

Research Article

Finite-Time Synchronization of Uncertain Complex Dynamic Networks with Nonlinear Coupling

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The finite-time synchronization control is studied in this paper for a class of nonlinear uncertain complex dynamic networks. The uncertainties in the network are unknown but bounded and satisfy some matching conditions. The coupling relationship between network nodes is described by a nonlinear function satisfying the Lipchitz condition. By introducing a simple Lyapunov function, two main results regarding finite-time synchronization of a class of complex dynamic networks with parameter uncertainties are derived. By employing some analysis techniques like matrix inequalities, suitable controllers can be designed based on the obtained synchronization criteria. Moreover, with the obtained control input, the time instant required for the system to achieve finite-time synchronization can be estimated if a set of LMIs are feasible or an assumption on the eigenvalues of some matrices can be satisfied. Finally, the effectiveness of the proposed results is verified by numerical simulation.

1. Introduction

Synchronization is a very important nonlinear phenomenon in nature. It means that two or more time-varying quantities maintain a certain relative relationship during the change process. As we know, a large number of systems, such as neural networks [1], social systems, electrical power systems [2], and the Internet, can be described by complex networks in real life. Synchronization, as one of the most significant characteristics of a network, has been a hot topic in the field of complex networks [3–6]. Due to its difficulty, the study of synchronization of a complex network remains challenging [7]. Up to now, fruitful results have emerged on this issue, see, e.g., [8–11].

Establishing an appropriate network model in accordance with the physical system's network characteristics is the basis of the study of complex network. In some actual complex network models, the internal coupling between network nodes is assumed to be a linear relation [8]. The relationship among the factors within the nodes of a complex network system is likely to be nonlinear. For example, the inner relationship among users, as the nodes of a

mobile user network, is nonlinear. Therefore, the research on the inner coupling and nonlinear relationship between the nodes of a complex network is of great importance. At present, some research results about this problem are available [9, 10]. However, many problems still require further study. Moreover, the synchronization state may be affected and even destroyed due to external disturbances, fluctuations in system parameters, and other uncertain factors [12–15]. In a word, the research on the synchronization control of uncertain complex networks not only supports the theoretical results but also expands the value of such method in practical applications. To our knowledge, relevant research results are abundant [16–20]. Wang et al. [16] studied the exponential synchronization problem of a class of Markov jumping complex dynamic networks with partially uncertain transition rate and stochastic disturbances using the containment control method. In [17], Rakkiyappan et al. realized the exponential pulse projection and lag synchronization between uncertain complex networks by controlling some nodes and reduced the time required to achieve synchronization. Zhao and Cai [18] researched the exponential synchronization of complex

delayed dynamic networks with uncertain parameters by using the method of intermittent control. Wu and Liu [19] obtained some sufficient conditions that can be used to allow uncertain complex dynamic networks to achieve exponential synchronization through adaptive control. Lu et al. [20] proposed two types of sufficient conditions for the global asymptotical synchronization of delayed complex dynamic networks with switching topology based on the single Lyapunov and multiple Lyapunov function methods. Syed Ali et al. [21] studied a class of Markovian jump complex dynamic networks with parameter uncertainties. However, those works only investigated the conditions of synchronization of uncertain complex network systems in infinite time.

Exponential synchronization has a high convergence rate, but it reaches the required synchronization state in an infinite time. In many practical applications, systems are expected to realize synchronization in a finite time to ensure their superior performance. Hence, the study of the finite-time synchronization of complex networks has received extensive attention. In recent years, the finite-time synchronization of complex networks has been studied by various researchers and important results have been attained. For example, Zhao et al. [22] proposed the finite-time synchronization for multilink complex networks via discontinuous control. Zhang et al. [23, 24] obtained several sufficient conditions for the finite-time synchronization of complex networks with nonidentical nodes under impulsive disturbances and stochastic noise perturbations. In [25], the problem of finite-time synchronization of complex dynamic networks with time-varying and nonidentical nodes was researched. Sun et al. [26] analyzed the finite-time synchronization problem of two complex chaotic systems with unknown parameters, which mainly used a nonsingular terminal sliding control method. Sheng et al. [27] studied the finite-time outer-synchronization of complex networks with Markov jump topology by using the method that combines random jump control and deterministic switch control. However, the results of the comprehensive analysis of finite-time synchronization are generally obtained on the basis of deterministic models of complex network systems.

In summary, the research on the finite-time synchronization of uncertain complex networks is relevant for theoretical development and practical applications. However, few studies explore the finite-time synchronization of uncertain complex networks. Xiao et al. [28] studied the finite-time cluster synchronization of a complex dynamic network with time-varying delay by using the hybrid control strategy; literature [29] used the average dwell time method to obtain some sufficient conditions for realizing finite-time synchronization of sampled-data T-S fuzzy complex dynamic networks. But, both of them considered the case where the relationship between the nodes is linearly coupled, and they do not consider the uncertainties that may be contained in the system. Zhang and Han [30] researched the finite-time synchronization of uncertain complex networks with nonidentical nodes on the basis of special unilateral

coupling control; however, the method requires the estimation of uncertain parameters and the assumption of a topology that satisfies conditions automatically. Mei et al. [31] discussed a class of finite-time synchronization for drive-response systems with structure identification and uncertain parameters. Xu et al. [32] studied the finite-time synchronization of Markovian jump complex networks with generally uncertain transition rates. Ma et al. [33] studied the robust and nonfragile finite-time H_∞ synchronization control for complex networks with uncertain inner coupling. Zhao et al. [34] obtained new results on finite-time parameter identification and synchronization of uncertain complex dynamic networks with perturbations and impulsive effects on the basis of the finite time stability theorem, in which a system's uncertainty is caused by uncertain parameters. Moreover, in [31–34], the nonlinearity of the internal coupling of systems was not considered. Sun et al. [35] realized the finite-time synchronization of complex chaotic systems with three unknown parameter terms by means of sliding mode control. In [36], the finite-time synchronization between two complex networks without uncertainty was proposed. However, the individual couplings were considered as linear, and the state vector of the i th dynamic node was not included. Li et al. [37] investigated the finite-time synchronization for a class of complex networks with nonidentical nodes and uncertain disturbance, but the system studied was linearly coupled, and the disturbance terms were assumed to satisfy the upper bound condition, without considering the possible disturbance factors of the nodes themselves.

Motivated by the abovementioned analysis, we study the problem of finite-time synchronization control for nonlinear uncertain complex dynamic networks. In the real world, the coupling within many network nodes is nonlinear. For example, when a network node is a neuron, the state components within the neuron share a typical nonlinear relationship. Therefore, this study considers a complex network model, in which the coupling between the nodes is nonlinear and the nonlinear vector function satisfies the Lipschitz condition. The innovations of this work are described as follows:

- (1) A finite-time synchronization problem for a complex network model with uncertain parameters and nonlinear coupling inside the nodes is studied. The parameter uncertainties satisfy the matching condition. This assumption is based on practical problems and is nontrivial.
- (2) A finite-time synchronization control strategy is proposed for this type of model and sufficient conditions for the finite-time synchronization are obtained. First, under an LMI condition, the time required for the system to achieve finite-time synchronization can be obtained with a given input. Second, under another assumption on the eigenvalues of some matrices, an upper bound for the synchronization time is obtained under another input.

