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Research Article

Nonfragile Integral-Based Event-Triggered Control of Uncertain Cyber-Physical Systems under Cyber-Attacks

Shen Yan,[1] **Sing Kiong Nguang**,[2] **and Liruo Zhang**[2]

[1]*College of Mechanical & Electronic Engineering, Nanjing Forestry University, Nanjing 210037, China*
[2]*Department of Electrical, Computer and Software Engineering, The University of Auckland, Auckland 1142, New Zealand*

Correspondence should be addressed to Shen Yan; yanshenzdh@gmail.com

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This article studies the problem of nonfragile integral-based event-triggered control for uncertain cyber-physical systems under cyber-attacks. An integral-based event-triggered scheme is proposed to reduce the data transmissions and save the limited network resources. The triggering condition is related to the mean of system state over a finite time interval instead of instant system state. Random cyber-attacks in a communication channel are taken into account and described by a stochastic variable subject to Bernoulli distribution. A novel Lyapunov–Krasovskii functional based on Legendre polynomials is constructed, and the Bessel–Legendre inequality technique is employed to handle the integral term induced by the integral-based event-triggered scheme. Resorting to these treatments, sufficient conditions are established via a set of linear matrix inequalities to guarantee the asymptotic mean-square stability of the closed-loop system. Finally, a numerical example shows that the presented method is effective.

1. Introduction

Cyber-physical systems (CPSs) are systems where computational and physical resources are tightly integrated via a shared network. They have been applied in industrial fields, such as medical devices, chemical process industries, and transportation systems [1–3]. Security is a critical issue in CPSs, as a great deal of communication signals are exchanged over the network channel that is open to potential cyber-attacks. Cyber-attacks, introduced by external attackers, could cause negative impact on the communication channels and deteriorate system stability performance. Thus, it has attracted much attention to study the control and filtering problems for CPSs under the threats of network attacks [4–7]. The two representative kinds of cyber-attacks are, namely, deception attacks and denial-of-service (DoS) attacks. The deception attacks aim at injecting deceitful information into the transmitted signal, and the DoS attacks could block or break the communication link among system nodes. Some recent results on the study of cyber-attacks are reported in [8, 9]. The distributed recursive filtering problem of stochastic systems with time delays and deception attacks is investigated in [8]. The finite-horizon tracking control issue of a stochastic system subject to hybrid attacks is developed in [9], where both the stochastic deception attacks and stochastic DoS attacks are taken into consideration.

Note that, in the above results, communication signals are transmitted through the network without considering some practical constraints, such as limited network bandwidth and computation burden. Some effective manners such as event-triggered scheme (ETS) [10], signal quantization [11], and real-time scheduling [12, 13] have been proposed to save communication resources and reduce computation burden. Recently, ETS has gained growing attention and interests in existing research studies [14–23] for its simplicity of execution and effectiveness of reducing data transmissions. Due to this fact, the idea of ETS is introduced into security control for CPSs and has produced many interesting results [24–29]. To be specific, in [24], a resilient H_∞ load frequency controller is designed to stabilize the multiarea power systems with cyber-attacks. The issue of event-triggered controller design for networked

systems under the resilient event-triggered mechanism and periodic DoS jamming attacks is developed in [25]. For multiagent systems with deception attacks, [26] is concerned with the event-triggered output consensus control issue. Zha et al. [27] investigate the design of decentralized event-triggered H_∞ controller of delayed neural networks under deception attacks. Liu et al. [28] address the distributed event-triggered control issue for networked control systems with stochastic cyber-attacks, and a decentralized triggering strategy is utilized to release the network communication burden. In [29], the resilient load frequency control problem for multiarea power systems with cyber-attacks is investigated by using an event-triggered communication mechanism. In these studies, only the instant system information, such as current system state and the last triggered signal, is applied to design the ETS conditions. In practical systems, the system state could have abrupt fluctuations over some time interval incurred by external disturbances, system uncertainties, or environment noises. If such state with abrupt fluctuations is triggered to controller, it is negative for stabilizing the system performance. In [30], the idea of introducing the accumulation error information or system dynamics over a finite time interval into the design of triggering rule is proposed. Considering systems with stochastic measurement noises, an integral-based ETS utilizing the average state over a finite time interval is proposed to reduce the effect of measurement noises in [31], where an approximation approach is used to deal with the integral triggering condition. This method to treat the integral term will result in approximation error, which needs further investigation to exclude such approximation error and obtain less conservative results. In addition, the controllers in these works are assumed to be implemented perfectly. However, uncertainties caused by execution errors or unknown noises are usually observed in engineerings, which may lead to system performance deterioration. To conquer this problem, Sakthivel et al. [32] handle the uncertainties by norm-bounded variations to design the dissipative analysis-based nonfragile controller for network-based singular systems with ETS. In [33], the issue of nonfragile event-triggered H_∞ control of linear systems under unreliable communication links is addressed. For linear systems against actuator saturation and disturbances, Liu and Yang [34] study the design of nonfragile dynamic output feedback controller via an event-triggered communication scheme. It is noted that the cyber-attacks are not considered in communication channels, and the mean of system state is not taken into account for ETS design in [32–34]. To our best knowledge, few results have been investigated for integral-based event-triggered control problem of uncertain systems with cyber-attacks and gain uncertainties, which inspire the current study.

This paper focuses on the co-design of integral-based ETS and nonfragile controller for uncertain CPSs with cyber-attacks and gain uncertainties. The main contributions of this paper are summarized as follows:

(1) An integral-based ETS that utilizes the mean of system state over a fixed time interval is proposed to reduce the wastage of limited network resources. It covers the conventional ETS [10] as a special case as the fixed time interval approaches to zero and can further reduce unnecessary data transmissions.

(2) The closed-loop system with the integral-based ETS, cyber-attacks, and gain variations is established as a distributed delay system, in which a Bernoulli variable is adopted to describe the stochastic cyber-attacks. A novel augmented Lyapunov–Krasovskii functional (LKF) based on Legendre polynomials is constructed. With the help of the novel LKF and Bessel–Legendre inequality, the integral term induced by integral-based ETS is handled without the approximation error in [31].

The organization of this paper is given as follows. Section 2 presents the problem statement and some preliminaries. The main results of stability analysis and control synthesis are derived in Section 3. To show the effectiveness of the proposed approach, a numerical example is given in Section 4. Some conclusions and future research topics are shown in Section 5.

1.1. Notation. In this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. $\mathbb{E}\{\cdot\}$ means the mathematical expectation. The notation $X>0$ (<0) represents that X is symmetric and positive (negative) definite. The superscript "T" stands for the transpose of a vector or matrix. $X^\perp$ denotes the kernel of X. $He(X)$ equals to X^T+X. The notation $\begin{pmatrix} k \\ j \end{pmatrix}$ means the binomial coefficients given by $k!/(k-j)!j!$. $\otimes$ refers to the Kronecker product. $\mathbb{N}$ is the set of nonnegative integers.

2. Preliminaries

Consider the following uncertain continuous-time system as

$$\dot{x}(t) = (A+\Delta A(t))x(t) + Bu(t), \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the system state, $u(t)\in\mathbb{R}^m$ is system input, A and B are known system matrices, and $\Delta A(t) = \varsigma(t)\widehat{A}$, where $\varsigma(t)$ is a stochastic variable with expectation $\overline{\varsigma}$ and variance $\overline{\varsigma}^2$ and $\widehat{A}$ means the deviation from the nominal model. Different from the commonly used uncertainty model $\Delta A(t) = DF(t)E$ with exactly known bounds, we focus on the statistic of the uncertainty deviating from the nominal part.

