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Research Article

Evolutionary Cooperation in Networked Public Goods Game with Dependency Groups

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Either in microlevel organizations or macrolevel societies, the individuals acquire benefits or payoffs by forming interdependency groups linked by common interests. Conducting research on the effects of interdependency groups on the evolution of cooperation could have a better understanding of the social dilemma problem. In this paper, we studied a spatial public goods game with nonlocal interdependency groups where each of participants is located in a two-dimensional square lattice or Watts–Strogatz small-world network with payoffs obtaining from the interactions with nearest neighbors. In terms of the enhancement factor, the effects of group density on the evolutionary cooperation can be quite different. For a low enhancement factor, the cooperation level is a nonmonotonic function with the varying density of interdependency groups in the system, which means a proper density of interdependency groups can best promote the cooperative level. For a moderate enhancement factor, a higher density of interdependency groups can always correspond to a higher cooperative level. However, if the enhancement factor is too high, a high density of interdependency groups can impede the evolutionary cooperation. We give the explanations for the different roles of group density of interdependency by using the transition probabilities of *C* players into *D* players as well as the reverse. Our findings are very helpful for the understanding of emergence cooperation as well as the cooperation regulation in the selfish individuals.

1. Introduction

Cooperation is a ubiquitous phenomenon in biological societies, referring the process that individuals or groups working together for common or mutual benefits, such as marital relations and alliances. However, it is still a challenge to understand the emergence and maintenance of cooperation. Evolutionary game theory provides an abundant framework to address this issue and attract attention from many researchers, including physicists, biologists, and economists [1–3]. The results of this study can promote recognitions on the mechanism of cooperative emergence and imply that interdependency is an important mechanism to enhance cooperation. The prisoner's dilemma game (PDG) has been used to conduct extensive exploration on the characteristics of conflict among different groups with various interest preferences, and it can be used to explain cooperative behaviors among interactions between two-player groups [4, 5].

Taking a large number of interacting players into consideration, the same conflict is also able to be seized by the public goods game (PGG) [6–11]. Based on the classical PGG, N players are able to determine whether an amount c should be invested into a common pool (cooperate) or not (defect) concurrently and separately. Multiplied by an enhancement level r with $r > 1$, the obtained investment is able to be regarded as the groups production and management degree or the synergy impacts of collaboration. The obtained investment is then distributed evenly again among all participants, regardless of their factual strategy. Based on these processes, the whole group will obtain the most benefit when all players put all their property into the public pool. However, defectors do not have to take up any costs when they get the same benefits as the cooperators. Therefore, defection is frequently the first choice and seems to be the natural strategy to select for selfish players when $r < N$.

Nowak and May have indicated that spatial structure is capable of promoting the progress of collaboration on the basis of the well-renowned PDG model which represents network reciprocity [12]. Subsequently, based on the seminal work, evolutionary game has been extended by researchers to diverse topology structures, finding that it is possible to introduce agents' heterogeneity in the form of heterogeneous interaction networks [13–21], which is also demonstrated to play a vital role in the smooth evolution of collaboration [15, 22–31]. Then, other core mechanisms responsible for stimulating cooperation, including migration [32], time-scale heterogeneity [33, 34], multilayered networks [35, 36] or groups [37, 38], aspirations [39–43], punishment [44], group intelligence [45], tolerance [46], and dynamic groupings [47], have been shown in a number of innovative studies. Besides, coevolutionary games, in which, the communications or several particular properties of players are also influenced by evolution, have helped us understand the occurrence of system properties facilitating the evolutionary collaboration more deeply [48].

What should be noted is that when it comes to Fermi function, Szabó and Tőke have introduced the noise in strategy adoption [49] representing players' bounded rationality, finding that appropriate noise degree [50, 51] or players' bounded rationality in strategy restriction [28] is able to promote collaboration. Besides, Perc has explored the effects imposed by stochastic payoff changes on the development of collaboration in the spatial PDG, finding that Gaussian noise is able to promote collaboration resonantly, which is similar to classical coherence resonance [52]. Later, the PDG framework has investigated the correlation between the "dynamical" coherence resonances induced by the payoff noise and the so-called "evolutionary" coherence resonance induced by the noise of strategy application [53]. Nevertheless, in terms of interactions by multiple players, it remains unclear in terms of the impacts imposed by the group-payoff variation on the evolutionary collaboration or the accurate mechanism.