- (3) A simple and feasible controller is designed for the model studied. Under the input of the controller, the time required for the system to reach the finite-time synchronization state is only related to the system parameters.

The remainder of this paper is organized as follows: In Section 2, the model of an uncertain complex dynamic network with nonlinear coupling and some preliminaries are introduced. In Section 3, some sufficient conditions for the system to realize finite-time synchronization are obtained. In Section 4, several numerical examples are given to verify the effectiveness of the theoretical results.

2. System Description and Preliminaries

In this paper, we consider a time-varying uncertain complex dynamic network composed of N nonlinear decentralized coupling nodes, where each node is an n -dimensional dynamic system. The equation of state for node i is described as follows:

$$\begin{aligned} \dot{x}_i(t) = & (A + \Delta A(t))x_i(t) + (B + \Delta B(t))f(x_i(t)) \\ & + \sum_{j=1}^N c_{ij}g(x_j(t)) + u_i(t), \quad i = 1, 2, \dots, N, \end{aligned} \quad (1)$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in R^n$ and $u_i(t)$ are the state vector and the control input vector of the i th node, respectively; $f(x) \in R^n$ is a continuously differentiable nonlinear vector function describing the dynamic characteristics of nodes; $g(x) \in R^n$ is a smooth nonlinear vector function representing the internal coupling between the state variables of each node; $C = (c_{ij})_{N \times N}$ is the parameter matrix of external coupling structure, which represents the topology and coupling strength of the network. If a connection exists between node i and node j , then $c_{ij} \neq 0$; otherwise, $c_{ij} = 0$. The diagonal elements of the matrix C are given by $c_{ii} = -\sum_{j=1, j \neq i}^N c_{ij}$. The matrices A and B are constant matrices, and $\Delta A(t)$ and $\Delta B(t)$ are the norm bounded uncertainty matrices of the system with the following form:

$$[\Delta A(t), \Delta B(t)] = HF(t)[E_1, E_2], \quad (2)$$

where H , E_1 , and E_2 are real known constant matrices of appropriate dimensions and $F(t)$ is the unknown time-varying matrix function satisfying $F^T(t)F(t) \leq I$.

Remark 1. Many studies on nonlinear complex network models assume that the dynamic behavior function of nodes is a noncoupling nonlinear function. However, in some networks, the dynamic characteristics between their nodes are interconnected. A typical example is a neural network. The coefficient matrix of the dynamic behavior function is not an identity matrix. This type of model is studied in the literature [17, 38, 39].

Remark 2. The complex dynamic network model in the current study involves a few cases in which the parameters are uncertain. These cases are not widely studied in the literature [17, 33, 38–41]. For example, the authors in [17, 33, 40] considered that the internal coupling matrix of a complex network model is uncertain. Although the literature [35] considers the parameter uncertainty, the model is a kind of neural networks model. In [36], a case of incomplete transmission factors was presented. In fact, the coupling of the nodes of a complex network model has a nonlinear relationship, and the findings of the literature [17, 33, 40] can be combined with the special case of the model in the present study.

Definition 1 (see [42]). The complex dynamic network (1) is said to be globally synchronized, if there is a control input $u_i(t)$, $i = 1, 2, \dots, N$, such that

$$\|x_i(t) - s(t)\| = 0, \quad t \longrightarrow \infty, \quad i = 1, 2, \dots, N, \quad (3)$$

where $s(t) \in R^n$ is the solution of the isolated node system $\dot{s}(t) = (A + \Delta A(t))s(t) + (B + \Delta B(t))f(s(t))$.

Furthermore, if there exists a constant $t^* > 0$, such that $\lim_{t \rightarrow t^*} \|e_i(t)\| = 0$, $\|e_i(t)\| \equiv 0$ for any $t \geq t^*$, where $e_i(t) = x_i(t) - s(t)$, we can say that system (1) is finite-time synchronized.

Definition 2 (see [43]). Consider $A = (a_{ij}) \in C^{m \times n}$ and $B = (b_{ij}) \in C^{p \times q}$. Then, $A \otimes B = C^{m \times p \times n \times q}$ is called the Kronecker product of matrix A and B .

Next, we list a number of key properties which could be occurred in the proof:

- (1) $(A \otimes B)^T = A^T \otimes B^T$
- (2) Suppose that $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{s \times r}$, $C = (c_{ij})_{n \times p}$ and $D = (d_{ij})_{r \times t}$, then $(A \otimes B)(C \otimes D) = AC \otimes BD$

Let $e_i(t) = x_i(t) - s(t)$ be the synchronization error vectors $i = 1, 2, \dots, N$. Then, the synchronization error system can be given by

$$\begin{aligned} \dot{e}_i(t) = & (A + \Delta A(t))e_i(t) + (B + \Delta B(t))(f(x_i(t)) - f(s(t))) \\ & + \sum_{j=1}^N c_{ij}\phi(e_j)(t) + u_i(t). \end{aligned} \quad (4)$$

Set $\phi(e_i)(t) = g(x_i(t)) - g(s(t))$. This study aims to design a suitable feedback controller $u_i(t)$ of the following form such that the system (1) achieves finite-time synchronization:

$$u_i(t) = -w_i e_i(t) - k \text{sign}(e_i(t)) |e_i(t)|^\beta, \quad (5)$$

where w_i ($i = 1, 2, \dots, N$) and k are real constants and $0 < \beta < 1$, and

$$|e_i(t)|^\beta = \left(|e_{i1}(t)|^\beta, |e_{i2}(t)|^\beta, \dots, |e_{in}(t)|^\beta \right)^T,$$

$$\text{sign}(e_i(t)) = \begin{cases} 1, & e_i(t) > 0, \\ 0, & e_i(t) = 0, \\ -1, & e_i(t) < 0. \end{cases} \quad (6)$$

Assumption 1. For any $x(t), s(t) \in R^n$, there exists a positive constant L such that

$$\|f(x(t)) - f(s(t))\| \leq L\|x(t) - s(t)\|. \quad (7)$$

Assumption 2. The nonlinear function $g(x_i)$ satisfies the Lipschitz condition, i.e., there exists a constant σ_i such that

$$0 \leq \frac{g_i(\varepsilon_1) - g_i(\varepsilon_2)}{\varepsilon_1 - \varepsilon_2} \leq \sigma_i, \quad \forall \varepsilon_1, \varepsilon_2 \in R, \varepsilon_1 \neq \varepsilon_2, i = 1, 2, \dots, N. \quad (8)$$

To end this section, we introduce some lemmas for the proof of the main results in the paper.

Lemma 1 (see [8]). Let $b_i > 0 (i = 1, 2, \dots, n)$ and $0 < m < n$, then

$$\left(\sum_{i=1}^p b_i^n \right)^{1/n} \leq \left(\sum_{i=1}^p b_i^m \right)^{1/m}. \quad (9)$$

Lemma 2 (see [44]). Let H, E , and F be real matrices of appropriate dimensions such that $F^T F \leq I$. Then, the following inequalities holds:

For any scalar $\gamma > 0$ and vectors $X_1, X_2 \in R^n$, $2X_1^T H F E X_2 \leq \gamma X_1^T H H^T X_1 + \gamma^{-1} X_2^T E^T E X_2$.

