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Research Article

Finite-Time Adaptive Tracking Control for a Class of Pure-Feedback Nonlinear Systems with Disturbances via Decoupling Technique

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This paper addresses the finite-time adaptive tracking control problem for a class of pure feedback nonlinear systems whose nonaffine functions may not be differentiable. By properly modeling the nonaffine function, the design difficulty of the pure feedback structure is overcome without using the median value theorem. In our design procedure, an finite-time adaptive controller is elaborately developed using the decoupling technology, which eliminates the limitation assumption on the partial derivatives of nonaffine functions. Furthermore, the constructed controller can stabilize the system within a finite-time so that all signals in the closed-loop system are semiglobally uniformly finite-time bounded (SGUFB), while ensuring the tracking performance. Finally, the simulation results prove the effectiveness of the proposed method.

1. Introduction

In the past few decades, there have been various research results on the nonfinite time stability of nonlinear systems [1–10], and these results are widely applied to practical systems. However, in actual engineering, the control goal is always expected to be achieved within a finite time. Nonfinite time stable schemes cannot accomplish such control objective because nonfinite time stable control often requires a long transient response. Therefore, the definition of finite-time stability was first proposed in [11, 12] and has received great attention. The finite-time stability can ensure that the system state variables quickly converge to equilibrium within a limited time. At present, the finite-time control of nonlinear systems has become a new research hot spot [13–15]. At the same time, there are many challenging problems which needs hard work to overcome.

On the contrary, the rapid development of computer technology has made great progress in the research of adaptive control [16–23]. It is worth mentioning that when there exist completely unknown nonlinear functions in a nonlinear system with a strict feedback structure, radial basis function neural networks (RBF NNs) and fuzzy logic systems play an important role in its adaptive control [24–35]. Using the approximation ability of RBF NNs or fuzzy logic systems, there have been many meaningful research results on adaptive intelligent control for strict feedback nonlinear systems [36–39]. Despite great success in the research on adaptive intelligent control for strict feedback nonlinear systems have been achieved, the research on finite-time control for the nonlinear system is not fully considered [40–42].

In recent years, the finite-time adaptive control schemes for strict-feedback nonlinear systems have been developed in [43–47]. However, the finite-time control strategies in [43–47] are only applicable to the strict feedback nonlinear systems, but not applicable to the pure feedback nonlinear systems. It is worth noting that the study on the finite-time tracking control of pure feedback systems has achieved some results [48, 49], but almost all the results are obtained based

on the use of differential median theorem, which can convert the pure feedback structure into the strict feedback structure. This requires us to make restrictive assumptions for the partial derivatives of nonaffine functions. However, it is well known that nonsmooth nonlinearities, such as dead zones and hysteresis, exist in a wide range of practical control systems. Thus, not all system functions are differentiable in actual control, which requires exploring a new design technique to deal with the pure feedback structure.

In order to meet the actual requirements better, we consider the finite-time adaptive control problem for a class of nonlinear systems with the pure-feedback structure as well as external disturbances. A finite-time adaptive control method based on the decoupling technology is proposed to make the system have better transient response. By selecting the design parameters appropriately, the generated tracking error can converge to a smaller neighborhood of the origin so that the system output follows the desired trajectory within a limited time. The main contributions of this article are as follows. First, we consider a more general class of pure-feedback systems with nonaffine nonlinear functions that may not be differentiable. In order to make our method more practical in industrial control systems than the existing methods [50–59], the limitation of using the differential median theorem in the study of pure feedback systems is eliminated. Second, we construct a suitable controller to stabilize the system in a finite time, which not only ensures that the system state variables quickly converge to equilibrium within a limited time but also improves the robustness of the system and reduces the effects of approximation errors. Third, the appropriate scaling technique is applied to reduce the number of adaptive parameters in the process of designing the controller such that the developed result is more suitable for the actual operation process, which also reduces the complexity of the design procedure. In the end, even if the control direction of the system is unknown, our method can still make all signals in the closed-loop system which are SGUFB.

2. Mathematical Preliminaries

Consider a class of pure-feedback nonlinear systems given by

$$\begin{cases} \dot{\tau}_i = g_i(\overline{\tau}_i, \tau_{i+1}) + r_i(t), & 1 \le i \le n-1, \\ \dot{\tau}_n = g_n(\overline{\tau}_n, u) + r_n(t), \\ y = \tau_1, \end{cases} \tag{1}$$

where $\overline{\tau}_i = [\tau_1, \tau_2, \ldots, \tau_i]^T \in R^i$, $y \in R$, and $u \in R$ are the system state, output, and control input, respectively, $g_i(\overline{\tau}_i, \tau_i + 1), 1 \le i \le n$ are unknown nonaffine nonlinear functions, and $r_i(t)$ are the unknown external disturbances.

Definition 1 (see [60]). *The equilibrium $\zeta = 0$ of nonlinear system $\dot{\zeta} = g(\zeta)$ is semiglobal practical finite-time stable (SGPFS) if for all $\zeta(t_0) = \zeta_0$, there exists $\varepsilon > 0$ and a settling time $T(\varepsilon, \zeta_0) < \infty$ to make $\|\zeta(t)\| < \varepsilon$, for all $t \ge t_0 + T$.*

Lemma 1 (see [60]). *Consider the system $\dot{\zeta} = f(\zeta)$. If there is a smooth positive definite function $V(\zeta)$ and scalars $\kappa > 0$, $0 < \eta < 1$, and $\rho > 0$ such that*

$$\dot{V}(\zeta) \le -\kappa V^{\eta}(\zeta) + \rho, \quad t \ge 0, \tag{2}$$

then this nonlinear system $\dot{\zeta} = f(\zeta)$ is SGPFS.

Proof. For $\forall 0 < \theta \le 1$, from (2), one has

$$\dot{V}(\zeta) \le -\theta\kappa V^{\eta}(\zeta) - (1-\theta)\kappa V^{\eta}(\zeta) + \rho. \tag{3}$$

Let $\Omega_\zeta = \{\zeta \mid V^{\eta}(\zeta) \le \rho/(1-\theta)\kappa\}$ and $\overline{\Omega}_\zeta = \{\zeta \mid V^{\eta}(\zeta) > \rho/(1-\theta)\kappa\}$, if $\zeta(t) \in \overline{\Omega}_\zeta$, one yields $\dot{V}(\zeta) \le -\theta\kappa V^{\eta}(\zeta)$. By solving differential equations, we can obtain that

$$T_{\text{reach}} = \frac{1}{(1-\eta)\theta\kappa}\left[V^{1-\eta}(\zeta(0)) - \left(\frac{\rho}{(1-\theta)\kappa}\right)^{(1-\eta)/\eta}\right]. \tag{4}$$

So, $\zeta(t) \in \Omega_\zeta$ is held for $\forall T \ge T_{\text{reach}}$, otherwise, the trajectory of $\zeta(t)$ does not exceed the set Ω_ζ. This means that the time to reach the set Ω_ζ is bounded as T_{reach}. In other words, the solution of $\dot{\zeta} = f(\zeta)$ is bounded in a finite time.

