Hindawi
Complexity
Volume 2019, Article ID 8789438, 8 pages
https://doi.org/10.1155/2019/8789438

Research Article

New Results on the Control for a Kind of Uncertain Chaotic Systems Based on Fuzzy Logic

Bo Wang [1,2] and L. L. Chen [1]

[1] *School of Electrical Engineering and Electronic Information, Xihua University, Chengdu 610039, China*
[2] *School of Applied Mathematics, University Electronic Science and Technology of China, Chengdu 610054, China*

Correspondence should be addressed to Bo Wang; coolbie@163.com

Received 3 October 2018; Revised 7 January 2019; Accepted 26 February 2019; Published 27 March 2019

Guest Editor: Carlos-Arturo Loredo-Villalobos

Copyright © 2019 Bo Wang and L. L. Chen. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In this paper, the problem of the control for an uncertain nonlinear chaotic system has been studied; based on fuzzy logic, a kind of single-dimensional controller is constructed for the control of the chaotic systems in the situation that uncertainties and unknowns exist; at last some typical numerical simulations are carried out, and corresponding results illuminate the effectiveness of the controller.

1. Introduction

Nonlinear systems exist in real engineering widely. Since the pioneering work from Lurie in 1944, the research on nonlinear system control has become the challenging issue, and many techniques, such as differential geometry technique [1, 2], sliding mode technique [3–6] and so on, have been proposed to deal with this problem. It can be noted that these approaches are based on multidimensional control. However, in some cases, the single-dimensional controller is more cherished for its simpler structure and more convenient application in practice.

As an important branch of nonlinear systems, chaotic system and its control received many attentions, and a lot of related results have been reported so far [7–14]. For instance, in [7], based on output feedback control strategy, a method was presented to realize the control for unified chaotic systems; in [8], the synchronization control for Lü systems with unknown parameters was investigated; in [9], the adaptive control for the synchronization of hyperchaotic systems was studied; in [10], the fuzzy control for Arneodo chaotic system is discussed. However most of these researches focused on just one typical chaotic system. In addition, it is well known that there exist many kinds of uncertainties in practical control system, and the following chaotic system model is studied.

$$\begin{aligned} \dot{x}_i &= (b_i + \Delta b_i)\, x_{i+1} + f_i(\overline{x}_i), \quad 1 \le i \le n-1 \\ \dot{x}_n &= h_n(x) + \Delta h_n(x) + f_n(x) + b_n u \\ y &= x_1 \end{aligned} \tag{1}$$

where $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in \mathbf{R}^n$, $\overline{x}_i = [x_1, x_2, \cdots, x_i]^{\mathrm{T}} \in \mathbf{R}^i$, b_i are the known system parameters and satisfy $0 < b_{im} \le |b_i| \le b_{iM}$, where b_{im} and b_{iM} are the positive scalars, $f_n(x)$ and $f_i(\overline{x}_i)$ are the unknown terms, Δb_i and Δh_n are the uncertainties, h_n is the known term, b_n is the control parameter, y is the system output, and u is the single-dimensional control input. A lot of chaotic systems can be transformed into the system with the form (1) through topological mapping.

As an important technique, fuzzy techniques are very suitable for the research of nonlinear and complex systems (see [15–23] and references therein), and they will be introduced to design the single-dimensional controller for system (1) in this paper. Some simulations will be included to illuminate the effectiveness of the constructed controller.

2. Model Description and Preliminaries

It is well known that fuzzy logic system can approximate the nonlinear function. Let $f(x)$ denote the smooth function and $\varphi(x)$ denote the fuzzy logic system. There exists the optimal parameter $\theta^* = \arg\min_{\theta\in\Omega_0}[\sup_{x\in\Omega}|f(x)-\varphi(x)|]$ for the least approximation error, where Ω_0 and Ω are bounded sets of θ and x.

Define fuzzy rules as

$$\begin{aligned}&\text{IF } x_1 \text{ is } \mathrm{F}_1^j \text{ and} \ldots \text{and } x_n \text{ is } \mathrm{F}_n^j,\\ &\quad\text{then } \varphi(x) \text{ is } \mathrm{B}^j \ \left(j = 1, 2, \cdots, N\right)\end{aligned} \tag{2}$$

Define the following fuzzy logic system [16]

$$\varphi(x) = \frac{\sum_{j=1}^{N} \theta_j \prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{N} \prod_{i=1}^{n} \mu_{F_i^j}(x_i)} \tag{3}$$

where $x = [x_1, x_2, \cdots, x_n]^{\mathrm{T}} \in \mathbf{R}^n$, $\mu_{F_i^j}(x_i)$ is the fuzzy membership function, $\theta_j = \max_{\varphi(x)\in\mathbf{R}} B^j(\varphi(x))$.

Let $\xi(x) = [\xi_1(x), \xi_2(x), \cdots, \xi_N(x)]^{\mathrm{T}}$ and $\theta = [\theta_1, \theta_2, \cdots, \theta_N]^{\mathrm{T}}$; one can get $\varphi(x) = \xi^{\mathrm{T}}(x)\theta$.

Hence, if $\overline{f}_k$ is the continuous function from a compact set, $\varphi_k(\overline{x}_k)$ can approximate $\overline{f}_k$, which means that there exist $\theta_k = [\theta_1{}^k, \theta_2{}^k, \cdots, \theta_N{}^k]^{\mathrm{T}}$ and $\varepsilon_k > 0$, such that

$$\left|\overline{f}_k(\overline{x}_k) - \xi_k{}^{\mathrm{T}}(\overline{x}_k)\theta_k\right| \le \varepsilon_k, \quad k = 1, 2, \cdots, n. \tag{4}$$

where $\overline{x}_k = [x_1, x_2, \cdots, x_k]^{\mathrm{T}}, k = 1, 2, \cdots, n$.

In the paper, the following lemmas are concerned.

