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Research Article

Results on a Novel Piecewise-Linear Memristor-Based Chaotic System

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The paper studies a kind of piecewise-linear memristor-based chaotic system. Based on a novel and complicated piecewise-linear memristor model, a chaotic system is constructed; then a random sequence extraction approach on the basis of the given memristor-based chaotic system is designed; at last the random sequence test is carried out to show the potential application value in encryption field of the new memristor-based chaotic system.

1. Introduction

Chaos phenomenon in circuit was first reported by Leon O. Chua at Waseda University in 1983. Since then, chaos gets a number of attentions from scholars for its potential application in chaotic synchronization [1, 2], secret communication [3, 4], image's encryption [5, 6], and so on.

In 1971, Chua first proposed the concept of memristor [7], which was regarded as the fourth element in electronic circuits. Although the technique to produce the real memristor is not mature so far, memristor model can be used to construct chaotic circuit system. At present, a number of investigations on memristor-based chaotic system have been carried out [8–12]. For example, the chaotic circuit with one memristor is studied in [13, 14], the chaotic circuit with two memristors is concerned in [15, 16], the integer order chaotic memristor circuit is investigated in [17, 18], and the fractional chaotic memristor circuit is researched in [19, 20]. However for the existing piecewise-linear memristive chaotic circuits, the nonlinear device usually adopts Chua's piecewise-linear memristor model, and there are seldom other kinds of piecewise-linear memristor model for selection.

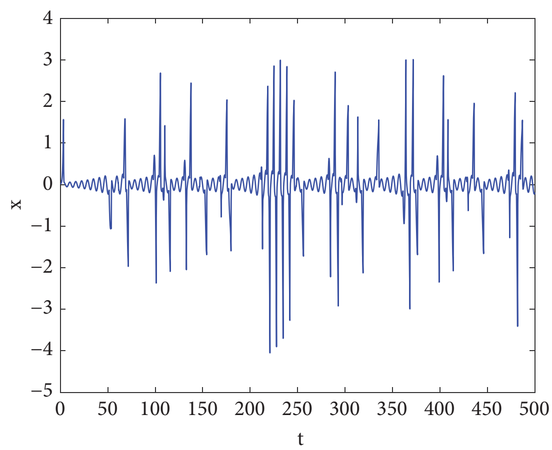
Random number generator is one important application field of chaotic system [21, 22], and how to generate a random sequence with better performance has always been a challenging issue. Although some approaches to extract random sequence based on memristive chaotic system have been proposed up to now, corresponding random sequences are easy to analyze by attackers with the well-known prior knowledge of memristive chaotic system. Hence it is meaningful to build the memristive chaotic system on the basis of the new and complicated memristor model. All these motive our research.

The organization of the paper is as follows: a chaotic system based on a novel piecewise-linear memristor model will be proposed in Section 2, and corresponding analysis will also be provided; then the random sequence test will be carried out in Section 3; the conclusion will be presented in Section 4.

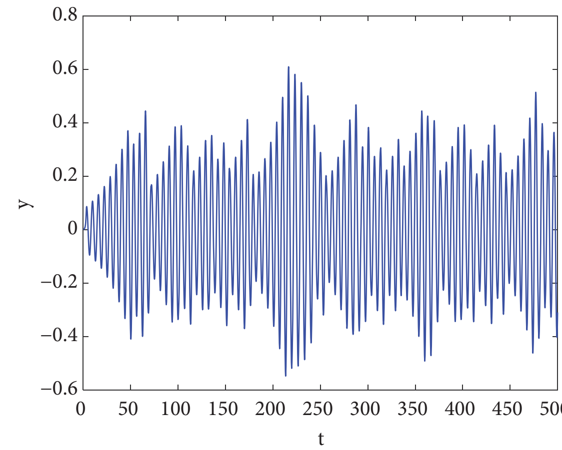
2. Memristor-Based Chaotic System

In this section, a novel memristor-based system is proposed as follows:

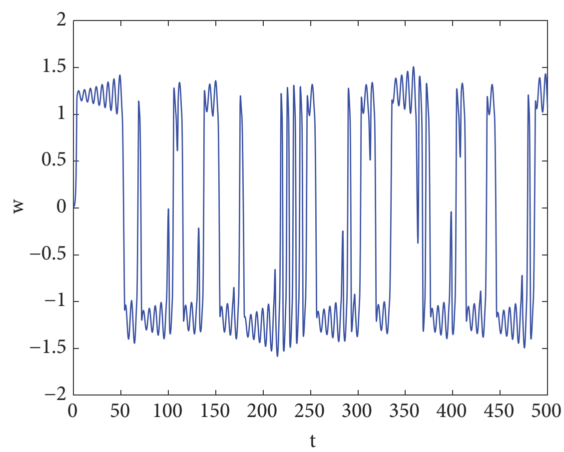
$$\begin{aligned} \dot{x} &= \alpha\left(y - x - \overline{W}(w)\,x\right) \\ \dot{y} &= \beta x + (\kappa + \gamma - \beta)\,y - z \\ \dot{z} &= \kappa y - z \\ \dot{w} &= x \end{aligned} \tag{1}$$



(a) Time response of state variable x(t)



(b) Time response of state variable y(t)



(c) Time response of state variable w(t)

FIGURE 1: Time response of state variables of the memristor-based system.

where $\overline{W}(w)$ is the constructed memristor model, which satisfies

$$\overline{W}(w) = \begin{cases} A, & |w| \le m,\ |x| \le n \\ B, & |w| > m,\ |x| \le n \\ C, & |w| \le m,\ |x| > n \\ D, & |w| > m,\ |x| > n \end{cases} \tag{2}$$

When $\alpha = 4.7, \beta = 0.061, \gamma = 1, \kappa = 2.1, A = -1.29, B = 0.75, C = -1.17, D = 0.93, m = 1, n = 1.56$, initial value is $[0.01, 0, 0, 0.01]$, and the system dynamics are displayed in Figures 1 and 2.

Remark 1. Figure 1(a) displays the time response of state variable x(t). Figure 1(b) displays the time response of state variable y(t). Figure 1(c) displays the time response of state variable w(t). It can be seen that the state variables of the memristor-based system look chaotic and disorganized visually.

Remark 2. Figure 2(a) displays the phase diagram of y-w. Figure 2(b) displays the phase diagram of x-w. Figure 2(c) displays the phase diagram of x-z. It can be seen that the state variables of the memristor-based system keep moving in a certain attractor and do not converge to a point or diverge to infinity.

Firstly, consider the system dissipativity:

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} \tag{3}$$

When $|w| \le m, |x| \le n$, one can get

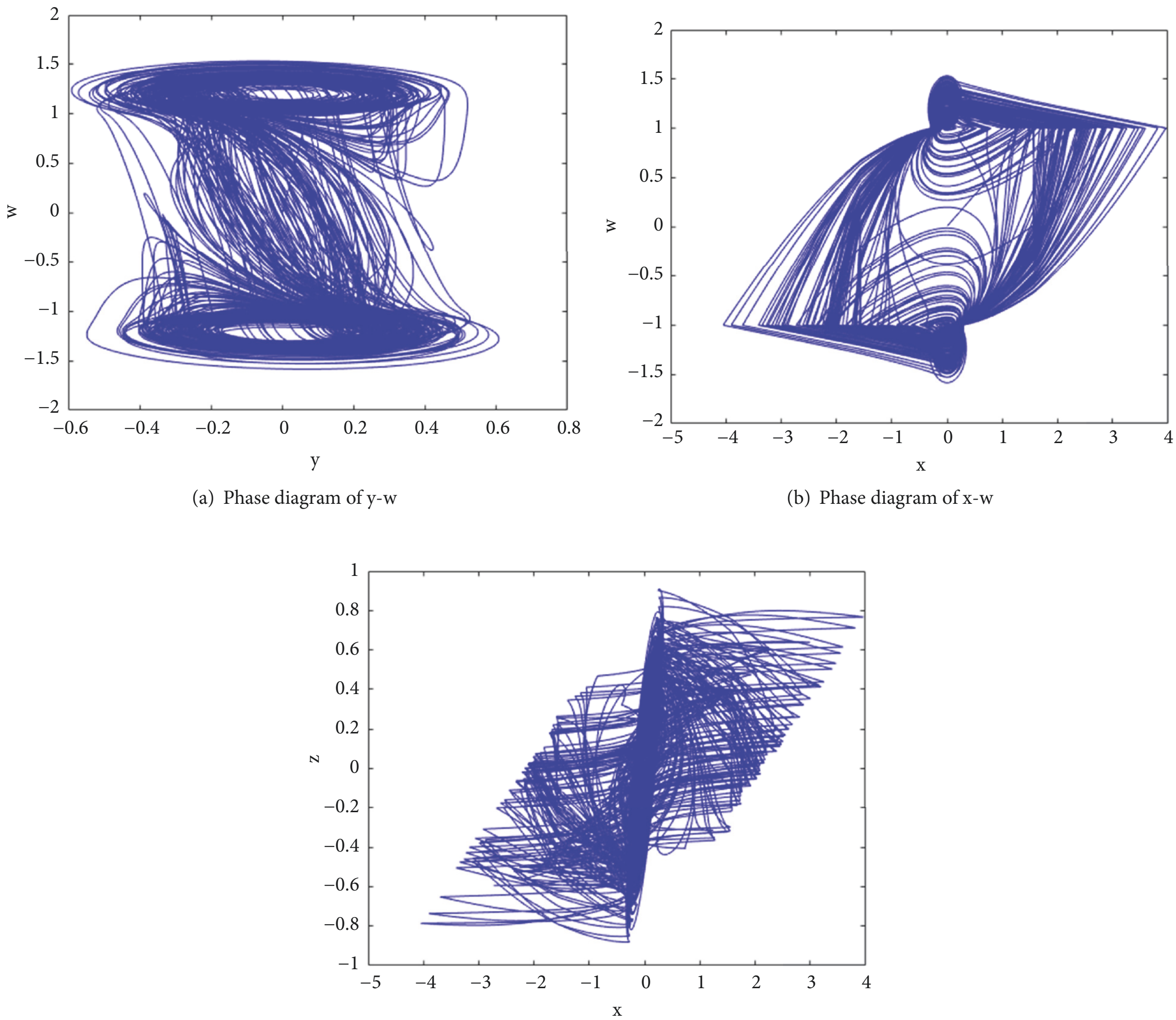
$$\nabla V = -1.37 \tag{4}$$

When $|w| > m, |x| \le n$, one can get

$$\nabla V = -3.41 \tag{5}$$

When $|w| \le m, |x| > n$, one can get

$$\nabla V = -1.49 \tag{6}$$



(a) Phase diagram of y-w

(b) Phase diagram of x-w

(c) Phase diagram of x-z

FIGURE 2: Phase diagram of state variables of the memristor-based system.

When $|w| > m, |x| > n$, one can get

$$\nabla V = -3.59 \tag{7}$$

From the above it can be concluded that $\nabla V < 0$, which means that the memristor-based system is dissipative.

Next, by adjusting parameter α, consider the dynamics of the given memristor-based system. The simulations are displayed in Figure 3.

Remark 3. Figure 3(a) displays the Lyapunov exponent diagram of the memristor-based system. Figure 3(b) displays the bifurcation diagram of the memristor-based system. It can be seen that the given memristor-based system is in the chaotic state when $\alpha \in [3.5, 7]$.