A lot of attention has been paid for the evolutionary cooperation in terms of interdependency network reciprocity [35, 54–56], where players can acquire payoffs from the interactions in each layer of network or share information about strategic choices between players residing in different network layers. Besides, Zhang and Yang studied the effects of random partnerships on the evolution of cooperation in spatial game theory on a square lattice, in terms of SPDG (Spatial Prisoner Dilemma Game) [57]. Poncela et al. explored the effect of limiting the number of interactions that a node can establish per round of a prisoner's dilemma game [58]. It has been shown that the diverse utilities of players maintain healthy public cooperativeness even in the face of adverse conditions [54], and for a proper interdependence level across network layers, the cooperation is promoted best [35]. Similarly, the information sharing across layers also reinforce the cooperation level significantly [55]. In real human or animal societies, individuals not only share information but also share payoffs or material well-being by the interdependent relationships (such as marriage or other connections with common interests) among them. In this case, the gains that individuals made in the social or economic activities are shared with the rest of the dependent group. Here, we explore the density of payoff-sharing groups on the evolution of cooperation by using the public goods game. We find that the density of payoff-sharing groups can play both positive and negative role for the evolutionary cooperation depending on the enhancement level of the game. We give explanations for the different roles of group density of interdependency by using the transition probabilities of C players into D players as well as the reverse.

Hereafter, we proceed to describe in detail the public goods game with payoff-sharing groups, followed by the presentation of the main results. We round off the discussion with concluding remarks.

2. Model

We applied the simplified version of the PGG, in which, the core factors of the social plight are reserved, whose strength, however, is decided by the enhancement degree [9]. We take this simplified version of PGG into consideration, in which, the player is in a two-dimensional lattice of four neighbors with periodic boundary conditions, with individual player and their nearest neighbor forming a $G = 5$ group. At first, with equal likability of random choice, a collaborator or defector takes up each position. At every time step, the payoff of a focal player i is determined by its strategy s_i and the quantity of partners n_c in the neighborhood. Thus, the player i's payoff is

$$P_i = \frac{r(n_c + s_i)c}{G} - s_i c. \tag{1}$$

s_i is 1 if the player i is a collaborator. Otherwise, it is 0. In our study, in the convenience of simplicity, the investment c of a collaborator is set to 1, with the parameter r denoting the enhancement degree of the group. A defective strategy, compared with a collaborative strategy, will obtain more benefits when $r < G$, characterizing the social dilemma. Nevertheless, for $r > G$, it will be a better choice for all individuals to use a collaborative strategy.

Here, we introduce the payoff-sharing groups in the public goods game. We randomly choose a fraction p of players and match them one by one as paired interdependent groups, and therefore, the parameter p controls the density of payoff-sharing groups in the system. At each round of game, we reset the payoff of players in each interdependency group to the average of group payoff due to interdependencies, i.e., $P'_i = P'_j = (P_i + P_j)/2$ for two interdependent players i and j.

It is possible for every player to imitate the strategy of a freely selected neighbor in updating strategies, and a probability is determined by the payoff disparity synchronously, i.e., the probability for one player i to use the neighbor j's strategy can be expressed as

$$W_{ij} = \frac{1}{1 + \exp\left[\left(P'_i - P'_j\right)/\kappa\right]}, \tag{2}$$

where κ features the bounded rationality of a person [49], indicating the unsureness or mistakes in the strategy application. (In the $\kappa \longrightarrow 0$ restrain, the stochastic impacts are able to be ignored and people are of excellent rationality, while for $\kappa \longrightarrow \infty$ restrain, the stochasticity is the largest and people turn to be irrational comprehensively.)

It should be noted that the parameter p serves as a core parameter in this model, and we will systematically discuss its influence on evolutionary collaboration in the next section. We perform the simulations on both regular square lattice and Watts–Strogatz (WS) small-world network with rewiring probability 0.1. Simulations are conducted for a population of $N = 100 \times 100$ people. The core number of cooperator concentration ρ_C is studied in the steady condition. Equilibrium frequencies of collaborators are acquired by more than 20,000 Monte Carlo time steps from 120,000 steps in total on average, with each data point outcome derived from more than 200 realizations on average.