Since the network bandwidth is limited, unnecessary data transmission will lead to the wastage of limited network resources. In order to decrease communication frequency, an integral-based event-triggered scheme, drawn in Figure 1, is presented as

$$s_{k+1} = \min_s \left\{ s\ge s_k \,\middle|\, e^T(s)\Phi e(s) \ge \delta\widehat{x}^T(s_k)\Phi\widehat{x}(s_k) \right\}, \tag{2}$$

where $\delta\in[0,1)$; $e(s) = 1/\tau\int_{s-\tau}^{s} x(v)\mathrm{d}v - \widehat{x}(s_k)$, $1/\tau\int_{s-\tau}^{s} x(v)\mathrm{d}v$ is the mean of system state; $\widehat{x}(s_k) = 1/\tau\int_{s_k-\tau}^{s_k} x(v)\mathrm{d}v$ and $\widehat{x}(s_{k+1}) = 1/\tau\int_{s_{k+1}-\tau}^{s_{k+1}} x(v)\mathrm{d}v$, the mean of the current and

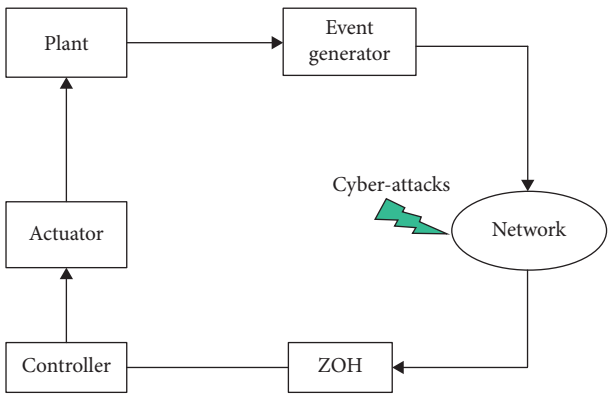

Figure 1: Framework of event-triggered control for CPSs against cyber-attacks.

next triggered control signal, respectively; τ is the length of the mean of system state over $[s-\tau, s)$; and Φ is the positive weighting matrix.

Remark 1. The proposed triggering condition (2) takes into account the mean of system state over $[s-\tau, s)$, formulated as $1/\tau \int_{s-\tau}^{s} x(v)\mathrm{d}v$, instead of the instant system state $x(s)$. The triggering condition is then formed by integration, which motivates the name of integral-based ETS.

Remark 2. When $\tau \longrightarrow 0, e(s) = \int_{s-\tau}^{s} (1/\tau)x(v)\mathrm{d}v - \hat{x}(s_k)$ is equivalent to $x(s) - x(s_k)$. Then, the triggering mechanism (2) is rewritten as

$$s_{k+1} = \min_{s}\left\{s \geq s_k \,\middle|\, (x(s) - x(s_k))^T \cdot \Phi(x(s) - x(s_k)) \geq \delta x(s_k)^T \Phi x(s_k)\right\}, \tag{3}$$

which reduces to the normal ETS proposed in [10].

In real systems, the network transmission delay is a critical problem, which may cause system performance degradation. It is usually modeled by constant delay and time-varying delay. Here, the constant network transmission delay is considered and denoted by τ_1, which meets

$$t_k = s_k + \tau_1 < s_{k+1} + \tau_1 = t_{k+1}, \quad k \in \mathbb{N}. \tag{4}$$

Therefore, combining the transmission delay and zero-order holder (ZOH) with the triggering condition (2), it gives that

$$\varepsilon^T(t)\Phi\varepsilon(t) < \delta\hat{x}^T(s_k)\Phi\hat{x}(s_k), \quad t \in [t_k, t_{k+1}), \tag{5}$$

where $\varepsilon(t) \triangleq 1/\tau \int_{t-\tau_2}^{t-\tau_1} x(v)\mathrm{d}v - \hat{x}(s_k)$, $\tau_2 = \tau_1 + \tau$, $\varepsilon(t)$ is the error term, and $1/\tau \int_{t-\tau_2}^{t-\tau_1} x(v)\mathrm{d}v$ will be used to represent the control signal $\hat{x}(s_k)$ to establish the closed-loop system.

It is noted that the data communication over the network channel between event generator and controller is easy to be attacked by hackers. The random cyber-attacks on communication signals could lead to inaccurate control operations. Moreover, taking into account the controller gain variations, the control input reflecting the gain variations and cyber-attacks is modeled below:

$$\begin{aligned} u(t) &= \alpha(s_k)(K + \Delta K)\hat{x}(s_k) \\ &\quad + (1 - \alpha(s_k))(K + \Delta K))f(\hat{x}(s_k)), \quad t \in [t_k, t_{k+1}), \end{aligned} \tag{6}$$

where $\alpha(s_k) \in \{0, 1\}$ is a Bernoulli variable satisfying the probability: $\mathrm{Prob}\{\alpha(s_k) = 1\} = \overline{\alpha}$ and $\mathrm{Prob}\{\alpha(s_k) = 0\} = 1 - \overline{\alpha}$, respectively; the triggered data is attacked by external hacker with the cyber-attack function $f(\hat{x}(s_k))$ for $\alpha(s_k) = 0$; K is the controller gain to be designed; and $\Delta K = E_a G(t) E_b$ is the gain uncertainty satisfying the norm-bounded condition $G^T(t)G(t) \leq I$.

Remark 3. The impact of cyber-attacks on the transmitted data is described by a stochastic variable $\alpha(s_k)$ obeying Bernoulli distribution. When $\alpha(s_k) = 1$, it represents the triggered data sent to the controller successfully. For $\alpha(s_k) = 0$, the triggered data is attacked by deception signals as transmitting over the network channel.

Remark 4. In this paper, the controller is designed considering only the stochastic occurring deception attacks whose purpose is to degrade the system performance and stability by corrupting or saturating the control inputs.

Therefore, substituting (6) and $\varepsilon(t)$ defined in (5) into (1), the closed-loop system is modeled as a system with a distributed input delay:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + \alpha(s_k)B(K + \Delta K) \\ &\quad \cdot \left(\frac{1}{\tau}\int_{t-\tau_2}^{t-\tau_1} x(v)\mathrm{d}v - \varepsilon(t)\right) \\ &\quad + (1 - \alpha(s_k))B(K + \Delta K)f(\hat{x}(s_k)). \end{aligned} \tag{7}$$

This aim of this paper is to design nonfragile state feedback controller (6) such that the closed-loop system (2) under the integral-based ETS and cyber-attacks is asymptotically stable.

The following assumption and technical lemmas will be utilized in achieving the main results.

Assumption 1. The cyber-attack function $f(\cdot)$ in (6), for $i = 1, 2, \ldots, n$, is assumed to meet the sector-bounded condition

$$(f(x) - F_1 x)^T (f(x) - F_2 x) \leq 0, \tag{8}$$

where F_1 and F_2 are two real constant matrices and satisfy $F_2 - F_1 \geq 0$.

Remark 5. The cyber-attack function $f(\cdot)$ that satisfies Assumption 1 can be signal saturation $f(x(t)) = \mathrm{sat}(x(t))$, time-varying percentage function $f(x(t)) = (I + \Delta(t))x(t)$, where $\Delta(t)$ is bounded less than 1 and so on.

Lemma 1 (see [35]). *Let the compound function $x \in \mathscr{L}_2([-\vartheta_2, -\vartheta_1] \longrightarrow \mathbb{R}^n)$, symmetric matrix $\mathscr{R} > 0$, and $\vartheta_2 > \vartheta_1 > 0$. The Legendre polynomials considered over the interval $[-\vartheta_2, -\vartheta_1]$ are defined by*

$$\mathbb{L}_k(u) = (-1)^k \sum_{l=0}^{k} \mathscr{P}_l^k \left(\frac{u+\vartheta_2}{\vartheta_2-\vartheta_1} \right)^l, \quad \forall k \in \mathbb{N}, \tag{9}$$

with $\mathscr{P}_l^k = (-1)^l \binom{k}{j}\binom{k+j}{j}$, *where* $k = 0, 1, \ldots, N$.

