Research Article

Global Robust Exponential Synchronization of Multiple Uncertain Neural Networks Subject to Event-Triggered Strategy

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This paper proposes the event-triggered strategy (ETS) for multiple neural networks (NNs) with parameter uncertainty and time delay. By establishing event-triggered mechanism and using matrix inequality techniques, several sufficient criteria are obtained to ensure global robust exponential synchronization of coupling NNs. In particular, the coupling matrix need not be the Laplace matrix in this paper. In addition, the lower bounds of sampling time intervals are also found by the established event-triggered mechanism. Eventually, three numerical examples are offered to illustrate the obtained results.

1. Introduction

Multisystem network is a complex dynamical network, which has received great attraction due to its many applications in various fields, such as secure communications and biological systems. The dynamical characteristics of multisystem networks have been paid more and more attention in the fields of science and engineering (see [1–4]). As we know, synchronization of a coupling system means that its all subsystems produce common behavior under different initial values. The coupling scheme describing the interaction rules between subsystems plays an important role in ensuring the synchronization. In particular, the synchronization of multisystem networks has become a hotspot of nonlinear scientific research because the synchronization of multisystem networks can better describe many observed natural phenomena and can create ordered multisystem networks. As a kind of control, coupling is the key to ensure the synchronization of complex dynamical networks; hence, several coupling schemes are proposed to realize the synchronization. However, most of the existing works require that the coupling matrix is always the Laplace matrix; that is, the row sums of the coupling matrix all are zero and the nondiagonal elements are nonnegative, which greatly restrict us to design a good performance coupling controller.

In fact, we notice that there are some non-Laplacian coupling in complex dynamical networks, so we can try to achieve the synchronization of multisystem networks by constructing non-Laplacian coupling matrix. In addition, in recent decades, NNs have become a hot research topic because of its rich content and wide application; therefore, there are many results in the research of NNs, such as in [1, 5–12]. The authors in [1] proposed a new ETS to achieve the synchronization of multiple NNs (MNNs). The dynamical characteristics of nonautonomous fractional-order delay NNs were studied in [5]. Wu and Zeng [9] derived two anti-synchronization algorithms to realize exponential anti-synchronization of memristive recurrent NNs. In [10], some results were established to ensure the Lagrange stability of NNs with memristive synapses. Wu and Zeng [11] discussed a class of memristive NNs, and several sufficient criteria were established for exponential stabilization by using matrix inequality techniques. Wu et al. [12] proposed a novel and efficacious method to study the periodic NNs, and some new results for the periodic NNs were obtained. Therefore, it is an interesting and far-reaching research topic to study the synchronization of MNNs by constructing a proper non-Laplacian coupling matrix.

Parametric uncertainty arises from a partial understanding of mathematical models, for instance, empirical

quantities and constitutive laws (see [13–22]). The uncertainty of parameters must be considered in actual system because the parameters of the model in the process of industrial control are often uncertain. It is fortunate that uncertain parameters have been considered in many models in order to describe practical problems more accurately. In [13], the dynamical characteristics of stochastic nonlinear systems with parametric uncertainty were concerned. Maharajan et al. [14] investigated the problem of enhanced results on robust finite-time passivity for uncertain discrete-time Markovian jumping BAM delayed NNs. The authors in [17] provided several novel delay-dependent stability criteria to ensure robust stability of uncertain stochastic systems. Huang et al. [20] investigated robust state estimation of uncertain neural networks by designing robust state estimator and using new bounding technique. Zhu et al. [22] discussed the stability of uncertain neutral systems, and some new criteria were provided to guarantee the stability of the model. Moreover, fractional-order system is a charming research field, which describes the real world more accurately than integer-order system. Recently, Zhang [21] studied uncertain fractional-order system and its application. The uncertain parameters of the above models are required to be norm-bounded; it is a common way to deal with these uncertain parameters by using inequality techniques. With the mature of the technology, some problems in MNNs have been well solved; however, it is still a great challenge to design a feedback controller with good performance when we analyze multisystem networks. There are many papers about the topic of parametric uncertainty, but the issue about trusted control in the MNNs with uncertain parameters still has a long way to go and much further work is worth studying. In addition, time delay is often inevitable in practical systems, which may lead to instability of the system; in other words, time delay is the important reason for instability of the model. Although parameter uncertainty and time delay bring difficulties to the theoretical analysis of dynamical characteristics of system, in practical application, the analysis of model is often unreasonable without considering uncertain parameter and time delay. Hence, theoretical knowledge and practical experience urge us to study the dynamics problems of MNNs with parameter uncertainty and time delay.

Event-triggered mechanism is a very important sampling mechanism, that is, the emergence of certain events rely on the state of the system and this is also the difference between event-triggered feedback control and the traditional sampling mechanism. Because continuous-time control requires the continuous information, it is expensive and unrealistic in the real world. Unlike continuous-time control, event-triggered mechanism is a discontinuous-time control, which only requires the local communication data. Considering the sampling period, the sampling data control scheme usually adopts zero-order hold to keep the last sampling system state and control signal which is sent to the next event. In event-triggered mechanism, the sampling will not start and the controller will not be updated, unless its size reaches the specified threshold; therefore, the greatest advantage of event-triggered mechanism is to reduce communication data. Furthermore, event-triggered mechanism has been widely

studied due to its effectiveness in practical systems, for example, [23–26] and references therein. Dolk and Heemels [23] investigated a networked control system subject to event-triggered control and its application to packet losses. In [24], a new decentralized event-triggered item for distributed networked systems was proposed to reduce the waste of network resources. The event-triggered rule was proposed to solve the problem of excessive use of communication resources in [25]. The authors in [26] studied neural networks by the proposed event-triggered rule, and the rule effectively solved a large number of computational problems. It is not difficult to find that event-triggered mechanism is very useful and meaningful sampling mechanism because many practical problems can be solved via ETS, but there are few research results on the combination of event-triggered mechanism and MNNs.

Based on the above discussion, the synchronization of MNNs with parameter uncertainty and time delay is studied and emphasized. By establishing the event-triggered mechanism, this paper derives several sufficient criteria to guarantee the synchronization of the systems; it shows that these results are different from the previous ones. Furthermore, the lower bounds of sampling time intervals are also obtained; that is, if the triggering time t_k is known, we can predict the next triggering time t_{k+1} . Roughly speaking, this paper has three highlights: (1) Most existing works on multisystem networks require that the coupling matrix is always the Laplace matrix, in which row sums of coupling matrix all are zero and nondiagonal elements are non-negative. Because of the particularity of Laplacian matrix, the design of coupling controller is greatly limited. In fact, we have found some non-Laplacian coupling in coupled dynamic networks. In this paper, the coupling matrix need not be the Laplace matrix; that is to say, row sums of coupling matrix can be nonzero constant and nondiagonal elements are arbitrary. (2) Some criteria are proposed to ensure the synchronization of MNNs with parameter uncertainty and time delay by establishing event-triggered mechanism and using matrix inequality techniques. Compared with the most of the existing papers, the results obtained are more simple and convenient in this paper. (3) We can find the lower bounds of the sampling time intervals by the established event-triggered mechanism, and then, we can know that the Zeno behavior will not happen.

The remaining part of this paper is designed as follows. Section 2 introduces the model of MNNs with parameter uncertainty and time delay and some preliminary knowledge. Section 3 gives several sufficient criteria to achieve the global robust exponential synchronization, and the lower bounds of the sampling time intervals are also found. Section 4 provides three numerical examples to demonstrate the obtained results. In the end, Section 5 summarizes the paper's relevant conclusion.

2. Preliminaries and Problem Formulation

Throughout the paper, R^n represents the n -dimensional Euclidean space. I stands for an identity matrix with proper dimensionality. Let $C_\tau := C([- \tau, 0], R^n)$ be a Banach space that composed of all the continuous functions $\varphi : [- \tau, 0]$ to

R^n . $\|\cdot\|_2$ and $\|\cdot\|_{\inf}$ stand for 2-norm and inf-norm, respectively, and let us write $\|\cdot\|_2$ in terms of $\|\cdot\|$ for the sake of simplicity. For a matrix $A = (a_{ij})_{n \times n}$, $a_{\max} = \max\{|a_{ij}|\}$, and $a_{\min} = \min\{|a_{ij}|\}$; furthermore, A^T stands for its transpositive, and $A > 0$, $A < 0$, $A \geq 0$, and $A \leq 0$ mean that A are positive definite, negative definite, positive semidefinite, and negative semidefinite, respectively. $\text{diag}\{\cdot\}$ is representative of diagonal matrix. ϕ refers to empty set. If no otherwise specified, matrices are supposed to have compatible dimensions.

Consider a group of r NNs with parameter uncertainty and time delay, and the model of k th NNs with parameter uncertainty and time delay is given as follows:

$$\begin{aligned} \frac{dz_k(t)}{dt} = & -(D + \Delta D)z_k(t) + (B + \Delta B)f(z_k(t)) \\ & + (C + \Delta C)g(z_k(t - \tau)) + J(t) + u_k(t), \end{aligned} \quad (1)$$

where $k \in \mathcal{J} = \{1, 2, \dots, r\}$; $t \geq 0$; $z_k(t) = (z_{k1}(t), z_{k2}(t), \dots, z_{kn}(t))^T \in R^n$ refers to the state vector; $D = \text{diag}\{d_1, d_2, \dots, d_n\}$ is the real diagonal positive-definite matrix standing for the neuron self-inhibitions; $J(t) \in R^n$ is the input or bias; $\tau > 0$ represents the transmission delay; $f(z_k(t)) = (f_1(z_{k1}(t)), f_2(z_{k2}(t)), \dots, f_n(z_{kn}(t)))^T$ and $g(z_k(t - \tau)) = (g_1(z_{k1}(t - \tau)), g_2(z_{k2}(t - \tau)), \dots, g_n(z_{kn}(t - \tau)))^T$ are the neuron activation functions; $B = (b_{km})_{n \times n}$ is the weight matrix and $C = (c_{km})_{n \times n}$ is the delay weight matrix; ΔB , ΔC , and ΔD are the norm-bounded uncertainty terms; and $u_k(t) \in R^n$ denotes the control input.

