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Research Article

Robust Control and Synchronization of 3-D Uncertain Fractional-Order Chaotic Systems with External Disturbances via Adding One Power Integrator Control

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This paper investigates the control and synchronization of a class of 3-D uncertain fractional-order chaotic systems with external disturbances. The adding one power integrator control scheme, which is the generalization of the traditional backstepping method, is used to investigate the global stability of the control and synchronization manifold. As a result, several criteria for chaos control and synchronization are obtained. Compared with the previous results, the presented strategies can not only be applied to a class of strict-feedback systems but also be applied to more general class of fractional-order chaotic systems. In addition, the proposed controllers are robust against uncertain parameters and external disturbances. To validate the effectiveness of the proposed criteria, two illustrative examples are given.

1. Introduction

Chaotic systems, which are characterized by possessing at least one positive Lyapunov exponent, are a special case of nonlinear deterministic systems with unpredictable behaviors. In general, chaotic systems can be classified into two categories: integer-order and fractional-order. Due to its potential applications in secure communications, biological systems [1] and so on, the control and synchronization of integer-order chaotic systems have been extensively studied for more than twenty years. Recently, chaos control and synchronization of fractional-order chaotic systems have attracted extensive attention of researchers and many results have been proposed by employing different control methods such as backstepping control method [2], sliding mode control method [3, 4], PID control approach [5], periodically intermittent control strategy [6], adaptive fuzzy backstepping control [7], and adaptive fuzzy prescribed performance control [8]. The backstepping method is a systematic design approach. By designing virtual controllers and partial Lyapunov functions step by step one can finally derive a common Lyapunov function for the overall system. The main advantage of this method is that it can guarantee the global stability, tracking, and transient performance of nonlinear systems [9]. Various excellent backstepping strategies for the control and synchronization of fractional-order chaotic system have been investigated and proposed in recent studies. For example, the control fractional-order ferroresonance system, which can show chaotic phenomenon, was considered in paper [10] by using the adaptive backstepping control. Papers [11, 12] investigated the stabilization and synchronization of a class of fractional-order chaotic systems via backstepping approach. Paper [13] proposed a new chaotic system with no equilibrium point and discussed its fractional-order backstepping control. The stabilization problem of a class of fractional-order chaotic systems with unknown parameters by using adaptive backstepping strategy was considered in [14]. However, this schemes presented in papers [10–14] can only be used for nonlinear systems with strict-feedback structure. In addition, the external perturbations have not been taken into account here. Since in practical situations the external disturbance unavoidably exists and it may destroy the stabilization of chaotic systems, it is necessary to consider the effect of external disturbances in designing an adaptive controller.

In 2000, paper [15] presented a new control method: adding a power integrator control. Similar to the traditional backstepping method, this new control method is also a systematic design approach and recursive Lyapunov-based scheme. In order to obtain a common Lyapunov function for the overall system, one needs to design power integrators and partial Lyapunov functions step by step. Since the power integrator is the generalization of the virtual controller, thus adding a power integrator control can be viewed as the generalization of the traditional backstepping method. The remarkable superiorities of adding a power integrator control are its robustness to parameter uncertainties and good transient response of nonlinear systems. Therefore, the adding a power integrator control method is applicable for the control and synchronization problem of chaotic system.

Motivated by the above discussions this paper considers the control and synchronization of a class of 3-D fractional-order chaotic systems. This chaotic system with uncertain parameters is assumed to be affected by external disturbances. By using the adding one power integrator control scheme, several criteria for chaos control and synchronization are presented. The theoretical results are validated by two numerical simulations.

Compared with the mentioned prior papers [10–14], there are two advantages which make our control scheme attractive and meaningful. (a) The schemes presented in papers [10–14] are only valid for a class of strict-feedback chaotic systems. However, the technique proposed in this paper can be applied for any 3-D fractional-order chaotic systems. (b) The controllers constructed in papers [10–14] are under the same assumption that systems are free from external perturbations. However, the external perturbations are taken into consideration in our paper.

The subsequent sections of this paper are organized as follows: the system description and related preliminaries are introduced in Section 2. The control and synchronization schemes are presented in Sections 3 and 4, respectively. Some illustrative examples to verify the effectiveness of the obtained theoretical result are given in Section 5. Finally, conclusions are drawn in Section 6.

2. Preliminaries and System Description

Definition 1 (see [16]). The Caputo's fractional derivative of order α of function $f(t)$ is

$$D_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau, \tag{1}$$

where $t \geq t_0, n \in N^{+}, n-1 < \alpha \leq n$ and

$$f^{(n)}(\tau) = \frac{d^n (f(\tau))}{d\tau^n}. \tag{2}$$

When $0 < \alpha \leq 1$, then the Caputo fractional derivative of order α of function $f(t)$ reduces to

$$D_{t_0}^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} \frac{f'(\tau)}{(t-\tau)^{\alpha}} d\tau. \tag{3}$$

Lemma 2 (see [17]). *Suppose $x(t) \in R$ is a derivable function, then*

$$\frac{1}{2} D_{t_0}^{\alpha} x^2(t) \leq x(t) D_{t_0}^{\alpha} x(t), \quad \forall \alpha \in (0,1), \ t_0 \geq 0. \tag{4}$$

In the sequel, for the sake of simplicity, we use $D^{\alpha}x$ to denote $D_0^{\alpha}x$.

The fractional-order chaotic system which is considered in this paper is

$$\begin{aligned} D^{\alpha} x_1 &= f_{11}(x) + a_1 g_{11}(x) + d_{11}, \\ D^{\alpha} x_2 &= f_{12}(x) + b_1 g_{12}(x) + d_{12}, \\ D^{\alpha} x_3 &= f_{13}(x) + c_1 g_{13}(x) + d_{13}, \end{aligned} \tag{5}$$

where $0 < \alpha \leq 1$, $x = (x_1, x_2, x_3)^T \in R^{3\times 1}$ is the state vector of the system (5), $f_{1i}(x), g_{1i}(x)$ are all known continuous functions, $i = 1,2,3$. a_1, b_1, c_1 are unknown parameters, d_{11}, d_{12}, d_{13} are bounded external disturbances.

Assumption 3. Suppose there exist known constants $\overline{d}_{1i} > 0$ such that $|d_{1i}| \leq \overline{d}_{1i}$, $i = 1,2,3$.

3. The Control Scheme

The control scheme of system (5) is presented in this section.

Based on system (5), the controlled system is rewritten as

$$\begin{aligned} D^{\alpha} x_1 &= f_{11}(x) + a_1 g_{11}(x) + d_{11} + u_1, \\ D^{\alpha} x_2 &= f_{12}(x) + b_1 g_{12}(x) + d_{12} + u_2, \\ D^{\alpha} x_3 &= f_{13}(x) + c_1 g_{13}(x) + d_{13} + u_3, \end{aligned} \tag{6}$$

where u_1, u_2, u_3 are controllers.

The purpose of this section is to find suitable controllers u_1, u_2, u_3, such that the equilibrium point $(0,0,0)$ of system (6) is asymptotically stable.

Let us introduce some new variables: $\xi_1 = x_1$, $x_2^* = -\alpha_1 x_1$, $\xi_2 = x_2 - x_2^*$, $x_3^* = -\alpha_2 \xi_2$, $\xi_3 = x_3 - x_3^*$, where $\alpha_1 > 0, \alpha_2 > 0$.