Lemma 1 (see [24]). *If $f(t) \in L_\infty \cap L_2$ and $\dot{f}(t) \in L_\infty$, one has*

$$\lim_{t\longrightarrow+\infty} f(t) = 0 \tag{5}$$

3. Main Results

For convenience, let $\Delta g_i(\overline{x}_{i+1}) = \Delta b_i x_{i+1}$, $1 \le i \le n-1$, and $\Delta g_n = \Delta h_n(x)$.

Step 1. Define the tracking error $e_1 = y - y_{\mathrm{d}}$; y_{d} is the desired trajectory.

For the first subsystem of system (1), the virtual variable α_1 is introduced, such that

$$\begin{aligned}\dot{e}_1 &= \dot{x}_1 - \dot{y}_{\mathrm{d}}\\ &= b_1x_2 - b_1\alpha_1 + b_1\alpha_1 + f_1(\overline{x}_1) + \Delta g_1(\overline{x}_2) - \dot{y}_{\mathrm{d}}\\ &= b_1e_2 + b_1\alpha_1 + f_1(\overline{x}_1) + \Delta g_1(\overline{x}_2) - \dot{y}_{\mathrm{d}}\end{aligned} \tag{6}$$

where $e_2 = x_2 - \alpha_1$.

Step 2. For the second subsystem of system (1), the virtual variable α_2 is introduced, such that

$$\begin{aligned}\dot{e}_2 &= \dot{x}_2 - \dot{\alpha}_1\\ &= b_2x_3 - b_2\alpha_2 + b_2\alpha_2 - \dot{\alpha}_1 + f_2(\overline{x}_2) + \Delta g_2(\overline{x}_3)\\ &= b_2e_3 + b_2\alpha_2 - \dot{\alpha}_1 + f_2(\overline{x}_2) + \Delta g_2(\overline{x}_3)\end{aligned} \tag{7}$$

where $e_3 = x_3 - \alpha_2$.

Step k ($k < n$). For k-th subsystem of system (1), the virtual variable α_k is introduced, such that

$$\begin{aligned}\dot{e}_k &= \dot{x}_k - \dot{\alpha}_{k-1}\\ &= b_kx_{k+1} - b_k\alpha_k + b_k\alpha_k - \dot{\alpha}_{k-1} + f_k(\overline{x}_k)\\ &\quad + \Delta g_k(\overline{x}_{k+1})\\ &= b_ke_{k+1} + b_k\alpha_k - \dot{\alpha}_{k-1} + f_k(\overline{x}_k) + \Delta g_k(\overline{x}_{k+1})\end{aligned} \tag{8}$$

where $e_{k+1} = x_{k+1} - \alpha_k$

Step n. For the n-th subsystem of system (1), one can get

$$\begin{aligned}\dot{e}_n &= \dot{x}_n - \dot{\alpha}_{n-1}\\ &= b_nu + h_n(x) + f_n(x) + \Delta g_n(x) - \dot{\alpha}_{n-1}\end{aligned} \tag{9}$$

where $e_n = x_n - \alpha_{n-1}$

Then, the following tracking error dynamic system can be derived

$$\begin{aligned}\dot{e}_i &= b_ie_{i+1} + b_i\alpha_i - \dot{\alpha}_{i-1} + f_i(\overline{x}_i) + \Delta g_i(\overline{x}_{i+1}(t)),\\ &\qquad 1 \le i \le n-1\\ \dot{e}_n &= -\dot{\alpha}_{n-1} + h_n(x) + f_n(x) + \Delta g_n(x) + b_nu\end{aligned} \tag{10}$$

where $\alpha_0 = y_{\mathrm{d}}$

The object of this paper is to design a controller, such that

$$\lim_{t\longrightarrow+\infty} e(t) = 0 \tag{11}$$

Choose the first Lyapunov function as

$$V_1 = \frac{1}{2b_1}e_1^2 \tag{12}$$

then

$$\begin{aligned}\dot{V}_1 &= \frac{1}{b_1}e_1\dot{e}_1 = \frac{1}{b_1}e_1(b_1e_2 + b_1\alpha_1 + f_1 + \Delta g_1 - \dot{y}_{\mathrm{d}})\\ &= e_1(e_2 + \alpha_1) + \frac{1}{b_1}e_1(f_1 + \Delta g_1 - \dot{y}_{\mathrm{d}})\\ &= e_1\left(e_2 + \alpha_1 + \overline{f}_1\right)\end{aligned} \tag{13}$$

where

$$\overline{f}_1 = \frac{f_1 + \Delta g_1 - \dot{y}_{\mathrm{d}}}{b_1}. \tag{14}$$

Let $\alpha_1 = -\lambda_1e_1 - \varphi_1$, $\lambda_1 > 0$, where $\varphi_1 = \xi_1{}^{\mathrm{T}}(\overline{x}_2)\theta_1$ is used to approximate the nonlinear function $\overline{f}_1$, then

$$\dot{V}_1 = -\lambda_1 e_1^2 + e_1 e_2 + e_1\left(\overline{f}_1 - \varphi_1\right) \tag{15}$$

Choose the second Lyapunov function as

$$V_2 = V_1 + \frac{1}{2b_2} e_2^2 \tag{16}$$

Let $\alpha_2 = -\lambda_2 e_2 - e_1 - \varphi_2$, $\lambda_2 > 0$, where $\varphi_2 = \xi_2{}^{\mathrm{T}}(\overline{x}_3)\theta_2$ is used to approximate the nonlinear function $\overline{f}_2$, then

$$\begin{aligned}
\dot{V}_2 &= \dot{V}_1 + \frac{1}{b_2} e_2 \dot{e}_2 \\
&= \dot{V}_1 + e_2\left(e_3 + \alpha_2 + \frac{f_2 + \Delta g_2 - \dot{\alpha}_1}{b_2}\right) \\
&= -\lambda_1 e_1^2 + e_2\left(e_1 + e_3 + \alpha_2 + \overline{f}_2\right) + e_1\left(\overline{f}_1 - \varphi_1\right) \\
&= -\lambda_1 e_1^2 + e_2 e_3 - \lambda_2 e_2^2 + e_1\left(\overline{f}_1 - \varphi_1\right) \\
&\quad + e_2\left(\overline{f}_2 - \varphi_2\right) \\
&= -\sum_{i=1}^{2} \lambda_i e_i^2 + \sum_{i=1}^{2} e_i\left(\overline{f}_i - \varphi_i\right) + e_2 e_3
\end{aligned} \tag{17}$$

