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Research Article

Optimal Financing and Dividend Strategies with Time Inconsistency in a Regime Switching Economy

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This paper incorporates a manager's time-inconsistent preferences into the unified dynamic q-theoretic framework to investigate their impact on the optimal external financing and dividend payout strategies in a regime switching economy. We find that with a higher degree of time inconsistency, either in a favorable market condition or in a financial crisis, dividends are paid out earlier and the equity issues are smaller in size in each occurrence; in a favorable market condition equity financing occurs particularly early. Hence, time inconsistency would result in a decreasing of a firm's precautionary savings, which may directly cause capital chain rupture and make liquidation more likely. It also implies that corporate external financing and dividend payout are highly dependent on the degree of the manager's time-inconsistent preferences in a regime switching economy.

1. Introduction

Recent studies show that a firm's financing and investment behavior changes dramatically during financial crises. For instance, Campello et al. [1] and Campello et al. [2] show that more financially constrained firms planned more cuts in investment spending, expended more cash, drew more credit from banks, and engaged in more asset sales during a crisis. A stochastic model of regime switching which is proposed by Hamilton [3] has been applied previously in the literature (e.g., Guo et al. [4], Sotomayor and Cadenillas [5], and Jang and Kim [6]). These studies illustrate that firms' policies, including the optimal investment policy, consumption-investment policy, optimal reinsurance, and asset allocation strategies, are affected by regime shifts. On economic grounds, there are indeed reasons to believe that regime shifts contain the possibility of significant impact on firms' policy choices. For example, business cycle expansion and contraction "regimes" potentially have sizable effects on the profitability or riskiness of investment and, hence, on firms' willingness to invest in physical or human capital. We still, however, know little about the implications of changes in financial conditions on external financing and payout decisions in a regime switching economy.

There are some studies of external financing and payout strategies, such as Bolton et al. [7, 8], Chen et al. [9], and Chen et al. [10]. Bolton et al. [7] proposes a q-theoretic model of investment and financing for financially constrained firms, and shareholders' dividend payments are discounted exponentially at a constant discount rate which means that shareholders have time-consistent preferences. Bolton et al. [8] investigate external financing and payout strategies under regime switching and time-consistent preferences. They find that the transition intensity out of favorable market condition affects the firm's market-timing behavior, including the firm's investment, external financing, and payout strategies. Loewenstein and Prelec [11] and Rabin [12] use experimental studies of time preferences suggesting that the assumption of time-consistent preferences is unrealistic, and decision-makers often have time-inconsistent preferences (also called present-biased preferences). Hence, Chen et al. [9] and Chen et al. [10] characterize the optimal dividend strategy of an insurance company whose manager has time-inconsistent preferences.

Most economic decisions are intertemporal in nature and involve tradeoffs between current and future rewards. An important component of any intertemporal model is the discount function that discounts delayed rewards to the present for decision-making. Overwhelming evidence has been documented in the psychology and behavioral science literature that suggests time inconsistency is prevalent in human preferences (e.g., Thaler and Shefrin [13], Ainslie and Herrnstein [14], Kirby and Herrnstein [15], Myerson and Green [16], Mcclure et al. [17], and Dellavigna and Malmendier [18]). In pursuing immediate gratification, individuals often exhibit a reversal of preferences when choosing between a smaller, earlier reward and an alternative larger but later reward. That is, when two rewards are both far away in time, a larger-later reward is preferred to a smaller-sooner reward (e.g., subjects prefer 101 dollars in 31 days to 100 dollars in 30 days). When both rewards are brought forward in time, however, the sooner-smaller reward becomes preferred (e.g., subjects prefer 50 dollars today to 51 dollars tomorrow). Such preferences exhibit present biases and highlight a conflict between today's preferences and future preferences. They also show that an individual's impatience decreases with time, meaning that a shareholder or manager also has time-inconsistent preferences.

The most common way to model time inconsistency is the quasi-hyperbolic discount function, in which the discount rate decreases in the horizon (see Harris and Laibson [19]). By this means, many researchers show that time inconsistency leads to the tendency to choose actions inducing short-run benefits over those inducing long-run benefits, such as Grenadier and Wang [20], Lien and Yu [21], and Li et al. [22]. Time inconsistency can be interpreted as an intrinsic property of preferences in research on the market-timing motive for external financing/payout decisions. Therefore, an important question is the correlation of time inconsistency and external financing/payout decisions in a regime switching economy.

In this paper, following Bolton et al. [8] (hereafter, BCW), we extend the unified dynamic q-theoretic framework to construct a time-inconsistent model in which the manager has time-inconsistent preferences. Suppose that the manager has time-varying impatience modeled with a quasi-hyperbolic discount function, and her objective is to choose the firm's optimal investment and payout strategies to maximize firm value. Intuitively, since the manager is working on behalf of shareholders and makes decisions that appeal to shareholders, when shareholders have time-inconsistent preferences, the manager should also exhibit time-inconsistent preferences. This is also assumed in Grendier and Wang [20] and Chen et al. [10]. Shareholders are aware that the manager's time preferences will change in the future and they would write contracts with the manager to mitigate the problem of inconsistency and maximize shareholder value. However, before writing the contract, shareholders need to know how corporate decisions are influenced by the manager's time-inconsistent preferences.

In a regime switching economy, the firm can be in one of two states: a favorable market condition or a financial crisis, that is, state G or state B. Based on the classical BCW model, we treat changes in financing conditions as exogenous. In this paper, we assume that the favorable market condition and financial crisis have a low financing cost and a high financing cost, respectively. There are several reasons for the expensive financing cost in a financial crisis, for example, changes in financial intermediation costs, changes in investors' risk attitudes, changes in market sentiment, or changes in aggregate uncertainty and information asymmetry. We make predictions about how the degree of time-inconsistent preferences influences optimal external financing and dividend payout strategies and how much difference the manager's time-inconsistent preferences make in a regime switching economy.

The main results of our analysis are as follows. First, firm value, the firm's investment, and external financing and payout strategies are affected by two major factors: time inconsistency and regime switching. For instance, the impact of time inconsistency on firm value and the firm's investment is much higher in a favorable market condition than in a financial crisis. In a favorable market condition, the manager with time-inconsistent preferences pays out dividends and issues equity earlier even without immediate financing needs but raises less equity capital than her time-consistent counterpart. Second, with a higher degree of time inconsistency, dividends are paid out earlier and less equity is issued each time. In particular, the firm chooses to issue equity early in a favorable market condition.

