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Research Article

A Study on Lump and Interaction Solutions to a (3 + 1)-Dimensional Soliton Equation

Xi-zhong Liu, [1] **Zhi-Mei Lou**, [1] **Xian-Min Qian**, [2] **and Lamine Thiam** [1]

[1] *Institute of Nonlinear Science, Shaoxing University, Shaoxing 312000, China*
[2] *Yuanpei College, Shaoxing University, Shaoxing 312000, China*

Correspondence should be addressed to Xi-zhong Liu; liuxizhong123@163.com

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Based on bilinear formulation of a $(3+1)$-dimensional soliton equation, lump solution and related interaction solutions are investigated. The lump solutions of the soliton equation are classified into three cases with nonsingularity conditions being given. The interaction solutions between lump and a stripe soliton are obtained in eight cases, which have interesting fusing and fission behaviors with changing time. The interaction solutions of the soliton equation between a lump and a resonant pair of stripe solitons are also given, and we find that the lump just exist for a finite period during the interaction process.

1. Introduction

As we know, in nonlinear systems, soliton solutions are usually exponentially localized in certain directions. In recent years, rational solutions to nonlinear systems have attracted a great attention. In particular, a kind of special rational solution called lump solution which localized rationally in all directions of the space has been studied a lot. Since lump solution was first discovered [1] many integrable nonlinear systems are found to have lump solutions, including the KPI equation [2, 3], the BKP equation [4], the Ishimori-I equation [5], and so on [6–12].

For a nonlinear equation, having Hirota bilinear formation is an indication of being integrable in this sense. Various solutions of bilinear equations, including solitons, positons, and complexitons, can be obtained via Wronskian formulation [13], while lump solutions can be obtained by taking long wave limit of solitons [3]. Lump is stable as soliton, the main difference between them lies in the fact that soliton decay exponentially in certain directions while a lump is a localized wave that decay rationally in all directions in space and moves with a uniform velocity [14]. In addition, soliton have a relation between amplitude and width but lump waves have no such relation. Another fascinating feature of lump solutions is that the interaction phase shifts between lump waves are exactly zero [1]. In another aspect, the interactions between lump and various solitons, such as stripe soliton and resonant stripe solitons, have been studied for many integrable nonlinear systems [15–19]. It reveals that most of the interaction solutions between lump and various solitons are completely inelastic [20], while a few of them are elastic [21]. The interaction between lump wave and rogue wave solutions, which have great potential applications in the field of nonlinear optics [22] and oceanography [23] can also be generated by interaction between lump and a pair of resonant kink stripe solitons [8].

The study of lump solution mainly focuses on $(2+1)$-dimensional integrable systems including some reduction equations from some $(3+1)$-dimensional systems. Some attempts are also made to find rational solutions to nonintegrable $(3+1)$-dimensional KP I and KP II by tanh-function method and G/G'-expansion method. In this paper, we search for lump solution and abundant interaction solutions between lump and some kinds of solitons for the $(3+1)$-dimensional soliton equation based on its Hirota bilinear form.

The $(3+1)$-dimensional soliton equation takes the form

$$3u_{xz} - \left(2u_t + u_{xxx} - 2uu_x\right)_y + 2\left(u_x\partial_x^{-1}u_y\right)_x = 0, \quad (1)$$

where ∂_x^{-1} stands for an inverse operator of the partial differential operator ∂_x. As we know, the AKNS (Ablowitz–Kaup–Newell–Segur) system, which can reduce to the nonlinear Schrödinger equation, is one of the most important physical models. In reference [24], equation (1) was derived and decomposed into systems of ordinary differential equations with the help of $(1+1)$-dimensional AKNS equations, from which algebraic-geometrical solutions were obtained in terms of Riemann theta functions. Also, solutions of the $(3+1)$-dimensional soliton equation (1) can be derived from the solutions of the first three members of the AKNS hierarchy (see Proposition 2.4 in reference [24]). In reference [25], an N-soliton solution of equation (1) was obtained based on its bilinear form by using perturbation method and constructing an N-th order Wronskian determinant of solutions by introducing four linear differential equations.

The paper is organized as follows: In Section 2, we give general form of lump solutions of the soliton equation (1) in quadratic function form, which can be classified into three cases. The nonsingularity conditions of these lump solutions are analyzed, and two special solutions are explicitly given and analyzed graphically. In Section 3, interaction solutions between a lump and a stripe soliton are given in eight cases, which have rich dynamic behaviors with different time. In Section 4, interaction solutions between a lump and a resonant stripe soliton are given in two cases with some discussions of the interaction dynamical behaviors.

2. Lump Solutions to the (3 + 1)-Dimensional Soliton Equation

To search for lump solutions of equation (1), we first changed it into a bilinear form [25]:

$$2\left(3f_{xx}f-3f_x^2-2f_{yt}f+2f_yf_t-f_{xxxy}f+f_{xxx}f_y+3f_{xxy}f_x-3f_{xx}f_{xy}\right)=0, \tag{2}$$

under the transformation

$$u=-3(\ln f)_{xx}. \tag{3}$$

Equation (2) can also be written in the form of

$$\left(3D_xD_z-2D_yD_t-D_yD_x^3\right)f\cdot f=0, \tag{4}$$

with Hirota's bilinear derivatives D_x, D_y, D_z, D_t being defined as follows [26]:

$$D_x^\alpha D_y^\beta D_z^\gamma D_t^\delta(g(x,y,z,t)\cdot f(x,y,z,t)) =\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^\alpha\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial y'}\right)^\beta\left(\frac{\partial}{\partial z}-\frac{\partial}{\partial z'}\right)^\gamma\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^\delta g(x,y,z,t)f(x',y',z',t')|_{x=x',y=y',z=z',t=t'}. \tag{5}$$

The quadratic function solutions of the soliton equation (2) can in general be assumed in the form

$$\begin{aligned} f&=g^2+h^2+a_{11},\\ g&=a_1x+a_2y+a_3z+a_4t+a_5,\\ h&=a_6x+a_7y+a_8z+a_9t+a_{10}, \end{aligned} \tag{6}$$

with a_i, $(i=1,\cdots,11)$ being real constants, the rational solutions of equation (1) can be generated by substituting equation (6) into equation (3).

