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Research Article

Searching for Analytical Solutions of the (2+1)-Dimensional KP Equation by Two Different Systematic Methods

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In this paper, we derive analytical solutions of the (2+1)-dimensional Kadomtsev-Petviashvili (KP) equation by two different systematic methods. Using the $\exp(-\psi(z))$-expansion method, exact solutions of the mentioned equation including hyperbolic, exponential, trigonometric, and rational function solutions have been obtained. Based on the work of Yuan *et al.*, we proposed the extended complex method to seek exact solutions of the (2+1)-dimensional KP equation. The results demonstrate that the applied methods are efficient and direct methods to solve the complex nonlinear systems.

1. Introduction

The (2+1)-dimensional KP equation [1] is given by

$$u_{xt} - 6u_x^2 - 6uu_{xx} + u_{xxxx} + 3u_{yy} = 0, \qquad (1)$$

which is a universal nonlinear integrable system in two spatial and one temporal coordinates and can be utilized to describe the law of motion of water waves in (2+1)-dimensional spaces and plasmas in magnetic fields [2–4]. For example, in the study of water waves, this equation appears in the description of a tsunami wave travelling in the inhomogeneous zone on the bottom of the ocean [2], and it also appears in the study of nonlinear ion acoustic waves in magnetized dusty plasma [4]. Over the past few years, many research results for the (2+1)-dimensional KP equation have been generated. As to this equation, traveling wave solutions [5, 6], rogue wave, and a pair of resonance stripe solitons [7] are discovered. Symmetry reductions [8] and conservation laws [9] are also investigated. Using the Hirota bilinear form of the (2+1)-dimensional KP equation, mixed lump-kink solutions are presented under the help of Maple [10]. By the positive quadratic function and exponential function, rational lump solutions and line soliton pairs to the (2+1)-dimensional KP equation are established [11].

It is well known that nonlinear differential equations (NLDEs) are universally applied in plasma physics, solid state physics, nonlinear optics, fluid dynamics, biology, chemistry, etc. For instance, the singular behaviors [12, 13] and impulsive phenomena [14, 15] often show some blow-up properties [16, 17] which happen in lots of complex physical processes. In order to solve various differential equations, some analytical tools as well as symbolic calculation techniques were established, such as fixed-point theorems [18, 19], variational methods [20, 21], topological degree method [22–25], iterative techniques [26, 27], bilinear method [28–31], modified simple equation method [32], $\exp(-\psi(z))$-expansion method [33–38], Lie group method [39, 40], and complex method [41–50].

The $\exp(-\psi(z))$-expansion method is an efficient method for finding the exact solutions of NLDEs. Many researchers, such as Roshid, Khan, and Jafari, have used this method to study NLDEs [35–37]. The complex method, proposed by Yuan *et al.* [41, 42], is established via complex differential equations and complex analysis. It is a useful tool to find exact solutions of NLDEs which are Briot-Bouquet equations or satisfy $\langle p, q\rangle$ condition [41]. Based on the work of Yuan *et al.*, we introduce the extended complex method to seek exact solutions of NLDEs which are not Briot-Bouquet equations or do not satisfy $\langle p, q\rangle$ condition.

The extended complex method can solve more differential equations in mathematical physics than the complex method. In this paper, two different systematic methods which are the $\exp(-\psi(z))$-expansion method and extended complex method are employed to search analytical solutions of the (2+1)-dimensional KP equation. Computer simulations are given to illustrate our main results. Comparisons and conclusions are presented in the last section.

2. The $\exp(-\psi(z))$-Expansion Method

Consider a nonlinear PDE as follows:

$$F\left(u, u_x, u_y, u_t, u_{xx}, u_{yy}, u_{tt}, \ldots\right) = 0, \tag{2}$$

where F is a polynomial consisting of the unknown function $u(x, y, t)$, the partial derivatives of $u(x, y, t)$, the highest-order partial derivatives of $u(x, y, t)$, and some nonlinear terms.

Step 1. Substitute traveling wave transformation,

$$\begin{aligned} u(x, y, t) &= u(z), \\ z &= kx + Ly + \lambda t, \end{aligned} \tag{3}$$

into (2) to convert it to the ODE,

$$P\left(u, u', u'', u''', \ldots\right) = 0, \tag{4}$$

where P is a polynomial of u and its derivatives.

Step 2. Suppose that (4) has the following exact solutions:

$$u(z) = \sum_{j=0}^{n} B_j \left(\exp(-\psi(z))\right)^j, \tag{5}$$

where B_j $(0 \le j \le n)$ are constants to be determined later, such that $B_n \neq 0$ and $\psi = \psi(z)$ satisfies the ODE as follows:

$$\psi'(z) = \gamma + \exp(-\psi(z)) + \mu \exp(\psi(z)). \tag{6}$$

Equation (6) has the solutions as follows.

When $\gamma^2 - 4\mu > 0$, $\mu \neq 0$,

$$\psi(z) = \ln\left(\frac{-\sqrt{(\gamma^2 - 4\mu)}\tanh\left(\left(\sqrt{\gamma^2 - 4\mu}/2\right)(z+a)\right) - \gamma}{2\mu}\right), \tag{7}$$

$$\psi(z) = \ln\left(\frac{-\sqrt{(\gamma^2 - 4\mu)}\coth\left(\left(\sqrt{\gamma^2 - 4\mu}/2\right)(z+a)\right) - \gamma}{2\mu}\right). \tag{8}$$

When $\gamma^2 - 4\mu < 0$, $\mu \neq 0$,

$$\psi(z) = \ln\left(\frac{\sqrt{(4\mu - \gamma^2)}\tan\left(\left(\sqrt{(4\mu - \gamma^2)}/2\right)(z+a)\right) - \gamma}{2\mu}\right), \tag{9}$$

$$\psi(z) = \ln\left(\frac{\sqrt{(4\mu - \gamma^2)}\cot\left(\left(\sqrt{(4\mu - \gamma^2)}/2\right)(z+a)\right) - \gamma}{2\mu}\right). \tag{10}$$

When $\gamma^2 - 4\mu > 0$, $\gamma \neq 0$, $\mu = 0$,

$$\psi(z) = -\ln\left(\frac{\gamma}{\exp(\gamma(z+a)) - 1}\right). \tag{11}$$

When $\gamma^2 - 4\mu = 0$, $\gamma \neq 0$, $\mu \neq 0$,

$$\psi(z) = \ln\left(-\frac{2(\gamma(z+a) + 2)}{\gamma^2(z+a)}\right). \tag{12}$$

When $\gamma^2 - 4\mu = 0$, $\gamma = 0$, $\mu = 0$,

$$\psi(z) = \ln(z+a), \tag{13}$$

where a is an arbitrary constant and $B_n \neq 0, \gamma, \mu$ are constants in (7)- (13). We determine the positive integer n through considering the homogeneous balance between the highest-order derivatives and nonlinear terms of (4).

