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Research Article

Optimal Policies for the Pricing and Replenishment of Fashion Apparel considering the Effect of Fashion Level

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Fashion apparel, with short product lifecycles and highly volatile demand, requires careful attention during both the initial ordering periods before the selling season and during the selling season, with its decisions regarding price and replenishment. Using Pontryagin's maximum principle method, this study investigates the problem of the dynamic pricing strategy and replenishment cycle for fashion apparel by considering the effect of fashion level on demand. First, we provide a framework for fashion apparel by formulating a model that includes both price and demand at different fashion levels. We then provide an algorithm to derive the optimal dynamic pricing strategy and replenishment cycle. Numerical examples and sensitivity analyses of the main system parameters are provided to demonstrate the obtained results, which form the basis for managerial insights. It is shown that the apparel retailer has three types of optimal dynamic pricing strategies and that the optimal strategy is independent of the replenishment cycle. The apparel retailer is able to realize the profit advantage of a continuously variable price policy by adjusting the sales price periodically.

1. Introduction

Fast fashion is an industrial practice widely applied in fashion retailing. For example, famous international retail brands such as H&M, Top Shop, and Zara have all implemented fast fashion [1]. The sales of fashion garments are not only influenced by price, but also by their fashion level, which largely depends on the elements of style, color, and material. Fashion products are quite different from other traditional products. In general, the higher the fashion level, the higher their market value. However, fashion level drops over time, resulting in a continuous decline in market value, a rapid reduction of demand, and product backlogs. The latest data showed that the fashion industry faces excess inventory. For example, H&M, the famous fast fashion brand company, announced at the end of March 2018 that the total value of its unsold accumulation of clothes was $4 billion. In 2016, the total inventory of 79 textile and apparel quoted companies in China amounted to $81 billion.

In the fashion retail industry, inventory planning decisions are crucial. Having a short product lifecycle and highly volatile demand, fashion items require careful attention in the initial ordering process before selling season and in the decisions relating to price and replenishment during the selling season [2]. Some apparel firms apply dynamic pricing strategies based on adjusting prices during the selling season to promote demand, lower inventory levels, and realize greater profit. [3]. Such strategies involve charging high prices when the fashion apparel's value is relatively high and, later, when the value decreases, the items are sold at a discount price. For example, Uniqlo, GAP, Nike, and Zara use dynamic pricing strategies, which affect consumers' selection. Uniqlo, a Japanese fashion retailer, successfully implemented these strategies, thus demonstrating that it is better to reduce prices at the beginning of the year and sell its apparel at a 60% discount in a season than to raise prices by 30% at the end of the year [3].

However, the abovementioned pricing strategy ignores certain attributes of the fashion apparel industry. Over time, the market value of fashion apparel continuously decreases. In recent years, due to the rapid evolution of information technologies, it has become increasingly easier to implement dynamic pricing. Therefore, apparel retailers should reexamine their pricing policies and explore different means of

implementing dynamic pricing, making them more responsive to consumer demand. According to the time-varying characteristics of fashion clothing, how to implement a time-varying dynamic pricing strategy has obvious practical significance for managing inventory replenishment and improving net profits. In this light, this study aims to investigate the joint optimal dynamic pricing and replenishment policies of a fashion apparel retailer by considering fashion level.

Nevertheless, few research efforts have explicitly considered the inventory decisions of fashion apparel from the perspective of pricing and replenishment cycles. Fisher et al. [4] studied how replenishment order quantity may minimize the cost of lost sales, back orders, and obsolete inventory in a two-stage model. Kogan and Spiegel [5] developed a model with time and price-dependent demand and an initial inventory level. Tsao [6] developed an analytical model for the inventory control of deteriorating fashion goods under the conditions of trade credit and partial backlogs. While they identified the optimal pricing and replenishment strategy, price remained a static variable. Due to the market value of fashion apparel decreasing over time, the abovementioned pricing models are not fit for the fashion apparel industry. Fashion apparel has a short lifecycle and products show changes in market value similar to instantaneous deteriorating items. Here, deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility, or the loss of a commodity's marginal value, all of which decrease usefulness. Most physical goods, such as medicine, volatile liquids, and fashion goods, undergo a process of decay or deterioration [7]. Therefore, decision-making regarding the joint dynamic pricing strategy and replenishment cycle for deteriorating items can be used as a reference for inventory management in the fashion apparel industry.

For firms, managing the inventories of deteriorating items represents an important challenge that has understandably attracted widespread attention from both researchers and practitioners. Ghare and Schrader [8] were the first to establish an economic order quantity model to study the optimal ordering policy for deteriorating items. Wu et al. [9] formulated an inventory model with deteriorating items and price-sensitive demand and obtained the retailer's optimal selling price and the length of replenishment cycle. As demand for deteriorating items is affected by both price and time that remains before a product's expiration, it is essential to model these simultaneous effects and establish a joint optimal pricing and inventory replenishment policy. However, few researchers have explicitly considered the dependency of demand on both price and time. Maihami et al. [10] established joint pricing and inventory control for deteriorating items, with demand dependent on time and price and with partial backlogging allowed. Avinadav et al. [11] also used a price- and time-dependent demand function to develop a mathematical model that calculates the optimal price and replenishment period of perishable items. Li et al. [12] considered the problem of joint pricing, inventories, and investment preservation by considering the general price-dependent demand rate. In the abovementioned inventory models, price remains static over the time horizon.

In recent years, due to the rapid evolution of information technologies, dynamic pricing has become increasingly viable. Consequently, firms' optimal dynamic pricing strategy has attracted scholarly attention. Gallego and Ryzin [13] formulated an inventory model to investigate the problem of dynamic pricing in which demand was price sensitive and stochastic. There is abundant research on pricing models that explicitly address dynamic pricing [14–21]. However, the abovementioned models rarely regard pricing strategy as a process of dynamic change. Wang et al. [22] developed an inventory model to identify the optimal dynamic pricing strategy for noninstantaneous deteriorating items. Li et al. [23] considered the problem of dynamic pricing and periodic ordering for deteriorating items with a stochastic inventory level that depends on stock-dependent demand and the selling price. Herbon et al. [24] formulated an inventory model with deteriorating items and price-sensitive demand and obtained the retailer's optimal dynamic selling price and the length of replenishment cycle. Tashakkor et al. [25] addressed the variability of the deterioration rate in a joint dynamic pricing and replenishment cycle problem, where the deterioration rate is an increasing step function. Zhang et al. [26] used Pontryagin's maximum principle to obtain both the optimal dynamic pricing strategy and the replenishment cycle for noninstantaneous deteriorating items, with demand dependent on a store's sales price and the quantity of the items displayed.