3. Simulation and Analysis

Firstly, the evolution of cooperation affected by the density of payoff-sharing groups is studied. Clearly, it can be found in results indicated in the right panel of Figure 1 that the equilibrium frequencies ρ_C as functions of the density of payoff-sharing groups p for diversified r when $\kappa = 0.1$. No matter what value of p is, the cooperator concentration ρ_C increases continuously from 0 to 1, whereby we can find that a larger value of p always corresponds to a higher cooperator concentration ρ_C for some lower values of r, and a larger value of p can correspond to a lower cooperator concentration ρ_C for some high values of r. This result implies that the density of payoff-sharing groups can play both positive and negative roles for the persistence of cooperation. In order to account for this phenomenon more clearly, we have plot the cooperator concentration ρ_C as functions of p for different enhancement level of r in Figure 2. For some small values of r (i.e., $r = 5.4$), the cooperator concentration ρ_C versus p is a nonmonotonic function and there exists one optimal density of payoff-sharing groups that can best support the emergence of cooperation. For some moderate values of r, the cooperator concentration ρ_C increases monotonously with the increase of p, and the cooperator concentration ρ_C decreases monotonously with the increase of p for some high values of p. Similar results for small-world networks can be also found in Figure 2(b). All these results indicate that the effects of the density of payoff-sharing groups on the cooperation level depends on the enhancement level of the system for both regular square lattice and WS small-world networks.

We aim to explore the underlying mechanism of the nonmonotonic phenomenon of ρ_C versus p for some small values of r. We studied the transition probability $P_{C\longrightarrow D}$ of C players into D players and the reverse probability $P_{D\longrightarrow C}$ as functions of p in Figure 3. For a lower value of p, it is benefit for the change of D players into C players and thus leads the emergence of cooperation; however, it also provides the possibilities for the C players changing into D players. Therefore, the system maintains a low level of cooperation when the number of D players that changed from C players and the number of C players that changed from D players reach equilibrium. With the increase of p, the transition probabilities $P_{C\longrightarrow D}$ and $P_{D\longrightarrow C}$ increase collectively, and the cooperation reaches its highest level when $P_{C\longrightarrow D}$ equals $P_{D\longrightarrow C}$.

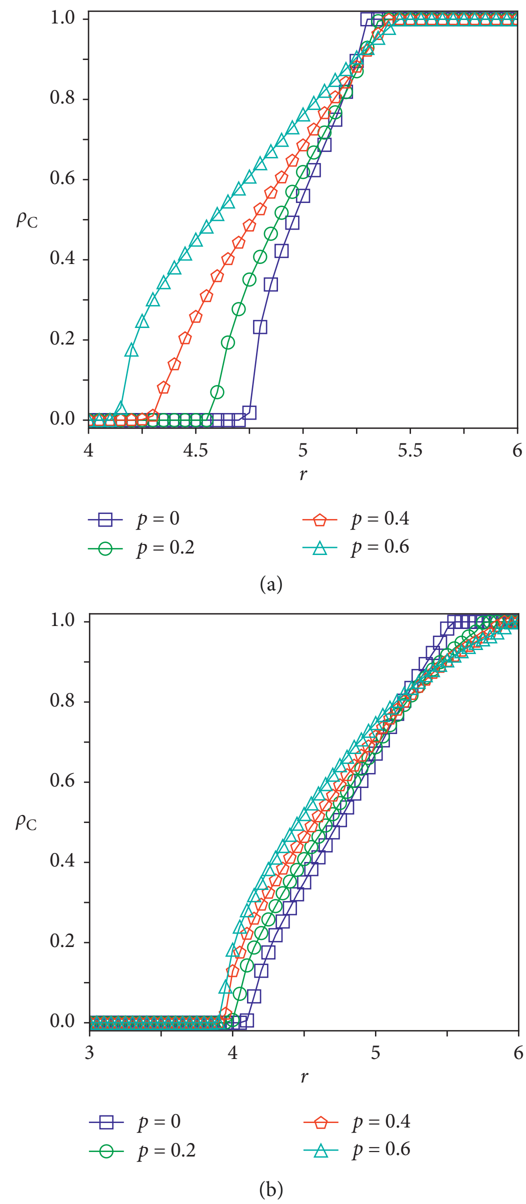

Figure 1: The cooperation level ρ_C as functions of r for different group density p of players on regular square lattice (a) and WS small-world network (b), respectively.

Figure 4 shows the evolution of cooperation frequency, the payoff advantage of cooperators (the payoff of cooperators minus the payoff of defectors), and the payoffs of players engaged in the interdependency groups is studied over time. In the early stages of evolution, cooperators and defectors are evenly distributed around each player, and defectors are highly profitable. Since the introduction of interdependency groups in the system, a defector and a cooperator can form an interdependent group and the cooperators can share some payoffs from their defective partners, which avoid the elimination of cooperators in the early stage of evaluation. After that,

