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Research Article

Adaptive Asymptotic Tracking Control for a Class of Uncertain Switched Systems via Dynamic Surface Technique

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A novel adaptive tracking control scheme is proposed for a class of uncertain nonlinear switched systems with perturbations in this paper. The common Lyapunov function method is introduced to handle the switched system in the design process of the desired adaptive controller. In addition, a dynamic surface control method is proposed by employing a nonlinear filter such that the "explosion of complexity" problem existing in the conventional backstepping design can be overcome. Under the presented adaptive controller, all the closed-loop signals are semiglobally bounded, and especially the output signal of the controlled system can follow the given reference signal asymptotically. To show the availability of the presented control scheme, a simulation is given in this paper.

1. Introduction

Switched systems, as a typical class of hybrid systems, have attracted many researchers' sights in the past decades (e.g., see [1–14]) since many physical systems can be mathematically modeled by the switched systems. Owing to the large-scale applications, the research on switched systems never stops and great achievements have been made (e.g., see [1–6]). The stabilization problem for a class of slowly switched systems based on unstable subsystems is studied in [6]. The optimal control scheme of the switching positive system is presented in [7]. The stabilization problem of switched linear systems with mode-dependent average dwell time is addressed in [8]. Most significant issues on switched systems have been acquired for multitudinous switched systems under arbitrary switching or constraint switching (e.g., see [8–11]), and references therein.

Adaptive control of uncertain nonlinear systems has achieved significant research results (e.g., see [15–25]) including adaptive feedback linearization [15], adaptive backstepping [16], immersion and invariance adaptive control [17], and adaptive neural network/fuzzy-logic control [21, 25]. Very recently, some novel adaptive control schemes have been established for uncertain systems with the unknown control directions or the parameter estimator triggering [26–28]. However, the "explosion of complexity" problem exists in the repeated differentiations of virtual control variables [18, 19] in the backstepping approach. Thus, with the system order increasing, the computation of the backstepping controller will be more complicate.

To overcome the "explosion of complexity" problem in the backstepping design process, for uncertain strict-feedback/nonstrict-feedback nonlinear systems, the dynamic surface control (DSC) technique is established (e.g., see [29–38]). In literature studies [29, 30], a virtual control law is designed in each design step by using a low-pass filter, thus it can avoid the derivative of the virtual controllers. Another advantage of the DSC approach is that it can reduce the requirement on the smoothness of plant functions and desired signal obviously. Therefore, the DSC technique has a large-scale application in the process of designing simplified adaptive controllers for uncertain nonlinear systems. The adaptive robust DSC with composite adaptation laws is designed for the uncertain system with the semistrict feedback form in [31]. Based on multiple models, the result is applied to enhance the transient response of an adaptive DSC system in [32]. An adaptive DSC scheme is presented by using composite learning to ensure parameter convergence without the persistent excitation condition in [33]. Various adaptive backstepping schemes using neural networks (NNs) or fuzzy logic systems as approximators are proposed to deal with the time delays, dynamic uncertainties, and output dead zone for stochastic large-scale nonlinear systems in [34–37]. Very recently, a novel DSC method with the nonlinear filter is proposed for a class of uncertain systems, and the asymptotic tracking control performance is achieved in [38].

In the above discussions, some adaptive DSC schemes have been developed for uncertain switched systems. Nevertheless, these control schemes can just guarantee the semiglobal boundedness for the controlled systems, and the asymptotic stability cannot be achieved since the stability analysis cannot be completed by the conventional DSC technique with a linear low-pass filter. In this paper, we try to address this issue by employing the DSC method with a nonlinear filter. The main contributions of this work are summarized as following.

(i) As far as we know now, this is the first work to address the asymptotic tracking control problem for uncertain switched systems by using the DSC method to solve the "explosion of complexity" problem existing in the conventional backstepping design.

(ii) By introducing the DSC method with a nonlinear filter proposed in [38], the desired controller is developed based on the common Lyapunov function, and then the stability analysis of the closed-loop switching control system is completed according to Barbalat's lemma.

This paper is organized as follows. The problem statement and some preliminaries are introduced and adaptive DSC scheme with a detailed stability analysis are presented in Section 2. Then, a simulation example is given in Section 3. A conclusion is drawn in Section 4.

2. Problem Statement and Main Result

2.1. Problem Statement and Some Preliminaries. Taking the following class of uncertain strict-feedback nonlinear switched systems into account,

$$\begin{cases} \dot{x}_i = x_{i+1} + \theta_i f_{i,\varrho(t)}\left(\overline{x_i}\right) + d_i(t), \quad i = 1,\dots,n-1, \\ \dot{x}_n = u + \theta_n f_{n,\varrho(t)}\left(\overline{x_n}\right) + d_n(t), \\ y = x_1, \end{cases} \tag{1}$$

where $\overline{x}_i = [x_1,\dots,x_i]^T \in R^i$, $i = 1,\dots,n$ are the states of the system, $u \in R$ is the control input, and $y \in R$ is the system output. For $i = 1,\dots,n$, θ_i are the unknown constants, $d_i(t)$ are the perturbations, and $f_i(\cdot) : R^n \longrightarrow R$ are the known continuously differentiable functions. $\varrho(t) : [0,\infty)$ is the switching signal, and p denotes the number of subsystems in the switched system.