Substituting equation (6) into (2), a direct computation yields the following three types of solutions:

Case 1 $(a_1\neq 0)$:

$$\begin{aligned} a_1&=a_1,\\ a_2&=-\frac{a_6a_7}{a_1},\\ a_3&=-\frac{2a_4a_6a_7}{3a_1^2},\\ a_4&=a_4,\\ a_5&=a_5,\\ a_6&=a_6,\\ a_7&=a_7,\\ a_8&=\frac{2a_4a_7}{3a_1},\\ a_9&=\frac{a_6a_4}{a_1},\\ a_{10}&=a_{10},\\ a_{11}&=a_{11}. \end{aligned} \tag{7}$$

Case 2 $(a_1a_7\neq 0)$:

$$\begin{aligned} a_1&=a_1,\\ a_2&=-\frac{a_6a_7}{a_1},\\ a_3&=-\frac{a_6a_8}{a_1},\\ a_4&=\frac{3a_1a_8}{2a_7},\\ a_5&=a_5,\\ a_6&=a_6,\\ a_7&=a_7,\\ a_8&=a_8,\\ a_9&=\frac{3a_6a_8}{2a_7},\\ a_{10}&=\frac{a_5a_6}{a_1},\\ a_{11}&=a_{11}. \end{aligned} \tag{8}$$

Case 3 $((a_1a_9 - a_4a_6)(a_1a_7 - a_2a_6) \neq 0)$:

$$\begin{aligned} a_1 &= a_1, \\ a_2 &= a_2, \\ a_3 &= \frac{2\left[(a_2a_4 - a_7a_9)a_1 + (a_2a_9 + a_4a_7)a_6\right]}{3\left(a_1^2 + a_6^2\right)}, \\ a_4 &= a_4, \\ a_5 &= a_5, \\ a_6 &= a_6, \\ a_7 &= a_7, \\ a_8 &= \frac{2\left[(a_2a_9 + a_4a_7)a_1 - (a_2a_4 - a_7a_9)a_6\right]}{3\left(a_1^2 + a_6^2\right)}, \\ a_9 &= a_9, \\ a_{10} &= a_{10}, \\ a_{11} &= -\frac{3\left(a_1^2 + a_6^2\right)^2 (a_1a_2 + a_6a_7)}{2(a_1a_9 - a_4a_6)(a_1a_7 - a_2a_6)}. \end{aligned} \tag{9}$$

To require analyticity of the solution (3) with (6), a_{11} in the first two solutions of equations (7) and (8) should satisfy $a_{11} > 0$. While for the solution (9), to ensure $a_{11} > 0$, we assume the parameters of a_i $(i = 1, 2, 6, 7)$ are positive, and also

$$\frac{a_1}{a_6} \in \left(\frac{a_2}{a_7}, \frac{a_4}{a_9}\right) \text{ or } \frac{a_1}{a_6} \in \left(\frac{a_4}{a_9}, \frac{a_2}{a_7}\right), \tag{10}$$

with the condition of $(a_2/a_7) \neq (a_4/a_9)$.

To give some concrete descriptions of the lump solutions, we choose the parameters of Case 1 as follows:

$$\begin{aligned} a_i &= 1, \ (i = 1, 4, 5, 6, 7, 9, 10, 11), \\ a_2 &= -1, \\ a_3 &= -\frac{2}{3}, \\ a_8 &= \frac{2}{3}, \end{aligned} \tag{11}$$

which leads to a lump solution for the soliton equation (1):

$$u = \frac{108\left(18t^2 + 36tx + 18x^2 - 18y^2 - 24yz - 8z^2 + 36t + 36x + 9\right)}{\left(18t^2 + 36tx + 18x^2 + 18y^2 + 24yz + 8z^2 + 36t + 36x + 27\right)^2}. \tag{12}$$

For Case 3, we choose the parameters being fixed as follows:

$$\begin{aligned} a_i &= 1, \ (i = 2, 5, 9, 10), \\ a_3 &= a_8 = \frac{65}{6}, \\ a_4 &= 8, \\ a_6 &= 2, \\ a_7 &= 8, \\ a_{11} &= \frac{432}{49}, \end{aligned} \tag{13}$$

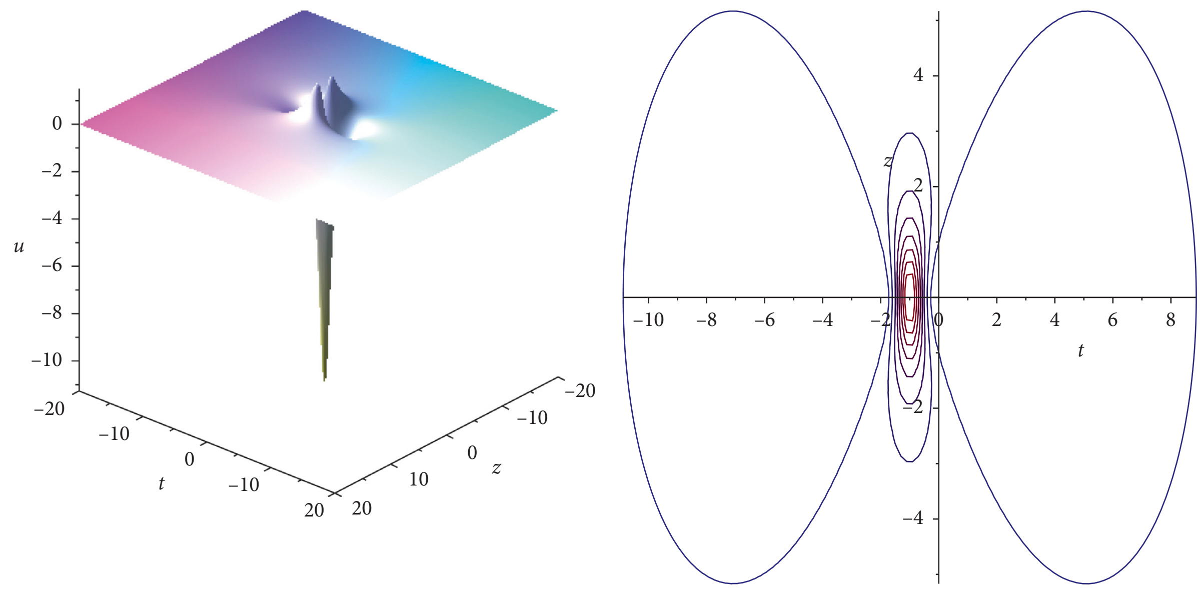
which leads to a lump solution

$$\begin{aligned} u &= \frac{E_1}{F_1}, \\ E_1 &= \big(42336\big(14112t^2 + 31752tx + 114660ty + 171990tz \\ &\quad + 7056x^2 + 31752xy + 76440xz + 14112y^2 \\ &\quad + 171990yz + 207025z^2 + 15876t + 7056x + 15876y \\ &\quad + 38220z - 6012\big)\big), \\ F_1 &= \big(57330t^2 + 31752tx + 28224ty + 171990tz + 7056x^2 \\ &\quad + 31752xy + 76440xz + 57330y^2 + 171990yz \\ &\quad + 207025z^2 + 15876t + 7056x + 15876y + 38220z + 9540\big)^2. \end{aligned} \tag{14}$$

Figure 1(a) gives the three-dimensional structure of the solution (12) with $x = y = 0$, and Figure 1(b) is the corresponding contour plot. From Figure 1, it is obvious that the maximal value is 3/2 which dwells at two points $(z = 0, t = -1 + (\sqrt{6}/2))$ and $(z = 0, t = -1 - (\sqrt{6}/2))$, while the minimal value is -12, which corresponds to the point $(z = 0, t = -1)$. Similarly, Figure 2 reveals the structure of the lump solution (14) in the case of $z = t = 0$. The maximal value 49/72 corresponds to the points of $(x = -(1/2) + (9/7)\sqrt{2}, y = 0)$ and $(x = -(1/2) - (9/7)\sqrt{2}, y = 0)$, while the minimal value $-(49/9)$ corresponds to the point $(x = -(1/2), y = 0)$.