The study by Tsao [6] is the only one that analyzed the joint pricing and replenishment cycle decision-making problem for fashion goods. The studies that are closest to ours are [6, 26]. Our work mainly differs from theirs in three aspects. First, in their approach, the quantity of deteriorating fashion goods continuously decreases during the replenishment period [6, 26]. In contrast, in our study, while fashion apparel's market value decreases over time its quantity does not. Second, in our study, the fashion level decreases over time and exerts a negative effect on the demand rate. This factor, however, is not considered in the problem of pricing fashion goods in Tsao's model [6]. In our model of fashion apparel, we assume that the demand rate depends on both sales price and fashion level, which is a more realistic assumption. To the best of our knowledge, our study is the first to operate under this assumption when analyzing the joint optimal dynamic pricing strategy and replenishment cycle for fashion apparel. Finally, in Tsao's model price was a static variable [6]. In this study on fashion apparel, we use optimal control theory to identify the optimal dynamic pricing strategy and replenishment cycle that maximizes average profit per unit time. This optimization problem can be solved hierarchically, such that, at a lower level of the hierarchy, we derive the dynamic pricing strategy for the given replenishment cycle by applying Pontryagin's maximum principle. Then, at a higher level of the hierarchy, we derive the optimal replenishment cycle for the given pricing strategy. Furthermore, we design an algorithm to derive the optimal dynamic pricing strategy and replenishment cycle. Numerical examples and sensitivity analyses of main system parameters are provided to demonstrate the obtained results, and some managerial insights are offered.

Table 1: Notations and descriptions.

variables	
T	length of replenishment cycle
$p(t)$	selling price per unit
$p_{\max}(t)$	reservation price
$\omega(t)$	fashion level of apparel
$I(t)$	inventory level at time t
$MR(t)$	marginal profit
Q	order quantity
D	demand rate
π	average profit per unit time
parameters	
$T_{\max}$	maximum sales period
a	initial demand rate
b	price parameter
c	purchasing cost per unit
h	inventory holding cost per unit
η	deterioration rate of the fashion level
A	ordering cost per order

The study includes six sections. In Section 2, the notations and assumptions of the problem are described. Section 3 provides the theoretical results of the dynamic pricing strategies, and Section 4 provides the theoretical results of the fixed price strategies. Numerical results and sensitivity analyses are represented in Section 5. Finally, Section 6 offers a conclusion and discusses implications for management.

2. Modeling Assumptions

2.1. Problem Formulation. Consider a revenue-maximizing retailer that orders and sells fashion apparel in a finite replenishment cycle $[0, T]$, where the replenishment cycle T needs to be determined. In addition to the common decision variables of when and how much to order, we also consider selling price as a decision variable; this price is dynamically determined throughout the replenishment period.

The dynamic pricing strategy and the fixed price strategy are both commonly followed in the fashion apparel industry. We make a performance comparison (including average profit and price trajectory) of these two pricing strategies by undertaking a numerical analysis. The related notations used throughout this study are detailed in Table 1.

2.2. Demand Function. In this study, we formulate the demand function based on the following assumptions.

Assumption 1. The demand function $D(p(t))$ explicitly depends separately on both the apparel's selling price and fashion level. According to the additive combination of functions $d(p(t))$ and $\omega(t)$, respectively, the demand function can be expressed as follows:

$$D(p(t)) = d(p(t)) + \omega(t) \tag{1}$$

The function $d(p(t))$ is strictly decreasing and differentiable with respect to $p(t)$. We note that the dependence of demand is explicit in dynamic pricing and implicit in the time factor. Just as the fashion level of clothing deteriorates over time, so too is demand implicitly affected by fashion level. In general, when $p(t)$ tends to infinity, demand is zero, i.e., $\lim_{p(t)\longrightarrow\infty} D(p(t)) = 0$, $\forall t > 0$, and $\omega(t) = -\lim_{p(t)\longrightarrow\infty} d(p(t))$. Thus, $\omega(t)$ is a constant. This contradicts Assumption 1, which requires the existence of a time effect on fashion level. A consistent definition of $D(p(t))$ depends on an additional assumption regarding the fashion level of apparel, which has the property of decreasing over time.

Assumption 2. For fashion apparel, there exists a consumer's reservation price, $p_{\max}(t)$, defined as the lowest price at t, where the demand of fashion clothing is zero. In other words, if the selling price is higher than the reservation price, the demand rate must vanish.

The reservation price satisfies the following conditions:

(i) The reservation price is higher than purchasing cost, $p_{\max}(0) > c$, and $p_{\max}(t)$ continuously decreases over $0 \le t \le T_{\max}$;

(ii) $p_{\max}(t) = 0$ for $t \ge T_{\max}$ and $\partial p_{\max}(t)\partial t < 0$. It should be noted that the reservation price decreases with time t because the market value of fashion apparel decreases with time;

(iii) $d(p_{\max}(t)) + \omega(t) = 0$, $0 \le t \le T_{\max}$, which means that if the clothing price is higher than the reservation price during the sales period, consumers do not buy anything. From $d(p_{\max}(t)) + \omega(t) = 0$, we have

$$p_{\max}(t) = d^{-1}(\omega(t)) \tag{2}$$

The reservation price is the inverse function of fashion level. Assumption 2 resolves the inconsistency between $D(p(t)) = d(p(t)) + \omega(t)$ and $\lim_{p(t)\longrightarrow\infty} D(p(t)) = 0$, $\forall t > 0$.

From the above assumptions, the demand function of the dynamic scenario is as follows:

$$D(p(t)) = \begin{cases} d(p(t)) + \omega(t), & 0 \le t \le T_{\max},\ p(t) \le p_{\max}(t) \\ 0 & otherwise \end{cases} \tag{3}$$

Assumption 3. The demand function of the static scenario is similar to the dynamic pricing strategy. Also, this demand function depends separately on both the apparel's selling price and fashion level, but the price does not change over time: $D(p, t) = d(p) + \omega(t)$, where $d(p)$ represents the demand rate under a fixed price and $\omega(t)$ is the fashion level that is affected by the time factor.

2.3. Fashion Level Function. From the above assumptions, the demand function depends on the price and fashion level of clothing. To illustrate the effect of fashion level on demand, we offer the following proposition.

Proposition 4. *The fashion level function $\omega(t)$ is strictly decreasing with t.*

From Proposition 4, we can see that the fashion level function $\omega(t)$ is strictly decreasing with time t. Therefore, we can deduce that demand decreases with time t, which is

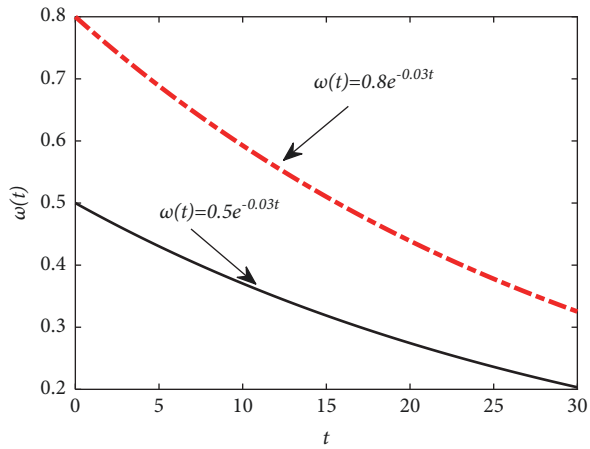

FIGURE 1: The effect of the initial fashion level on fashion level.