The remainder of this paper proceeds as follows. Section 2 sets up the model. Section 3 presents the model solution. Section 4 provides quantitative analysis. Section 5 concludes.

2. The Model

Our model incorporates a manager's time-inconsistent preferences into the BCW model of dynamic framework. In this section, we specify the manager's time preferences, regime switching, production technology, and stochastic financing opportunities for a financially constrained company.

2.1. Time Preferences of the Manager. Based on Grenadier and Wang [20] and Harris and Laibson [19], we use a continuous-time version of the quasi-hyperbolic discount function to reflect the empirically documented declining discount rate. We model the manager as a sequence of successive decision-makers (called self 1,2,...). $D_n(t,\varepsilon)$ denotes self n's discount function for any time $t \in [t_n, t_{n+1})$, so shareholders value \$1 paid in the future as $D_n(t,\varepsilon)$, where $D_n(t,\varepsilon)$ is as follows:

$$D_n(t,\varepsilon) = \begin{cases} e^{-\eta(\varepsilon-t)} & if\ \varepsilon \in [t_n, t_{n+1}), \\ \beta e^{-\eta(\varepsilon-t)} & if\ \varepsilon \in [t_{n+1}, \infty), \end{cases} \tag{1}$$

for all $\varepsilon \geq t$. Here, the planning horizon of self n is divided into the present period (from t_n to $t_{n+1}(> t_n)$) and the future period (from t_{n+1} to ∞). When the current self n dies at time t_{n+1}, the next self $n+1$ is born. The preferences of self $n+1$ are again divided into a present $[t_{n+1}, t_{n+2})$ and a future $[t_{n+2}, \infty)$, and so on. This discount function means that the manager is more impatient in making short-term decisions than long-term decisions.

All periods consisting of present and future are discounted exponentially with discount factor $0 < \eta < 1$. The future period is further discounted with uniform weight $0 < \beta \leq 1$. The duration of the present period is exponentially distributed with parameter λ. The parameters λ and β determine the degree of time-inconsistent preferences. β determines how much the future period is valued relative to the present period. λ determines the arrival rate of the future selves and thus how often preferences change. When β is lower, the current self is more impatient. When $\beta = 0$, the current self is only concerned about her actions in the present and completely disregards her future selves. With a greater λ, the degree of time-inconsistent preferences changes more quickly and the arrival rate of the future period increases. Moreover, when $\lambda = 0$ or $\beta = 1$, discount function (1) degenerates into the traditional exponential discount function which represents time-consistent preferences.

It is important to note the following: first, discount function (1) is also called the "present-biased" or "quasi-geometric" discount function in the literature (see Krusell and Smith Jr. [23] and Hsiaw [24]) and, second, self n can also be viewed as the n^{th} manager of the firm. In this case, t_n and t_{n+1} can be seen as the beginning and the end of manager n's term of office. The manager cares more about the decisions made during her term in office and pays out dividends to the shareholders. The manager will overvalue her career achievements and undervalue the achievements of her successors by discounting them with an additional factor β. Her career achievements are measured by the dividends that the shareholders receive.

2.2. Regime Switching. We model regime switching by switching the firm's external financing conditions. Namely, suppose that the firm has two aggregate states, denoted by $s_t=\{G, B\}$, where state G (B) is the state with favorable external financing condition (financial crisis). A risk-averse manager requires a risk premium to compensate for the risk of the economy switching states. We characterize this risk premium through the wedge between the transition intensity under the physical probability measure and that under the risk-neutral probability measure. Over time increment dt, the state switches from G to B (or from B to G) with a probability $\zeta_G dt$ (or $\zeta_B dt$). Let $\widehat{\zeta}_G dt$ and $\widehat{\zeta}_B dt$ denote the risk-neutral transition intensities from G to B and from B to G, respectively. Then

$$\begin{aligned} \widehat{\zeta}_G &= e^{\pi_G}\zeta_G, \\ \widehat{\zeta}_B &= e^{\pi_B}\zeta_B, \end{aligned} \tag{2}$$

where π_G and π_B capture the risk adjustment for the change of states G and B, respectively. In other words, $\pi_G(\pi_B)$ can be interpreted as the price of risk with respect to financing shocks in state G (B). Let $\pi_G > 0$ and $\pi_B < 0$, so we have $e^{\pi_G} > 1$ and $0 < e^{\pi_B} < 1$. Then $\widehat{\zeta}_G > \zeta_G$ ($\widehat{\zeta}_B < \zeta_B$) which implies that the transition intensity out of state G (B) is higher (lower) under the risk-neutral probability measure than under the physical measure. For simplicity, we set $\pi_G = -\pi_B$. Intuitively, it reflects the fact that a manager's risk aversion towards a bad state is captured by making it more likely to switch to the bad state and less likely to leave it. In short, it is as if a risk-averse investor were uniformly more "pessimistic" than a risk-neutral investor; she thinks "good times" are likely to be shorter and "bad times" longer.

2.3. Production Technology. The firm employs capital and cash as the only factors of production. We normalize the price of capital to one and denote by K and I the firm's capital stock and gross investment, respectively. The firm's capital stock K evolves according to

$$dK_t = \left(I_t - \delta_s K_t\right) dt, \quad t \geq 0, \tag{3}$$

where $\delta_s \geq 0$ is the rate of depreciation in state s.

The firm's operating revenue is proportional to its capital stock K_t and is given by $K_t dA_t$, where dA_t is the firm's revenue shock over time increment dt. We assume that

$$dA_t = \mu_s dt + \sigma_s dZ_t, \quad t \geq 0, \tag{4}$$

where Z_t is a standard Brownian motion under the risk-neutral measure. Meanwhile, $\mu_s > 0$ and $\sigma_s > 0$ denote the drift and volatility of the risk-neutral productivity shock in state s, respectively. The firm's operating profit dY_t over time increment dt is then given by

$$dY_t = K_t dA_t - I_t dt - \Gamma\left(I_t, K_t, s_t\right) dt, \quad t \geq 0, \tag{5}$$

where $K_t dA_t$ is the firm's operating revenue, $I_t dt$ is the investment cost over time, and $\Gamma(I_t, K_t, s_t)dt$ is the additional adjustment cost that the firm incurs in the investment process.

Based on the neoclassical investment literature (Hayashi [25]), we assume that the firm's adjustment cost is homogeneous of degree one in I and K. In other words, the adjustment cost takes the form $\Gamma(I, K, s) = g_s(i)K$, where i is the firm's investment-capital rate ($i = I/K$) and $g_s(i)$ is increasing and convex in i. We assume that $g_s(i)$ is quadratic:

$$g_s(i) = \frac{\theta_s \left(i - \nu_s\right)^2}{2}, \tag{6}$$

where θ_s is the adjustment cost parameter and ν_s is a constant in state s. In the literature, ν_s is usually considered as the rate of deprecation δ_s (or zero), which implies a zero adjustment cost when net investment is zero (or zero adjustment cost for zero gross investment).