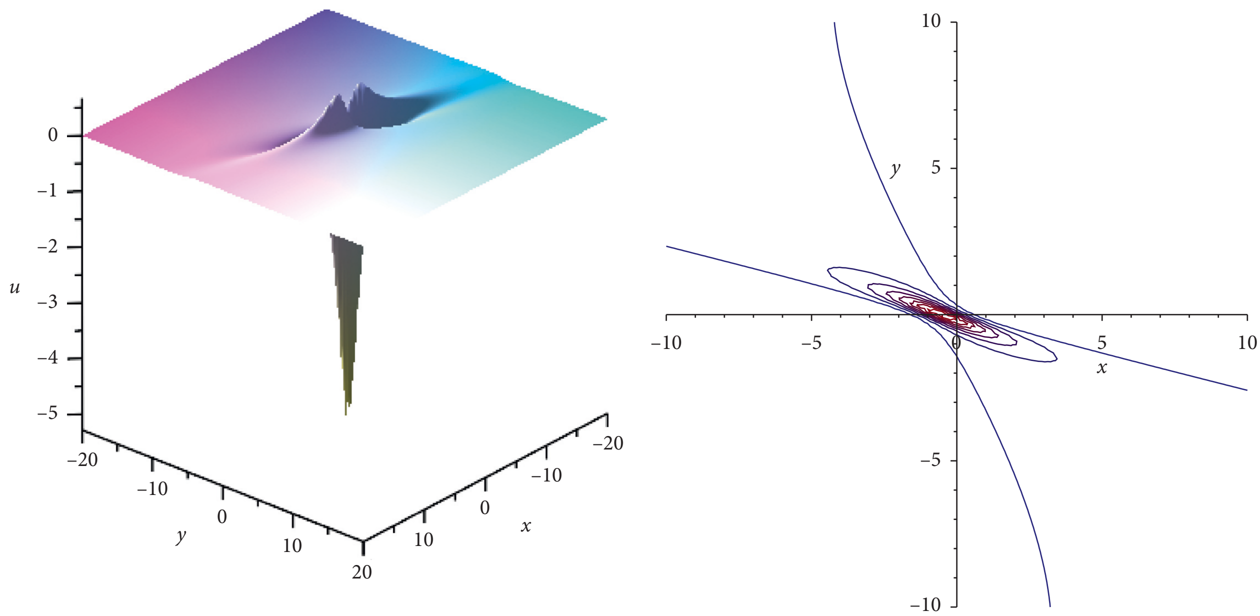
3. Interaction Solutions between a Lump and a Stripe Soliton

To search for the interaction solutions between a lump and a stripe soliton of the soliton equation (1), we assume f in equation (2) to have the form



(a) (b)

Figure 1: Structure of the lump solution (12) with $x = y = 0$: (a) the three-dimensional plot; (b) the density plot.



(a) (b)

Figure 2: Structure of the lump solution (14) with $z = t = 0$: (a) the three-dimensional plot; (b) the density plot.

$$
\begin{aligned}
f &= m_1^2 + n_1^2 + a_{11} + l_1,\\
m_1 &= a_1 x + a_2 y + a_3 z + a_4 t + a_5,\\
n_1 &= a_6 x + a_7 y + a_8 z + a_9 t + a_{10},\\
l_1 &= k e^{k_1 x + k_2 y + k_3 z + k_4 t},
\end{aligned} \tag{15}
$$

with a_i $(i = 1, \cdots, 11)$, k, and k_i $(i = 1, 2, 3)$ being real constants.

Substituting equation (15) into equation (2), after some routine computations, we have the following eight types of solutions:

Case 1 ($k_1 \neq 0$):

$$\begin{aligned} a_1 &= 0, \\ a_2 &= 0, \\ a_3 &= \frac{2a_4k_2}{3k_1}, \\ a_4 &= a_4, \\ a_5 &= a_5, \\ a_6 &= 0, \\ a_7 &= 0, \\ a_8 &= \frac{2a_9k_2}{3k_1}, \\ a_9 &= a_9, \\ a_{10} &= a_{10}, \\ a_{11} &= a_{11}, \\ k_1 &= k_1, \\ k_2 &= k_2, \\ k_3 &= \frac{k_2\left(k_1^3+2k_4\right)}{3k_1}. \end{aligned} \tag{16}$$

Case 2 ($k_1 \neq 0$):

$$\begin{aligned} a_1 &= 0, \\ a_2 &= 0, \\ a_3 &= \frac{a_2\left(k_1^3+2k_4\right)}{3k_1}, \\ a_4 &= 0, \\ a_5 &= a_5, \\ a_6 &= 0, \\ a_7 &= a_7, \\ a_8 &= \frac{\left(k_1^3+2k_4\right)a_7}{3k_1}, \\ a_9 &= 0, \\ a_{10} &= a_{10}, \\ a_{11} &= a_{11}, \\ k_1 &= k_1, \\ k_2 &= k_2, \\ k_3 &= \frac{k_2\left(k_1^3+2k_4\right)}{3k_1}. \end{aligned} \tag{17}$$

Case 3 ($k_1 a_6 \neq 0$):

$$\begin{aligned} a_1 &= 0, \\ a_2 &= a_2, \\ a_3 &= \frac{a_2\left(3a_6^2k_1^4+6a_6^2k_1k_4-4a_4^2\right)}{9a_6^2k_1^2}, \\ a_4 &= a_4, \\ a_5 &= a_5, \\ a_6 &= a_6, \\ a_7 &= -\frac{2a_2a_4}{3a_6k_1^2}, \\ a_8 &= -\frac{4a_2a_4\left(2k_1^3+k_4\right)}{9a_6k_1^3}, \\ a_9 &= \frac{a_6\left(k_1^3+2k_4\right)}{2k_1}, \\ a_{10} &= a_{10}, \\ a_{11} &= \frac{a_6^2}{k_1^2}, \\ k_1 &= k_1, \\ k_2 &= -\frac{2a_2a_4}{3a_6^2k_1}, \\ k_3 &= -\frac{2a_2a_4\left(k_1^3+2k_4\right)}{9a_6^2k_1^2}. \end{aligned} \tag{18}$$