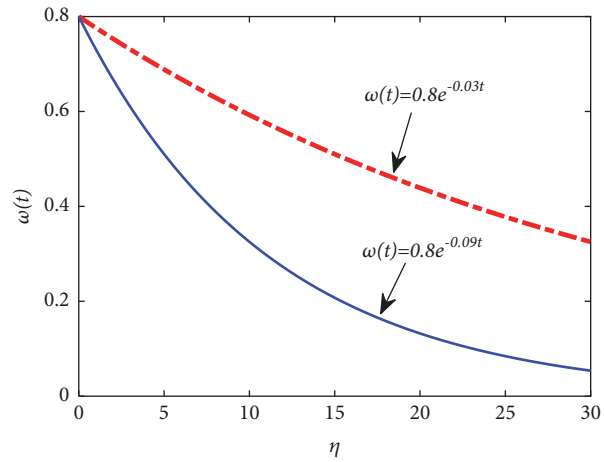

FIGURE 2: The effect of the deterioration rate on fashion level.

consistent with the actual operation of garment enterprises, so the assumption of the demand function in (3) is reasonable.

To further describe the effect of fashion level on consumer demand, we assume that fashion level can be expressed as follows:

$$\omega(t) = \omega_0 e^{-\eta t} \tag{4}$$

Here, ω_0 ($\omega_0 > 0$) is the listed clothing's initial fashion level. Also, η ($0 < \eta < 1$) is the fashion level's deterioration rate. If ω_0 is constant, the fashion level $\omega(t)$ decreases exponentially over time t. This model results from assuming that the fashion level of clothing is a state variable with the negative exponential distribution having the parameter η. This characterizes the fashion level's attenuation property with time t.

Considering the effect of the initial fashion level ω_0 on the fashion level $\omega(t)$, as shown in Figure 1, we can see that $\omega(t)$ decreases as time t increases. The higher the initial fashion level ω_0, the slower the attenuation of fashion level $\omega(t)$ at time t. Therefore, it is obvious that the initial fashion level has a delayed effect. In practice, to satisfy consumers' individualized demand for fashion products, apparel retailers often try their best to consider consumer psychology and enhance consumers' feelings, specifically by providing them with service experiences such as trial-wear periods, fashion displays, and clothes matching, thus raising the initial fashion level.

Similarly, the fashion level's deterioration rate has an effect on fashion level $\omega(t)$. As shown in Figure 2, the smaller the deterioration rate η, the slower the lowering of fashion level $\omega(t)$ because the fashion level largely depends on the elements of style, color, and material, all of which influence the deterioration rate. Therefore, suppliers should increase the investment in the creative design of fashion elements to make η smaller and raise the fashion level of clothing.

3. The Optimal Strategies under the Dynamic Scenario

3.1. The Objective Function. In the replenishment period, the apparel retailer's cost includes inventory holding costs and the ordering cost per replenishment. Thus, those costs are defined as $C_1 = h\int_0^T I(p(t),t)dt$, and $C_2 = c\int_0^T D(p(t),t)dt + A$, respectively, where h is the inventory holding cost per unit, c is the purchasing cost per unit, and A is ordering cost per order.

The sales revenue of the apparel retailer is $SR = \int_0^T p(t)D(p(t),t)dt$. Therefore, the total profit of the retailer per replenishment period, represented by Π_1, is given by

$$\begin{aligned}\Pi_1(p(t),T) &= SR - C_1 - C_2 \\ &= \int_0^T (p(t)-c)\,D(p(t))\,dt \\ &\quad - h\int_0^T I(p(t),t)\,dt - A\end{aligned} \tag{5}$$

The average profit per unit time, represented by π_1, is given by

$$\pi_1(p(t),T) = \frac{1}{T}\Pi_1(p(t),T) \tag{6}$$

The goal of this study is to determine the optimal dynamic pricing strategy and replenishment cycle while maximizing average profit per unit time, which can be described by the following optimization problem:

$$\begin{aligned}&\max_{p(t),T} \quad \pi_1(p(t),T) \\ &s.t. \quad \begin{cases}\dfrac{\partial I(t)}{\partial t} = -D(p(t),t), & 0 \le t \le T \\ I(T) = 0, & 0 \le p \le p_{\max}(t)\end{cases}\end{aligned} \tag{7}$$

In this study, note that the average profit per unit time is the objective function because if the objective function is the total profit of a period $\Pi_1(p(t),T)$ and the decision variable T is the upper bound of the integral, which should theoretically be a large T to maximize the objective function. However, this is contrary to the actual operation of garment enterprises, which is characterized by a shorter selling lifecycle and by

having a longer replenishment cycle T, leading to extremely high inventory costs. Therefore, by considering the average profit per replenishment period, we not only compare the operational performance of the same clothing enterprise across different sales periods, but we also compare the operational performance of different enterprises in the same sales period. Similar studies are available [24–28].

The optimization problem of formula (7) can be solved hierarchically. First, the optimal price $p^*(t)$ is obtained for a given replenishment period T. Then, by substituting the optimal price into the objective function, the optimal replenishment period T^* is obtained. This analysis of the hierarchical sequence covers all possible combinations of the two decision variables, so the optimal solution of the objective function can be obtained as follows:

$$\max_{p(t),T} \quad \pi_1\left(p\left(t\right),T\right) = \max_{T\le T_{\max}}\left\{\max_{p(t)\le p_{\max}(t)} \pi_1\left(p\left(t\right),T\right)\right\} \tag{8}$$

3.2. The Optimal Dynamic Price Strategy Given the Replenishment Period. Based on the hierarchical level discussed above, we first consider a given replenishment period, T, and then identify a dynamic price function $p(t)$ that maximizes the retailer's average profit in the period. The objective function is $\max_{p(t)}\{\pi_1(p(t),T)\} = (1/T)\max_{p(t)}\Pi_1(p(t))$ because T is a constant, and then the optimization problem presented in (7) is converted into another optimization problem as follows:

$$\begin{aligned}
&\max_{p(t)} \quad \Pi_1\left(p\left(t\right)\right) \\
&\qquad = \max_{p(t)} \int_0^T \left(\left(p\left(t\right)-c\right)D\left(p\left(t\right),t\right)-hI\left(t\right)\right)dt \\
&\qquad\quad - A \\
&s.t. \quad \begin{cases} \dot{I}\left(t\right) = -D\left(p\left(t\right),t\right), & 0\le t\le T \\ I\left(T\right)=0, & 0\le p\le p_{\max}\left(t\right)\end{cases}
\end{aligned} \tag{9}$$