Finally, the firm can liquidate capital at any moment and obtain a liquidation value $l_s K_t$, where $l_s > 0$ denotes the recovery value per unit of capital in state s. Let T denote the optimal liquidation time.

2.4. Stochastic Financing Opportunities. For simplicity, we only research external equity financing as the source of external funds for the firm. When the firm increases external equity financing, it incurs a fixed cost $\phi_s K$, where ϕ_s is the fixed cost parameter in state s. Besides the fixed cost $\phi_s K$, it also causes a variable cost $\gamma_s > 0$ for each incremental dollar it raises. Let H_t, X_t, and U_t denote the process for the firm's cumulative external financing, the firm's cumulative issuance costs, and the firm's cumulative nondecreasing payout process to shareholders up to time t, respectively. Hence, dH_t denotes the net proceeds from external financing over time interval $(t, t + dt)$, dX_t denotes the financing costs to raise net proceeds from external financing, and dU_t denotes the payout over time interval $(t, t + dt)$.

The manager distributes cash to shareholders in order to avoid paying a carry cost on the firm's retained cash holdings. We assume that cash inside the firm earns a below-market riskless return and incurs the carry cost denoted by $\kappa_s > 0$ in state s. Then the dynamics for the firm's cash W evolves as follows:

$$dW_t = dY_t + (r_s - \kappa_s) W_t dt + dH_t - dU_t, \tag{7}$$

where r_s is the risk-free interest rate in state s. Observe that dH_t and dU_t are endogenously determined by the firm.

3. The Solution

This section investigates the firm optimality in a regime switching economy when the manager has time-consistent or time-inconsistent preferences. Let $F(K, W, s)$ and $P(K, W, s)$ denote firm value with time consistency and time inconsistency in state s, respectively. The manager would choose to pay out cash once its stock grows sufficiently large in order to avoid the cash-carrying cost. Hence, the payout boundary with time consistency (time inconsistency) is denoted by $\overline{W}_s$ ($\overline{W}_s^P$). Similarly, if the firm's cash holdings are low, it could choose to issue equity. Let $\underline{W}_s$ and $\underline{W}_s^P$ denote the issuance boundary with time consistency and time inconsistency, respectively. We first characterize the solution for the case in which a manager has time-consistent preferences. In this case, the firm's decision-making and firm value rely on the following three regions: an external financing/liquidation region ($W \le \underline{W}_s$), an internal financing region ($\underline{W}_s < W < \overline{W}_s$), and a payout region ($W \ge \overline{W}_s$).

3.1. Time-Consistent Model. Suppose that the manager with time-consistent preferences chooses firm's investment I, cumulative payout policy U, cumulative external financing H, and liquidation time T to maximize firm value defined at time $t = 0$ (under the risk-neutral measure):

$$\mathbb{E}_0 \left[\int_0^T e^{-r\varepsilon} \left(dU_\varepsilon - dH_\varepsilon - dX_\varepsilon \right) + e^{-rT} \left(l_s K_T + W_T \right) \right]. \tag{8}$$

The first term is the discounted value of net payouts to shareholders; the second term is the discounted value upon liquidation. Optimality could imply that the firm never liquidates, so we have $T = \infty$.

When the firm's cash flow W is in the internal financing region, i.e., $W \in (\underline{W}_s, \overline{W}_s)$, $F(K, W, s)$ satisfies the following system of Hamilton-Jacobi-Bellman (HJB) equations (under the risk-neutral measure):

$$\begin{aligned} & r_s F(K, W, s) \\ & = \max_{I_s} \left\{ (I_s - \delta_s K) F_K(K, W, s) + \frac{\sigma_s^2 K^2}{2} F_{WW}(K, W, s) + \left[(r_s - \kappa_s) W + \mu_s K - I_s - \Gamma(I, K, s) \right] F_W(K, W, s) + \hat{\zeta}_s \left(F(K, W, s^*) - F(K, W, s) \right) \right\}, \end{aligned} \tag{9}$$

where s^* denotes the state that is different from s. The first term on the right side of (9) represents the impact of capital stock changes on firm value. The second term represents the effects of the volatility of W on firm value. And the third term represents the effects of the expected change in the firm's cash holdings W on firm value. The last term captures the expected change of firm value when the state changes from s to s^*.

We use the scale invariance of the firm's technology to write firm value as $F(K, W, s) = f_s(w) K$ and investment as $I_s = i_s(w) K$ in the case of time consistency, where $w = W/K$ is the cash-capital ratio. Substituting these terms into (9), we obtain the following ordinary differential equations (ODEs) for $f_s(w)$:

$$\begin{aligned} & r_s f_s(w) \\ & = \max_{i_s(w)} \left\{ (i_s(w) - \delta_s) \left(f_s(w) - w f_s'(w) \right) + \frac{\sigma_s^2}{2} f_s''(w) + \left[(r_s - \kappa_s) w + \mu_s - i_s(w) - g_s(i_s(w)) \right] f_s'(w) + \hat{\zeta}_s \left(f_{s^*}(w) - f_s(w) \right) \right\}. \end{aligned} \tag{10}$$

The first-order condition for the investment-capital ratio $i_s(w)$ is given by

$$i_s(w) = \frac{1}{\theta_s} \left(\frac{f_s(w)}{f_s'(w)} - w - 1 \right) + \nu_s. \tag{11}$$

Taking the derivative of investment-capital ratio $i_s(w)$ in (11) with respect to w, we get

$$i_s'(w) = -\frac{1}{\theta_s} \frac{f_s(w) f_s''(w)}{\left(f_s'(w) \right)^2}. \tag{12}$$

Equation (12) shows that the firm's investment increases with w if and only if firm value is concave.

Firstly, we specify the payout boundary. When the marginal value of cash flow held by the firm is less than the marginal value of cash flow held by shareholders, the manager starts paying out the excess cash. Thus the endogenous payout boundary $\overline{w}_s = \overline{W}_s/K$ satisfies the following value matching and super contact conditions (Dumas [26]):

$$\begin{aligned} f_s'(\overline{w}_s) &= 1, \\ f_s''(\overline{w}_s) &= 0. \end{aligned} \tag{13}$$

On the other hand, if $f_s'(w) < 1$, the firm is better off distributing the excess cash as a lump sum and reducing its cash holdings to $\overline{w}_s$. So we have

$$f_s(w) = f_s(\overline{w}_s) + (w - \overline{w}_s), \quad w > \overline{w}_s. \tag{14}$$

Intuitively, this condition reflects the fact that the firm holds too much cash or the firm's cash holdings in state s^* are such that $\overline{w}_s < w < \overline{w}_{s^*}$ and the state of the economy could suddenly switch from s^* to s.