The control objective of this paper for system (1) is to design an adaptive DSC law u such that the output $y(t)$ asymptotically tracks a desired trajectory $y_r(t)$, and the boundedness of all the signals in the closed-loop system is guaranteed.

Assumption 1. The desired trajectory $y_r(t)$ and its derivatives $\dot{y}_r(t)$ and $\ddot{y}_r(t)$ are bounded and available.

Assumption 2. The perturbations $d_i(t)$, $i = 1,\dots,n$ are bounded, i.e., $|d_i(t)| \le W_i$ with the constants $W_i > 0$.

Lemma 1 (see [39]). *For any $\varepsilon > 0$ and $z \in R$, the following inequality can be obtained:*

$$0 \le |z| - \frac{z^2}{\sqrt{z^2 + \varepsilon^2}} < \varepsilon. \tag{2}$$

2.2. Adaptive Controller Design. The presented adaptive DSC scheme is similar to the backstepping technique, and it contains n steps as follows. Define the estimate error $\tilde{*} = * - \hat{*}$, where $\hat{*}$ is the estimate of $\hat{*}$. Let $\varepsilon_i = |\theta_i|$, $i = 1,\dots,n$.

Step 1. The first surface error is defined as $z_1 = x_1 - y_r$, and the time derivative of z_1 is

$$\dot{z}_1 = \dot{x}_1 - \dot{y}_r = x_2 + \theta_1 f_{1,\varrho(t)}\left(\overline{x_1}\right) + d_1(t) - \dot{y}r. \tag{3}$$

Then, the Lyapunov function candidate V_1 is defined as

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2\gamma_1}\tilde{\varepsilon}_1^2 + \frac{1}{2\beta_1}\widetilde{W}_1^2, \tag{4}$$

where γ_1 and β_1 are positive design parameters.

Considering equations (1)–(4), the following equation can be obtained:

$$\begin{aligned} \dot{V}_1 &= z_1\left(x_2 + \theta_1 f_{1,\varrho(t)}\left(\overline{x_1}\right) + d_1(t) - \dot{y}_r\right) - \frac{1}{\gamma_1}\tilde{\varepsilon}_1\dot{\hat{\varepsilon}} - \frac{1}{\beta_1}\widetilde{W}_1\dot{\hat{W}}_1 \\ &= z_1\left(x_2 + \theta_1 f_{1,\varrho(t)}\left(\overline{x_1}\right) + d_1(t) - \dot{y}_r + \alpha_1 - \alpha_1\right) - \frac{1}{\gamma_1}\tilde{\varepsilon}_1\dot{\hat{\varepsilon}} \\ &\quad - \frac{1}{\beta_1}\widetilde{W}_1\dot{\hat{W}}_1. \end{aligned} \tag{5}$$

On the basis of $f_{i,\varrho(t)}$, we can obtain the following inequality:

$$\left|f_{i,\varrho(t)}\right| \leq \sqrt{\sum_{j=1}^{p} f_{i,j}^2}. \tag{6}$$

The above inequality is introduced to design the desired controller by employing the common Lyapunov function method. In order to develop an adaptive controller without the switching signal $\varrho(t)$, the subsystem function $f_{i,\varrho(t)}$ is bounded by an upper bound function which is without the switching signals.

Let $D_i = \sqrt{\sum_{j=1}^{p} f_{i,j}^2}$. The following inequality can be obtained:

$$\begin{aligned} z_1\theta_1 f_{1,\varrho(t)} &\leq |z_1||\theta_1||f_{i,\varrho(t)}| \\ &\leq |z_1||\theta_1| D_1 \\ &\leq \varepsilon_1 \frac{z_1^2 D_1^2}{\sqrt{z_1^2 D_1^2 + \sigma(t)^2}} + \varepsilon_1 \sigma(t), \end{aligned} \tag{7}$$

and the following inequality can be obtained by using Assumption 2

$$\begin{aligned} z_1 d_1(t) &\leq |z_1||d_1(t)| \leq |z_1| W_1 = |z_1|\left(\hat{W}_1 + \hat{W}_1\right) \\ &\leq \frac{z_1^2 \hat{W}_1^2}{\sqrt{z_1^2 \hat{W}_1^2 + \sigma(t)^2}} + \sigma(t) + |z_1| \widetilde{W}_1. \end{aligned} \tag{8}$$

On designing the virtual control law α_1 as

$$\alpha_1 = -k_1 z_1 + \dot{y}_r - \hat{\varepsilon}_1 \frac{z_1 D_1^2}{\sqrt{z_1^2 D_1^2 + \sigma(t)^2}} - \frac{z_1 \hat{W}_1^2}{\sqrt{z_1^2 \hat{W}_1^2 + \sigma(t)^2}}, \tag{9}$$

the update laws of $\hat{\varepsilon}_1$ and $\hat{W}_1$ can be designed as

$$\dot{\hat{\varepsilon}}_1 = \gamma_1 \frac{z_1^2 D_1^2}{\sqrt{z_1^2 D_1^2 + \sigma(t)^2}}, \tag{10}$$