For the given replenishment cycle T, to obtain the dynamic pricing strategy, we use Pontryagin's maximum principle stated in [29] to solve the optimal control problem expressed in (9). The Hamiltonian function $H(p(t), I(t), \lambda(t), t)$ is formulated as follows:

$$\begin{aligned}
&H\left(p\left(t\right),I\left(t\right),\lambda\left(t\right),t\right) \\
&\quad = \left(p\left(t\right)-c\right)D\left(p\left(t\right)\right)-hI\left(t\right)-\lambda\left(t\right)D\left(p\left(t\right)\right)
\end{aligned} \tag{10}$$

where $\lambda(t)$ is the costate variable. Also, $\lambda(t)$ denotes the marginal value or shadow price of the inventory level. The maximum principle states that the necessary conditions for $p^*(t)$ with the corresponding state trajectory $I^*(t)$ to be an optimal control are the existence of the continuous and piecewise continuously differentiable function $\lambda(t)$ such that the following conditions hold. By applying the necessary conditions of the maximum principle, we obtain the following:

$$\begin{aligned}
\dot{\lambda}\left(t\right) &= -\frac{\partial H}{\partial I} \\
\dot{I}\left(t\right) &= -\frac{\partial H}{\partial \lambda} \\
\lambda\left(0\right) &= 0 \\
\frac{\partial H}{\partial p} &= 0
\end{aligned} \tag{11}$$

According to (10) and (11), the optimal dynamic price can be obtained as follows:

$$p^*\left(t\right) = \begin{cases} \psi\left(t\right), & 0\le\psi\left(t\right)\le p_{\max}\left(t\right) \\ p_{\max}\left(t\right), & \psi\left(t\right)\ge p_{\max}\left(t\right)\end{cases} \tag{12}$$

where $\psi(t)$ is the implicit function that satisfies the following:

$$d'\left(\psi\left(t\right)\right)\left(\psi\left(t\right)-c-ht\right)+D\left(\psi\left(t\right)\right)=0 \tag{13}$$

The partial derivative of the second order is obtained from the Hamilton function H:

$$\begin{aligned}
\frac{\partial^2 H}{\partial p^2} &= 2d'\left(p^*\left(t\right)\right)+\left(p^*\left(t\right)-c-ht\right)d''\left(p^*\left(t\right)\right) \\
&\le 0
\end{aligned} \tag{14}$$

It is shown from (14) that $p^*(t)$ is the optimal price. According to the formula (12) and (13), we can see that the expression $p^*(t)$ does not contain T. That is, the optimal price does not depend on the length of replenishment. Given T, the dependence of optimal price on time t can be equivalently reformulated into the dependence on the time remaining until replenishment, $T-t$. This property is intuitive, and the apparel retailer can decide on the optimal price according to how much time is left before the next replenishment period.

As discussed above, the optimal dynamic pricing and the reservation price vary with time. The next proposition compares the optimal dynamic pricing $p^*(t)$ with the reservation price $p_{\max}(t)$ and the marginal cost denoted by MC, where $MC(t) = c + ht$.

Proposition 5. *There exists a unique replenishment time denoted by T_1 such that*

(i) $MC(t) < p^*(t) < p_{\max}(t)$ *for* $0 \le t < T_1$;
(ii) $MC(t) = p^*(t) = p_{\max}(t)$ *for* $t = T_1$;
(iii) $MC(t) > p^*(t) = p_{\max}(t)$ *for* $t \ge T_1$.

Proposition 5 provides an important insight; namely, the optimal dynamic price is always less than the reservation price, but the optimal dynamic price may be either more than or less than the marginal cost when considering the fashion level of apparel and the reservation price of consumers. In

particular, if the reservation price is equal to the marginal cost, i.e., $p_{\max}(T_1) = c + hT_1$, the replenishment cycle T_1 can be obtained. Note that the specific case of the functions described in Proposition 5 is schematically plotted in Figure 3.

In the following sections, we specifically investigate the optimal dynamic price path. With the influence of fashion level, the optimal dynamic price of apparel has different paths of change. Let the demand function $d(p(t))$ be linear, i.e., $d(p(t)) = a - bp(t)$, $(a > 0\ b > 0)$. Substituting $d(p(t))$ into the $d(p_{\max}(t)) + \omega(t) = 0$ in Assumption 2, we obtain the reservation price:

$$p_{\max}(t) = \frac{1}{b}(a + \omega(t)) \tag{15}$$

Then substituting $d(p(t))$ into (13), we have $-b(p^*(t) - c - ht) + a - bp^*(t) + \omega(t) = 0$. Thus, the equivalent form of (12) is obtained:

$$p^*(t) = \begin{cases} \frac{1}{2}(p_{\max}(t) + c + ht), & 0 \le t \le T_1 \\ p_{\max}(t), & t > T_1 \end{cases} \tag{16}$$

Consider that the second derivative of the Hamilton function H to p is $\partial^2 H/\partial p^2 = -2b < 0$, which illustrates that $p^*(t)$ is optimal. Equation (16) shows that the optimal price $p^*(t)$ monotonically increases in h. Next, we differentiate both sides of (16) with respect to t, which yields the following:

$$\frac{\partial p^*(t)}{\partial t} = \frac{b}{2}(\omega'(t) + bh), \quad \text{for } 0 \le t \le T_1. \tag{17}$$

The next proposition illustrates the effect of fashion level on the optimal dynamic price path.

Proposition 6. *In terms of the property of the fashion level function $\omega(t) = \omega_0 e^{-\eta t}$ being convex, i.e., $\omega''(t) > 0$, the optimal price $p^*(t)$ has three possible paths:*

(i) *$p^*(t)$ monotonically decreases in time t for $0 \le t < T_1$, $\omega'(t) < -bh$;*

(ii) *$p^*(t)$ monotonically increases in time t for $0 \le t < T_1$, $\omega'(t) > -bh$;*

(iii) *$p^*(t)$ first decreases and then increases for $0 \le t < T_1$.*

Note that the specific case of optimal dynamic price described in Proposition 6 is schematically plotted in Figure 3. We take case (iii) of Proposition 6 as an example because in the beginning $\omega'(t) < -bh$, $p^*(t)$ decreased, until $\omega'(t) = -bh$, and $p^*(t)$ then increased with the increase of the derivative of the fashion level function $\omega'(t)$, the red curve shown in Figure 3. Figure 3 also shows that the optimal dynamic price $p^*(t)$ consists of two parts, and the first part is $p^*(t) = \psi(t)$ for $0 < t < T_1$. The second part is $p^*(t) = p_{\max}(t)$, and when $t = T_1$ the optimal dynamic price begins to switch to another path, i.e., $p^*(t) = p_{\max}(t)$ for $t \ge T_1$, the green curve shown in Figure 3.