Secondly, we give the issuance boundary. The firm chooses to raise external funds by issuing shares so as to bring its cash stock back into the interior region when it is sufficiently valuable. In this case, the firm could suddenly change from the state s^* with the financing boundary $\underline{w}_{s^*}$ into the other state s with a higher financing boundary $\underline{w}_s$. And its cash holdings could be midway between the two lower financing boundaries ($\underline{w}_{s^*} < w < \underline{w}_s$). The firm optimally chooses to issue equity financing when its going-concern value is higher than its liquidation value or the cost of external financing is low. Let M_s denote the firm's cash target level after equity issuance. We define $m_s = M_s/K$ and $\underline{w}_s = \underline{W}_s/K$. Hence, when $w \leq \underline{w}_s$, firm value $f_s(w)$ satisfies

$$f_s(w) = f_s(m_s) - \phi_s - (1+\gamma_s)(m_s - w), \quad w \leq \underline{w}_s. \tag{15}$$

We have the following value matching and smooth pasting conditions:

$$\begin{aligned} f_s(\underline{w}_s) &= f_s(m_s) - \phi_s - (1+\gamma_s)(m_s - \underline{w}_s), \\ f_s'(m_s) &= 1 + \gamma_s. \end{aligned} \tag{16}$$

Intuitively, if the firm chooses to issue equity before it runs out of cash, it must be the case that the marginal value of cash at the issuance boundary $\underline{w}_s > 0$ is equal to the marginal issuance cost $1 + \gamma_s$. So we have

$$f_s'(\underline{w}_s) = 1 + \gamma_s, \quad \underline{w}_s > 0. \tag{17}$$

Finally, if (17) fails to hold, the firm does not raise funding until it runs out of cash. Then the optimal liquidation boundary with time consistency is $\underline{w}_s = 0$ and firm value is given by

$$f_s(0) = l_s. \tag{18}$$

3.2. Time-Inconsistent Model. In this section, we assume that the time-inconsistent manager chooses the firm's investment I, cumulative payout policy U, cumulative external financing H, and liquidation time T to maximize firm value defined as follows (under the risk-neutral measure) at any time $t \in [t_n, t_{n+1})$:

$$\begin{aligned} \mathbb{E}_t \Bigg[& \int_t^T D_n(t,\varepsilon)(dU_\varepsilon - dH_\varepsilon - dX_\varepsilon) \\ & + D_n(t,T)(l_s K_T + W_T) \Bigg]. \end{aligned} \tag{19}$$

If $T = \infty$, it could display that the firm's optimality is to never choose liquidation. The firm's decision-making and firm value also rely on the following three regions: an external financing/liquidation region ($W \leq \underline{W}_s^P$), an internal financing region ($\underline{W}_s^P < W < \overline{W}_s^P$), and a payout region ($W \geq \overline{W}_s^P$).

Let $P(K,W,s)$ and I_s^P denote firm value and investment in the case of time inconsistency, respectively. Hence, in the internal financing region, using Ito's generalized lemma and the dynamic programming method, firm value with time inconsistency satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} &\eta P(K,W,s) \\ &= \max_{I_s^P} \Big\{ (I_s^P - \delta_s K) P_K(K,W,s) + \frac{\sigma_s^2 K^2}{2} P_{WW}(K,W,s) + \left[(r_s - \kappa_s) W + \mu_s K - I_s^P - G(I_s^P, K) \right] P_W(K,W,s) + \widehat{\zeta}_s \left[P(K,W,s^*) - P(K,W,s) \right] + \lambda \left[\beta H(K,W,s) - P(K,W,s) \right] \Big\}, \end{aligned} \tag{20}$$

where the function $H(K,W,s)$ is defined by

$$\begin{aligned} &\eta H(K,W,s) \\ &= \Big\{ (I_s^P - \delta_s K) H_K(K,W,s) + \frac{\sigma_s^2 K^2}{2} H_{WW}(K,W,s) + \left[(r_s - \kappa_s) W + \mu_s K - I_s^P - G(I_s^P, K) \right] H_W(K,W,s) + \widehat{\zeta}_s \left[H(K,W,s^*) - H(K,W,s) \right] \Big\}. \end{aligned} \tag{21}$$

Note that the last term on the right side of (20) captures the effect of the manager's time-inconsistent preferences on firm value. It is intuitive. Once the current self (who decides on I_s^P during the interval $[t_n, t_{n+1})$) loses control at rate λ, firm value has a sudden reduction $\beta H(K,W,s) - P(K,W,s)$. The function $H(K,W,s)$ means that the current self expects her successor (who makes decisions during the interval $[t_{n+1}, t_{n+2})$) will also apply nonexponential discounting. Hence, the succeeding self will use the investment strategy I_s^P. In a word, the last term represents the average changes in firm value due to the random changes in the manager's time preferences modeled by a Poisson process with λ.

We also can write firm value as $P(K,W,s) = p_s(w)K$, $H(K,W,s) = h_s(w)K$ and investment as $I_s^P = i_s^P(w)K$, where $w = W/K$ is the cash-capital ratio. Hence, we obtain the following ordinary differential equations (ODEs) for $p_s(w)$:

$$\eta p_s(w) = \max_{i_s^P(w)} \left\{ \left(i_s^P(w) - \delta_s\right)\left(p_s(w) - w p_s'(w)\right) + \frac{\sigma_s^2}{2} p_s''(w) + \left[\left(r_s - \kappa_s\right)w + \mu_s - i_s^P(w) - g_s\left(i_s^P(w)\right)\right] p_s'(w) + \widehat{\zeta}_s\left[p_{s^*}(w) - p_s(w)\right] + \lambda\left[\beta h_s(w) - p_s(w)\right] \right\}, \tag{22}$$

where the function $h_s(w)$ is defined by

$$\eta h_s(w) = \left\{ \left(i_s^P(w) - \delta_s\right)\left(h_s(w) - w h_s'(w)\right) + \frac{\sigma_s^2}{2} h_s''(w) + \left[\left(r_s - \kappa_s\right)w + \mu_s - i_s^P(w) - g_s\left(i_s^P(w)\right)\right] h_s'(w) + \widehat{\zeta}_s\left[h_{s^*}(w) - h_s(w)\right] \right\}. \tag{23}$$