Lemma 2 (see [61]). *For real variables z and ς and any positive constants β, μ, and ι, the following relation holds:*

$$|z|^{\mu}|\varsigma|^{\beta} \le \frac{\mu}{\mu+\beta}\iota|z|^{\mu+\beta} + \frac{\beta}{\mu+\beta}\iota^{-\mu/\beta}|\varsigma|^{\mu+\beta}. \tag{5}$$

Lemma 3 (see [62]). *For $z_i \in R, i = 1, \ldots, n, 0 \le p \le 1$, the following inequality is true:*

$$\left(\sum_{i=1}^{n}|z_i|\right)^p \le \sum_{i=1}^{n}|z_i|^p \le n^{1-p}\left(\sum_{i=1}^{n}|z_i|\right)^p. \tag{6}$$

Remark 1. It is worth noting that system function $g_i(\overline{\tau}_i, \tau_{i+1})$ is always assumed to satisfy $0 \le \underline{p}_i \le \partial g_i(\overline{\tau}_i, \tau_{i+1})/\partial\tau_{i+1} \le \overline{p}_i$, $(\underline{p}_i, \overline{p}_i \in R)$ in existing articles [52, 54, 55]. However, it is well known that not all system functions are differentiable in actual control, which requires exploring a new design technique to deal with the pure feedback structures. Next, we will introduce a decoupling technique to deal with the unknown nonaffine nonlinear functions of the pure feedback system (1) rather than the median theorem.

Lemma 4 (see [63]). *In order to effectively design the control input of the system, the decoupling technology is utilized to deal with the nonaffine terms. After a series of processing, the following formula can be obtained:*

$$g_i(\overline{\tau}_i, \tau_{i+1}) = g_i(\overline{\tau}_i, 0) + P_i(\overline{\tau}_{i+1})\tau_{i+1} + \Gamma_i(\overline{\tau}_{i+1}), \tag{7}$$

where $P_i(\overline{\tau}_{i+1})$ and $\Gamma_i(\overline{\tau}_{i+1})$ are defined immediately below.

Proof. Define $G_i(\overline{\tau}_i, \tau_{i+1}) = g_i(\overline{\tau}_i, \tau_{i+1}) - g_i(\overline{\tau}_i, 0), i = 1, 2, \ldots, n$, where $\tau_{n+1} = u$ and $\overline{\tau}_{n+1} = [\overline{\tau}_n^T, u]^T$. We assume that the function $G_i(\overline{\tau}_i, \tau_{i+1})$ satisfies

$$\begin{cases} \underline{G}_i\tau_{i+1} + \Delta_{1i} \le G_i(\overline{\tau}_i, \tau_{i+1}) \le \overline{G}_i\tau_{i+1} + \Delta_{2i}, & \tau_{i+1} \ge 0, \\ \underline{G}_i'\tau_{i+1} + \Delta_{3i} \le G_i(\overline{\tau}_i, \tau_{i+1}) \le \overline{G}_i'\tau_{i+1} + \Delta_{4i}, & \tau_{i+1} \le 0, \end{cases} \tag{8}$$

where $\underline{G}_i, \overline{G}_i, \underline{G}_i'$, and $\overline{G}_i'$ are unknown positive constants and $\Delta_{1i}, \Delta_{2i}, \Delta_{3i}$, and $\Delta_{4i}, i = 1, \ldots, n$, are unknown constants.

It can be shown that there exist functions $\vartheta_{i1}(\overline{\tau}_{i+1})$ and $\vartheta_{i2}(\overline{\tau}_{i+1})$, taking values in the closed interval [0, 1] and satisfying

$$\begin{cases} G_i(\overline{\tau}_i, \tau_{i+1}) = (1 - \vartheta_{i1}(\overline{\tau}_{i+1}))(\underline{G}_i\tau_{i+1} + \Delta_{1i}) + \vartheta_{i1}(\overline{\tau}_{i+1})(\overline{G}_i\tau_{i+1} + \Delta_{2i}), & \tau_{i+1} \ge 0, \\ G_i(\overline{\tau}_i, \tau_{i+1}) = (1 - \vartheta_{i2}(\overline{\tau}_{i+1}))(\underline{G}_i'\tau_{i+1} + \Delta_{3i}) + \vartheta_{i2}(\overline{\tau}_{i+1})(\overline{G}_i'\tau_{i+1} + \Delta_{4i}), & \tau_{i+1} \le 0. \end{cases} \tag{9}$$

To facilitate the controller design, we have defined the following simplified symbols $P_i(\overline{\tau}_{i+1})$ and $\Gamma_i(\overline{\tau}_{i+1})$ as

$$P_i(\overline{\tau}_{i+1}) = \begin{cases} (1 - \vartheta_{i1}(\overline{\tau}_{i+1}))\underline{G}_i + \vartheta_{i1}(\overline{\tau}_{i+1})\overline{G}_i, & \tau_{i+1} \ge 0, \\ \vartheta_{i2}(\overline{\tau}_{i+1})\overline{G}_i' + (1 - \vartheta_{i2}(\overline{\tau}_{i+1}))\underline{G}_i', & \tau_{i+1} \le 0, \end{cases} \tag{10}$$

$$\Gamma_i(\overline{\tau}_{i+1}) = \begin{cases} (1 - \vartheta_{i1}(\overline{\tau}_{i+1}))\Delta_{1i} + \vartheta_{i1}(\overline{\tau}_{i+1})\Delta_{2i}, & \tau_{i+1} \ge 0, \\ \vartheta_{i2}(\overline{\tau}_{i+1})\Delta_{4i} + (1 - \vartheta_{i2}(\overline{\tau}_{i+1}))\Delta_{3i}, & \tau_{i+1} \le 0. \end{cases} \tag{11}$$

We can infer from the above definition that $P_i(\overline{\tau}_{i+1})$ and $\Gamma_i(\overline{\tau}_{i+1})$ are bounded. Then, we can model the nonaffine terms $G_i(\overline{\tau}_i, \tau_i + 1)$ as

$$G_i(\overline{\tau}_i, \tau_i + 1) = P_i(\overline{\tau}_{i+1})\tau_{i+1} + \Gamma_i(\overline{\tau}_{i+1}). \tag{12}$$