The distributed event-triggered controller is given below:

$$u_k(t) = \lambda \sum_{m=1}^r y_{km} z_m(t_{ms}), \quad t \in [t_{ks}, t_{k(s+1)}), \quad (2)$$

for $k \in \mathcal{J}$ and $s \geq 1$, where $\{t_{ks}\}$ refers to the sampling time sequence, $\lambda > 0$ represents coupling gain, and $K = (y_{km})_{r \times r}$ stands for the coupling matrix such that

$$\sum_{m=1}^r y_{km} = \varrho \in R. \quad (3)$$

Next, we define the following distributed event-triggered function:

$$H_{ks}(t) = \|z_k(t) - z_k(t_{ks})\| - \alpha_k \sum_{m \in \mathcal{J}_k} \|z_k(t_{ms}) - z_k(t_{ks})\|, \quad (4)$$

and the sampling time sequence $\{t_{ks}\}$ satisfies ETS:

$$t_{k(s+1)} = \inf\{t > t_{ks}, H_{ks} > 0\}, \quad (5)$$

for $k \in \mathcal{J}$ and $s \geq 1$, where $\alpha_k > 0$ refers to the control parameter and $\mathcal{J}_k \subseteq \mathcal{J}$ with $\mathcal{J}_k = \{m, m \neq k, y_{km} \neq 0\}$.

Remark 1. Let $\Phi_{km} = \alpha_k \sum_{m \in \mathcal{J}_k} \|z_k(t_{ms}) - z_k(t_{ks})\|$ be the sth threshold on the k th neuron. It is not difficult to find that Φ_{km} determines sampling time of the k th neuron, and Φ_{km} is closely related to the control parameter α_k and subset $\mathcal{J}_k$. In particular, if $\alpha_k = 0$ or $\mathcal{J}_k = \phi$, the k th neuron will have sampling time at any $t \geq t_0$; that is to say, the Zeno behavior does happen when $\alpha_k = 0$ or $\mathcal{J}_k = \phi$. It should be noted that

the transmission event is triggered and the controller is updated when the measurement error $\|z_k(t) - z_k(t_{ks})\|$ exceeds the threshold Φ_{km} in the event-triggered controller; unlike the event-triggered controller, the impulse controller samples on the determined impulse time sequence $\{t_{ks}\}$. Thus, we can know that the sampling time sequence $\{t_{ks}\}$ satisfies $t_{k(s+1)} = \inf\{t > t_{ks}, H_{ks} > 0\}$ in the event-triggered controller, and the sampling time sequence $\{t_{ks}\}$ is known in the impulse controller.

Then, we can rewrite model (1) as follows:

$$\begin{aligned} \frac{dz_k(t)}{dt} = & -(D + \Delta D)z_k(t) + (B + \Delta B)f(z_k(t)) \\ & + (C + \Delta C)g(z_k(t - \tau)) + J(t) \\ & + \lambda \sum_{m=1}^r y_{km} z_m(t_{ms}), \quad t \in [t_{ks}, t_{k(s+1)}). \end{aligned} \quad (6)$$

The initial conditions of model (6) are assumed to be

$$z_k(t_0 + \vartheta) = \varphi_k(\vartheta), \quad (7)$$

where $\varphi_k \in C_\tau$, $\vartheta \in [-\tau, 0]$, and $k = 1, 2, \dots, n$.

Let $z(t, t_0, \varphi)$ stands for the solution of model (6); in order to avoid this paper being too long, we assume that there exists a unique solution of system (6).

System (6) is called to be globally robustly exponentially synchronized, namely, each subsystem in (6) can achieve global robust exponential synchronization if there are constants $M \geq 1$ and $h > 0$ satisfying

$$\|z_k(t) - z_m(t)\| \leq M \max_{i,j \in \mathcal{J}} \|\varphi_i - \varphi_j\| e^{-h(t-t_0)}, \quad (8)$$

for any $t \geq 0$ and $k, m \in \mathcal{J}$, where h stands for convergence rate.

For the sake of discussion, we give two basic assumptions.

Assumption 1

$$\begin{aligned} \Delta D &= M_1 R_1(t) N_1, \\ \Delta B &= M_2 R_2(t) N_2, \\ \Delta C &= M_3 R_3(t) N_3, \end{aligned} \quad (9)$$

where M_i and N_i ($i = 1, 2, 3$) are constant matrices with proper dimensionality and $R_i(t)$ are the unknown matrix with $R_i^T(t) R_i(t) \leq I$ ($i = 1, 2, 3$).

Assumption 2. For any $u, v \in R$ and $k \in \mathcal{J}$, the activation functions $f_k(u)$ and $g_k(u)$ are all continuous in R and satisfy

$$\begin{aligned} 0 < \frac{f_k(u) - f_k(v)}{u - v} &\leq p_k, \\ 0 < \frac{g_k(u) - g_k(v)}{u - v} &\leq q_k, \end{aligned} \quad (10)$$

where $p_k > 0$ and $q_k > 0$ represent Lipschitz constants. Let $P = \text{diag}\{p_1^2, p_2^2, \dots, p_n^2\}$ and $Q = \text{diag}\{q_1^2, q_2^2, \dots, q_n^2\}$.

Remark 2. Assumption 1 guarantees the boundedness of ΔB , ΔC , and ΔD . Assumption 2 ensures the speciality of the activation functions $f_k(u)$ and $g_k(u)$. The Lipschitz constants $p_k > 0$ and $q_k > 0$ are dependent on the activation functions $f_k(u)$ and $g_k(u)$ rather than fixed. Lipschitz continuity is a smoother condition than uniform continuity; intuitively, Lipschitz continuity restricts the speed of function change. Furthermore, the slope of the function satisfying Lipschitz condition must be less than a real number called Lipschitz constant. In differential equation theory, Lipschitz condition is a core condition in the existence and uniqueness theorem of solutions under initial conditions.

Next, two important lemmas are introduced as follows.

Lemma 1 (see [19]). *Let F , G , and $S(t)$ be real matrices and $\eta > 0$ is a constant; if $S^T(t)S(t) \leq I$, then*

$$F^T S^T(t) G^T + G S(t) F \leq \frac{1}{\eta} F^T F + \eta G G^T. \quad (11)$$

Lemma 2 (see [18]). *Let F , G , P , and $Q(t)$ be real matrices; if $P > 0$ and $Q^T(t)Q(t) \leq I$, then*

- (1) For scalar $\xi > 0$ and vectors z_1 and z_2 of proper dimensionality

$$2z_1^T F^T Q(t) G z_2 \leq \frac{1}{\xi} z_1^T F^T F z_1 + \xi z_2^T G^T G z_2. \quad (12)$$

- (2) For matrix $P > 0$ of suitable dimensions:

$$2z_1^T z_2 \leq z_1^T P^{-1} z_1 + z_2^T P z_2. \quad (13)$$

3. Main Results

Let $\varepsilon_{km}(t) = z_k(t) - z_m(t)$ be the synchronization error and $\mu_k(t) = z_k(t) - z_k(t_{ks})$ be the measurement error for $t \in [t_{ks}, t_{k(s+1)})$. Then, by establishing event-triggered mechanism and using matrix inequality techniques, we obtain several novel conditions to achieve global robust exponential synchronization of system (6), and the lower bounds of sampling time interval are also found.

Theorem 1. *Under ETS (5), for model (6), if Assumptions 1 and 2 hold, $\eta > 0$ and $\xi_i > 0$ ($i = 1, 2, \dots, 5$) are constants, $\alpha_k > 0$, and $\mathcal{J}_k \neq \emptyset$ for $k \in \mathcal{J}$, then $\inf_{s \geq 1} \{t_{k(s+1)} - t_{ks}\} > 0$ for any $s \geq 1$.*

Proof. From $\mu_k(t) = z_k(t) - z_k(t_{ks})$, we have

$$\begin{aligned} \frac{d\mu_k(t)}{dt} &= -(D + \Delta D)z_k(t) + (B + \Delta B)f(z_k(t)) \\ &\quad + (C + \Delta C)g(z_k(t - \tau)) + u_k(t), \quad t \in [t_{ks}, t_{k(s+1)}), \end{aligned} \quad (14)$$

then

$$\begin{aligned} \frac{d\|\mu_k(t)\|^2}{dt} &= 2\mu_k^T(t) [-(D + \Delta D)z_k(t) + (B + \Delta B)f(z_k(t)) \\ &\quad + (C + \Delta C)g(z_k(t - \tau)) + u_k(t)] \\ &= 2\mu_k^T(t)(D + \Delta D)z_k(t_{ks}) - 2\mu_k^T(t)(D + \Delta D)\mu_k(t) \\ &\quad + 2\mu_k^T(t)(B + \Delta B)[f(z_k(t)) - f(z_k(t_{ks}))] \\ &\quad + 2\mu_k^T(t)(C + \Delta C)[g(z_k(t - \tau)) - g(z_k(t_{k\bar{s}}))] \\ &\quad + 2\mu_k^T(t)u_k(t) + 2\mu_k^T(t)(B + \Delta B)f(z_k(t_{ks})) \\ &\quad + 2\mu_k^T(t)(C + \Delta C)g(z_k(t_{k\bar{s}})). \end{aligned} \quad (15)$$