where

$$\overline{f}_2 = \frac{f_2 + \Delta g_2 - \dot{\alpha}_1}{b_2}. \tag{18}$$

Let $\alpha_{k-1} = -\lambda_{k-1} e_{k-1} - e_{k-2} - \varphi_{k-1}$, $\lambda_{k-1} > 0$, where $\varphi_{k-1} = \xi_{k-1}{}^{\mathrm{T}}(\overline{x}_k)\theta_{k-1}$ is used to approximate the nonlinear function $\overline{f}_{k-1}$, then

$$\dot{V}_{k-1} = -\sum_{i=1}^{k-1} \lambda_i e_i^2 + \sum_{i=1}^{k-1} e_i\left(\overline{f}_i - \varphi_i\right) + e_{k-1} e_k \tag{19}$$

Choose the k-th Lyapunov function ($k < n$) as

$$V_k = V_{k-1} + \frac{1}{2b_k} e_k^2 \tag{20}$$

Hence

$$\begin{aligned}
\dot{V}_k &= \dot{V}_{k-1} + \frac{1}{b_k} e_k \dot{e}_k \\
&= \dot{V}_{k-1} + e_k\left(e_{k+1} + \alpha_k + \frac{1}{b_k}\left(f_k + \Delta g_k - \dot{\alpha}_{k-1}\right)\right) \\
&= -\sum_{i=1}^{k-1} \lambda_i e_i^2 + \sum_{i=1}^{k-1} e_i\left(\overline{f}_i - \varphi_i\right) \\
&\quad + e_k\left(e_{k+1} + \alpha_k + e_{k-1} + \frac{1}{b_k}\left(f_k + \Delta g_k - \dot{\alpha}_{k-1}\right)\right) \\
&= -\sum_{i=1}^{k-1} \lambda_i e_i^2 + \sum_{i=1}^{k-1} e_i\left(\overline{f}_i - \varphi_i\right) \\
&\quad + e_k\left(e_{k+1} + \alpha_k + e_{k-1} + \overline{f}_k\right)
\end{aligned} \tag{21}$$

where

$$\overline{f}_k = \frac{1}{b_k}\left(f_k + \Delta g_k - \dot{\alpha}_{k-1}\right). \tag{22}$$

Let $\alpha_k = -\lambda_k e_k - e_{k-1} - \varphi_k$, $\lambda_k > 0$, where $\varphi_k = \xi_k{}^{\mathrm{T}}(\overline{x}_{k+1})\theta_k$ is used to approximate the nonlinear function $\overline{f}_k$, then

$$\dot{V}_k = -\sum_{i=1}^{k} \lambda_i e_i^2 + \sum_{i=1}^{k} e_i\left(\overline{f}_i - \varphi_i\right) + e_k e_{k+1} \tag{23}$$

It is consistent with our notation that $\alpha_{n-1} = -\lambda_{n-1} e_{n-1} - e_{n-2} - \varphi_{n-1}$, $\lambda_{n-1} > 0$, where $\varphi_{n-1} = \xi_{n-1}{}^{\mathrm{T}}(\overline{x}_n)\theta_{n-1}$ is used to approximate the nonlinear function $\overline{f}_{n-1}$, then

$$\dot{V}_{n-1} = -\sum_{i=1}^{n-1} \lambda_i e_i^2 + \sum_{i=1}^{n-1} e_i\left(\overline{f}_i - \varphi_i\right) + e_{n-1} e_n \tag{24}$$

Choose the n-th Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2b_n} e_n^2 \tag{25}$$

Hence

$$\begin{aligned}
\dot{V}_n &= \dot{V}_{n-1} + e_n\left(u + \frac{1}{b_n}\left(\overline{f}_n - \dot{\alpha}_{n-1}\right)\right) \\
&= -\sum_{i=1}^{n-1} \lambda_i e_i^2 + \sum_{i=1}^{n-1} e_i\left(\overline{f}_i - \varphi_i\right) + e_{n-1} e_n \\
&\quad + e_n\left(u + \frac{1}{b_n}\left(f_n + \Delta g_n - \dot{\alpha}_{n-1}\right)\right) \\
&= -\sum_{i=1}^{n-1} \lambda_i e_i^2 + \sum_{i=1}^{n-1} e_i\left(\overline{f}_i - \varphi_i\right) \\
&\quad + e_n\left(u + e_{n-1} + \frac{1}{b_n}\left(f_n + \Delta g_n - \dot{\alpha}_{n-1}\right)\right)
\end{aligned} \tag{26}$$

where

$$\overline{f}_n = \frac{1}{b_n}\left(f_n + \Delta g_n - \dot{\alpha}_{n-1}\right) \tag{27}$$

Suppose that $\varphi_n = \xi_n{}^{\mathrm{T}}(x)\theta_n$ approximate the nonlinear function $\overline{f}_n$ and that is based on Lyapunov theory, then the following theoretical result can be obtained.

Theorem 2. *For $\lambda_n > 0$, $r_i > 0$ and $k_i > 0$, based on the controller*

$$u = -\lambda_n e_n - e_{n-1} - \frac{h_n(x)}{b_n} - \xi_n{}^{\mathrm{T}}(x)\,\theta_n \tag{28}$$

and the adaptive law

$$\dot{\theta}_i = r_i e_i \xi_i - 2k_i \theta_i, \quad i = 1, 2, \cdots, n \tag{29}$$

then the output of chaotic system (10) can track the desired trajectory.