Step 3. Inserting (5) into (4) and then considering the function $\exp(-\psi(z))$ yields a polynomial of $\exp(-\psi(z))$. Let the coefficients of the same power about $\exp(-\psi(z))$ equal zero; then, we get a set of algebraic equations. We solve the algebraic equations to obtain the values of $B_n \neq 0, \gamma, \mu$ and then we put these values into (5) along with (7)-(13) to finish the determination of the solutions for the given PDE.

3. Application of the $\exp(-\psi(z))$-Expansion Method to the (2+1)-Dimensional KP Equation

Substitute

$$\begin{aligned} u(x, y, t) &= u(z), \\ z &= kx + Ly + \lambda t \end{aligned} \tag{14}$$

into (1), and we get

$$k^4 u'''' - 6k^2 u u'' - 6k^2 \left(u'\right)^2 + \left(k\lambda + 3L^2\right) u'' = 0. \tag{15}$$

Take the homogeneous balance between u'''' and $(u')^2$ in (15) to yield

$$u(z) = B_0 + B_1 \exp(-\psi(z)) + B_2 \left(\exp(-\psi(z))\right)^2, \tag{16}$$

where $B_2 \neq 0$ and B_1 and B_0 are constants.

Substituting u'''', uu'', $(u')^2$, u'' into (15) and equating the coefficients about $\exp(-\psi(z))$ to zero, we get

$$\begin{aligned}
e^{0(-\psi(z))}&: k^4B_1\gamma^3\mu + 14k^4B_2\mu^2\gamma^2 + 8k^4B_1\gamma\mu^2 \\
&\quad + 16k^4B_2\mu^3 - 6k^2B_0B_1\mu\gamma - 12k^2B_0B_2\mu^2 \\
&\quad - 6k^2{B_1}^2\mu^2 + 3B_1L^2\mu\gamma + B_1k\lambda\mu\gamma + 6B_2L^2\mu^2 \\
&\quad + 2B_2k\lambda\mu^2 = 0, \\
e^{1(-\psi(z))}&: B_1\gamma^4k^4 + 30B_2\gamma^3k^4\mu + 22B_1\gamma^2k^4\mu \\
&\quad + 120B_2\gamma k^4\mu^2 + 16B_1k^4\mu^2 - 6B_0B_1\gamma^2k^2 \\
&\quad - 36B_0B_2\gamma k^2\mu - 18{B_1}^2\gamma k^2\mu - 36B_1B_2k^2\mu^2 \\
&\quad - 12k^2B_0B_1\mu + 3B_1L^2\gamma^2 + B_1\gamma^2k\lambda + 18B_2L^2\gamma\mu \\
&\quad + 6B_2\gamma k\lambda\mu + 6B_1L^2\mu + 2B_1k\lambda\mu = 0, \\
e^{2(-\psi(z))}&: 16B_2\gamma^4k^4 + 15k^4B_1\gamma^3 + 232B_2\gamma^2k^4\mu \\
&\quad + 60B_1\gamma k^4\mu + 136k^4B_2\mu^2 - 24B_0B_2\gamma^2k^2 \\
&\quad - 12{B_1}^2\gamma^2k^2 - 90B_1B_2\gamma k^2\mu - 36{B_2}^2k^2\mu^2 \\
&\quad - 18B_0B_1\gamma k^2 - 48B_0B_2k^2\mu - 24{B_1}^2k^2\mu \\
&\quad + 12B_2L^2\gamma^2 + 4B_2\gamma^2k\lambda + 9B_1L^2\gamma + 3B_1\gamma k\lambda \\
&\quad + 24B_2L^2\mu + 8B_2k\lambda\mu = 0, \\
e^{3(-\psi(z))}&: 130B_2\gamma^3k^4 + 50B_1\gamma^2k^4 + 440B_2\gamma k^4\mu \\
&\quad - 54B_1B_2\gamma^2k^2 + 40B_1k^4\mu - 84{B_2}^2\gamma k^2\mu \\
&\quad - 60B_0B_2\gamma k^2 - 30{B_1}^2\gamma k^2 - 108B_1B_2k^2\mu \\
&\quad - 12B_0B_1k^2 + 30B_2L^2\gamma + 10B_2\gamma k\lambda + 6B_1L^2 \\
&\quad + 2B_1k\lambda = 0, \\
e^{4(-\psi(z))}&: 330B_2\gamma^2k^4 + 60B_1\gamma k^4 - 48{B_2}^2\gamma^2k^2 \\
&\quad + 240B_2k^4\mu - 126B_1B_2\gamma k^2 - 96{B_2}^2k^2\mu \\
&\quad - 36B_0B_2k^2 - 18{B_1}^2k^2 + 18B_2L^2 + 6B_2k\lambda = 0, \\
e^{5(-\psi(z))}&: 336B_2\gamma k^4 + 24B_1k^4 - 108{B_2}^2\gamma k^2 \\
&\quad - 72B_1B_2k^2 = 0, \\
e^{6(-\psi(z))}&: 120B_2k^4 - 60{B_2}^2k^2 = 0.
\end{aligned} \tag{17}$$

Solving the above algebraic equations yields

$$\begin{aligned}
B_2 &= 2k^2, \\
B_1 &= 2k^2\gamma, \\
B_0 &= \frac{\gamma^2k^4 + 8k^4\mu + 3L^2 + k\lambda}{6k^2},
\end{aligned} \tag{18}$$

where γ and μ are arbitrary constants.