Lemma 3 (see [45]). Assume that a continuous, positive definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -\omega V^\rho(t), \quad \forall t \geq t_0, V(t_0) \geq 0, \quad (10)$$

where $\omega > 0, 0 < \rho < 1$ are constants. Then, for any given t_0 , $V(t)$ satisfies the following inequality:

$$\begin{cases} V^{1-\rho}(t) \leq V^{1-\rho}(t_0) - \omega(1-\rho)(t-t_0), & t_0 \leq t \leq t^*, \\ V(t) = 0, & t \geq t^*, \end{cases} \quad (11)$$

with t^* given by $t^* = t_0 + (V^{1-\rho}(t_0)/\omega(1-\rho))$.

Lemma 4 (see [46]). For any n -dimension vector X and vector Y , the following inequality is satisfied:

$$2X^T Y \leq \frac{1}{\gamma} X^T X + \gamma Y^T Y, \quad (12)$$

where γ is a suitable positive constant.

Lemma 5 (see [47]). Suppose there is a Lyapunov function $V(x)$ defined on a neighborhood $v \in \mathfrak{R}^n$ of the origin, and

$$\dot{V}(x) \leq -\alpha V^\eta(x) + \theta V(x), \quad \forall x \in v \setminus \{0\}. \quad (13)$$

Then, the origin of (1) is finite-time stable. The set

$$\Omega = \left\{ x \mid V(x)^{1-\eta} < \frac{\alpha}{\theta} \right\} \cap v, \quad (14)$$

is contained in the domain of attraction of the origin. The settling time satisfies $T_s \leq ((\ln(1 - (\theta/\alpha)V^{1-\eta}(x_0)))/\theta(\eta - 1)), x_0 \in \Omega$.

3. Main Result

In this section, several sufficient conditions of finite-time synchronization for the uncertain complex network system under the controller (5) are provided by establishing a Lyapunov function. In the following analysis process, the time required for the system to achieve finite-time synchronization under different conditions or the upper bound of time required to be satisfied will be calculated.

Theorem 1. Under Assumptions 1 and 2, there exists a positive definite symmetric matrix W , so that the following inequality holds:

$$\begin{bmatrix} I_N \otimes \Pi - 2W \otimes I_n & C \otimes I_n \\ * & 0 \end{bmatrix} \leq 0,$$

$$\begin{aligned} \Pi &= A + A^T + HH^T + E_1^T E_1 + LB \\ &\quad + L(HH^T + E_2^T E_2) + LB^T, \\ W &= \text{diag}\{w_1, w_2, \dots, w_N\}, \end{aligned} \quad (15)$$

then the controlled complex dynamic network (1) is synchronized with the given smooth dynamic $s(t)$ in a finite-time t^* with

$$t^* = \frac{V^{1-\delta}(0)}{2k(1-\delta)}, \quad (16)$$

where $V(0) = \sum_{i=1}^N e_i^T(0)e_i(0)$ and $\delta = (1 + \beta)/2$.

Proof. Let $e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$, and construct the following Lyapunov function:

$$V(t) = \sum_{i=1}^N e_i^T(t)e_i(t). \quad (17)$$

Calculating the time derivative of $V(t)$ along the trajectory of the error system (4) leads to

$$\begin{aligned}
\dot{V}(t) &= \sum_{i=1}^N \left[\dot{e}_i^T(t) e_i(t) + e_i^T(t) \dot{e}_i(t) \right] \\
&= \sum_{i=1}^N \left[(A + \Delta A(t)) e_i(t) + (B + \Delta B(t)) (f(x_i(t)) - f(s(t))) + \sum_{j=1}^N c_{ij} \phi(e_j)(t) - w_i e_i(t) - k \text{sign}(e_i(t)) |e_i(t)|^\beta \right]^T e_i(t) \\
&\quad + \sum_{i=1}^N e_i^T(t) \left[(A + \Delta A(t)) e_i(t) + (B + \Delta B(t)) (f(x_i(t)) - f(s(t))) + \sum_{j=1}^N c_{ij} \phi(e_j)(t) - w_i e_i(t) - k \text{sign}(e_i(t)) |e_i(t)|^\beta \right] \\
&= \sum_{i=1}^N \left[e_i^T(t) \left[(A + \Delta A(t))^T + (A + \Delta A(t)) - 2w_i I_n \right] e_i(t) + (f(x_i(t)) - f(s(t)))^T (B + \Delta B(t))^T e_i(t) \right. \\
&\quad \left. + e_i^T(t) (B + \Delta B(t)) (f(x_i(t)) - f(s(t))) \right] + \sum_{i=1}^N \sum_{j=1}^N \phi^T(e_j)(t) c_{ij}^T e_i(t) + \sum_{i=1}^N \sum_{j=1}^N e_i^T(t) c_{ij} \phi(e_j)(t) \\
&\quad - \sum_{i=1}^N k \text{sign}^T(e_i(t)) |e_i(t)|^\beta e_i(t) - \sum_{i=1}^N k e_i^T(t) \text{sign}(e_i(t)) |e_i(t)|^\beta.
\end{aligned} \tag{18}$$

From Assumption 1 and Lemma 2, we get

$$\begin{aligned}
e_i^T(t) \Delta A(t) e_i(t) &= e_i^T(t) H F(t) E_1 e_i(t) \leq \frac{1}{2} (e_i^T(t) H H^T e_i(t) + e_i^T(t) E_1^T E_1 e_i(t)), \\
e_i^T(t) \Delta A^T(t) e_i(t) &= e_i^T(t) E_1^T F^T(t) H^T e_i(t) \leq \frac{1}{2} (e_i^T(t) E_1^T E_1 e_i(t) + e_i^T(t) H H^T e_i(t)), \\
e_i^T(t) \left[(A + \Delta A(t))^T + (A + \Delta A(t)) \right] e_i(t) &\leq e_i^T(t) \left[A + A^T + H H^T + E_1^T E_1 \right] e_i(t),
\end{aligned} \tag{19}$$

Similarly, we can get that

$$\begin{aligned}
&(f(x_i(t)) - f(s(t)))^T (B + \Delta B(t))^T e_i(t) + e_i^T(t) (B + \Delta B(t)) (f(x_i(t)) - f(s(t))) \\
&\leq L e_i^T(t) \left[B + B^T + H H^T + E_2^T E_2 \right] e_i(t), \\
\dot{V}(t) &\leq \sum_{i=1}^N e_i^T(t) \left[A + H H^T + E_1^T E_1 + A^T + L B + L (H H^T + E_2^T E_2) + L B^T \right] e_i(t) + \sum_{i=1}^N \sum_{j=1}^N \phi^T(e_j)(t) c_{ij}^T e_i(t) \\
&\quad + \sum_{i=1}^N \sum_{j=1}^N e_i^T(t) c_{ij} \phi(e_j)(t) - 2 \sum_{i=1}^N e_i^T(t) w_i e_i(t) - \sum_{i=1}^N k \text{sign}^T(e_i(t)) |e_i(t)|^\beta e_i(t) - \sum_{i=1}^N k e_i^T(t) \text{sign}(e_i(t)) |e_i(t)|^\beta \\
&= \sum_{i=1}^N e_i^T(t) \left[A + H H^T + E_1^T E_1 + A^T + L B + L (H H^T + E_2^T E_2) + L B^T \right] e_i(t) - 2 \sum_{i=1}^N e_i^T(t) w_i e_i(t) \\
&\quad + \sum_{i=1}^N \sum_{j=1}^N \phi^T(e_j)(t) c_{ij}^T e_i(t) + \sum_{i=1}^N \sum_{j=1}^N e_i^T(t) c_{ij} \phi(e_j)(t) - 2 \sum_{i=1}^N k |e_i(t)|^{1+\beta} \\
&= \zeta^T(t) \Omega \tilde{\zeta}(t) - 2k \sum_{i=1}^N |e_i(t)|^{\beta+1},
\end{aligned} \tag{20}$$

where I is identity matrix with appropriate dimensions.

