Hindawi
Complexity
Volume 2019, Article ID 8012890, 13 pages
https://doi.org/10.1155/2019/8012890

Research Article

Weighted Couple-Group Consensus Analysis of Heterogeneous Multiagent Systems with Cooperative-Competitive Interactions and Time Delays

Xingcheng Pu,[1,2] **Chaowen Xiong**,[3] **Lianghao Ji**,[4] **and Longlong Zhao**[4]

[1] *Key Laboratory of Intelligent Analysis and Decision on Complex Systems, Chongqing University of Posts and Telecommunications, Chongqing 400065, China*
[2] *School of Science, Chongqing University of Posts and Telecommunications, Chongqing 400065, China*
[3] *School of Automation, Chongqing University of Posts and Telecommunications, Chongqing 400065, China*
[4] *School of Computer Science and Technology, Chongqing University of Posts and Telecommunications, Chongqing 400065, China*

Correspondence should be addressed to Xingcheng Pu; puxc@cqupt.edu.cn and Chaowen Xiong; s170331052@stu.cqupt.edu.cn

Received 9 November 2018; Revised 23 February 2019; Accepted 28 February 2019; Published 17 March 2019

Guest Editor: Marin Lujak

In this paper, the weighted couple-group consensus of continuous-time heterogeneous multiagent systems with input and communication time delay is investigated. A novel weighted couple-group consensus protocol based on cooperation and competition interaction is designed, which can relax the in-degree balance condition. By using graph theory, general Nyquist criterion and Gerschgorin disc theorem, the time delay upper limit that the system may allow is obtained. The conclusions indicate that there is no relationship between weighted couple-group consensus and communication time delay. When the agents input time delay, the coupling weight between the agents, and the systems control parameters are satisfied, the multiagent system can converge to any given weighted coupling group consistent state. The experimental simulation results verify the correctness of the conclusion.

1. Introduction

As an important branch of distributed system, multiagent systems (MASs) have been paid great attention by many scholars due to their wide application in many fields [1–6], such as multirobot system, wireless sensor network, and distributed target tracking. For example, in [5], the distributed formation control problem for multiple nonholonomic wheeled mobile robots would be solved by using a variable transformation, algebraic graph theory, matrix theory, and Lyapunov control approach.

Consensus or synchronization, as one of the most important problems of MASs, is to design an appropriately distributed protocol to make different agents achieve a common state. Group consensus, as an extension of consensus, is very suitable for multitasks and large-scale problems. Up to now, there are many results for consensus or group consensus [7–18]. On the other hand, the controllability problem of multiagent systems has attracted great interests and concern since Tanner proposed it in 2004. In the past decades, many controllability criteria have been given for multiagent systems [19–24]. However, most of these results focused on single time scale. In [25], the group controllability of two-time-scale multiagent networks was firstly proposed and some easy-to-use criteria were proposed for group controllability of two-time-scale multiagent networks compared with the rank criterion. In [26], Long et al. further investigated second-order controllability of two-time-scale multiagent systems, and some more effective second-order controllability conditions would be determined only by the eigenvalues of system matrices. In [27], a new format of time-varying formation shape was proposed, and a new class of distributed adaptive observer-based controllers was designed under the mild assumption that both leaders and followers were introspective. As we know, most existing results have been obtained mainly based on the nodes of the network system. In some other real situations,

each agent cannot obtain the neighbors state information in a real networked system. Therefore, in [28], the authors studied the discrete-time nonnegative edge synchronization of networked systems based on neighbors output information, which gives us a novelty and interesting synchronization method.

1.1. Related Contributions. It should be noticed that all the aforementioned results are based on the common assumption that the multiagent systems are homogeneous. In this situation, all agents of the whole systems have the same dynamics. However, in real life, almost every agent has its own dynamics because of different external and interaction impacts. Hence, it is natural for us to model heterogeneous multiagent systems. In recent years, some heterogeneous multiagent systems models have been established [29–32]. In [29], dynamical consensus of heterogeneous multiagent systems which consist of the first-order and second-order agent dynamics has been discussed. In [30], a consensus protocol is proposed for high-ordered heterogeneous systems with uncertain communication delays. Furthermore, more and more scholars pay much attention to the group consensus of heterogeneous systems. For example, in [31], a heterogeneous system consisting of first-order and second-order agents has been studied on the basis of fixed and switching topologies. In [32], some sufficient group consensus conditions have been obtained for a kind of heterogeneous system with diverse input time delays based on frequency-domain analysis method and matrix theory. In [33], some sufficient couple-group consensus conditions have been derived for a kind of discrete-time heterogeneous systems consisting of first-order and second- order agents under the influence of communication and input time delays. In [34], Li et al. studied the consensus problem in heterogeneous linear discrete-time MASs. In [35], Cui et al. discussed the consensus problem of heterogeneous chaotic network systems with or without delay. In [36], the consensus problems of linear systems and nonlinear systems were studied separately. In [37], Liu et al. studied the consensus problem of heterogeneous MASs under certain assumptions. In [38], Goldin et al. studied the consensus of heterogeneous networks with undirected topology.

At the same time, some achievements have been made in the research of weighted consensus. For example, in [39], the concept of weighted consensus was proposed, and the multiagent weighted average consensus is studied. In [40], Shi et al. studied the robust consensus control for a class of MASs by PID algorithm and weighted edge dynamics. In addition, MASs based on cooperation-competition interactions are also receiving more and more attention. In [41], Hu et al. studied the second-order consensus problem of heterogeneous MASs. In [42], Hu et al. studied the swarming behavior of multiple Euler-Lagrange systems with cooperation-competition interactions.

1.2. The Main Motivation. It is obvious that heterogeneous systems are more complex than homogeneous systems and it is more difficult for us to deal with the relevant crucial topics. Inspired by the recent developments for heterogeneous multiagent systems, this paper will further investigate the weighted group consensus. To the best of our knowledge, most of existing literatures only discuss homogeneous systems, the multiagent systems in which all agent share a common value.

In this paper, we mainly investigate the weighted group consensus for a class of continuous-time heterogeneous multiagent systems with input and communication time delay. In recent years, although group consensus of multiagent systems has derived many significant results. It is worth mentioning that most of the existing results only discussed the situation where all agents possessed a fixed weighted-value, even most of the obtained results mainly focused on the consensus of heterogeneous multiagent systems, and few results were proposed for group consensus of heterogeneous networks with input and communication time delay. Furthermore, all these related conclusions were based only on agents' competitive or cooperative relation. However, in complicated practical situation, the consensus protocol needs to be adjusted with circumstances, cooperative tasks, or other constraint conditions. All these reasons incite us to study the weighted group consensus for heterogeneous multiagent systems with input and communication time delay.

1.3. Statement of Contributions. There are three main contributions in this paper. Firstly, the model is different from cooperative or competitive heterogeneous networks, both cooperative and competitive interactions are considered, it extends the scope of the existing research, and a kind of weighted couple-group consensus agreement based on cooperation-competition relationship is introduced, which is quite different from the literature [31, 32, 35, 37, 38]. Relying on the new protocol control, the agents can receive neighbor information more reasonably and speed up the system to achieve group consensus. Secondly, in order to simplify the analysis process, we remove the dynamic virtual speed of the first-order agent, such as in [29, 31, 32, 37]. A novel weighted couple-group consensus protocol is designed, which relaxes the in-degree balance condition and the results are also applicable to directed and undirected graphs. On the other hand, we turn the weighted matrix into a dynamic form, which makes the designed controller more flexible. Thirdly, some sufficient conditions have been obtained for the group consensus of this system by using graph theory, general Nyquist criterion, and Gerschgorin disc theorem. Unlike the [31, 32, 37], we do not require that the system satisfies the condition that the geometric versatility of the zero eigenvalues of the Laplacian matrix is not less than 2, which makes the system's topology more flexible. With the help of these conditions, the time delay upper limit of this system can be computed and the multiagent system can converge to any given state only if the weighted group consensus parameters are satisfied. The simulation results well verify the correctness of the conclusion.

The rest of this article is organized as follows. The second section lists some preliminary knowledge and problem description. The third section presents the main results and proof process of group consensus. The fourth section

verifies the correctness and effectiveness of the proposed method through simulation. Finally we come to a conclusion.

Note. In this context, $\mathbb{C}$ denotes a complex set and R denotes a real set. I_N represents a unit matrix, where N represents a dimension. $\mathrm{Re}(Z)$ is the real part, and $|Z|$ is the model, where $\forall Z \in \mathbb{C}$. $\lambda_i(A)$ represents the *ith* eigenvalue of matrix A, and $\det(A)$ represents the determinant of the matrix.

2. Problem Description and Preliminary Knowledge

In order to facilitate the follow-up work, we need introduce some preliminary knowledge of the graph theory.

2.1. Graph Theory and Interconnection Topology. Considering N agents, the topological relationship of the agent is represented by the graph $G = (V, E, A)$, where $V = \{v_1, v_2, \cdots, v_N\}$ represent the set of vertices of the graph. $E \subseteq V \times V$ and $A = (a_{ij})_{N\times N} \in R^{N\times N}$ represent the edge set and the adjacency matrix, respectively. In this article, the case of containing a self-loop is not considered.