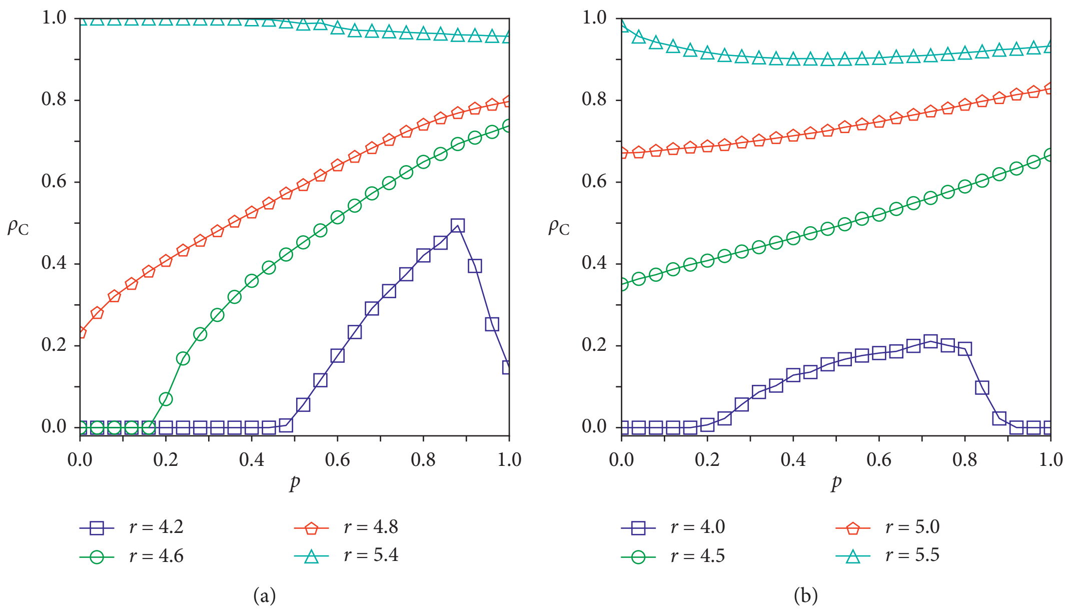

FIGURE 2: The cooperation level as functions of group density p for different enhancement levels of r on regular square lattice (a) and WS small-world network (b), respectively.

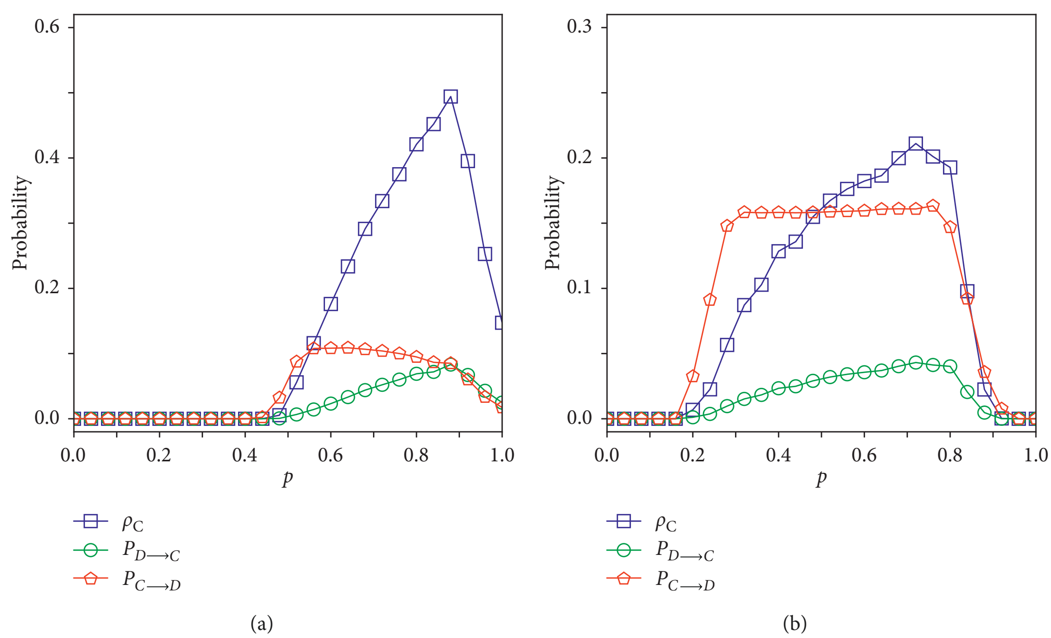

FIGURE 3: The cooperation level ρ_C, transition probability $P_{C\longrightarrow D}$ of C players into D players and the transition probability $P_{D\longrightarrow C}$ of D players into C players for regular square lattice, and WS small-world network, respectively.