Hence, (7) was established, and we can rewrite (1) as

$$\begin{cases} \dot{\tau}_i = g_i(\overline{\tau}_i, 0) + P_i(\overline{\tau}_{i+1})\tau_{i+1} + \Gamma_i(\overline{\tau}_{i+1}) + r_i(t), & 1 \le i \le n-1, \\ \dot{\tau}_n = g_n(\overline{\tau}_n, 0) + P_n(\overline{\tau}_{n+1})u + \Gamma_n(\overline{\tau}_{n+1}) + r_n(t), \\ y = \tau_1. \end{cases} \tag{13}$$

In backstepping design, the variable τ_{i+1} is usually taken as the virtual control input for the ith subsystem. So, the virtual control coefficient function $P_i(\overline{\tau}_{i+1})$ should not pass though the zero point. Therefore, the following assumption is pressed on the system (13).

Assumption 1. The desired trajectory y_d and its derivatives $\dot{y}_d$ and $\ddot{y}_d$ are continuous and bounded.

Assumption 2. Due to realistic considerations, for $i = 1, \ldots, n$, there exist unknown positive constants r_i^* such that $|r_i(t)| \le r_i^*$.

Remark 2. Define $P_m = \min_{i=1,2,\ldots,\rho}\{\underline{G}_i, \overline{G}_i, \underline{G}_i', \overline{G}_i'\}$, $P_M = \max_{i=1,2,\ldots,\rho}\{\underline{G}_i, \overline{G}_i, \underline{G}_i', \overline{G}_i'\}$, and $A_i^* = \max_{i=1,2,\ldots,\rho}\{|\Delta_{1i}| + |\Delta_{2i}|, |\Delta_{3i}| + |\Delta_{4i}|\}$. It can be inferred from definitions (10) and (11) that the functions $P_i(\overline{\tau}_{i+1})$ and $\Gamma_i(\overline{\tau}_{i+1})$ satisfy

$$0 \le P_m \le P_i(\overline{\tau}_{i+1}) \le P_M, \tag{14}$$

$$0 \le \left|\Gamma_i(\overline{\tau}_{i+1})\right| \le A_i^*. \tag{15}$$

RBFNNs: in the design of this article, the following radial basis function neural networks (RBF NNs) is used to approximate the continuous function $h(Z)$: $R^n \longrightarrow R$:

$$h_{nn}(Z) = W^T\xi(Z), \tag{16}$$

where $W = [W_1, W_2, \ldots, W_l] \in R^l$ is the weight vector and the neural network node number $l > 1$. $\xi(Z) = [\xi_1(Z), \ldots, \xi_l(Z)]^T$ is the basic vector being chosen as the commonly used Gaussian functions, which has the form:

$$\xi_i(Z) = \exp\left[\frac{-(Z - \mu_i)^T(Z - \mu_i)}{\kappa^2}\right], \tag{17}$$

where $Z \in \Omega_Z \subset R^n$ is the input vector, $\mu_i = [\mu_{i1}, \ldots, \mu_{in}]^T$ is the center of the respective field, and κ is the width of the Gaussian function.

As shown in [64], the neural network can approximate any continuous function on the compact set $\Omega_Z \subset R^q$ to any desired accuracy ε^* as follows:

$$h_{nn}(Z) = W^{*T}\xi(Z) + \varepsilon(Z), \quad Z \in \Omega_Z \subset R^q, \tag{18}$$

where W^* is the ideal constant weight vector and $\varepsilon(Z)$ is the approximation error satisfying $|\varepsilon(Z)| \le \varepsilon^*$, $\varepsilon^* > 0$ is a very small constant.

3. Adaptive State-Feedback Controller Design

In this section, the finite-time adaptive controller is proposed for the backstepping control of system (13). To start, consider the following change of coordinates:

$$\Xi_i = \tau_i - \alpha_{i-1}, \quad i = 1, 2, 3, \ldots, n, \tag{19}$$

where α_{i-1} is the virtual control signal constructed in step $i-1$ and $\alpha_0 = y_d$.

Step 1: differentiating Ξ_1 through the first system of (13), we have

$$\dot{\Xi}_1 = \dot{\tau}_1 - \dot{y}_d = g_1(\overline{\tau}_1, 0) + P_1(\overline{\tau}_2)\tau_2 + \Gamma_1(\overline{\tau}_2) + r_1(t) - \dot{y}_d. \tag{20}$$

Choose Lyapunov function candidate to construct the virtual control signal of this system as

$$V_1 = \frac{1}{2}\Xi_1^2 + \frac{P_m}{2\gamma_1}\widetilde{\psi}_1^2. \tag{21}$$

By substituting (20), we can get the time derivative of V_1 as

$$\begin{aligned}\dot{V}_1 = {} & \Xi_1\left[g_1(\overline{\tau}_1, 0) + P_1(\overline{\tau}_2)(\Xi_2 + \alpha_1) + \Gamma_1(\overline{\tau}_2)\right. \\ & \left. + r_1(t) - \dot{y}_d\right] - \frac{P_m}{\gamma_1}\widetilde{\psi}_1\dot{\hat{\psi}}_1,\end{aligned} \tag{22}$$

where $\Xi_2 = \tau_2 - \alpha_1$. Now, we define a new function as $g_1' = g_1(\overline{\tau}_1, 0) + (1/2)P_M\Xi_1 - \dot{y}_d + k_1\Xi_1^{2\eta-1} + (3/2)\Xi_1$ with $\eta = (2n-1)/(2n+1)$, where n is a natural number and $k_1 > 0$ is a constant. Then, (22) can be rewritten as

$$\begin{aligned}\dot{V}_1 = {} & \Xi_1\left[g_1' - k_1\Xi_1^{2\eta-1} - \frac{1}{2}P_M\Xi_1 - \frac{3}{2}\Xi_1 + P_1(\overline{\tau}_2)(\Xi_2 + \alpha_1)\right. \\ & \left. + \Gamma_1(\overline{\tau}_2) + r_1(t)\right] - \frac{P_m}{\gamma_1}\widetilde{\psi}_1\dot{\hat{\psi}}_1 \\ = {} & \Xi_1\left[g_1' + P_1(\overline{\tau}_2)(\Xi_2 + \alpha_1)\right] + \Xi_1\left(\Gamma_1(\overline{\tau}_2)\right. \\ & \left. + r_1(t)\right) - \frac{1}{2}P_M\Xi_1^2 - k_1\Xi_1^{2\eta} \\ & - \frac{3}{2}\Xi_1^2 - \frac{P_m}{\gamma_1}\widetilde{\psi}_1\dot{\hat{\psi}}_1.\end{aligned} \tag{23}$$