By using these new variables, the controllers u_1, u_2, u_3 in system (6) are designed as

$$\begin{aligned} u_1 &= x_2 - f_{11}(x) - \overline{a}_1 g_{11}(x) + \beta_1, \\ u_2 &= x_3 - f_{12}(x) - \overline{b}_1 g_{12}(x) + \beta_2, \\ u_3 &= -\alpha_6 \xi_3 - f_{13}(x) - \overline{c}_1 g_{13}(x) + \beta_3, \end{aligned} \tag{7}$$

where $\overline{a}_1, \overline{b}_1, \overline{c}_1$ are the estimated values of a_1, b_1, c_1, respectively, $\beta_1 = -\overline{d}_{11} \operatorname{sign}(x_1 + \alpha_2 \xi_2 + \alpha_1 \alpha_2 \xi_3)$, $\beta_2 = -\overline{d}_{12} \operatorname{sign}(\xi_2 + \alpha_2 \xi_3)$, $\beta_3 = -\overline{d}_{13} \operatorname{sign}(\xi_3)$, $\alpha_6 > 0$ is a constant.

After plugging controllers (7) into system (6), one derives

$$\begin{aligned} D^{\alpha} x_1 &= x_2 + \left(a_1 - \overline{a}_1\right) g_{11}(x) + d_{11} + \beta_1, \\ D^{\alpha} x_2 &= x_3 + \left(b_1 - \overline{b}_1\right) g_{12}(x) + d_{12} + \beta_2, \\ D^{\alpha} x_3 &= -\alpha_6 \xi_3 + \left(c_1 - \overline{c}_1\right) g_{13}(x) + d_{13} + \beta_3, \end{aligned} \tag{8}$$

Obviously, the control of system (6) is transformed into the stability problem of equilibrium point $(0,0,0)$ of system (8). To solve the stability problem of system (8), we propose the following Theorem 4.

Theorem 4. *Suppose the update laws of* $\overline{a}_1, \overline{b}_1, \overline{c}_1$ *are designed as*

$$\begin{aligned} D^{\alpha}\overline{a}_1 &= (\xi_1 + \alpha_2\xi_2 + \alpha_1\alpha_2\xi_3)\, g_{11}(x), \\ D^{\alpha}\overline{b}_1 &= (\xi_2 + \alpha_2\xi_3)\, g_{12}(x), \\ D^{\alpha}\overline{c}_1 &= \xi_3 g_{13}(x). \end{aligned} \tag{9}$$

If there exist constants $\alpha_i > 0$ $(i = 1,2,3,4,5,6)$, *such that the following inequalities are satisfied:*

$$\begin{aligned} -\alpha_1 + \alpha_3 + \alpha_5 &< 0, \\ \alpha_1 - \alpha_2 + \frac{\left(-\alpha_1^2 + 1\right)^2}{4\alpha_3} + \alpha_4 &< 0, \\ -\alpha_6 + \alpha_2 + \frac{\left(\alpha_1\alpha_2 - \alpha_1^2 + 1\right)^2}{4\alpha_4} + \frac{\alpha_1^4\alpha_2^2}{4\alpha_5} &< 0, \end{aligned} \tag{10}$$

then the equilibrium point of system (8) is asymptotically stable.

Proof. We use the adding one power integrator control scheme to prove Theorem 4. Suppose the first power integrator is

$$V_1 = \int_0^{x_1} (s - 0)\, ds = \frac{1}{2}x_1^2. \tag{11}$$

Clearly, V_1 can be viewed as a Lyapunov function.

Now, by using Lemma 2 we can calculate the derivative of V_1 and derive that

$$\begin{aligned} D^{\alpha}V_1 &\le x_1 D^{\alpha}x_1 \\ &= x_1\left(x_2 + (a - \overline{a}_1)\, g_{11}(x) + \beta_1 + d_{11}\right) \\ &= x_1\left(x_2 - x_2^*\right) + x_1x_2^* + (a - \overline{a}_1)\, x_1 g_{11}(x) \\ &\quad + x_1(\beta_1 + d_{11}). \end{aligned} \tag{12}$$

Note that $\xi_1 = x_1, x_2^* = -\alpha_1 x_1 = -\alpha_1\xi_1, \xi_2 = x_2 - x_2^*$, then we get

$$\begin{aligned} D^{\alpha}V_1 &\le \xi_1\xi_2 - \alpha_1\xi_1^2 + (a - \overline{a}_1)\, x_1 g_{11}(x) \\ &\quad + x_1(\beta_1 + d_{11}). \end{aligned} \tag{13}$$

The second power integrator is constructed as

$$W_1 = \int_{x_2^*}^{x_2} \left(s - x_2^*\right) ds = \frac{1}{2}\left(x_2 - x_2^*\right)^2, \tag{14}$$

we have

$$\begin{aligned} \dot{W}_1 &\le \left(x_2 - x_2^*\right) D^{\alpha}x_2 - \left(x_2 - x_2^*\right) D^{\alpha}x_2^* \\ &\le \xi_2\left(x_3 + \left(b - \overline{b}_1\right) g_{12}(x) + \beta_2 + d_{12}\right) \\ &\quad - \xi_2 D^{\alpha}(-\alpha_1 x_1) \\ &= \xi_2x_3 + \xi_2\left(b - \overline{b}_1\right) g_{12}(x) + \xi_2(\beta_2 + d_{12}) \\ &\quad + \alpha_1\xi_2\left(x_2 + (a - \overline{a}_1)\, g_{11}(x) + \beta_1 + d_{11}\right) \\ &= \xi_2x_3 + \xi_2\left(b - \overline{b}_1\right) g_{12}(x) + \alpha_1\xi_2\left(x_2 - x_2^*\right) \\ &\quad + \alpha_1\xi_2x_2^* + (a - \overline{a}_1)\,\alpha_1\xi_2 g_{11}(x) + \xi_2(\beta_2 + d_{12}) \\ &\quad + \alpha_1\xi_2(\beta_1 + d_{11}) \\ &= \xi_2x_3 + \alpha_1\xi_2^2 - \alpha_2^2\xi_1\xi_2 + \xi_2\left(b - \overline{b}_1\right) g_{12}(x) \\ &\quad + (a - \overline{a}_1)\,\alpha_1\xi_2 g_{11}(x) + \xi_2(\beta_2 + d_{12}) \\ &\quad + \alpha_1\xi_2(\beta_1 + d_{11}). \end{aligned} \tag{15}$$

Choosing $V_2 = V_1 + W_1$, we obtain

$$\begin{aligned} D^{\alpha}V_2 &= D^{\alpha}V_1 + D^{\alpha}W_1 \\ &\le -\alpha_1\xi_1^2 + \xi_1\xi_2 + (a - \overline{a}_1)\, x_1 g_{11}(x) \\ &\quad + x_1(\beta_1 + d_{11}) + \xi_2x_3 + \alpha_1\xi_2^2 - \alpha_1^2\xi_1\xi_2 \\ &\quad + \xi_2\left(b - \overline{b}_1\right) g_{12}(x) + (a - \overline{a}_1)\,\alpha_1\xi_2 g_{11}(x) \\ &\quad + \xi_2(\beta_2 + d_{12}) + \alpha_1\xi_2(\beta_1 + d_{11}) \\ &= -\alpha_1\xi_1^2 + \alpha_1\xi_2^2 + \left(1 - \alpha_1^2\right)\xi_1\xi_2 + \xi_2x_3 \\ &\quad + (a - \overline{a}_1)\left(x_1 g_{11}(x) + \alpha_1\xi_2 g_{11}(x)\right) \\ &\quad + \left(b - \overline{b}_1\right)\xi_2 g_{12}(x) + (x_1 + \alpha_1\xi_2)(\beta_1 + d_{11}) \\ &\quad + \xi_2(\beta_2 + d_{12}) \\ &= -\alpha_1\xi_1^2 + \alpha_1\xi_2^2 + \left(1 - \alpha_1^2\right)\xi_1\xi_2 + \xi_2\left(x_3 - x_3^*\right) \\ &\quad + \xi_2x_3^* + (a - \overline{a}_1)\left(x_1 g_{11}(x) + \alpha_1\xi_2 g_{11}(x)\right) \\ &\quad + \left(b - \overline{b}_1\right)\xi_2 g_{12}(x) + (x_1 + \alpha_1\xi_2)(\beta_1 + d_{11}) \\ &\quad + \xi_2(\beta_2 + d_{12}). \end{aligned} \tag{16}$$