$$\begin{aligned}\zeta(t) &= [e^T(t) \ \phi^T(e(t))]^T, \tilde{\zeta}(t) = [e(t) \ \phi(e(t))]^T, \\ \Omega &= \begin{pmatrix} I_N \otimes \Pi - 2W \otimes I_n & C \otimes I_n \\ * & 0 \end{pmatrix}, \\ \Pi &= A + HH^T + E_1^T E_1 + A^T + LB + L(HH^T + E_2^T E_2) + LB^T, \\ W &= \text{diag}(w_1, w_2, \dots, w_N).\end{aligned}\tag{21}$$

If the inequality (15) holds, then

$$\dot{V}(t) \leq \zeta^T(t) \Omega \tilde{\zeta}(t) - 2k \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)|^{\beta+1} \leq -2k \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)|^{\beta+1}.\tag{22}$$

Using Lemma 1, one has

$$\begin{aligned}\left(\sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)|^{\beta+1} \right)^{1/(1+\beta)} &\geq \left(\sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)|^2 \right)^{1/2}, \\ \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)|^{\beta+1} &\geq \left(\sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)| \right)^{(1+\beta)/2} = \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{(1+\beta)/2}.\end{aligned}\tag{23}$$

In conclusion, we have

$$\begin{aligned}\dot{V}(t) &\leq -2k \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)|^{\beta+1} \leq -2k \left(\sum_{i=1}^N e_i^T(t) e_i(t) \right)^{(1+\beta)/2} \\ &= -2kV^{(1+\beta)/2}(t).\end{aligned}\tag{24}$$

From Lemma 3, $V(t)$ converges to zero in a finite time t^* , where t^* is given by

$$t^* = \frac{V^{1-\delta}(0)}{2k(1-\delta)},\tag{25}$$

where $V(0) = \sum_{i=1}^N e_i^T(0) e_i(0)$ and $\delta = (1 + \beta)/2$. \square

Remark 3. The results of the finite-time synchronization of the complex dynamic network model studied are abundant, but the results of the finite-time synchronization of complex network models with uncertain parameters are scant [17, 33, 38–41]. In [17, 33, 40], infinite-time synchronization was studied with consideration of the incoherent matrix of complex network models being uncertain. In [30, 41], the finite-time synchronization of uncertain complex network systems was investigated. In [30], the dynamic behavior function of a single node containing unknown parameters was considered, and the internal coupling matrix was linearly uncertain. In [41], switching control was studied, and in both of the abovementioned references, eigenvalue characterization was used. Our models and research methods are different.

Corollary 1. *If Assumptions 1 and 2 are established and there exist a positive definite symmetric matrix W such that*

$$\lambda_{\max}(M) - \lambda_{\min}(2W) \leq 0,\tag{26}$$

where

$$\begin{aligned}M &= I_N \otimes \left(A + HH^T + E_1^T E_1 + A^T + LB + L(HH^T + E_2^T E_2) + LB^T \right) \\ &\quad + \left((C\Sigma)^T (C\Sigma) + I_N \right) \otimes I_n, \\ \Sigma &= \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_N), W = \text{diag}(w_1, w_2, \dots, w_N),\end{aligned}\tag{27}$$

where $\lambda_{\max}(M)$ is the maximum eigenvalue of the matrix M and $\lambda_{\min}(W)$ is the minimum eigenvalue of the matrix W , then the system (1) can achieve finite-time synchronization under the controller $u_i(t)$. The synchronization time t^* is obtained as

$$t^* = \frac{V^{1-\delta}(0)}{2k(1-\delta)},\tag{28}$$

where $V(0) = \sum_{i=1}^N e_i^T(0) e_i(0)$ and $\delta = (1 + \beta)/2$.

Proof. The proof is similar to Theorem 2 and it is omitted.

If the uncertainties of the complex dynamic networks are not considered, the complex dynamic network model is given as follows:

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t) + Bf(x_i(t)) + \sum_{j=1}^N c_{ij}g(x_j(t)) + u_i(t), \\ i &= 1, 2, \dots, N.\end{aligned}\tag{29}$$

In this situation, the error system becomes

$$\dot{e}_i(t) = Ae_i(t) + B(f(x_i(t)) - f(s(t))) + \sum_{j=1}^N c_{ij}\phi(e_j(t)) + u_i(t).\tag{30}$$

For the complex network model (29), we can obtain the following result. \square

Corollary 2. Under Assumptions 1 and 2, if there exists a positive definite symmetric matrix W , so that the inequality (15) is satisfied, where $\Pi = A + A^T + LB + LB^T$, then we can conclude that the system (29) can achieve finite-time synchronization under the controller $u_i(t)$. The settling time is obtained as

$$t^* = \frac{V^{1-\delta}(0)}{2k(1-\delta)}, \quad (31)$$

where $V(0) = \sum_{i=1}^N e_i^T(0)e_i(0)$ and $\delta = (1 + \beta)/2$.

Remark 4. At present, the research results of complex dynamic networks without uncertainties are abundant. However, these results are different from those of previous research. For example, the state variable of the i th node was not considered in [25], and only the linear coupling relationship between nodes was recognized. A study [?] used nonperiodic intermittent control to enable a complex network system to reach finite-time synchronization; however, each node in the system under consideration was in the same state. In [48], the complex network model and research method are different from that in the present study.

If Γ represents the internal coupling between the state variables of each node, the complex dynamic network model is as follows:

$$\begin{aligned} \dot{x}_i(t) &= (A + \Delta A(t))x_i(t) + (B + \Delta B(t))f(x_i(t)) \\ &+ c \sum_{j=1}^N d_{ij}\Gamma x_j(t) + u_i(t), \end{aligned} \quad (32)$$

where the real number c is the coupling strength; $\Gamma \in R^{n \times n}$ is the inner connecting matrix between nodes; the constant matrix $D = (d_{ij})_{N \times N}$ represents the system's linear coupling matrix, and the definition of d_{ij} is as follows: if a connection exists between node i and node j ($i \neq j$), then $d_{ij} > 0$; otherwise, $d_{ij} = 0$. The diagonal elements of matrix D are defined as follows: $d_{ii} = -\sum_{j=1, j \neq i}^N d_{ij}$. Then, the error system reads

$$\begin{aligned} \dot{e}_i(t) &= (A + \Delta A(t))e_i(t) + (B + \Delta B(t))(f(x_i(t)) - f(s(t))) \\ &+ c \sum_{j=1}^N d_{ij}\Gamma e_j(t) + u_i(t). \end{aligned} \quad (33)$$

For the complex dynamic model (32), we can make the following results.