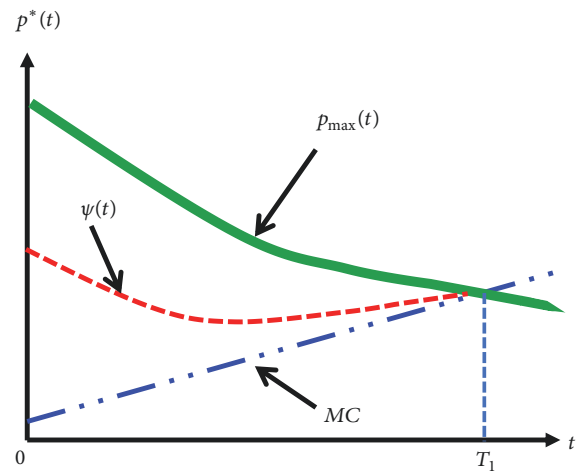

Figure 3: The optimal dynamic price paths.

3.3. The Optimal Replenishment Period Given Dynamic Price. The problem at the higher hierarchical level of the decomposition discussed above is stated as follows: given the optimal dynamic price function, $p^*(t)$, maximize the average profit under the replenishment period, T, as a single decision variable. Then the optimization problem presented in (9) is converted into another optimization problem as follows:

$$\begin{aligned} \max_T \quad & \pi_1(p^*(t), T) \\ s.t. \quad & T \le T_{\max} \end{aligned} \tag{18}$$

In Section 3.2, we obtain that there exists a unique time T_1, such that beyond that time point the optimal price equals the reservation price $p_{\max}(t)$, and there can be no demand. Therefore, the optimal replenishment time must be no later than T_1, i.e., $T^* < T_1$.

In addition, $MC(t)$ is the apparel retailer's marginal cost at time t. Now we can define instantaneous marginal profit, MR, as the rate of contribution to profit at time t exclusive of fixed cost:

$$MR(t) = p^*(t) - MC(t) \tag{19}$$

According to (19), when $t = T$, the retailer's marginal profit is $MR(T) = p^*(T) - M(T)$.

Proposition 7. *Given the optimal dynamic price $p^*(t)$, if $\pi_1(p^*(t), T_2) > 0$, the optimal replenishment period $T^* = \min\{T_{\max}, T_2\}$, where T_2, is unique solution of:*

$$\pi_1(p^*(t), T) = D(p^*(t))\, MR(T), \quad 0 \le T \le T_1 \tag{20}$$

Otherwise, (20) has no solution, and the optimal replenishment period $T^* = \min\{T_{\max}, T_1\}$.

On the basis of $p^*(t)$ and T^*, the optimal order quantity can be calculated as follows:

$$Q^* = \int_0^{T^*} (a - bp^*(t) + \omega(t))\, dt \tag{21}$$

3.4. Algorithm. Based on the propositions above, we propose the following algorithm to obtain the optimal solution.

Step 1. Let $d(p(t)) = a - bp(t)$ and $\omega(t) = \omega_0 e^{-\eta t}$.

Step 2. Determine $p_{\max}(t)$ and $\psi(t)$ from (2) and (13), respectively.

Step 3. According to Proposition 5, determine T_1 by solving $MC(T_1) = p_{\max}(T_1)$ and define $p^*(t) = \psi(t)$ for $0 \le t \le T$ and $p^*(t) = p_{\max}(t)$ for $t > T_1$.

Step 4. Verify the second-order condition (14).

Step 5. Compute T^* from Proposition 7.

Step 6. Compute the optimal order quantity Q^* and the average profit per unit time π_1^* from (21) and (6), respectively.

4. The Optimal Strategies under the Static Scenario

The dynamic pricing strategy discussed above is one in which demand is affected by fashion level. By means of a dynamic pricing strategy it is easy to distinguish the consumers who are sensitive and insensitive to fashion, thus maximizing the retailer's profit.

Besides the dynamic pricing strategy, retailers often follow the fixed price strategy because of its long-run benefits. Some famous retailers, such as Macy's, often use a fixed price strategy. So, for fashion clothing, the determination of which particular strategy is more beneficial, dynamic pricing or fixed pricing, becomes important. To facilitate a comparison between dynamic pricing and fixed pricing strategies, this section discusses the decision-making problem of fashion retailers under fixed price conditions.

Section 2.2 showed that the demand function under the fixed pricing strategy is as follows:

$$D(p,t) = d(p) + \omega(t) = a - bp + \omega_0 e^{-\eta t} \tag{22}$$

It is worth noting that price does not change with time. The retailer's total profit per replenishment period, represented by Π_2, is given by the following:

$$\Pi_2(p,T) = \int_0^T (p-c) D(p,t)\,dt - h\int_0^T I(p,t)\,dt - A \tag{23}$$

The goal of Section 4 is to determine the fixed pricing strategy and replenishment cycle while maximizing average profit per unit time, which can be described by the following optimization problem:

$$\max_{p,T} \quad \pi_2(p,T) = \frac{1}{T}\Pi_2(p,T) \tag{24}$$

Similar to the dynamic pricing scenario, the method of the sequential hierarchical optimization can still be used to solve the objective function under fixed price conditions. The problem at the lower hierarchical level of the decomposition discussed above is stated as follows: given the replenishment period, T, identify a fixed price denoted by p_F that maximizes the total profit for the period,

$$\begin{aligned}\max_{p} \quad & \pi_2(p,T) \\ &= \max_{p}\left\{\frac{1}{T}\int_0^T (p-c) D(p,t)\,dt - h\int_0^T I(p,t)\,dt - A\right\}\end{aligned} \tag{25}$$

Proposition 8. *For any given replenishment period $T \in [0, T_{\max}]$, $\pi_2(p,T)$ is strictly concave in p; and the optimal price is given by $p_F^* = (1/2)\{c + a/b + hT/2 - (\omega_0/bT\eta)(e^{-\eta T} - 1)\}$.*

The problem at the higher hierarchical level of the decomposition discussed above is stated as follows: given the optimal dynamic price function, p_F^*, maximize the average profit under the replenishment period, T, as a single decision variable. Then, the optimization problem presented in (24) is converted into another optimization problem as follows:

$$\begin{aligned}\max_{T} \quad & \pi_2(p_F^*,T) \\ &= \max_{T}\left\{\frac{1}{T}\int_0^T (p_F^*-c) D\,dt - h\int_0^T I\,dt - A\right\}\end{aligned} \tag{26}$$

According to p_F^* and T^*, the optimal order quantity can be calculated as follows:

$$Q^* = \int_0^{T^*} (a - bp_F^* + \omega(t))\,dt \tag{27}$$

Due to the mathematical complexity involved, it is difficult for us to propose an explicit solution for (26). Thus, in the following, we present the numerical results between the fixed price model and the dynamic price model.

5. Numerical Example

In this section, we conduct numerical analyses to gain managerial insights. Set $a = 20$, $b = 2$, $c = 2$, $h = 0.5$, $\eta = 0.2$, $A = 250$, $T_{\max} = 20$, and $\omega_0 = 20$. In Section 5.1, we use the solution algorithm described in Section 3 to find the optimal solution. Also, we compare the operational performances of the dynamic and static scenarios, focusing on the average profit and selling price. In Section 5.2, we examine the effects of the deterioration rate of fashion level and purchasing cost on the optimal solution and obtain some useful insights.