Since $x_3^* = -\alpha_2\xi_2, \xi_3 = x_3 - x_3^*$, we get

$$\begin{aligned} D^{\alpha}V_2 &\le -\alpha_1\xi_1^2 + \alpha_1\xi_2^2 - \alpha_2\xi_2^2 + \left(1 - \alpha_1^2\right)\xi_1\xi_2 + \xi_2\xi_3 \\ &\quad + (a - \overline{a}_1)\left(x_1 g_{11}(x) + \alpha_1\xi_2 g_{11}(x)\right) \\ &\quad + \left(b - \overline{b}_1\right)\xi_2 g_{12}(x) + (x_1 + \alpha_1\xi_2)(\beta_1 + d_{11}) \\ &\quad + \xi_2(\beta_2 + d_{12}). \end{aligned} \tag{17}$$

The third power integrator is similarly defined as

$$W_2 = \int_{x_3^*}^{x_3} (s - x_3^*)\, ds = \frac{1}{2}(x_3 - x_3^*)^2, \tag{18}$$

then we derive

$$\begin{aligned}
D^\alpha W_2 &= (x_3 - x_3^*) D^\alpha x_3 - (x_3 - x_3^*) D^\alpha x_3^* \\
&\le \xi_3 (-\alpha_6 \xi_3 + (c - \overline{c}_1) g_{13}(x) + \beta_3 + d_{13}) \\
&\quad - \xi_3 D^\alpha (-\alpha_2 \xi_2) \\
&= \xi_3 (-\alpha_6 \xi_3 + (c - \overline{c}_1) g_{13}(x) + \beta_3 + d_{13}) \\
&\quad + \alpha_2 \xi_3 D^\alpha (x_2 - x_2^*) \\
&= \xi_3 (-\alpha_6 \xi_3 + (c - \overline{c}_1) g_{13}(x) + \beta_3 + d_{13}) \\
&\quad + \alpha_2 \xi_3 (x_2 + \alpha_1 x_1) \\
&= \xi_3 (-\alpha_6 \xi_3 + (c - \overline{c}_1) g_{13}(x) + \beta_3 + d_{13}) \\
&\quad + \alpha_2 \xi_3 \left(x_3 + \left(b - \overline{b}_1\right) g_{12}(x) + \beta_2 + d_{12}\right) \\
&\quad + \alpha_1 \alpha_2 \xi_3 (x_2 + (a - \overline{a}_1) g_{11}(x) + \beta_1 + d_{11}) \\
&= \xi_3 (-\alpha_6 \xi_3) + (c - \overline{c}_1) \xi_3 g_{13}(x) + \alpha_2 \xi_3 x_3 \\
&\quad + \alpha_1 \alpha_2 \xi_3 x_2 + \left(b - \overline{b}_1\right) \alpha_2 \xi_3 g_{12}(x) \\
&\quad + \alpha_1 \alpha_2 \xi_3 (a - \overline{a}_1) g_{11}(x) + \xi_3 (\beta_3 + d_{13}) \\
&\quad + \alpha_2 \xi_3 (\beta_2 + d_{12}) + \alpha_1 \alpha_2 \xi_3 (\beta_1 + d_{11}) \\
&= \xi_3 (-\alpha_6 \xi_3) + \alpha_2 \xi_3 (x_3 - x_3^*) + \alpha_2 \xi_3 x_3^* \\
&\quad + \alpha_1 \alpha_2 \xi_3 (x_2 - x_2^*) + \alpha_1 \alpha_2 \xi_3 x_2^* \\
&\quad + (c - \overline{c}_1) \xi_3 g_{13}(x) + \left(b - \overline{b}_1\right) \alpha_2 \xi_3 g_{12}(x) \\
&\quad + (a - \overline{a}_1) \alpha_1 \alpha_2 \xi_3 g_{11}(x) + \xi_3 (\beta_3 + d_{13}) \\
&\quad + \alpha_2 \xi_3 (\beta_2 + d_{12}) + \alpha_1 \alpha_2 \xi_3 (\beta_1 + d_{11}) \\
&= \xi_3 (-\alpha_6 \xi_3) + \alpha_2 \xi_3^2 + \alpha_2 \xi_3 (-\alpha_2 \xi_2) + \alpha_1 \alpha_2 \xi_3 \xi_2 \\
&\quad + \alpha_1 \alpha_2 \xi_3 (-\alpha_1 \xi_1) + (c - \overline{c}_1) \xi_3 g_{13}(x) \\
&\quad + \left(b - \overline{b}_1\right) \alpha_2 \xi_3 g_{12}(x) \\
&\quad + (a - \overline{a}_1) \alpha_1 \alpha_2 \xi_3 g_{11}(x) + \xi_3 (\beta_3 + d_{13}) \\
&\quad + \alpha_2 \xi_3 (\beta_2 + d_{12}) + \alpha_1 \alpha_2 \xi_3 (\beta_1 + d_{11}) \\
&= \xi_3 (-\alpha_6 \xi_3) + \alpha_2 \xi_3^2 + \left(\alpha_1 \alpha_2 - \alpha_2^2\right) \xi_2 \xi_3 \\
&\quad - \alpha_1^2 \alpha_2 \xi_1 \xi_3 + (c - \overline{c}_1) \xi_3 g_{13}(x) \\
&\quad + \left(b - \overline{b}_1\right) \alpha_2 \xi_3 g_{12}(x) \\
&\quad + (a - \overline{a}_1) \alpha_1 \alpha_2 \xi_3 g_{11}(x) + \xi_3 (\beta_3 + d_{13}) \\
&\quad + \alpha_2 \xi_3 (\beta_2 + d_{12}) + \alpha_1 \alpha_2 \xi_3 (\beta_1 + d_{11}).
\end{aligned} \tag{19}$$

Now, the final Lyapunov function is chosen as

$$\begin{aligned}
V_3 &= V_2 + W_2 + \frac{1}{2}(a_1 - \overline{a}_1)^2 + \frac{1}{2}\left(b_1 - \overline{b}_1\right)^2 \\
&\quad + \frac{1}{2}(c_1 - \overline{c}_1)^2.
\end{aligned} \tag{20}$$

The derivative of V_3 is

$$\begin{aligned}
D^\alpha V_3 &\le D^\alpha V_2 + D^\alpha W_2 - (a_1 - \overline{a}_1) D^\alpha \overline{a}_1 \\
&\quad - \left(b_1 - \overline{b}_1\right) D^\alpha \overline{b}_1 - (c_1 - \overline{c}_1) D^\alpha \overline{c}_1 \\
&\le -\alpha_1 \xi_1^2 + \alpha_1 \xi_2^2 - \alpha_2 \xi_2^2 + \left(1 - \alpha_1^2\right) \xi_1 \xi_2 + \xi_2 \xi_3 \\
&\quad + (a - \overline{a}_1)(x_1 g_{11}(x) + \alpha_1 \xi_2 g_{11}(x)) \\
&\quad + \left(b - \overline{b}_1\right) \xi_2 g_{12}(x) + \xi_3 (-\alpha_6 \xi_3) + \alpha_2 \xi_3^2 \\
&\quad + \left(\alpha_1 \alpha_2 - \alpha_2^2\right) \xi_2 \xi_3 - \alpha_1^2 \alpha_2 \xi_1 \xi_3 \\
&\quad + (a - \overline{a}_1) \alpha_1 \alpha_2 \xi_3 g_{11}(x) \\
&\quad + \left(b - \overline{b}_1\right) \alpha_2 \xi_3 g_{12}(x) + (c - \overline{c}_1) \xi_3 g_{13}(x) \\
&\quad + (x_1 + \alpha_1 \xi_2 + \alpha_1 \alpha_2 \xi_3)(\beta_1 + d_{11}) \\
&\quad + (\xi_2 + \alpha_2 \xi_3)(\beta_2 + d_{12}) + \xi_3 (\beta_3 + d_{13}) \\
&\quad - (a_1 - \overline{a}_1) D^\alpha \overline{a}_1 - \left(b_1 - \overline{b}_1\right) D^\alpha \overline{b}_1 \\
&\quad - (c_1 - \overline{c}_1) D^\alpha \overline{c}_1.
\end{aligned} \tag{21}$$