Corollary 3. Suppose that Assumptions 1 and 2 hold, and a positive definite symmetric matrix W exists such that the following inequality is satisfied:

$$\begin{aligned} I_N \otimes [A + A^T + HH^T + E_1^T E_1 + LB + L(HH^T + E_2^T E_2) + LB^T] \\ - 2W \otimes I_n + c(D \otimes \Gamma + D^T \otimes \Gamma^T) \leq 0, \end{aligned} \quad (34)$$

then system (32) can reach the finite-time synchronization state under the controller (5), and the settling time is obtained as follows:

$$t^* = \frac{V^{1-\delta}(0)}{2k(1-\delta)}, \quad (35)$$

where $V(0) = \sum_{i=1}^N e_i^T(0)e_i(0)$ and $\delta = (1 + \beta)/2$.

Remark 5. Existing studies on the finite-time synchronization of complex dynamic networks considered the coupling relationship between nodes as a linear one [28–30, 32, 34, 41, 49]. While regarding the uncertainty and dynamic behavior of a system, the results for a prefunction matrix as a nonidentity matrix are few. In [25, 30, 32], the uncertainty of a system and the dynamic matrix of the dynamic behavior function were not considered as the identity matrix. A neural network model was used in [34].

If the uncertainties of complex dynamic networks are not considered, the error system (33) can be rewritten as follows:

$$\begin{aligned} \dot{e}_i(t) &= Ae_i(t) + B(f(x_i(t)) - f(s(t))) \\ &+ c \sum_{j=1}^N d_{ij}\Gamma e_j(t) + u_i(t). \end{aligned} \quad (36)$$

Corollary 4. Assume that Assumptions 1 and 2 hold, and the following inequality is satisfied:

$$\begin{aligned} I_N \otimes (A + A^T + LB + LB^T) - 2W \otimes I_n \\ + c(D \otimes \Gamma + D^T \otimes \Gamma^T) \leq 0, \end{aligned} \quad (37)$$

then the error system (36) can achieve finite-time synchronization under the controller $u_i(t)$. Also, the settling time is as follows:

$$t^* = \frac{V^{1-\delta}(0)}{2k(1-\delta)}, \quad (38)$$

where $V(0) = \sum_{i=1}^N e_i^T(0)e_i(0)$ and $\delta = (1 + \beta)/2$.

Theorem 2. Under Assumptions 1 and 2, if there exists a positive definite symmetric matrix W such that

$$\lambda_{\max}(M) - \lambda_{\min}(2W) > 0, \quad (39)$$

where

$$\begin{aligned} M &= I_N \otimes \left[A + HH^T + E_1^T E_1 + A^T + HH^T + E_1^T E_1 + LB \right. \\ &\quad \left. + L(HH^T + E_2^T E_2) + LB^T \right] + [(C\Sigma)^T (C\Sigma) + I_N] \otimes I_n, \\ \Sigma &= \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_N), W = \text{diag}(w_1, w_2, \dots, w_N), \end{aligned} \quad (40)$$

with I_N being the N -dimension identity matrix, then the controlled complex dynamic network (1) is synchronized with the given smooth dynamic $s(t)$ in a finite time less than T_s as follows:

$$T_s \leq \frac{2 \ln(1 - (1/2k)(\lambda_{\max}(M) - \lambda_{\min}(2W))V^{(1-\beta)/2}(0))}{(\lambda_{\max}(M) - \lambda_{\min}(2W))(\beta - 1)}, \quad (41)$$

where $V(0) = (1/2)\sum_{i=1}^N e_i^T(0)e_i(0)$.

Proof. Let $e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T$, and construct the following Lyapunov function:

$$V(t) = \sum_{i=1}^N e_i^T(t)e_i(t). \quad (42)$$

Calculating the time derivative of $V(t)$ along the trajectory of the error system (4) leads to

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N [\dot{e}_i^T(t)e_i(t) + e_i^T(t)\dot{e}_i(t)] \\ &= \sum_{i=1}^N \left[(A + \Delta A(t))e_i(t) + (B + \Delta B(t))(f(x_i(t)) - f(s(t))) + \sum_{j=1}^N c_{ij}\phi(e_j)(t) - w_i e_i(t) - k \text{sign}(e_i(t))|e_i(t)|^\beta \right]^T e_i(t) \\ &\quad + \sum_{i=1}^N e_i^T(t) \left[(A + \Delta A(t))e_i(t) + (B + \Delta B(t))(f(x_i(t)) - f(s(t))) + \sum_{j=1}^N c_{ij}\phi(e_j)(t) - w_i e_i(t) - k \text{sign}(e_i(t))|e_i(t)|^\beta \right]. \end{aligned} \quad (43)$$

By Assumption 2 and Lemma 4, one gets

$$\begin{aligned} \phi(e_i(t)) &= g(x_i(t)) - g(s(t)) \leq \sigma_i e_i(t), \\ \phi^T(e(t))(C^T \otimes I_n)e(t) + e^T(t)(C \otimes I_n)\phi(e(t)) &\leq \frac{1}{2} [\phi^T(e(t))((C^T C) \otimes I_n)\phi(e(t)) + e^T(t)e(t)] \\ &\quad + \frac{1}{2} [\phi^T(e(t))((C^T C) \otimes I_n)\phi(e(t)) + e^T(t)e(t)] \\ &\leq \phi^T(e(t))((C^T C) \otimes I_n)\phi(e(t)) + e^T(t)e(t) \leq e^T(t) \left([(C\Sigma)^T(C\Sigma) + I_N] \otimes I_n \right) e(t). \end{aligned} \quad (44)$$

Thus,

$$\begin{aligned} \dot{V}(t) &\leq \sum_{i=1}^N e_i^T(t) \left[A + HH^T + E_1^T E_1 + A^T + LB + L(HH^T + E_2^T E_2) + LB^T - 2w_i I_n \right] e_i(t) + \sum_{i=1}^N \sum_{j=1}^N c_{ij}^T \phi^T(e_j)(t)e_i(t) \\ &\quad + \sum_{i=1}^N \sum_{j=1}^N e_i^T(t)c_{ij}\phi(e_j)(t) - \sum_{i=1}^N k \text{sign}^T(e_i(t))|e_i(t)|^\beta e_i(t) - \sum_{i=1}^N k e_i^T(t) \text{sign}(e_i(t))|e_i(t)|^\beta \\ &= e^T(t) \left[I_N \otimes [A + HH^T + E_1^T E_1 + A^T + LB + L(HH^T + E_2^T E_2) + LB^T] + [(C\Sigma)^T(C\Sigma) + I_N] \otimes I_n - 2W \otimes I_n \right] e(t) \\ &\quad - 2k \sum_{i=1}^N |e_i(t)|^{1+\beta} \\ &= e^T(t) (M - 2W \otimes I_n) e(t) - 2k \sum_{i=1}^N |e_i(t)|^{1+\beta}, \end{aligned} \quad (45)$$

where C is a symmetric matrix, and

$$\begin{aligned} M &= I_N \otimes [A + HH^T + E_1^T E_1 + A^T + HH^T + E_1^T E_1 + LB + L(HH^T + E_2^T E_2) + LB^T] + [(C\Sigma)^T (C\Sigma) + I_N] \otimes I_n, \\ \Sigma &= \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_N\}, W = \text{diag}\{w_1, w_2, \dots, w_N\}. \end{aligned} \quad (46)$$

According to the inequality (39), one has

$$\begin{aligned} \dot{V}(t) &\leq e^T(t) (\lambda_{\max}(M) - \lambda_{\min}(2W)) e(t) - 2k \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)|^{\beta+1} \\ &= \sum_{i=1}^N e_i^T(t) (\lambda_{\max}(M) - \lambda_{\min}(2W)) e_i(t) - 2k \sum_{i=1}^N \sum_{j=1}^N |e_{ij}(t)|^{\beta+1}. \end{aligned} \quad (47)$$

From (23), we can get that

$$\begin{aligned} \dot{V}(t) &\leq (\lambda_{\max}(M) - \lambda_{\min}(2W)) V(t) - 2k V^{(1+\beta)/2}(t) \\ &= -\alpha (V(t))^{(1+\beta)/2} + \theta V(t), \end{aligned} \quad (48)$$

where $\alpha = 2k$ and $\theta = (\lambda_{\max}(M) - \lambda_{\min}(2W))$.