Then, for all $N \in \mathbb{N}$, the integral inequality

$$\int_{-\vartheta_2}^{-\vartheta_1} x^T(u)\mathscr{R}x(u)\mathrm{d}u \geq \frac{1}{\vartheta_2-\vartheta_1}\Theta^T(x)(\mathscr{R}\otimes\mathscr{F})\Theta(x), \tag{10}$$

holds with $\mathscr{F} = \mathrm{diag}\{1, 3, \ldots, 2N+1\}$, $\Theta^T(x) = \left[\Theta_0^T(x)\cdots\Theta_k^T(x)\cdots\Theta_N^T(x)\right]$, and $\Theta_k = \int_{-\vartheta_2}^{-\vartheta_1} \mathbb{L}_k(u)x(u)\mathrm{d}u$.

Lemma 2 (see [11]). *Let* $\nu(t) \in \mathbb{R}^n$, $\mathscr{P} = \mathscr{P}^T \in \mathbb{R}^{n\times n}$, *and* $\mathscr{H} \in \mathbb{R}^{m\times n}$ *such that* $rank(\mathscr{H}) = g < n$, *then the following statements are equivalent:*

(a) $\nu^T(t)\mathscr{P}\nu(t) < 0$, for any $\nu(t) \neq 0$, $\mathscr{H}\nu(t) = 0$

(b) $\mathscr{H}^{\perp T}\mathscr{P}\mathscr{H}^{\perp} < 0$

(c) $\exists \mathscr{M} \in \mathbb{R}^{n\times m}$ such that $\mathscr{P} + He(\mathscr{M}\mathscr{H}) < 0$

Lemma 3 (see [36]). *For matrices* $\mathscr{E}_a$, $\mathscr{G}(t)$, *and* $\mathscr{E}_b$ *with compatible dimensions and* $\mathscr{G}^T(t)\mathscr{G}(t) \leq I$, *the inequality*

$$\mathscr{E}_a\mathscr{G}(t)\mathscr{E}_b + \mathscr{E}_a^T\mathscr{G}^T(t)\mathscr{E}_b^T \leq \beta\mathscr{E}_a^T\mathscr{E}_a + \beta^{-1}\mathscr{E}_b^T\mathscr{E}_b, \tag{11}$$

holds for any $\beta > 0$.

3. Main Results

Before providing the main results, first, we construct the following LKF related to Legendre polynomials as

$$V(t) = V_0(t) + V_1(t) + V_2(t), \tag{12}$$

where

$$\begin{aligned}
V_0(t) &= \zeta^T(t)P_N\zeta(t),\\
V_1(t) &= \int_{t-\tau_1}^{t} x^T(\nu)\left[S_1 + (\nu - t + \tau_1)R_1\right]x(\nu)\mathrm{d}\nu,\\
V_2(t) &= \int_{t-\tau_2}^{t-\tau_1} x^T(\nu)\left[S_2 + (\nu - t + \tau_2)R_2\right]x(\nu)\mathrm{d}\nu,\\
\zeta^T(t) &= \left[x^T(t)\ \ \Sigma^T(x)\ \ \Gamma^T(x)\right]^T,\\
\Sigma^T(x) &= \left[\Sigma_0^T(x)\ \cdots\ \Sigma_{k_1}^T(x)\ \cdots\ \Sigma_{d_1}^T(x)\right],\\
\Upsilon^T(x) &= \left[\Upsilon_0^T(x)\ \cdots\ \Upsilon_{k_2}^T(x)\ \cdots\ \Upsilon_{d_2}^T(x)\right],\\
\Sigma_{k_1}(x) &= \int_{-\tau_1}^{0} \mathbb{L}_{1,k_1}(\nu)x(\nu)\mathrm{d}\nu, \quad k_1 = 0, 1, \ldots, d_1+1,\\
\Upsilon_{k_2}(x) &= \int_{-\tau_2}^{-\tau_1} \mathbb{L}_{2,k_2}(\nu)x(\nu)\mathrm{d}\nu, \quad k_2 = 0, 1, \ldots, d_2+1.
\end{aligned} \tag{13}$$

Next, Theorem 1 gives the sufficient conditions for guaranteeing the closed-loop system (2) to be asymptotically stable in mean square sense. Based on Theorem 1 and Lemma 3, sufficient co-design conditions of controller gain and triggering matrix are derived via a set of LMIs in Theorem 2.

Theorem 1. *For given scalars* d_1, d_2, τ, τ_1, τ_2, ρ, δ, *and* $\overline{\alpha}$, *under the proposed integral-based ETS (2) and controller gain* K, *the closed-loop system (2) is asymptotically stable in mean square sense if there exist symmetric matrices* P_N, $\Phi > 0$, $S_1 > 0$, $S_2 > 0$, $R_1 > 0$, *and* $R_2 > 0$ *and matrices* U, X_1, *and* X_2 *such that*

$$\widehat{P}_N > 0, \tag{14}$$

$$\Pi + He(\mathscr{W}\mathscr{V}) < 0, \tag{15}$$

where

$$\widehat{P}_N = P_N + \mathrm{diag}\left\{0, \frac{(\mathscr{F}_1\otimes S_1)}{\tau_1}, \frac{(\mathscr{F}_2\otimes S_2)}{\tau}\right\},$$

$$\begin{aligned}
\Pi = {} & \mathrm{He}\left(H_N^T P_N M_N\right) + \delta\mathscr{I}_1^T\Phi\mathscr{I}_1 - \rho\mathscr{I}_2^T\overline{F}\mathscr{I}_2\\
& + \mathrm{diag}\{0_n, \Pi_{12}, 0_n, \Pi_{14}, \Pi_{15}, \Pi_{16}, \Pi_{17}, \Pi_{18}\},
\end{aligned}$$

$$\mathscr{I}_1 = \begin{bmatrix} 0_n & 0_n & 0_n & 0_n & 0_n & 0_{n\times(d_1+1)n} & \frac{1}{\tau}I_n & 0_{n\times d_2 n} & -I_n \end{bmatrix},$$

$$\mathscr{I}_2^T = \begin{bmatrix} 0_n & 0_n & 0_n & 0_n & 0_n & 0_{n\times(d_1+1)n} & \frac{1}{\tau}I & 0_{n\times d_2 n} & -I_n & 0_n & 0_n & I_n & 0_n & 0_n & 0_{n\times(d_1+1)n} & 0_n & 0_{n\times d_2 n} & 0_n \end{bmatrix},$$

$$\overline{F} = \begin{bmatrix} \overline{F}_1 & \overline{F}_2 \\ \overline{F}_2^T & I \end{bmatrix},$$

$$\overline{F}_1 = \frac{F_1^T F_2 + F_2^T F_1}{2},$$

$$\overline{F}_2 = \frac{F_1^T + F_2^T}{2},$$

$$\Pi_{12} = S_1 + \tau_1 R_1,$$

$$\Pi_{14} = -S_1 + S_2 + \tau R_2,$$

$$\Pi_{15} = -S_2,$$

$$\Pi_{16} = -\frac{(\mathscr{F}_1 \otimes R_1)}{\tau_1},$$

$$\Pi_{17} = -\frac{(\mathscr{F}_2 \otimes R_2)}{\tau},$$

$$\Pi_{18} = -\Phi,$$

$$\mathscr{W} = \begin{bmatrix} W_1^T & W_2^T & 0_n & 0_n & 0_n & 0_{n\times(d_1+1)n} & 0_{n\times(d_2+1)n} & 0_n \end{bmatrix}^T,$$

$$\mathscr{V} = \begin{bmatrix} -I & A + \overline{\varsigma}\widehat{A} & (1-\overline{\alpha})B(K+\Delta K) & 0_n & 0_n & 0_{n\times(d_1+1)n} & \frac{\overline{\alpha}}{\tau}B(K+\Delta K) & 0_{n\times d_2 n} & -\overline{\alpha}B(K+\Delta K) \end{bmatrix},$$