For each $t \in [t_{ks}, t_{k(s+1)})$, there exists a $\bar{s} \in [0, s]$ satisfying $t - \tau \in [t_{k\bar{s}}, t_{k(\bar{s}+1)}]$; according to the event-triggered function, for any $t \in [t_{ks}, t_{k(s+1)})$, there exists a $\bar{s} \in [0, s]$ satisfying

$$\|\mu_k(t - \tau)\| \leq \alpha_k \sum_{m \in \mathcal{J}_k} \|z_m(t_{m\bar{s}}) - z_k(t_{k\bar{s}})\|. \quad (16)$$

According to Lemmas 1 and 2, we get

$$\begin{aligned} 2\mu_k^T(t)\Delta D z_k(t_{ks}) &= 2\mu_k^T(t)M_1 R_1(t)N_1 z_k(t_{ks}) \\ &\leq \frac{1}{\xi_1} \mu_k^T(t)M_1 M_1^T \mu_k(t) \\ &\quad + \xi_1 z_k^T(t_{ks})N_1^T N_1 z_k(t_{ks}), \\ -2\mu_k^T(t)\Delta D \mu_k(t) &\leq \mu_k^T(t) \left(\eta M_1 M_1^T + \frac{1}{\eta} N_1^T N_1 \right) \\ &\quad \cdot \mu_k(t), \end{aligned} \quad (17)$$

$$\begin{aligned} 2\mu_k^T(t)\Delta B[f(z_k(t)) - f(z_k(t_{ks}))] &= 2\mu_k^T(t)M_2 R_2(t)N_2[f(z_k(t)) - f(z_k(t_{ks}))] \\ &\leq \frac{1}{\xi_2} \mu_k^T(t)M_2 M_2^T \mu_k(t) + \xi_2[f(z_k(t)) \\ &\quad - f(z_k(t_{ks}))]^T N_2^T N_2[f(z_k(t)) - f(z_k(t_{ks}))], \\ 2\mu_k^T(t)\Delta C[g(z_k(t - \tau)) - g(z_k(t_{k\bar{s}}))] &= 2\mu_k^T(t)M_3 R_3(t)N_3[g(z_k(t - \tau)) - g(z_k(t_{k\bar{s}}))] \\ &\leq \frac{1}{\xi_3} \mu_k^T(t)M_3 M_3^T \mu_k(t) + \xi_3[g(z_k(t - \tau)) - g(z_k(t_{k\bar{s}}))]^T \\ &\quad \cdot N_3^T N_3[g(z_k(t - \tau)) - g(z_k(t_{k\bar{s}}))], \end{aligned} \quad (18)$$

$$\begin{aligned} 2\mu_k^T(t)\Delta B f(z_k(t_{ks})) &= 2\mu_k^T(t)M_2 R_2(t)N_2 f(z_k(t_{ks})) \\ &\leq \frac{1}{\xi_4} \mu_k^T(t)M_2 M_2^T \mu_k(t) + \xi_4 f^T(z_k(t_{ks})) \\ &\quad \cdot N_2^T N_2 f(z_k(t_{ks})), \end{aligned} \quad (19)$$

(21)

$$\begin{aligned}
2\mu_k^T(t)\Delta Cg(z_k(t_{k\bar{s}})) &= 2\mu_k^T(t)M_3R_3(t)N_3g(z_k(t_{k\bar{s}})) \\
&\leq \frac{1}{\xi_5}\mu_k^T(t)M_3M_3^T\mu_k(t) + \xi_5g^T(z_k(t_{k\bar{s}})) \\
&\quad \cdot N_3^TN_3g(z_k(t_{k\bar{s}})).
\end{aligned} \tag{22}$$

Combining (15)–(22), we obtain

$$\begin{aligned}
\frac{\|d\mu_k(t)\|^2}{dt} &\leq 2\mu_k^T(t)Dz_k(t_{ks}) + \frac{1}{\xi_1}\mu_k^T(t)M_1M_1^T\mu_k(t) + \xi_1z_k^T(t_{ks})N_1^TN_1z_k(t_{ks}) - 2\mu_k^T(t)D\mu_k(t) + \mu_k^T(t)\left[\eta M_1M_1^T + \frac{1}{\eta}N_1^TN_1\right]\mu_k^T(t) \\
&\quad + 2\mu_k^T(t)B[f(z_k(t)) - f(z_k(t_{ks}))] + \frac{1}{\xi_2}\mu_k^T(t)M_2M_2^T\mu_k(t) + \xi_2[f(z_k(t)) - f(z_k(t_{ks}))]^TN_2^TN_2[f(z_k(t)) - f(z_k(t_{ks}))] \\
&\quad + 2\mu_k^T(t)C[g(z_k(t-\tau)) - g(z_k(t_{k\bar{s}}))] + \frac{1}{\xi_3}\mu_k^T(t)M_3M_3^T\mu_k(t) + \xi_3[g(z_k(t-\tau)) - g(z_k(t_{k\bar{s}}))]^TN_3^TN_3 \\
&\quad \cdot [g(z_k(t-\tau)) - g(z_k(t_{k\bar{s}}))] + 2\mu_k^T(t)u_k(t) + 2\mu_k^T(t)Bf(z_k(t_{ks})) + 2\mu_k^T(t)Cg(z_k(t_{k\bar{s}})) + \frac{1}{\xi_4}\mu_k^T(t)M_2M_2^T\mu_k(t) \\
&\quad + \xi_4f^T(z_k(t_{ks}))N_2^TN_2f(z_k(t_{ks})) + \frac{1}{\xi_5}\mu_k^T(t)M_3 \cdot M_3^T\mu_k(t) + \xi_5g^T(z_k(t_{k\bar{s}}))N_3^TN_3g(z_k(t_{k\bar{s}})) \\
&\leq \left[\|D\| + \frac{1}{\xi_1}\|M_1M_1^T\| - 2\|D\| + \eta\|M_1M_1^T\| + \frac{1}{\eta}\|N_1^TN_1\| + \|B\| + \frac{1}{\xi_2}\|M_2M_2^T\| + \|C\| + \frac{1}{\xi_3}\|M_3M_3^T\| + \|B\| + \frac{1}{\xi_4}\|M_2M_2^T\| \right. \\
&\quad \left. + \|C\| + \frac{1}{\xi_5}\|M_3M_3^T\| + 1\right]\|\mu_k(t)\|^2 + [\|D\| + \xi_1\|N_1^TN_1\|]\|z_k(t_{ks})\|^2 + [\|B\| + \xi_2\|N_2^TN_2\|]\|f(z_k(t)) - f(z_k(t_{ks}))\|^2 \\
&\quad + [\|C\| + \xi_3\|N_3^TN_3\|]\|g(z_k(t-\tau)) - g(z_k(t_{k\bar{s}}))\|^2 + [\|B\| + \xi_4\|N_2^TN_2\|]\|f(z_k(t_{ks}))\|^2 + [\|C\| + \xi_5\|N_3^TN_3\|]\|g(z_k(t_{k\bar{s}}))\|^2 \\
&\quad + \|\mu_k(t)\|^2 \\
&\leq [-\|D\| + 2\|B\| + 2\|C\| + \left(\frac{1}{\xi_1} + \eta\right)\|M_1M_1^T\| + \frac{1}{\eta}\|N_1^TN_1\| + \left(\frac{1}{\xi_2} + \frac{1}{\xi_4}\right)\|M_2M_2^T\| + \left(\frac{1}{\xi_3} + \frac{1}{\xi_5}\right)\|M_3M_3^T\| \\
&\quad + \|B\|\|P\| + \xi_2\|N_2^TN_2\|\|P\| + 1]\|\mu_k(t)\|^2 + [\|C\|\|Q\| + \xi_3\|N_3^TN_3\|\|Q\|] \cdot \|\mu_k(t-\tau)\|^2 + [\|D\| + \xi_1\|N_1^TN_1\|]\|z_k(t_{ks})\|^2 \\
&\quad + [\|B\| + \xi_4\|N_2^TN_2\|]\|f(z_k(t_{ks}))\|^2 + [\|C\| + \xi_5\|N_3^TN_3\|]\|g(z_k(t_{k\bar{s}}))\|^2 + \|u_k(t)\|^2.
\end{aligned} \tag{23}$$

Thus,

$$\frac{d\|\mu_k(t)\|^2}{dt} \leq \Lambda\|\mu_k(t)\|^2 + \Omega_{ks}, \tag{24}$$

where

$$\begin{aligned}
\Omega_{ks} &= [\|C\|\|Q\| + \xi_3\|N_3^TN_3\|\|Q\|]\left(\alpha_k \sum_{m \in \mathcal{J}_k} \|z_m(t_{m\bar{s}}) - z_k(t_{k\bar{s}})\|\right)^2 \\
&\quad + [\|D\| + \xi_1\|N_1^TN_1\|]\|z_k(t_{ks})\|^2\|M_2M_2^T\| + [\|B\| + \xi_4\|N_2^TN_2\|] \\
&\quad \cdot \|f(z_k(t_{ks}))\|^2 + [\|C\| + \xi_5\|N_3^TN_3\|]\|g(z_k(t_{k\bar{s}}))\|^2 \\
&\quad + \left\|\lambda \sum_{m=1}^r y_{km}z_m(t_{ms})\right\|^2,
\end{aligned} \tag{25}$$

and $\Lambda > 0$ such that

$$\begin{aligned}
\Lambda &\geq -\|D\| + 2\|B\| + 2\|C\| + \left(\frac{1}{\xi_1} + \eta\right)\|M_1M_1^T\| + \frac{1}{\eta}\|N_1^TN_1\| \\
&\quad + \left(\frac{1}{\xi_2} + \frac{1}{\xi_4}\right)\|M_2M_2^T\| + \left(\frac{1}{\xi_3} + \frac{1}{\xi_5}\right)\|M_3M_3^T\| \\
&\quad + \|B\|\|P\| + \xi_2\|N_2^TN_2\|\|P\| + 1.
\end{aligned} \tag{26}$$