Note that the undirected graph can be thought as a special directed graph, and we assume $a_{ij} > 0$ if $e_{ij} \in E$ in this paper. That is, if and only if the node (agent) is able to receive information from the node (agent), $a_{ij} > 0$. At the same time, $N_i = \{j \in V : e_{ij} \in E\}$ represents the set of neighbor nodes, and $D_i = \deg_{in}(i) = \sum_{j=1}^{N} a_{ij}$ represents the set of nodes within the degree, where the in-degree matrix D can also be expressed as $\mathrm{diag}\{d_1, d_2, \cdots d_N\}$. Therefore, $L = D - A$ is defined as a Laplacian matrix. Note. The adjacency matrix A is a symmetric matrix if and only if the graph is an undirected graph.

2.2. Problem Statement. Based on the above-mentioned preliminary knowledge of graph theory, in this paper we propose a heterogeneous multiagent system with N agents, which contains second-order and first-order dynamics. In order not to lose generality, it is assumed that the first n agents are second-order dynamics, and the last m agents are first-order dynamics, where $N = m + n$. The specific system model can be designed as follows:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \\ & \qquad i \in o_1 \\ \dot{x}_i(t) &= u_i(t), \quad i \in o_2, \end{aligned} \tag{1}$$

where $o_1 = \{1, 2, \ldots, n\}$, $o_2 = \{n+1, n+2, \ldots, n+m\}$, $o = o_1 \cup o_2$, $x_i(t)$, $v_i(t)$, and $u_i(t) \in R$, where $x_i(t)$ is the location of the agent i, $u_i(t)$ is the control rule of the i agent, and $v_i(t)$ is the speed. Obviously, since each agent's neighbors can be first-order or second-order, they are divided into $N_{i,s}$ and $N_{i,f}$. So the neighbor node set $N_i = N_{i,f} \cup N_{i,s}$. Because the dynamics of the agent in the system are heterogeneous, Its adjacency matrix can be expressed as

$$A = \begin{bmatrix} A_s & A_{sf} \\ A_{fs} & A_f \end{bmatrix} \tag{2}$$

where $A_s \in R^{n\times n}$ is an adjacency matrix composed of second-order agents, A_{sf} is composed of coupling weights from second-order agents to first-order agents, A_{fs} is composed of first-order to second-order coupling weights, and $A_f \in R^{m\times m}$ is an adjacency matrix composed of first-order agents. Therefore, we can further write the Laplacian matrix as follows.

$$L = D - A = \begin{bmatrix} L_s + D_{sf} & -A_{sf} \\ -A_{fs} & L_f + D_{fs} \end{bmatrix} \tag{3}$$

The matrix L represents the interaction between only the second-order agents, and the matrix L_f represents the interaction between only the first-order agents. It should be noted that both of the matrices are Laplacian matrices, where $D_{sf} = \mathrm{diag}\{\sum_{j\in N_{i,f}} a_{ij},\ i \in o_1\}$ and $D_{fs} = \mathrm{diag}\{\sum_{j\in N_{i,s}} a_{ij},\ i \in o_2\}$ are the in-degree matrix of the agent i, which represents the neighbor information received from different orders.

To facilitate the follow-up work, here are some definitions and lemmas.

Definition 1. For the heterogeneous MASs to progressively implement the weighted couple-group consensus, the system should satisfy the following two conditions:

$$\begin{aligned} \lim_{t\longrightarrow+\infty} \left\| x_i(t) - x_j(t) \right\| &= 0, \quad \text{if } i, j \in o_k,\ k = 1, 2, \\ \lim_{t\longrightarrow+\infty} \left\| v_i(t) - v_j(t) \right\| &= 0, \quad \text{if } i, j \in o_k,\ k = 1. \end{aligned} \tag{4}$$

Definition 2. For the bipartite graph $G = (V, E)$, the vertex set V can be split into two disjoint subsets V_1 and V_2, where $V_1 \cap V_2 = \emptyset$, and at the same time $\forall e = (w, q) \in E$, where $w \in V_1$ and $q \in V_2$.

Lemma 3 (see [15]). *For an undirected bipartite graph, $\lambda_i(L) \in R$. At the same time, it should be noted that directed bipartite graphs containing directed spanning trees have the following two properties: (1) $\mathrm{rank}(L) = n - 1$, (2) when $\lambda_i(L) \neq 0$, $\mathrm{Re}(\lambda_i(L)) > 0$, where n is the number of system agents, matrix $L = D + A$.*

3. Main Results

Most existing works are based on the competition or cooperation relationship of agents. At the same time, only a

single form of delay is considered. For example, in [38], the grouping of heterogeneous systems with the same input delay is studied. Its system is described as follows:

$$\begin{aligned}\dot{x}_i(t) &= v_i(t),\\ \dot{v}_i(t) &= \sum_{j\in o_1} a_{ij}\left[x_j(t-\tau)-x_i(t-\tau)\right]\\ &\quad+\sum_{j\in o_2} a_{ij}x_j(t-\tau)\\ &\quad+\sum_{j\in o_1} a_{ij}\left[v_j(t-\tau)-v_i(t-\tau)\right]\\ &\quad+\sum_{j\in o_2} a_{ij}v_j(t-\tau),\\ &\qquad i\in o_1.\end{aligned} \tag{5}$$

And

$$\begin{aligned}\dot{x}_i(t) &= v_i(t-\tau)+\sum_{j\in o_2} a_{ij}\left[x_j(t-\tau)-x_i(t-\tau)\right]\\ &\quad+\sum_{j\in o_1} a_{ij}x_j(t-\tau),\\ \dot{v}_i(t) &= \sum_{j\in o_2} a_{ij}\left[x_j(t)-x_i(t)\right]+\sum_{j\in o1} a_{ij}x_j(t),\\ &\qquad i\in o_2.\end{aligned} \tag{6}$$

In (5) and (6), it is not difficult to see that the agents rely on cooperative relationships for information exchange, and there are also speed estimates in the first-order agents. Considering that in practical applications, competitive interactions are inevitable. Therefore, we design a weighted couple-group consensus protocol that utilizes the competition-cooperative interaction of agents. The specific form is as follows:

$$\begin{aligned}\dot{x}_i(t) &= v_i(t),\\ \dot{v}_i(t) &= \alpha_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)-x_i(t-\tau)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)+x_i(t-\tau)\right]\right]-\beta_i v_i(t-\tau),\\ &\qquad i\in o_1.\end{aligned} \tag{7}$$

And

$$\begin{aligned}\dot{x}_i(t) &= \gamma_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)-x_i(t-\tau)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)+x_i(t-\tau)\right]\right],\quad i\in o_2.\end{aligned} \tag{8}$$

Here τ_{ij} indicates communication delay between agent j and agent i, and τ represents the identical input delay of the agents. N_{si} denotes a neighbor of the same dynamic as the agent i. Similarly, N_{di} denotes a neighbor of a different dynamic from the agent i. Meanwhile, α_i, β_i, and $\gamma_i > 0$, where $\alpha_i = \{\alpha_1, \alpha_2, \ldots, \alpha_N\}$, $\beta_i = \{\beta_1, \beta_2, \ldots, \beta_N\}$, $\gamma_i = \{\gamma_1, \gamma_2, \ldots, \gamma_N\}$, N is the number of agents.

Remark 4. This paper designs a controller with weighted coefficients. By adjusting the weighted coefficient of the controller, the state of many agents can be globally converged to any given weighted state. Compared with the original controller, the designed controller is more flexible and more adaptable to different states. At the same time, when the agent j and the agent i have the same dynamic, we adopt a cooperative approach. When the agents j and i have different dynamic, we use a competitive approach. By using cooperation-competition relationship, we ensure that heterogeneous MASs can achieve weighted couple-group consensus.