5.1. Optimal Solutions. Applying the algorithm presented in Section 3, we obtain the optimal price paths and replenishment period under the dynamic scenario. In Figure 4, the path of optimal dynamic price $p^*(t)$ consists of two parts. The first part is the path between the marginal cost $MC(t)$ and reservation price $p_{\max}(t)$. The optimal price decreases for $0 \le t < 6.93$ and then increases to $t = T_1 = 16.7$. Afterwards, demand vanishes, and the optimal price decreases and equals the reservation price, i.e., $p^*(t) = p_{\max}(t)$.

As shown in Figure 5, the objective $\pi_1(T)$ is plotted as a function of the replenishment period. Furthermore,

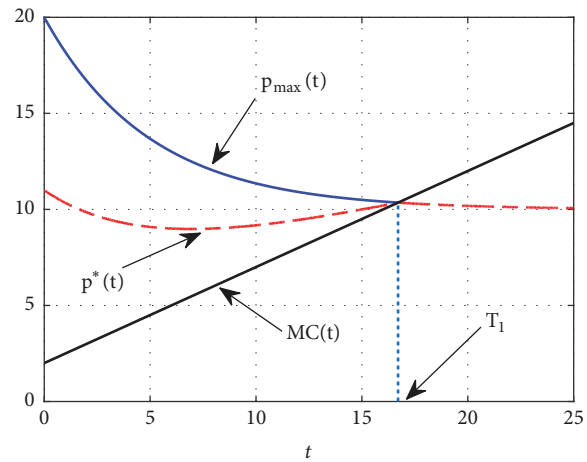

FIGURE 4: The optimal dynamic pricing strategy.

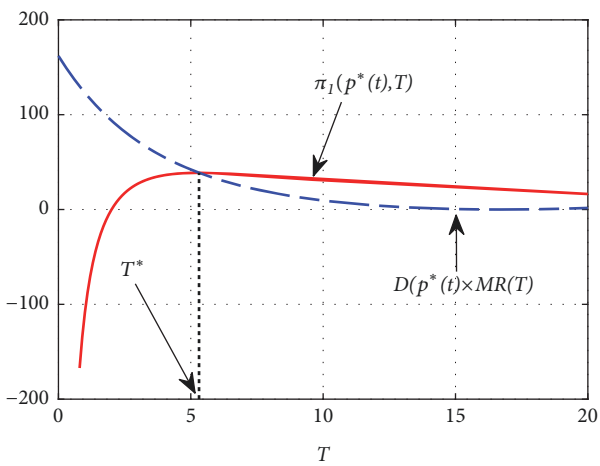

FIGURE 5: The optimal replenishment period.

$D(p^*(t))MR(T)$ is plotted as a function of the replenishment period. The optimal replenishment period, determined from $\pi_1(p^*(t), T) = D(p^*(t))MR(T)$, is $T^* = T_2 = 5.31$.

That is, the optimal replenishment period is determined by the point of intersection of the average profit and marginal profit curve.

Through numerical simulation, we also compare the corresponding price trajectories, as shown in Figure 6. Here, the corresponding optimal dynamic pricing strategy $p^*(t)$ is

$$p^*(t) = \begin{cases} 5e^{-0.2t} + 0.25t + 6, & 0 \le t \le 16.7 \\ 10e^{-0.2t} + 10, & t > 16.7, \end{cases} \tag{28}$$

and the average profit unit time is $\pi_1^* = 38.726$ (from (6)). In contrast, if the apparel retailer cannot vary the price, the optimal fixed price is $p_F^* = 9.75$ and the average profit unit time is $\pi_2^* = 38.104 < \pi_1^*$.

Furthermore, we investigate the effects of parameter changes on the profits obtained from the proposed method. Table 2 shows that the difference in the profits $\Delta\pi = \pi_1^* - \pi_2^*$ increases as the deterioration rate of fashion level increases

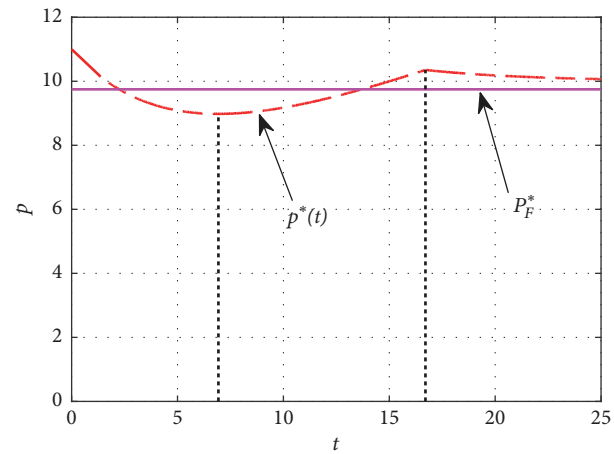

FIGURE 6: Comparison of dynamic pricing strategy and fixed price strategy.

because the faster the deterioration rate, the less fashionable the garments, and hence consumers have less incentive to buy. Consequently, the apparel retailer dynamically adjusts prices during different selling periods. The retailer charges a high price when the fashion apparel's value is elevated and, later, when its value decreases, such items are sold at a discount price. Thus, by adjusting the sales price periodically, the apparel retailer is able to realize the profit advantage of a continuously variable price policy.

5.2. Sensitivity Analysis

5.2.1. Impact of the Deterioration Rate (η). As discussed above, the deterioration rate affects the optimal dynamic price, the optimal replenishment period, the order quantity, and the apparel retailer's profit. Figure 7 shows that the curve of the optimal dynamic price $p^*(t)$ with different $\eta \in \{0.1, 0.2, 0.3\}$ and the other parameters is kept unchanged. A higher value of η represents a greater attenuation rate for the fashion level. It is shown from Figure 7 that, for a smaller value of η, the apparel retailer sets a higher selling price due to the greater marketing demand during the sales period. This higher price is set because the smaller value of η, the more fashionable the garments. As a result, greater marketing demand consumes inventory items more quickly. Overall, a smaller η implies both greater demand and greater order quantity, thus shortening the replenishment period, and Table 3 shows that T^* decreases as η increases. Finally, as shown in Table 3, average profit increases as η decreases because fashion level depends largely on stylistic elements. Consequently, taking measures to lower η, such as increasing investment in the creative design of fashion elements, is beneficial for the apparel retailer.

5.2.2. Impact of the Purchasing Cost (c). Table 4 shows the replenishment period T^*, the order quantity Q^*, and the average profit π_1^* with different $c \in \{1, 2, 3\}$ and the other parameters are kept unchanged. Figure 8 shows the curve of the optimal dynamic price $p^*(t)$. From the figure and table, we can see that $p^*(t)$ and T^* are increasing in c, while

Table 2: Profit comparison under different η.