$$\dot{\hat{W}}_1 = \beta_1 |z_1|. \tag{11}$$

In view of (5)–(11), one has

$$\begin{aligned} \dot{V}_1 &= z_1\Bigg(x_2 + \theta_1 f_{1,\varrho(t)}\overline{(x_1)} + d_1(t) - \dot{y}_r + \Bigg(-k_1 z_1 + \dot{y}_r \\ &\quad - \hat{\varepsilon}_1 \frac{z_1 D_1^2}{\sqrt{z_1^2 D_1^2 + \sigma(t)^2}} - \frac{z_1 \widehat{W}_1^2}{\sqrt{z_1^2 \widehat{W}_1^2 + \sigma(t)^2}}\Bigg) - \alpha_1\Bigg) \\ &\quad - \frac{1}{\gamma_1}\widetilde{\varepsilon}_1\left(\gamma_1 \frac{z_1^2 D_1^2}{\sqrt{z_1^2 D_1^2 + \sigma(t)^2}}\right) - \frac{1}{\beta_1}\hat{W}_1\left(\beta_1 |z_1|\right) \\ &\leq -k_1 z_1^2 + (x_2 - \alpha_1) z_1 + (1 + \varepsilon_1)\sigma(t). \end{aligned} \tag{12}$$

Design the second error signal in the backstepping as $x_2 - \alpha_1$ such that it can avoid the "explosion of complexity" problem. Let α_1 pass through the following novel nonlinear filter to obtain a filtered virtual controller s_1:

$$\begin{aligned} \tau_1 \dot{s}_1 &= -e_1 - \frac{\tau_1 \widehat{M}_1^2 e_1}{\sqrt{\widehat{M}_1^2 e_1^2 + \sigma^2(t)}} - \tau_1 z_1, \\ s_1(0) &= \alpha_1(0), \end{aligned} \tag{13}$$

where $e_1 = s_1 - \alpha_1$ is the first boundary layer error, $\tau_1 > 0$ is a design parameter, $\widehat{M}_1$ is the estimate of M_1 and it will be clarified later, and $\sigma(t)$ is any positive uniform continuous and bounded function, which satisfies

$$\begin{aligned} \lim_{t \to \infty} \int_0^t \sigma(\tau) d\tau &\leq \sigma_1 < +\infty, \\ |\dot{\sigma}(t)| &\leq \sigma_2 < +\infty, \end{aligned} \tag{14}$$

where σ_1 and σ_2 are any positive constants.

Remark 1. In this paper, the nonlinear filter (13) is introduced to construct the desired controller. Compared with the reported results on adaptive DSC with a linear low-pass filter (e.g., see [30–37]), the advantage of our deign is that the asymptotic tracking control performance can be guaranteed, and the stability analysis can be completed successfully under the proposed controller.

Step i $(i = 2, \ldots, n-1)$. The ith surface error is designed as $z_i = x_i - s_{i-1}$; then, we get the following equation:

$$\dot{z}_i = x_{i+1} + \theta_i f_{i,\varrho(t)} + d_i(t) + \frac{\widehat{M}_{i-1}^2 e_{i-1}}{\sqrt{\widehat{M}_{i-1}^2 e_{i-1}^2 + \sigma^2(t)}} + z_{i-1} + \frac{e_{i-1}}{\tau_{i-1}}. \tag{15}$$

Design the virtual control law α_i and the update laws $\widehat{\varepsilon}_i$ and $\widehat{W}_i$ as follows:

$$\begin{aligned} \alpha_i &= -k_i z_i - 2z_{i-1} - \hat{\varepsilon}_i \frac{z_i D_i^2}{\sqrt{z_i^2 D_i^2 + \sigma^2(t)}} \\ &\quad - \frac{z_i \hat{W}_i^2}{\sqrt{z_i^2 \hat{W}_i^2 + \sigma^2(t)}} - \frac{\hat{M}_{i-1}^2 e_{i-1}}{\sqrt{\hat{M}_{i-1}^2 e_{i-1}^2 + \sigma^2(t)}} - \frac{e_{i-1}}{\tau_{i-1}}, \end{aligned} \tag{16}$$

$$\dot{\hat{\varepsilon}}_1 = \gamma_i \frac{z_i^2 D_i^2}{\sqrt{z_i^2 D_i^2 + \sigma^2(t)}}, \tag{17}$$

$$\dot{\hat{W}}_1 = \beta_i |z_i|, \tag{18}$$

where k_i, γ_i, and β_i are the positive design parameters.

Design the Lyapunov function candidate V_i as

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{1}{2\gamma_i}\widetilde{\varepsilon}_i^2 + \frac{1}{2\beta_i}\widetilde{W}_i^2, \tag{19}$$

where γ_i and β_i are the positive design parameters.