Theorem 5. *Based on system (7) and (8), and the undirected bipartite graph is assumed to be the topology of the system. if these conditions hold:* $\beta_i^2 > 2\alpha_i D_i$ *and* $\tau \in \{0, \min[1/2\beta_i, 1/2\gamma_i \max\{\widetilde{D}_i\}]\}$, *where* $D_i = \sum_{j\in N_i} a_{ij}$, $i \in o_1$ *and* $\widetilde{D}_i = \sum_{j\in N_i} a_{ij}$, $i \in o_2$, *then the system can progressively achieve weighted couple-group consensus.*

Proof. By performing the Laplace transform on (7) and (8), we can get the following expression:

$$\begin{aligned}sx_i(s) &= v_i(s),\\ sv_i(s) &= \alpha_i\left[\sum_{j\in N_{si}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)-e^{-\tau s}x_i(s)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)+e^{-\tau s}x_i(s)\right]\right]-\beta_i e^{-\tau s}v_i(s),\\ &\qquad i\in o_1.\end{aligned} \tag{9}$$

$$\begin{aligned}sx_i(s) &= \gamma_i\left[\sum_{j\in N_{si}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)-e^{-\tau s}x_i(s)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)+e^{-\tau s}x_i(s)\right]\right],\quad i\in o_2.\end{aligned} \tag{10}$$

Transform $x_i(t)$ and $v_i(t)$ into Laplace forms $x_i(s)$ and $v_i(s)$, respectively. From the (9), we have

$$\begin{aligned}s^2x_i(s) &= \alpha_i\left[\sum_{j\in N_{si}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)-e^{-\tau s}x_i(s)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s)+e^{-\tau s}x_i(s)\right]\right]-\beta_i s e^{-\tau s}x_i(s),\\ &\qquad i\in o_1.\end{aligned} \tag{11}$$

After transformation, we can get the following formula:

$$sx_i(s) = \frac{-s^2 x_i(s) + \alpha_i \left[\sum_{j\in N_{si}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s) - e^{-\tau s}x_i(s)\right] - \sum_{j\in N_{di}} a_{ij}\left[e^{-\tau_{ij}s}x_j(s) + e^{-\tau s}x_i(s)\right]\right]}{\beta_i e^{-\tau s}}, \quad i \in o_1. \tag{12}$$

Next, we define $x_s(s) = [x_1(s), x_2(s), \ldots, x_n(s)]^T$, $x_f(s) = [x_{n+1}(s), x_{n+2}(s), \ldots, x_{n+m}(s)]^T$, and

$$\widehat{L} = \left(\widehat{l}_{ij}\right)_{(n+m)\times(n+m)} = \begin{cases} e^{-\tau_{ij}s}a_{ij}, & i \neq j \\ \sum_{j\in N_i} a_{ij}e^{-\tau s}, & i = j. \end{cases} \tag{13}$$

According to (10) and (12), we can get

$$\begin{aligned} &sx_s(s) \\ &= \frac{-s^2 C_2 x_s(s) + C_2 C_1^{-1}\left(\widehat{L}_s + \widehat{D}_{sf}\right)x_s(s) - C_2 C_1^{-1}\widehat{A}_{sf}x_f(s)}{e^{-\tau s}}, \\ &sx_f(s) = -C_3^{-1}\widehat{A}_{fs}x_s(s) - C_3^{-1}\left(\widehat{L}_f + \widehat{D}_{fs}\right)x_f(s). \end{aligned} \tag{14}$$

Here

$$\begin{aligned} C_1 &= \begin{pmatrix} \frac{1}{\alpha_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\alpha_N} \end{pmatrix}, \\ C_2 &= \begin{pmatrix} \frac{1}{\beta_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{\beta_N} \end{pmatrix}, \\ C_3 &= \begin{pmatrix} \frac{1}{\gamma_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/\gamma_N \end{pmatrix}. \end{aligned} \tag{15}$$

Next, we define $y(s) = [x_s^T(s), x_f^T(s)]^T$, and we have

$$sy(s) = \widetilde{\Upsilon}(s)\, y(s). \tag{16}$$

Here

$$\widetilde{\Upsilon}(s) = \begin{bmatrix} \frac{-C_2 s^2 - C_2 C_1^{-1}\left(\widehat{L}_s + \widehat{D}_{sf}\right)}{e^{-\tau s}} & \frac{-C_2 C_1^{-1}\widehat{A}_{sf}}{e^{-\tau s}} \\ -C_3^{-1}\widehat{A}_{fs} & -C_3^{-1}\left(\widehat{L}_f + \widehat{D}_{fs}\right) \end{bmatrix}. \tag{17}$$

According to (16), we can get $\widetilde{\Theta}(s) = \det(sI - \widetilde{\Upsilon}(s))$. According to the Lyapunov stability criterion, when the $\mathrm{Re}(\lambda_i(\widetilde{\Theta}(s))) < 0$, or $s = 0$, the system achieves group consensus. Next, using general Nyquist criteria, we discuss these two situations.

When $s = 0$, it can be clearly seen that 0 is a characteristic value of the matrix $D + A$, so one root of the formula can be obtained when $s = 0$. At the same time, when $s = 0$, $\widetilde{\Theta}(0) = \det(D + A)(\prod_{i=1}^n \alpha_i / \prod_{i=1}^n \beta_i)\prod_{i=1}^m \gamma_i$.

When $s \neq 0$, set $\widetilde{\Theta}(s) = \det(\Phi(s) + I)$ and

$$\Phi(s) = \begin{bmatrix} \frac{s^2 C_2 + C_2 C_1^{-1}\left(\widehat{L}_s + \widehat{D}_{sf}\right)}{se^{-\tau s}} & \frac{C_2 C_1^{-1}\alpha_i \widehat{A}_{sf}}{se^{-\tau s}} \\ \frac{C_3^{-1}\widehat{A}_{fs}}{s} & \frac{C_3^{-1}\left(\widehat{L}_f + \widehat{D}_{fs}\right)}{s} \end{bmatrix} \tag{18}$$

where $s = j\omega$. In order for the system to achieve group consensus, the general Nyquist criterion, if and only if the point $(-1, j0)$ is not surrounded by the Nyquist curve, $\widetilde{\Theta}(s)'s$ root is located on the left half of the complex field. Based on the Gerschgorin disk theorem, we can get

$$\lambda(\Phi(j\omega)) \in \{\Phi_i, i \in o_1\} \cup \{\Phi_i, i \in o_2\}. \tag{19}$$

When $i \in o_1$, we have the following.

$$\begin{aligned} \Phi_i = \Bigg\{ x : x \in \mathbb{C}, &\left| x - \frac{\alpha_i}{j\omega\beta_i}\sum_{j\in N_i} a_{ij} - \frac{j\omega}{\beta_i}e^{j\omega\tau} \right| \\ &\leq \sum_{j\in N_i}\left|\frac{\alpha_i a_{ij}}{j\omega\beta_i}e^{-j\omega(\tau_{ij}-\tau)}\right| \Bigg\} \end{aligned} \tag{20}$$

For the convenience of calculation, we set $D_i = \sum_{j\in N_i} a_{ij}$, $i \in o_1$. At the same time, according to the general criteria, since the point $(-a, j0)$, $a \geq 1$, cannot be encircled in Φ_i, $i \in o_1$, we can further transform the inequality into the following form:

$$\left| -a - \frac{\alpha_i D_i}{j\omega\beta_i} - \frac{j\omega}{\beta_i}e^{j\omega\tau} \right| > \sum_{j\in N_i}\left|\frac{\alpha_i a_{ij}}{j\omega\beta_i}e^{-j\omega(\tau_{ij}-\tau)}\right|. \tag{21}$$

According to the Euler formula and from (21), we can get

$$\left|-a-\frac{\alpha_i D_i}{\omega\beta_i}j-\frac{j\omega}{\beta_i}\left(\cos\omega\tau+j\sin\omega\tau\right)\right| > \left|\frac{\alpha_i D_i}{j\omega\beta_i}\left(\cos\omega\left(\tau_{ij}-\tau\right)-j\sin\omega\left(\tau_{ij}-\tau\right)\right)\right|. \tag{22}$$

After some transformation, we can get the following.

$$a^2-\frac{2a\omega}{\beta_i}\sin\omega\tau+\frac{\omega^2}{\beta_i^2}-\frac{2\alpha_i D_i}{{\beta_i}^2}\cos\omega\tau>0 \tag{23}$$

It is easy to see from (23) that when $a \geq 1$, $a^2-(2a\omega/\beta_i)\sin\omega\tau$ is monotonically increasing.

$$1-\frac{2\omega}{\beta_i}\sin\omega\tau+\frac{\omega^2}{\beta_i^2}-\frac{2\alpha_i D_i}{{\beta_i}^2}\cos\omega\tau>0 \tag{24}$$

Since β_i is a positive number, we can transform (24) into the following form.

$$\beta_i^2-2\omega\beta_i\sin\omega\tau+\omega^2-2\alpha_i D_i\cos\omega\tau>0 \tag{25}$$

According to (24), it is obvious that the following two inequalities are true:

$$2\alpha_i D_i\cos\omega\tau-\beta_i^2<0 \tag{26}$$

and

$$2\omega\beta_i\sin\omega\tau-\omega^2<0. \tag{27}$$

According to (26), we can get $\beta_i^2>2\alpha_i D_i$, because $\cos\omega\tau\leq 1$. According to (27), we can change it to the following form.

$$1-2\beta_i\tau\left(\frac{\sin\omega\tau}{\omega\tau}\right)>0 \tag{28}$$

Since $(\sin\omega\tau/\omega\tau)\leq 1$, (27) is established if and only if $\tau\leq(1/2\beta_i)$.