The first-order condition for the investment-capital ratio $i_s^P(w)$ is given by

$$i_s^P(w) = \frac{1}{\theta_s}\left(\frac{p_s(w)}{p_s'(w)} - w - 1\right) + \nu_s. \tag{24}$$

Firstly, we specify the payout boundary. As with time inconsistency, the endogenous payout boundary $\overline{w}_s^P = \overline{W}_s^P/K$ satisfies the following value matching and super contact conditions:

$$p_s'\left(\overline{w}_s^P\right) = 1,$$
$$p_s''\left(\overline{w}_s^P\right) = 0. \tag{25}$$

On the other hand, if $p_s'(w) < 1$, we have

$$p_s(w) = p_s\left(\overline{w}_s^P\right) + \left(w - \overline{w}_s^P\right), \quad w > \overline{w}_s^P. \tag{26}$$

Additionally, the upper boundary condition of $h_s(w)$ is defined by

$$h_s'\left(\overline{w}_s^P\right) = 1, \tag{27}$$

where $\overline{w}_s^P$ depends on (25). When $w > \overline{w}_s^P$, it also has $h_s(w) = h_s(\overline{w}_s^P) + (w - \overline{w}_s^P)$.

Secondly, we give the issuance boundary. Similar to the case of time consistency, if the firm is sufficiently valuable it then chooses to raise external funds through an equity issuance so as to bring its cash stock back into the interior region. Let M_s^P denote the firm's cash target level after equity issuance. We can write $m_s^P = M_s^P/K$ and $\underline{w}_s^P = \underline{W}_s^P/K$. Hence, when $w \le \underline{w}_s^P$, firm value $p_s(w)$ satisfies

$$p_s(w) = p_s\left(m_s^P\right) - \phi_s - \left(1 + \gamma_s\right)\left(m_s^P - w\right), \quad w \le \underline{w}_s^P. \tag{28}$$

We have the following value matching and smooth pasting conditions:

$$p_s\left(\underline{w}_s^P\right) = p_s\left(m_s^P\right) - \phi_s - \left(1 + \gamma_s\right)\left(m_s^P - \underline{w}_s^P\right),$$
$$p_s'\left(m_s^P\right) = 1 + \gamma_s. \tag{29}$$

Intuitively, if the firm chooses to issue equity before it runs out of cash, it must be the case that the marginal value of cash at the issuance boundary $\underline{w}_s^P > 0$ is equal to the marginal issuance cost $1 + \gamma_s$. So we have

$$p_s'\left(\underline{w}_s^P\right) = 1 + \gamma_s, \quad \underline{w}_s^P > 0. \tag{30}$$

Of course, the lower boundary condition of $h_s(w)$ is defined by

$$h_s\left(\underline{w}_s^P\right) = h_s\left(m_s^P\right) - \phi_s - \left(1 + \gamma_s\right)\left(m_s^P - \underline{w}_s^P\right), \tag{31}$$

where m_s^P and $\underline{w}_s^P$ depend on (29) and (30), respectively.

Finally, if condition (30) fails to hold, the firm does not raise funding until it runs out of cash. Then liquidation could be preferred; we have

$$p_s(0) = l_s,$$
$$h_s(0) = l_s. \tag{32}$$

Table 1: Summary of the parameter values.

Parameters	Symbol	State G	State B
Risk-free rate/discount factor	r/η	5%	
Volatility of productivity shock	σ	12%	
Rate of depreciation	δ	15%	
Risk-neutral mean productivity shock	μ	20.78%	
Adjustment cost parameter	θ	1.8	
Center of adjustment cost parameter	ν	15%	
Proportional cash-carrying cost	κ	1.5%	
Proportional financing cost	γ	6%	
State transition intensity	ζ_s	0.125	0.5
Capital liquidation value	l_s	1	0.3
Fixed financing cost	ϕ_s	0.5%	50%
Price of risk for financing shocks	π_s	ln(3)	– ln(3)

4. Quantitative Analysis

This section provides quantitative analysis of the time-inconsistent model and time-consistent model in a regime switching economy. We firstly exhibit our choice of parameters and then illustrate the model's solutions in the case of time inconsistency and time consistency. In order to make a reliable comparison, we select some parameter values following the BCW model.

The parameters remain the same in both states: $r = 5\%$, $\sigma = 12\%$, $\delta = 15\%$, $\nu = 15\%$, $\theta = 1.8$, and $\mu = 20.78\%$ (under the risk-neutral probability measure). We rely on the technology parameters estimated by Eberly et al. [27] for these parameter choices. The cash-carrying cost is set to $\kappa = 1.5\%$. We do not take a firm stand on the precise interpretation of the cash-carrying cost; it can be due to a tax disadvantage of cash or to agency frictions.

We set the marginal cost of issuance in both states to be $\gamma = 6\%$. We keep this parameter constant across the two states for simplicity and focus only on changes in the fixed cost of equity issuance to capture changes in the firm's financing opportunities. The fixed cost of equity issuance in state G is set at $\phi_G = 0.5\%$. As for the issuance costs in state B, we choose $\phi_B = 50\%$. Although in reality these parameter values can also change with the aggregate state, we keep them fixed in this paper to isolate the effects of changes in external financing conditions.

We assume that the transition intensity out of state G is set to $\zeta_G = 0.125$ to reflect an average duration of eight years for state G. Our paper chooses the price of risk with respect to financing shocks in state G to be $\pi_G = \ln(3)$. So, the risk-adjusted transition intensity out of states G and B is $\widehat{\zeta}_G = 0.375$ and $\widehat{\zeta}_B = 0.167$, respectively. In addition, we assume that the discount factor η equals the risk-free rate r. Table 1 summarizes the parameter values in the model.