Since $\mu_k(t_{ks}) = 0$, according to (24), we have

$$\|\mu_k(t_{k(s+1)})\|^2 + \frac{\Omega_{ks}}{\Lambda} \leq \frac{\Omega_{ks}}{\Lambda} e^{\Lambda(t_{k(s+1)} - t_{ks})}. \tag{27}$$

From ETS (5), we get

$$e^{\Lambda(t_{k(s+1)} - t_{ks})} \geq 1 + \frac{\Lambda(\alpha_k \sum_{m \in \mathcal{J}_k} \|z_m(t_{m\bar{s}}) - z_k(t_{k\bar{s}})\|)^2}{\Omega_{ks}}, \tag{28}$$

then

$$t_{k(s+1)} - t_{ks} \geq \frac{1}{\Lambda} \ln \left[1 + \frac{\Lambda (\alpha_k \sum_{m \in \mathcal{J}_k} \|z_m(t_{m\bar{s}}) - z_k(t_{k\bar{s}})\|)^2}{\Omega_{ks}} \right]; \quad (29)$$

hence, we can know $\inf_{s \geq 1} \{t_{k(s+1)} - t_{ks}\} > 0$ when $\mathcal{J}_k \neq \emptyset$. The proof is finished. $\square$

Remark 3. By Theorem 1, we can know $\alpha_k > 0$ and $\mathcal{J}_k \neq \emptyset$ implying that the Zeno behavior does not happen for sampling time sequence $\{t_{ks}\}$. In addition, if $\alpha_k > 0$ and $\mathcal{J}_k \neq \emptyset$, there is no Zeno behavior for model (6) under ETS (5).

Theorem 2. Let condition (5), Assumption 1, and Assumption 2 hold; there is $l \in \mathcal{J}$ satisfying $\alpha_k > 0$ and $\mathcal{J}_k \neq \emptyset$ ($k \in \mathcal{J} \setminus \{l\}$). Then, model (6) is globally robustly exponentially synchronized under ETS (5) if there exist $r - 1$ constants $\xi_k > 0$ ($k \in \mathcal{J} \setminus \{l\}$) and a constant $\nu > 0$ satisfying

$$\frac{\Psi K_l + K_l^T \Psi}{2} + \nu I \leq 0, \quad (30)$$

and there is $0 < \alpha_{\max} < 1/(r + r_{\max} - 1)$ and three constants $\eta > 0$, $\xi_6 > 0$, and $\xi_7 > 0$ satisfying

$$\begin{aligned} & -2d_{\min} - 2\lambda \left(\frac{\nu}{\xi_{\max}} - \frac{\alpha_{\max}(r-1)(r+r_{\max}-1)\xi_{\max}\nu_{l\max}}{(1-\alpha_{\max}(r+r_{\max}-1))\xi_{\min}} \right) \\ & + \|B\| + \|C\| + \|B\|\|P\| + \eta \|M_1 M_1^T\| + \frac{1}{\eta} \|N_1^T N_1\| \\ & + \frac{1}{\xi_6} \|M_2 M_2^T\| + \xi_6 \|N_2^T N_2\| \|P\| + \frac{1}{\xi_7} \|M_3 M_3^T\| \\ & + \|C\|\|Q\| + \xi_7 \|N_3^T N_3\| \|Q\| < 0, \end{aligned} \quad (31)$$

where $d_{\min} = \min_{1 \leq k \leq n} \{d_k\}$, $K_l = (y_{km} - y_{lm})_{k, m \in \mathcal{J} \setminus \{l\}}$, $\Psi = \text{diag}\{\zeta_{l_1}, \zeta_{l_2}, \dots, \zeta_{l_{r-1}}\}$, $\{l_1, l_2, \dots, l_{r-1}\}$ denotes for rearrangement of $\mathcal{J} \setminus \{l\}$ with $l_1 < l_2 < \dots < l_{r-1}$, $\nu_{l\max} = \max_{k, m \in \mathcal{J} \setminus \{l\}} \{|y_{km} - y_{lm}|\}$, r_k stands for the number of elements in $\mathcal{J}_k$, and $r_{\max} = \max_{1 \leq k \leq r} \{r_k\}$.

Proof. By Theorem 1, for any $k \in \mathcal{J} \setminus \{l\}$, $t_{ks} \rightarrow \infty$ as $s \rightarrow \infty$, then, there exists a $s \geq 1$ satisfying $t \in [t_{ks}, t_{k(s+1)})$:

$$\begin{aligned} \frac{d\varepsilon_{kl}(t)}{dt} &= \frac{dz_k(t)}{dt} - \frac{dz_l(t)}{dt} \\ &= -(D + \Delta D)(z_k(t) - z_l(t)) + (B + \Delta B)[f(z_k(t)) \\ &\quad - f(z_l(t))] + (C + \Delta C)[g(z_k(t - \tau)) \\ &\quad - g(z_l(t - \tau))] + u_k(t) - u_l(t), \end{aligned} \quad (32)$$

where

$$u_k(t) - u_l(t) = \lambda \sum_{m=1, m \neq l}^r (y_{km} - y_{lm})(z_m(t_{ms}) - z_l(t)), \quad (33)$$

since

$$\begin{aligned} u_k(t) &= \lambda \varrho z_l(t) + \lambda \sum_{m=1, m \neq l}^r y_{km}(z_m(t_{ms}) - z_l(t)), \\ u_l(t) &= \lambda \varrho z_l(t) + \lambda \sum_{m=1, m \neq l}^r y_{lm}(z_m(t_{ms}) - z_l(t)). \end{aligned} \quad (34)$$

Let $W_k(t) = \varepsilon_{kl}^T(t) \varepsilon_{kl}(t)$ and $W(t) = \sum_{k \in \mathcal{J} \setminus \{l\}} \zeta_k W_k(t)$, then

$$\begin{aligned} \frac{dW_k(t)}{dt} &= 2\varepsilon_{kl}^T(t) \frac{d\varepsilon_{kl}(t)}{dt} \\ &\leq -2\varepsilon_{kl}^T(t)(D + \Delta D)\varepsilon_{kl}(t) + 2\varepsilon_{kl}^T(t)(B + \Delta B) \\ &\quad \cdot [f(z_k(t)) - f(z_l(t))] + 2\varepsilon_{kl}^T(t)(C + \Delta C) \\ &\quad \cdot [g(z_k(t - \tau)) - g(z_l(t - \tau))] \\ &\quad + 2\varepsilon_{kl}^T(t)(u_k(t) - u_l(t)). \end{aligned} \quad (35)$$

By Lemma 2, we have

$$\begin{aligned} & 2\varepsilon_{kl}^T(t) \Delta B [f(z_k(t)) - f(z_l(t))] \\ & \leq \frac{1}{\xi_6} \varepsilon_{kl}^T(t) M_2 M_2^T \varepsilon_{kl}(t) + \xi_6 [f(z_k(t)) - f(z_l(t))]^T N_2^T N_2 \\ & \quad \cdot [f(z_k(t)) - f(z_l(t))], \end{aligned} \quad (36)$$

$$\begin{aligned} & 2\varepsilon_{kl}^T(t) \Delta C [g(z_k(t - \tau)) - g(z_l(t - \tau))] \\ & \leq \frac{1}{\xi_7} \varepsilon_{kl}^T(t) M_3 M_3^T \varepsilon_{kl}(t) + \xi_7 [g(z_k(t - \tau)) - g(z_l(t - \tau))]^T \\ & \quad \cdot N_3^T N_3 [g(z_k(t - \tau)) - g(z_l(t - \tau))]. \end{aligned} \quad (37)$$

Substituting (36) and (37) into (35), we get

$$\begin{aligned} \frac{dW(t)}{dt} &\leq \left[-2d_{\min} + \eta \|M_1 M_1^T\| + \frac{1}{\eta} \|N_1^T N_1\| + \|B\| \right. \\ &\quad + \|B\|\|P\| + \frac{1}{\xi_6} \|M_2 M_2^T\| + \xi_6 \|N_2^T N_2\| \|P\| + \|C\| \\ &\quad + \frac{1}{\xi_7} \|M_3 M_3^T\| \left. \right] W(t) + [\|C\|\|Q\| + \xi_7 \|N_3^T N_3\| \|Q\|] W(t - \tau) \\ &\quad + 2\varepsilon_{kl}^T(t)(u_k(t) - u_l(t)). \end{aligned} \quad (38)$$

According to (2) and (5), we obtain

$$\begin{aligned}
\varepsilon_{kl}^T(t)(u_k(t) - u_l(t)) &= \varepsilon_{kl}^T(t)\lambda \sum_{m=1, m \neq l}^r (y_{km} - y_{lm}) \\
&\quad \cdot (z_m(t_{ms}) - z_l(t)) \\
&= \varepsilon_{kl}^T(t)\lambda \sum_{m=1, m \neq l}^r (y_{km} - y_{lm}) \\
&\quad \cdot (\varepsilon_{ml}(t) - \mu_m(t)) \\
&= \lambda \sum_{m=1, m \neq l}^r (y_{km} - y_{lm}) \varepsilon_{kl}^T(t) \varepsilon_{ml}(t) \\
&\quad - \lambda \varepsilon_{kl}^T(t) \sum_{m=1, m \neq l}^r (y_{km} - y_{lm}) \mu_m(t).
\end{aligned} \tag{39}$$