Similarly, when $i\in o_2$, we can get the following inequalities according to the Gerschgorin theorem:

$$\Phi_i=\left\{x:x\in\mathbb{C},\left|x-\frac{\gamma_i}{j\omega}\sum_{j\in N_i}a_{ij}e^{-j\omega\tau}\right| \leq \sum_{j\in N_i}\left|\frac{\gamma_i a_{ij}}{j\omega}e^{-j\omega\tau_{ij}}\right|\right\} \tag{29}$$

so, the point $(-a, j0)$, $a\geq 1$, cannot be encircled in Φ_i, $i\in o_2$, and then the following inequality is obtained.

$$\left|-a-\frac{\gamma_i}{j\omega}\sum_{j\in N_i}a_{ij}e^{-j\omega\tau}\right|>\sum_{j\in N_i}\left|\frac{\gamma_i a_{ij}}{j\omega}e^{-j\omega\tau_{ij}}\right| \tag{30}$$

Next, we define $\widetilde{D}_i=\sum_{j\in N_i}a_{ij}$, $i\in o_2$; then from (30), we have the following.

$$\left|-a+\frac{\gamma_i\widetilde{D}_i}{j\omega}\sum_{j\in N_i}\left(j\cos\omega\tau+\sin\omega\tau\right)\right| > \sum_{j\in N_i}\left|\frac{\gamma_i\widetilde{D}_i}{j\omega}\left(-j\cos\omega\tau_{ij}-\sin\omega\tau_{ij}\right)\right| \tag{31}$$

After some calculations, we can get the following simplified formula.

$$a^2-\frac{2a\gamma_i\widetilde{D}_i}{\omega}\sin\omega\tau>0 \tag{32}$$

From (32), we know that $a^2-(2a\gamma_i\widetilde{D}_i/\omega)\sin\omega\tau$ will gradually increase as a increases. Here we set $a=1$. Obviously, we have the following.

$$1-\frac{2\gamma_i\widetilde{D}_i}{\omega}\sin\omega\tau>0 \tag{33}$$

Since $(\sin\omega\tau/\omega\tau)\leq 1$, (32) is established if and only if $\tau\leq(1/2\gamma_i\widetilde{D}_i)$.

Obviously, we have completed the proof of Theorem 5. □

Corollary 6. *Based on system (7) and (8), a bipartite digraph containing a directed spanning tree is assumed to be the topology of the system. If these conditions hold: $\beta_i^2>2\alpha_i D_i$ and $\tau\in[0,\min\{1/2\beta_i, 1/2\gamma_i\max\{\widetilde{D}_i\}\}]$, where $D_i=\sum_{j\in N_i}a_{ij}$, $i\in o_1$, and $\widetilde{D}_i=\sum_{j\in N_i}a_{ij}$, $i\in o_2$, then the system can progressively achieve weighted couple-group consensus.*

Combined with the previous analysis, it is clear that the theorem is completed.

Using Proof and Lemma 3, it is clear that Corollary 6 is true.

Theorem 5 is proved.

Remark 7. From Theorem 5, we can see that the control parameters α_i, β_i, γ_i and coupling weight of the system are the key parameters affecting the consensus of the weighted couple-group, and the input time delay is determined by the coupling weight and the control parameters. However, we can see that communication delay has no effect on group consensus.

Remark 8. The proposed system (7) and (8) is constructed by using the cooperation-competitive interaction between agents in this paper. Since most of the agents currently working rely on the cooperation or competitive relationship, such as in [17, 18, 29, 31, 32, 34–39], this paper studies the group consensus of heterogeneous complex systems from a new perspective. At the same time, it should be noted that in the proposed protocol, the first-order agent does not contain virtual speed estimation, which can make more rational use of resources and reduce computational cost, for example, in [29, 31, 32, 37].

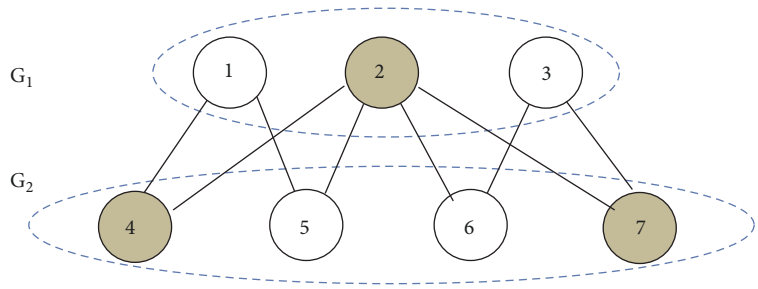


Figure 1: The bipartite digraph topology of the heterogeneous MASs.

Remark 9. Different from the works in [31, 32, 37], we have relaxed the condition of intra-degree balance, which facilitates communication between agents. In real life, there are many limitations in in-degree balance, because it will result in no actual communication between subsystems [13]. In other words, it will cause the interaction between agents in different subsystems to be offset. At the same time, we do not require that the system satisfies the condition that the geometric versatility of the zero eigenvalues of the Laplacian matrix is not less than 2, which makes the system's topology more flexible.

Remark 10. Most of the works are weighted by a fixed value. We use dynamic weighted methods here, namely, α_i, β_i, and γ_i. The weighted coefficients corresponding to each agent are different, which enables the MASs state to converge globally to any given weighted state. Compared with the original controller, the designed controller is more flexible and more adaptable to different states. In addition, in most of the existing works, the consideration of the delay problem is relatively simple. Only the effects of either input delays [36] or time delays are not considered, such as [31, 32, 36].

To discuss the effect of different input delays and communication delays on the multiagent implementation of group consensus, we rewrite (7) and (8) as follows:

$$\begin{aligned}\dot{x}_i(t) &= v_i(t),\\ \dot{v}_i(t) &= \alpha_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)-x_i\left(t-\tau_i\right)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)+x_i\left(t-\tau_i\right)\right]\right]-\beta_i v_i(t\\ &\quad-\tau_i), \quad i\in o_1.\end{aligned} \tag{34}$$

$$\begin{aligned}\dot{x}_i(t) &= \gamma_i\left[\sum_{j\in N_{si}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)-x_i\left(t-\tau_i\right)\right]\right.\\ &\quad\left.-\sum_{j\in N_{di}} a_{ij}\left[x_j\left(t-\tau_{ij}\right)+x_i\left(t-\tau_i\right)\right]\right], \quad i\in o_2.\end{aligned} \tag{35}$$

Here τ_{ij} represents the communication delay between the agents i and j, and τ_i represents the input time delay of the agent i.

Theorem 11. *Based on Protocol (34) and (35), the undirected bipartite graph is assumed to be the topology of the system. If these conditions hold: $\beta_i^2 > 2\alpha_i D_i$ and if $i \in o_1$, $\tau_i \in [0, 1/2\beta_i]$ or, otherwise, $\tau_i \in [0, 1/2\gamma_i \widetilde{D}_i]$, $i \in o_2$, where $D_i = \sum_{j\in N_i} a_{ij}$, $i \in o_1$, and $\widetilde{D}_i = \sum_{j\in N_i} a_{ij}$, $i \in o_2$, then the system can progressively achieve weighted couple-group consensus.*

Corollary 12. *Based on Protocol (34) and (35), a bipartite digraph containing a directed spanning tree is assumed to be the topology of the system. If these conditions hold: $\beta_i^2 > 2\alpha_i D_i$ and if $i \in o_1$, $\tau_i \in [0, 1/2\beta_i]$ or, otherwise, $\tau_i \in [0, 1/2\gamma_i \widetilde{D}_i]$, $i \in o_2$, where $D_i = \sum_{j\in N_i} a_{ij}$, $i \in o_1$, and $\widetilde{D}_i = \sum_{j\in N_i} a_{ij}$, $i \in o_2$, then the system can progressively achieve weighted couple-group consensus.*

The conclusion here is obvious.

Remark 13. From Theorem 11, the communication delay of the agent has no effect on the group consensus of the system. At the same time, the upper limit of the input time delay is controlled by the control parameters and coupling weights with the same dynamics, and the delay conditions between different dynamics are different. Communication delay has no effect on the group consensus of the system.

Remark 14. Since the system needs some other external conditions when implementing group consensus, our assumed topology is not a specific topology. For example, in [31, 32, 36, 37], the topology of the system is also an undirected graph or a graph containing a directed spanning tree. At the same time, in order to achieve group consensus, some additional assumptions are needed, mentioned in Remarks 7, 8, and 9.