η	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
π_1^*	70.822	64.576	59.074	54.163	49.740	45.729	42.073	38.726
π_2^*	70.821	64.548	58.980	53.982	49.457	45.336	41.566	38.104
$\Delta\pi$	0.001	0.029	0.094	0.182	0.283	0.393	0.507	0.623

Table 3: The impact of η on decisions of retailer.

η	T^*	Q^*	π_1^*
0.20	5.312	68.16	38.73
0.19	5.315	68.92	40.36
0.18	5.324	69.76	42.07
0.17	5.339	70.67	43.86
0.16	5.359	71.67	45.73
0.15	5.385	72.77	47.69
0.14	5.419	73.98	49.74
0.13	5.460	75.32	51.90
0.12	5.509	76.80	54.16
0.11	5.569	78.44	56.55

Table 4: The impact of c on decisions of retailer.

c	T^*	Q^*	π_1^*
1	4.97	70.04	52.19
2	5.31	68.16	38.73
3	5.75	66.18	26.55
4	6.37	64.08	15.76
5	7.37	61.82	6.50

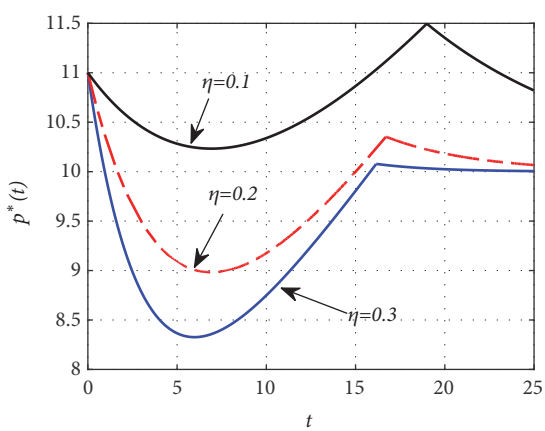

Figure 7: The optimal dynamic pricing strategy with different η.

Q^* and π_1^* are decreasing in c. This implies that when the purchasing cost c increases, the apparel retailer adds this cost to the selling price $p^*(t)$. Facing a large c, the apparel retailer compensates for the increased purchasing costs by lengthening the replenishment cycle, reducing orders, and raising the selling price.

6. Conclusions

Considering how both fashion level and changes in reservation price influence the performance of apparel retailers, this study proposes two pricing strategies for such enterprises: the dynamic pricing strategy and the fixed pricing strategy. We formulate a garment inventory model with demand being dependent on price and fashion level and determine the optimal prices and the optimal replenishment cycle with the objective of maximizing average profits in a finite time horizon. We characterize the properties of the optimal solution and propose an algorithm. Our research advances the following conclusions and managerial insights. First, a relatively rapid deterioration rate of fashion level means that the garment is of a relatively poor design. For the fashion apparel retailer, a dynamic price strategy is better than a fixed price strategy. This implies that the apparel retailer, by periodically adjusting sales price, can realize the profit advantage of a continuously variable price policy. Second, the apparel retailer has three types of optimal dynamic pricing strategies from which to choose, with the optimal dynamic pricing strategy being independent of replenishment cycle T. The optimal price strategy depends on time t and is dependent on remaining time $T - t$. This is intuitive, and the apparel retailer can decide on the optimal price of fashion clothing according to how much time is left before the next replenishment period. In contrast, the optimal fixed price is dependent on the replenishment period. Third, numerical analysis shows that both the deterioration rate of fashion level and the purchasing cost exert significant effects on both the optimal joint policy and the profit, thus providing fashion apparel retailers with guidance that can improve their profitability.

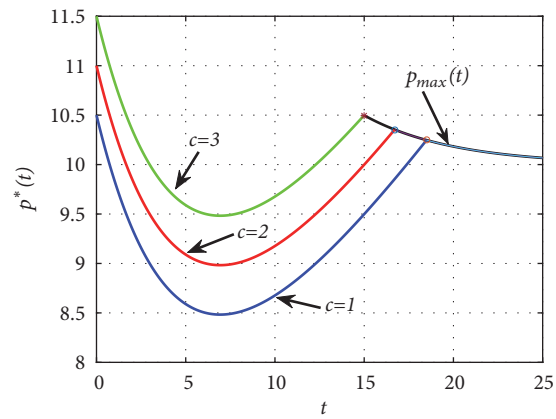

Figure 8: The optimal dynamic pricing strategy with different c.

Appendix

Proof of Proposition 4. From Assumption 2, differentiating $d(p_{\max}(t)) + \omega(t) = 0$ with respect to t yields $\partial\omega(t)/\partial t =$

$-(\partial d(p_{\max}(t))/\partial p_{\max}(t))(\partial p_{\max}(t)/\partial t)$. From Assumption 1, we have $\partial d(p_{\max}(t))/\partial p_{\max}(t) < 0$ and $\partial p_{\max}(t)/\partial t < 0$ for $0 \le t \le T_{\max}$. We can obtain $\partial\omega(t)/\partial t < 0$. This completes the proof. □

Proof of Proposition 5. From Assumption 2, the reservation price $p_{\max}(t)$ decreases monotonically in time, $p_{\max}(0) > c$. If $t > T_{\max}$, then $p_{\max}(t) = 0$. Therefore, there exists a unique replenishment time denoted by T_1, such that $p_{\max}(t) > MC(t)$ for $t < T_1$ and $p_{\max}(t) < MC(t)$ for $t > T_1$. From Assumption 1, we have $d'(\psi(t)) < 0$. From (13), we can obtain $(c + ht - \psi(t)) < 0$, that is, $MC(t) < p^*(t)$, therefore $MC(t) < p^*(t) < p_{\max}(t)$.

According to the demand function of Assumption 1, if $\psi(t) > p_{\max}(t)$, then $D(p(t)) = 0$, and if it is substituted into 10), then $p^*(t) = p_{\max}(t)$. (Thus, considering consumer reservation price and marginal cost, there exists the unique replenishment period T_1 that makes $MC(t) < p^*(t) < p_{\max}(t)$ for $0 \le t \le T_1$, $MC(t) = p^*(t) = p_{\max}(t)$ for $t = T_1$, and $MC(t) > p^*(t) = p_{\max}(t)$ for $t \ge T_1$.) □

Proof of Proposition 6. We differentiate both sides of (16) with respect to t, which yields $\partial p^*(t)/\partial t = (b/2)(\omega'(t)+bh)$, for $0 \le t \le T_1$. If $\omega'(t) < -bh$, then $\partial p^*(t)/\partial t < 0$, and we obtain line (i) in Proposition 6. If $\omega'(t) > -bh$, then $\partial p^*(t)/\partial t > 0$, and we obtain line (ii) in Proposition 6. This proves Proposition 6 and completes the proof. □