the cooperators that are connected together are able to earn a higher payoff than defectors. However the payoffs of defectors will be reduced gradually because of mutual betrayal, and the payoff advantage of cooperation will increase. A cooperator and a cooperator form an interdependent group with a probability $\rho_C\rho_C$, and a defector and a cooperator form an interdependency group with a probability $2\rho_C(1-\rho_C)$. Therefore, most cooperators are exploited by defectors, and their returns are reduced when the cooperation level is increased. Cooperators who form CC dependent groups have stable returns. DD and CD are not stable in the evolution. Therefore, in the early stage of early evolution, the cooperation level is low and the interdependency group will lead to a relatively stable cooperative cluster formed by CC interdependent groups. Later on, CD interdependent group will reduce the stability of cooperative benefits and accelerate the collapse of cooperative clusters, which always has a negative impact on the formation of cooperative clusters. See Figures 4 and 5. This is the very reason why the cooperation frequency is not monotonous when r is small.

With the increase of r, such as closing to 5, the payoff gap between cooperators and defectors decreases and the

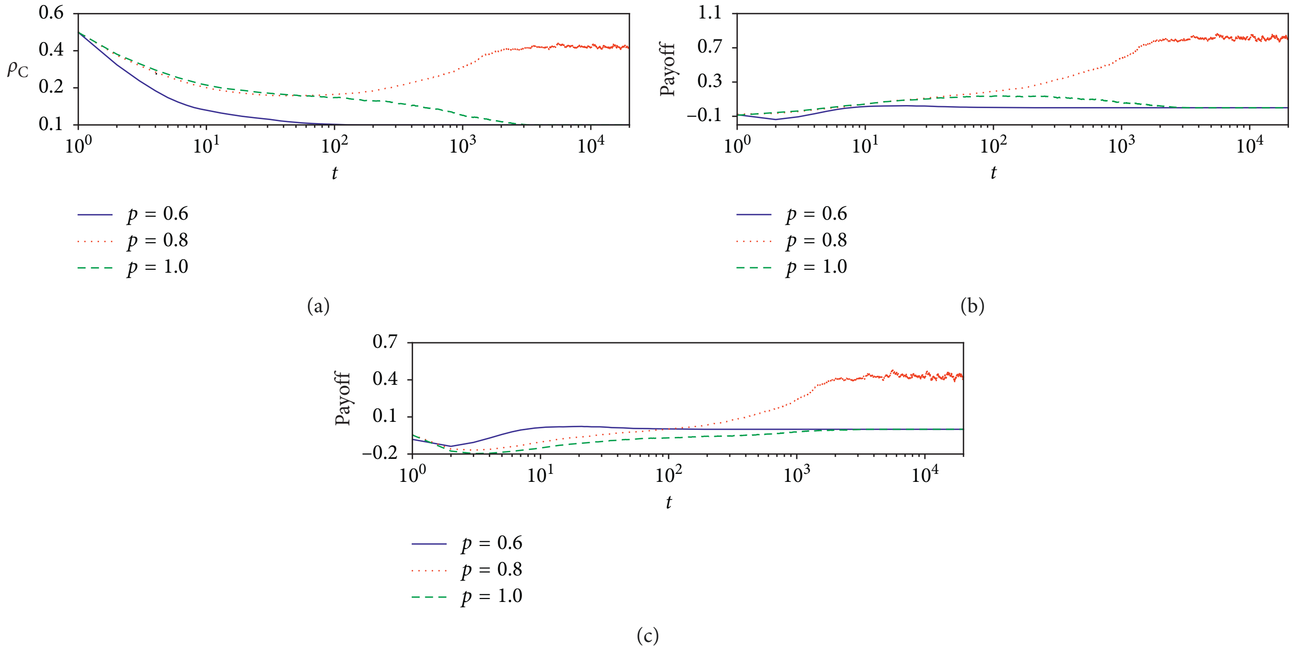

FIGURE 4: Time evolution of ρ_C, f_{dc}, and f_{cd} for different values of p as shown in panels (a), (b), and (c), respectively.