In view of equations (15)–(19), consider the time derivative of V_i as

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + z_i\dot{z}_i - \frac{1}{\gamma_i}\widetilde{\varepsilon}_i\dot{\hat{\varepsilon}}_i - \frac{1}{\beta_i}\widetilde{W}_i\dot{\hat{W}}_i \\ &\le -\sum_{j=1}^{i}k_jz_j^2 + \sum_{j=1}^{i-1}z_je_j + (x_{i+1}-\alpha_i)z_i + \left(i + \sum_{j=1}^{i}\varepsilon_j\right)\sigma(t).\end{aligned} \tag{20}$$

Let α_i pass through the following nonlinear filter to obtain a filtered virtual controller s_i

$$\begin{aligned}\tau_i\dot{s}_i &= -e_i - \frac{\tau_i\hat{M}_i^2e_i}{\sqrt{\hat{M}_i^2e_i^2 + \sigma^2(t)}} - \tau_iz_i, \\ s_i(0) &= \alpha_i(0),\end{aligned} \tag{21}$$

and define

$$e_i = s_i - \alpha_i, \tag{22}$$

where e_i means the ith boundary layer error and τ_i is a filter time constant.

Step n. Considering the nth surface error as $z_n = x_n - s_{n-1}$, one has

$$\dot{z}_n = u + \theta_nf_{n,\varrho(t)} + d_n(t) + \frac{\widehat{M}_{n-1}^2e_{n-1}}{\sqrt{\widehat{M}_{n-1}^2e_{n-1}^2 + \sigma^2(t)}} + z_{n-1} + \frac{e_{n-1}}{\tau_{n-1}}. \tag{23}$$

Design the actual control law u as

$$\begin{aligned}u = &-k_nz_n - 2z_{n-1} - \hat{\varepsilon}_n\frac{z_nD_n^2}{\sqrt{z_n^2D_n^2 + \sigma^2(t)}} \\ &- \frac{z_n\hat{W}_n^2}{\sqrt{z_n^2\hat{W}_n^2 + \sigma^2(t)}} - \frac{\hat{M}_{n-1}^2e_{n-1}}{\sqrt{\hat{M}_{n-1}^2e_{n-1}^2 + \sigma^2(t)}} - \frac{e_{n-1}}{\tau_{n-1}},\end{aligned} \tag{24}$$

and consider the update laws for $\widehat{\varepsilon}_i$ and $\widehat{W}_n$ as follows:

$$\dot{\hat{\varepsilon}}_n = \gamma_n\frac{z_n^2D_n^2}{\sqrt{z_n^2D_n^2 + \sigma^2(t)}}, \tag{25}$$

$$\dot{\hat{W}}_n = \beta_n|z_n|, \tag{26}$$

where k_n, β_n, and γ_n are the positive design parameters.

Design the Lyapunov function candidate V_n as follows:

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2\gamma_n}\widetilde{\varepsilon}_n^2 + \frac{1}{2\beta_n}\widetilde{W}_n^2, \tag{27}$$

where γ_n and β_n are the positive design parameters.

Then, considering equations (23)–(27), we have

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + z_n\dot{z}_n - \frac{1}{\gamma_n}\widetilde{\varepsilon}_n\dot{\hat{\varepsilon}}_n - \frac{1}{\beta_n}\widetilde{W}_n\dot{\hat{W}}_n \\ &\le -\sum_{j=1}^{n}k_jz_j^2 + \sum_{j=1}^{n-1}z_je_j + \left(n + \sum_{j=1}^{n}\varepsilon_j\right)\sigma(t).\end{aligned} \tag{28}$$

2.3. Stability Analysis. Based on inequality (28), the main result of this paper is presented by the following theorem.

Theorem 1. *Consider the closed-loop system consisting of the plant (1), the nonlinear filters (13), (22), the actual controller (24), and the adaptive laws (10), (11), (17), (18), (25), and(26). Suppose that Assumptions 1-2 hold, for any initial conditions satisfying $V(0) \le q$, where q is a given constant, there exit design parameters k_i, β_i, γ_i, $i = 1, \ldots, n$, τ_j, and η_j, $j = 1, \ldots, n-1$, such that the following statements hold:*

(i) All the resulting closed-loop signals are semiglobally bounded

(ii) The tracking error $z_1 = y - y_r$ converges to zero asymptotically

Proof. The compact sets are defined as

$$\begin{aligned}\Omega_1 &= \left\{[y_r, \dot{y}_r, \ddot{y}_r]^T : y_r^2 + \dot{y}_r^2 + \ddot{y}_r^2 \le B_0\right\}, \\ \Omega_2 &= \{V(t) \le q\},\end{aligned} \tag{29}$$

where B_0 is a known positive constant. Note that set $\Omega_1 \times \Omega_2$ is also a compact in R^{4n+1}. $\sigma(t)$ and $\dot{\sigma}(t)$ are the bounded functions.

Differentiating the boundary layer errors $e_i = s_i - \alpha_i$ yields

$$\begin{aligned}\dot{e}_i = &\frac{-e_i}{\tau_i} - \frac{\hat{M}_i^2e_i}{\sqrt{\hat{M}_i^2e_i^2 + \sigma^2(t)}} - z_i + B_i(z_1, \ldots, z_{i+1}, e_1, \ldots, \\ &e_i, \hat{\varepsilon}_1, \ldots, \hat{\varepsilon}_i, \hat{M}_1, \ldots, \hat{M}_i, \hat{W}_1, \ldots, \hat{W}_i, y_r, \ddot{y}_r, \ddot{y}_r, \dddot{y}_r, \sigma(t), \\ &\dot{\sigma}(t)), \quad i = 1, \ldots, n-1,\end{aligned} \tag{30}$$