Note that

$$\begin{aligned}
(x_1 + \alpha_1 \xi_2 + \alpha_1 \alpha_2 \xi_3)(\beta_1 + d_{11}) &\le 0, \\
(\xi_2 + \alpha_2 \xi_3)(\beta_2 + d_{12}) &\le 0, \\
\xi_3 (\beta_3 + d_{13}) &\le 0.
\end{aligned} \tag{22}$$

We get

$$\begin{aligned}
D^\alpha V_3 &\le -\alpha_1 \xi_1^2 + \alpha_1 \xi_2^2 - \alpha_2 \xi_2^2 + \left(1 - \alpha_1^2\right) \xi_1 \xi_2 + \xi_2 \xi_3 \\
&\quad + \xi_3 (-\alpha_6 \xi_3) + \alpha_2 \xi_3^2 + \left(\alpha_1 \alpha_2 - \alpha_2^2\right) \xi_2 \xi_3 \\
&\quad - \alpha_1^2 \alpha_2 \xi_1 \xi_3 \\
&\quad + (a - \overline{a}_1)(x_1 + \alpha_1 \xi_2 + \alpha_1 \alpha_2 \xi_3) g_{11}(x) \\
&\quad + \left(b - \overline{b}_1\right)(\xi_2 + \alpha_2 \xi_3) g_{12}(x) \\
&\quad + (c - \overline{c}_1) \xi_3 g_{13}(x) - (a_1 - \overline{a}_1) D^\alpha \overline{a}_1 \\
&\quad - \left(b_1 - \overline{b}_1\right) D^\alpha \overline{b}_1 - (c_1 - \overline{c}_1) D^\alpha \overline{c}_1.
\end{aligned} \tag{23}$$

Substituting $D^\alpha \overline{a}_1, D^\alpha \overline{b}_1, D^\alpha \overline{c}_1$ into $D^\alpha V_3$ yields

$$\begin{aligned}
D^\alpha V_3 &\le -\alpha_1 \xi_1^2 + (\alpha_1 - \alpha_2) \xi_2^2 + \left(1 - \alpha_1^2\right) \xi_1 \xi_2 + \alpha_2 \xi_3^2 \\
&\quad + \left(\alpha_1 \alpha_2 - \alpha_2^2 + 1\right) \xi_2 \xi_3 - \alpha_1^2 \alpha_2 \xi_1 \xi_3 \\
&\quad + \xi_3 (-\alpha_6 \xi_3).
\end{aligned} \tag{24}$$

By using the inequality: $ab \le \epsilon a^2 + b^2/4\epsilon$ $(\epsilon > 0)$, we have

$$\begin{aligned}
&D^{\alpha}V_3 \\
&\le -\alpha_1\xi_1^2 + (\alpha_1 - \alpha_2)\xi_2^2 + \alpha_2\xi_3^2 + \alpha_3\xi_1^2 \\
&\quad + \frac{\left(1-\alpha_1^2\right)^2}{4\alpha_3}\xi_2^2 + \alpha_4\xi_2^2 + \frac{\left(1+\alpha_1\alpha_2-\alpha_1^2\right)^2}{4\alpha_4}\xi_3^2 \\
&\quad + \alpha_5\xi_1^2 + \frac{\alpha_1^4\alpha_2^2}{4\alpha_5}\xi_3^2 + \xi_3\left(-\alpha_6\xi_3\right) \\
&= \left(-\alpha_1 + \alpha_3 + \alpha_5\right)\xi_1^2 \\
&\quad + \left(\alpha_1 - \alpha_2 + \frac{\left(1-\alpha_1^2\right)^2}{4\alpha_3} + \alpha_4\right)\xi_2^2 \\
&\quad + \left(-\alpha_6 + \alpha_2 + \frac{\left(1+\alpha_1\alpha_2-\alpha_1^2\right)^2}{4\alpha_4} + \frac{\alpha_1^4\alpha_2^2}{4\alpha_5}\right)\xi_3^2.
\end{aligned} \tag{25}$$

In view of that

$$\begin{aligned}
-\alpha_1 + \alpha_3 + \alpha_5 &< 0, \\
\alpha_1 - \alpha_2 + \frac{\left(-\alpha_1^2+1\right)^2}{4\alpha_3} + \alpha_4 &< 0, \\
-\alpha_6 + \alpha_2 + \frac{\left(\alpha_1\alpha_2-\alpha_1^2+1\right)^2}{4\alpha_4} + \frac{\alpha_1^4\alpha_2^2}{4\alpha_5} &< 0,
\end{aligned} \tag{26}$$

we obtain $D^{\alpha}V_3 < 0$, which implies that $\lim_{t\longrightarrow\infty}\xi_1 = \lim_{t\longrightarrow\infty}\xi_2 = \lim_{t\longrightarrow\infty}\xi_3 = 0$. According to the definition of ξ_1, ξ_2, ξ_3 we conclude that $\lim_{t\longrightarrow\infty}x_1 = \lim_{t\longrightarrow\infty}x_2 = \lim_{t\longrightarrow\infty}x_3 = 0$. The proof of Theorem 4 is completed. □

Remark 5. From the first inequality of (10), it is easy to see that for any given values α_3 and α_5 we can choose proper α_1 such that $-\alpha_1 + \alpha_3 + \alpha_5 < 0$. Based on the second inequality of (10) one can know that for any given value α_1 and α_4 we can take suitable α_2 such that $\alpha_1 - \alpha_2 + (-\alpha_1^2+1)^2/4\alpha_3 + \alpha_4 < 0$. Similarly, according to the third inequality of (10) one can derive that for any given values $\alpha_1, \alpha_2, \alpha_4$ and α_5 one can select appropriate α_6 such that $-\alpha_6 + \alpha_2 + (\alpha_1\alpha_2 - \alpha_1^2 + 1)^2/4\alpha_4 + \alpha_1^4\alpha_2^2/4\alpha_5 < 0$. Therefore, there are many feasible solutions of equation (10). For example, $\alpha_1 = 2, \alpha_2 = 8, \alpha_3 = \alpha_4 = \alpha_5 = 1/2, \alpha_6 = 586$ is one of the feasible solutions. In addition, from (25) it is not difficult to know that the smaller the values of $-\alpha_1 + \alpha_3 + \alpha_5, \alpha_1 - \alpha_2 + (-\alpha_1^2 + 1)^2/4\alpha_3 + \alpha_4$ and $-\alpha_6 + \alpha_2 + (\alpha_1\alpha_2 - \alpha_1^2 + 1)^2/4\alpha_4 + \alpha_1^4\alpha_2^2/4\alpha_5$, the faster the convergence speed of V_3.