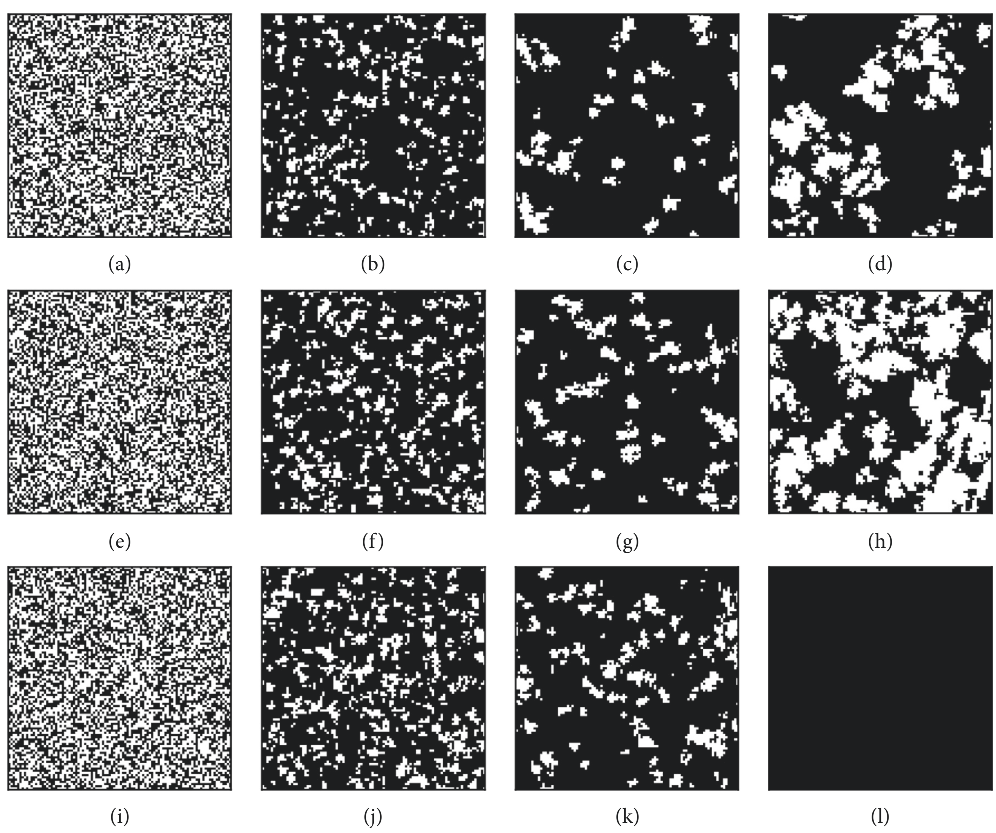

FIGURE 5: Characteristic snapshots of C and D players on a 100×100 square lattice evolve from a random initial state. Here, C (D) is denoted as white (black). (a)–(d) show the results with the setting $p = 0.6$ for $t = 1$, $t = 10$, $t = 50$, and $t = 100$, respectively. (e)–(h) show the result with the setting $p = 0.8$ for $t = 1$, $t = 10$, $t = 100$, and $t = 10,000$, respectively. (i)–(l) show the result with the setting $p = 0.8$ for $t = 1$, $t = 10$, $t = 100$, and $t = 10,000$, respectively.

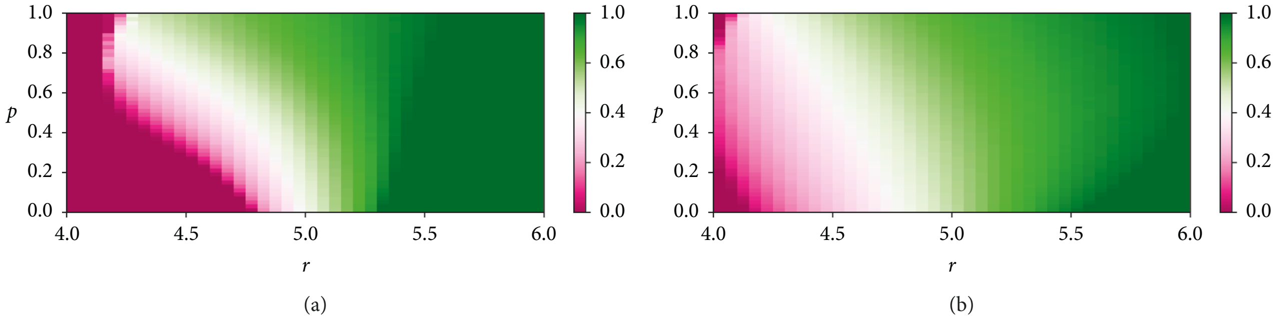

FIGURE 6: The cooperative level ρ_C versus the parameter space (p, r) for random networks (top panel) and WS small-world networks with a rewiring probability 0.1 (bottom panel), respectively.