where

$$\begin{aligned}B_1(\cdot) &= -\dot{\alpha}_1 \\ &= -\frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 - \frac{\partial\alpha_1}{\partial\widehat{\varepsilon}_1}\dot{\hat{\varepsilon}}_1 - \frac{\partial\alpha_1}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_1}{\partial\dot{y}_r}\ddot{y}_r - \frac{\partial\alpha_1}{\partial\widehat{W}_1}\dot{\hat{W}}_1, \\ B_i(\cdot) &= -\dot{\alpha}_i \\ &= -\sum_{j=1}^{i}\frac{\partial\alpha_i}{\partial x_j}\dot{x}_j - \frac{\partial\alpha_i}{\partial\widehat{\varepsilon}_j}\dot{\hat{\varepsilon}}_j - \frac{\partial\alpha_i}{\partial e_{i-1}}\dot{e}_{i-1} \\ &\quad - \frac{\partial\alpha_i}{\partial\widehat{M}_{i-1}}\dot{\hat{M}}_{i-1} - \frac{\partial\alpha_i}{\partial\widehat{W}_i}\dot{\hat{W}}_i - \frac{\partial\alpha_i}{\partial y_r}\dot{y}_r - \frac{\partial\alpha_i}{\partial\dot{y}_r}\ddot{y}_r - \frac{\partial\alpha_i}{\partial\sigma(t)}\dot{\sigma}(t),\end{aligned} \tag{31}$$

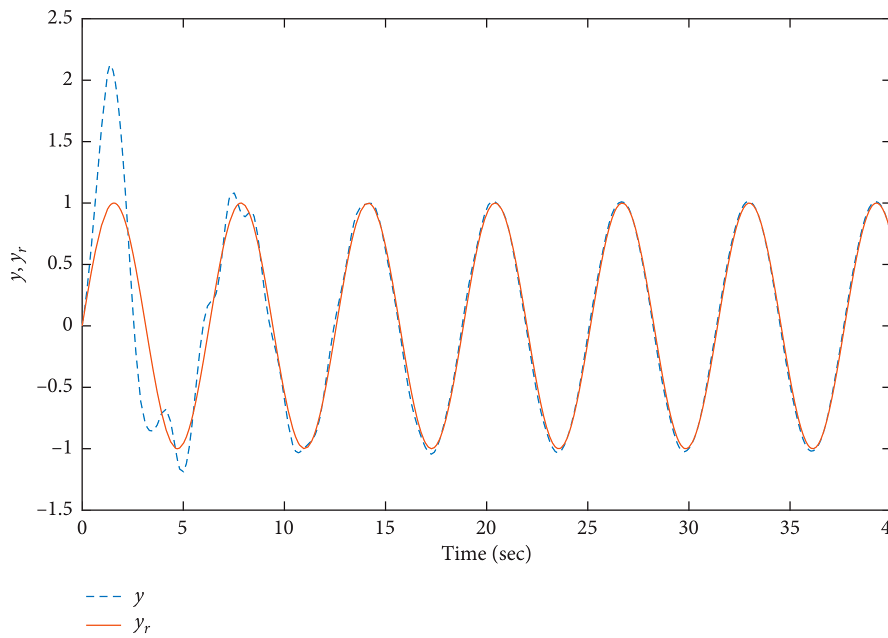

Figure 1: The reference signal y_r and the output signal y.

are the continuous functions on $\Omega_1 \times \Omega_2$. As a consequence, a positive constant M_i can be obtained such that $|B_i(\cdot)| \leq M_i$, where $M_i > 0$ is an unknown constant.

We consider the Lyapunov function candidate as follows:

$$V = V_n + \sum_{i=1}^{n-1} \frac{1}{2} e_i^2 + \sum_{i=1}^{n-1} \frac{1}{2\eta_i} \widetilde{M}_i^2, \tag{32}$$

where η_i, $i = 1, \ldots, n-1$ are the positive design parameters.

The time derivative of V is

$$\begin{aligned} \dot{V} &= \dot{V}_n + \sum_{i=1}^{n-1} e_i \dot{e}_i - \sum_{i=1}^{n-1} \frac{1}{\eta_i} \widetilde{M}_i \dot{\widehat{M}}_i \\ &\leq -\sum_{i=1}^{n} k_i z_i^2 - \sum_{i=1}^{n-1} \frac{e_i^2}{\tau_i} - \sum_{i=1}^{n-1} \frac{\widehat{M}_i^2 e_i}{\sqrt{\widehat{M}_i^2 e_i^2 + \sigma^2(t)}} \\ &\quad + \sum_{i=1}^{n-1} M_i |e_i| - \sum_{i=1}^{n-1} \frac{1}{\eta_i} \widetilde{M}_i \dot{\widehat{M}}_i + \left(n + \sum_{j=1}^{n} \varepsilon_j \right) \sigma(t). \end{aligned} \tag{33}$$

We can obtain the following inequality in view of Lemma 1:

$$\begin{aligned} M_i |e_i| &= \widehat{M}_i |e_i| + \widehat{M}_i |e_i| \\ &\leq \frac{\widehat{M}_i^2 e_i^2}{\sqrt{\widehat{M}_i^2 e_i^2 + \sigma^2(t)}} + \sigma(t) + \widetilde{M}_i |e_i|, \end{aligned} \tag{34}$$

and then, we have

$$\dot{V} \leq -\sum_{i=1}^{n} k_i z_i^2 - \sum_{i=1}^{n-1} \frac{e_i^2}{\tau_i} - \sum_{i=1}^{n-1} \frac{1}{\eta_i} \widetilde{M}_i \left(\dot{\widehat{M}}_i - \beta_i |e_i| \right) + \varpi \sigma(t), \tag{35}$$

where $\varpi := 2n + \sum_{j=1}^{n} \varepsilon_j - 1$ is a constant.