Case 4 ($k_1 a_6 a_4 \neq 0$):

$$\begin{aligned} a_1 &= 0, \\ a_2 &= a_2, \\ a_3 &= \frac{a_2\left(3a_6^2k_1^4+24a_6^2k_1k_4-16a_4^2\right)}{36a_6^2k_1^2}, \\ a_4 &= a_4, \\ a_5 &= a_5, \\ a_6 &= a_6, \\ a_7 &= \frac{\left(3a_6k_1^2-4a_4\right)\left(3a_6k_1^2+4a_4\right)a_2}{24a_6k_1^2a_4}, \\ a_8 &= -\frac{a_2\left(9a_6^2k_1^7-36a_6^2k_1^4k_4+80a_4^2k_1^3+64a_4^2k_4\right)}{144k_1^3a_6a_4}, \\ a_9 &= -\frac{a_6\left(k_1^3-4k_4\right)}{4k_1}, \\ a_{10} &= a_{10}, \\ a_{11} &= -\frac{a_6^2\left(3a_6k_1^2-4a_4\right)\left(3a_6k_1^2+4a_4\right)}{16k_1^2a_4^2}, \\ k_1 &= k_1, \\ k_2 &= -\frac{a_2\left(9a_6^2k_1^4+16a_4^2\right)}{24a_6^2k_1a_4}, \\ k_3 &= -\frac{a_2\left(9a_6^2k_1^4+16a_4^2\right)\left(k_1^3+2k_4\right)}{72a_6^2k_1^2a_4}. \end{aligned} \tag{19}$$

Case 5 $(k_1 a_1 \neq 0)$:

$$
\begin{aligned}
a_1 &= a_1,\\
a_2 &= -\frac{a_6 a_7}{a_1},\\
a_3 &= -\frac{a_7 a_6\left(k_1^3 + 2k_4\right)}{3a_1 k_1},\\
a_4 &= \frac{a_1\left(k_1^3 + 2k_4\right)}{2k_1},\\
a_5 &= a_5,\\
a_6 &= a_6,\\
a_7 &= a_7,\\
a_8 &= \frac{\left(k_1^3 + 2k_4\right)a_7}{3k_1},\\
a_9 &= \frac{a_6\left(k_1^3 + 2k_4\right)}{2k_1},\\
a_{10} &= a_{10},\\
a_{11} &= a_{11},\\
k_1 &= k_1,\\
k_2 &= 0,\\
k_3 &= 0.
\end{aligned}
\tag{20}
$$

Case 6 $(k_1 a_1 a_6 \neq 0)$:

$$
\begin{aligned}
a_1 &= a_1,\\
a_2 &= 0,\\
a_3 &= \frac{(a_1^2 + a_6^2)k_1 k_2}{a_1},\\
a_4 &= \frac{a_1^2 k_1^3 + 3a_6^2 k_1^3 + 2a_1^2 k_4}{2a_1 k_1},\\
a_5 &= a_5,\\
a_6 &= a_6,\\
a_7 &= \frac{k_2\left(a_1^2 + a_6^2\right)}{a_6 k_1},\\
a_8 &= \frac{k_2\left(a_1^2 k_1^3 + a_6^2 k_1^3 + 2a_1^2 k_4 + 2a_6^2 k_4\right)}{3a_6 k_1^2},\\
a_9 &= -\frac{a_6\left(k_1^3 - k_4\right)}{k_1},\\
a_{10} &= a_{10},\\
a_{11} &= \frac{a_1^2 + a_6^2}{k_1^2},\\
k_1 &= k_1,\\
k_2 &= k_2,\\
k_3 &= \frac{k_2\left(k_1^3 + 2k_4\right)}{3k_1}.
\end{aligned}
\tag{21}
$$

Case 7 $(k_2 k_1 b_1 \neq 0)$:

$$
\begin{aligned}
a_1 &= \frac{b_1}{k_2},\\
a_2 &= 0,\\
a_3 &= \frac{a_7 k_1^2\left(a_6 k_2 + a_7 k_1\right)}{2b_1},\\
a_4 &= \frac{\left(a_6 k_2 + a_7 k_1\right)\left[4a_6\left(k_1^3 - k_4\right)k_2 - a_7 k_1\left(k_1^3 - 4k_4\right)\right]}{4k_2 k_1 b_1},\\
a_5 &= a_5,\\
a_6 &= a_6,\\
a_7 &= a_7,\\
a_8 &= -\frac{\left(k_1^3 - 4k_4\right)a_7}{6k_1},\\
a_9 &= -\frac{4a_6 k_1^3 k_2 + 3a_7 k_1^4 - 4a_6 k_2 k_4}{4k_2 k_1},\\
a_{10} &= a_{10},\\
a_{11} &= \frac{2a_6 a_7^2}{\left(a_6 k_2 + a_7 k_1\right)k_2},\\
k_1 &= k_1,\\
k_2 &= k_2,\\
k_3 &= \frac{k_2\left(k_1^3 + 2k_4\right)}{3k_1}.
\end{aligned}
\tag{22}
$$

with b_1 satisfying $a_6^2 k_2^2 - a_7^2 k_1^2 + b_1^2 = 0$.

Case 8 $(k_1 a_2 a_7 a_9 \neq 0)$:

$$
\begin{aligned}
a_1 &= -\frac{2a_9 a_7}{3a_2 k_1^2},\\
a_2 &= a_2,\\
a_3 &= \frac{2\left(2k_1^3 + k_4\right)a_2}{3k_1},\\
a_4 &= -\frac{a_9 a_7\left(k_1^3 + 2k_4\right)}{3a_2 k_1^3},\\
a_5 &= a_5,\\
a_6 &= 0,\\
a_7 &= a_7,\\
a_8 &= -\frac{3a_2^2 k_1^3 - a_7^2 k_1^3 - 2a_7^2 k_4}{3a_7 k_1},\\
a_9 &= a_9,\\
a_{10} &= a_{10},\\
a_{11} &= \frac{4a_7^2 a_9^2}{9a_2^2 k_1^6},\\
k_1 &= k_1,\\
k_2 &= -\frac{3a_2^2 k_1^3}{2a_7 a_9},\\
k_3 &= -\frac{a_2^2 k_1^2\left(k_1^3 + 2k_4\right)}{2a_7 a_9}.
\end{aligned}
\tag{23}
$$

To give more detailed analysis of these interaction solutions, we take the Case 6 as an example with the parameters being fixed as follows:

$$
\begin{aligned}
a_1 &= a_5 = a_6 = a_9 = a_{10} = k = k_1 = k_2 = 1,\\
a_2 &= 0,\\
a_3 &= 2,\\
a_4 &= 4,\\
a_7 &= 2,\\
a_8 &= \frac{10}{3},\\
a_{11} &= 2,\\
k_3 &= \frac{5}{3},\\
k_4 &= 2,
\end{aligned}
\tag{24}
$$

which leads to

$$
\begin{aligned}
u &= -\frac{E_2}{F_2},\\
E_2 &= 27\Big\{\Big[18x^2 + (90t + 36y + 96z - 36)x + 36y^2\\
&\quad + (36t + 120z - 36)y + 136z^2\\
&\quad + (204t - 96)z + 153t^2 - 90t\Big]e^{2t+x+y+(5/3)z}\\
&\quad - 72x^2 - (360t + 144y + 384z + 144)x\\
&\quad - (576t + 288z + 144)y - 480z^2\\
&\quad - (1104t + 384)z - 288t^2 - 360t\Big\},\\
F_2 &= \Big[9e^{2t+x+y+(5/3)z} + 18x^2 + (90t + 36y + 96z + 36)x\\
&\quad + 36y^2 + (36t + 120z + 36)y + 136z^2\\
&\quad + (204t + 96)z + 153t^2 + 90t + 36\Big]^2.
\end{aligned}
\tag{25}
$$