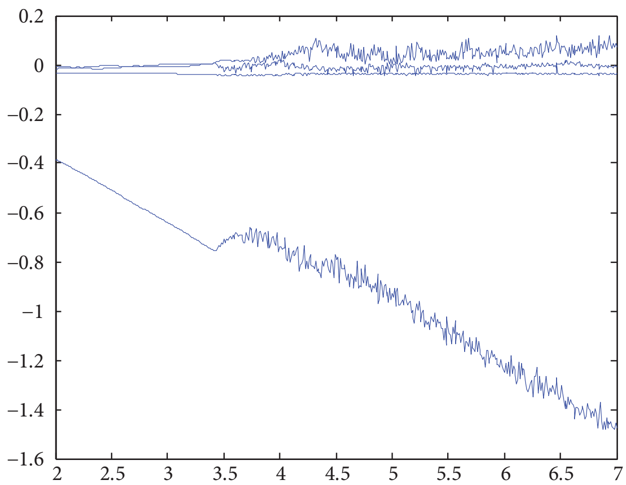
3. Random Sequence

In this section, we design a new random sequence extraction approach on the basis of the given memristor-based system as follows.

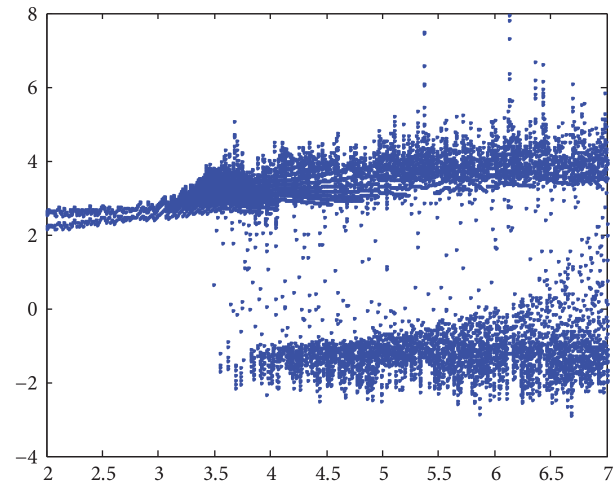
Step 1. Let X=[x,y,z,w]. Set the initial variable of the given memristor-based chaotic system X_0=[0.01, 0, 0, 0.01], the threshold value m=4, and the sequence label j=0.

Step 2. Carry out iterations 1000 times based on the given memristor-based chaotic system, corresponding system state variable is denoted as X(j), and set the sequence label j=1.

Step 3. Carry out an iterative operation based on the given memristor-based chaotic system, and set the sequence label j=j+1.



(a) Lyapunov exponent diagram



(b) Bifurcation diagram

FIGURE 3: Dynamics of given memristor-based system with different a.

TABLE 1: NIST test on random sequences.

Chaos Systems	New system		Chua's system [23]	
Statistical test	P value	Proportion	P value	Proportion
Frequency	0.9735	0.9900	0.7981	0.9800
Block Frequency	0.7362	1.0000	0.6786	1.0000
Cumulative Sums	0.5478	0.9900	0.1222	0.9800
Runs	0.9176	1.0000	0.8343	0.9900
Longest Run	0.5261	0.9900	0.2896	0.9800
Rank	0.6858	1.0000	0.4372	1.0000
Approximate Entropy	0.5247	0.9800	0.1916	0.9800
Serial	0.8653	1.0000	0.5955	1.0000
Linear Complexity	0.4275	0.9900	0.0668	0.9700

Step 4. Determine i:

i = REM (z(j),2),

where REM (.) is the remainder operation.

Step 5. Determine $P_{t1}(j)$:

when $w_i(j) > 4$, $P_{t1}(j)=1$;

otherwise, $P_{t1}(j)=0$;

where $w_i(j)$ denotes the value of ith decimal place of w(j).

Step 6. Determine the mapping variable M(j) according to i:

if i=0, M(j)=x(j);

if i=1, M(j)=y(j).

Step 7. Determine $P_{s1}(j)$:

when $M_i(j) > 4$,$P_{s1}(j)=1$;

otherwise, $P_{s1}(j)=0$.

where $M_i(j)$ denotes the value of ith decimal place of $M_i(j)$.

Step 8. Determine P_j

P_j = XOR ($P_{t1}(j)$, $P_{s1}(j)$);

where XOR (.) is the exclusive-or operation.