$$M_N = \begin{bmatrix} I_n & 0_n & 0_n & 0_n & 0_n & 0_{n\times(d_1+1)n} & 0_{n\times(d_2+1)n} & 0_n \\ 0_{(d_1+1)n\times n} & \mathscr{L}_{d_1}(0) & 0_{(d_1+1)n\times n} & -\mathscr{L}_{d_1}(-\tau_1) & 0_{(d_1+1)n\times n} & \Lambda_{d_1} & 0_{(d_1+1)n\times(d_2+1)n} & 0_{(d_1+1)n\times n} \\ 0_{(d_2+1)n\times n} & 0_{(d_2+1)n\times n} & 0_{(d_2+1)n\times n} & \mathscr{L}_{d_2}(-\tau_1) & -\mathscr{L}_{d_2}(-\tau_2) & 0_{(d_2+1)n\times(d_1+1)n} & \Lambda_{d_2} & 0_{(d_2+1)n\times n} \end{bmatrix},$$

$$H_N = \begin{bmatrix} 0_n & I_n & 0_{n\times 3n} & 0_{n\times(d_1+1)n} & 0_{n\times(d_2+1)n} & 0_n \\ 0_{(d_1+1)n\times n} & 0_{(d_1+1)n\times n} & 0_{(d_1+1)n\times 3n} & I_{(d_1+1)n} & 0_{(d_1+1)n\times(d_2+1)n} & 0_{(d_1+1)n\times n} \\ 0_{(d_2+1)n\times n} & 0_{(d_2+1)n\times n} & 0_{(d_2+1)n\times 3n} & 0_{(d_1+1)n\times(d_2+1)n} & I_{(d_2+1)n} & 0_{(d_2+1)n\times n} \end{bmatrix},$$

$$\mathscr{L}_{d_1}(0) = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix},$$

$$\mathscr{L}_{d_1}(-\tau_1) = \begin{bmatrix} I \\ -I \\ \vdots \\ (-1)^{d_1} I \end{bmatrix},$$

$$\mathscr{L}_{d_2}(-\tau_1) = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix},$$

$$\mathscr{L}_{d_2}(-\tau_2) = \begin{bmatrix} I \\ -I \\ \vdots \\ (-1)^{d_2} I \end{bmatrix},$$

$$\Lambda_{d_1} = \begin{bmatrix} \lambda_{d_1 0}^{0} I & \cdots & \lambda_{d_1 0}^{d_1} I \\ \vdots & \ddots & \vdots \\ \lambda_{d_1 d_1}^{0} I & \cdots & \lambda_{d_1 d_1}^{d_1} I \end{bmatrix},$$

$$\lambda_{d_1 k_1}^{i} = \begin{cases} \dfrac{-(2i+1)\left(1-(-1)^{k_1+i}\right)}{\tau_1}, & i \le k_1, \\ 0, & i > k_1, \end{cases}$$

$$\Lambda_{d_2} = \begin{bmatrix} \lambda_{d_2 0}^{0} I & \cdots & \lambda_{d_2 0}^{d_2} I \\ \vdots & \ddots & \vdots \\ \lambda_{d_2 d_2}^{0} I & \cdots & \lambda_{d_2 d_2}^{d_2} I \end{bmatrix},$$

$$\lambda_{d_2 k_2}^{i} = \begin{cases} \dfrac{-(2i+1)\left(1-(-1)^{k_2+i}\right)}{\tau}, & i \le k_2, \\ 0, & i > k_2. \end{cases} \tag{16}$$

Proof. Applying Lemma 1 to the constructed LKF (12), it gives that

$$\int_{t-\tau_1}^{t} x^T(v) S_1 x(v) \mathrm{d}v \ge \frac{1}{\tau_1} \Omega^T(x) \left(\mathscr{F}_1 \otimes S_1\right) \Omega(x), \tag{17}$$

$$\int_{t-\tau_2}^{t-\tau_1} x^T(v) S_2 x(v) \mathrm{d}v \ge \frac{1}{\tau} \Gamma^T(x) \left(\mathscr{F}_2 \otimes S_2\right) \Gamma(x). \tag{18}$$

According to (12), (17), and (18), it yields

$$V(t) \ge \zeta^T(t) \widehat{P}_N \zeta(t) + \int_{t-\tau_1}^{t} x^T(v)(v-t+\tau_1) R_1 x(v) \mathrm{d}v \cdot \int_{t-\tau_2}^{t-\tau_1} x^T(v)(v-t+\tau_2) R_2 x(v) \mathrm{d}v. \tag{19}$$

From $R_1 > 0$, $R_2 > 0$, and $\widehat{P}_N > 0$, the LKF $V(t)$ is ensured to be positive.

Now, we calculate the time derivative of $V(t)$ as

$$\begin{aligned} \dot{V}_1(t) &= 2\zeta^T(t) P_N \dot{\zeta}(t) = 2\xi^T(t) H^T P_N M \xi(t), \\ \dot{V}_2(t) &= x^T(t)(S_1 + \tau_1 R_1) x(t) - x^T(t-\tau_1) S_1 x(t-\tau_1) \\ &\quad - \int_{t-\tau_1}^{t} x^T(v) R_1 x(v) \mathrm{d}v, \\ \dot{V}_3(t) &= x^T(t-\tau_1)(S_2 + \tau_2 R_2) x(t-\tau_1) \\ &\quad - x^T(t-\tau_2) S_2 x(t-\tau_2) - \int_{t-\tau_2}^{t-\tau_1} x^T(v) R_2 x(v) \mathrm{d}v, \end{aligned} \tag{20}$$

where

$$\xi^T(t) = \left[\dot{x}^T(t) \;\; x^T(t) \;\; f^T(x(s_k)) \;\; x^T(t-\tau_1) \;\; x^T(t-\tau_2) \;\; \Sigma^T(x) \;\; \Upsilon^T(x) \;\; \varepsilon^T(t)\right]. \tag{21}$$

The derivatives of $\Sigma(x)$ and $\Upsilon(x)$ introduced in (12) are also computed as

$$\begin{aligned}\dot{\Sigma}(x) &= \left[\dot{\Sigma}_0^T(x) \ \cdots \ \dot{\Sigma}_{k_1}^T(x) \ \cdots \ \dot{\Sigma}_{d_1}^T(x)\right]^T,\\ \dot{\Upsilon}(x) &= \left[\dot{\Upsilon}_0^T(x) \ \cdots \ \dot{\Upsilon}_{k_2}^T(x) \ \cdots \ \dot{\Upsilon}_{d_2}^T(x)\right]^T,\end{aligned} \tag{22}$$

where

$$\begin{aligned}\dot{\Sigma}_{k_1}(x) &= \mathbb{L}_{1,k_1}(0)x(t) - \mathbb{L}_{1,k_1}(-\tau_1)x(t-\tau_1)\\ &\quad - \int_{-\tau_1}^{0} \dot{\mathbb{L}}_{1,k_1}(v)x(t+v)\mathrm{d}v,\\ \dot{\Upsilon}_{k_2}(x) &= \mathbb{L}_{2,k_2}(-\tau_1)x(t-\tau_1) - \mathbb{L}_{2,k_2}(-\tau_2)x(t-\tau_2)\\ &\quad - \int_{-\tau_2}^{-\tau_1} \dot{\mathbb{L}}_{2,k_2}(v)x(t+v)\mathrm{d}v.\end{aligned} \tag{23}$$

Based on the Legendre polynomial properties presented in [35], one has

$$\begin{aligned}\dot{\Sigma}_{k_1}(x) &= x(t) - (-1)^{k_1}x(t-\tau_1) - \sum_{i=0}^{k_1-1}\lambda_{d_1k_1}^{i}\Sigma_i(x),\\ \dot{\Upsilon}_{k_2}(x) &= x(t-\tau_1) - (-1)^{k_2}x(t-\tau_2) - \sum_{i=0}^{k_2-1}\lambda_{d_2k_2}^{i}\Upsilon_i(x),\end{aligned} \tag{24}$$

which lead to

$$\begin{aligned}\dot{\Sigma}(x) &= \mathscr{L}_{d_1}(0)x(t) - \mathscr{L}_{d_1}(-\tau_1)x(t-\tau_1) + \Lambda_{d_1}\Sigma(x),\\ \dot{\Upsilon}(x) &= \mathscr{L}_{d_2}(-\tau_1)x(t) - \mathscr{L}_{d_2}(-\tau_2)x(t-\tau_2) + \Lambda_{d_2}\Upsilon(x).\end{aligned} \tag{25}$$