Figure 3 in three-dimensional and Figure 4 in contour lines describe the interactional dynamical behaviors of the solution (25) with $z = 0$ and $t = -40, -20, 0, 20$, respectively. It can be easily seen from these figures that when time is taken as minus ones lump soliton fuses with the stripe soliton and the amplitude of the lump soliton become smaller even to vanish when time approaching to minus infinity. On the other hand, when time is taken as positive the lump separates with the stripe soliton.

4. Interaction between a Lump Soliton and a Pair of Stripe Solitons

In this section, we search for interaction solution between a lump soliton and a pair of stripe solitons. To this end, we assume the solution of equation (2) being a combination of positive quadratic function and hyperbolic cosine function, i.e.,

$$
\begin{aligned}
f &= m_2^2 + n_2^2 + a_{11} + l_2,\\
m_2 &= a_1 x + a_2 y + a_3 z + a_4 t + a_5,\\
n_2 &= a_6 x + a_7 y + a_8 z + a_9 t + a_{10},\\
l_2 &= k\cosh(k_1 x + k_2 y + k_3 z + k_4 t),
\end{aligned}
\tag{26}
$$

where a_i $(i = 1, \cdots, 11)$, k, and k_i $(i = 1, \cdots, 4)$ are real constants.

Substituting equation (26) into equation (2), after some routine work, we obtain two types of solutions:

Case 1 $(k_1 k_2 k_4 a_7 \neq 0)$:

$$
\begin{aligned}
a_1 &= \frac{a_7 k_1}{k_2},\\
a_2 &= -\frac{a_7(k_1^3 - 4k_4)}{3k_1^3},\\
a_3 &= \frac{a_7(5k_1^6 - 4k_1^3 k_4 + 8k_4^2)}{9k_1^4},\\
a_4 &= 0,\\
a_5 &= a_5,\\
a_6 &= \frac{a_7(k_1^3 - 4k_4)}{3k_1^2 k_2},\\
a_7 &= a_7,\\
a_8 &= 0,\\
a_9 &= -\frac{a_7(5k_1^6 - 4k_1^3 k_4 + 8k_4^2)}{6k_1^3 k_2},\\
a_{10} &= a_{10},\\
a_{11} &= \frac{9k^2 k_1^6 k_2^2}{4a_7^2(5k_1^6 - 4k_1^3 k_4 + 8k_4^2)},\\
k_2 &= k_2,\\
k_3 &= \frac{k_2(k_1^3 + 2k_4)}{3k_1},\\
k &= k.
\end{aligned}
\tag{27}
$$

Case 2 $(k_1 k_2 k_4 a_7 \neq 0)$:

$$
\begin{aligned}
a_1 &= -\frac{a_7 k_1}{k_2},\\
a_2 &= \frac{a_7(k_1^3 - 4k_4)}{3k_1^3},\\
a_3 &= -\frac{a_7(5k_1^6 - 4k_1^3 k_4 + 8k_4^2)}{9k_1^4},\\
a_4 &= 0,\\
a_5 &= a_5,\\
a_6 &= \frac{a_7(k_1^3 - 4k_4)}{3k_1^2 k_2},\\
a_7 &= a_7,\\
a_8 &= 0,\\
a_9 &= -\frac{a_7(5k_1^6 - 4k_1^3 k_4 + 8k_4^2)}{6k_1^3 k_2},\\
a_{10} &= a_{10},\\
a_{11} &= \frac{9k^2 k_1^6 k_2^2}{4a_7^2(5k_1^6 - 4k_1^3 k_4 + 8k_4^2)},\\
k_2 &= k_2,\\
k_3 &= \frac{k_2(k_1^3 + 2k_4)}{3k_1},\\
k &= k.
\end{aligned}
\tag{28}
$$

