Research Article

A New Two-Lane Lattice Hydrodynamic Model considering the Traffic Interruption Probability under Honk Environment

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By accounting for the traffic interruption probability on two-lane highway under honk environment, an extended lattice hydrodynamic model is presented in the paper. In view of the novel model, a series of researches are carried out. The neutral stability condition and the stable condition can be derived through linear analysis. Then, the mKdV equation near the critical point can be obtained by applying nonlinear analysis, which describes the traffic jams according to the kink-antikink density waves. In addition, numerical simulation is performed, which confirms that traffic interruption probability on two-lane highway under honk environment can develop traffic jams by the change of density waves. Also, the phenomenon is consistent with the results of previous theoretical analysis. It shows that accounting for the traffic interruption probability on two-lane highway under honk environment can stabilize the traffic flow efficiently.

1. Introduction

In recent years, with the development of social science and technology and the improvement of living standards, the proportion of transportation is getting higher and higher in people's daily life. Therefore, it is not difficult to find that the phenomenon of traffic congestion has become general and severe. From the perspective of theoretical research, the situation as lots of traffic congestion is imbalanced for the traffic system. To explain the evolution traffic congestion formation and improve the stability of traffic flow, scholars at home and abroad have proposed a series of traffic models in recent decades, such as the car-following models [1, 2], lattice hydrodynamic models [3, 4], and cellular automaton models [5–9].

According to the phenomenon that each vehicle changes the optimal velocity through the following distance of the preceding vehicle, Bando et al. [10] proposed the original car-following model. Furthermore, with the idea of the car-

following model, Nagatani [11] put forward the lattice hydrodynamic model for the first time. Different from the car-following models, the research direction of lattice hydrodynamic model is the change of traffic flow and density between adjacent interval vehicles. In fact, it is a shift in research direction from micro to macro. On this basis, the study of lattice hydrodynamic model has become more and more colorful, and a lot of factors affecting the traffic flow have been considered, such as driver's anticipation [12–16], driver's visual angle [17], partial reduced lane [18–20], and so on.

In addition, intricate road environment is also one of the most important factors affecting traffic flow. Besides the usual straight roads [21], the current research also includes curved roads [22–25], gradient highway [26], and two-lane highway [27–35]. Among them, the research of two-lane highway is developed recently, which is more coincident with the actual traffic conditions. However, it is relatively few due to the complexity. In 2019, Zhu et al. [27] came up with a

new lattice hydrodynamic model in two-lane system, considering the empirical lane-changing rate. Qi et al. [28] put forward a lattice hydrodynamic model in 2019, accounting for “backward looking” effect and relative flow information in two-lane highway. Also, Rong and Wen [29] proposed an extended delayed feedback control method for the two-lane traffic flow. And in 2017, Wang et al. [30] proposed a microscopic traffic model, which attempted to reproduce the empirical phenomena of congested traffic flow with an on-ramp. Following, a coupled map car-following model has been presented, which was extended with feedback control scheme for traffic jam and energy consumption to two-lane traffic system [31]. According to the above analysis, the situation of two-lane highway plays an important role in improving traffic flow. There is no doubt that driver’s characteristic effect is also one of the significant factors affecting traffic flow in a complex traffic system, such as honking effect. In the previous research, considering that there is no similar work about it, we propose an improved two-lane lattice hydrodynamic model, which accounts for the traffic interruption probability under honk environment. This paper mainly discusses the impact of lane-changing rate, honking effect, and different driving personalities of drivers on easing traffic congestion, hoping that it can provide some theoretical basis for the implementation of relevant measures in the actual traffic system in the future.

This paper is organized as follows: Section 2 is developed to discuss the new two-lane lattice hydrodynamic model accounting for the traffic interruption probability under honk environment. In Section 3, the stability condition is derived by the use of linear analysis. Following, the mKdV equation can be obtained by applying nonlinear analysis in Section 4. In Section 5, numerical simulation is performed. Finally, the conclusion is provided in Section 6.

2. The Modified Lattice Hydrodynamic Model

In 1999, Nagatani [4] presented a new two-lane lattice hydrodynamic model firstly, which considered lane-changing behaviors compared with the single-lane lattice hydrodynamic model. Figure 1 shows the original two-lane lattice hydrodynamic model of traffic flow, which mainly considers the change of density wave between different lattice caused by vehicle lane change. The above situation can be decreased by the physical models as follows:

$$\begin{aligned} \partial_t \rho_{1,j} + \rho_0 (\rho_{1,j} v_{1,j} - \rho_{1,j-1} v_{1,j-1}) \\ = \gamma |\rho_0^2 V'(\rho_0)| (\rho_{2,j+1} - 2\rho_{1,j} + \rho_{2,j-1}), \end{aligned} \quad (1)$$

$$\begin{aligned} \partial_t \rho_{2,j} + \rho_0 (\rho_{2,j} v_{2,j} - \rho_{2,j-1} v_{2,j-1}) \\ = \gamma |\rho_0^2 V'(\rho_0)| (\rho_{1,j+1} - 2\rho_{2,j} + \rho_{1,j-1}), \end{aligned} \quad (2)$$

where γ is the coefficient constant of lane-changing rate. When the density at site j on lane 1 is lower than the density at site $j-1$ on lane 2, $\gamma |\rho_0^2 V'(\rho_0)| (\rho_{2,j-1} - \rho_{1,j})$ means the lane-changing rate from lane 2 to lane 1. From what has been discussed above, when the density at site $j+1$ on lane 2 is lower than that at site j on lane 1, the lane-changing rate is similar to $\gamma |\rho_0^2 V'(\rho_0)| (\rho_{1,j} - \rho_{2,j+1})$ from lane 1 to lane 2.

