time estimation method in Theorem 13 cannot reflect that the switching of coupling matrix $A^{l,m}$ is how to impact finite-time synchronization dynamic of the network (1). This further testifies there still exists weakness of the application of the proposed method in Theorem 13. Besides this, by using the above similar simulation method, the simulation results of the other cases in Corollaries 27–30 can also be obtained. All these further show that the derived results in this paper are reasonable and valuable.

5. Conclusions

Different from the existing earlier works about synchronization problems for nonlinear coupling complex networks and neural networks, this paper mainly emphasizes the impact of nonlinear coupling function and outer-coupling matrix switching on global synchronization dynamics for a class of NCMWCNs with switching topology in finite time. According to the existing and new finite-time synchronization theories, two finite-time synchronization controllers are, respectively, designed to achieve finite-time synchronization of NCMWCNs with switching topology. Furthermore, based on the obtained controllers, sufficient conditions of the impact of nonlinear coupling function and outer-coupling matrix switching on finite-time synchronization dynamics for NCMWCNs with switching topology are derived. By comparing the results of synchronization convergence time for NCMWCNs, it is testified that synchronization finite time estimation approach built on the new finite-time synchronization theory can more effectively reflect that nonlinear coupling function is how to impact finite-time synchronization dynamics. Numerical simulations further demonstrate the correctness and usefulness of the proposed results.

It should be noted that, in the addressed network of this paper, time delay is not considered. Actually, due to information transmission and finite processing speed, in many real practical systems, time delay is inevitable. Inspired by the delayed consensus analysis of multi-agent networked systems [53], in the future, we will propose nonlinear coupling delayed multi-weighted complex networks with switching topology and investigate its finite-time/fixed-time synchronization dynamics. Besides this, in the network (1), if $A^{l,m} = (a_{ij}^{l,m})_{N\times N}$ is a complex-valued connection outer-coupling matrix [54], how to get the related results is still an open problem.

Data Availability

No data were used to support this study. The reason is that this study is about how to prove stability of equation from mathematics aspect.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Xin Wang carried out the main part of this manuscript. Bin Yang participated in the discussion, corrected the derived results, and finished the simulation part. All authors read and approved the final manuscript.

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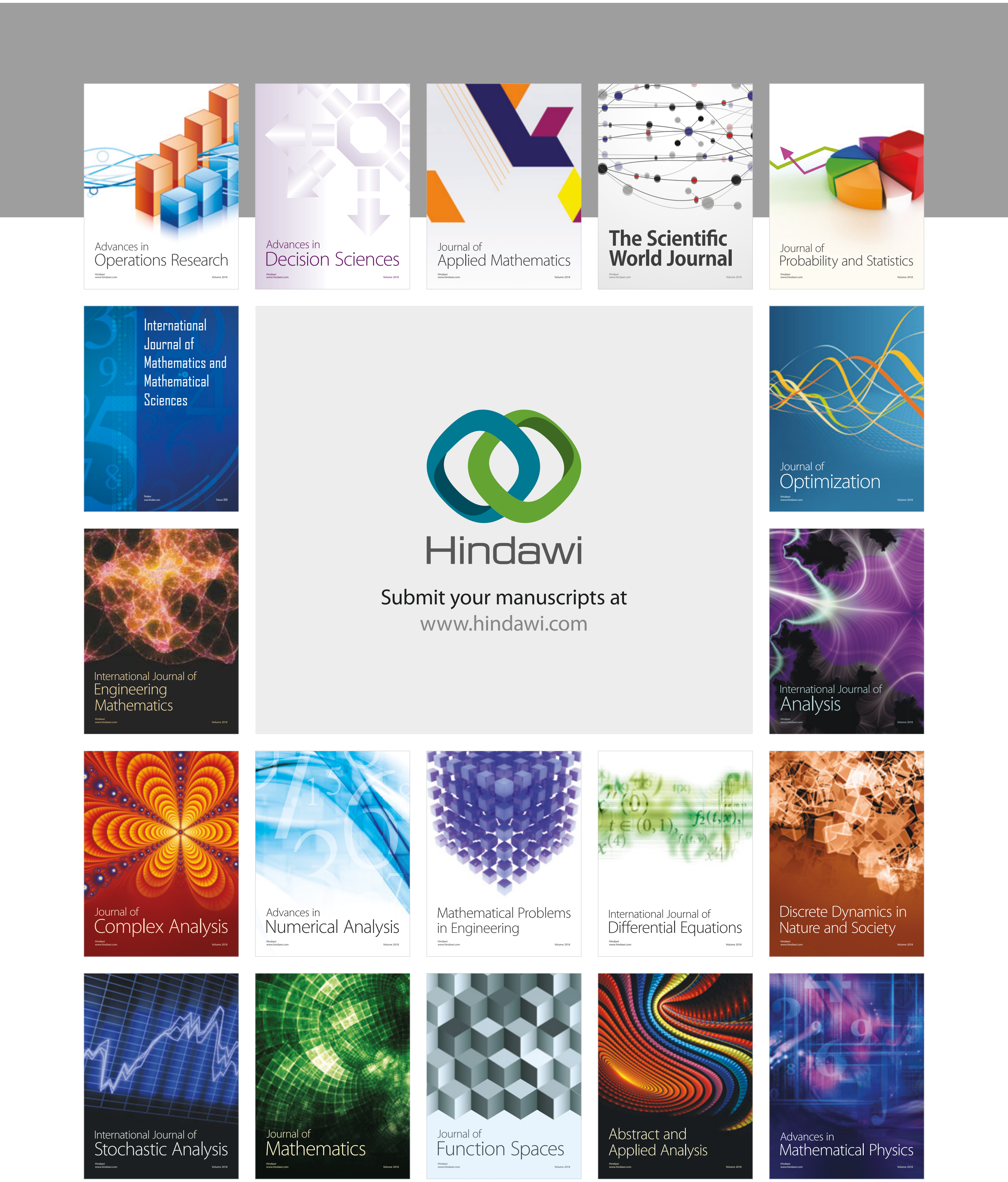
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