To ensure the closed-loop system (2) to be asymptotically stable, one just needs

$$\begin{aligned}\dot{V}(t) &= 2\xi^T(t)H^TP_NM\xi(t) + x^T(t)(S_1+\tau_1R_1)x(t)\\ &\quad - x^T(t-\tau_1)S_1x(t-\tau_1) + x^T(t-\tau_1)(S_2+\tau R_2)x\\ &\quad \cdot (t-\tau_1) - x^T(t-\tau_2)S_2x(t-\tau_2)\\ &\quad - \int_{t-\tau_1}^{t} x^T(v)R_1x(v)\mathrm{d}v - \int_{t-\tau_2}^{t-\tau_1} x^T(v)R_2x(v)\mathrm{d}v < 0.\end{aligned} \tag{26}$$

Revisiting the triggering condition (5), one can get

$$\varepsilon^T(t)\Phi\varepsilon(t) < \delta\xi^T(t)\mathscr{I}_1^T\Phi\mathscr{I}_1\xi(t). \tag{27}$$

The integral terms in (26) are handled by Lemma 1, which yield

$$-\int_{t-\tau_1}^{t} x^T(v)R_1x(v)\mathrm{d}v \le -\frac{1}{\tau_1}\Sigma^T(x)(\mathscr{F}_1\otimes R_1)\Sigma(x), \tag{28}$$

$$-\int_{t-\tau_2}^{t-\tau_1} x^T(v)R_2x(v)\mathrm{d}v \le -\frac{1}{\tau}\Upsilon^T(x)(\mathscr{F}_2\otimes R_2)\Upsilon(x). \tag{29}$$

Then, according to Assumption 1, one has

$$\begin{aligned}&\begin{bmatrix}\hat{x}(s_k)\\ f(\hat{x}(s_k))\end{bmatrix}^T\begin{bmatrix}\overline{F}_1 & \overline{F}_2\\ \overline{F}_2^T & I\end{bmatrix}\begin{bmatrix}\hat{x}(s_k)\\ f(\hat{x}(s_k))\end{bmatrix}\\ &= \begin{bmatrix}\int_{t-\tau_2}^{t-\tau_1}\frac{1}{\tau}x(v)\mathrm{d}v - \varepsilon(t)\\ f(\hat{x}(s_k))\end{bmatrix}^T\begin{bmatrix}\overline{F}_1 & \overline{F}_2\\ \overline{F}_2^T & I\end{bmatrix}\\ &\quad \cdot\begin{bmatrix}\int_{t-\tau_2}^{t-\tau_1}\frac{1}{\tau}x(v)\mathrm{d}v - \varepsilon(t)\\ f(\hat{x}(s_k))\end{bmatrix} \le 0,\end{aligned} \tag{30}$$

which is rewritten as

$$-\rho\xi^T(t)\mathscr{I}_2^T\overline{F}\mathscr{I}_2\xi(t) \ge 0, \tag{31}$$

for any $\rho > 0$.

Combining (27)–(29), (3) holds if the following inequality is satisfied:

$$\xi^T(t)\Pi\xi(t) < 0. \tag{32}$$

According to the description of system (2) and Lemma 2, we have

$$\mathbb{E}\left\{\xi^T(t)(\Pi + \mathrm{He}(\bar{\mathscr{W}}\bar{\mathscr{V}}))\xi(t)\right\} < 0, \tag{33}$$

where

$$\begin{aligned}\bar{\mathscr{W}} &= \left[W_1 \ \ W_2 \ \ 0_n \ \ 0_n \ \ 0_n \ \ 0_{n\times(d_1+1)n} \ \ 0_{n\times(d_2+1)n} \ \ 0_n\right]^T,\\ \bar{\mathscr{V}} &= \left[-I \ \ A+\Delta A(t) \ \ (1-\alpha(s_k))B(K+\Delta K) \ \ 0_n \ \ 0_n \ \ 0_{n\times(d_1+1)n} \ \ \frac{\alpha(s_k)}{\tau}B(K+\Delta K) \ \ 0_{n\times d_2n} \ \ -\alpha(s_k)B(K+\Delta K)\right].\end{aligned} \tag{34}$$

Noting that

$$\mathbb{E}\left\{\xi^T(t)\mathrm{He}(\bar{\mathscr{W}}\bar{\mathscr{V}})\xi(t)\right\} = \xi^T(t)\mathrm{He}(\mathscr{W}\mathscr{V})\xi(t) = 0, \tag{35}$$

it is clear that (33) is equivalent to (15). □

Remark 6. In the existing result [31] concerning integral-based event-triggered control issue, an approximation method is adopted to handle the integral term induced by the triggering condition. Based on this method, approximation error is an inevitable factor, which could introduce conservativeness in controller design. Nevertheless, with the help of Legendre polynomials and their properties, such approximation error is removed in this paper, and less conservative results could be obtained.

Theorem 2. *For given scalars d_1, d_2, τ, τ_1, τ_2, ρ, δ, and $\overline{\alpha}$, under the proposed integral-based ETS (2), the closed-loop system (2) is asymptotically stable in mean square sense if there exist symmetric matrices P_N, $\Phi > 0$, $S_1 > 0$, $S_2 > 0$, $R_1 > 0$, and $R_2 > 0$ and matrices U, W_1, W_2, Z, and N such that*

$$\begin{bmatrix} \widehat{\Pi} & \beta \mathscr{W} B \mathscr{E}_a & \mathscr{E}_b^T \\ * & -\beta I & 0 \\ * & * & -\beta I \end{bmatrix} < 0, \tag{36}$$

where

$$\begin{aligned}
\widehat{\Pi} &= \begin{bmatrix} \Pi + He(\widetilde{\mathscr{W}}\widetilde{\mathscr{V}}) & \Pi_2^T \\ * & -\mu_3 He(N) \end{bmatrix}, \\
\mathscr{E}_a &= E_a, \\
\mathscr{E}_b &= \begin{bmatrix} 0_n & 0_n & (1-\overline{\alpha})E_b & 0_n & 0_n & 0_{n\times(d_1+1)n} & \frac{\overline{\alpha}}{\tau}E_b & 0_{n\times d_2 n} & -\overline{\alpha}E_b \end{bmatrix}, \\
\Pi_2 &= \begin{bmatrix} \Pi_{21} & \Pi_{22} & \Pi_{23} & 0_n & 0_n & 0_n & 0_{n\times(d_1+1)n} & \Pi_{24} & 0_{n\times d_2 n} & \Pi_{25} \end{bmatrix}, \\
\Pi_{21} &= (\mu_1 BN - W_1 B)^T, \\
\Pi_{22} &= (\mu_2 BN - W_2 B)^T, \\
\Pi_{23} &= (1-\overline{\alpha})\mu_3 Z, \\
\Pi_{24} &= \frac{\overline{\alpha}}{\tau}\mu_3 Z, \\
\Pi_{25} &= -\overline{\alpha}\mu_3 Z, \\
\widetilde{\mathscr{W}} &= \begin{bmatrix} I & 0_n & 0_n & 0_n & 0_n & 0_{n\times(d_1+1)n} & 0_{n\times(d_2+1)n} & 0_n & 0_n \\ 0_n & I & 0_n & 0_n & 0_n & 0_{n\times(d_1+1)n} & 0_{n\times(d_2+1)n} & 0_n & 0_n \end{bmatrix}^T, \\
\widetilde{\mathscr{V}} &= \begin{bmatrix} -W_1 & W_1(A+\overline{\varsigma}\widehat{A}) & (1-\overline{\alpha})\mu_1 BZ & 0_n & 0_n & 0_{n\times(d_1+1)n} & \frac{\overline{\alpha}}{\tau}\mu_1 BZ & 0_{n\times d_2 n} & -\overline{\alpha}\mu_1 BZ & 0_n \\ -W_2 & W_2(A+\overline{\varsigma}\widehat{A}) & (1-\overline{\alpha})\mu_2 BZ & 0_n & 0_n & 0_{n\times(d_1+1)n} & \frac{\overline{\alpha}}{\tau}\mu_2 BZ & 0_{n\times d_2 n} & -\overline{\alpha}\mu_2 BZ & 0_n \end{bmatrix}.
\end{aligned} \tag{37}$$

Moreover, the controller gain is obtained as $K = N^{-1}Z$.