From ETS (5), one has

$$\begin{aligned}
\|\mu_k(t)\| &\leq \alpha_k \sum_{m \in \mathcal{J}_k} \|z_m(t_{ms}) - z_k(t_{ks})\| \\
&\leq \alpha_k \sum_{m \in \mathcal{J}_k} \|\mu_k(t) - \mu_m(t) + \varepsilon_{ml}(t) - \varepsilon_{kl}(t)\| \\
&\leq \alpha_{\max} \left[r_k \|\mu_k(t)\| + \sum_{m \in \mathcal{J}_k} \|\mu_m(t)\| + \sum_{m \in \mathcal{J}_k} \|\varepsilon_{ml}(t)\| \right. \\
&\quad \left. + r_k \|\varepsilon_{kl}(t)\| \right] \\
&\leq \alpha_{\max} [r_{\max} \|\mu_k(t)\| + (r-1) \|\mu_m(t)\| \\
&\quad + (r-1) \|\varepsilon_{ml}(t)\| + r_{\max} \|\varepsilon_{kl}(t)\|],
\end{aligned} \tag{40}$$

then

$$\sum_{m \in \mathcal{J}_k} \|\mu_m(t)\| \leq G_l \sum_{m \in \mathcal{J}_k} \|\varepsilon_{ml}(t)\|, \quad k \in \mathcal{J}, \tag{41}$$

where

$$G_l = \frac{\alpha_{\max}(r + r_{\max} - 1)}{1 - \alpha_{\max}(r + r_{\max} - 1)}. \tag{42}$$

By (41), we have

$$\begin{aligned}
&\left| \sum_{k, m \in \mathcal{J} \setminus \{l\}} \zeta_k (y_{km} - y_{lm}) \varepsilon_{kl}(t) \mu_m(t) \right| \\
&\leq \zeta_{\max} y_{l \max} \sum_{k, m \in \mathcal{J} \setminus \{l\}} \|\varepsilon_{kl}(t)\| \|\mu_m(t)\| \\
&\leq G_l \zeta_{\max} y_{l \max} \sum_{k, m \in \mathcal{J} \setminus \{l\}} \|\varepsilon_{kl}(t)\| \|\varepsilon_{ml}(t)\| \\
&\leq G_l \zeta_{\max} y_{l \max} (r-1) \sum_{k, m \in \mathcal{J} \setminus \{l\}} \|\varepsilon_{kl}(t)\|^2 \\
&\leq \frac{G_l (r-1) \zeta_{\max} y_{l \max}}{\zeta_{\min}} W(t).
\end{aligned} \tag{43}$$

According to (18), one gets

$$\begin{aligned}
\sum_{k, m \in \mathcal{J} \setminus \{l\}} \zeta_k (y_{km} - y_{lm}) \varepsilon_{kl}^T(t) \varepsilon_{ml}(t) &= \sum_{k, m \in \mathcal{J} \setminus \{l\}} \frac{1}{2} [y_{km} \zeta_k + y_{mk} \zeta_m \\
&\quad - y_{lm} \zeta_k - y_{lk} \zeta_m] \varepsilon_{kl}^T(t) \varepsilon_{ml}(t) \\
&\leq -\nu \sum_{k \in \mathcal{J} \setminus \{l\}} \varepsilon_{kl}^T(t) \varepsilon_{kl}(t) \\
&\leq -\frac{\nu}{\zeta_{\max}} W(t).
\end{aligned} \tag{44}$$

Combining (39), (43), and (44), we obtain

$$\begin{aligned}
&\sum_{k, m \in \mathcal{J} \setminus \{l\}} \zeta_k \varepsilon_{kl}^T(t) (u_k(t) - u_l(t)) \\
&\leq \lambda \left[-\frac{\nu}{\zeta_{\max}} + \frac{G_l (r-1) \zeta_{\max} y_{l \max}}{\zeta_{\min}} \right] W(t).
\end{aligned} \tag{45}$$

From the definition of $W(t)$, one has

$$\begin{aligned}
\frac{dW(t)}{dt} &\leq -2d_{\min} - 2\lambda \left(\frac{\nu}{\zeta_{\max}} - \frac{G_l (r-1) \zeta_{\max} y_{l \max}}{\zeta_{\min}} \right) \\
&\quad + \|B\| + \|C\| + \|B\| \|P\| + \eta \|M_1 M_1^T\| + \frac{1}{\eta} \|N_1^T N_1\| \\
&\quad + \frac{1}{\xi_6} \|M_2 M_2^T\| + \xi_6 \|N_2^T N_2\| \|P\| + \frac{1}{\xi_7} \|M_3 M_3^T\| W(t) \\
&\quad + [\|C\| \|Q\| + \xi_7 \|N_3^T N_3\| \|Q\|] W(t - \tau).
\end{aligned} \tag{46}$$

By (31), there is a constant $h > 0$ such that

$$\begin{aligned}
2h - 2d_{\min} - 2\lambda \left(\frac{\nu}{\zeta_{\max}} - \frac{G_l (r-1) \zeta_{\max} y_{l \max}}{\zeta_{\min}} \right) \\
+ \|B\| + \|C\| + \|B\| \|P\| + \eta \|M_1 M_1^T\| + \frac{1}{\eta} \|N_1^T N_1\| \\
+ \frac{1}{\xi_6} \|M_2 M_2^T\| + \xi_6 \|N_2^T N_2\| \|P\| + \frac{1}{\xi_7} \|M_3 M_3^T\| \\
+ (1 + e^{2h\tau}) (\|C\| \|Q\| + \xi_7 \|N_3^T N_3\| \|Q\|) \leq 0.
\end{aligned} \tag{47}$$

Let $V(t) = e^{2h(t-t_0)} W(t)$, then

$$\begin{aligned}
\frac{dV(t)}{dt} \leq & \left[2h - 2d_{\min} - 2\lambda \left(\frac{\nu}{\zeta_{\max}} - \frac{G_l(r-1)\zeta_{\max}y_{l\max}}{\zeta_{\min}} \right) \right. \\
& + \|B\| + \|C\| + \|B\|\|P\| + \eta\|M_1M_1^T\| + \frac{1}{\eta}\|N_1^TN_1\| \\
& + \frac{1}{\xi_6}\|M_2M_2^T\| + \xi_6\|N_2^TN_2\|\|P\| + \frac{1}{\xi_7}\|M_3M_3^T\| \Big] V(t) \\
& + e^{2hr} \left[\|C\|\|Q\| + \xi_7\|N_3^TN_3\|\|Q\| \right] V(t-\tau) \\
\leq & [2h - 2d_{\min} - 2\lambda \left(\frac{\nu}{\zeta_{\max}} - \frac{G_l(r-1)\zeta_{\max}y_{l\max}}{\zeta_{\min}} \right) \\
& + \|B\| + \|C\| + \|B\|\|P\| + \eta\|M_1M_1^T\| + \frac{1}{\eta}\|N_1^TN_1\| \\
& + \frac{1}{\xi_6}\|M_2M_2^T\| + \xi_6\|N_2^TN_2\|\|P\| + \frac{1}{\xi_7}\|M_3M_3^T\| \\
& + (1 + e^{2hr})(\|C\|\|Q\| + \xi_7\|N_3^TN_3\|\|Q\|)] V(t) \leq 0,
\end{aligned} \tag{48}$$

when $V(t + \vartheta) \leq V(t)$ for any $\vartheta \in [-\tau, 0]$.

Let

$$\begin{aligned}
\tilde{V}(t) &= \sup_{\vartheta \in [-\tau, 0]} V(t + \vartheta), \\
\tilde{W}(t) &= \sup_{\vartheta \in [-\tau, 0]} W(t + \vartheta),
\end{aligned} \tag{49}$$

then, we have

$$\frac{d\tilde{V}(t)}{dt} \leq 0, \quad \forall t \geq t_0; \tag{50}$$

hence,

$$W(t) \leq \sum_{m \in \mathcal{S} \setminus \{l\}} \zeta_m \|\tilde{\varepsilon}_{lm}(t_0)\|^2 e^{-2h(t-t_0)}, \tag{51}$$

which implies

$$\|\varepsilon_{kl}(t)\| \leq \sqrt{\frac{\zeta_{\max}}{\zeta_{\min}}} \|\tilde{\varepsilon}_l(t_0)\| e^{-h(t-t_0)}, \tag{52}$$

for all $k \in \mathcal{S} \setminus \{l\}$, where $\varepsilon_l(t) = (\varepsilon_{l_1 l}^T(t), \varepsilon_{l_2 l}^T(t), \dots, \varepsilon_{l_{r-1} l}^T(t))$ and

$$\begin{aligned}
\|\tilde{\varepsilon}_{kl}(t)\| &= \sup_{\vartheta \in [-\tau, 0]} \|\varepsilon_{kl}(t + \vartheta)\|, \\
\|\tilde{\varepsilon}_l(t)\| &= \sup_{\vartheta \in [-\tau, 0]} \|\varepsilon_l(t + \vartheta)\|.
\end{aligned} \tag{53}$$

Then, for any $k, m \in \mathcal{S}$ and $t \geq t_0$, we obtain

$$\|\varepsilon_{km}(t)\| \leq \|\varepsilon_{kl}(t)\| + \|\varepsilon_{ml}(t)\| \leq 2\sqrt{\frac{\zeta_{\max}}{\zeta_{\min}}} \|\tilde{\varepsilon}_l(t_0)\| e^{-h(t-t_0)}. \tag{54}$$

Thus, model (6) is globally robustly exponentially synchronized with convergence rate h . The proof is finished. $\square$

Theorem 3. Let condition (5), Assumption 1, and Assumption 2 hold; there is $l \in \mathcal{S}$ satisfying $\alpha_k > 0$ and $\mathcal{S}_k \neq \emptyset$ ($k \in \mathcal{S} \setminus \{l\}$). Moreover, if there exist $r-1$ constants $\zeta_k > 0$ satisfying

$$\beta_k = -[y_{kk} - y_{lk}] - \frac{1}{\zeta_k} \sum_{m=1, m \neq k, l}^r \zeta_m |y_{km} - y_{lm}| > 0, \tag{55}$$

and there is $0 < \alpha_{\max} < 1/(2r_{\max})$ and three constants $\eta > 0$, $\xi_6 > 0$, and $\xi_7 > 0$ satisfying

$$\begin{aligned}
& -2d_{\min} - 2\lambda \left(\beta_{\min} - \frac{2\alpha_{\max} r_{\max} \zeta_{\max} \|K_l\|_{\inf}}{(1 - 2\alpha_{\max} r_{\max}) \zeta_{\min}} \right) + \|B\| + \|C\| \\
& + \|B\|\|P\| + \eta\|M_1M_1^T\| + \frac{1}{\eta}\|N_1^TN_1\| + \frac{1}{\xi_6}\|M_2M_2^T\| \\
& + \xi_6\|N_2^TN_2\|\|P\| + \frac{1}{\xi_7}\|M_3M_3^T\| + \|C\|\|Q\| \\
& + \xi_7\|N_3^TN_3\|\|Q\| < 0,
\end{aligned} \tag{56}$$

then model (6) is globally robustly exponentially synchronized under ETS (5).