Next, based on (15) and Assumption 2, we can obtain

$$\begin{aligned}\dot{V}_1 \le {} & -\frac{P_M}{2}\Xi_1^2 - k_1\Xi_1^{2\eta} + \Xi_1\left(g_1' + P_1(\overline{\tau}_2)(\Xi_2 + \alpha_1)\right) \\ & + |\Xi_1|(A_1^* + r_1^*) - \frac{3}{2}\Xi_1^2 - \frac{P_m}{\gamma_1}\widetilde{\psi}_1\dot{\hat{\psi}}_1.\end{aligned} \tag{24}$$

Because the unknown function g_1' cannot be used for the controller design, we can infer from (18) that

$$g_1' = W_1^{*T}\xi_1(\tau_1') + \varepsilon_1(\tau_1'), |\varepsilon_1(\tau_1')| \le \varepsilon_1^*, \tag{25}$$

where $\tau_1' = [\tau_1, y_d, \dot{y}_d]^T$. For simplicity, we use ξ_1 and ε_1 instead of $\xi_1(\tau_1')$ and $\varepsilon_1(\tau_1')$, respectively. Define $\psi_1 = \|W_1^*\|^2/P_m$, and $\widetilde{\psi}_1 = \psi_1 - \hat{\psi}_1$ is the parameter estimation error; then, using Yang's inequality and Remark 2, one yields

$$\Xi_1 P_1(\overline{\tau}_2)\Xi_2 \le \frac{P_M}{2}\Xi_1^2 + \frac{P_M}{2}\Xi_2^2, \tag{26}$$

$$|\Xi_1|(A_1^* + r_1^*) \le \Xi_1^2 + \frac{1}{2}A_1^{*2} + \frac{1}{2}r_1^{*2}, \tag{27}$$

$$\begin{aligned}\Xi_1 g_1' = \Xi_1 W_1^{*T}\xi_1 + \Xi_1\varepsilon_1 \le {} & \frac{P_m}{2a_1^2}\Xi_1^2\psi_1\xi_1^T\xi_1 \\ & + \frac{1}{2}a_1^2 + \frac{1}{2}\Xi_1^2 + \frac{1}{2}\varepsilon_1^{*2},\end{aligned} \tag{28}$$

where $a_1 > 0$ is the design positive constant. Substituting (26) and (28) into (24) produces

$$\begin{aligned}\dot{V}_1 \le {} & -\frac{c_1}{(1+P_m)}\Xi_1^{2\eta} + \Xi_1\left(\frac{P_m}{2a_1^2}\Xi_1\psi_1\xi_1^T\xi_1 + P_1(\overline{\tau}_2)\alpha_1\right) \\ & + \frac{1}{2}\delta_1^{*2} - \frac{P_m}{\gamma_1}\widetilde{\psi}_1\dot{\hat{\psi}}_1 + \frac{P_M}{2}\Xi_2^2,\end{aligned} \tag{29}$$

where $c_1 = k_1(1 + P_m)$ and $\delta_1^{*2} = a_1^2 + \varepsilon_1^{*2} + A_1^{*2} + r_1^{*2}$. Next, we construct a virtual signal as

$$\alpha_1 = -k_1\Xi_1^{2\eta-1} - \frac{\hat{\psi}_1}{2a_1^2}\Xi_1\xi_1^T\xi_1. \tag{30}$$

Substituting (30) into (29) yields

$$\dot{V}_1 \le -c_1\Xi_1^{2\eta} + \frac{P_m}{\gamma_1}\widetilde{\psi}_1\left(\frac{\gamma_1}{2a_1^2}\Xi_1^2\xi_1^T\xi_1 - \dot{\hat{\psi}}_1\right) + \frac{1}{2}\delta_1^{*2} + \frac{P_M}{2}\Xi_2^2. \tag{31}$$

Next, we construct the adaptive rate $\dot{\hat{\psi}}_1$ as

$$\dot{\hat{\psi}}_1 = \frac{\gamma_1}{2a_1^2}\Xi_1^2\xi_1^T\xi_1 - \Upsilon_1\hat{\psi}_1, \quad \hat{\psi}_1(0) \geq 0, \tag{32}$$

where $\gamma_1 > 0$ and $\Upsilon_1 > 0$ are two design constants. As a result, one can obtain the following formula:

$$\dot{V}_1 \leq -c_1\Xi_1^{2\eta} + \frac{P_m\Upsilon_1}{\gamma_1}\tilde{\psi}_1\hat{\psi}_1 + \frac{1}{2}\delta_1^{*2} + \frac{P_M}{2}\Xi_2^2. \tag{33}$$

Step $i\,(i = 2, \ldots, n-1)$: from (19), we can know that $\Xi_i = \tau_i - \alpha_{i-1}$. Next, we use P_i and Γ_i instead of $P_i(\overline{\tau}_{i+1})$ and $\Gamma_i(\overline{\tau}_{i+1})$ for simplicity. Then, the dynamic equation of Ξ_i is constructed as follows:

$$\dot{\Xi}_i = g_i(\overline{\tau}_i, 0) + P_i\tau_{i+1} + \Gamma_i + r_i(t) - \dot{\alpha}_{i-1}, \tag{34}$$

where

$$\dot{\alpha}_{i-1} = \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\tau_k}\left[g_k(\overline{\tau}_k, 0) + P_k\tau_{k+1} + \Gamma_k\right] + \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat{\psi}_k}\dot{\hat{\psi}}_k + \sum_{k=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial y_d^{(k)}}y_d^{(k+1)}. \tag{35}$$

Choose a Lyapunov function candidate as

$$V_i = V_{i-1} + \frac{1}{2}\Xi_i^2 + \frac{P_m}{2\gamma_i}\tilde{\psi}_i^2. \tag{36}$$