Consider the update laws for $\widehat{M}_i$ as

$$\dot{\widehat{M}}_i = \eta_i |e_i|, \quad i = 1, \ldots, n-1. \tag{36}$$

From equation (35), the following inequality can be obtained:

$$\dot{V} \leq -\sum_{i=1}^{n} k_i z_i^2 - \sum_{i=1}^{n-1} \frac{e_i^2}{\tau_i} + \varpi \sigma(t). \tag{37}$$

Integrating inequality (37) over $[0, t]$ yields

$$\begin{aligned} V(t) &\leq V(0) - \int_0^t \left(\sum_{i=1}^{n} k_i z_i^2(\kappa) + \sum_{i=1}^{n} \frac{e_i^2(\kappa)}{\tau_i} \right) d\kappa + \varpi \int_0^t \sigma(\kappa) d\kappa \\ &\leq V(0) + \varpi \sigma_1, \end{aligned} \tag{38}$$

which means that z_i, z_n, $\tilde{\varepsilon}_i$, $\hat{\varepsilon}_n$, $\widehat{W}_i$, $\widehat{W}_n$, e_i, and $\widehat{M}_i$,$i = 1, \ldots, n-1$ are bounded. Therefore, we also conclude that x_i, x_n, s_i, α_i, and u, $i = 1, \ldots, n-1$ are bounded. From inequality (38), we can obtain the following inequality:

$$\int_0^t \sum_{i=1}^{n} k_i z_i^2(\kappa) d\kappa \leq V(0) + \varpi \sigma_1. \tag{39}$$

By applying Barbalat's lemma [40, 41] to inequality (39), it is concluded that

$$\lim_{t \longrightarrow \infty} z_1 = 0, \tag{40}$$

that is to say, the asymptotic stability is accomplished.

Remark 2. For uncertain switched systems (e.g., see [21–23]), lots of adaptive control schemes have been developed by the conventional backstepping method. This paper

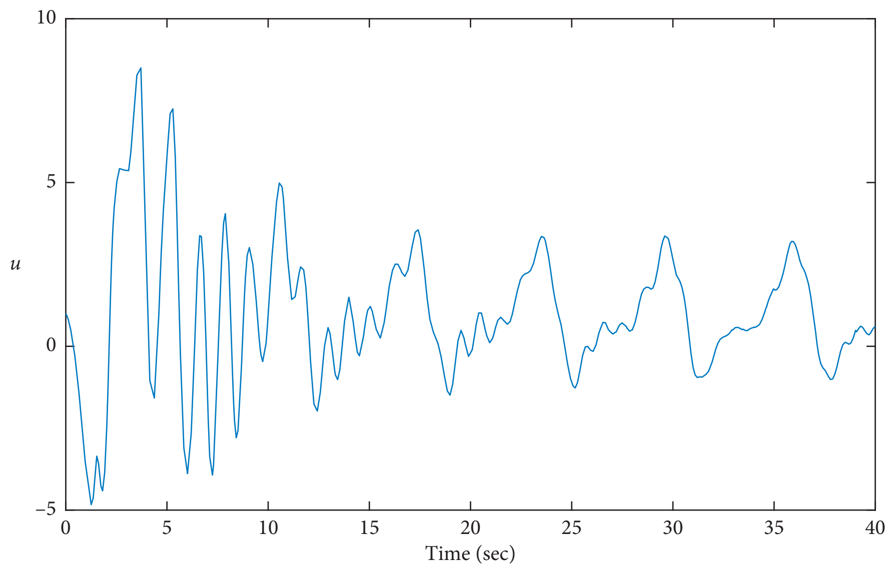

Figure 2: The control signal u.