Proof. Let $\overline{W}(t) = \max_{k \in \mathcal{S} \setminus \{l\}} \{(1/\zeta_k) \|\varepsilon_{kl}(t)\|^2\}$, and fixing $k \in \mathcal{S} \setminus \{l\}$ satisfying

$$\overline{W}(t) = \frac{1}{\zeta_k} \|\varepsilon_{kl}(t)\|^2, \tag{57}$$

then

$$\begin{aligned}
\frac{d\overline{W}(t)}{dt} &= \frac{2}{\zeta_k} \|\varepsilon_{kl}(t)\| \frac{d\|\varepsilon_{kl}(t)\|}{dt} \\
&= \frac{2}{\zeta_k} \|\varepsilon_{kl}(t)\| \text{sign}(\varepsilon_{kl}(t))^T \frac{d\varepsilon_{kl}(t)}{dt} \\
&\leq \left[-2d_{\min} + \eta\|M_1M_1^T\| + \frac{1}{\eta}\|N_1^TN_1\| \right. \\
&\quad + \|B\| + \|B\|\|P\| + \frac{1}{\xi_6}\|M_2M_2^T\| \\
&\quad + \xi_6\|N_2^TN_2\|\|P\| + \|C\| + \frac{1}{\xi_7}\|M_3M_3^T\| \Big] \overline{W}(t) \\
&\quad + [\|C\|\|Q\| + \xi_7\|N_3^TN_3\|\|Q\|] \overline{W}(t-\tau) \\
&\quad + \frac{2}{\zeta_k} \|\varepsilon_{kl}(t)\| \text{sign}(\varepsilon_{kl}(t))^T (u_k(t) - u_l(t)),
\end{aligned} \tag{58}$$

where

$$u_k(t) - u_l(t) = \lambda \sum_{m \in \mathcal{F} \setminus \{l\}} (y_{km} - y_{lm})(z_m(t_{ms}) - z_l(t)). \quad (59)$$

Then, one gets

$$\begin{aligned} \text{sign}(\varepsilon_{kl}(t))^T (u_k(t) - u_l(t)) &= \lambda \text{sign}(\varepsilon_{kl}(t))^T \\ &\cdot \sum_{m \in \mathcal{F} \setminus \{l\}} (y_{km} - y_{lm})(\varepsilon_{ml}(t) - \mu_m(t)) \\ &\leq \lambda (y_{kk} - y_{lk}) \|\varepsilon_{kl}(t)\| \\ &\quad + \lambda \sum_{m \in \mathcal{F} \setminus \{l,k\}} |y_{km} - y_{lm}| \|\varepsilon_{ml}(t)\| \\ &\quad + \lambda \sum_{m \in \mathcal{F} \setminus \{l\}} |y_{km} - y_{lm}| \|\mu_m(t)\|. \end{aligned} \quad (60)$$

By Theorem 1, we obtain

$$\|\mu(t)\|_{\max} \leq \tilde{G}_l \|\varepsilon_l(t)\|_{\max}, \quad (61)$$

where $\tilde{G}_l = (2\alpha_{\max} r_{\max}) / (1 - 2\alpha_{\max} r_{\max})$, $\|\mu(t)\|_{\max} = \max_{k \in \mathcal{F} \setminus \{l\}} \{\|\mu_k(t)\|\}$, and $\|\varepsilon_l(t)\|_{\max} = \max_{m \in \mathcal{F} \setminus \{l\}} \{\|\varepsilon_{kl}(t)\|\}$.

By (60) and (61), we have

$$\begin{aligned} &\frac{1}{\zeta_k} \|\varepsilon_{kl}(t)\| \text{sign}(\varepsilon_{kl}(t))^T (u_k(t) - u_l(t)) \\ &\leq \lambda \left[y_{kk} - y_{lk} + \frac{1}{\zeta_k} \sum_{m \in \mathcal{F} \setminus \{l,k\}} \zeta_m |y_{km} - y_{lm}| \right] \overline{W}(t) \\ &\quad + \frac{1}{\zeta_k} \|\varepsilon_{kl}(t)\| \lambda \sum_{m \in \mathcal{F} \setminus \{l\}} |y_{km} - y_{lm}| \|\mu_m(t)\|_{\max} \\ &\leq -\lambda \beta_k \overline{W}(t) + \lambda \tilde{G}_l \frac{\zeta_{\max} \|K_l\|_{\inf}}{\zeta_{\min}} \overline{W}(t); \end{aligned} \quad (62)$$

thus, one has

$$\begin{aligned} \frac{d\overline{W}(t)}{dt} &\leq \left[-2d_{\min} - 2\lambda \left(\beta_{\min} - \tilde{G}_l \frac{\zeta_{\max} \|K_l\|_{\inf}}{\zeta_{\min}} \right) \right. \\ &\quad + \eta \|M_1 M_1^T\| + \frac{1}{\eta} \|N_1^T N_1\| + \|B\| + \|B\| \|P\| \\ &\quad + \frac{1}{\xi_6} \|M_2 M_2^T\| + \xi_6 \|N_2^T N_2\| \|P\| + \|C\| \\ &\quad \left. + \frac{1}{\xi_7} \|M_3 M_3^T\| \right] \overline{W}(t) \\ &\quad + [\|C\| \|Q\| + \xi_7 \|N_3^T N_3\| \|Q\|] \overline{W}(t - \tau). \end{aligned} \quad (63)$$

From (56), we can know that there is a $h > 0$ such that

$$\begin{aligned} &2h - 2d_{\min} - 2\lambda \left(\beta_{\min} - \frac{2\alpha_{\max} r_{\max} \zeta_{\max} \|K\|_{\inf}}{(1 - 2\alpha_{\max} r_{\max}) \zeta_{\min}} \right) \\ &\quad + \|B\| + \|C\| + \|B\| \|P\| + \eta \|M_1 M_1^T\| + \frac{1}{\eta} \|N_1^T N_1\| \\ &\quad + \frac{1}{\xi_6} \|M_2 M_2^T\| + \xi_6 \|N_2^T N_2\| \|P\| + \frac{1}{\xi_7} \|M_3 M_3^T\| \\ &\quad + (1 + e^{2h\tau}) (\|C\| \|Q\| + \xi_7 \|N_3^T N_3\| \|Q\|) \leq 0. \end{aligned} \quad (64)$$

Similar to Theorem 2, by (63), we get

$$\overline{W}(t) \leq \frac{1}{\zeta_{\min}} \|\tilde{\varepsilon}_l(t_0)\|^2 e^{-2h(t-t_0)}, \quad (65)$$

then

$$\|\varepsilon_{kl}(t)\| \leq \sqrt{\frac{\zeta_{\max}}{\zeta_{\min}}} \|\tilde{\varepsilon}_l(t_0)\| e^{-h(t-t_0)}, \quad k \in \mathcal{F} \setminus \{l\}. \quad (66)$$

Thus, model (6) is globally robustly exponentially synchronized with convergence rate h under ETS (5). The proof is finished. $\square$

Remark 4. Comparing Theorem 2 to Theorem 3, it is not hard to see that when r is large enough and $r_{\max}$ is relatively small, condition (30) is better than condition (55), but condition (31) is not as good as condition (56).

Theorem 4. Let condition (4), Assumption 1, and Assumption 2 hold; $\alpha_k > 0$ and $\mathcal{F}_k \neq \emptyset$. Then model (6) is globally robustly exponentially synchronized under ETS (5) if there exist r constants $\zeta_k > 0$ ($k \in \mathcal{F}$) and a constant $\nu > 0$ satisfying

$$\frac{\overline{\Psi}K + K^T \overline{\Psi}}{2} + \nu I \leq 0, \quad (67)$$

and there is $0 < \alpha_{\max} < 1/(r + r_{\max})$ and three constants $\eta > 0$, $\xi_8 > 0$, and $\xi_9 > 0$ satisfying

$$\begin{aligned} &-2d_{\min} - 2\lambda \left(\frac{\nu}{\zeta_{\max}} - \frac{\alpha_{\max} (r + r_{\max}) r \zeta_{\max} y_{\max}}{(1 - \alpha_{\max} (r + r_{\max})) \zeta_{\min}} \right) + \|B\| + \|C\| \\ &\quad + \|B\| \|P\| + \eta \|M_1 M_1^T\| + \frac{1}{\eta} \|N_1^T N_1\| + \frac{1}{\xi_8} \|M_2 M_2^T\| \\ &\quad + \xi_8 \|N_2^T N_2\| \|P\| + \frac{1}{\xi_9} \|M_3 M_3^T\| + \|C\| \|Q\| \\ &\quad + \xi_9 \|N_3^T N_3\| \|Q\| < 0, \end{aligned} \quad (68)$$

where $d_{\min} = \min_{1 \leq k \leq n} \{d_k\}$, $K = (y_{km})_{k,m \in \mathcal{F}}$, $\overline{\Psi} = \text{diag}\{\zeta_1, \zeta_2, \dots, \zeta_r\}$, and $y_{\max} = \max_{k,m \in \mathcal{F}} \{|y_{km}|\}$.