Remark 6. In Theorem 4 we assume that each equation in system (6) has one uncertain parameter. However, in practical application each equation may have more than one uncertain parameter. In fact, Theorem 4 can be easily modified to deal with the case which has more than one uncertain parameter. For example, if the first equation of system (6) has two uncertain parameters:

$$D^{\alpha}x_1 = f_{11}(x) + ag_1(x) + bg_2(x) + d_{11} + u_1, \tag{27}$$

where a and b are two uncertain parameters, then based on Theorem 4 we can take

$$u_1 = x_2 - f_{11}(x) - \overline{a}g_1(x) - \overline{b}g_2(x) + \beta_1, \tag{28}$$

and the update laws of $\overline{a}, \overline{b}$ are designed as

$$\begin{aligned}
D^{\alpha}\overline{a} &= \left(\xi_1 + \alpha_2\xi_2 + \alpha_1\alpha_2\xi_3\right)g_1(x), \\
D^{\alpha}\overline{b} &= \left(\xi_1 + \alpha_2\xi_2 + \alpha_1\alpha_2\xi_3\right)g_2(x),
\end{aligned} \tag{29}$$

where $\overline{a}$ and $\overline{b}$ are the estimated values of a and b, respectively.

Remark 7. In system (5), $f_{1i}(x)$ and $g_{1i}(x)$ can be any continuous functions which means that the technique proposed in this paper can be applied for any 3-D fractional-order chaotic systems. However, the schemes presented in papers [10–14] are only valid for a class of strict-feedback chaotic systems. In addition, the bounded external disturbances d_{11}, d_{12}, d_{13} are taken into consideration in system (5). However, the controllers presented in papers [10–14] are under the same assumption that systems are free from external perturbations.

4. The Synchronization Scheme

Based on the results obtained in Section 3, in this section the drive-response synchronization scheme is investigated.

Suppose system (5) is the drive system, then the response system is given as

$$\begin{aligned}
D^{\alpha}y_1 &= f_{21}(y) + a_2g_{21}(y) + d_{21} + u_1, \\
D^{\alpha}y_2 &= f_{22}(y) + b_2g_{22}(y) + d_{22} + u_2, \\
D^{\alpha}y_3 &= f_{23}(y) + c_2g_{23}(y) + d_{23} + u_3,
\end{aligned} \tag{30}$$

where $0 < \alpha \le 1$, $y = (y_1, y_2, y_3)^T \in R^{3\times1}$ is the state vector of system (30), $f_{2i}(y), g_{2i}(y)$ are all known functions, $i = 1, 2, 3$. a_2, b_2, c_2 are unknown parameters, d_{21}, d_{22}, d_{23} are bounded external disturbances, u_1, u_2, u_3 are controllers.

Assumption 8. Suppose there exist known constants $\overline{d}_{2i} > 0$ such that $|d_{2i}| \le \overline{d}_{2i}$, $i = 1, 2, 3$.

The synchronization error is defined as $e_i = y_i - x_i$, $i = 1,, 2, 3$. Then the error dynamic system is given as

$$\begin{aligned}
D^{\alpha}e_1 &= f_{21}(y) + a_2g_{21}(y) + d_{21} \\
&\quad - \left(f_{11}(x) + a_1g_{11}(x) + d_{11}\right) + u_1, \\
D^{\alpha}e_2 &= f_{22}(y) + b_2g_{22}(y) + d_{22} \\
&\quad - \left(f_{12}(x) + b_1g_{12}(x) + d_{12}\right) + u_2, \\
D^{\alpha}e_3 &= f_{23}(y) + c_2g_{23}(y) + d_{23} \\
&\quad - \left(f_{13}(x) + c_1g_{13}(x) + d_{13}\right) + u_3,
\end{aligned} \tag{31}$$

Now we introduce some new variables: $\eta_1 = e_1$, $e_2^* = -\alpha_1 e_1$, $\eta_2 = e_2 - e_2^*$, $e_3^* = -\alpha_2\eta_2$, $\eta_3 = e_3 - e_3^*$, where $\alpha_1 > 0, \alpha_2 > 0$.

By using these new variables, the controllers u_1, u_2, u_3 in system (31) are designed as

$$\begin{aligned} u_1 &= e_2 + g_{11}(x) - f_{21}(y) + \overline{a}_1 g_{11}(x) - \overline{a}_2 g_{21}(y) \\ &\quad + \gamma_1, \\ u_2 &= e_3 - f_{22}(y) + g_{12}(x) + \overline{b}_1 g_{12}(x) - \overline{b}_2 g_{22}(y) \\ &\quad + \gamma_2, \\ u_3 &= -\alpha_6\eta_3 - f_{23}(y) + g_{13}(x) + \overline{c}_1 g_{13}(x) \\ &\quad - \overline{c}_2 g_{23}(y) + \gamma_3, \end{aligned} \tag{32}$$

where $\overline{a}_1, \overline{b}_1, \overline{c}_1, \overline{a}_2, \overline{b}_2, \overline{c}_2$ are the estimated values of $a_1, b_1, c_1, a_2, b_2, c_2$, respectively, $\gamma_1 = -(\overline{d}_{11} + \overline{d}_{21})\text{sign}(e_1 + \alpha_2\eta_2 + \alpha_1\alpha_2\eta_3)$, $\gamma_2 = -(\overline{d}_{12} + \overline{d}_{22})\text{sign}(\eta_2 + \alpha_2\eta_3)$, $\gamma_3 = -(\overline{d}_{13} + \overline{d}_{23})\text{sign}(\eta_3)$, $\alpha_6 > 0$ is a constant.

After plugging controllers (32) into system (31), one derives

$$\begin{aligned} D^\alpha e_1 &= e_2 - (a_1 - \overline{a}_1) g_{11}(x) + (a_2 - \overline{a}_2) g_{21}(y) \\ &\quad - d_{11} + d_{21} + \gamma_1, \\ D^\alpha e_2 &= e_3 - (b_1 - \overline{b}_1) g_{12}(x) + (b_2 - \overline{b}_2) g_{22}(y) \\ &\quad - d_{12} + d_{22} + \gamma_2, \\ D^\alpha e_3 &= -\alpha_6\eta_3 - (c_1 - \overline{c}_1) g_{13}(x) + (c_2 - \overline{c}_2) g_{23}(y) \\ &\quad - d_{13} + d_{23} + \gamma_3, \end{aligned} \tag{33}$$

Obviously, the synchronization problem between systems (5) and (30) is transformed into the stability problem of the equilibrium point $(0, 0, 0)$ of system (33). To solve the stability problem of system (33), we propose Theorem 9 whose proof is omitted.

Theorem 9. *Suppose the update laws of $\overline{a}_1, \overline{b}_1, \overline{c}_1, \overline{a}_2, \overline{b}_2, \overline{c}_2$ are designed as*

$$\begin{aligned} D^\alpha \overline{a}_1 &= -(\eta_1 + \alpha_2\eta_2 + \alpha_1\alpha_2\eta_3) g_{11}(x), \\ D^\alpha \overline{b}_1 &= -(\eta_2 + \alpha_2\eta_3) g_{12}(x), \\ D^\alpha \overline{c}_1 &= -\eta_3 g_{13}(x), \\ D^\alpha \overline{a}_2 &= (\eta_1 + \alpha_2\eta_2 + \alpha_1\alpha_2\eta_3) g_{21}(y), \\ D^\alpha \overline{b}_2 &= (\eta_2 + \alpha_2\eta_3) g_{22}(y), \\ D^\alpha \overline{c}_2 &= \eta_3 g_{23}(y). \end{aligned} \tag{34}$$

If there exist constants $\alpha_i > 0$ $(i = 1, 2, 3, 4, 5, 6)$, such that the inequalities (10) are satisfied, then the equilibrium point of system (33) is asymptotically stable which means that the synchronization between systems (5) and (30) is achieved.