Step 9. Carry out circulation from Step 3 to Step 8 10 million times; one can obtain the binary random sequence:

$P = \{P_1, P_2, \ldots, P_N\}$

where j=N, and N=10000000 is the required sequence length.

Set the sampling time T as 0.001s, and $h = 5, i = 4$. Extract a binary random sequence with 10 million bits, which is divided into 100 groups and put into a software test kit, NIST, and the NIST test results of random sequences are displayed in Table 1.

Remark 4. Table 1 displays NIST test results of random sequences. The significance level of each test in NIST is set to 0.01, and P value ⩾ 0.01 means that the sequence would be random with a confidence of 0.99. It can be noted that P values of all tests on the proposed sequence are all bigger than 0.01, which means our sequence can be regarded as random. In addition, it can be seen that, compared with Chua's system [23], the sequence based on the new memristor-based chaotic system has bigger P value in Frequency test, Block Frequency test, Cumulative Sums test, Runs test, Longest Run test,

Table 2: Statistic test on sequences based on Beker and Piper method.

	Frequency test	Serial test	Poker test			Run test
			m=1	m=2	m=3	
Chua's system [23]	0.23	1.07	0.73	3.24	5.03	1.14
New system	0.08	0.76	0.56	2.87	4.35	0.52
Threshold	3.84	5.99	7.81	14.07	26.00	1.96

Rank test, Approximate Entropy test, Serial test, and Linear Complexity test; hence the proposed sequence has the better overall random statistic characteristics.

Next, the statistic test on 100000 bits of sequence based on Beker and Piper method [24] is carried out. Corresponding test results are shown in Table 2.

Remark 5. Table 2 shows the statistic test results of sequences based on Beker and Piper method. Frequency test, Serial test, Poker test, and Runs test are included in Beker and Piper approach to measure the relative frequencies of '0' and '1' in a section of sequence. The confidence level is set as 0.95, and threshold is introduced to determine the confidence of test results. It can be seen that all results are less than the preset threshold, which means that all tests are passed successfully, and the sequences can be regarded as random signal. In addition, it can be seen that the numerical values of Frequency test, Serial test, Poker test, and Runs test based on the proposed sequence are less than the ones based on Chua's system. More importantly, the proposed sequence is on the basis of chaotic system with the novel piecewise-linear memristor model, which will increase the security of corresponding cryptosystem for its unusual mechanism.

4. Conclusion

In this paper, based on a proposed complicated piecewise-linear memristor model, a new chaotic system has been constructed; then an extraction method on the basis of the given memristor-based system has been designed to obtain the random sequence; finally the random sequence test is carried out to show the potential application value of the new memristor-based chaotic system in encryption field.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declares that there are no conflicts of interests regarding the publication of this paper.

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References

[1] H. Handa and B. B. Sharma, "Novel adaptive feedback synchronization scheme for a class of chaotic systems with and without parametric uncertainty," *Chaos, Solitons & Fractals*, vol. 86, pp. 50–63, 2016.

[2] B. Wang and L. L. Chen, "New Results on Fuzzy Synchronization for a Kind of Disturbed Memristive Chaotic System," *Complexity*, vol. 2018, Article ID 3079108, p. 9, 2018.

[3] H. Huijberts, H. Nijmeijer, and R. Willems, "System identification in communication with chaotic systems," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 47, no. 6, pp. 800–808, 2000.

[4] F. Wang, X. Wang, C.-G. Yuan, and J. Ma, "Generating a prion with bacterially expressed recombinant prion protein," *Science*, vol. 327, no. 5969, pp. 1132–1135, 2010.

[5] D. S. Laiphrakpam and M. S. Khumanthem, "Cryptanalysis of symmetric key image encryption using chaotic Rossler system," *Optik - International Journal for Light and Electron Optics*, vol. 135, pp. 200–209, 2017.