Proof. A reference system is given by

$$\begin{aligned} \frac{dz_0(t)}{dt} &= -(D + \Delta D)z_0(t) + (B + \Delta B)f(z_0(t)) + (C + \Delta C) \\ &\quad \cdot g(z_0(t - \tau)) + J(t) + \gamma \rho z_0(t), \quad t \geq t_0, \end{aligned} \quad (69)$$

where $u_0 = \lambda q z_0(t)$, and there exists a $s \geq 1$ satisfying $t \in [t_{ks}, t_{k(s+1)})$.

According to (6) and (69), we have

$$\begin{aligned} \frac{d\varepsilon_{k0}(t)}{dt} &= \frac{dz_k(t)}{dt} - \frac{dz_0(t)}{dt} \\ &= -(D + \Delta D)(z_k(t) - z_0(t)) + (B + \Delta B) \\ &\quad [f(z_k(t)) - f(z_0(t))] + (C + \Delta C) \\ &\quad [g(z_k(t - \tau)) - g(z_0(t - \tau))] + u_k(t) - u_0(t), \end{aligned} \quad (70)$$

where

$$u_k(t) - u_0(t) = \lambda \sum_{m=1, m \neq l}^r y_{km}(z_m(t_{ms}) - z_0(t)). \quad (71)$$

Let $\hat{W}_k(t) = \varepsilon_{k0}^T(t) \varepsilon_{k0}(t)$ and $\hat{W}(t) = \sum_{k \in \mathcal{J}} \zeta_k \hat{W}_k(t)$, then

$$\begin{aligned} \frac{d\hat{W}_k(t)}{dt} &= 2\varepsilon_{k0}^T(t) \frac{d\varepsilon_{k0}(t)}{dt} \\ &\leq -2\varepsilon_{k0}^T(t) (D + \Delta D) \varepsilon_{k0}^T(t) + 2\varepsilon_{k0}^T(t) (B + \Delta B) \\ &\quad \cdot [f(z_k(t)) - f(z_0(t))] + 2\varepsilon_{k0}^T(t) (C + \Delta C) \\ &\quad \cdot [g(z_k(t - \tau)) - g(z_0(t - \tau))] \\ &\quad + 2\varepsilon_{k0}^T(t) (u_k(t) - u_0(t)). \end{aligned} \quad (72)$$

By Lemma 2, we have

$$\begin{aligned} &2\varepsilon_{k0}^T(t) \Delta B [f(z_k(t)) - f(z_0(t))] \\ &\leq \frac{1}{\xi_8} \varepsilon_{k0}^T(t) M_2 M_2^T \varepsilon_{k0}(t) \\ &\quad + \xi_8 [f(z_k(t)) - f(z_0(t))]^T N_2^T N_2 \\ &\quad \cdot [f(z_k(t)) - f(z_0(t))], \end{aligned} \quad (73)$$

$$\begin{aligned} &2\varepsilon_{k0}^T(t) \Delta C [g(z_k(t - \tau)) - g(z_0(t - \tau))] \\ &\leq \frac{1}{\xi_9} \varepsilon_{k0}^T(t) M_3 M_3^T \varepsilon_{k0}(t) \\ &\quad + \xi_9 [g(z_k(t - \tau)) - g(z_0(t - \tau))]^T \\ &\quad \cdot N_3^T N_3 [g(z_k(t - \tau)) - g(z_0(t - \tau))]. \end{aligned} \quad (74)$$

Substituting (73) and (74) into (72), we get

$$\begin{aligned} \frac{d\hat{W}(t)}{dt} &\leq \left[-2d_{\min} + \eta \|M_1 M_1^T\| + \frac{1}{\eta} \|N_1^T N_1\| \right. \\ &\quad + \|B\| + \|B\| \|P\| + \frac{1}{\xi_8} \|M_2 M_2^T\| \\ &\quad + \xi_8 \|N_2^T N_2\| \|P\| + \|C\| + \frac{1}{\xi_9} \|M_3 M_3^T\| \left. \right] \hat{W}(t) \\ &\quad + \left[\|C\| \|Q\| + \frac{1}{\xi_9} \|N_3^T N_3\| \|Q\| \right] \\ &\quad \cdot \hat{W}(t - \tau) + 2\zeta_k \varepsilon_{k0}^T(t) (u_k(t) - u_0(t)). \end{aligned} \quad (75)$$

Similar to Theorem 2, we obtain

$$\|\varepsilon_{k0}(t)\| \leq \sqrt{\frac{\zeta_{\max}}{\zeta_{\min}}} \|\tilde{\varepsilon}_0(t_0)\| e^{-h(t-t_0)}, \quad (76)$$

for all $k \in \mathcal{J}$, where $\varepsilon_0(t) = (\varepsilon_{10}^T, \dots, \varepsilon_{r0}^T)^T$, $\|\tilde{\varepsilon}_0(t_0)\| = \sup_{\vartheta \in [-\tau, 0]} \|\varepsilon_0(t + \vartheta)\|$, and h is a position constant such that

$$\begin{aligned} &2h - 2d_{\min} - 2\lambda \left(\frac{\gamma}{\zeta_{\max}} - \frac{\alpha_{\max}(r + r_{\max})r\zeta_{\max}\gamma_{\max}}{(1 - \alpha_{\max}(r + r_{\max}))\zeta_{\min}} \right) \\ &\quad + \|B\| + \|C\| + \|B\| \|P\| + \eta \|M_1 M_1^T\| + \frac{1}{\eta} \|N_1^T N_1\| \\ &\quad + \frac{1}{\xi_8} \|M_2 M_2^T\| + \xi_8 \|N_2^T N_2\| \|P\| + \frac{1}{\xi_9} \|M_3 M_3^T\| \\ &\quad + (1 + e^{2h\tau}) (\|C\| \|Q\| + \xi_9 \|N_3^T N_3\| \|Q\|) \leq 0. \end{aligned} \quad (77)$$

Then, for any $k, m \in \mathcal{J}$ and $t \geq t_0$, we get

$$\|\varepsilon_{km}(t)\| \leq \|\varepsilon_{k0}(t)\| + \|\varepsilon_{m0}(t)\| \leq 2\sqrt{\frac{\zeta_{\max}}{\zeta_{\min}}} \|\tilde{\varepsilon}_0(t_0)\| e^{-h(t-t_0)}. \quad (78)$$

Thus, the proof is finished. $\square$

Theorem 5. Let condition (4), Assumption 1, and Assumption 2 hold; $\alpha_k > 0$ and $\mathcal{J}_k \neq \emptyset$ ($k \in \mathcal{J}$), and if there exist r constants $\zeta_k > 0$ ($k \in \mathcal{J}$) satisfying

$$\beta_k = -\gamma_{kk} - \frac{1}{\zeta_k} \sum_{m=1}^r \zeta_m |\gamma_{km}| > 0, \quad (79)$$

and there is $0 < \alpha_{\max} < 1/(2r_{\max})$ and three constants $\eta > 0$, $\xi_8 > 0$, and $\xi_9 > 0$ satisfying

$$\begin{aligned} &-2d_{\min} - 2\lambda \left(\beta_{\min} - \frac{2\alpha_{\max} r_{\max} \zeta_{\max} \|K\|_{\inf}}{(1 - 2\alpha_{\max} r_{\max}) \zeta_{\min}} \right) \\ &\quad + \|B\| + \|C\| + \|B\| \|P\| + \eta \|M_1 M_1^T\| \\ &\quad + \frac{1}{\eta} \|N_1^T N_1\| + \frac{1}{\xi_8} \|M_2 M_2^T\| + \xi_8 \|N_2^T N_2\| \|P\| \\ &\quad + \frac{1}{\xi_9} \|M_3 M_3^T\| + \|C\| \|Q\| + \xi_9 \|N_3^T N_3\| \|Q\| < 0, \end{aligned} \quad (80)$$

then model (6) is globally robustly exponentially synchronized under ETS (5).

Remark 5. From Theorems 2–5, it is easy to find that they are closely related. In Theorems 2 and 3, the coupling matrix can be unstable. However, in Theorems 4 and 5, the coupling matrix must be stable. Theorems 4 and 5 can be derived from Theorems 2 and 3, respectively, when the coupling matrix is stable, but Theorems 3 and 4 are more simpler and convenient than Theorems 2 and 3.

Remark 6. Most existing works on multisystem networks (see [2–4]) require that the coupling matrix is always the Laplace matrix, in which row sums of coupling matrix all are zero and nondiagonal elements are nonnegative. In this paper, the coupling matrix need not be the Laplace matrix; that is to say, row sums of coupling matrix can be nonzero constant and nondiagonal elements are arbitrary. Compared with the previous works, the advantage of this paper is that the coupling matrix need not be the Laplace matrix, and the criteria obtained are more simple and convenient.

Remark 7. Actually, if we consider MNNs with non-Laplace coupling matrix and norm-unbounded uncertainty terms in this paper, we can get Theorem 1 by changing (17)–(22) and get Theorems 2 and 3 by changing (36) and (37). Similarly, we also get Theorems 4 and 5 by changing (72) and (74). In addition, if we consider time-varying delay instead of constant delay into system (6), when the time-varying delay $\tau_{ks}(t)$ is bounded, namely, there exists a constant σ satisfying $0 < \tau_{ks}(t) < \sigma$, and several similar conclusions can be obtained.