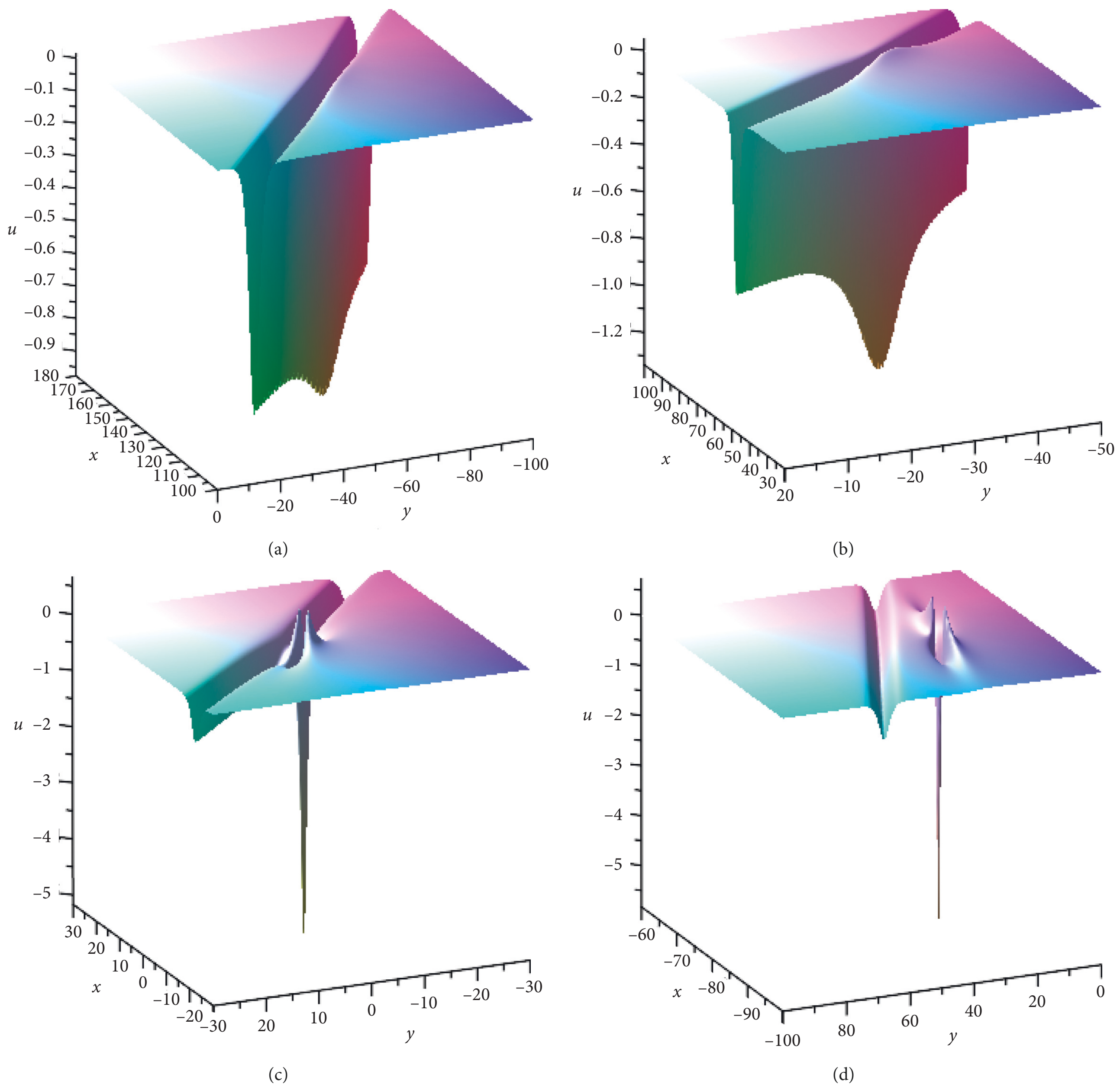


Figure 3: The three-dimensional structure of solution (25) with $z = 0$ and different time: (a) $t=-40$; (b) $t=-20$; (c) $t=0$; (d) $t=20$.

It can be easily verified that the constant a_{11} in equations (27) and (28) is positive for any nonzero constants k, k_1, k_2, k_4, a_7, which guarantees nonsingularity of the solution (3) with equation (26).

Choosing the parameters of Case 1 in (27) as follows:

$$\begin{aligned} &a_1 = k_1 = k_4 = -1,\\ &a_{10} = 1,\\ &a_{11} = \frac{1}{4},\\ &a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = a_8 = k = k_2 = k_3 = 1,\\ &a_9 = \frac{3}{2}, \end{aligned} \tag{29}$$

leads to a new solution of the soliton equation (1)

$$\begin{aligned} u &= -\frac{E_3}{F_3},\\ E_3 &= 12\Big\{4\cosh(t+x-y-z)^2 - (8t-32x)\\ &\quad\cdot \sinh(t+x-y-z) - 4\sinh(t+x-y-z)^2\\ &\quad+ \Big[8x^2 + 4tx + 8y^2 + (20t+16z+16)y + 8z^2 + (20t+16)z\\ &\quad+ 13t^2 + 20t + 25\Big]\cosh(t+x-y-z) - 32x^2 - 16tx + 32y^2\\ &\quad+ (80t+64z+64)y + 32z^2 + (80t+64)z + 48t^2 + 80t + 36\Big\},\\ F_3 &= \Big(8x^2 + 4tx + 8y^2 + (20t+16z+16)y + 8z^2\\ &\quad+ (20t+16)z + 13t^2 + 4\cosh(t+x-y-z) + 20t + 9\Big)^2. \end{aligned} \tag{30}$$

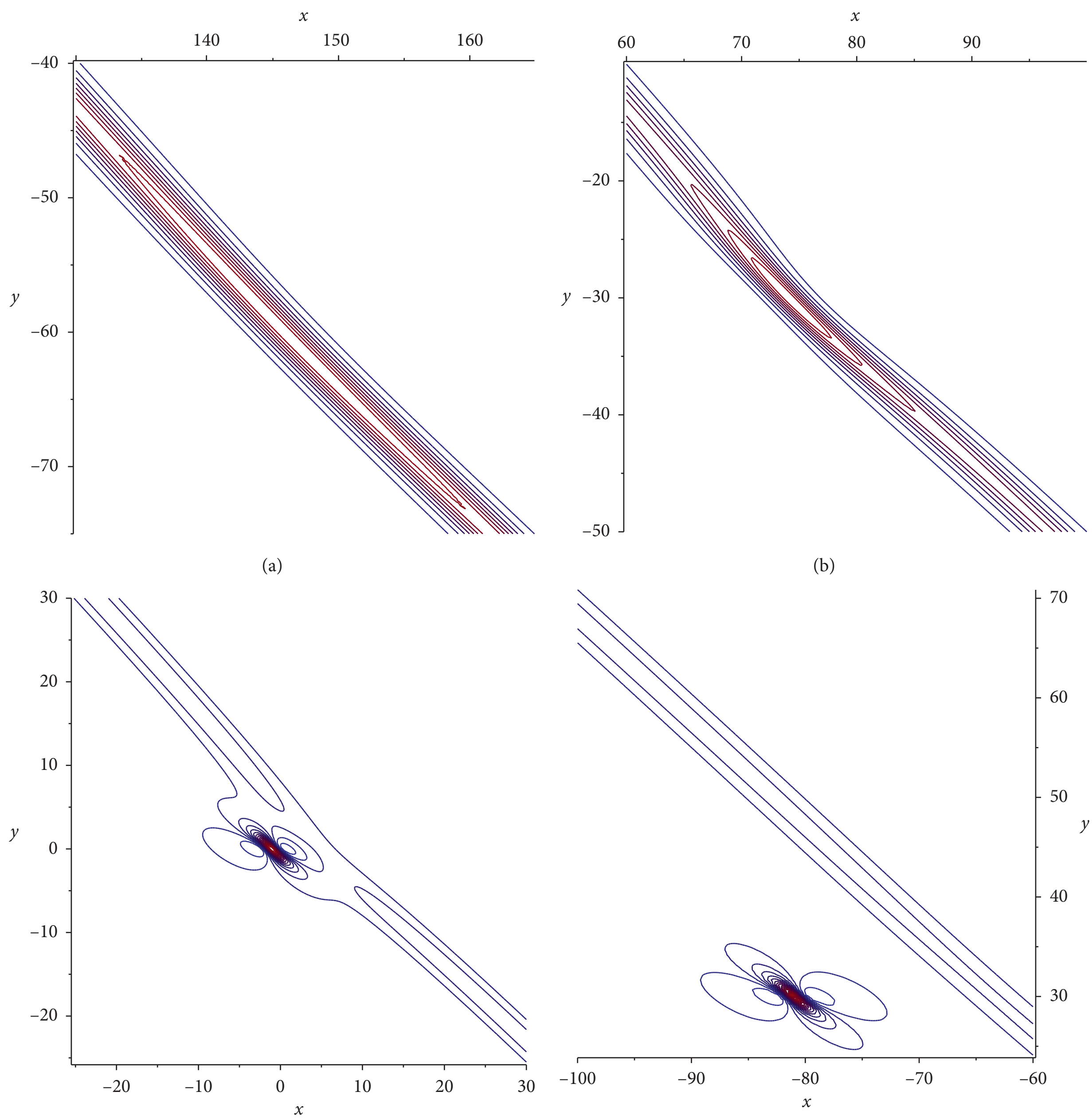


FIGURE 4: The contour plot of solution (25) with $z = 0$ and different time t: (a) $t = -40$; (b) $t = -20$; (c) $t = 0$; (d) $t = 20$.