We substitute (18) into (16), and then

$$\begin{aligned}
u(z) &= \frac{\gamma^2k^4 + 8k^4\mu + 3L^2 + k\lambda}{6k^2} + 2k^2\gamma\exp(-\psi(z)) \\
&\quad + 2k^2(\exp(-\psi(z)))^2.
\end{aligned} \tag{19}$$

Using (7) to (13) into (19), respectively, we obtain exact solutions of the (2+1)-dimensional KP equation as follows.

When $\gamma^2 - 4\mu > 0$, $\mu \neq 0$,

$$\begin{aligned}
u_1(z) &= \frac{\gamma^2k^4 + 8k^4\mu + 3L^2 + k\lambda}{6k^2} \\
&\quad - \frac{4k^2\gamma\mu}{\sqrt{(\gamma^2 - 4\mu)}\tanh\left(\left(\sqrt{\gamma^2 - 4\mu}/2\right)(z+a)\right) + \gamma} \\
&\quad + \frac{8k^2\mu^2}{\left(\sqrt{(\gamma^2 - 4\mu)}\tanh\left(\left(\sqrt{\gamma^2 - 4\mu}/2\right)(z+a)\right) + \gamma\right)^2}, \\
u_2(z) &= \frac{\gamma^2k^4 + 8k^4\mu + 3L^2 + k\lambda}{6k^2} \\
&\quad - \frac{4k^2\gamma\mu}{\sqrt{(\gamma^2 - 4\mu)}\coth\left(\left(\sqrt{\gamma^2 - 4\mu}/2\right)(z+a)\right) + \gamma} \\
&\quad + \frac{8k^2\mu^2}{\left(\sqrt{(\gamma^2 - 4\mu)}\coth\left(\left(\sqrt{\gamma^2 - 4\mu}/2\right)(z+a)\right) + \gamma\right)^2}.
\end{aligned} \tag{20}$$

When $\gamma^2 - 4\mu < 0$, $\mu \neq 0$,

$$\begin{aligned}
u_3(z) &= \frac{\gamma^2k^4 + 8k^4\mu + 3L^2 + k\lambda}{6k^2} \\
&\quad + \frac{4k^2\gamma\mu}{\sqrt{(4\mu - \gamma^2)}\tan\left(\left(\sqrt{4\mu - \gamma^2}/2\right)(z+a)\right) - \gamma} \\
&\quad + \frac{8k^2\mu^2}{\left(\sqrt{(4\mu - \gamma^2)}\tan\left(\left(\sqrt{4\mu - \gamma^2}/2\right)(z+a)\right) - \gamma\right)^2}, \\
u_4(z) &= \frac{\gamma^2k^4 + 8k^4\mu + 3L^2 + k\lambda}{6k^2} \\
&\quad + \frac{4k^2\gamma\mu}{\sqrt{(4\mu - \gamma^2)}\cot\left(\left(\sqrt{4\mu - \gamma^2}/2\right)(z+a)\right) - \gamma} \\
&\quad + \frac{8k^2\mu^2}{\left(\sqrt{(4\mu - \gamma^2)}\cot\left(\left(\sqrt{4\mu - \gamma^2}/2\right)(z+a)\right) - \gamma\right)^2}.
\end{aligned} \tag{21}$$

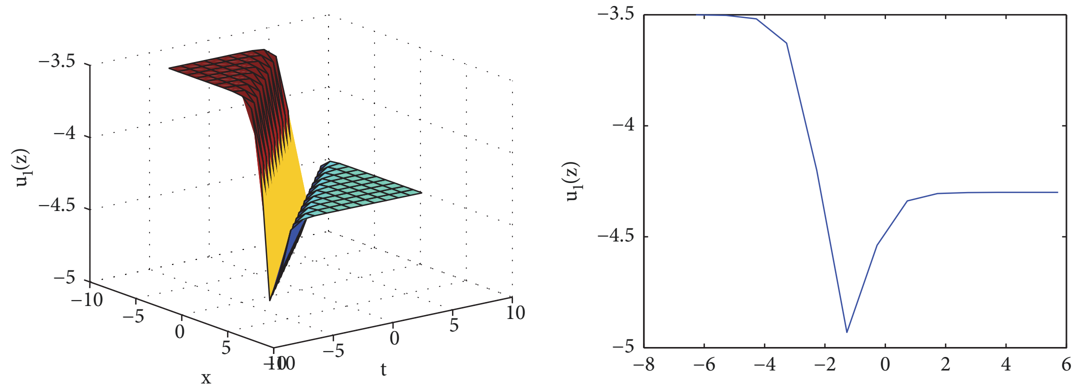

Figure 1: The 3D and 2D surfaces of $u_1(z)$ by considering the values $\gamma = 4$, $\mu = 3$, $k = 1$, $L = 1$, $\lambda = -43$, $y = 0$, $a = 1$, and $t = 0$ for the 2D graphic.

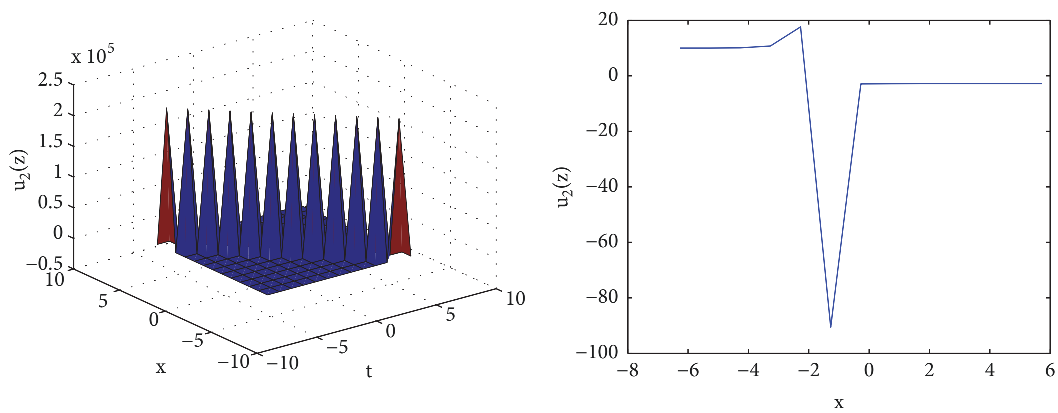

Figure 2: The 3D and 2D surfaces of $u_2(z)$ by considering the values $\gamma = 4$, $\mu = 3$, $k = 1$, $L = 1$, $\lambda = -43$, $y = 0$, $a = 1$, and $t = 0$ for the 2D graphic.