[6] B. Wang, L. L. Chen, and J. Cheng, "New result on maximum entropy threshold image segmentation based on P system," *Optik - International Journal for Light and Electron Optics*, vol. 163, pp. 81–85, 2018.

[7] L. O. Chua, "Memristor—the missing circuit element," *IEEE Transactions on Circuit Theory*, vol. 18, no. 5, pp. 507–519, 1971.

[8] B. Wang, J. Cheng, and F. Zou, "Stochastic finite-time H∞ filtering for nonlinear Markovian jump systems with partly known transition probabilities," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, 2018.

[9] Y. Babacan and F. Kaçar, "Memristor emulator with spike-timing-dependent-plasticity," *AEÜ - International Journal of Electronics and Communications*, vol. 73, pp. 16–22, 2017.

[10] B. Wang, L. Chen, and Y. Liu, "New results on contrast enhancement for infrared images," *Optik - International Journal for Light and Electron Optics*, vol. 178, pp. 1264–1269, 2019.

[11] J. Wu, L. Wang, G. Chen, and S. Duan, "A memristive chaotic system with heart-shaped attractors and its implementation," *Chaos, Solitons & Fractals*, vol. 92, pp. 20–29, 2016.

[12] B. Wang, D. Zhang, J. Cheng, and J. H. Park, "Fuzzy model-based nonfragile control of switched discrete-time systems," *Nonlinear Dynamics*, vol. 93, no. 4, pp. 2461–2471, 2018.

[13] H. H. Iu, D. S. Yu, A. L. Fitch, V. Sreeram, and H. Chen, "Controlling chaos in a memristor based circuit using a twin-T notch filter," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 58, no. 6, pp. 1337–1344, 2011.

[14] B. Muthuswamy, "Implementing memristor based chaotic circuits," *International Journal of Bifurcation and Chaos*, vol. 20, no. 5, pp. 1335–1350, 2010.

[15] R. K. Budhathoki, M. P. Sah, C. Yang, H. Kim, and L. Chua, "Transient behaviors of multiple memristor circuits based on flux charge relationship," *International Journal of Bifurcation and Chaos*, vol. 24, no. 2, 1430006, 14 pages, 2014.

[16] A. Buscarino, L. Fortuna, M. Frasca, and L. V. Gambuzza, "A chaotic circuit based on Hewlett-Packard memristor," *Chaos: An Interdisciplinary Journal of Nonlinear Science*, vol. 22, no. 2, Article ID 023136, 2012.

[17] R. Riaza, "First order mem-circuits: modeling, nonlinear oscillations and bifurcations," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 6, pp. 1570–1583, 2013.

[18] A. Buscarino, L. Fortuna, M. Frasca, and L. V. Gambuzza, "A gallery of chaotic oscillators based on HP memristor," *International Journal of Bifurcation and Chaos*, vol. 23, no. 5, Article ID 1330015, 2013.

[19] I. Petráš, "Fractional-order memristor-based Chua's circuit," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 57, no. 12, pp. 975–979, 2010.

[20] L. Teng, H. H. C. Iu, and X. Wang, "Chaotic behavior in fractional-order memristor-based simplest chaotic circuit using fourth degree polynomial," *Nonlinear Dynamics*, vol. 77, no. 1-2, pp. 231–241, 2014.

[21] H. Hu, L. Liu, and N. Ding, "Pseudorandom sequence generator based on the Chen chaotic system," *Computer Physics Communications*, vol. 184, no. 3, pp. 765–768, 2013.

[22] F. Özkaynak and S. Yavuz, "Security problems for a pseudorandom sequence generator based on the Chen chaotic system," *Computer Physics Communications*, vol. 184, no. 9, pp. 2178–2181, 2013.

[23] T. Matsumoto, "A Chaotic Attractor from Chua's Circuit," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 31, no. 12, pp. 1055–1058, 1984.

[24] H. J. Beker and F. C. Piper, Eds., *Cryptography and coding*, vol. 20 of *The Institute of Mathematics and its Applications Conference Series. New Series*, The Clarendon Press, Oxford University Press, New York, 1989.