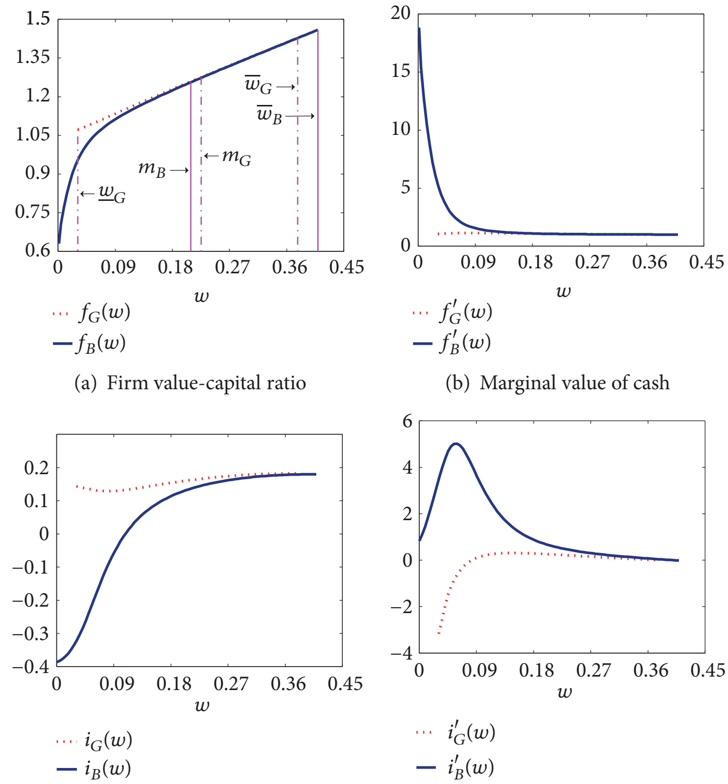
4.1. Time-Consistent Benchmark. Now, we consider that the manager has time-consistent preferences in both states. In state G, the firm can enter the crisis with a 12.5% transition intensity.

4.1.1. Firm Value and Investment with Time Consistency. Figure 1 exhibits the firm value-capital ratio and investment-capital ratio for states G and B as well as their sensitivities with respect to the cash-capital ratio w. Figure 1(a) shows that the optimal external financing boundary is $\underline{w}_G = 0.031$ in state G. At this point, raising external funds is optimal even though the firm still has sufficient cash to continue operating. It reflects that the time-consistent manager is concerned about the risk that the favorable financing opportunities disappear.

In state B, the parameter ϕ_B reflects that raising external financing becomes extremely costly and only firms which are desperate for cash are forced to raise new funds; then there is no favorable financing opportunity. Hence, the time-consistent manager issues equity only when the firm runs out of cash, $\underline{w}_B = 0$. Then, $f_B(0) = f_B(m_B) - \phi_B - (1 + \gamma_B)m_B$ but $f_B{}'(0) \neq 1 + \gamma_B$. As Figure 1(a) shows, the amount of equity issuance is $m_B - \underline{w}_B = 0.209$ in state B which is higher than the amount of equity issuance $m_G - \underline{w}_G = 0.191$ in state G. Since the fixed external financing cost is higher in state B than in state G, the lumpy size of the issuance is efficient to help the firm save on the fixed financing cost in state B. Additionally, the issuance boundary is lower in state B than in state G, $\underline{w}_B = 0 < \underline{w}_G = 0.031$, meaning that the firm in state B with cash holding $w \in (0, 0.031)$ will raise external funds $(m_G - w)$ through an equity issuance so that the state switches from B to G.

Figure 1(a) also indicates that the payout boundary $\overline{w}_B = 0.409$ in state B is higher than $\overline{w}_G = 0.377$ in state G. This reflects that the manager's precautionary motive is stronger in state B than in state G, so that the time-consistent manager expects to hold more cash hoarding in state B.

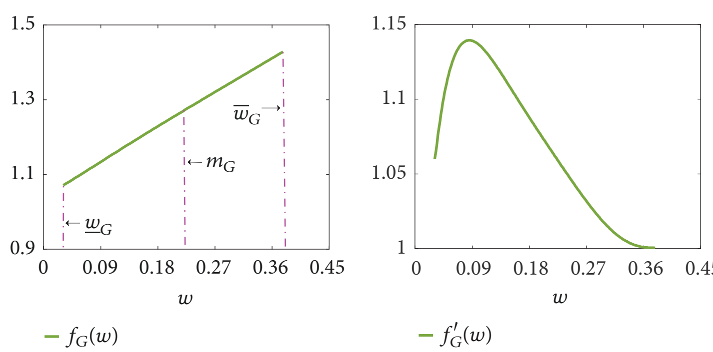
Figure 1(b) indicates that financing constraints have significant influences on the marginal value of cash in state B even though state B is not permanent. In our setting, when the firm runs out of cash, the marginal value of cash $f_B'(0)$ reaches 19. Remarkably, to avoid incurring costly external financing in state B, the time-consistent manager engages in large asset sales and divestment. Besides, in order to observe the change of firm value-capital ratio in state G, we plot the firm value-capital ratio and its sensitivity in Figure 2. It shows that $f_G(w)$



(a) Firm value-capital ratio (b) Marginal value of cash

(c) Investment-capital ratio (d) Investment-cash sensitivity

Figure 1: Comparing state G with state B: firm value-capital ratio and investment-capital ratio as well as their sensitivities.



(a) Firm value-capital ratio (b) Marginal value of cash

Figure 2: Firm value-capital ratio and its sensitivity in state G.

is not globally concave in w. For $0.085 < w < \overline{w}_G = 0.377$, $f_G(w)$ is concave.

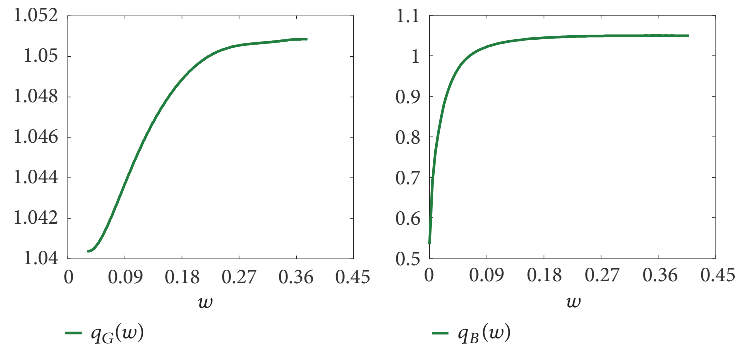
Figure 1(c) describes that corporate investment in state G is higher than in state B for a given w, and again the difference is extremely large when w is low. Also, investment is much more variable with respect to w in state B than in state G. And Figure 1(d) plots that the cash sensitivity of investment is much larger in state B than in state G. It also can be seen that corporate investment in state G is nonmonotonic with w and increases when $0.085 < w < \overline{w}_G = 0.377$. Unlike in state G, investment in state B is monotonic with w because the time-consistent manager is risk-averse.