Proof. It is noted that (15) can be written as

$$\Pi + He(\mathscr{W}\hat{\mathscr{V}}) + He(\mathscr{W}\Delta\mathscr{V}) < 0, \tag{38}$$

where

$$\begin{aligned}
\hat{\mathscr{V}} &= \begin{bmatrix} -I & A+\overline{\varsigma}\widehat{A} & (1-\overline{\alpha})BK & 0_n & 0_n & 0_{n\times(d_1+1)n} & \frac{\overline{\alpha}}{\tau}BK & 0_{n\times d_2 n} & -\overline{\alpha}BK \end{bmatrix}, \\
\Delta\mathscr{V} &= \begin{bmatrix} 0_n & 0_n & (1-\overline{\alpha})B\Delta K & 0_n & 0_n & 0_{n\times(d_1+1)n} & \frac{\overline{\alpha}}{\tau}B\Delta K & 0_{n\times d_2 n} & -\overline{\alpha}B\Delta K \end{bmatrix}.
\end{aligned} \tag{39}$$

Based on Lemma 3, one has

$$\begin{aligned}
He(\mathscr{W}\Delta\mathscr{V}) &= He(\mathscr{W}B\mathscr{E}_a G(t)\mathscr{E}_b) \le \beta\mathscr{W}B\mathscr{E}_a(\mathscr{W}B\mathscr{E}_a)^T \\
&\quad + \beta^{-1}\mathscr{E}_b^T\mathscr{E}_b.
\end{aligned} \tag{40}$$

With the help of Lemma 2, (15) is ensured by

$$\mathscr{H}^{\perp^T}\begin{bmatrix} \Pi + He(\mathscr{W}\hat{\mathscr{V}}) + \beta\mathscr{W}B\mathscr{E}_a(\mathscr{W}B\mathscr{E}_a)^T + \beta^{-1}\mathscr{E}_b^T\mathscr{E}_b & 0 \\ 0 & 0 \end{bmatrix}\mathscr{H}^{\perp} < 0, \tag{41}$$

where

$$\mathscr{H}^{\perp} = \begin{bmatrix} \mathscr{H}_1^{\perp} \\ \mathscr{H}_2^{\perp} \end{bmatrix},$$

$$\mathscr{H}_1^{\perp} = \operatorname{diag}\left\{ I, I, I, I, I, \underbrace{I, \ldots, I}_{d_1+d_2+2}, I \right\},$$

$$\mathscr{H}_2^{\perp} = \begin{bmatrix} 0_{n\times 2n} & (1-\overline{\alpha})BK & 0_{n\times 2n} & 0_{n\times(d_1+1)n} & \dfrac{\overline{\alpha}}{\tau}BK & 0_{n\times d_2 n} & -\overline{\alpha}BK \end{bmatrix}. \tag{42}$$

By applying Lemma 2 to (41), it leads to

$$\begin{bmatrix} \Pi + He(\mathscr{W}\hat{\mathscr{V}}) + \beta\mathscr{W}B\mathscr{E}_a(\mathscr{W}B\mathscr{E}_a)^T + \beta^{-1}\mathscr{E}_b^T\mathscr{E}_b & 0 \\ 0 & 0 \end{bmatrix} + He(\mathscr{M}\mathscr{H}) < 0, \tag{43}$$

where

$$\begin{aligned} \mathscr{M} &= \begin{bmatrix} \mathscr{M}_1 & \mathscr{M}_2 & 0_{m\times 3n} & 0_{m\times(d_1+1)n} & 0_{m\times(d_2+1)n} & 0_{m\times n} & \mu_3 N^T \end{bmatrix}^T, \\ \mathscr{H} &= \begin{bmatrix} 0_{m\times 2n} & (1-\overline{\alpha})K & 0_{m\times 2n} & 0_{m\times(d_1+1)n} & \dfrac{\overline{\alpha}}{\tau}K & 0_{m\times d_2 n} & -\overline{\alpha}K & -I_m \end{bmatrix}, \\ \mathscr{M}_1 &= (\mu_1 BN - W_1 B)^T, \\ \mathscr{M}_2 &= (\mu_2 BN - W_2 B)^T. \end{aligned} \tag{44}$$

Using the Schur complement, it results in

$$\begin{bmatrix} \widehat{\Pi} & \beta\mathscr{W}B\mathscr{E}_a & \mathscr{E}_b^T \\ * & -\beta I & 0 \\ * & * & -\beta I \end{bmatrix} < 0, \tag{45}$$

where

$$\widehat{\Pi} = \begin{bmatrix} \Pi + He(\mathscr{W}\hat{\mathscr{V}}) & 0 \\ 0 & 0 \end{bmatrix} + He(\mathscr{M}\mathscr{H}). \tag{46}$$

Therefore, (36) is ensured by (45). □

Remark 7. To deal with the cyber-attacks satisfying Assumption 1, the well-known S-procedure is also used in [27], which will cause the difficulty of controller design by the conventional pre- and post-multiplying technique. In [27], the control matrix B is assumed to be full column rank to solve this problem. However, a decomposition technique on the basis of Lemma 3 is adopted in Theorem 2, which removes such rank constraint.

Remark 8. The similar way in [10] can be utilized to derive the positive lower bound of inner-event time, which implies that the infinite triggering events over finite time interval are excluded via the proposed integral-based ETS (2).

When cyber-attacks are not considered, the closed-loop system (2) is rewritten as

$$\dot{x}(t) = (A + \Delta A(t))x(t) + B(K + \Delta K)\left(\frac{1}{\tau}\int_{t-\tau_2}^{t-\tau_1} x(v)\mathrm{d}v - \varepsilon(t)\right), \quad t \in [t_k, t_{k+1}). \tag{47}$$

Following the similar way to derive Theorem 2, the terms related with $f(x(s_k))$ are removed, and the control synthesis conditions are produced in Corollary 1.

Corollary 1. *For given scalars d_1, d_2, τ, τ_1, τ_2, ρ, and δ, considering the event-triggered scheme (2), the closed-loop system (3) is asymptotically stable if there exist symmetric matrices P_N, $\Phi > 0$, $S_1 > 0$, $S_2 > 0$, $R_1 > 0$, and $R_2 > 0$ and matrices W_1, W_2, Z, and N such that*

$$\begin{aligned} &\widehat{P}_N > 0, \\ &\begin{bmatrix} \widehat{\Xi} & \beta\widehat{\mathscr{W}}B\widehat{\mathrm{E}}_a & \widehat{\mathrm{E}}_b^T \\ * & -\beta I & 0 \\ * & * & -\beta I \end{bmatrix} < 0, \end{aligned} \tag{48}$$

where

$$\widehat{\Xi} = \begin{bmatrix} \Xi & \Xi_2 \\ \Xi_2^T & -\mu_3 He(W) \end{bmatrix} + He(\breve{\mathscr{W}}\breve{\mathscr{V}}),$$

$$\widehat{\mathscr{E}}_a = E_a,$$

$$\widehat{\mathscr{W}} = \begin{bmatrix} W_1^T & W_2^T & 0_n & 0_n & 0_{n\times(d_1+1)n} & 0_{n\times(d_2+1)n} & 0_n \end{bmatrix}^T,$$

$$\widehat{\mathscr{E}}_b = \begin{bmatrix} 0_{n\times 4n} & 0_{n\times(d_1+1)n} & \frac{1}{\tau}E_b & 0_{n\times d_2 n} & -E_b \end{bmatrix},$$

$$\Xi = He\left(\widehat{H}_N^T P_N \widehat{M}_N\right) + \delta\mathscr{I}_4^T \Phi \mathscr{I}_4 + \mathrm{diag}\{0_n, \Xi_{12}, \Xi_{13}, \Xi_{14}, \Xi_{15}, \Xi_{16}, \Xi_{17}\},$$