Differentiating V_i results in

$$\dot{V}_i = \dot{V}_{i-1} + \Xi_i\left[g_i(\overline{\tau}_i, 0) + P_i\tau_{i+1} + \Gamma_i + r_i(t) - \dot{\alpha}_{i-1}\right] - \frac{P_m}{\gamma_i}\tilde{\psi}_i\dot{\hat{\psi}}_i, \tag{37}$$

where $\psi_i = \|W_i^{*2}\|/P_m$, $\tilde{\psi}_i = \psi_i - \hat{\psi}_i$, represents the parameter estimation error. Similar to the processing in the first step, we need to define a new function as $g_i' = g_i(\overline{\tau}_i, 0) + P_M\Xi_i - \dot{\alpha}_{i-1} + k_i\Xi_i^{2\eta-1} + (3/2)\Xi_i$, with $k_i > 0$ is a design constant. Then, (37) can be rewritten as

$$\dot{V}_i = \dot{V}_{i-1} + \Xi_i\left[g_i' + P_i(\Xi_{i+1} + \alpha_i)\right] + \Xi_i\left[\Gamma_i + r_i(t)\right] - P_M\Xi_i^2 - k_i\Xi_i^{2\eta} - \frac{3}{2}\Xi_i^2 - \frac{P_m}{\gamma_i}\tilde{\psi}_i\dot{\hat{\psi}}_i. \tag{38}$$

Now, based on Assumptions 2 and Remark 2, one can obtain

$$\dot{V}_i \leq \dot{V}_{i-1} + \Xi_i\left[g_i' + P_i(\Xi_{i+1} + \alpha_i)\right] + \left|\Xi_i\right|\left[A_i^* + r_i^*\right] - P_M\Xi_i^2 - k_i\Xi_i^{2\eta} - \frac{3}{2}\Xi_i^2 - \frac{P_m}{\gamma_i}\tilde{\psi}_i\dot{\hat{\psi}}_i. \tag{39}$$

According to (18), we can choose the following neural network system:

$$g_i' = W_i^* \wedge \{\wedge * T\}\xi_i(\tau_i') + \varepsilon_i(\tau_i'), \left|\varepsilon_i(\tau_i')\right| \leq \varepsilon_i^*, \tag{40}$$

where $\tau \leq_i' = [\overline{\tau}_i^T, \overline{\hat{\psi}}_{i-1}^T, \overline{y}_d^{(i)^T}]^T \in \Omega_{\tau_i} \subset R^{3i}$ and $\overline{\hat{\psi}}_{i-1} = [\hat{\psi}_1, \hat{\psi}_2, \ldots, \hat{\psi}_{i-1}]^T$. For simplicity, we use ξ_i and ε_i to represent $\xi_i(\tau_i')$ and $\varepsilon_i(\tau_i')$, respectively. Thus, we can obtain

$$\Xi_i P_i \Xi_{i+1} \leq \frac{P_M}{2}\Xi_i^2 + \frac{P_M}{2}\Xi_{i+1}^2, \tag{41}$$

$$\left|\Xi_i\right|\left(A_i^* + r_i^*\right) \leq \Xi_i^2 + \frac{1}{2}A_i^{*2} + \frac{1}{2}r_i^{*2}, \tag{42}$$

$$\Xi_i g_i' = \Xi_i W_i^{*T}\xi_i + \Xi_i\varepsilon_i \leq \frac{P_m}{2a_i^2}\Xi_i^2\psi_i\xi_i^T\xi_i + \frac{1}{2}a_i^2 + \frac{1}{2}\Xi_i^2 + \frac{1}{2}\varepsilon_i^{*2}, \tag{43}$$

where $a_i > 0$ is the design positive constant. Like the first step, substituting (41)–(43) into (39), the following inequality holds:

$$\dot{V}_i \leq -\frac{c_i}{(1+P_m)}\Xi_i^{2\eta} + \Xi_i\left[\frac{P_m}{2a_i^2}\Xi_i\psi_i\xi_i^T\xi_i + P_i\alpha_i\right] + \frac{1}{2}\delta_i^{*2} - \frac{P_m}{\gamma_i}\tilde{\psi}_i\dot{\hat{\psi}}_i + \frac{P_M}{2}\Xi_{i+1}^2 - \frac{P_M}{2}\Xi_i^2 + \dot{V}_{i-1}, \tag{44}$$

where $c_i = k_i(1 + P_m)$ and $\delta_i^{*2} = a_i^2 + \varepsilon_i^{*2} + A_i^{*2} + r_i^{*2}$. Next, we construct the virtual signal α_i as well as the adaptive rate $\hat{\psi}_i$ as follows:

$$\alpha_i = -k_i\Xi_i^{2\eta-1} - \frac{\hat{\psi}_i}{2a_i^2}\Xi_i\xi_i^T\xi_i, \tag{45}$$

$$\dot{\hat{\psi}}_i = \frac{\gamma_i}{2a_i^2}\Xi_i^2\xi_i^T\xi_i - \Upsilon_i\hat{\psi}_i, \hat{\psi}_i(0) \geq 0, \tag{46}$$

where $\gamma_i > 0$ and $\Upsilon_i > 0$ are two design constants. As a result, substituting (45) and (46) into (44), one can get the following formula:

$$\dot{V}_i \le \dot{V}_{i-1} - \frac{P_M}{2}\Xi_i^2 - c_i\Xi_i^{2\eta} + \frac{P_m\Upsilon_i}{\gamma_i}\tilde{\psi}_i\hat{\psi}_i + \frac{1}{2}\delta_i^{*2} + \frac{P_M}{2}\Xi_{i+1}^2. \tag{47}$$

Comparing (33) and (47), we can get the following formula by mathematical induction:

$$\dot{V}_i \le -\sum_{k=1}^{i} c_k\Xi_k^{2\eta} + \sum_{k=1}^{i}\frac{P_m\Upsilon_k}{\gamma_k}\tilde{\psi}_k\hat{\psi}_k + \sum_{k=1}^{i}\frac{1}{2}\delta_k^{*2} + \frac{P_M}{2}\Xi_{i+1}^2. \tag{48}$$

Remark 3. As can be seen from formulas (28) and (43), we used Yang's inequality to obtain $\psi_i = \|W_i^{*2}\|/P_m$ in advance such that only one adaptive parameter should be estimated in each step of the controller design. However, multidimensional vectors (weight vectors) are directly estimated in some literatures such as $\dot{\hat{W}}_i = \Gamma_i((1-m(Z_i))z_iS_i(Z_i) - \sigma_i\hat{W}_i)$ in [65], which makes the design of the adaptive rate more difficult. Therefore, the method we adopt can reduce the number of adaptive parameters compared to the previous method in [65].