When $\gamma^2 - 4\mu > 0$, $\gamma \neq 0$, $\mu = 0$,

$$u_5(z) = \frac{\gamma^2 k^4 + 3L^2 + k\lambda}{6k^2} + \frac{2k^2\gamma^2}{\exp(\gamma(z+a)) - 1} + \frac{2k^2\gamma^2}{(\exp(\gamma(z+a)) - 1)^2}. \tag{22}$$

When $\gamma^2 - 4\mu = 0$, $\gamma \neq 0$, $\mu \neq 0$,

$$u_6(z) = \frac{4k^4\mu + 3L^2 + k\lambda}{6k^2} - \frac{k^2\gamma^3(z+a)}{\gamma(z+a)+2} + \frac{k^2\gamma^4(z+a)^2}{2(\gamma(z+a)+2)^2}. \tag{23}$$

When $\gamma^2 - 4\mu = 0$, $\gamma = 0$, $\mu = 0$,

$$u_7(z) = \frac{3L^2 + k\lambda}{6k^2} + \frac{2k^2}{(z+a)^2}. \tag{24}$$

The properties of the solutions are shown in Figures 1–5.

Remark. The $\exp(-\psi(z))$-expansion method is an efficient method. Khan and Akbar [6] used this method to obtain the exact solutions of the KP equation. We still give the details of solving KP equation with the $\exp(-\psi(z))$-expansion method in this paper for several reasons. First of all, we consider fourth-order ODE after the reduction instead of the second-order ODE of [6], so it is also a good example to show the use of the $\exp(-\psi(z))$-expansion method. Secondly, we obtain more generalized results compared with [6]. If we take $k = 1$, $L = 1$, and $\lambda = -\omega$, then the results of [6] can be obtained. Thirdly, we give some computer simulations to show the properties of the solutions.

4. The Extended Complex Method

Step 1. Substituting the transform $T : u(x,y,t) \longrightarrow U(z)$, $(x,y,t) \longrightarrow z$ into the given PDE yields

$$W\left(U, U', U'', U''', \ldots\right) = 0. \tag{25}$$

Step 2. Determine the weak $\langle p, q\rangle$ condition.

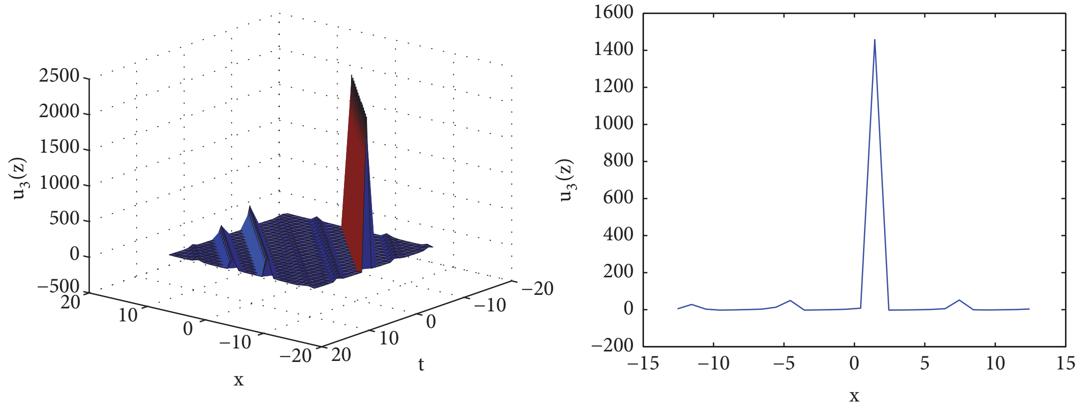

FIGURE 3: The 3D and 2D surfaces of $u_3(z)$ by considering the values $\gamma = 3$, $\mu = 2.5$, $k = 1$, $L = 1$, $\lambda = -32$, $y = 0$, $a = 1$, and $t = 0$ for the 2D graphic.

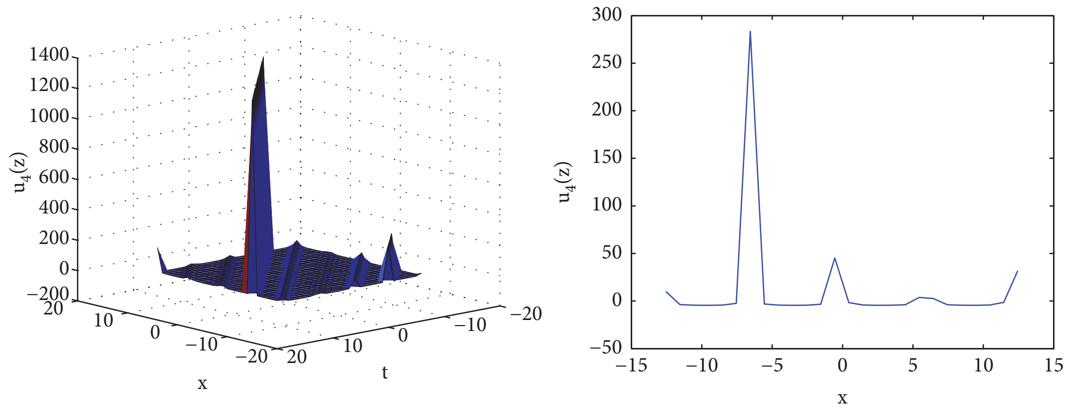

FIGURE 4: The 3D and 2D surfaces of $u_4(z)$ by considering the values $\gamma = 3$, $\mu = 2.5$, $k = 1$, $L = 1$, $\lambda = -32$, $y = 0$, $a = 1$, and $t = 0$ for the 2D graphic.

Let $p, q \in \mathbb{N}$, and suppose that the meromorphic solutions U of (25) have at least one pole. Substituting the Laurent series,

$$U(z) = \sum_{k=-q}^{\infty} A_k z^k, \quad q > 0, \ A_{-q} \neq 0, \tag{26}$$

into (25), if it is determined p distinct Laurent singular parts

$$\sum_{k=-q}^{-1} A_k z^k, \tag{27}$$

then the weak $\langle p, q \rangle$ condition of (25) holds.