4.1.2. Average q with Time Consistency. Now, we define the enterprise value as firm value net of the value of short-term liquid assets based on Bolton et al. [8]. The enterprise value captures the value created from productive illiquid capital and it equals $F(K, W, s) - W$. Therefore, average q, defined as the ratio between the enterprise value and capital stock, is denoted by

$$q_s(w) = \frac{F(K, W, s) - W}{K} = f_s(w) - w. \tag{33}$$

Then, the sensitivity of average q is

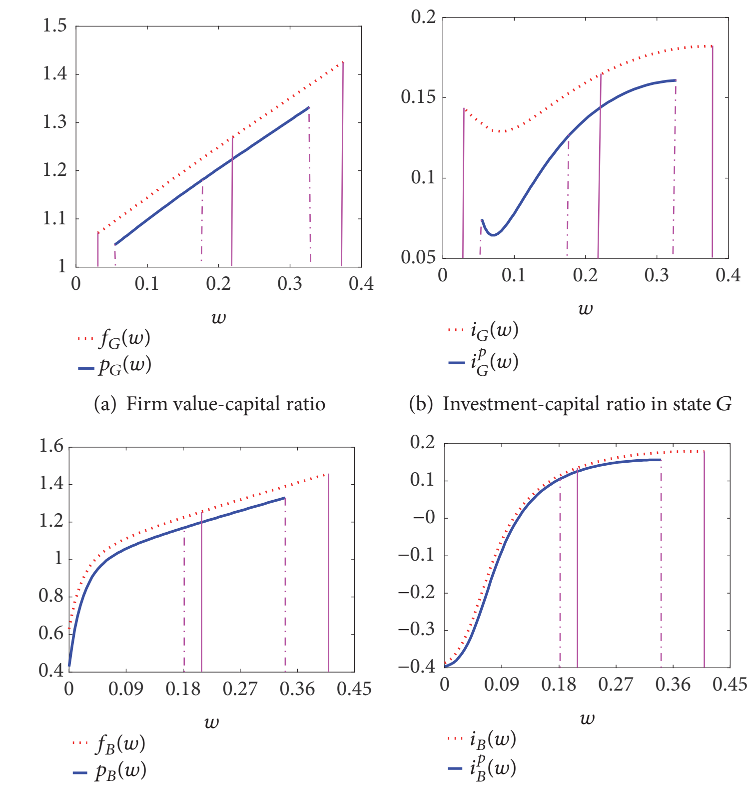
$$q'_s(w) = f'_s(w) - 1, \tag{34}$$



(a) Average q in state G

(b) Average q in state B

FIGURE 3: Average q with time consistency in both states.



(a) Firm value-capital ratio

(b) Investment-capital ratio in state G

(c) Firm value-capital ratio in state B

(d) Investment-capital ratio in state B

FIGURE 4: Firm value-capital ratio and investment-capital ratio for two preferences in a regime switching economy.

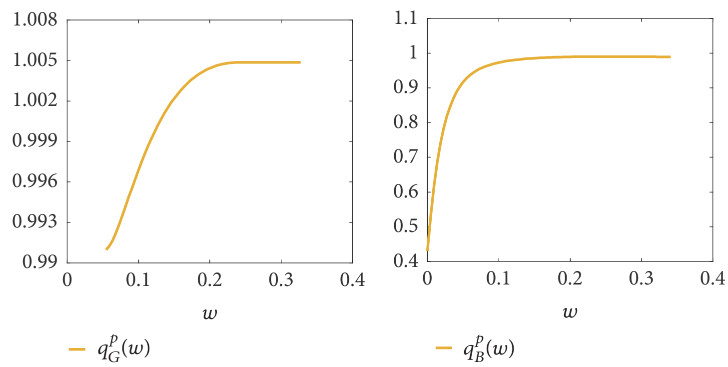
which measures how much the enterprise value changes with an extra dollar of internal cash. Equation (34) shows that the concavity and convexity of $q_s(w)$ and $f_s(w)$ are the same.

Figure 3 plots average q in both states. It shows that $q_G(w)$ is also not globally concave and $q_B(w)$ is concave in w, meaning that $f_G(w)$ is also not globally concave and $f_B(w)$ is concave in w, respectively. Particularly, for $0.085 < w < \overline{w}_G = 0.377$, $q_G(w)$ and $f_G(w)$ are concave.

In general, the firm's investment, firm value, optimal financing, and dividend payout strategies with time consistency have quite a difference in both states. Next, this paper considers the time-inconsistent model based on the time-consistent (BCW) model.

4.2. Time-Inconsistent Preferences. Next, this paper studies the effects of time-inconsistent preferences on firm value and investment in a regime switching economy.

4.2.1. Firm Value and Investment with Time Inconsistency. Figure 4 plots firm value-capital ratio and investment-capital ratio for both preferences (time-inconsistent and time-consistent preferences) in the two states, where our paper fixes the parameters $\lambda = 0.3$ and $\beta = 0.95$. Figure 4 shows that firm value and investment for both states are lower in the case of time inconsistency than in the case of time consistency for a given level of cash. It is interesting to note that the differences of firm value and investment between time consistency and



(a) Average q in state G (b) Average q in state B

FIGURE 5: Average q with time inconsistency in both states.

time inconsistency are higher in state G than in state B for given cash holdings w. That is, time-inconsistent preferences have more influences on firm value and investment in state G than in state B. It is intuitively obvious that irrational behavior of the manager is more outstanding in a favorable market condition than in a financial crisis. Because the financial condition is bad in a financial crisis, the manager becomes more cautious and rational when making the firm's decisions.

As Figure 4 shows, in state G, the external financing boundary $\underline{w}_G^p = 0.055$ is higher than $\underline{w}_G = 0.031$, the target cash-capital ratio $m_G^p = 0.177$ is lower than $m_G = 0.222$, and the amount of equity issuance $m_G^p - \underline{w}_G^p = 0.122$ is lower than $m_G - \underline{w}_G = 0.191$. Also, in state B, there is no favorable financing opportunity and then the time-inconsistent manager issues equity only when the firm exhausts its cash flow $\underline{w}_B^p = 0$. Namely, the amount of equity issuance $(m_B^p - \underline{w}_B^p)$ equals the target cash-capital ratio m_B^p. So the amount of equity is $m_B^p = 0.182$ which is lower than $m_B = 0.209$. The results indicate that the time-inconsistent manager issues equity earlier in state G and raises a less amount of equity issuance in the two states than her time-consistent counterpart. This is intuitive. Firstly, in state G, since the time-inconsistent manager is more impatient and worries more about the disappearing of favorable financing market condition, meaning that the state switches from G to B, she chooses to issue equity earlier than the time-consistent manager. Secondly, the less amount of issuance would help the firm economize on the financing costs and on subsequent cash-carrying costs for both states.