Step n: define $\psi_n = \|W_n^*\|^2/P_m$, where W_n^* is the ideal weight vector, and $\tilde{\psi}_n = \psi_n - \hat{\psi}_n$ is the parameter estimation error. Choose the Lyapunov function candidate for system (13) as follows:

$$V_n = V_{n-1} + \frac{1}{2}\Xi_n^2 + \frac{P_m}{2\gamma_n}\tilde{\psi}_n^2, \tag{49}$$

where $\gamma_n > 0$ is a design positive constant. It can be seen from the previous $n-1$ step that the virtual control signal α_{n-1} can be constructed such that the following inequality can be obtained:

$$\dot{V}_{n-1} \le -\sum_{k=1}^{n-1} c_k\Xi_k^{2\eta} + \sum_{k=1}^{n-1}\frac{P_m\Upsilon_k}{\gamma_k}\tilde{\psi}_k\hat{\psi}_k + \sum_{k=1}^{n-1}\frac{1}{2}\delta_k^{*2} + \frac{P_M}{2}\Xi_n^2. \tag{50}$$

As we all know, the dynamic equation of Ξ_n is as follows:

$$\dot{\Xi}_n = g_n(\bar{\tau}_n, 0) + P_n u + \Gamma_n + r_n(t) - \dot{\alpha}_{n-1}, \tag{51}$$

with

$$\dot{\alpha}_{n-1} = \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\tau_k}\left[g_k(\bar{\tau}_k, 0) + P_k\tau_{k+1} + \Gamma_k\right] + \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\psi}_k}\dot{\hat{\psi}}_k + \sum_{k=0}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k)}}y_d^{(k+1)}. \tag{52}$$

From (49), one can get the time derivative of V_n along (51) as

$$\dot{V}_n = \dot{V}_{n-1} + \Xi_n\left[g_n(\bar{\tau}_n, 0) + P_n u + \Gamma_n + r_n(t) - \dot{\alpha}_{n-1}\right] - \frac{P_m}{\gamma_n}\tilde{\psi}_n\dot{\hat{\psi}}_n. \tag{53}$$

Similar to the processing in the above steps, we define a new function as $g_n' = g_n + (1/2)P_M\Xi_n - \dot{\alpha}_{n-1} + k_n\Xi_n^{2\eta-1} + (3/2)\Xi_n$, where $k_n > 0$ is a design constant. Then, (53) can be rewritten as

$$\dot{V}_n = \dot{V}_{n-1} + \Xi_n\left[g_n' + P_n u\right] + \Xi_n\left[\Gamma_n + r_n(t)\right] - \frac{1}{2}P_M\Xi_n^2 - k_n\Xi_n^{2\eta} - \frac{3}{2}\Xi_n^2 - \frac{P_m}{\gamma_n}\tilde{\psi}_n\dot{\hat{\psi}}_n. \tag{54}$$

We designed the actual controller and the adaptation law as follows:

$$u = -k_n\Xi_n^{2\eta-1} - \frac{\hat{\psi}_n}{2a_n^2}\Xi_n\xi_n^T\xi_n, \tag{55}$$

$$\dot{\hat{\psi}}_n = \frac{\gamma_n}{2a_n^2}\Xi_n^2\xi_n^T\xi_n - \Upsilon_n\hat{\psi}_n, \quad \hat{\psi}_n(0) \ge 0, \tag{56}$$

where $\gamma_n > 0$ and $\Upsilon_n > 0$ are two design constants. Like (38)–(48), it is easy to obtain

$$\dot{V}_n \le -\sum_{j=1}^{n} c_j\Xi_j^{2\eta} + \sum_{j=1}^{n}\frac{P_m\Upsilon_j}{\gamma_j}\tilde{\psi}_j\hat{\psi}_j + \sum_{j=1}^{n}\frac{\delta_j^{*2}}{2}, \tag{57}$$

where $c_j = k_j(1+P_m)$ and $\delta_j^{*2} = a_j^2 + \varepsilon_j^{*2} + A_j^{*2} + r_j^{*2}$.

4. Stability Analysis

In this section, the main result will be summarized in Theorem 1.

Theorem 1. *Consider system (1) satisfying Assumptions 1–2, and suppose that the finite-time adaptive controller (55) and the adaptive law (46) as well as (56) are constructed based on the decoupling technology. As long as the design parameters η, k, a, γ, and Υ are properly selected, it can be ensured that the system output y follows the desired trajectory y_d, and at the same time all the signals of the pure-feedback nonlinear systems (1) are SGUFB.*

Proof. For the Lyapunov function candidate $V = V_n$, define $c = \min\{c_j, \Upsilon_j, j = 1, 2, \ldots, n\}$. Then, it follows from (57) that

$$\dot{V} \le -c\sum_{j=1}^{n}\Xi_j^{2\eta} + \sum_{j=1}^{n}\frac{P_m\Upsilon_j}{\gamma_j}\tilde{\psi}_j\hat{\psi}_j + \sum_{k=1}^{n}\frac{\delta_j^{*2}}{2}. \tag{58}$$

From the definition of $\tilde{\psi}$, we can get $\tilde{\psi}_j\hat{\psi}_j \le (1/2)\psi_j^2 - (1/2)\tilde{\psi}_j^2$; further rewrite (58) as

$$\dot{V} \leq -2^{\eta} c \sum_{j=1}^{n} \left(\frac{\Xi_j^2}{2}\right)^{\eta} - c \sum_{j=1}^{n} \frac{P_m}{2\gamma_j} \tilde{\psi}_j^2 + \sum_{k=1}^{n} \frac{\delta_j^{*2}}{2} + \sum_{j=1}^{n} \frac{P_m \Upsilon_j}{2\gamma_j} \psi_j^2. \tag{59}$$

For Lemma 2, we choose the appropriate parameters for $z = 1, \varsigma = c^{1/\eta} \sum_{j=1}^{n} (P_m/2\gamma_j)\tilde{\psi}_j^2$, and $\mu = 1 - \eta, \beta = \eta, \iota = (c^{\eta}/\eta)^{-\eta/(1-\eta)}$. Then, one can obtain

$$c\left(\sum_{j=1}^{n} \frac{P_m}{2\gamma_j} \tilde{\psi}_j^2\right)^{\eta} \leq (1-\eta)\iota + c \sum_{j=1}^{n} \frac{P_m}{2\gamma_j} \tilde{\psi}_j^2. \tag{60}$$