Figures 5 and 6 display the dynamical behaviors of interaction between a lump and a pair of stripe solitons with different time at $t = -6, -3, 0, 3, 6$. It can be seen from these figures that when the absolute value of time $|t|$ becomes larger the amplitude of the lump soliton becomes smaller even to vanish, also the position of minimum value of the lump transfers from one stripe to the other one when time goes from negative to positive. In fact, when the time $|t|$ is taken a large value the amplitude of lump tends to zero, which meets the characteristic of rogue waves. This kind of mechanism of generating rogue waves has been discussed in some other literatures (see e.g. [8]).

5. Conclusion

In summary, the soliton equation is studied in terms of lump solution and related interaction solutions based on its Hirota's bilinear form. Three cases of lump solutions of the soliton equations are obtained with the nonsingularity conditions being given. Interactions between a lump and a stripe soliton are given in eight cases, one of which is explicitly expressed and plotted. Figures 3 and 4 reveal that the amplitude of the lump and the relative position of the lump and the stripe depends on the variable time. The interaction between a lump and a resonant pair of stripe solitons are also obtained in two cases. It is interesting to find that in this type

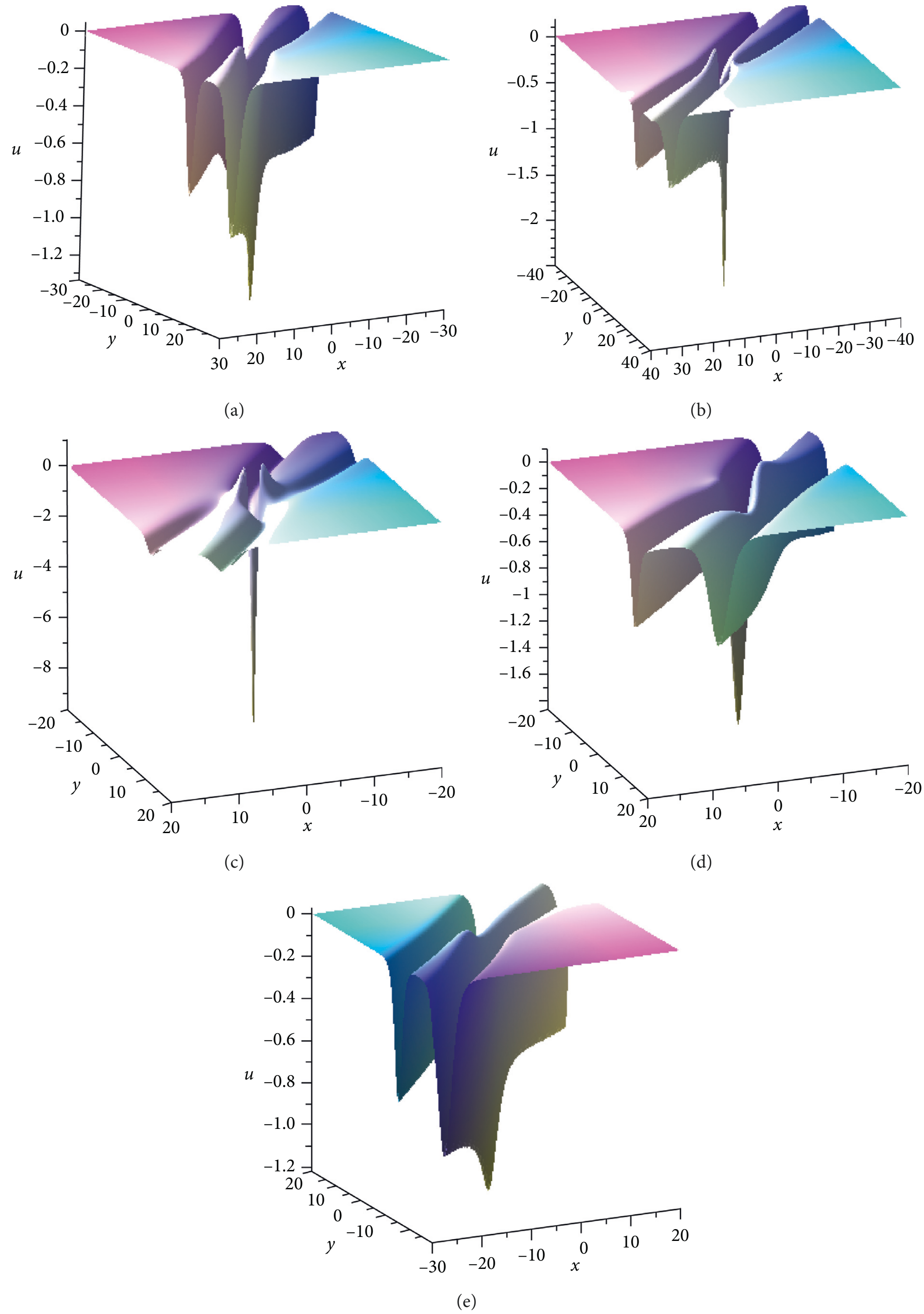


FIGURE 5: The structure of the interaction solution between lump and stripe solitons (30) with $z = 0$ and different time t: (a) $t = -6$; (b) $t = -3$; (c) $t = 0$; (d) $t = 3$; (e) $t = 6$.