According to Lemma 5, the system (1) can reach the state of synchronization in finite time $t^* \leq T_s$ as

$$T_s \leq \frac{2 \ln(1 - (1/2k) (\lambda_{\max}(M) - \lambda_{\min}(2W)) V^{(1-\beta)/2}(0))}{(\lambda_{\max}(M) - \lambda_{\min}(2W)) (\beta - 1)}, \quad (49)$$

where $V(0) = \sum_{i=1}^N e_i^T(0) e_i(0)$. \square

Remark 6. In the proof of Corollary 1, under the condition that the inequality (26) is assumed to be established, considering the influence of $M - 2W$ on the system is not necessary. The sufficient conditions for the system to achieve finite-time synchronization are obtained. In the proof of Theorem 2, we consider the influence of matrix $M - 2W$ on the conditions that the system must satisfy to achieve finite-time synchronization and reduce conservativeness.

Remark 7. In the proof result of Theorem 1, we obtain the exact time required for the uncertain complex dynamic network (1) to achieve the finite-time synchronization. In Theorem 2, we find the upper bound of the time required for the system to reach the synchronization state under other conditions. For some cases where the exact synchronization time needs to be known, the conclusion of Theorem 1 is more in line with the requirements. For others where there is no strict requirement for synchronization time or the eigenvalue of matrix is easier to be obtained, the conclusion of Theorem 2 has stronger applicability.

If the uncertainties of time-varying complex dynamic networks are not considered, the complex dynamic network model is shown in (29). According to the network model, we can obtain the following conclusion.

Corollary 5. Under Assumptions 1 and 2, if a positive definite symmetric matrix W exists and results in inequality

(39), where $M = I_N \otimes [A + A^T + LB + LB^T] + [(C\Sigma)^T (C\Sigma) + I_N] \otimes I_n$, then the system (29) can reach finite-time synchronization under the controller (5). The finite-time T_s is as follows:

$$T_s \leq \frac{2 \ln(1 - (1/2k) (\lambda_{\max}(M) - \lambda_{\min}(2W)) V^{(1-\beta)/2}(0))}{(\lambda_{\max}(M) - \lambda_{\min}(2W)) (\beta - 1)}. \quad (50)$$

When the coupling relationship between state variables of each node in the complex network system is linear, the state equation and error equation of the system are shown in (32) and (33). For this system, we can draw the following conclusions.

Corollary 6. Under Assumptions 1 and 2, assume that there exists a positive definite symmetric matrix W such that

$$\lambda_{\Theta} + \epsilon \lambda_{D+D^T} - 2\omega > 0, \quad (51)$$

where $\Theta = A + A^T + HH^T + E_1^T E_1 + LB + L(HH^T + E_2^T E_2) + LB^T$, $\epsilon = c \| \Gamma \|_2$, $\lambda_{\Theta} = \lambda_{\max}(\Theta)$, $\lambda_{D+D^T} = \lambda_{\max}(D + D^T)$, and $\omega = \min\{w_i\}$. Then, under the control of u_i , the system (32) can reach the finite-time synchronization state, and the settling time $t^* \leq T_s$ is obtained as

$$T_s \leq \frac{2 \ln(1 - (1/2k) (\lambda_{\Theta} + \epsilon \lambda_{D+D^T} - 2\omega) V^{(1-\beta)/2}(0))}{(\lambda_{\Theta} + \epsilon \lambda_{D+D^T} - 2\omega) (\beta - 1)}. \quad (52)$$

If the uncertainties of time-varying complex dynamic networks are not considered and the internal coupling between the state variables of each node is linear, the error state equation of the complex network system is shown as in (36). In this time, the following can concluded:

Corollary 7. Under Assumptions 1 and 2, if there exists a positive definite symmetric matrix W satisfying the inequality

$$\lambda_{\Lambda} + \epsilon \lambda_{D+D^T} - 2\omega > 0, \quad (53)$$

where $\Lambda = A + A^T + LB + LB^T$ and $\epsilon = c \| \Gamma \|_2$, then the system (36) can achieve the finite-time synchronization. Also, the settling time is obtained as follows:

$$T_s \leq \frac{2 \ln(1 - (1/2k) (\lambda_{\Lambda} + \epsilon \lambda_{D+D^T} - 2\omega) V^{(1-\beta)/2}(0))}{(\lambda_{\Lambda} + \epsilon \lambda_{D+D^T} - 2\omega) (\beta - 1)}, \quad (54)$$