Proof. Based on (25), construct Lyapunov function as

$$V = V_n + \sum_{i=1}^{n} \frac{1}{2r_i} \overline{\theta}_i^{\mathrm{T}} \overline{\theta}_i \tag{30}$$

where $\overline{\theta}_i = \theta_i^* - \theta_i$.

Combined with (28), it can be concluded that

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^{n} \lambda_i e_i^2 + \sum_{i=1}^{n} e_i \left(\overline{f}_i - \varphi_i\right) - \sum_{i=1}^{n} \frac{1}{r_i} \overline{\theta}_i^{\mathrm{T}} \dot{\theta}_i \\ &= -\sum_{i=1}^{n} \lambda_i e_i^2 + \sum_{i=1}^{n} e_i \left(\overline{f}_i - \theta_i^{*\mathrm{T}} \xi_i\right) \\ &\quad + \sum_{i=1}^{n} e_i \left(\theta_i^{*\mathrm{T}} \xi_i - \theta_i^{\mathrm{T}} \xi_i\right) - \sum_{i=1}^{n} \frac{1}{r_i} \overline{\theta}_i^{\mathrm{T}} \dot{\theta}_i \\ &= -\sum_{i=1}^{n} \lambda_i e_i^2 + \sum_{i=1}^{n} e_i \left(\overline{f}_i - \theta_i^{*\mathrm{T}} \xi_i\right) + \sum_{i=1}^{n} e_i \overline{\theta}_i^{\mathrm{T}} \xi_i \\ &\quad - \sum_{i=1}^{n} \frac{1}{r_i} \overline{\theta}_i^{\mathrm{T}} \dot{\theta}_i \\ &\le -\sum_{i=1}^{n} \lambda_i e_i^2 + \sum_{i=1}^{n} \overline{\theta}_i^{\mathrm{T}} \left(e_i \xi_i - \frac{1}{r_i} \dot{\theta}_i\right) + \sum_{i=1}^{n} \left|e_i \varepsilon_i\right| \end{aligned} \tag{31}$$

Define

$$S = -\sum_{i=1}^{n} \lambda_i e_i^2 + \sum_{i=1}^{n} \overline{\theta}_i^{\mathrm{T}} \left(e_i \xi_i - \frac{1}{r_i} \dot{\theta}_i\right) + \sum_{i=1}^{n} \left|e_i \varepsilon_i\right| \tag{32}$$

Supposing that $a_i = \lambda_i - 1/2$, it can be derived

$$\lambda_i = a_i + \frac{1}{2} \tag{33}$$

Hence

$$S = -\sum_{i=1}^{n} a_i e_i^2 - \frac{1}{2} \sum_{i=1}^{n} e_i^2 + \sum_{i=1}^{n} \overline{\theta}_i^{\mathrm{T}} \left(e_i \xi_i - \frac{1}{r_i} \dot{\theta}_i\right) + \sum_{i=1}^{n} \left|e_i \varepsilon_i\right| \tag{34}$$

Consider

$$-\frac{1}{2} \sum_{i=1}^{n} e_i^2 + \sum_{i=1}^{n} \left|e_i \varepsilon_i\right| \le -\frac{1}{2} \sum_{i=1}^{n} \varepsilon_i^2 \tag{35}$$

and with adaptive law (29), one can get

$$\begin{aligned} S &\le -\sum_{i=1}^{n} a_i e_i^2 + \sum_{i=1}^{n} \overline{\theta}_i^{\mathrm{T}} \left(e_i \xi_i - \frac{1}{r_i} \left(r_i e_i \xi_i - 2k_i \theta_i\right)\right) \\ &\quad + \frac{1}{2} \sum_{i=1}^{n} \varepsilon_i^2 \\ &= -\sum_{i=1}^{n} a_i e_i^2 + \sum_{i=1}^{n} \frac{2k_i}{r_i} \left(\theta_i^* - \theta_i\right)^{\mathrm{T}} \theta_i + \frac{1}{2} \sum_{i=1}^{n} \varepsilon_i^2 \\ &= -\sum_{i=1}^{n} a_i e_i^2 + \sum_{i=1}^{n} \frac{k_i}{r_i} \left(2\theta_i^{*\mathrm{T}} \theta_i - 2\theta_i^{\mathrm{T}} \theta_i\right) + \frac{1}{2} \sum_{i=1}^{n} \varepsilon_i^2 \end{aligned} \tag{36}$$

Consider

$$\theta_i^{*\mathrm{T}} \theta_i^* + \theta_i^{\mathrm{T}} \theta_i \ge 2\theta_i^{*\mathrm{T}} \theta_i \tag{37}$$

Then

$$2\theta_i^{*\mathrm{T}} \theta_i - 2\theta_i^{\mathrm{T}} \theta_i \le \theta_i^{*\mathrm{T}} \theta_i^* - \theta_i^{\mathrm{T}} \theta_i \tag{38}$$

One can derive

$$\begin{aligned} \dot{V} &\le -\sum_{i=1}^{n} a_i e_i^2 + \sum_{i=1}^{n} \frac{k_i}{r_i} \left(-\theta_i^{\mathrm{T}} \theta_i + \theta_i^{*\mathrm{T}} \theta_i^*\right) + \frac{1}{2} \sum_{i=1}^{n} \varepsilon_i^2 \\ &= -\sum_{i=1}^{n} a_i e_i^2 + \sum_{i=1}^{n} \frac{k_i}{r_i} \left(-\theta_i^{\mathrm{T}} \theta_i - \theta_i^{*\mathrm{T}} \theta_i^*\right) + \sum_{i=1}^{n} \frac{2k_i}{r_i} \theta_i^{*\mathrm{T}} \theta_i^* \\ &\quad + \frac{1}{2} \sum_{i=1}^{n} \varepsilon_i^2 \end{aligned} \tag{39}$$

Consider

$$\begin{aligned} \overline{\theta}_i^{\mathrm{T}} \overline{\theta}_i &= \left(\theta_i^* - \theta_i\right)^{\mathrm{T}} \left(\theta_i^* - \theta_i\right) = \theta_i^{*\mathrm{T}} \theta_i^* - 2\theta_i^{*\mathrm{T}} \theta_i + \theta_i^{\mathrm{T}} \theta_i \\ &\le 2\theta_i^{*\mathrm{T}} \theta_i^* + 2\theta_i^{\mathrm{T}} \theta_i \end{aligned} \tag{40}$$