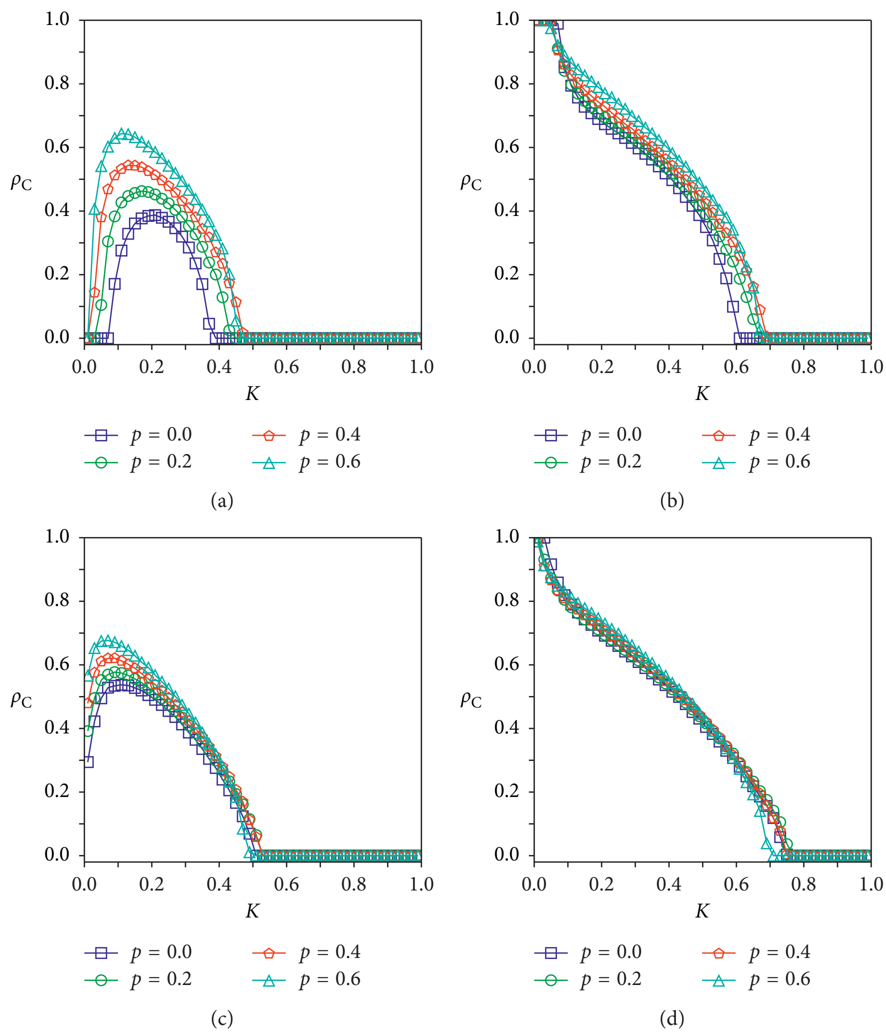

FIGURE 7: The cooperation level ρ_C as the functions of the noise level κ on the regular square lattice in the cases of $r = 4.8$ (a) and $r = 5.2$ (b) and on WS small-world network in the cases of $r = 4.8$ (c) and $r = 5.2$ (d), respectively.

cooperation becomes the dominant strategy in the system. In this case, CD interdependency groups have little influence on the stability of payoff collaborators. Thus, the early promotion of cooperation led the way. When the value of r is greater than 5, the best strategy in the system is cooperation, but the influence of the CD interdependency groups on the instability of the cooperator's payoff still exists, but the impact is small.

To verify the results reported above for various densities of interdependency groups in the strategy application, we figure out ρ_C depending on different p and r on regular square lattice and WS small-world network for a fixed $\kappa = 0.1$ in Figure 6.

It is found that there exists an optimal value of p leading to the highest cooperation level for some small values of r. In addition, we can also find that the cooperation level can be increased continuously for medium values of r, indicating that the cooperation can be impacted by p in different ways depending on the specific value of r.

Furthermore, we investigate the influence of noise level κ on the cooperation level ρ_C in Figure 7. In Figure 7, in spite of the minor differences in the specific curves, with the increase of the noise level κ, the cooperation level ρ_C identically changes nonmonotonically in the convex pattern, with $r = 4.8 < 5$ on both the regular square lattice (a) and the WS small-world network (c). Furthermore, the cooperation level ρ_C versus the noise level κ decreases monotonically with $r = 5.2 > 5$ on both the regular square lattice (b) and the WS small-world network (d). In addition, from Figure 7, we also can discover that the group density p of players influences the cooperation level ρ_C prominently for $r = 4.8 < 5$ and insignificantly for $r = 5.2 > 5$.