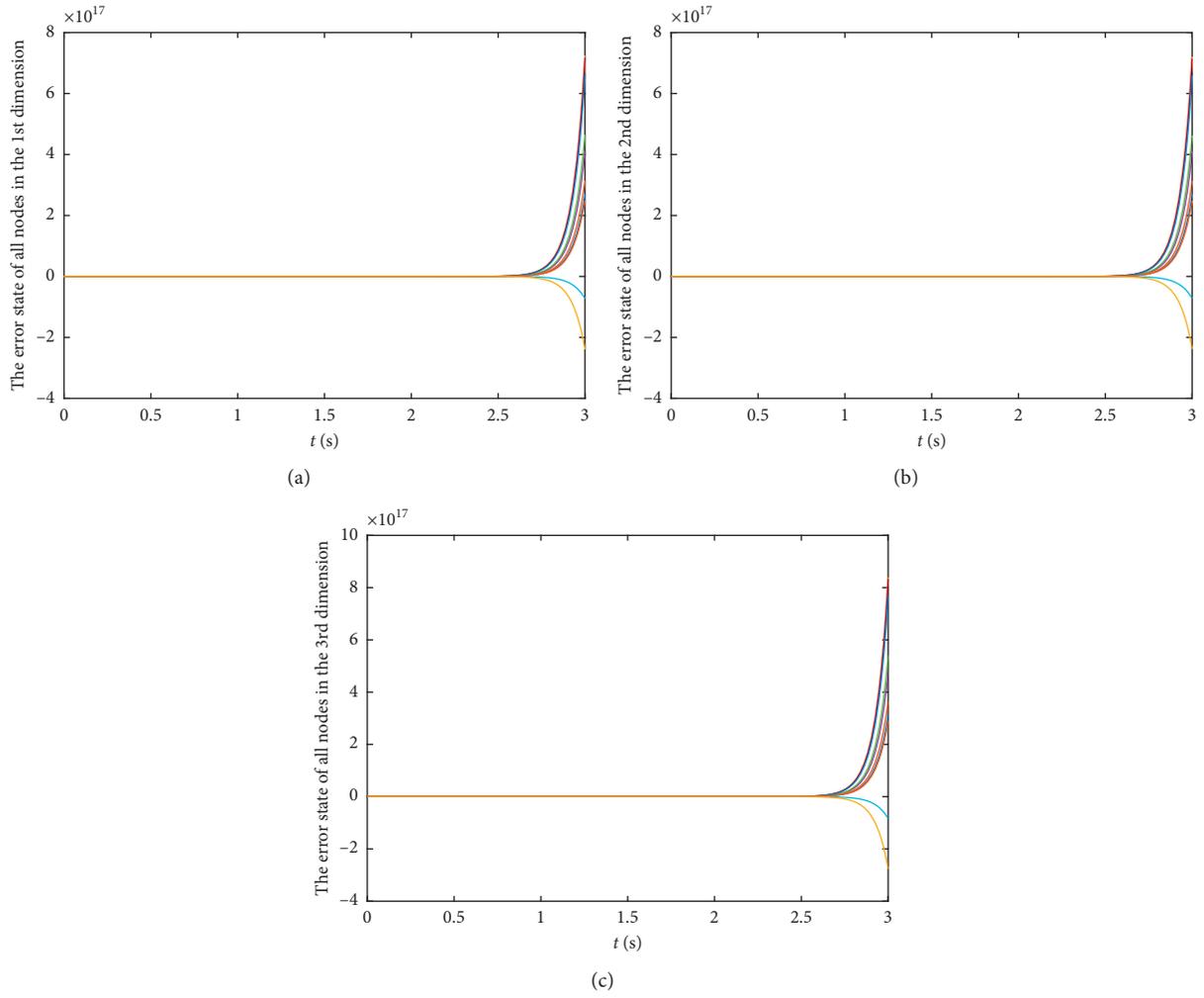


FIGURE 1: The state of all nodes in different dimensions without the controller.

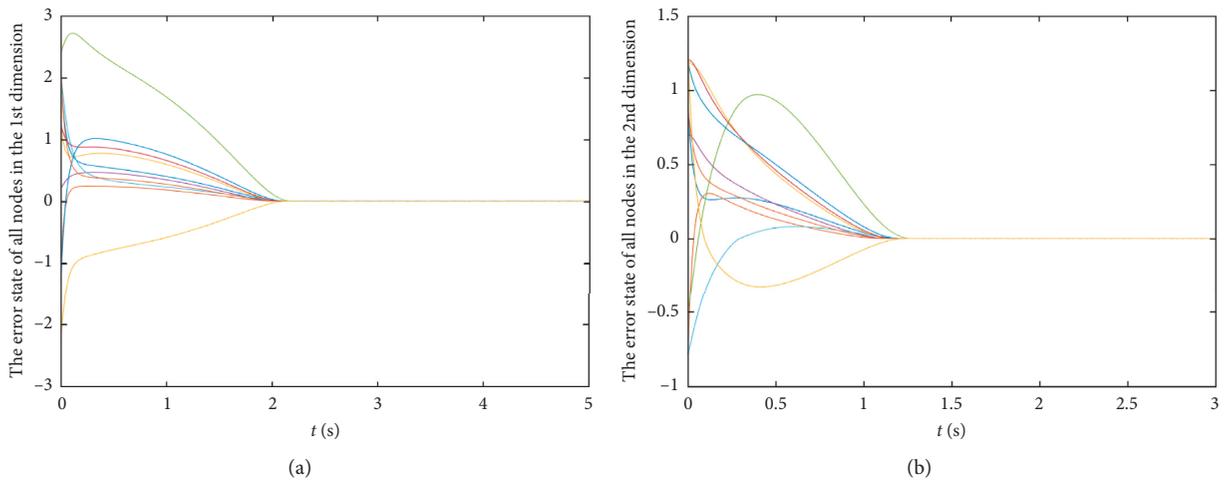


FIGURE 2: Continued.

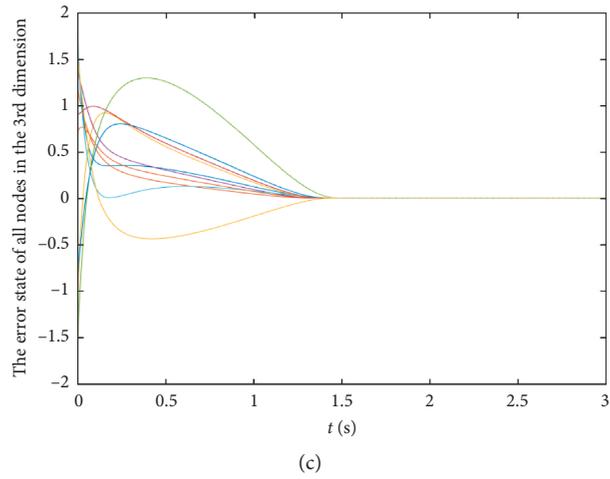


FIGURE 2: The error state of all nodes in different dimensions under the controller (5).