$$\mathscr{I}_4 = \begin{bmatrix} 0_{n\times 4n} & 0_{n\times(d_1+1)n} & \frac{1}{\tau}I & 0_{n\times d_2 n} & -I \end{bmatrix},$$

$$\Xi_{12} = S_1 + \tau_1 R_1,$$

$$\Xi_{13} = -S_1 + S_2 + \tau R_2,$$

$$\Xi_{14} = -S_2,$$

$$\Xi_{15} = -\frac{(\mathscr{F}_1 \otimes R_1)}{\tau_1}, \tag{49}$$

$$\Xi_{16} = -\frac{(\mathscr{F}_2 \otimes R_2)}{\tau},$$

$$\Xi_{17} = -\Phi,$$

$$\Xi_2^T = \begin{bmatrix} \Xi_{21} & \Xi_{22} & 0_n & 0_n & 0_{n\times(d_1+1)n} & \frac{\mu_3}{\tau}Z & 0_{n\times d_2 n} & -\mu_3 Z \end{bmatrix},$$

$$\Xi_{21} = (\mu_1 BN - W_1 B)^T,$$

$$\Xi_{22} = (\mu_2 BN - W_2 B)^T,$$

$$\breve{\mathscr{W}} = \begin{bmatrix} I & 0_n & 0_{n\times 2n} & 0_{n\times(d_1+1)n} & 0_{n\times(d_2+1)n} & 0_n & 0_n \\ 0_n & I & 0_{n\times 2n} & 0_{n\times(d_1+1)n} & 0_{n\times(d_2+1)n} & 0_n & 0_n \end{bmatrix}^T,$$

$$\breve{\mathscr{V}} = \begin{bmatrix} -W_1 & W_1(A + \bar{\varsigma}\widehat{A}) & 0_{n\times 2n} & 0_{n\times(d_1+1)n} & \frac{\mu_1}{\tau}BZ & 0_{n\times d_2 n} & -\mu_1 BZ & 0 \\ -W_2 & W_2(A + \bar{\varsigma}\widehat{A}) & 0_{n\times 2n} & 0_{n\times(d_1+1)n} & \frac{\mu_2}{\tau}BZ & 0_{n\times d_2 n} & -\mu_2 BZ & 0 \end{bmatrix}.$$

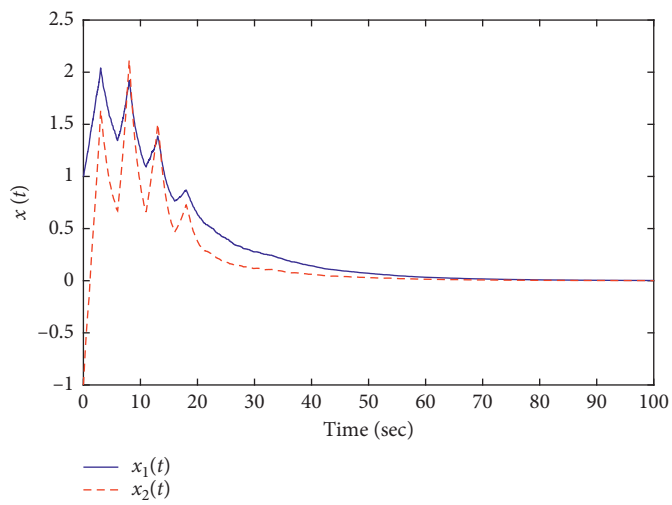

Figure 2: The system state responses under the controller solved by Theorem 2.

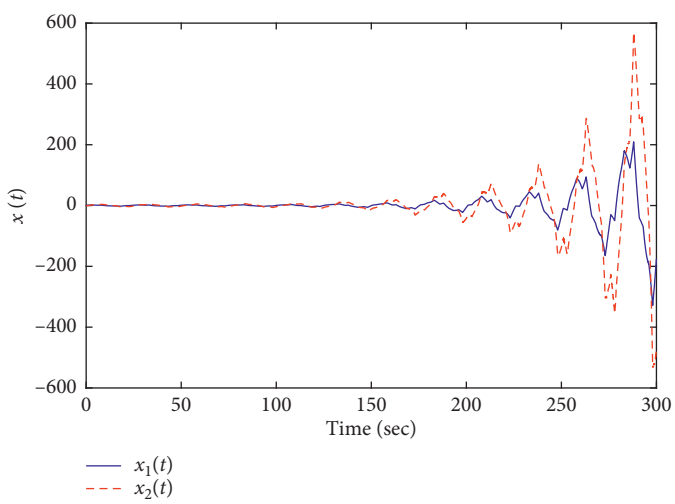

Figure 3: The system state responses under the controller solved by Corollary 1.

4. Numerical Example

Example 1. The parameters of system (1) and cyber-attack function are given as

$$A = \begin{bmatrix} 0.1 & 0 \\ 0.8 & -0.2 \end{bmatrix},$$
$$B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix},$$
$$\widehat{A} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix}, \tag{50}$$
$$\overline{\varsigma} = 0.8,$$
$$E_a = 0.02,$$
$$E_b = [0.2 \;\; 0.1].$$

Assume the cyber-attack function as

$$f(x(s_k)) = \operatorname{sat}(x(s_k)) = \begin{cases} \overline{x}_i, & x_i(s_k) > \overline{x}_i, \\ x_i(s_k), & -\overline{x}_i \le x_i(s_k) \le \overline{x}_i, i = 1, 2, \\ -\overline{x}_i, & x_i(s_k) < -\overline{x}_i, \end{cases} \tag{51}$$

where the saturation levels are $\overline{x}_1 = 0.5$ and $\overline{x}_2 = 0.4$. The cyber-attack function satisfies Assumption 1 with $F_1 = \operatorname{diag}\{0, 0\}$ and $F_2 = \operatorname{diag}\{1.2, 1.2\}$.

By choosing $\overline{\alpha} = 0.8$, $d_1 = d_2 = 1$, $\tau = 0.1$, $\tau_1 = 0.04$, $\tau_2 = 0.14$, $\rho = 15$, $\delta = 0.01$, $\mu_1 = \mu_2 = 1$, and $\mu_3 = 0.1$, the controller gain K and triggering parameter Φ solved by Theorem 2 are given as

$$K = [-0.4919 \;\; -0.6554],$$
$$\Phi = \begin{bmatrix} 284.6063 & -15.8585 \\ -15.8585 & 270.7435 \end{bmatrix}. \tag{52}$$

Then, by solving Corollary 1 where the cyber-attacks are not taken into account, it gives the corresponding parameters K and Φ as