4. Simulation

In this section, several simulation results will be used to illustrate the validity of the results obtained. Figure 1 shows a binary topology of a heterogeneous system. The entire system

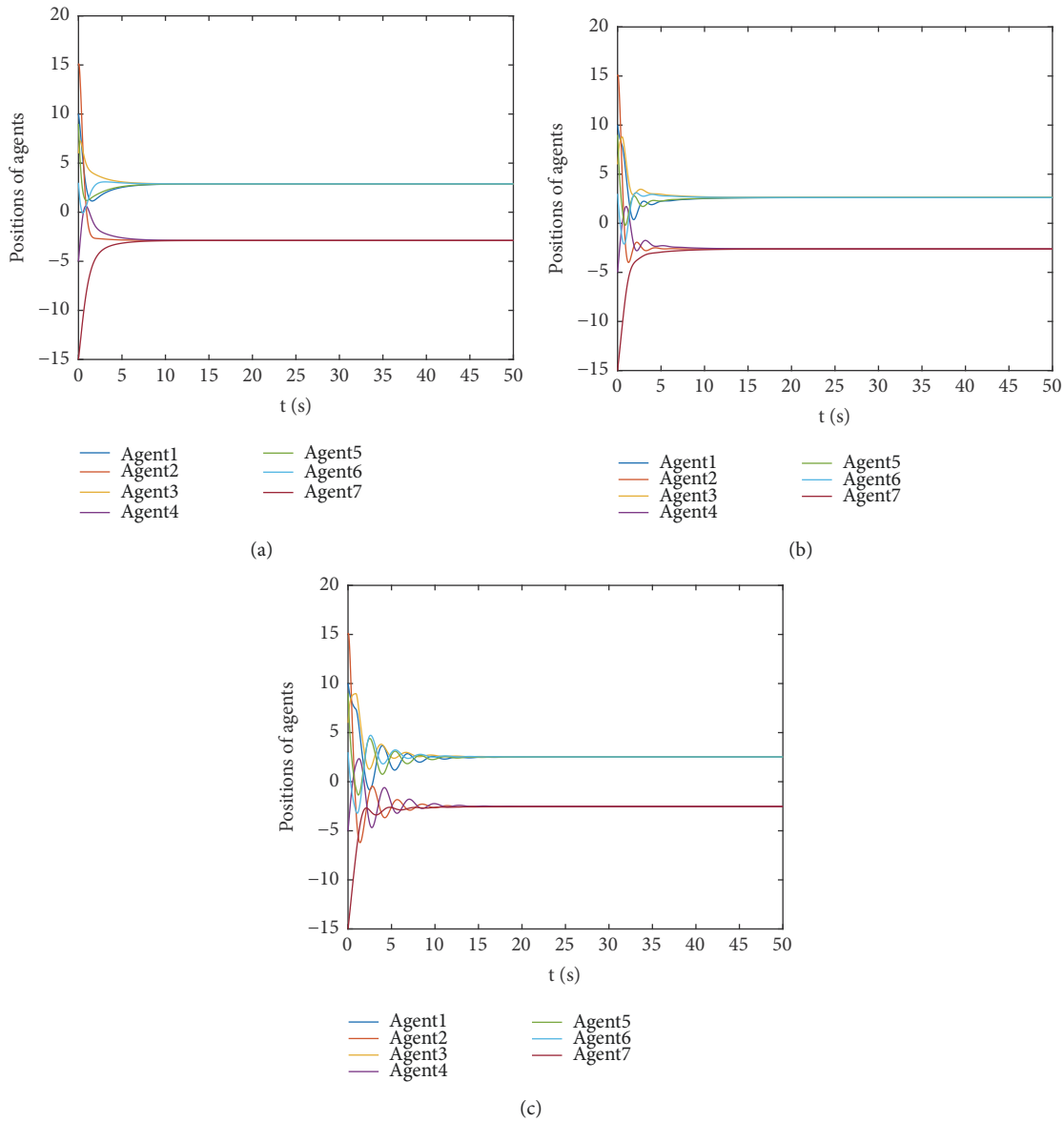


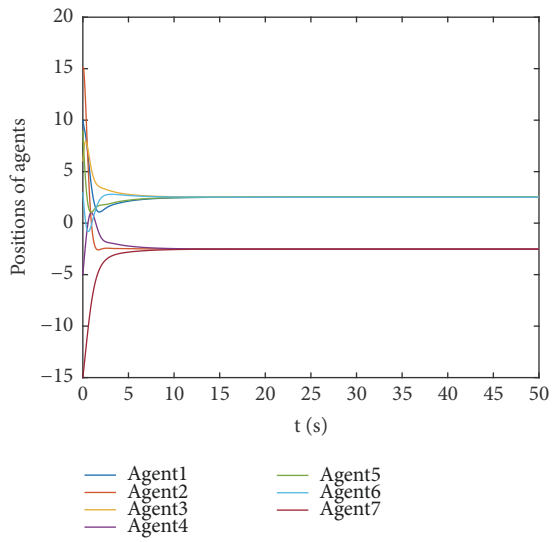
Figure 2: The agents position trajectories, where $\tau = 0.05$. (a) $\tau_{ij} = 0$, (b) $\tau_{ij} = 0.5$, and (c) $\tau_{ij} = 0.9$.

is divided into two subgroups, G_1 and G_2. The system contains agents 1, 2, 3, 4, 5, 6, and 7. In order not to lose generality, we denote 2, 4, and 7 as second-order agents, denoted by o_1. The first-order agent includes the remaining agents 1, 3, 5, and 6 and is represented by o_2. Obviously, subgroup G_1 and subgroup G_1 are heterogeneous in Figure 1.

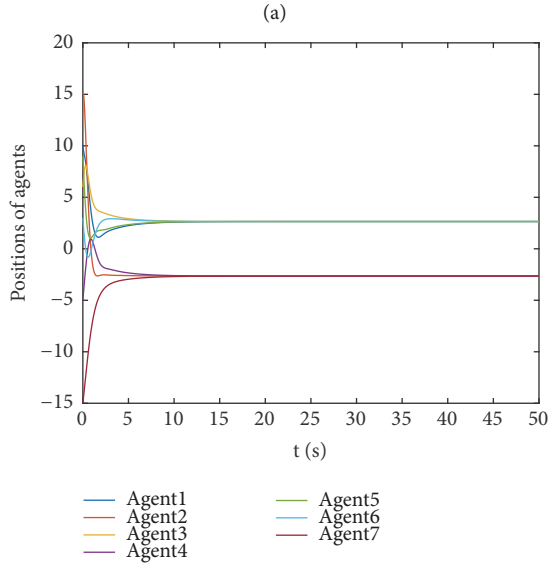
Remark 15. From Figure 1, the dynamics of the agents in subgroup G_1 and subgroup G_2 are heterogeneous. Obviously, we do not require that the dynamics of agents within the same subgroup be homogeneous, such as [32, 37].

Example 16. For convenience, we set $a_{ij} = 1$, $i, j \in [1, 7]$, and let $\alpha_i = \mathrm{diag}[1, 1.5, 2, 3, 0.9, 0.8, 0.5]$, $\beta_i = \mathrm{diag}[3, 4, 3, 4, 3, 3, 2]$, $\gamma_i = \mathrm{diag}[1, 1.5, 2, 3, 0.9, 0.8, 0.5]$. Since Figure 1 is an undirected bipartite graph, we can get $d_1 = 2$, $d_2 = 4$, $d_3 = 2$, $d_4 = 2$, $d_5 = 2$, $d_6 = 2$, $d_7 = 2$.

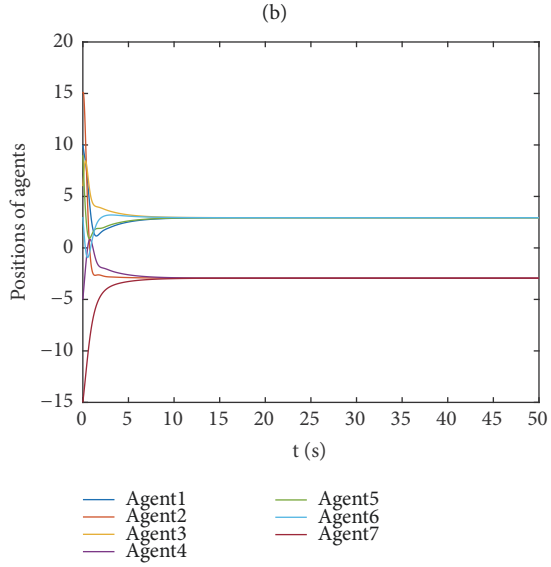
According to the qualification conditions proposed by Theorem 5, we can calculate the range of the input delay as $\tau = \min\{\tau_1, \tau_2, \tau_3, \tau_4, \tau_5, \tau_6, \tau_7\}$. In the simulation experiment, we assume $\tau = 0.05$. Obviously, τ at this time satisfies all the qualifications. To verify the impact of different delays on system group consensus, we assume different input delays and communication delays. In Figure 2, we assume an input delay of $\tau = 0.05$ and then input different communication delays to compare their effects on the system convergence rate. In Figure 3, we fixed the communication delay and then assumed different input delays.