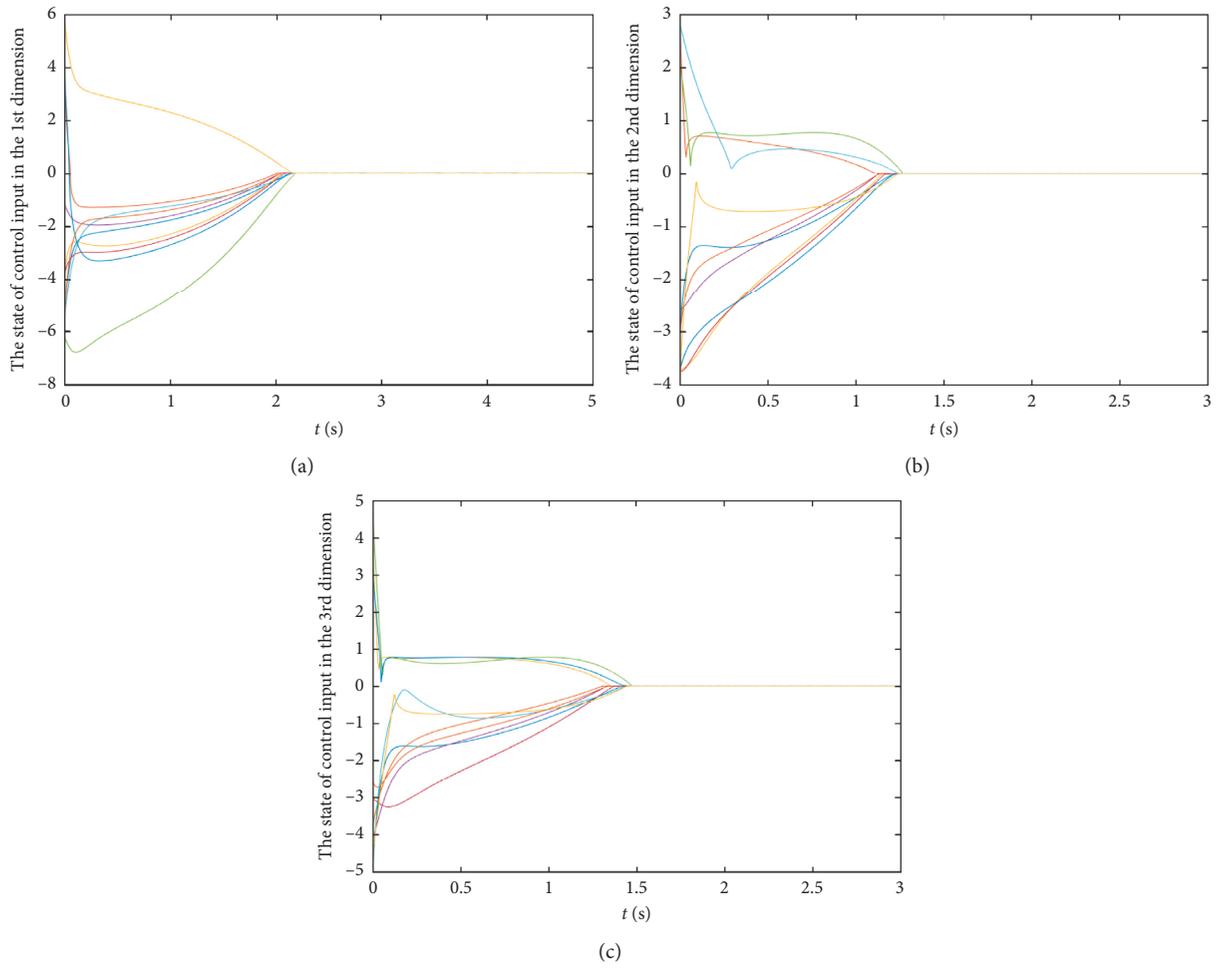


FIGURE 3: The state of the input u_i in different dimensions.

where $V(0) = \sum_{i=1}^N e_i^T(0)e_i(0)$.

Corollary 8. Assume that $\lambda_{\max}(M - 2W) > 0$, where $W = \text{diag}(w_1, w_2, \dots, w_N)$. If an appropriate matrix W exists and satisfies this inequality, then the system (1) can realize synchronization in finite time. The time used to realize synchronization is as follows:

$$T_s \leq \frac{2 \ln(1 - (1/2k)(\lambda_{\max}(M - 2W))V^{(1-\beta)/2}(0))}{(\lambda_{\max}(M - 2W))(\beta - 1)}, \quad (55)$$

where $V(0) = \sum_{i=1}^N e_i^T(0)e_i(0)$.

Remark 8. In the previous theorem and the corollary verification process, the eigenvalues of M are the maximum values, and the eigenvalues of W are the minimum values. In the present case, the eigenvalues of M and W are the maximum.

4. Numerical Simulation

In this section, we verify the theoretical results by numerical simulation.

Suppose that a network model consists of 10 nodes, the internal dimension of each node is 3, the coupling relationship between the nodes is nonlinear, and the system contains uncertainties. The state equation of each node is described as follows:

$$\begin{aligned} \dot{x}_i(t) &= (A + \Delta A(t))x_i(t) + (B + \Delta B(t))f(x_i(t)) \\ &+ \sum_{j=1}^{10} c_{ij}g(x_j(t)) + u_i(t), \quad i = 1, 2, \dots, 10, \end{aligned} \quad (56)$$

where A, B, C, H, E_1, E_2 , and Σ are the parameter values in the system given as

$$\begin{aligned} A &= \begin{bmatrix} -3 & 4 & 1 \\ 4 & -2 & 2 \\ 1 & 2 & -2 \end{bmatrix}, \\ B &= \begin{bmatrix} -5 & 1 & -1 \\ 1 & -2 & 1 \\ -1 & 1 & -3 \end{bmatrix}, \\ H &= \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix}, \\ E_1 &= \begin{bmatrix} -4 & 5 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 4 \end{bmatrix}, \\ E_2 &= \begin{bmatrix} 1 & 0 & 2 \\ -1 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix}, \end{aligned}$$

$$C = \begin{bmatrix} -3 & 1 & 1 & 0 & -1 & 1 & 2 & 0 & -3 & 2 \\ 1 & -2 & 1 & 1 & 2 & -1 & -3 & 0 & -1 & 2 \\ 1 & 2 & -5 & 2 & 0 & -1 & -3 & 0 & 2 & 2 \\ 0 & 1 & -2 & 0 & -2 & 2 & -1 & -1 & 0 & 3 \\ -1 & 2 & 1 & -2 & -1 & 1 & 0 & 0 & -1 & 1 \\ 1 & -1 & -1 & 2 & 1 & -2 & 0 & -3 & 2 & 1 \\ 2 & -3 & -1 & -1 & 0 & 0 & 2 & 1 & 2 & -2 \\ 1 & 1 & -3 & 1 & 3 & -3 & 1 & -2 & -1 & 2 \\ -3 & -1 & 1 & 3 & -3 & 2 & -2 & 1 & 3 & -1 \\ 0 & 2 & -2 & 1 & 1 & 0 & 1 & 2 & 0 & -5 \end{bmatrix},$$

$$\Sigma = \text{diag}([1, 1.5, 0.7, 0.8, 1, 1.2, 0.9, 0.8, 1.1, 1.4]),$$

$$I_N = \text{diag}([1, 1, 1, 1, 1, 1, 1, 1, 1, 1]),$$

$$I_n = \text{diag}([1, 1, 1]),$$

$$F(t) = \text{diag}([\sin(t), \sin(2t), \sin(3t)]). \quad (57)$$

The nonlinear vector-valued function in the complex network is described by $f(x) = \tanh(x)$, $g(x) = \sin(x)$. The matrix W and the controller gain k that satisfy the conditions in Theorem 1 can be found using the MATLAB toolbox. The motion trajectory of the error system can be obtained, as shown in the figures. Figure 1 shows the error state curve of all nodes in different dimensions without the controller, while Figure 2 represents the error state under the action of the controller (5). Meanwhile, Figure 3 shows the state of the control input u_i of all nodes. From these figures, it is clear that under the obtained controllers, each node of the system can achieve synchronization within the finite time.

According to the initial value data, when $k = 2$ and $\beta = 0.4$, the time required for the first, second, and third dimension of all nodes to reach the synchronization state is $t_1 = 2.157$, $t_2 = 1.209$, and $t_3 = 1.506$, respectively. Meanwhile, when MATLAB was used for simulation, the synchronization time corresponding to that mentioned above is $t_1^* = 2.133$, $t_2^* = 1.244$, and $t_3^* = 1.445$, respectively. By comparing the synchronization time of the two cases, it can be seen that the estimated value is basically consistent with the simulation value. Therefore, the feasibility of the theoretical method proposed in this paper has been verified.

5. Conclusion

This study investigates the finite-time synchronization control problem for a class of nonlinear uncertain complex dynamic networks. In the complex network model, the uncertainty of the system is considered, and the appropriate Lyapunov function is constructed under the assumption that the uncertain term satisfies the matching condition. An appropriate feedback control is designed on the basis of the linear matrix inequality and Lipschitz inequality. Some sufficient criterions are obtained for guaranteeing finite-time synchronization of a class of complex dynamic networks with parameters uncertainties under different conditions. According to this, an estimation of the upper bound of the

time required to synchronize the system is given. Finally, the feasibility and effectiveness of the proposed method are verified by numerical simulation.

In the upcoming research, we will focus on the following aspects:

- (1) The uncertainties considered in this paper are to satisfy both the norm bounded and matching conditions. In the following work, the case where the uncertain term only satisfies the norm bounded or only knows the upper or lower bounds of the system parameters will be studied.
- (2) The method of using the strategy of pinning control to realize finite-time synchronization of uncertain complex dynamic networks will be taken into account.
- (3) Event-triggered finite-time synchronization control for uncertain complex networks will be studied.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Both authors contributed equally to the writing of this paper. Furthermore, both authors also read carefully and approved the final manuscript.

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