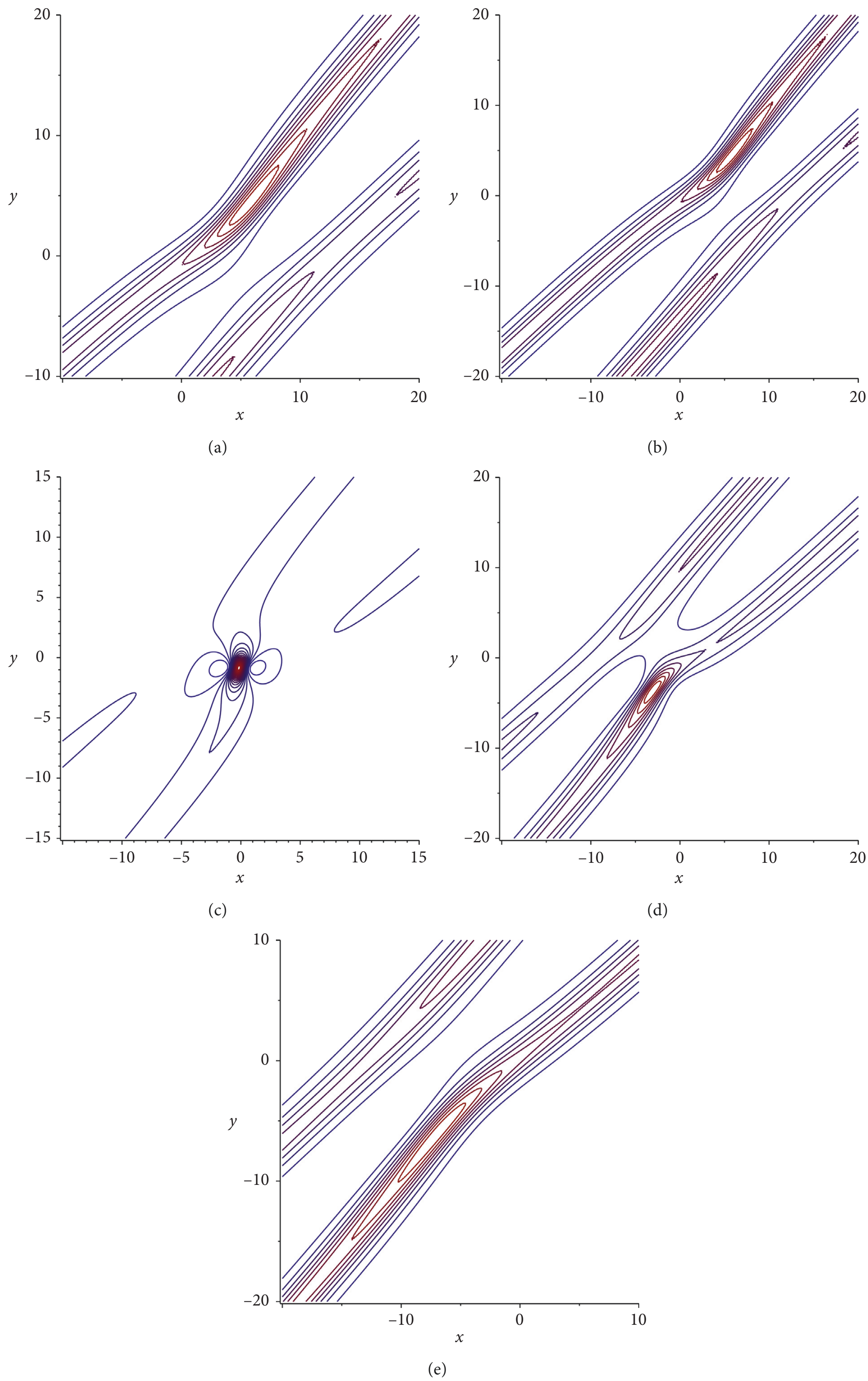


FIGURE 6: Density plot of the interaction solution between lump and stripe solitons (30) with $z = 0$ and different time t: (a) $t = -6$; (b) $t = -3$; (c) $t = 0$; (d) $t = 3$; (e) $t = 6$.

of interaction process the lump just exists in a finite period, which resembles the feature of a rogue wave "appear from nowhere and disappear without a trace."

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest to this work. There is no professional or other personal interest of any nature or kind in any product that could be construed as influencing the position presented in the manuscript entitled.

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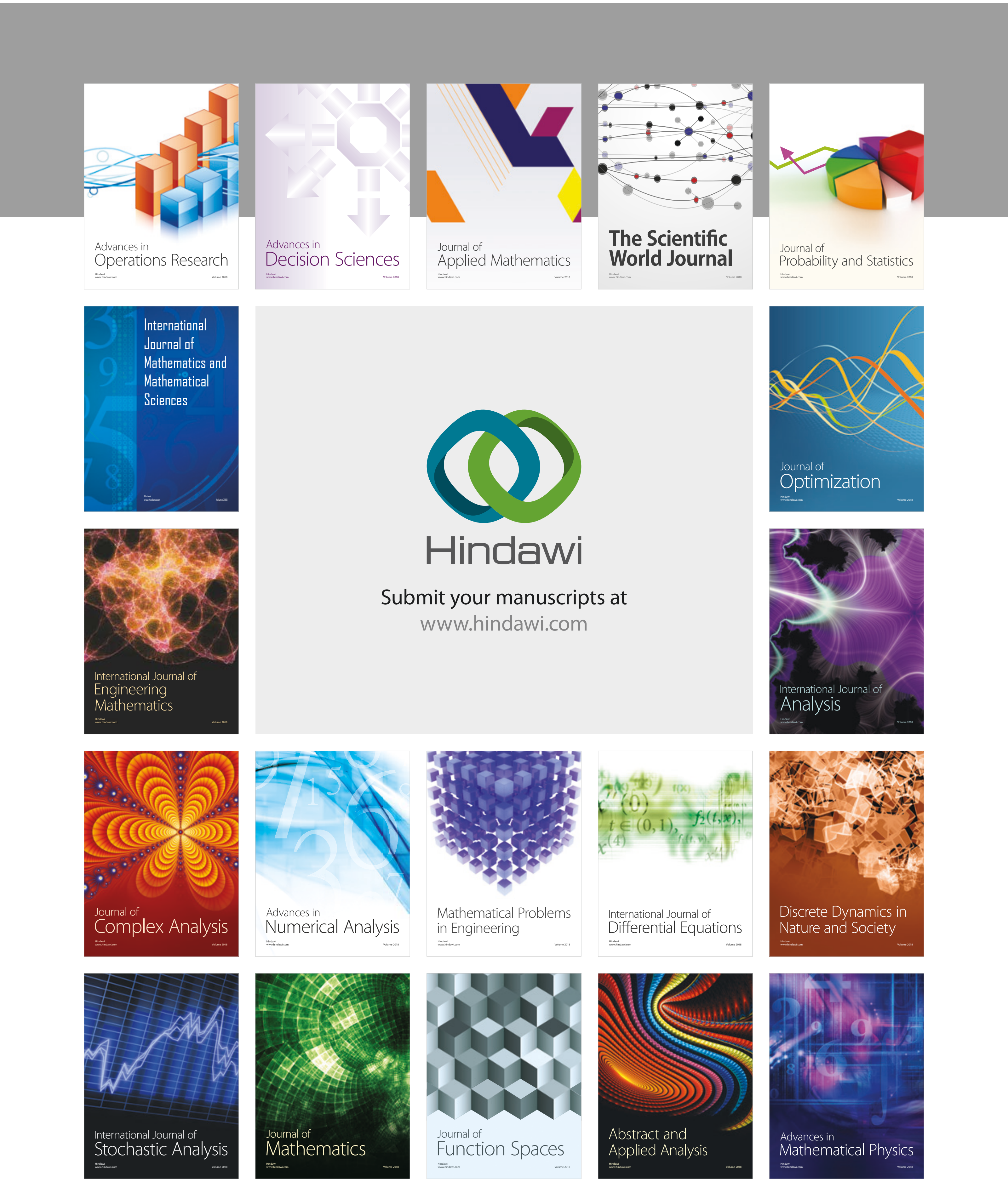
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