4. Discussion

In this paper, we have studied the effects of interdependency groups on the evolutionary collaboration in the public goods game. Based on the extensive simulations on regular square lattice and Watts–Strogatz (WS) small-world networks, we can find that the effects of interdependency groups on the evolutionary cooperation can be quite different for different values of enhancement levels. For a low enhancement factor, the cooperation level is a nonmonotonic function with the varying of density of interdependency groups in the system, which means a proper density of interdependency groups can best promote the cooperative level. For a moderate enhancement factor, a higher density of interdependency groups can always correspond to a higher cooperative level. However, if the enhancement factor is too high, a high density of interdependency groups can impede the evolutionary cooperation. We give explanations for the different roles of group density of interdependency by using the transition probabilities of C players into D players as well as the reverse. Our findings are very helpful for the understanding of emergence cooperation as well as the cooperation regulation in the selfish individuals.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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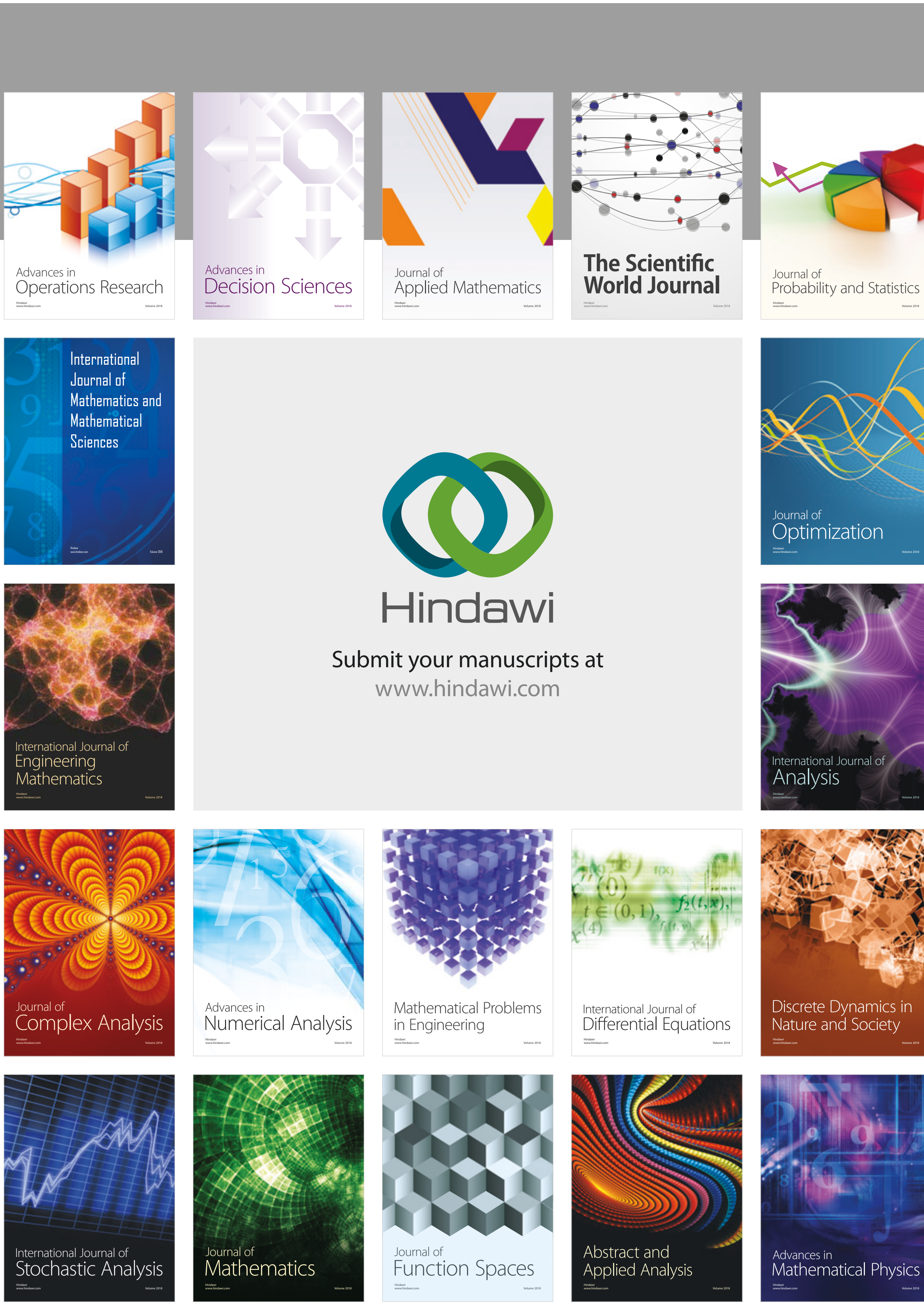
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