Weierstrass elliptic function $\wp(z) := \wp(z, g_2, g_3)$ with double periods satisfies the equation as follows:

$$\left(\wp'(z)\right)^2 = 4\wp(z)^3 - g_2\wp(z) - g_3 \tag{28}$$

and has the following addition formula [51]:

$$\wp(z - z_0) = -\wp(z) + \frac{1}{4}\left[\frac{\wp'(z) + \wp'(z_0)}{\wp(z) - \wp(z_0)}\right]^2 - \wp(z_0). \tag{29}$$

Step 3. Substitute the indeterminate forms

$$U(z) = \sum_{i=1}^{s-1}\sum_{j=2}^{q} \frac{(-1)^j \beta_{-ij}}{(j-1)!} \frac{d^{j-2}}{dz^{j-2}} \left(\frac{1}{4}\left[\frac{\wp'(z) + D_i}{\wp(z) - B_i}\right]^2 - \wp(z)\right) + \sum_{i=1}^{s-1} \frac{\beta_{-i1}}{2} \frac{\wp'(z) + D_i}{\wp(z) - B_i} + \sum_{j=2}^{q} \frac{(-1)^j \beta_{-sj}}{(j-1)!} \frac{d^{j-2}}{dz^{j-2}}\wp(z) + \beta_0, \tag{30}$$

$$U(z) = \sum_{i=1}^{s}\sum_{j=1}^{q} \frac{\beta_{ij}}{(z - z_i)^j} + \beta_0, \tag{31}$$

$$U(e^{\alpha z}) = \sum_{i=1}^{s}\sum_{j=1}^{q} \frac{\beta_{ij}}{(e^{\alpha z} - e^{\alpha z_i})^j} + \beta_0 \tag{32}$$

into (25), respectively, to yield the systems of algebraic equations, and solve the algebraic equations to obtain elliptic function solutions, rational function solutions, and simply periodic solutions with the pole at $z = 0$, where β_{-ij} are determined by (26), $D_i^2 = 4B_i^3 - g_2 B_i - g_3$ and $\sum_{i=1}^{s} \beta_{-i1} =$

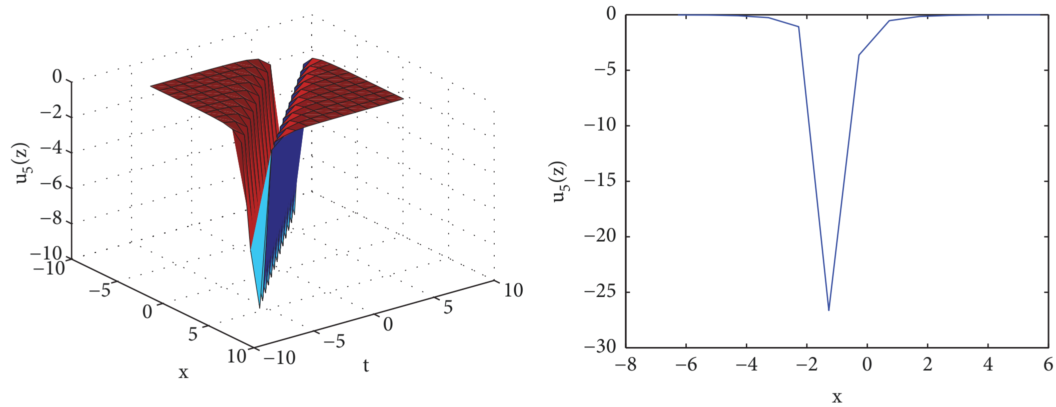

FIGURE 5: The 3D and 2D surfaces of $u_5(z)$ by considering the values $\gamma = 1$, $\mu = 0$, $k = 1$, $L = 1$, $\lambda = -4$, $y = 0$, $a = 1$, and $t = 0$ for the 2D graphic.

0, and $R(z)$, $R(e^{\alpha z})(\alpha \in \mathbb{C})$ have $s(\leq p)$ distinct poles of multiplicity q.

Step 4. Obtain the meromorphic solutions with arbitrary pole, and substitute the inverse transform T^{-1} into the meromorphic solutions to achieve the exact solutions to the original PDE.

5. Application of the Extended Complex Method to the (2+1)-Dimensional KP Equation

Inserting (26) into (15) yields $p = 1$, $q = 2$, and then the weak $\langle 1, 2\rangle$ condition of (15) holds.

By the weak $\langle 1, 2\rangle$ condition and (30), we have the form of the elliptic solutions of (15):

$$U_{10}(z) = \beta_{-12}\wp(z) + \beta_0, \tag{33}$$

with pole at $z = 0$.

Inserting $U_{10}(z)$ into (15), we have

$$\sum_{i=0}^{3} c_{1i}\wp^i(z) = 0, \tag{34}$$

where

$$\begin{aligned} c_{10} &= -12k^4\beta_{-12}g_3 + 3k^2\beta_{-12}\beta_0 g_2 + 6k^2{\beta_{-12}}^2 g_3 \\ &\quad - \frac{3}{2}\beta_{-12}L^2 g_2 - \frac{1}{2}\beta_{-12}k\lambda g_2, \\ c_{11} &= -18k^4\beta_{-12}g_2 + 9k^2{\beta_{-12}}^2 g_2, \\ c_{12} &= -36k^2\beta_0\beta_{-12} + 18L^2\beta_{-12} + 6k\lambda\beta_{-12}, \\ c_{13} &= 120k^4\beta_{-12} - 60k^2{\beta_{-12}}^2. \end{aligned} \tag{35}$$

Equating the coefficients of all powers about $\wp(z)$ in (34) to zero, we get a system of algebraic equations:

$$\begin{aligned} c_{10} &= 0, \\ c_{11} &= 0, \\ c_{12} &= 0, \\ c_{13} &= 0. \end{aligned} \tag{36}$$

Solving the above equations, we obtain

$$\begin{aligned} \beta_{-12} &= 2k^2, \\ \beta_0 &= \frac{3L^2 + k\lambda}{6k^2}, \end{aligned} \tag{37}$$

and then

$$U_{10}(z) = 2k^2\wp(z) + \frac{3L^2 + k\lambda}{6k^2}. \tag{38}$$

Therefore, the elliptic solutions of (15) with arbitrary pole are

$$U_1(z) = 2k^2\wp(z - z_0) + \frac{3L^2 + k\lambda}{6k^2}, \tag{39}$$

where $z_0 \in \mathbb{C}$.