4. Three Numerical Examples

We consider model (1) with $n = 2$, where activation function $f(z) = g(z)$ and $f(z) = (f_1(z_1), f_2(z_2))^T$ such that

$$f_m(z_m) = \frac{(|z_m + 1| - |z_m - 1|)}{2}, \quad m = 1, 2, \quad (81)$$

the weight matrices

$$\begin{aligned} D &= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \\ B &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}, \\ C &= \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & -0.2 \end{bmatrix}, \\ \Delta D &= \sin \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ \Delta B &= \cos \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \\ \Delta C &= \sin \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}. \end{aligned} \quad (82)$$

Example 1. Consider system (6) with $r = 4$. Let $\tau = 0.001$, $\eta = 0.5$, $\xi_6 = 0.5$, $\xi_7 = 0.5$, and coupling matrix

$$K = \begin{bmatrix} -1.19 & 0 & 0.4 & 0.8 \\ 0 & -1.19 & 0.2 & 1 \\ 0 & 0.2 & -1.19 & 1 \\ 1 & 0.2 & 0 & -1.19 \end{bmatrix}, \quad (83)$$

we know that $\varrho = 0.01$ and K has a positive eigenvalue 0.01. Choosing $l = 1$, then

$$K_1 = \begin{bmatrix} -1.19 & -0.2 & 0.2 \\ 0.2 & -1.59 & 0.2 \\ 0.2 & -0.4 & -1.99 \end{bmatrix}. \quad (84)$$

Let $\nu = 1.0546$, then

$$\frac{K_1 + K_1^T}{2} + \nu I \leq 0, \quad (85)$$

which implies that condition (30) holds, where $\zeta_i = 1$ ($2 \leq i \leq 4$); let $\lambda = 10$ and $\alpha_k = 0.012$, then

$$\begin{aligned} & -2d_{\min} - 2\lambda \left(\nu - \frac{\alpha_{\max}(r-1)(r+r_{\max}-1)g_{l\max}}{1-\alpha_{\max}(r+r_{\max}-1)} \right) \\ & + \|B\| + \|C\| + \|B\|\|P\| + \eta\|M_1M_1^T\| + \frac{1}{\eta}\|N_1^TN_1\| \\ & + \frac{1}{\xi_6}\|M_2M_2^T\| + \xi_6\|N_2^TN_2\|\|P\| + \frac{1}{\xi_7}\|M_3M_3^T\| \\ & + \|C\|\|Q\| + \xi_7\|N_3^TN_3\|\|Q\| = -0.8420 < 0; \end{aligned} \quad (86)$$

hence, condition (31) holds; furthermore, we choose the initial states $z_1(9) = (0.6, 0.5)^T$, $z_2(9) = (-1, 0.8)^T$, $z_3(9) = (0.7, -1)^T$, and $z_4(9) = (-1.2, 0.2)^T$ for $9 \in [-0.001, 0]$. Figure 1 describes the evolutions of the event-triggered controller and synchronization errors. Figure 2 depicts the threshold and the evolutions of measurement errors. By Theorem 2, we can know that system (6) is globally robustly exponentially synchronized under ETS (5).

Example 2. Let $\tau = 0.001$, $\eta = 0.5$, $\xi_8 = 0.5$, $\xi_9 = 0.5$, and coupling matrix

$$K = \begin{bmatrix} -1.06 & 0 & 0.02 & 0.04 \\ 0 & -1.03 & -0.02 & 0.06 \\ -0.02 & 0 & -1.08 & 0.01 \\ 0.02 & 0 & 0.04 & -1.06 \end{bmatrix}, \quad (87)$$

we know that $\varrho = -1$ and K is a stable matrix. Let $\nu = 1.25$, then

$$\frac{K + K^T}{2} + \nu I \leq 0; \quad (88)$$

hence, condition (67) holds, where $\zeta_i = 1$ ($1 \leq i \leq 4$); choosing $\lambda = 10$ and $\alpha_k = 0.021$, then

$$\begin{aligned} & -2d_{\min} - 2\lambda \left(\nu - \frac{\alpha_{\max}(r+r_{\max})ry_{\max}}{1-\alpha_{\max}(r+r_{\max})} \right) \\ & + \|B\| + \|C\| + \|B\|\|P\| + \eta\|M_1M_1^T\| + \frac{1}{\eta}\|N_1^TN_1\| \\ & + \frac{1}{\xi_8}\|M_2M_2^T\| + \xi_8\|N_2^TN_2\|\|P\| + \frac{1}{\xi_9}\|M_3M_3^T\| \\ & + \|C\|\|Q\| + \xi_9\|N_3^TN_3\|\|Q\| \\ & = -0.64 < 0, \end{aligned} \quad (89)$$

FIGURE 1: Time evolutions of the event-triggered controller and the synchronization errors in Example 1.

FIGURE 2: Time evolutions of measurement errors $\mu_k(t)$ and the thresholds Φ_{ks} ($k = 1, 2, 3, 4$) in Example 1.

FIGURE 3: Time evolutions of the event-triggered controller and the synchronization errors in Example 2.

FIGURE 4: Time evolutions of measurement errors $\mu_k(t)$ and the thresholds Φ_{ks} ($k = 1, 2, 3, 4$) in Example 2.

FIGURE 5: Time evolutions of the event-triggered controller and the synchronization errors in Example 3.

thus, condition (68) holds; furthermore, we choose the initial states $z_1(\vartheta) = (0.6, 0.5)^T$, $z_2(\vartheta) = (-1, 0.8)^T$, $z_3(\vartheta) = (0.7, -1)^T$, and $z_4(\vartheta) = (-1.2, 0.2)^T$ for $\vartheta \in [-0.001, 0]$. Figure 3 describes the evolutions of the event-triggered controller and synchronization errors. Figure 4 depicts the threshold and the evolutions of measurement errors. By Theorem 4, we can know that system (6) is globally robustly exponentially synchronized under ETS (5).

Example 3. Let $\tau = 0.001$, $\eta = 0.5$, $\xi_8 = 0.5$, $\xi_9 = 0.5$, and coupling matrix

$$K = \begin{bmatrix} -2.6 & 0 & 0.2 & 0 & 0 & 0.4 \\ 0 & -2.3 & -0.3 & 0 & 0 & 0.6 \\ 0 & -0.2 & -2.3 & 0 & 0 & 0.5 \\ 0 & -0.3 & 0 & -2.7 & 0 & 1 \\ 0 & 0 & 0.1 & 0 & -2.6 & 0.5 \\ 0.4 & 0 & 0 & 0 & 0.2 & -2.6 \end{bmatrix}, \quad (90)$$

we know that $\varrho = -2$ and K is a stable matrix. Let $\nu = 0.99$, then

$$\beta_k = -\gamma_{kk} - \frac{1}{\zeta_k} \sum_{m=1}^6 \zeta_m |y_{km}| \geq 0.4; \quad (91)$$

hence, condition (79) holds, where $\zeta_i = 1$ ($1 \leq i \leq 6$), choosing $\lambda = 18$ and $\alpha_k = 0.015$, then

$$\begin{aligned} & -2d_{\min} - 2\lambda \left(\beta_{\min} - \frac{2\alpha_{\max} r_{\max} \|K\|_{\inf}}{1 - 2\alpha_{\max} r_{\max}} \right) \\ & + \|B\| + \|C\| + \|B\|\|P\| + \eta \|M_1 M_1^T\| \\ & + \frac{1}{\eta} \|N_1^T N_1\| + \frac{1}{\xi_8} \|M_2 M_2^T\| \\ & + \xi_8 \|N_2^T N_2\| \|P\| + \frac{1}{\xi_9} \|M_3 M_3^T\| + \|C\|\|Q\| \\ & + \xi_9 \|N_3^T N_3\| \|Q\| = -0.2636 < 0, \end{aligned} \quad (92)$$

thus, condition (80) holds; in addition, we choose the initial states $z_1(\vartheta) = (0.6, 0.5)^T$, $z_2(\vartheta) = (-1, 0.8)^T$, $z_3(\vartheta) = (0.7, -1)^T$, $z_4(\vartheta) = (-1.2, 0.2)^T$, $z_5(\vartheta) = (-1.5, -0.8)^T$, and $z_6(\vartheta) = (-0.1, 0.6)^T$ for $\vartheta \in [-0.01, 0]$. Figure 5 describes the evolutions of the event-triggered controller and synchronization errors. Figure 6 depicts the threshold and the evolutions of measurement errors. By Theorem 5, we can know that system (6) is globally robustly exponentially synchronized under ETS (5).

Remark 8. It can be seen that the row sums of coupling matrices in Examples 1, 2, and 3 all are 0.01, -1 , and -2 , respectively, and there are nondiagonal elements in Examples 2 and 3. Thus, our coupling matrices all are non-Laplacian.

5. Concluding Remarks

This paper has introduced a novel ETS for MNNs with parameter uncertainty and time delay. By using matrix inequality techniques and establishing the event-triggered mechanism, some sufficient criteria are derived for judging global robust exponential synchronization of the system; in addition, the lower bounds of the sampling time intervals are also obtained. In particular, the most existing papers on multisystem networks require that the coupling matrix is Laplace matrix, in which row sums of coupling matrix all are zero and nondiagonal elements are nonnegative. However, the coupling matrix need not be the Laplace matrix in this paper; that is to say, row sums of coupling matrix can be nonzero constant and nondiagonal elements are arbitrary. In

FIGURE 6: Time evolutions of measurement errors $\mu_k(t)$ and the thresholds Φ_{ks} ($k = 1, 2, 3, 4, 5, 6$) in Example 3.

the future, we can do the following works: (1) MNNs will be extended to fractional-order MNNs. (2) It is meaningful to study MNNs with parameter uncertainty and time delay via centralized and decentralized data-sampling approaches. (3)

MNNs with parameter uncertainty and time-varying delay may be valuable research topic. (4) It is a challenge to study MNNs with non-Laplace coupling matrix and norm-unbounded uncertainty terms.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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