Then

$$-\frac{1}{2} \overline{\theta}_i^{\mathrm{T}} \overline{\theta}_i \ge -\theta_i^{\mathrm{T}} \theta_i - \theta_i^{*\mathrm{T}} \theta_i^* \tag{41}$$

One can derive

$$\begin{aligned} \dot{V} &\le -\sum_{i=1}^{n} a_i e_i^2 - \sum_{i=1}^{n} \frac{k_i}{2r_i} \overline{\theta}_i^{\mathrm{T}} \overline{\theta}_i + \sum_{i=1}^{n} \frac{2k_i}{r_i} \theta_i^{*\mathrm{T}} \theta_i^* + \frac{1}{2} \sum_{i=1}^{n} \varepsilon_i^2 \\ &\le -\sum_{i=1}^{n} a_i \frac{2b_{im}}{2b_i} e_i^2 - \sum_{i=1}^{n} \frac{k_i}{2r_i} \overline{\theta}_i^{\mathrm{T}} \overline{\theta}_i + \sum_{i=1}^{n} \frac{2k_i}{r_i} \theta_i^{*\mathrm{T}} \theta_i^* \\ &\quad + \frac{1}{2} \sum_{i=1}^{n} \varepsilon_i^2 \end{aligned} \tag{42}$$

Choosing $\lambda_i > 1/2$, one can obtain $a_i > 0$.

Let

$$\begin{aligned} a_0 &= \min \left\{2b_{im} a_i, k_i : i = 1, 2, \cdots, n\right\} \\ b_0 &= \sum_{i=1}^{n} \frac{2k_i}{r_i} \theta_i^{*\mathrm{T}} \theta_i^* + \frac{1}{2} \sum_{i=1}^{n} \varepsilon_i^2 \end{aligned} \tag{43}$$

Then

$$\dot{V} \le -a_0 \left(\sum_{i=1}^{n} \frac{1}{2b_i} e_i^2 + \sum_{i=1}^{n} \frac{1}{2r_i} \overline{\theta}_i^{\mathrm{T}} \overline{\theta}_i\right) + b_0 = -a_0 V + b_0 \tag{44}$$

The solution of differential equation $\dot{V} = -a_0 V + b_0$ is

$$\begin{aligned} V(t) &= V(0) \exp\left(-a_0 t\right) \\ &\quad + b_0 \exp\left(-a_0 t\right) \frac{\exp\left(a_0 t\right) - 1}{a_0} \end{aligned} \tag{45}$$

Considering (44), it can be derived that

$$V(t) \leq \left(V(0) - \frac{b_0}{a_0}\right)\exp(-a_0 t) + \frac{b_0}{a_0} \leq V(0)\exp(-a_0 t) + \frac{b_0}{a_0} \leq V(0) + \frac{b_0}{a_0} \quad (46)$$

Define

$$d_0 = \min\{a_i : i = 1, 2, \cdots, n\}. \quad (47)$$

From (42), one can obtain

$$\begin{aligned} \dot{V} &\leq -\sum_{i=1}^{n} a_i e_i^2 - \sum_{i=1}^{n} \frac{k_i}{2r_i}\bar{\theta}_i^{\mathrm{T}}\bar{\theta}_i + \sum_{i=1}^{n}\frac{2k_i}{r_i}\theta_i^{*\mathrm{T}}\theta_i^* + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 \\ &\leq -\min\{a_i\}\sum_{i=1}^{n} e_i^2 + \sum_{i=1}^{n}\frac{2k_i}{r_i}\theta_i^{*\mathrm{T}}\theta_i^* + \frac{1}{2}\sum_{i=1}^{n}\varepsilon_i^2 \\ &= -d_0\sum_{i=1}^{n} e_i^2 + b_0 \end{aligned} \quad (48)$$

Hence, when $\|e\| > (b_0/d_0)^{1/2}$, one can get $\dot{V} < 0$, which means $e(t) \in L_\infty$.

Integrating both sides of inequality (48) from 0 to T, one can get

$$\int_0^T \dot{V}(t)\,dt \leq -\int_0^T d_0\sum_{i=1}^{n} e_i^2(s)\,ds + Tb_0 \quad (49)$$

Consider

$$\int_0^T \dot{V}(t)\,dt = V(T) - V(0) \quad (50)$$

One can get

$$V(T) - V(0) \leq -d_0\sum_{i=1}^{n}\int_0^T e_i^2(s)\,ds + Tb_0 \quad (51)$$

Hence

$$\sum_{i=1}^{n}\int_0^T e_i^2(s)\,ds \leq \frac{1}{d_0}\left(V(0) - V(T) + Tb_0\right) \quad (52)$$

which means $e(t) \in L_2$. From error dynamic system (10), it can be concluded that $\dot{e}(t) \in L_\infty$. Accordingly based on Lemma 1, one can get $\lim_{t\longrightarrow+\infty} e(t) = 0$, which means the achievement of the track control. The proof of Theorem 2 is thus completed. □

4. Numerical Simulation

First the following uncertain Arneodo system is considered.