Apply the addition formula to $U_1(z)$, and then

$$\begin{aligned} U_1(z) &= -2k^2\wp(z) + \frac{k^2}{2}\left(\frac{\wp'(z) + D}{\wp(z) - C}\right)^2 \\ &\quad + \frac{3L^2 + k\lambda - 12k^4 C}{6k^2}, \end{aligned} \tag{40}$$

where $C^2 = 4D^3 - g_2 D - g_3$ and g_2 and g_3 are arbitrary constants.

By the weak $\langle 1, 2\rangle$ condition and (31), we have the indeterminate forms of rational solutions:

$$U_{20}(z) = \frac{\beta_{12}}{(z-1)^2} + \frac{\beta_{11}}{z-1} + \beta_{10}, \tag{41}$$

with pole at $z = 0$.

Inserting $U_{20}(z)$ into (15), we have

$$\sum_{i=1}^{4} c_{2i} z^{-i-2} = 0, \tag{42}$$

where

$$\begin{aligned}
c_{21} &= -12k^2\beta_{20}\beta_{11} + 6L^2\beta_{11} + 2k\lambda\beta_{11},\\
c_{22} &= -36k^2\beta_{20}\beta_{12} - 18k^2\beta_{11}^2 + 18L^2\beta_{12} + 6k\lambda\beta_{12},\\
c_{23} &= 24k^4\beta_{11} - 72k^2\beta_{12}\beta_{11},\\
c_{24} &= 120k^4\beta_{12} - 60k^2\beta_{12}^2.
\end{aligned} \tag{43}$$

Equating the coefficients of all powers about z in (42) to zero, we get a system of algebraic equations:

$$\begin{aligned}
c_{21} &= 0,\\
c_{22} &= 0,\\
c_{23} &= 0,\\
c_{24} &= 0.
\end{aligned} \tag{44}$$

Solving the above equations, we obtain

$$\begin{aligned}
\beta_{12} &= 2k^2,\\
\beta_{11} &= 0,\\
\beta_{10} &= \frac{3L^2 + k\lambda}{6k^2},
\end{aligned} \tag{45}$$

and then

$$U_{20}(z) = \frac{2k^2}{(z-1)^2} + \frac{3L^2 + k\lambda}{6k^2}. \tag{46}$$

Substitute $U(z) = R(\eta)$ into (15), and then

$$\begin{aligned}
& k^4\alpha^4\left(R^{(4)}\eta^4 + 6R'''\eta^3 + 7R''\eta^2 + R'\eta\right)\\
&\quad - 6k^2\alpha^2 R\left(\eta R' + \eta^2 R''\right) - 6k^2\left(\alpha R'\eta\right)^2\\
&\quad + \left(k\lambda + 3L^2\right)\alpha^2\left(\eta R' + \eta^2 R''\right) = 0,
\end{aligned} \tag{47}$$

where $\eta = e^{\alpha z}$ $(\alpha \in \mathbb{C})$.

Substituting

$$U_{30}(z) = \frac{b_{12}}{(e^{\alpha z}-1)^2} + \frac{b_{11}}{e^{\alpha z}-1} + b_{10} \tag{48}$$

into (47), we obtain that

$$\sum_{i=1}^{5} c_{3i}\alpha^2 e^{(6-i)\alpha z}\left(e^{\alpha z}-1\right)^{-6} = 0, \tag{49}$$

where

$$\begin{aligned}
c_{31} &= k^4\alpha^2 b_{11} - 6k^2 b_0 b_{11} + 3b_{11}L^2 + b_{11}k,\\
c_{32} &= 10k^4\alpha^2 b_{11} + 16k^4\alpha^2 b_{12} + 12k^2 b_0 b_{11} - 24k^2 b_0 b_{12}\\
&\quad - 12k^2 {b_{11}}^2 - 6b_{11}L^2 + 12b_{12}L^2 - 2b_{11}k\lambda\\
&\quad + 4b_{12}k\lambda,\\
c_{33} &= 66k^4\alpha^2 b_{12} + 36k^2 b_0 b_{12} + 18k^2 {b_{11}}^2 - 54k^2 b_{12} b_{11}\\
&\quad - 18b_{12}L^2 - 6b_{12}k\lambda,\\
c_{34} &= -10k^4\alpha^2 b_{11} + 36k^4\alpha^2 b_{12} - 12k^2 b_0 b_{11}\\
&\quad + 36k^2 b_{12} b_{11} - 48k^2 {b_{12}}^2 + 6b_{11}L^2 + 2b_{11}k\lambda,\\
c_{35} &= -k^4\alpha^2 b_{11} + 2k^4\alpha^2 b_{12} + 6k^2 b_0 b_{11} - 12k^2 b_0 b_{12}\\
&\quad - 6k^2 {b_{11}}^2 + 18k^2 b_{12} b_{11} - 12k^2 {b_{12}}^2 - 3b_{11}L^2\\
&\quad + 6b_{12}L^2 - b_{11}k\lambda + 2b_{12}k\lambda.
\end{aligned} \tag{50}$$

Equating the coefficients of all powers about $e^{\alpha z}$ in (49) to zero, we get a system of algebraic equations:

$$\begin{aligned}
c_{31} &= 0,\\
c_{32} &= 0,\\
c_{33} &= 0,\\
c_{34} &= 0,\\
c_{35} &= 0.
\end{aligned} \tag{51}$$

Solving the above equations, we obtain

$$\begin{aligned}
b_{12} &= 2k^2\alpha^2,\\
b_{11} &= 2k^2\alpha^2,\\
b_{10} &= \frac{k^4\alpha^2 + 3L^2 + k\lambda}{6k^2}.
\end{aligned} \tag{52}$$