5. Simulation Results

There are many new chaotic systems in the literature [18–20]. In this section the new fractional-order chaotic system which has been introduced in [20] is considered since it has many interesting properties.

Example 10 (the control of a new fractional-order chaotic system). The new fractional-order chaotic system is described by the following set of equations:

$$\begin{aligned} D^\alpha x_1 &= x_2 - a x_1 + b x_2 x_3, \\ D^\alpha x_2 &= c x_2 - x_1 x_3 + x_3, \\ D^\alpha x_3 &= d x_1 x_2 - h x_3, \end{aligned} \tag{35}$$

where α is the fractional-order. $x = (x_1, x_2, x_3)^T \in R^3$ is the state variable, parameters a, b, c, d and h are positive real constants. This system shows chaotic behaviors for the parameters $a = 3, b = 2.7, c \in (4.45, 4, 60) \cup (4.86, 4.94) \cup [5.17, 7), d = 2$ and $h = 9$. The chaotic attractor of system (35) with $\alpha = 0.995, a = 3, b = 2.7, c = 5, d = 2, h = 9$ with initial conditions $x_1(0) = -3, x_2(0) = 1.5, x_3(0) = 2$ is shown in Figure 1.

For the control purpose, system (35) can be rewritten as

$$\begin{aligned} D^\alpha x_1 &= x_2 - a x_1 + b x_2 x_3 + \sin t + u_1, \\ D^\alpha x_2 &= c x_2 - x_1 x_3 + x_3 + 2\cos t + u_2, \\ D^\alpha x_3 &= d x_1 x_2 - h x_3 + 2\sin t \cos t + u_3, \end{aligned} \tag{36}$$

where $\sin t, 2\cos t, 2\sin t\cos t$ are the external disturbances, u_1, u_2, u_3 are controllers.

Suppose a, b, c, d, h are unknown parameters, then $f_{11}(x) = x_2$, $f_{12}(x) = -x_1 x_3 + x_3$. Based on Theorem 4 the controllers u_1, u_2, u_3 are chosen as

$$\begin{aligned} u_1 &= \overline{a}_1 x_1 - \overline{b}_1 x_2 x_3 + \beta_1, \\ u_2 &= x_1 x_3 - \overline{c}_1 x_2 + \beta_2, \\ u_3 &= -\alpha_6\xi_3 - \overline{d} x_1 x_2 + \overline{h} x_3 + \beta_3, \end{aligned} \tag{37}$$

where $\overline{a}, \overline{b}, \overline{c}, \overline{d}, \overline{h}$ are the estimated values of a, b, c, d, h, respectively, $\beta_1 = -\text{sign}(x_1 + \alpha_2\xi_2 + \alpha_1\alpha_2\xi_3)$, $\beta_2 = -2\text{sign}(\xi_2 + \alpha_2\xi_3)$, $\beta_3 = -2\text{sign}(\xi_3)$.

The update laws of $\overline{a}, \overline{b}, \overline{c}, \overline{d}, \overline{h}$ are designed as

$$\begin{aligned} D^\alpha \overline{a} &= -(\xi_1 + \alpha_2\xi_2 + \alpha_1\alpha_2\xi_3) x_1, \\ D^\alpha \overline{b} &= (\xi_1 + \alpha_2\xi_2 + \alpha_1\alpha_2\xi_3) x_2 x_3, \\ D^\alpha \overline{c} &= (\xi_2 + \alpha_2\xi_3) x_2, \\ D^\alpha \overline{d} &= \xi_3 x_1 x_2, \\ D^\alpha \overline{h} &= -\xi_3 x_3. \end{aligned} \tag{38}$$

If we take $\alpha_1 = 2, \alpha_2 = 8, \alpha_3 = \alpha_4 = \alpha_5 = 1/2, \alpha_6 = 586$, then the conditions of Theorem 4 are satisfied.

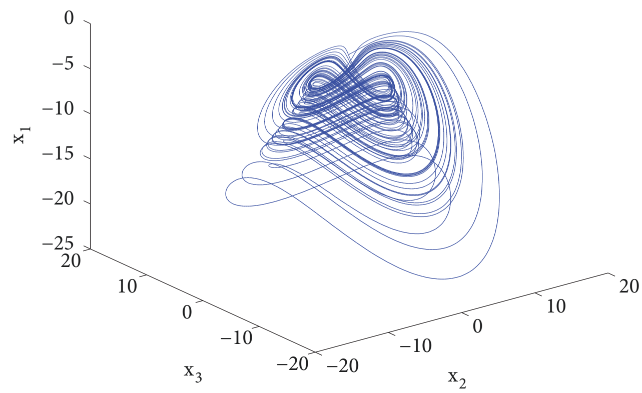

FIGURE 1: The chaos attractor of system (35) with $(x_1(0), x_2(0), x_3(0)) = (-3, 1.5, 2)$.

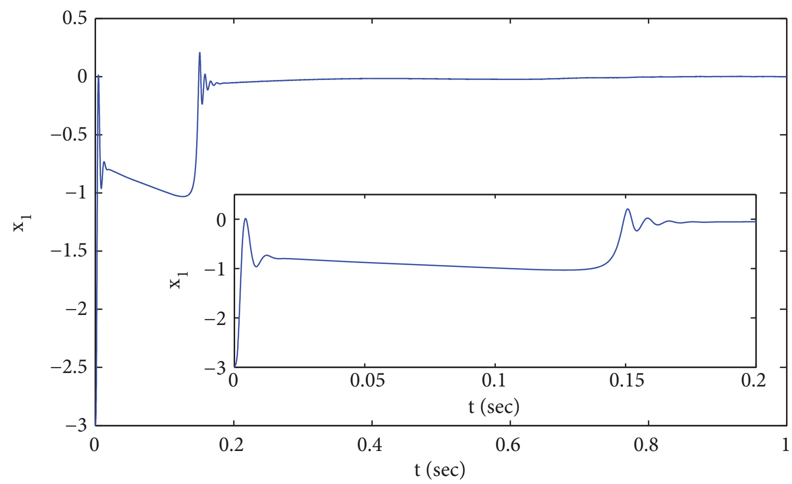

FIGURE 2: The time response of state x_1 of system (36) with controller (37).

According to Theorem 4 the equilibrium point of system (36) is asymptotically stable. The simulation results with $x_1(0) = -3, x_2(0) = 1.5, x_3(0) = 2$ and $\widehat{a}(0) = \widehat{b}(0) = \widehat{c}(0) = \widehat{d}(0) = \widehat{h}(0) = 1$ are shown in Figures 2–5. Figures 2–4 are the time response of states x_1, x_2, x_3 of system (36) with controller (37). Figure 5 displays the time response of states $\widehat{a}, \widehat{b}, \widehat{c}, \widehat{d}, \widehat{h}$ of system (36) with controller (37).

Example 11 (the synchronization between two new fractional-order chaotic systems). Another new chaotic system considered in this section is the fractional-order modified unified chaotic system [21] which is given as

$$\begin{aligned} D^\alpha y_1 &= (35 - 25\theta)(y_2 - y_1), \\ D^\alpha y_2 &= (-7 + 35\theta) y_1 - y_1 y_3 + (28 - 29\theta) y_2, \\ D^\alpha y_3 &= y_1 y_2 - \frac{1}{3}(9 - \theta) y_3, \end{aligned} \tag{39}$$

where α is the fractional-order. $y = (y_1, y_2, y_3)^T \in R^3$ is the state variable, parameter $\theta \in [0, 1]$. The chaotic attractor of system (39) with $\theta = 1$ with initial conditions $y_1(0) = 2, y_2(0) = -1.5, \ y_3(0) = 5$ is shown in Figure 6.