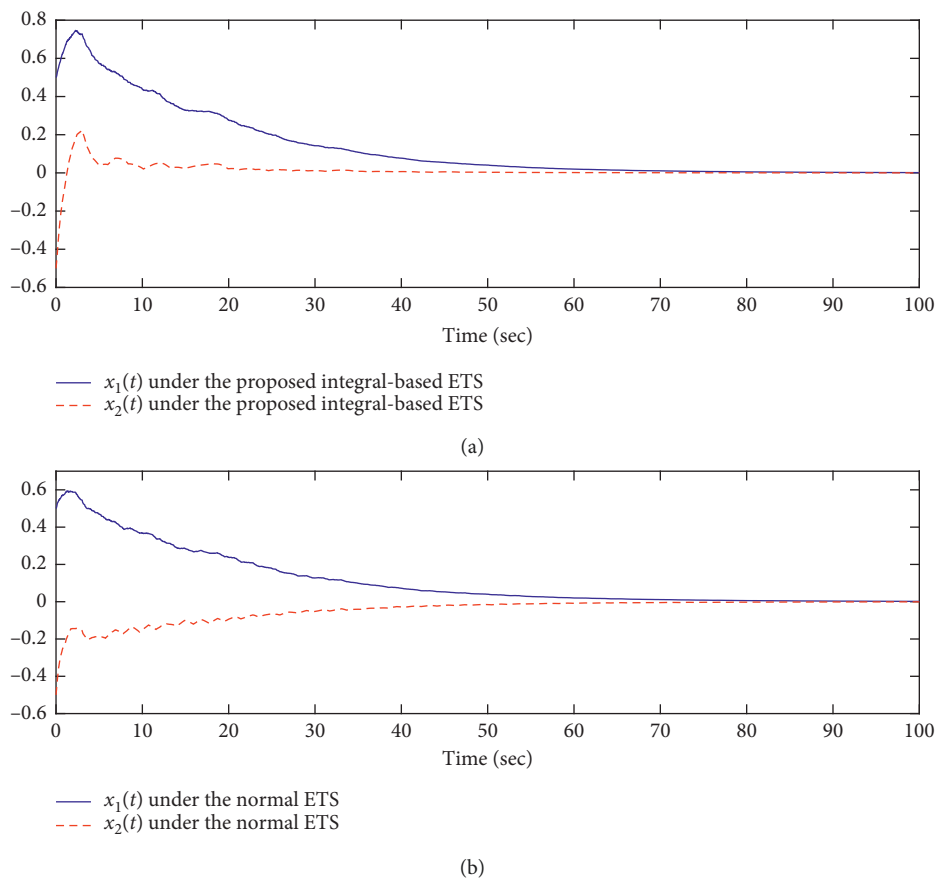

Figure 4: The comparison of system state $x(t)$.

$$K = [-0.3050 \quad -0.3878],$$
$$\Phi = \begin{bmatrix} 127.6124 & -2.9095 \\ -2.9095 & 125.5567 \end{bmatrix}. \tag{53}$$

In the simulation, we choose the initial condition as $x(0) = [1 \quad -1]^T$ and $G(t) = \sin(t)$. The system state responses under the controller solved by Theorem 2 and Corollary 1 are drawn in Figures 2 and 3, respectively.

From Figure 3, one can see that the cyber-attacks can cause system instability. When the cyber-attacks are considered in system analysis, the system state responses given in Figure 2 are stabilized by the designed controller solved by Theorem 2, which also shows the effectiveness of the proposed method.

Moreover, to illustrate the effectiveness of the proposed integral-based ETS, the following simulations are implemented. For $\overline{\alpha} = 0.7$ and the same parameters with above, the corresponding controller gain K and triggering parameter Φ solved by Theorem 2 are given as

$$K = [-0.6818 \quad -1.0267],$$
$$\Phi = \begin{bmatrix} 307.9407 & -31.9729 \\ -31.9729 & 270.2628 \end{bmatrix}. \tag{54}$$

In the simulation, initial condition is chosen as $x(0) = [0.5 \quad -0.5]^T$. For $d_1 = d_2 = 1$, with the above parameters, the compared trajectories of system state under the proposed integral-based ETS (2) and the normal ETS (3) are depicted in Figure 4. The triggered signal $x(s_k)$ and attacked signal $f(x(s_k))$ under these two ETSs are given in Figures 5 and 6, respectively. Figure 7 illustrates the comparison of release time intervals.

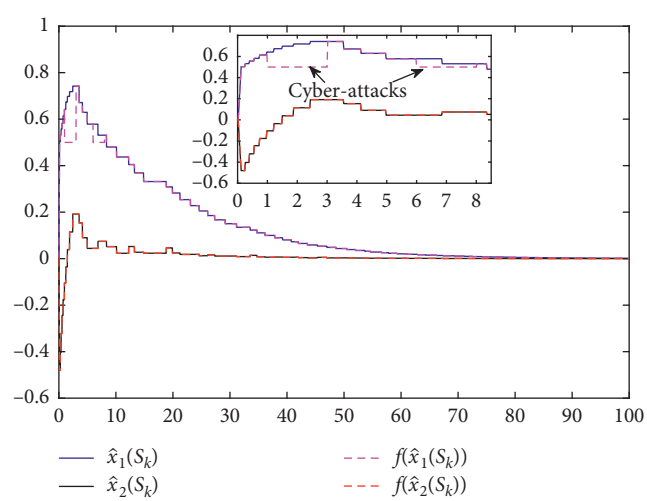

Figure 5: $x(s_k)$ and $f(x(s_k))$ under the integral-based ETS (2).

From Figure 4, it is seen that the similar convergence times are performed under these two different ETSs. However, from Figure 7 and Table 1, the proposed integral-

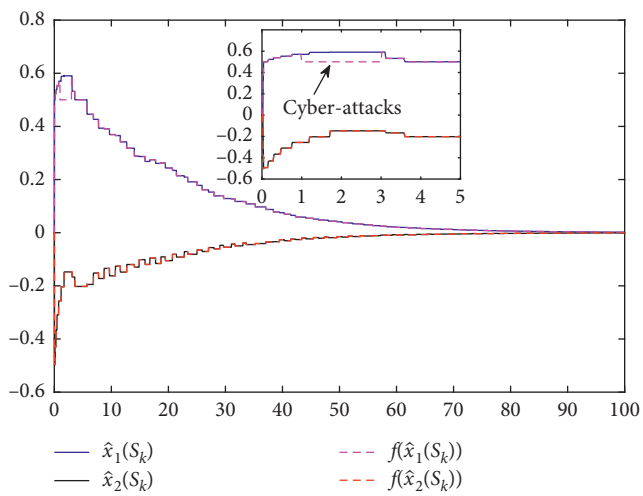

Figure 6: $x(s_k)$ and $f(x(s_k))$ under the normal ETS (3).

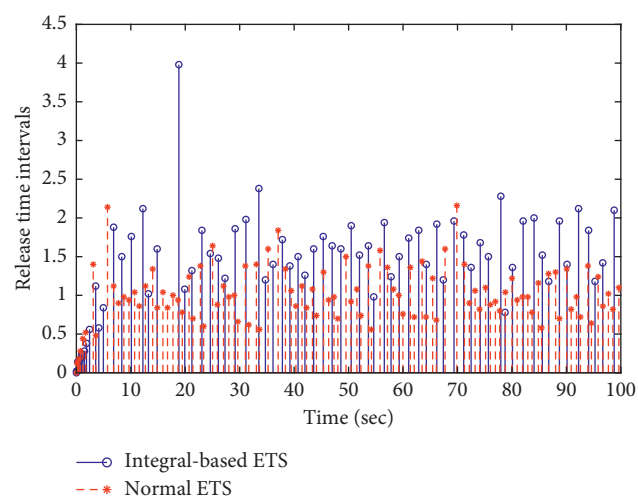

Figure 7: The comparison of release time intervals.

Table 1: Average triggering time intervals, $\overline{T}(s)$.

	Integral-based ETS	Normal ETS
$\overline{T}(s)$	1.37	1.01

based ETS increases 26.26% of the average triggering time interval compared with the normal ETS. Consequently, these results show that the proposed integral-based ETS generates fewer trigger events and saves more network resources than the normal ETS.

5. Conclusions

This paper focuses on the study of the nonfragile integral-based event-triggered control issue for linear CPSs with transmission delay, gain variations, and cyber-attacks. An integral-based ETS is proposed to decrease the unnecessary data transmissions, which utilizes the mean of system state instead of the instant system information. A stochastic variable is used to model the randomly launched cyber-attacks in the communication channel. To deal with the integral term induced by integral-based ETS, Legendre polynomials are introduced to construct a novel LKF. With the help of the LMI technique, some sufficient conditions are obtained to ensure the mean square stability of the closed-loop system. Finally, the validity of the presented results is shown through a numerical example. Since the controllers are usually distributed at different nodes in real CPSs, the same triggering conditions and the same cyber-attacks for all communication channels maybe conservative. Thus, how to apply our proposed event-triggered scheme to the issue of decentralized security control against hybrid attacks is one of our future investigations. Furthermore, extending the proposed integral-based ETS to the finite time control [37, 38] and the chance-constrained control [39] will be another interesting research topic.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

All authors declare that they have no conflicts of interest.

Acknowledgments

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