Proof of Proposition 7. For notational convenience, we denote $\pi_1(T) = \pi_1(p^*(t), T)$ and $D(t) = D(p^*(t))$. Differentiating $\pi_1(p^*(t), T)$ with respect to T yields

$$\begin{aligned}
\frac{\partial \pi_1(T)}{\partial T} &= \frac{\partial}{\partial T}\left\{\frac{1}{T}\Pi_1(p(t), T)\right\} \\
&= \frac{\partial}{\partial T}\left\{\frac{1}{T}\left\{\int_0^T (p^*(t) - c) D(p^*(t))\, dt \right.\right. \\
&\quad \left.\left. - h\int_0^T I(p^*(t), t)\, dt - A\right\}\right\} \\
&= -\frac{1}{T^2}\left\{\int_0^T (p^*(t) - c) D(p^*(t))\, dt \right. \\
&\quad \left. - h\int_0^T I(p^*(t), t)\, dt - A\right\} + \frac{1}{T}\left\{(p^*(T) - c)\right. \\
&\quad \left. \cdot D(T) - h\frac{d}{dT}\int_0^T\int_t^T D(p^*(s), s)\, ds\, dt\right\} \\
&= \frac{1}{T}\left\{(p^*(T) - c) D(T) - h\frac{d}{dT}\right. \\
&\quad \left. \cdot \int_0^T\int_t^T D(s)\, ds\, dt\right\} = -\frac{\pi(T)}{T} + \frac{1}{T}\{(p^*(T) \\
&\quad - c) D(T) - hTD(T)\} = -\frac{1}{T}\{\pi(T) - D(T) \\
&\quad \cdot (p^*(T) - c - hT)\}
\end{aligned} \tag{A.1}$$

Thus, the first derivative of $\pi_1(p^*(t), T)$ at time T_2 is

$$\begin{aligned}
&\frac{\partial \pi_1(p^*(t), T)}{\partial T} \\
&\quad = -\frac{1}{T}\{\pi_1(p^*(t), T) - D(p^*(t)) MR(T)\}
\end{aligned} \tag{A.2}$$

According to the property of the optimal dynamic price discussed in Section 3, $D(p^*(t))MR(T)$ is positive and decreases for $0 \le T \le T_1$ (see Proposition 5), until $T = T_1$, when it becomes zero. In addition, the limit value of the objective function is $\lim_{T\to 0}\pi_1(p^*(t), T) = -\infty$. That is to say, $\pi_1(p^*(t), T)$ is very small at the beginning, close to negative infinity. It can be easily shown that $\partial\pi_1(p^*(t), T)/\partial T > 0$, and $\pi_1(p^*(t), T)$ increases and reaches a maximum at $T = T_1$,. Thus, we can obtain $T^* = \min\{T_{\max}, T_1\}$. If $\pi_1(p^*(t), T)$ hits the zero level and becomes positive, then there exists an extreme point T_2 satisfying $\partial\pi_1(p^*(t), T)/\partial T|_{T=T_2} = 0$. Hence, we can obtain $\pi_1(p^*(t), T) = D(p^*(t))MR(T)$. Next, we prove the uniqueness of T_2. Taking the second-order derivative of $\pi_1(p^*(t), T)$ with respect to T yields

$$\begin{aligned}
\left.\frac{\partial^2 \pi_1(T)}{\partial T^2}\right|_{T=T_2} &= \frac{\partial}{\partial T}\left\{-\frac{\pi(T)}{T}\right. \\
&\quad \left. + \frac{1}{T}\{(p^*(T) - c) D(T) - hTD(T)\}\right\}\Bigg|_{T=T_2} \\
&= -\frac{1}{T_2}\{-D(T_2)[(p^*(T_2))' - h] - d'(T_2) \\
&\quad \cdot [p^*(T_2) - c - hT_2]\} = \frac{1}{T_2}\{d'(p^*(T_2)) \\
&\quad \cdot (p^*(T_2))' + \omega'(T_2)(p^*(T_2) - c - hT_2) \\
&\quad + D(T_2)[(p^*(T_2))' - h]\} = \frac{1}{T_2}\{d'(p^*(T_2)) \\
&\quad \cdot (p^*(T_2))'(p^*(T_2) - c - hT_2) + D(T_2) \\
&\quad - hD(T_2) + \omega'(T_2)(p^*(T_2) - c - hT_2)\}
\end{aligned} \tag{A.3}$$

Thus, the second derivative of $\pi_1(p^*(t), T)$ at time T_2 is

$$\begin{aligned}
\left.\frac{\partial^2 \pi_1(T)}{\partial T^2}\right|_{T=T_2} &= \frac{1}{T_2}\{d'((p^*(T_2)))(p^*(T_2))' \\
&\quad \cdot (p^*(T_2) - c - hT) + D(T_2) - hD(T_2) \\
&\quad + \omega'(T_2)(p^*(T_2) - c - hT_2)\}
\end{aligned} \tag{A.4}$$

From Proposition 4, the fashion level function decreases with time t, i.e., $\omega'(T_2) < 0$. Therefore, the last two terms on the right-hand side of (A.4) are less than zero. From $d'(\psi(t))(\psi(t) - c - ht) + D(\psi(t)) = 0$, the first term on the right-hand side of (A.4) is equal to zero. Thus, $(\partial^2\pi_1(T)/\partial T^2)|_{T=T_2} = (1/T_2)\{-hD(T_2) + \omega'(T_2)(p^*(T_2) - c - hT_2)\}$. From $(p^*(T_2) - c - hT_2) > 0$ and $\omega'(T_2) < 0$, we have

$\partial^2\pi_1(T)/\partial T^2|_{T=T_2} < 0$. Hence, there exists a unique point T_2 satisfying $\partial\pi_1(p^*(t),T)/\partial T|_{T=T_2} = 0$. This completes the proof. □

Proof of Proposition 8. Taking the first-order derivative of $\Pi_2(p,T)$ with respect to T, we obtain

$$\begin{aligned}\frac{\partial\Pi_2(p,T)}{\partial p} &= T\frac{\partial\pi_2(p,T)}{\partial p}\\ &= \frac{\partial}{\partial p}\int_0^T (p-c)\left(a-bp+\omega_0 e^{-\eta t}\right)dt\\ &\quad - h\frac{\partial}{\partial p}\int_0^T\int_t^T \left(a-bp+\omega_0 e^{-\eta s}\right)ds\,dt\\ &= \int_0^T \left\{a-bp+\omega_0 e^{-\eta t}-b(p-c)\right\}dt\\ &\quad - h\int_0^T -b(T-t)\,dt\\ &= aT-2bpT+bcT+\frac{bhT^2}{2}\\ &\quad -\frac{\omega_0}{\eta}\left(e^{-\eta T}-1\right)\end{aligned} \tag{A.5}$$

It can be easily shown that $\partial^2\pi_2(p,T)/\partial p^2 = -2b < 0$. Therefore, there exists a unique price p_F^*. satisfying $\partial\pi_2(p,T)/\partial p|_{P_F} = 0$, and we can obtain $p_F^* = (1/2)\{c + a/b + hT/2 - (\omega_0/bT\eta)(e^{-\eta T}-1)\}$. This completes the proof. □

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this study.

Acknowledgments

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