Suppose system (35) is the drive system and system (39) is the response system. The drive system with external disturbances is described as

$$\begin{aligned} D^\alpha x_1 &= x_2 - a x_1 + b x_2 x_3 + \sin t, \\ D^\alpha x_2 &= c x_2 - x_1 x_3 + x_3 + 2\cos t, \\ D^\alpha x_3 &= d x_1 x_2 - h x_3 + 2\sin t \cos t. \end{aligned} \tag{40}$$

The controlled response system with external disturbances is given as

$$\begin{aligned} D^\alpha y_1 &= (35 - 25\theta)(y_2 - y_1) + 2\cos t + u_1, \\ D^\alpha y_2 &= (-7 + 35\theta) y_1 - y_1 y_3 + (28 - 29\theta) y_2 + \sin t \\ &\quad + u_2, \\ D^\alpha y_3 &= y_1 y_2 - \mu y_3 + \cos t + u_3, \end{aligned} \tag{41}$$

where u_1, u_2, u_3 are controllers and $\mu = (1/3)(9 - \theta)$.

Now for simplicity, we assume parameters a and c in system (40) and parameter μ in system (41) are unknown.

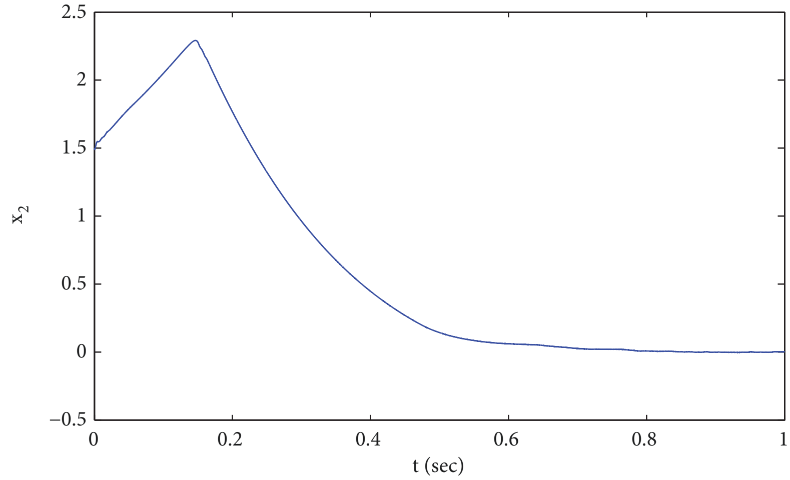

Figure 3: The time response of state x_2 of system (36) with controller (37).

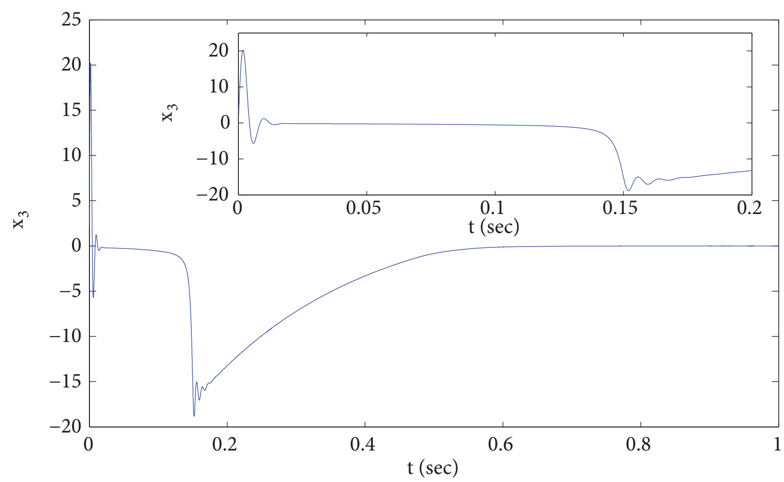

Figure 4: The time response of state x_3 of system (36) with controller (37).

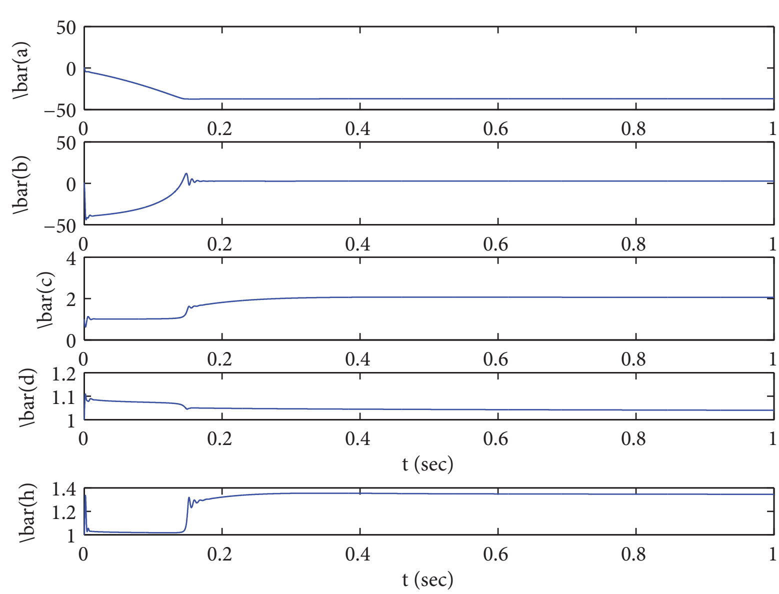

Figure 5: The time response of states $\overline{a}, \overline{b}, \overline{c}, \overline{d}, \overline{h}$ of system (36) with controller (37).

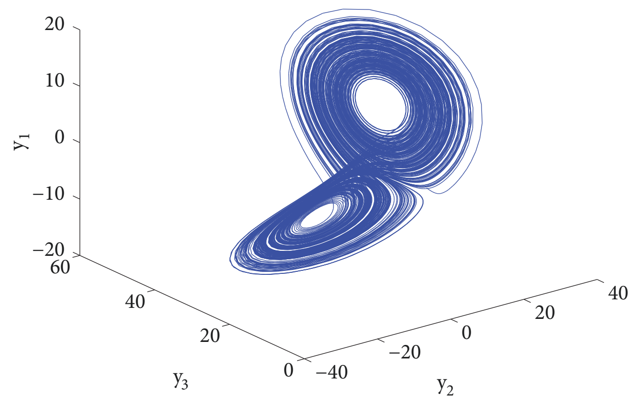

FIGURE 6: The chaos attractor of fractional-order modified unified chaotic system with $(y_1(0), y_2(0), y_3(0)) = (2, -1.5, 5)$.

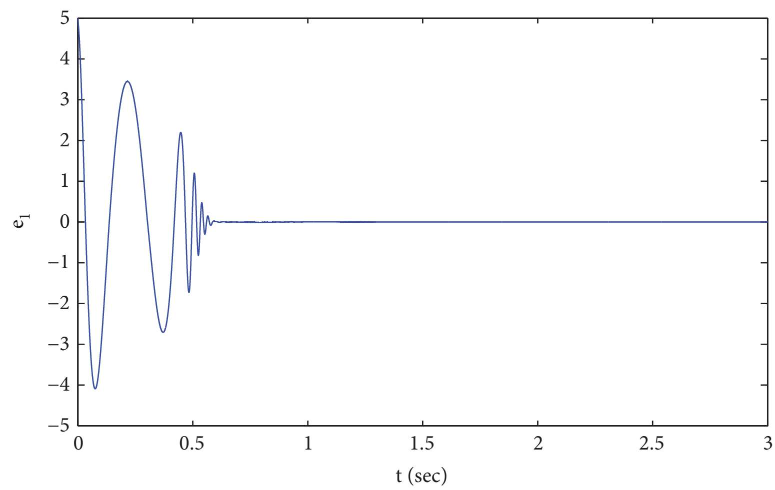

FIGURE 7: The time response of the error state e_1.