$$\begin{aligned} \dot{x}_1 &= b_1 x_2 + f_1 \\ \dot{x}_2 &= (b_2 + \Delta b_2)x_3 + f_2 \\ \dot{x}_3 &= h_3 + \Delta h_3 + f_3 + b_3 u \end{aligned} \quad (53)$$

where

$$\begin{aligned} b_1 &= 1, \\ f_1 &= 0.3\sin x_1, \\ b_2 &= 1, \\ \Delta b_2 &= 0.02, \\ f_2 &= 0.1\cos(x_1 x_2), \\ h_3 &= c_3 x_1{}^3 - c_0 x_1 - c_1 x_2 - c_2 x_3, \\ \Delta h_3 &= 0.1 x_1, \\ f_3 &= 0.2\cos(x_1)\sin(x_3), \\ b_3 &= 5, \\ c_0 &= -5.4, \\ c_1 &= 3.5, \\ c_2 &= 1, \\ c_3 &= -1, \\ k_1 &= 1, \\ k_2 &= 1.5, \\ k_3 &= 1.5, \\ r_1 &= 1.5, \\ r_2 &= 2, \\ r_3 &= 2, \\ \lambda_1 &= 2.5, \\ \lambda_2 &= 5, \\ \lambda_3 &= 5, \\ \alpha_1 &= -\lambda_1(x_1 - y_d) + \dot{y}_d, \\ \alpha_2 &= -\lambda_2(x_2 - \alpha_1) - (x_1 - y_d) - \xi_2^{\mathrm{T}}(\overline{x}_2)\theta, \\ u &= -\lambda_3(x_3 - \alpha_2) - (x_2 - \alpha_1) - \xi_3^{\mathrm{T}}(\overline{x}_3)\theta, \end{aligned}$$

$$\xi_{1j}(\overline{x}_1) = \frac{\mu F_1^j(x_1)}{\sum_{j=1}^{9}\mu F_1^j(x_1)},$$

$$\xi_{2j}(\overline{x}_2) = \frac{\mu F_1^j(x_1)\mu F_2^j(x_2)}{\sum_{j=1}^{9}\mu F_1^j(x_1)\mu F_2^j(x_2)},$$

$$\xi_{3j}(x) = \frac{\mu F_1^j(x_1)\mu F_2^j(x_2)\mu F_3^j(x_3)}{\sum_{j=1}^{9}\mu F_1^j(x_1)\mu F_2^j(x_2)\mu F_3^j(x_3)},$$

$$\begin{aligned} \mu F_i^1(x_i) &= \exp\left(-0.5(x_i + 2)^2\right], \\ \mu F_i^2(x_i) &= \exp\left(-0.5(x_i + 1.5)^2\right], \\ \mu F_i^3(x_i) &= \exp\left(-0.5(x_i + 1)^2\right], \\ \mu F_i^4(x_i) &= \exp\left(-0.5(x_i + 0.5)^2\right], \\ \mu F_i^5(x_i) &= \exp\left(-0.5x_i^2)\right], \\ \mu F_i^6(x_i) &= \exp\left(-0.5(x_i - 0.5)^2\right], \\ \mu F_i^7(x_i) &= \exp\left(-0.5(x_i - 1)^2\right], \\ \mu F_i^8(x_i) &= \exp\left(-0.5(x_i - 1.5)^2\right], \end{aligned}$$

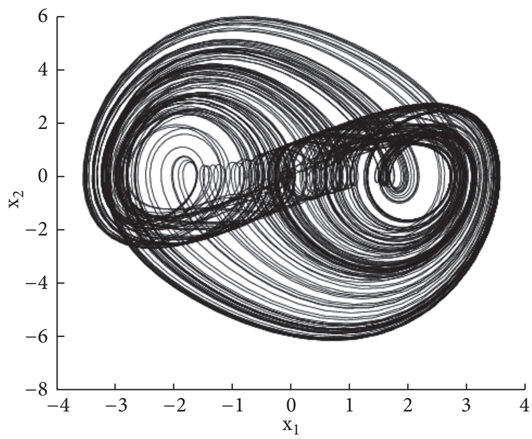

FIGURE 1: Chaotic attractor of Arneodo system.

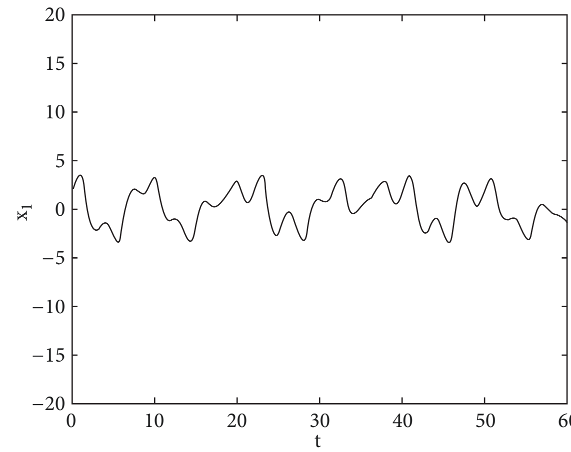

FIGURE 2: State response of x_1 of Arneodo system.

$$\mu F_i^9(x_i) = \exp\left(-0.5(x_i - 2)^2\right),$$
$$\xi_1(\overline{x}_1) = [\xi_{11}(\overline{x}_1), \xi_{12}(\overline{x}_1), \cdots, \xi_{19}(\overline{x}_1)]^T,$$
$$\xi_2(\overline{x}_2) = [\xi_{21}(\overline{x}_2), \xi_{22}(\overline{x}_2), \cdots, \xi_{29}(\overline{x}_2)]^T,$$
$$\xi_3(\overline{x}_3) = [\xi_{31}(\overline{x}_3), \xi_{32}(\overline{x}_3), \cdots, \xi_{39}(\overline{x}_3)]^T. \quad (54)$$

Let desired trajectory $y_d = \sin 2\pi t$, initial value $x(0) = [2, 0, 0]^T$, and the simulation results are displayed in Figures 1–7.

Remark 3. Figure 1 displays the chaotic attractor of Arneodo system. Figure 2 displays the state response of x_1 of Arneodo system. From Figures 1 and 2, it can be seen that Arneodo system has the complicated dynamical behavior. Figure 3 displays the state response of variable x_1 of uncertain Arneodo system. It can be seen that the existence of unknowns and uncertainties makes Arneodo system unstable.

Remark 4. Figure 4 displays the fuzzy membership function. Figure 5 displays the state response of control input. Figure 6 displays the state response of y_d and y. Figure 7 displays the state response of position tracking error. From Figures 4–7, it can be seen that for uncertain Arneodo system, the position tracking can be achieved during 0.5 second based on the designed controller.