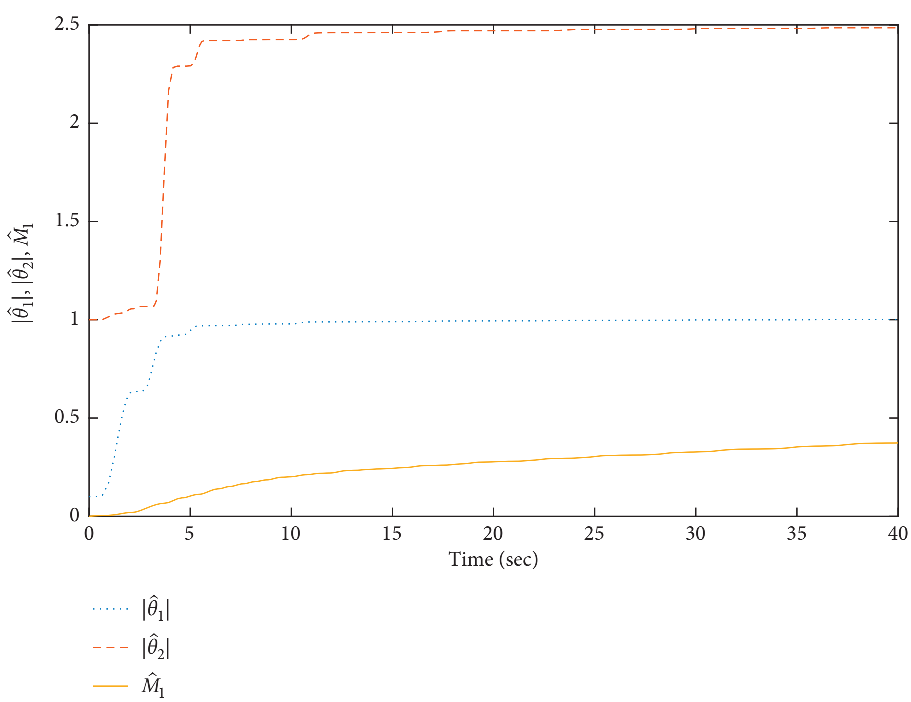

Figure 3: Adaptive parameters $|\hat{\theta}_1|$, $|\hat{\theta}_2|$, and $\hat{M}_1$.

introduces the adaptive DSC method to solve the "explosion of complexity" problem existing in the aforementioned literature studies, and the asymptotic tracking performance has been achieved under the developed adaptive controller.

Remark 3. Recently, some novel adaptive control schemes have been proposed for uncertain systems with time-varying performance bounds and actuator failures in [42, 43], where the "explosion of complexity" problem cannot be well solved and the asymptotic tracking performance is not achieved. Of course, this paper does not consider the actuator failure phenomenon of the controlled system, which is the further study program in the future.

3. Simulation Example

Consider the following nonlinear switched system:

$$\begin{cases} \dot{x}_1 = x_2 + \theta_1 f_{1,\varrho(t)}\overline{(x_1)} + d_1(t), \\ \dot{x}_2 = u + \theta_2 f_{2,\varrho(t)}\overline{(x_2)} + d_2(t), \\ y = x_1, \end{cases} \tag{41}$$

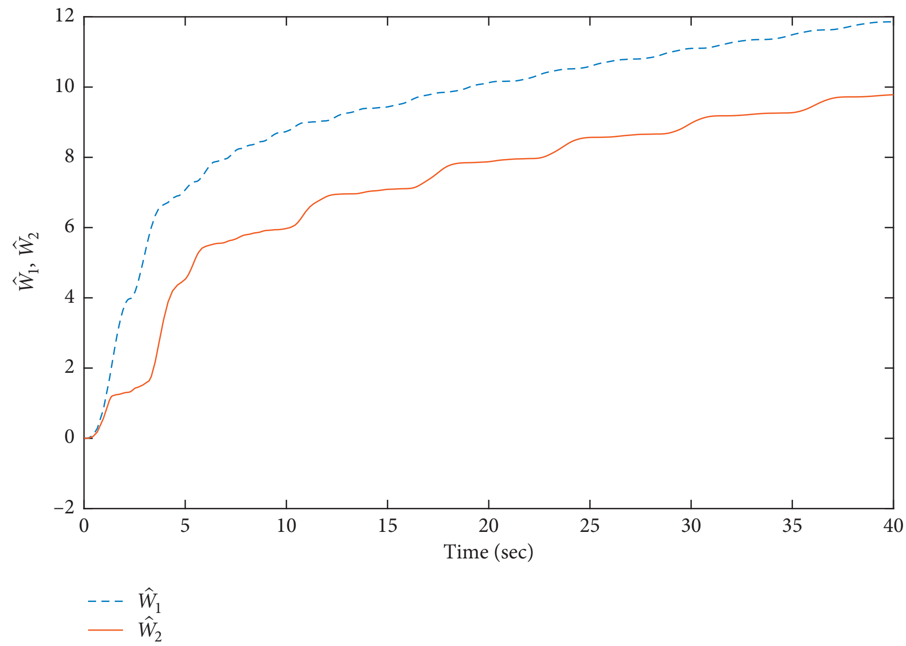

Figure 4: Adaptive parameters $\widehat{W}_1$ and $\widehat{W}_2$.