According to Theorem 9, the controllers u_1, u_2, u_3 in system (41) are designed as

$$\begin{aligned} u_1 &= e_2 + x_2 + bx_2x_3 - (35 - 25\theta)(y_2 - y_1) - \overline{a}_1 x_1 \\ &\quad + \gamma_1, \\ u_2 &= e_3 - x_1x_3 + x_3 \\ &\quad - ((-7 + 35\theta) y_1 - y_1y_3 + (28 - 29\theta) y_2) \\ &\quad + \overline{c}_1 x_2 + \gamma_2, \\ u_3 &= -\alpha_6\eta_3 + dx_1x_2 - hx_3 - y_1y_2 + \overline{\mu} y_3 + \gamma_3, \end{aligned} \tag{42}$$

where $\overline{a}_1, \overline{c}_1, \overline{\mu}$ are the estimated values of a_1, c_1, μ, respectively, $\gamma_1 = -3\text{sign}(e_1 + \alpha_2\eta_2 + \alpha_1\alpha_2\eta_3)$, $\gamma_2 = -3\text{sign}(\eta_2 + \alpha_2\eta_3)$, $\gamma_3 = -3\text{sign}(\eta_3)$, $\alpha_6 > 0$ is a constant.

The update laws of $\overline{a}_1, \overline{c}_1, \overline{\mu}$ are designed as

$$\begin{aligned} D^\alpha \overline{a} &= (\eta_1 + \alpha_2\eta_2 + \alpha_1\alpha_2\eta_3) x_1, \\ D^\alpha \overline{c} &= -(\eta_2 + \alpha_2\eta_3) x_2, \\ D^\alpha \overline{\mu} &= -\eta_3 y_3. \end{aligned} \tag{43}$$

Similarly, we take $\alpha_1 = 2, \alpha_2 = 8, \alpha_3 = \alpha_4 = \alpha_5 = 1/2, \alpha_6 = 586$. Then the conditions of Theorem 9 are satisfied. According to Theorem 9 the synchronization between systems (40) and (41) will be achieved. The simulation results with $x_1(0) = -3, x_2(0) = 1.5, x_3(0) = 2, (y_1(0), y_2(0), y_3(0)) = (2, -1.5, 5)$ and $\overline{a}(0) = \overline{c}(0) = \overline{\mu}(0) = 1$ are shown in Figures 7–10. Figures 7–9 are the time response of states e_1, e_2, e_3 of the error system. Figure 10 shows the time response of states $\overline{a}, \overline{c}, \overline{\mu}$ of the error system.

Remark 12. Although the chaotic systems investigated in our numerical simulations are not in strict-feedback form and subject to external disturbances, from Figures 2–5 and 7–10 one can find that the numerical results are completely consistent with the theoretical analysis presented in this paper which in turn demonstrates the validity, effectiveness, and good performance of the proposed schemes.

6. Conclusions

This paper deals with the control and synchronization of a class of 3-D uncertain fractional-order chaotic systems with external disturbances by using the adding one power

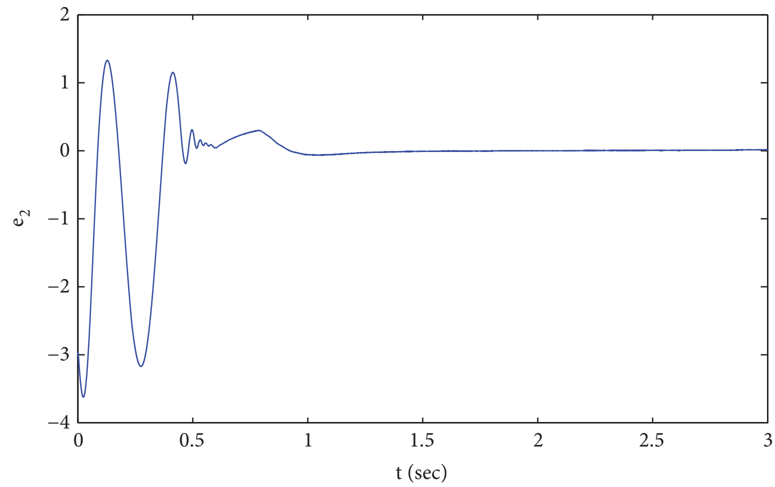

FIGURE 8: The time response of the error state e_2.

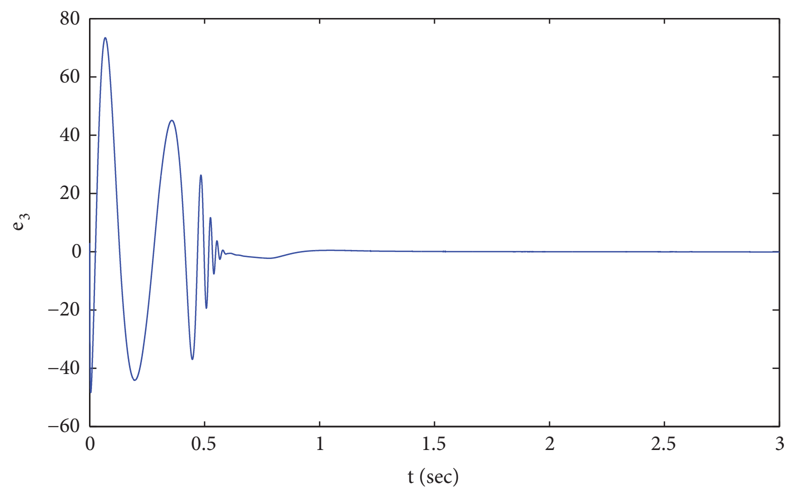

FIGURE 9: The time response of the error state e_3.

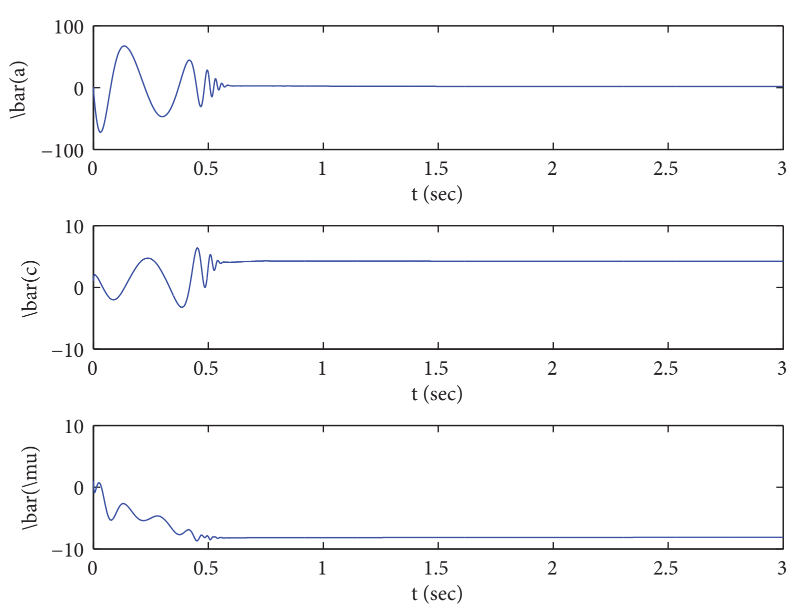

FIGURE 10: The time response of states $\overline{a}, \overline{c}, \overline{\mu}$.

integrator control method. Based on the systematic step by step control design procedure, some novel criteria are presented. These proposed control strategies can not only be applied to a class of strict-feedback systems, but also be applied to more general class of fractional-order chaotic systems. In addition, the presented controllers are robust against uncertain parameters and external disturbances. In the end, some illustrative examples and numerical simulation results are given to demonstrate the effectiveness of the proposed method.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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