Substituting (60) into (59) and using the zoom method of (6), the following inequality holds:

$$\begin{aligned}
\dot{V} \leq & -2^{\eta} c\left(\sum_{j=1}^{n} \frac{\Xi_j^2}{2}\right)^{\eta} - c\left(\sum_{j=1}^{n} \frac{P_m}{2\gamma_j} \tilde{\psi}_j^2\right)^{\eta} + c\left(\sum_{j=1}^{n} \frac{P_m}{2\gamma_j} \tilde{\psi}_j^2\right)^{\eta} \\
& - c \sum_{j=1}^{n} \frac{P_m}{2\gamma_j} \tilde{\psi}_j^2 + \sum_{k=1}^{n} \frac{\delta_j^{*2}}{2} + \sum_{j=1}^{n} \frac{P_m \Upsilon_j}{2\gamma_j} \psi_j^2, \\
\leq & -2^{\eta} c\left(\sum_{j=1}^{n} \frac{\Xi_j^2}{2}\right)^{\eta} - c\left(\sum_{j=1}^{n} \frac{P_m}{2\gamma_j} \tilde{\psi}_j^2\right)^{\eta} + (1-\eta)\iota \\
& + \sum_{k=1}^{n} \frac{\delta_j^{*2}}{2} + \sum_{j=1}^{n} \frac{P_m \Upsilon_j}{2\gamma_j} \psi_j^2.
\end{aligned} \tag{61}$$

Applying Lemma 3, we can further simplify the time derivative of V as

$$\dot{V} \leq -\kappa V^{\eta} + \rho, \tag{62}$$

where

$$\begin{aligned}
\kappa &= \min\{2^{\eta} c, c\}, \\
\rho &= (1-\eta)\iota + \sum_{k=1}^{n} \frac{\delta_j^{*2}}{2} + \sum_{j=1}^{n} \frac{P_m \Upsilon_j}{2\gamma_j} \psi_j^2.
\end{aligned} \tag{63}$$

Now, define $T^* = 1/((1-\eta)\theta\kappa)[V^{1-\eta}(\Xi(0), \Phi(0)) - (\rho/(1-\theta)\kappa)^{(1-\eta)/\eta}]$ with $\Xi(0) = (\Xi_1(0), \Xi_2(0), \ldots, \Xi_n(0))^T$, and $\Phi(0) = (\psi_1(0), \psi_2(0), \ldots, \psi_n(0))^T$. Based on Lemma 1, we can get $V^{\eta}(\zeta) \leq \rho/((1-\theta)\kappa)$ for $\forall T \geq T^*$. So, the solution of $\dot{\zeta} = f(\zeta)$ is bounded in a finite time and all the signals in the nonlinear system (1) are SGUFB. To be more precise, the finite-time controller proposed by us can converge the tracking error to a small neighborhood of zero and remains there after the finite time T^*. In order to be more intuitive, we will confirm the research results through a simulation example.

5. A Simulation Example

In this section, we will demonstrate the effectiveness of the proposed scheme through the following simulation example.

Let us consider a two-dimensional nonaffine pure-feedback nonlinear system with disturbance as follows:

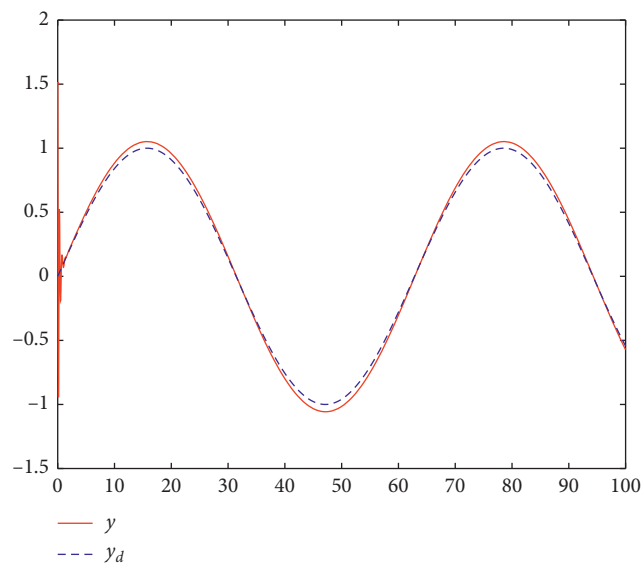

Figure 1: The responses of the system out $y(t)$ and reference signal y_d of the example.

$$\begin{cases}
\dot{\tau}_1 = \tau_1 + \tau_2 + \dfrac{\tau_2^3}{5} + 0.2\sin^2(\tau_1\tau_2), \\
\dot{\tau}_2 = \tau_1\tau_2 + u + 0.1\sin(\tau_1\tau_2), \\
y = \tau_1,
\end{cases} \tag{64}$$

where $\overline{\tau}_i = [\tau_1, \tau_2, \ldots, \tau_i]^T \in R^i, i = 1, \ldots, n$, $y \in R$ are the system state and output and $u \in R$ are the system control input, respectively. The reference signal of the system output is given as $y_d = 0.1\sin(t)$. According Lemma 3, we construct virtual control signal and actual controller as well as the adaptation law for system (64) as follows:

$$\begin{aligned}
\alpha_1 &= -k_1 \Xi_1^{2\eta-1} - \frac{\hat{\psi}_1}{2a_1^2} \Xi_1 \xi_1^T \xi_1, \\
\dot{\hat{\psi}}_1 &= \frac{\gamma_1}{2a_1^2} \Xi_1^2 \xi_1^T \xi_1 - \Upsilon_1 \hat{\psi}_1, \quad \hat{\psi}_1(0) \geq 0, \\
u &= -k_2 \Xi_2^{2\eta-1} - \frac{\hat{\psi}_2}{2a_2^2} \Xi_2 \xi_2^T \xi_2, \\
\dot{\hat{\psi}}_2 &= \frac{\gamma_2}{2a_2^2} \Xi_2^2 \xi_2^T \xi_2 - \Upsilon_2 \hat{\psi}_2, \quad \hat{\psi}_n(0) \geq 0.
\end{aligned} \tag{65}$$

Then, the initial conditions are given as $\tau_1(0) = 0, \tau_2(0) = 0$, and $\hat{\psi}(0) = 0$. We choose the design parameter in the simulation as follows: $\eta = 99/101, k_1 = 15, k_2 = 8, a_1 = 1, a_2 = 1, \gamma_1 = 10, \gamma_2 = 10$, $\Upsilon_1 = 0.5$, and $\Upsilon_2 = 0.5$. Finally, we get Figures 1–6. Figure 1 denotes the responses of the system out $y(t)$ and reference signal y_d of the example. The tracking error Ξ_1 of the example between the output of the system and the reference signal converges to a small neighborhood, which can be observed intuitively in Figure 2. Figure 3 shows the response of the state τ_2 variable.