(a)

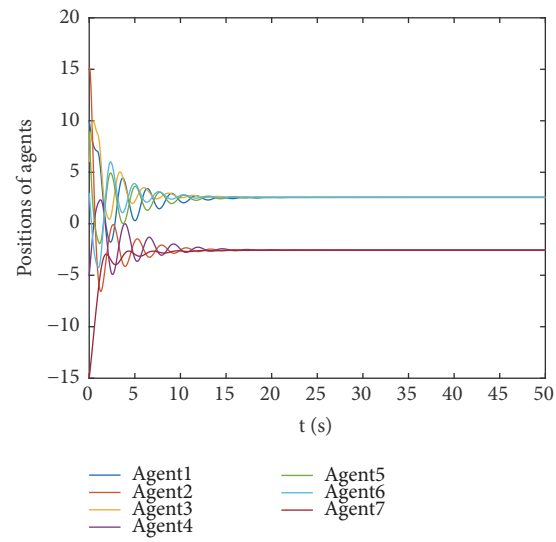


(b)

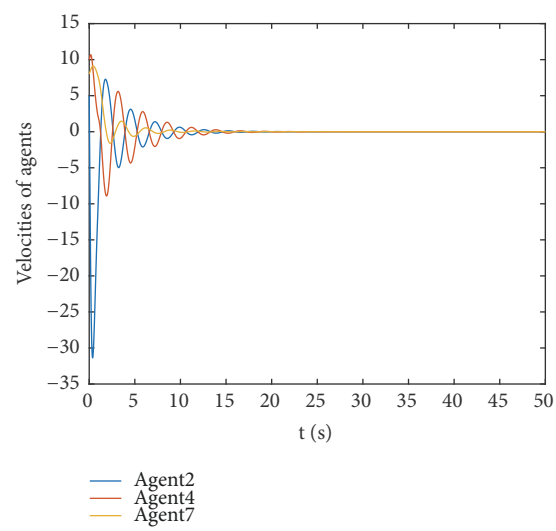


(c)

Figure 3: The agents position trajectories, where $\tau_{ij} = 0.2$. (a) $\tau = 0$, (b) $\tau = 0.03$, and (c) $\tau = 0.08$.



(a)



(b)

Figure 4: The state trajectories of the agents under undirected topology in Figure 1 with different input time delays $\tau_1 = 0.2$, $\tau_2 = 0.1$, $\tau_3 = 0.1$, $\tau_4 = 0.1$, $\tau_5 = 0.15$, $\tau_6 = 0.25$, $\tau_7 = 0.2$, communication delay $\tau_{ij} = 0.9$. (a) Positions. (b) Velocities.

Remark 17. It can be seen from Figures 2 and 3 that the input delay and communication delay will affect the convergence trajectory of the agent. When the input delay or the communication delay increases, the convergence speed of the agent decreases, so we can increase the convergence speed by reducing the delay.

From the qualification of Theorem 11, we can calculate the range of input delay for each agent: $\tau_1 = [0, 1/4]$, $\tau_2 = [0, 1/8]$, $\tau_3 = [0, 1/8]$, $\tau_4 = [0, 1/8]$, $\tau_5 = [0, 1/3.6]$, $\tau_6 = [0, 1/3.2]$, $\tau_7 = [0, 1/4]$. Here we take $\tau_1 = 0.2$, $\tau_2 = 0.1$, $\tau_3 = 0.1$, $\tau_4 = 0.1$, $\tau_5 = 0.15$, $\tau_6 = 0.25$, $\tau_7 = 0.2$. Obviously all τ are satisfied with Theorem 11. Figure 4 demonstrates that

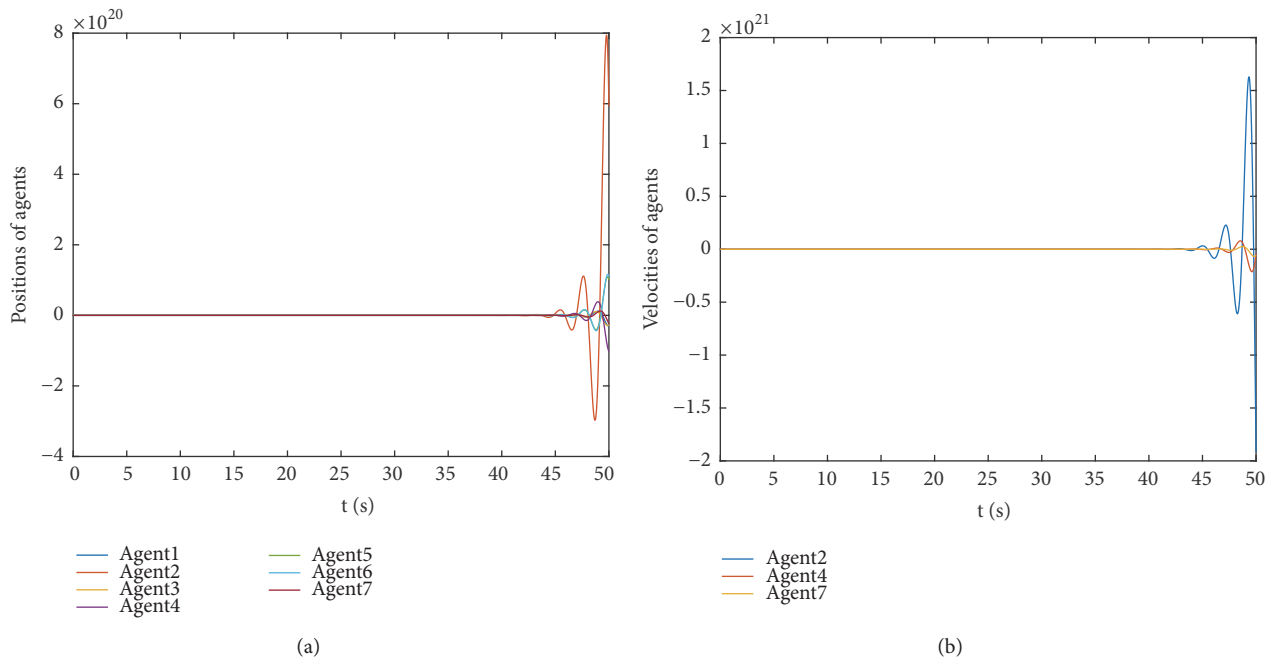


FIGURE 5: The state trajectories of the agents under undirected topology in Figure 1 with different input time delays $\tau_1 = 0.2$, $\tau_2 = 0.5$, $\tau_3 = 0.1$, $\tau_4 = 0.1$, $\tau_5 = 0.15$, $\tau_6 = 0.25$, $\tau_7 = 0.2$, communication delay $\tau_{ij} = 0.9$. (a) Positions. (b) Velocities.

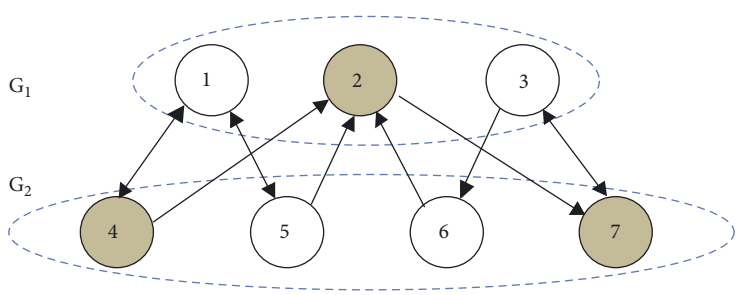


FIGURE 6: The directed graph topology of the heterogeneous MASs.

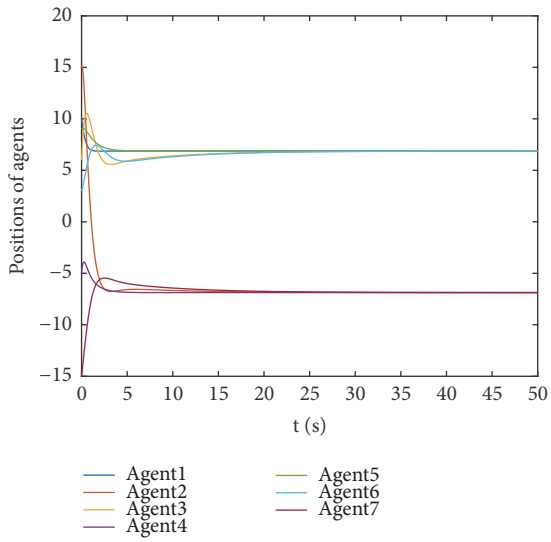
weighted couple-group consensus is achievable. At the same time, according to the upper bound calculated by Theorem 11, we assume $\tau_2 = 0.5$. As can be seen from Figure 5, the system is divergent at this time.

Next, we will testify Theorem 11 and Corollary 12.

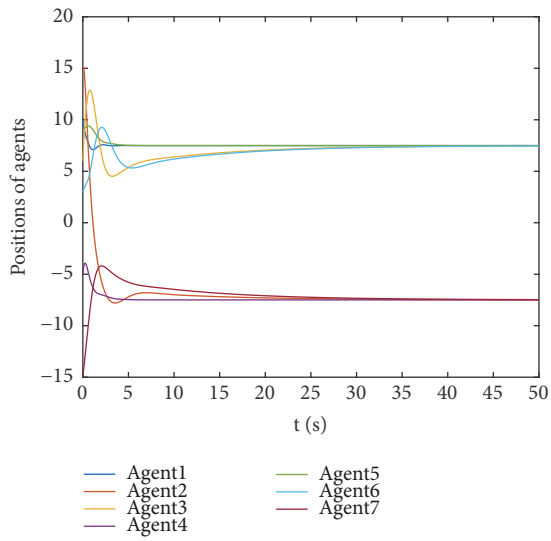
Example 18. We assume that the topology of a heterogeneous system contains a directed spanning tree, as shown in Figure 6. Since Figure 6 is an directed bipartite graph, we can get $d_1 = 2$, $d_2 = 3$, $d_3 = 1$, $d_4 = 1$, $d_5 = 1$, $d_6 = 1$, $d_7 = 2$. According to Corollary 6, we can assume that $\tau = 0.05$; obviously, τ satisfies all the qualifications. In Figure 7, we set the input delay $\tau = 0.05$ to a fixed value and enter different communication delays. According to the topology and Corollary 12 of Figure 6, we set $\tau_1 = 0.2$, $\tau_2 = 0.1$, $\tau_3 = 0.2$, $\tau_4 = 0.1$, $\tau_5 = 0.5$, $\tau_6 = 0.5$, $\tau_7 = 0.2$, and $\tau_{ij} = 0.9$, as shown in Figure 8. Obviously, from Figures 7 and 8, we can easily find that the system can progressively implement weighted couple-group consensus. When $\tau_4 = 0.5$, the system is divergent, as shown in Figure 9.