Additionally, in state G the payout boundary $\overline{w}_G^p = 0.327$ is lower than $\overline{w}_G = 0.377$, and in state B the payout boundary $\overline{w}_B^p = 0.341$ is lower than $\overline{w}_B = 0.409$. This is intuitive. On the one hand, since the time-inconsistent manager values the dividends delivered by her future selves less than those paid by her current self, she prefers to pay out the cash earlier by lowering the dividend payment in her present period. On the other hand, in order to reduce the cash-carrying cost, the time-inconsistent manager pays out cash earlier and keeps less cash reserve.

In short, these results illustrate that the time-inconsistent manager is more impatient in making short-term decisions than long-term decisions.

4.2.2. Average q with Time Inconsistency. Similar to average q with time consistency, we define the enterprise value with time inconsistency as firm value net of the value of short-term liquid assets. The enterprise value with time inconsistency equals $P(K, W, s) - W$. Therefore, average q with time inconsistency is denoted by

$$q_s^p(w) = \frac{P(K, W, s) - W}{K} = p_s(w) - w. \quad (35)$$

Then, the sensitivity of average q with time inconsistency is

$$\left(q_s^p\right)'(w) = p_s'(w) - 1, \quad (36)$$

which measures how much the enterprise value with time inconsistency changes with an extra dollar of internal cash. Equation (36) implies that the concavity and convexity of $q_s^p(w)$ and $p_s(w)$ are the same.

Figure 5 plots average q with time inconsistency in both states. It shows that $q_G^p(w)$ is not globally concave and $q_B^p(w)$ is concave in w. Hence, $p_G(w)$ is not globally concave and $p_B(w)$ is concave in w. Although the manager's time-inconsistent preferences reduce firm value, they do not change the concavity of firm value.

5. Conclusions

This paper extends the classical BCW model to account for a manager's time-inconsistent preferences. Using a continuous-time version of the quasi-hyperbolic discount function, we model the time-inconsistent preferences of the manager. This paper researches the effects of the degree of time inconsistency on the optimal equity financing and dividend payout strategies during the favorable market condition and financial crisis. We find that, with a higher degree of time inconsistency, the manager with time-inconsistent preferences prefers to issue less equity each time and pay out dividends earlier in the two states; in particular, external equity financing occurs earlier in a favorable market condition. Hence, time-inconsistent preferences have a significant impact on the optimal external financing and payout strategies of the firm. In addition, the impact of time inconsistency on firm value and investment is higher in a favorable market condition than in a financial crisis. It highlights that time inconsistency and regime switching are very important for firm policy. Shareholders can use this finding to write a more advantageous contract with the manager.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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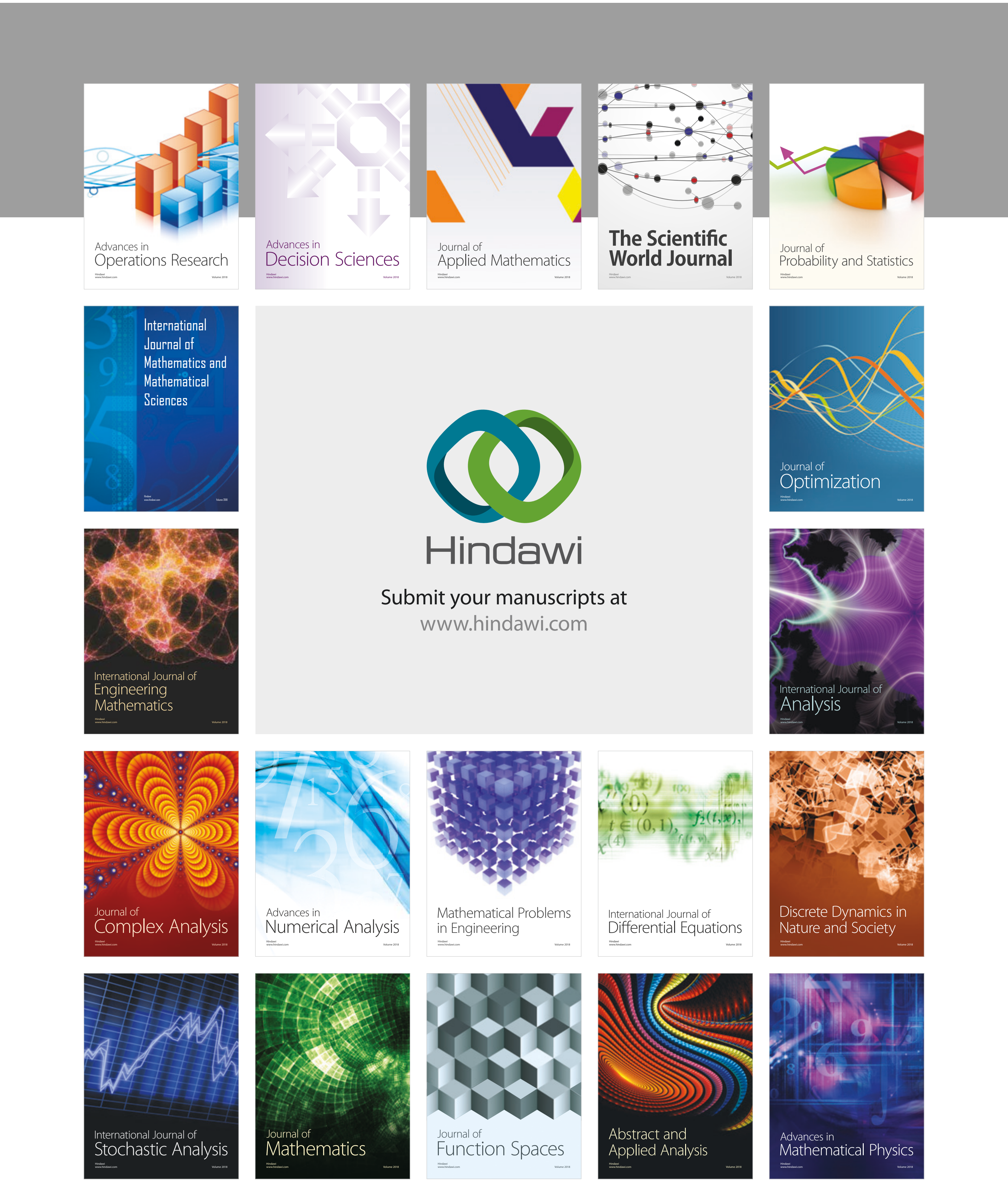

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