Therefore, simply periodic solutions to (15) with pole at $z = 0$ are

$$\begin{aligned}
U_{30}(z) &= \frac{2k^2\alpha^2}{(e^{\alpha z}-1)^2} + \frac{2k^2\alpha^2}{(e^{\alpha z}-1)} + \frac{k^4\alpha^2 + 3L^2 + k\lambda}{6k^2}\\
&= \frac{2k^2\alpha^2 e^{\alpha z}}{(e^{\alpha z}-1)^2} + \frac{k^4\alpha^2 + 3L^2 + k\lambda}{6k^2}\\
&= \frac{k^2\alpha^2}{2}\coth^2\frac{\alpha z}{2} + \frac{3L^2 + k\lambda - 2k^4\alpha^2}{6k^2}.
\end{aligned} \tag{53}$$

Similar to $U_{30}(z)$, we substitute

$$U_{40}(z) = \frac{b_{12}}{(e^{\alpha z}+1)^2} + \frac{b_{11}}{e^{\alpha z}+1} + b_{10} \tag{54}$$

into (47) to yield

$$\begin{aligned} b_{12} &= 2k^2\alpha^2, \\ b_{11} &= -2k^2\alpha^2, \\ b_{10} &= \frac{k^4\alpha^2 + 3L^2 + k\lambda}{6k^2}, \end{aligned} \tag{55}$$

and then

$$\begin{aligned} U_{40}(z) &= \frac{2k^2\alpha^2}{(e^{\alpha z}+1)^2} - \frac{2k^2\alpha^2}{(e^{\alpha z}+1)} + \frac{k^4\alpha^2 + 3L^2 + k\lambda}{6k^2} \\ &= -\frac{2k^2\alpha^2 e^{\alpha z}}{(e^{\alpha z}+1)^2} + \frac{k^4\alpha^2 + 3L^2 + k\lambda}{6k^2} \\ &= \frac{k^2\alpha^2}{2}\tanh^2\frac{\alpha z}{2} - \frac{2k^4\alpha^2 - 3L^2 - k\lambda}{6k^2}. \end{aligned} \tag{56}$$

Substituting

$$\begin{aligned} U_{50}(z) &= \frac{b_{14}}{(e^{\alpha z}-1)^2} + \frac{b_{13}}{(e^{\alpha z}-1)^2} + \frac{b_{12}}{e^{\alpha z}-1} \\ &\quad + \frac{b_{11}}{e^{\alpha z}+1} + b_{10} \end{aligned} \tag{57}$$

into (47) to yield

$$\begin{aligned} b_{14} &= 2k^2\alpha^2, \\ b_{13} &= 2k^2\alpha^2, \\ b_{12} &= 2k^2\alpha^2, \\ b_{11} &= -2k^2\alpha^2, \\ b_{10} &= \frac{k^4\alpha^2 + 3L^2 + k\lambda}{3k^2}, \end{aligned} \tag{58}$$

then

$$\begin{aligned} U_{50}(z) &= \frac{2k^2\alpha^2}{(e^{\alpha z}-1)^2} + \frac{2k^2\alpha^2}{(e^{\alpha z}+1)^2} + \frac{2k^2\alpha^2}{(e^{\alpha z}-1)} \\ &\quad - \frac{2k^2\alpha^2}{(e^{\alpha z}+1)} + \frac{k^4\alpha^2 + 3L^2 + k\lambda}{3k^2} \\ &= \frac{2k^2\alpha^2 e^{\alpha z}}{(e^{\alpha z}-1)^2} - \frac{2k^2\alpha^2 e^{\alpha z}}{(e^{\alpha z}+1)^2} + \frac{k^4\alpha^2 + 3L^2 + k\lambda}{3k^2} \\ &= \frac{k^2\alpha^2}{2}\coth^2\frac{\alpha z}{2} + \frac{k^2\alpha^2}{2}\tanh^2\frac{\alpha z}{2} \\ &\quad + \frac{3L^2 + k\lambda - 2k^4\alpha^2}{3k^2}. \end{aligned} \tag{59}$$

By the above procedures, we collect meromorphic solutions of (15) with arbitrary pole as follows:

$$\begin{aligned} U_1(z) &= -2k^2\wp(z) + \frac{k^2}{2}\left(\frac{\wp'(z) + D}{\wp(z) - C}\right)^2 \\ &\quad + \frac{3L^2 + k\lambda - 12k^4C}{6k^2}, \\ U_2(z) &= \frac{2k^2}{(z - z_0 - 1)^2} + \frac{3L^2 + k\lambda}{6k^2}, \\ U_3(z) &= \frac{k^2\alpha^2}{2}\coth^2\frac{\alpha(z - z_0)}{2} + \frac{3L^2 + k\lambda - 2k^4\alpha^2}{6k^2}, \\ U_4(z) &= \frac{k^2\alpha^2}{2}\tanh^2\frac{\alpha(z - z_0)}{2} - \frac{2k^4\alpha^2 - 3L^2 - k\lambda}{6k^2}, \\ U_5(z) &= \frac{k^2\alpha^2}{2}\coth^2\frac{\alpha(z - z_0)}{2} \\ &\quad + \frac{k^2\alpha^2}{2}\tanh^2\frac{\alpha(z - z_0)}{2} \\ &\quad + \frac{3L^2 + k\lambda - 2k^4\alpha^2}{3k^2}. \end{aligned} \tag{60}$$

where $z_0 \in \mathbb{C}$, $C^2 = 4D^3 - g_2 D - g_3$ and g_2 and g_3 are arbitrary constants.

The properties of the solutions are shown in Figures 6–8.

Remark. Based on the work of Yuan *et al.* [41, 42], we put forward the extended complex method for the first time. To our knowledge, the solutions obtained by this method have not been reported in former literature.

6. Comparisons and Conclusions

Borhanifar *et al.* [5] utilized the sine-cosine, the standard tanh, and the extended tanh methods to study the (2+1)-dimensional KP equation. We can observe that some important results to this equation have been obtained by using the above three methods. However, when we compare the results of this paper by applying two different systematic methods with the results of [5], the exponential function solutions and elliptic function solutions as a new aspect have been proposed to the literature. All these methods enrich the study of the (2+1)-dimensional KP equation.