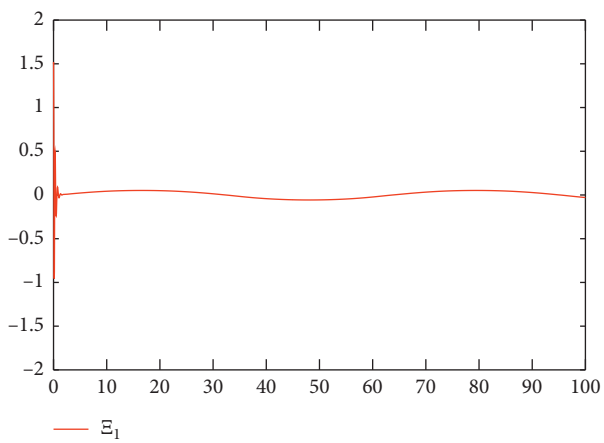

Figure 2: The response of the tracking error Ξ_1 of the example.

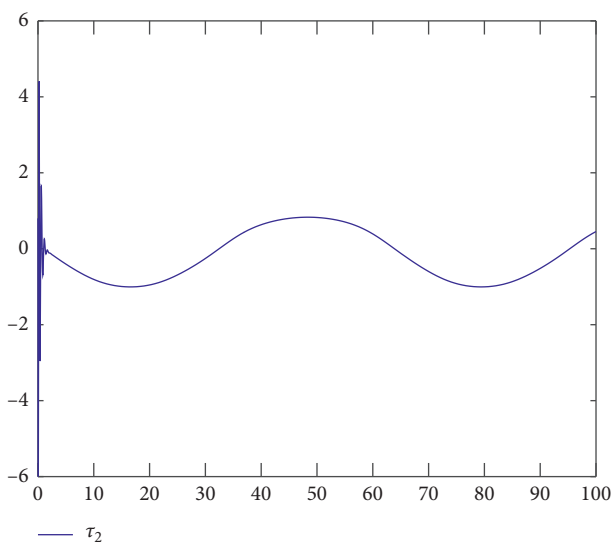

Figure 3: The response of the state variable τ_2 of the example.

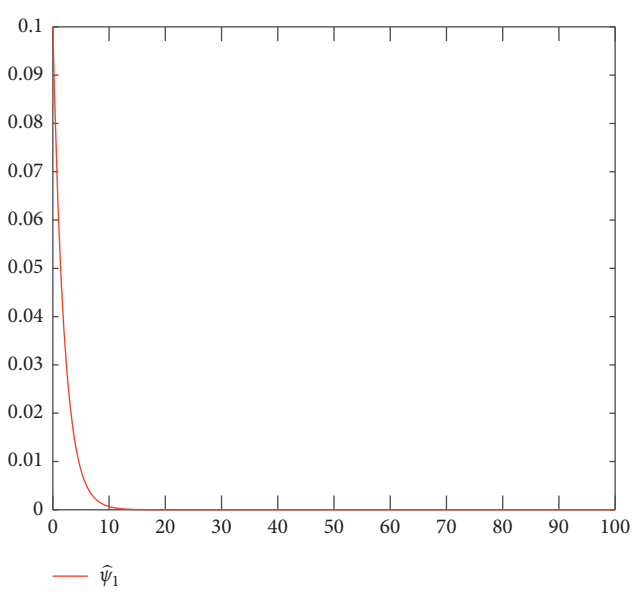

Figure 4: The response of the adaptive rate $\hat{\psi}_1$ of the example.

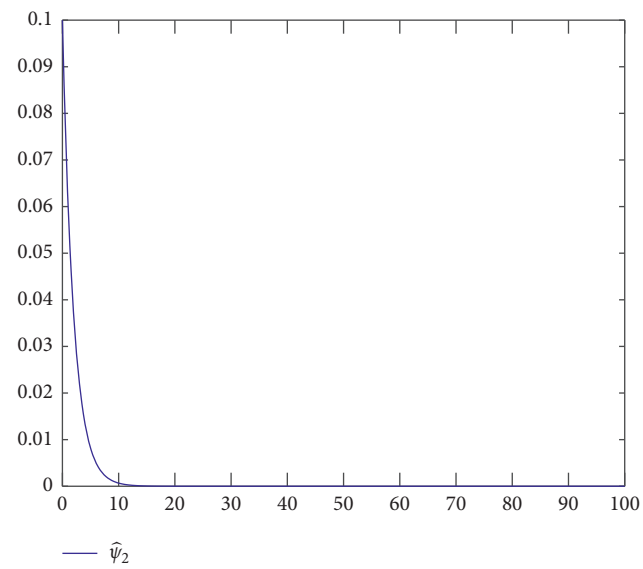

Figure 5: The response of the adaptive rate $\hat{\psi}_2$ of the example.

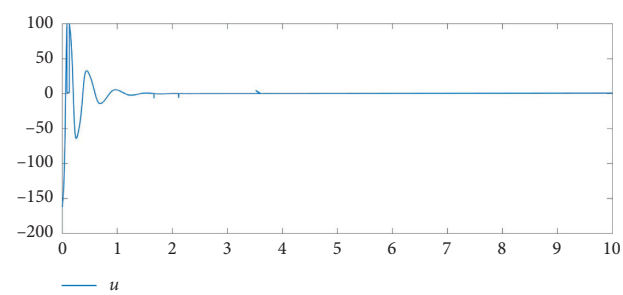

Figure 6: The response of the control law input u.

From the trends in Figures 4 and 5, it is clear to see that the boundedness of the adaptive rate $\dot{\hat{\psi}}_1$ and $\dot{\hat{\psi}}_2$. It can be seen from these results that even though the nonaffine function of our simulation system is not differentiable, it has achieved excellent control performance. Finally, the response of the control law u_f is shown in Figure 6.

6. Conclusions

A novel finite-time adaptive controller has been presented for the considered pure-feedback nonlinear system in this paper. The first design difficulty in this paper is to decouple the pure feedback system without using the median value theorem. The second design difficulty is the extremely complicated formula derivation when designing the finite-time controller, in order to make the system variables converge to the equilibrium quickly in a limited time. Compared with the existing results, the developed method addressed the finite-time adaptive tracking control problem for the pure feedback system whose nonaffine functions may not be differentiable. Furthermore, the decoupling technology has been used in our design frame to eliminate the restrictive assumption of partial derivatives of nonaffine functions, which makes the method more widely used. It is worth noting that the finite-time controller constructed by

us can not only ensure that the system state variables quickly converge to equilibrium within a limited time but also improve the robustness of the closed-loop system. In the future, the finite-time adaptive control of various types of complex switched nonlinear systems can be further discussed, such as multiple input multiple output stochastic switched nonlower triangular systems and stochastic switched nonlower triangular pure feedback nonlinear systems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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