5. Conclusion

In this paper, based on fuzzy logic, a single-dimensional controller has been constructed for the control of a kind of uncertain chaotic systems. Some typical examples have been employed and corresponding simulation results have illuminated the effectiveness of proposed controller.

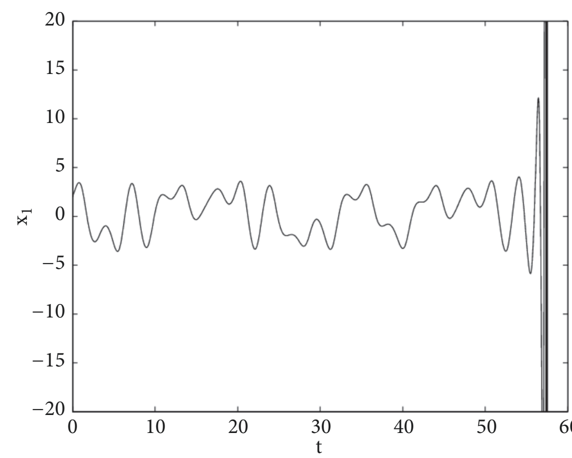

FIGURE 3: State response of x_1 of uncertain Arneodo system.

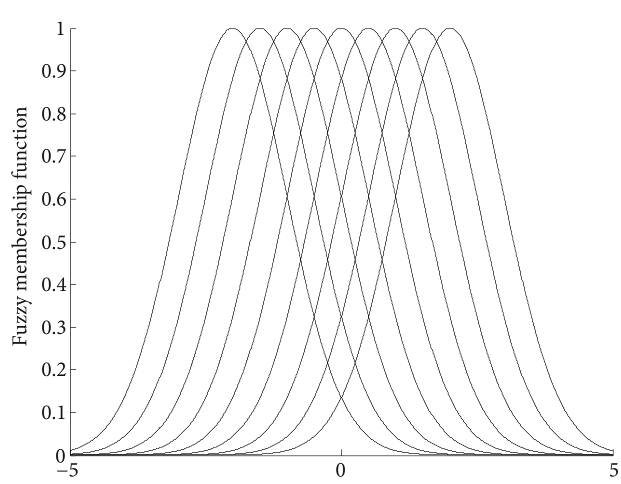

FIGURE 4: Fuzzy membership function.

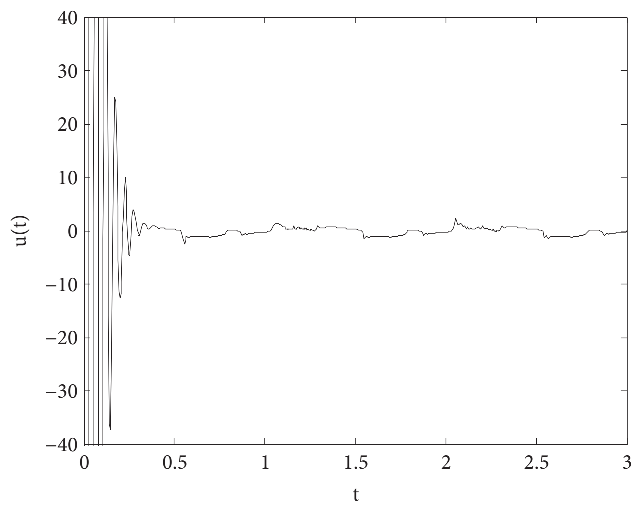

Figure 5: State response of control input.

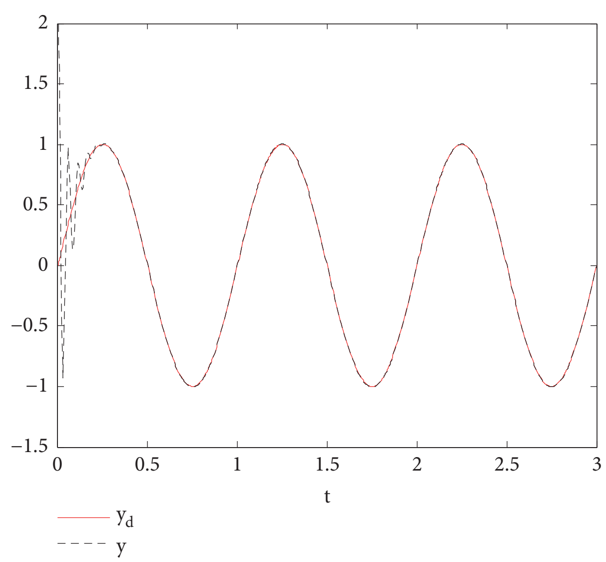

Figure 6: State response of y_d and y.

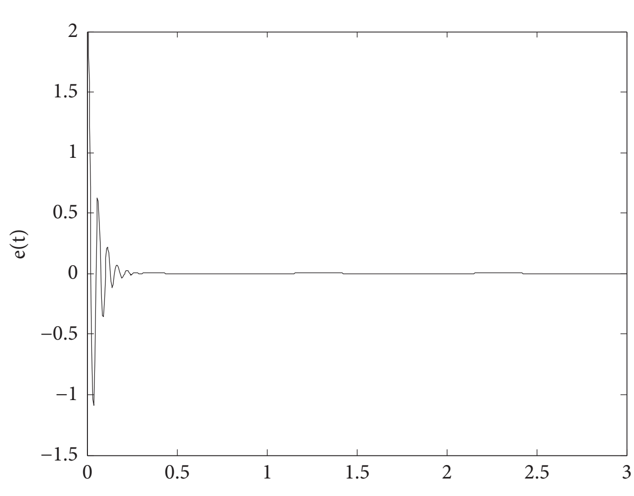

Figure 7: State response of position tracking error.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The work is supported by National Natural Science Foundation of China (11472297, 51475453) and Key Laboratory of Fluid and Power Machinery of the Ministry of Education.