5. Conclusion

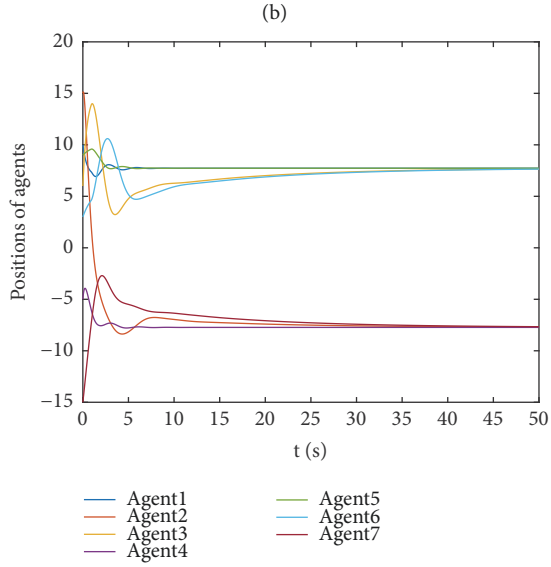
This paper studies the group consensus problem of heterogeneous MASs based on bipartite graph structure. The dynamic weighted couple-group consensus in the case of time delay is considered. A new weighted couple-group consensus protocol is designed by using cooperation and competition interaction between agents. Using graph theory, matrix theory, Gerschgorin disk theorem, and general Nyquist criterion,



(a)

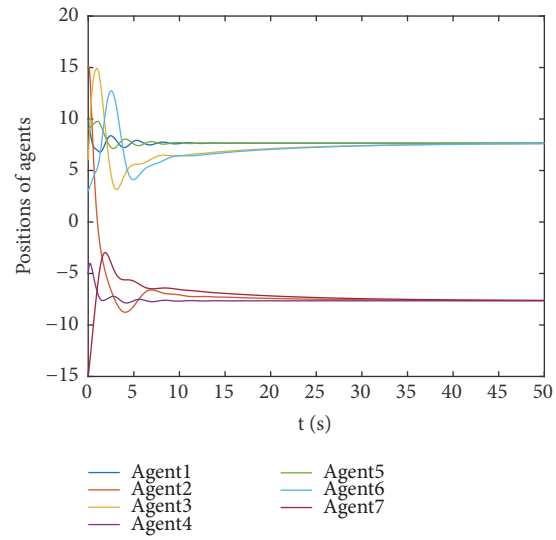


(b)

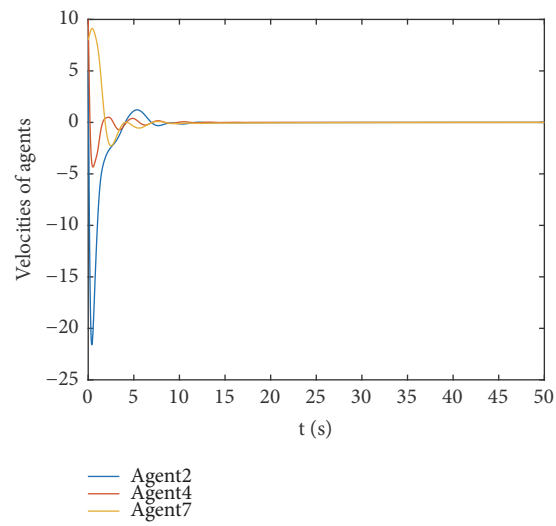


(c)

FIGURE 7: The agents position trajectories, where $\tau = 0.05$. (a) $\tau_{ij} = 0$, (b) $\tau_{ij} = 0.5$, and (c) $\tau_{ij} = 0.9$.



(a)



(b)

FIGURE 8: The state trajectories of the agents under directed topology in Figure 6 with different input time delays $\tau_1 = 0.2$, $\tau_2 = 0.1$, $\tau_3 = 0.1$, $\tau_4 = 0.1$, $\tau_5 = 0.5$, $\tau_6 = 0.5$, $\tau_7 = 0.2$, communication delay $\tau_{ij} = 0.9$. (a) Positions. (b) Velocities.

the upper bound of the maximum delay that can be tolerated when the system reaches convergence is obtained. It is not difficult to see from the theoretical results that the weighted couple-group consensus of the heterogeneous MASs is not directly related to the communication delay. The heterogeneous MASs implement weighted couple-group consensus, which is determined by the coupling weight between the agents, the input time delay, and the control parameters. In addition, in order to speed up the convergence of the system, we can reduce the communication delay or input delay, or both of them. The simulation example validated the results. In the future work, we will study the group consensus problems of more complex heterogeneous multiagent systems.

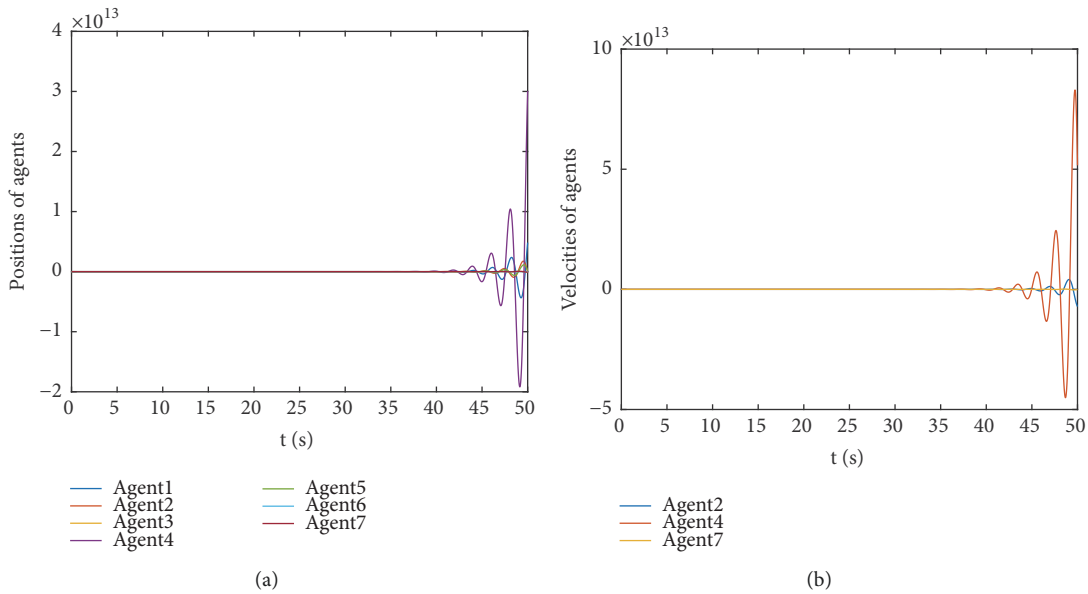


Figure 9: The state trajectories of the agents under directed topology in Figure 6 with different input time delays $\tau_1 = 0.2$, $\tau_2 = 0.1$, $\tau_3 = 0.1$, $\tau_4 = 0.5$, $\tau_5 = 0.5$, $\tau_6 = 0.5$, $\tau_7 = 0.2$, communication delay $\tau_{ij} = 0.9$. (a) Positions. (b) Velocities.

For example, we will consider switching topology or event driven.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant No. **61876200**, in part by the Natural Science Foundation Project of Chongqing Science and Technology Commission under Grant No. cstc2018jcyjAX0112, in part by the Educational Reform Project of Chongqing University of Posts and Telecommunications (Grant No. XJG1635), and in part by the Scientific and Technological Research Program of Chongqing Municipal Education Commission (Grant No. KJ1400432).

References

[1] X. Shi, J. Cao, and W. Huang, "Distributed parametric consensus optimization with an application to model predictive consensus problem," *IEEE Transactions on Cybernetics*, vol. 48, no. 7, pp. 2024–2035, 2018.

[2] M. H. Rezaei and M. B. Menhaj, "Stationary average consensus protocol for a class of heterogeneous high-order multi-agent systems with application for aircraft," *International Journal of Systems Science*, vol. 49, no. 10, pp. 1–15, 2018.

[3] X. Wu, Z. Yang, J. Huo, and J. Zhang, "UAV formation control based on consistency," in *Proceedings of the 2015 7th International Conference on Modelling, Identification and Control (ICMIC)*, pp. 1–5, Sousse, Tunisia, 2016.