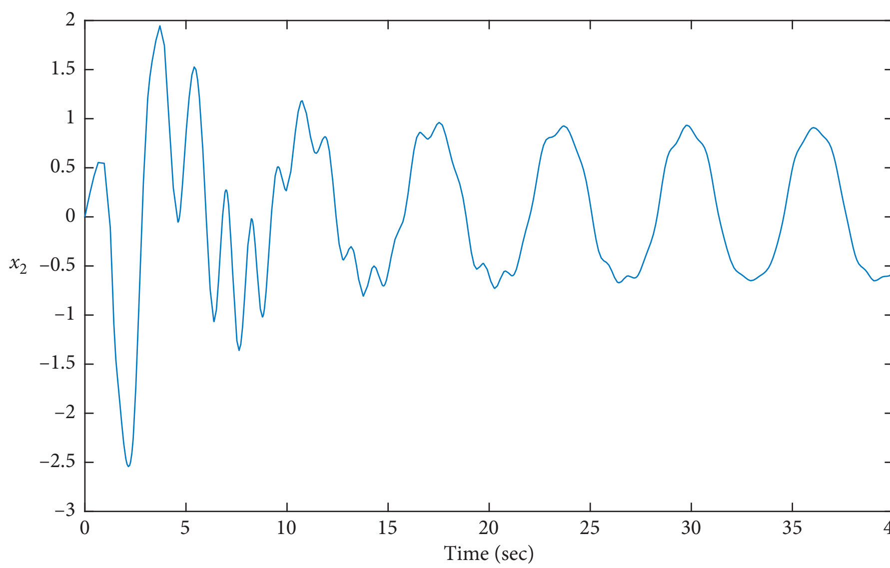

Figure 5: State x_2.

where θ_1 and θ_2 are the unknown parameters; $d_1(t) = \cos(t)$ and $d_2(t) = \sin(t)$; $\varrho(t) \in \{1, 2\}$; and the switched functions are designed as follows: $f_{1,1} = x_1 e^{-0.5x_1}$, $f_{1,2} = \sin(x_1)$, $f_{2,1} = \sin(x_1 x_2)$, and $f_{2,2} = x_1 x_2^2$. When the output y satisfies $y > 0$, $f_{i,\sigma(t)} = f_{i,1}$, $i = 1, 2$. Otherwise, $f_{i,\sigma(t)} = f_{i,2}$, $i = 1, 2$. For simulation purpose, it is assumed that $\theta_1 = 1$ and $\theta_2 = 2.5$. The objective of control is that the output $y(t)$ asymptotically tracks the desired trajectory $y_r(t) = \sin(t)$ via the proposed adaptive controller u.

The design parameters are selected as $k_1 = 1.2$, $k_2 = 2$, $\gamma_1 = 1.12$, $\gamma_2 = 1.02$, $\beta_1 = \beta_2 = 3$, $\eta_1 = 0.25$, $\tau_1 = 1$, and $\sigma(t) = 3e^{-0.0001t}$. The initial conditions of this switched system are chosen as $[x_1(0), x_2(0)]^T = [0, 0]^T$, $\hat{\varepsilon}_1(0) = 0.1$, $\hat{\varepsilon}_2(0) = 1$, $\hat{W}_1(0) = \hat{W}_2(0) = 0$, and $\hat{M}_1(0) = 0$. The simulation results are displayed in Figures 1–5. Figure 1 shows the output tracking performance under the presented DSC scheme, from which it can be analyzed that the asymptotic output tracking has been achieved. Figure 2 shows the control signal u. Figures 3 and 4 show the adaptive parameters. The state x_2 is shown in Figure 5. The switching signal is presented in Figure 6. All these simulation results show that the closed-loop signals are bounded.

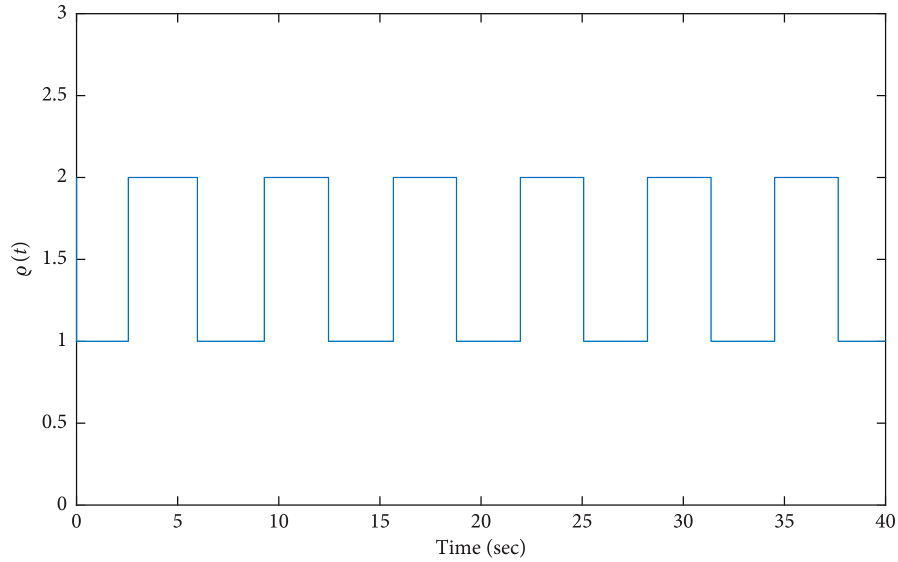

Figure 6: Switching signal $\varrho(t)$.

4. Conclusions

This paper studies the adaptive tracking control problem for a class of uncertain nonlinear systems with perturbations and switching signal. To deal with the switched signals of the considered system, the common Lyapunov function method is employed to deign the desired controller. Furthermore, according to a nonlinear filter, a DSC method is presented to overcome the "explosion of complexity" problem. Under the proposed controller, it has been expressed that all the closed-loop signals remain semiglobally bounded, and the tracking error converges to zero asymptotically. In the end of this paper, to checkout the validity of the control scheme, a simulation example is presented.

Data Availability

This paper is a theoretical study and no data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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