References

[1] T.-L. Chien, C.-C. Chen, and C.-Y. Hsu, "Tracking control of nonlinear automobile idle-speed time-delay system via differential geometry approach," *Journal of The Franklin Institute*, vol. 342, no. 7, pp. 760–775, 2005.

[2] R. Yu and Y. Xie, "Investigation on cubic chua's circuit via differential geometry," *Physics Procedia*, vol. 24, pp. 412–417, 2012.

[3] A. Levant and M. Livne, "Weighted homogeneity and robustness of sliding mode control," *Automatica*, vol. 72, pp. 186–193, 2016.

[4] Z. Liu, C. Gao, and Y. Kao, "Robust H-infinity control for a kind of neutral-type systems via sliding mode observer," *Applied Mathematics and Computation*, vol. 271, pp. 669–681, 2015.

[5] J. Hua, L.-X. An, and Y.-M. Li, "Bionic fuzzy sliding mode control and robustness analysis," *Applied Mathematical Modelling: Simulation and Computation for Engineering and Environmental Systems*, vol. 39, no. 15, pp. 4482–4493, 2015.

[6] J. Xu, M. Wang, and L. Qiao, "Dynamical sliding mode control for the trajectory tracking of underactuated unmanned underwater vehicles," *Ocean Engineering*, vol. 105, pp. 54–63, 2015.

[7] Z. Shen and J. Li, "Chaos control for a unified chaotic system using output feedback controllers," *Mathematics and Computers in Simulation*, vol. 132, pp. 208–219, 2017.

[8] H. Tirandaz and A. Hajipour, "Adaptive synchronization and anti-synchronization of TSUCS and Lü unified chaotic systems with unknown parameters," *Optik - International Journal for Light and Electron Optics*, vol. 130, pp. 543–549, 2017.

[9] B. Wang, P. Shi, H. R. Karimi, Y. Song, and J. Wang, "Robust H-infinity synchronization of a hyper-chaotic system with disturbance," *Nonlinear Analysis: Real World Applications*, vol. 14, no. 3, pp. 1487–1495, 2013.

[10] W. Wang and Y. Fan, "Synchronization of Arneodo chaotic system via backstepping fuzzy adaptive control," *Optik - International Journal for Light and Electron Optics*, vol. 126, no. 20, pp. 2679–2683, 2015.

[11] B. Wang, L. Chen, and Y. Liu, "New results on contrast enhancement for infrared images," *Optik - International Journal for Light and Electron Optics*, vol. 178, pp. 1264–1269, 2019.

[12] C. Ge, H. Wang, Y. Liu, and J. H. Park, "Stabilization of chaotic systems under variable sampling and state quantized controller," *Fuzzy Sets and Systems*, vol. 344, pp. 129–144, 2018.

[13] B. Wang, L. Chen, and Z. Zhang, “A novel method on the edge detection of infrared image,” *Optik - International Journal for Light and Electron Optics*, vol. 180, pp. 610–614, 2019.

[14] B. Wang, X. Zhang, and X. Dong, “Novel secure communication based on chaos synchronization,” *IEICE Transactions on Fundamentals of Electronics, Communications and Computer Sciences*, vol. E101A, no. 7, pp. 1132–1135, 2018.

[15] Y. Zheng, “Fuzzy prediction-based feedback control of fractional order chaotic systems,” *Optik - International Journal for Light and Electron Optics*, vol. 126, no. 24, pp. 5645–5649, 2015.

[16] B. Wang and L. L. Chen, “New results on fuzzy synchronization for a kind of disturbed memristive chaotic system,” *Complexity*, vol. 2018, Article ID 3079108, 9 pages, 2018.

[17] B. Wang, J. Cheng, and S. M. Zhong, “Bounded input bounded output stability for Lurie system with time-varying delay,” *Advances in Difference Equations*, Paper No. 57, 13 pages, 2018.

[18] B. Wang, J. Cheng, and F. Zou, “Stochastic finite-time H∞ filtering for nonlinear Markovian jump systems with partly known transition probabilities,” *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 233, no. 1, pp. 31–43, 2018.

[19] G. Lai, Z. Liu, Y. Zhang, and C. L. Philip Chen, “Adaptive fuzzy tracking control of nonlinear systems with asymmetric actuator backlash based on a new smooth inverse,” *IEEE Transactions on Cybernetics*, vol. 46, no. 6, pp. 1250–1262, 2016.

[20] B. Wang, “Results on a novel piecewise-linear memristor-based chaotic system,” *Complexity*, vol. 2019, Article ID 8948656, 6 pages, 2019.

[21] V. Nazari and B. Surgenor, “Improved position tracking performance of a pneumatic actuator using a fuzzy logic controller with velocity, system lag and friction compensation,” *International Journal of Control, Automation, and Systems*, vol. 14, no. 5, pp. 1376–1388, 2016.

[22] B. Wang, D. Zhang, J. Cheng, and J. H. Park, “Fuzzy model-based nonfragile control of switched discrete-time systems,” *Nonlinear Dynamics*, vol. 93, no. 4, pp. 2461–2471, 2018.

[23] L. X. Wang, *Adaptive Fuzzy Systems and Control*, Prentice Hall, Englewood Cliffs, NJ, USA, 1994.

[24] K. Narendra and A. Annaswamy, *Stable Adaptive Systems*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1989.


Advances in
Operations Research
Advances in
Decision Sciences
Journal of
Applied Mathematics
The Scientific
World Journal
Journal of
Probability and Statistics
International
Journal of
Mathematics and
Mathematical
Sciences
Hindawi
Submit your manuscripts at
www.hindawi.com
Journal of
Optimization
International Journal of
Engineering
Mathematics
International Journal of
Analysis
Journal of
Complex Analysis
Advances in
Numerical Analysis
Mathematical Problems
in Engineering
International Journal of
Differential Equations
Discrete Dynamics in
Nature and Society
International Journal of
Stochastic Analysis
Journal of
Mathematics
Journal of
Function Spaces
Abstract and
Applied Analysis
Advances in
Mathematical Physics