[4] F. Ye, H. Dong, Y. Lu, and W. Zhang, "Consensus controllers for general integrator multi-agent systems: analysis, design and application to autonomous surface vessels," *IET Control Theory & Applications*, vol. 12, no. 5, pp. 669–678, 2018.

[5] Z. Peng, G. Wen, S. Yang, and A. Rahmani, "Distributed consensus-based formation control for nonholonomic wheeled mobile robots using adaptive neural network," *Nonlinear Dynamics*, vol. 86, no. 1, pp. 605–622, 2016.

[6] S. Yang, Y. Cao, Z. Peng, G. Wen, and K. Guo, "Distributed formation control of nonholonomic autonomous vehicle via RBF neural network," *Mechanical Systems and Signal Processing*, vol. 87, part B, pp. 81–95, 2017.

[7] F. Xiao, T. Chen, and H. Gao, "Synchronous hybrid event- and time-driven consensus in multiagent networks with time delays," *IEEE Transactions on Cybernetics*, vol. 46, no. 5, pp. 1165–1174, 2016.

[8] H. Li, G. Chen, T. Huang, Z. Dong, W. Zhu, and L. Gao, "Event-triggered distributed average consensus over directed digital networks with limited communication bandwidth," *IEEE Transactions on Cybernetics*, vol. 46, no. 12, pp. 3098–3110, 2016.

[9] Y. Chen, H. Dong, J. Lu, X. Sun, and K. Liu, "Robust consensus of nonlinear multiagent systems with switching topology and bounded noises," *IEEE Transactions on Cybernetics*, vol. 46, no. 6, pp. 1276–1285, 2016.

[10] X. Wu, Y. Tang, J. Cao, and W. Zhang, "Distributed consensus of stochastic delayed multi-agent systems under asynchronous switching," *IEEE Transactions on Cybernetics*, vol. 46, no. 8, pp. 1817–1827, 2016.

[11] M. O. Oyedeji and M. S. Mahmoud, "Couple-group consensus conditions for general first-order multiagent systems with communication delays," *Systems & Control Letters*, vol. 117, pp. 37–44, 2018.

[12] Y. Han, W. Lu, and T. Chen, "Cluster consensus in discrete-time networks of multiagents with inter-cluster nonidentical inputs," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 24, no. 4, pp. 566–578, 2013.

[13] L. Ji, Q. Liu, and X. Liao, "On reaching group consensus for linearly coupled multi-agent networks," *Information Sciences*, vol. 287, pp. 1–12, 2014.

[14] X. Liao and L. Ji, "On pinning group consensus for dynamical multi-agent networks with general connected topology," *Neurocomputing*, vol. 135, no. 5, pp. 262–267, 2014.

[15] Q. Wang and Y. Wang, "Cluster synchronization of a class of multi-agent systems with a bipartite graph topology," *Science China Information Sciences*, vol. 57, no. 1, pp. 12203-012203, 2014.

[16] W. Xu and D. W. C. Ho, "Clustered Event-triggered consensus analysis: an impulsive framework," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 11, pp. 7133–7143, 2016.

[17] H.-X. Hu, W. Yu, G. Wen, Q. Xuan, and J. Cao, "Reverse group consensus of multi-agent systems in the cooperation-competition network," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 63, no. 11, pp. 2036–2047, 2016.

[18] B.-R. An, G.-P. Liu, and C. Tan, "Group consensus control for networked multi-agent systems with communication delays," *ISA Transactions*, vol. 76, pp. 78–87, 2018.

[19] H. G. Tanner, "On the controllability of nearest neighbor interconnections," in *Proceedings of the 43rd IEEE Conference on Decision and Control (CDC '04)*, vol. 3, pp. 2467–2472, December 2004.

[20] B. Liu, T. Chu, L. Wang, Z. Zuo, G. Chen, and H. Su, "Controllability of switching networks of multi-agent systems," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 6, pp. 630–644, 2012.

[21] B. Liu, T. Chu, L. Wang, and G. Xie, "Controllability of a leader-follower dynamic network with switching topology," *IEEE Transactions on Automatic Control*, vol. 53, no. 4, pp. 1009–1013, 2008.

[22] Bo Liu, H. Feng, L. Wang et al., "Controllability of second-order multiagent systems with multiple leaders and general dynamics," *Mathematical Problems in Engineering*, vol. 2013, Article ID 587569, 6 pages, 2013.

[23] B. Liu, H. Su, R. Li, D. Sun, and W. Hu, "Switching controllability of discrete-time multi-agent systems with multiple leaders and time-delays," *Applied Mathematics and Computation*, vol. 228, no. 9, pp. 571–588, 2014.

[24] L. Wang, F. Jiang, G. Xie, and Z. Ji, "Controllability of multi-agent systems based on agreement protocols," *Science China Information Sciences*, vol. 52, no. 11, pp. 2074–2088, 2009.

[25] M. Long, H. Su, and B. Liu, "Group controllability of two-time-scale multi-agent networks," *Journal of The Franklin Institute*, vol. 355, no. 13, pp. 6045–6061, 2018.

[26] M. Long, H. Su, and B. Liu, "Second-order controllability of two-time-scale multi-agent systems," *Applied Mathematics and Computation*, vol. 343, pp. 299–313, 2019.

[27] W. Jiang, G. Wen, Z. Peng, T. Huang, and R. A. Rahmani, "Fully distributed formation-containment control of heterogeneous linear multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 48, no. 12, pp. 2242–2250, 2018.

[28] H. Su, H. Wu, and X. Chen, "Observer-based discrete-time nonnegative edge synchronization of networked systems," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 10, pp. 2446–2455, 2017.

[29] C.-L. Liu and F. Liu, "Dynamical consensus seeking of heterogeneous multi-agent systems under input delays," *International Journal of Communication Systems*, vol. 26, no. 10, pp. 1243–1258, 2013.

[30] Y.-P. Tian and Y. Zhang, "High-order consensus of heterogeneous multi-agent systems with unknown communication delays," *Automatica*, vol. 48, no. 6, pp. 1205–1212, 2012.

[31] G. Wen, J. Huang, C. Wang, Z. Chen, and Z. Peng, "Group consensus control for heterogeneous multi-agent systems with fixed and switching topologies," *International Journal of Control*, vol. 89, no. 2, pp. 259–269, 2016.

[32] G. Wen, Y. Yu, Z. Peng, and H. Wang, "Dynamical group consensus of heterogenous multi-agent systems with input time delays," *Neurocomputing*, vol. 175, pp. 278–286, 2015.

[33] Y. Jiang, L. Ji, Q. Liu, S. Yang, and X. Liao, "Couple-group consensus for discrete-time heterogeneous multiagent systems with cooperative–competitive interactions and time delays," *Neurocomputing*, vol. 319, pp. 92–101, 2018.

[34] S. Li, G. Feng, X. Luo, and X. Guan, "Output consensus of heterogeneous linear discrete-time multiagent systems with structural uncertainties," *IEEE Transactions on Cybernetics*, vol. 45, no. 12, pp. 2868–2879, 2015.

[35] B. Cui, T. Ma, F. L. Lewis, C. Zhao, Y. Song, and C. Feng, "Distributed adaptive consensus control of heterogeneous multi-agent chaotic systems with unknown time delays," *IET Control Theory & Applications*, vol. 9, no. 16, pp. 2414–2422, 2015.

[36] J. Qin, Q. Ma, H. Gao, Y. Shi, and Y. Kang, "On group synchronization for interacting clusters of heterogeneous systems," *IEEE Transactions on Cybernetics*, vol. 99, pp. 1–12, 2016.

[37] C. Liu, Q. Zhou, and X. Hu, "Group consensus of heterogeneous multi-agent systems with fixed topologies," *International Journal of Intelligent Computing and Cybernetics*, vol. 8, no. 4, pp. 294–311, 2015.

[38] D. Goldin and J. Raisch, "Consensus for agents with double integrator dynamics in heterogeneous networks," *Asian Journal of Control*, vol. 16, no. 1, pp. 30–39, 2014.

[39] W. Yichang, *Weighted Average-Consensus for Directed Networks of Multi-Agent*, vol. 23, no. (2), 2007.

[40] C.-X. Shi and G.-H. Yang, "Robust consensus control for a class of multi-agent systems via distributed PID algorithm and weighted edge dynamics," *Applied Mathematics and Computation*, vol. 316, pp. 73–88, 2018.

[41] H. Hu, G. Chen, and Q. Xuan, "Second-order consensus for heterogeneous agents in the cooperation-competition network," in *Proceedings of the 2015 27th Chinese Control and Decision Conference (CCDC)*, pp. 267–272, Qingdao, China, May 2015.

[42] H.-X. Hu, G. Wen, W. Yu, Q. Xuan, and G. Chen, "Swarming behavior of multiple Euler-Lagrange systems with cooperation-competition interactions: an auxiliary system approach," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 11, pp. 5726–5737, 2018.