The $\exp(-\psi(z))$-expansion method allows us to express the exact solutions of NLDEs as a polynomial of $\exp(-\psi(z))$, in which $\psi(z)$ satisfies the ODE (6). We can determine the degree of the polynomial through the homogeneous balance and obtain the values of the undetermined coefficients of the polynomial via the calculations of computer software, and then we obtain the exact solutions. With this method, we obtained seven solutions to the mentioned equation in which rational solution $u_7(z)$ is equivalent to $U_2(z)$ if we consider $z_0 = -a - 1$.

With the extended complex method, we can derive meromorphic solutions of the differential equations which do not satisfy $\langle p, q\rangle$ condition or are not Briot-Bouquet equation

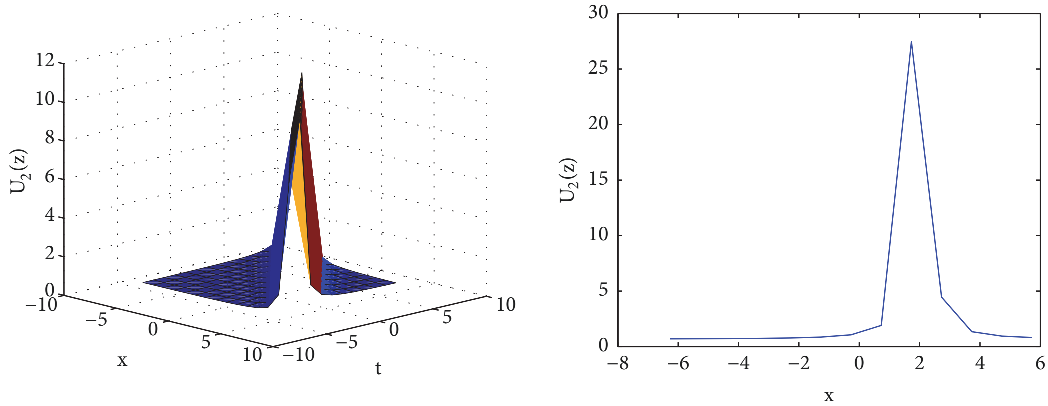

FIGURE 6: The 3D and 2D surfaces of $U_2(z)$ by considering the values $k = 1$, $L = 1$, $\lambda = 1$, $y = 0$, $z_0 = 1$, and $t = 0$ for the 2D graphic.

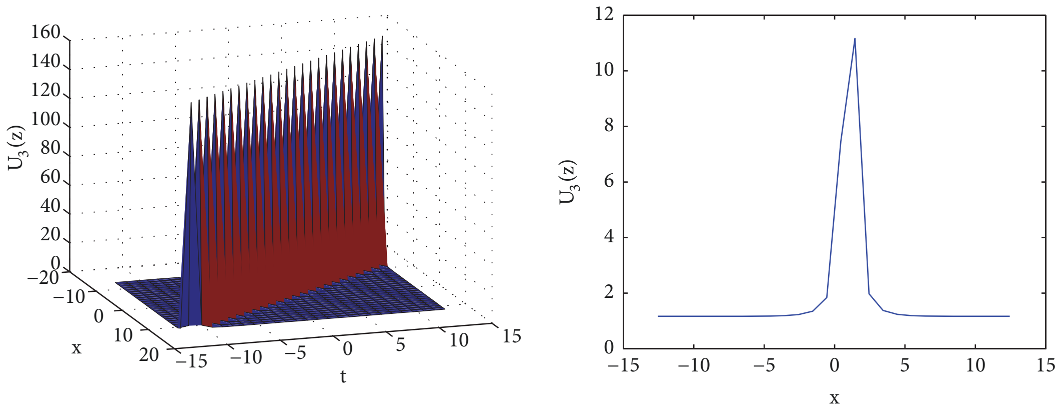

FIGURE 7: The 3D and 2D surfaces of $U_3(z)$ by considering the values $k = 1$, $L = 1$, $\lambda = 1$, $\alpha = 1$, $y = 0$, $z_0 = 1$, and $t = 0$ for the 2D graphic.

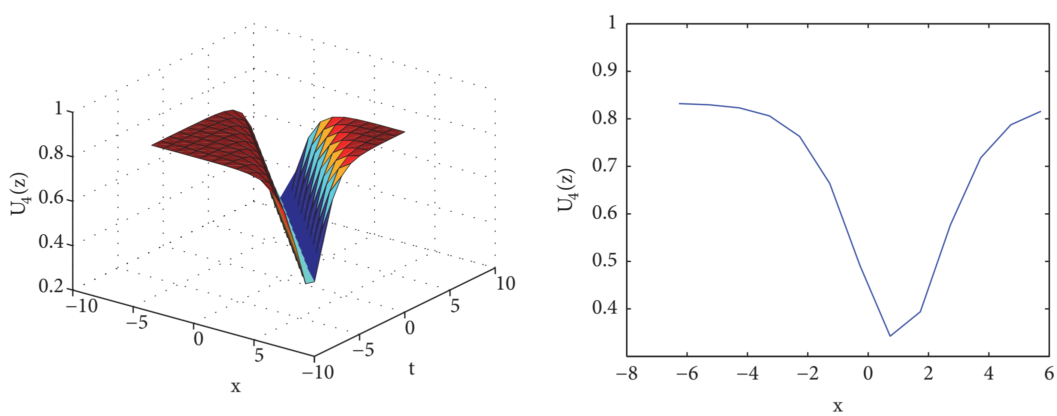

FIGURE 8: The 3D and 2D surfaces of $U_4(z)$ by considering the values $k = 1$, $L = 1$, $\lambda = 1$, $\alpha = 1$, $y = 0$, $z_0 = 1$, and $t = 0$ for the 2D graphic.

[41]. Therefore, more NLDEs in mathematical physics can be solved by the extended complex method. Using the indeterminate forms of the solutions, we are able to seek meromorphic solutions $U(z)$ for the differential equation with a pole at $z = 0$, and then we can derive meromorphic solutions $U(z - z_0)$, $z_0 \in \mathbb{C}$, for the differential equation with an arbitrary pole.

In this article, we search for analytical solutions of the (2+1)-dimensional KP equation by two different systematic methods. By the $\exp(-\psi(z))$-expansion method and extended complex method, we obtain five kinds of exact solutions. The results demonstrate that these two methods are efficient and direct methods allowing us to do complicated and tedious algebraic calculation.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no competing interests.

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