Then, we obtain the continuity equation as below by combining equations (1) and (2):

$$\partial_t \rho_j + \rho_0 (\rho_j v_j - \rho_{j-1} v_{j-1}) = \gamma |\rho_0^2 V'(\rho_0)| (\rho_{j+1} - 2\rho_j + \rho_{j-1}), \quad (3)$$

where ρ_0 means the average density of the entire traffic system, ρ_j and v_j are the local density and velocity at site j on lanes, respectively. In addition, in the two-lane model, we chose $\rho_j = (\rho_{1,j} + \rho_{2,j})/2$, $\rho_j v_j = (\rho_{1,j} v_{1,j} + \rho_{2,j} v_{2,j})/2$.

So, the evolution equation can be obtained as

$$\partial_t \rho_j(t) v_j(t) = a \rho_0 V(\rho_j(t)) - a \rho_j(t) v_j(t), \quad (4)$$

where a is the sensitivity of a driver.

The optimal velocity function is

$$V(\rho_j(t)) = \frac{v_{\max}}{2} \left[\tanh\left(\frac{2}{\rho_0} - \frac{\rho_j(t)}{\rho_0^2} - \frac{1}{\rho_c}\right) + \tanh\left(\frac{1}{\rho_c}\right) \right], \quad (5)$$

where $v_{\max}$ is the maximal velocity and ρ_c means the safety density.

In addition, we also found that the driver would be affected under honk environment. When the current speed of the vehicle is lower than the speed of the vehicle in front, the driver would honk. When the driver hears the horn, he often adjusts his speed according to the traffic conditions he observes. Different characteristics of drivers may have different responses to honking. Considering this, we can assume that the traffic flow would also change. The difference between the maximum traffic and aggressive and timid driving respectively is a probability event, and we assign the probability parameter value p to this event. Because this probability event is random, we call this phenomenon traffic interruption probability in this paper; then the effect can be presented:

$$\begin{aligned} \partial_t \rho_j v_j = a \rho_0 V(\rho_{j+1}) - a \rho_j v_j + a \lambda [p(Q_m - Q_j(t + \tau_1)) \\ + (1-p)(Q_m - Q_j(t - \tau_2))], \end{aligned} \quad (6)$$

where $Q_m = \rho_m v_m$, it means the maximum flux on the traffic system, λ is the reaction coefficient, p shows the parameter representing the characteristics strength of drivers in traffic flow, τ_1 and τ_2 are the reactive time, which correspond to aggressive and timid drivers, respectively. For simplicity, we take $\tau_1 = \tau_2 = \lambda \tau$.

FIGURE 1: The physical model of traffic flow on two-lane highway.

Eliminating speed v_j from (3) and (6), we can obtain the density evolution equation as follows:

$$\begin{aligned}
 & [1 + a\lambda\tau(2p-1)]\partial_t^2\rho_j - \gamma|\rho_0^2V'(\rho_0)|[1 + a\lambda\tau(2p-1)]\partial_t\rho_{j+1} \\
 & + [2\gamma|\rho_0^2V'(\rho_0)|(1 + a\lambda\tau(2p-1)) + a(\lambda+1)]\partial_t\rho_j \\
 & - \gamma|\rho_0^2V'(\rho_0)|[1 + a\lambda\tau(2p-1)]\partial_t\rho_{j-1} \\
 & - a(\lambda+1)\gamma|\rho_0^2V'(\rho_0)|(\rho_{j+1} - 2\rho_j + \rho_{j-1}) \\
 & + a\rho_0^2[V(\rho_{j+1}) - V(\rho_j)] = 0.
 \end{aligned} \tag{7}$$

3. Linear Stability Analysis

By applying linear analysis, the effect of the traffic interruption probability on two-lane highway under honk environment is investigated. In the steady state of uniform traffic flow, the density and the optimal velocity are both constant values, which are chosen as ρ_0 and $V(\rho_0)$, respectively. The steady-state solution is

$$\rho_j(t) = \rho_0, V(\rho_j(t)) = V(\rho_0). \tag{8}$$

A small deviation $y_j(t)$ is imposed on the steady-state solution, which is expressed as below:

$$\rho_j(t) = \rho_0 + y_j(t). \tag{9}$$

Inserting (8) and (9) into (7), the equation can be obtained:

$$\begin{aligned}
 & [1 + a\lambda\tau(2p-1)]\partial_t^2y_j(t) \\
 & - \gamma|\rho_0^2V'(\rho_0)|[1 + a\lambda\tau(2p-1)]\partial_t y_{j+1}(t) \\
 & + [2\gamma|\rho_0^2V'(\rho_0)|(1 + a\lambda\tau(2p-1)) + a(\lambda+1)]\partial_t y_j(t) \\
 & - \gamma|\rho_0^2V'(\rho_0)|[1 + a\lambda\tau(2p-1)]\partial_t y_{j-1}(t) \\
 & + a\rho_0^2V'(\rho_0)[y_{j+1}(t) - y_j(t)] \\
 & - a(\lambda+1)\gamma|\rho_0^2V'(\rho_0)|(\rho_{j+1}(t) - 2\rho_j(t) + \rho_{j-1}(t)) = 0.
 \end{aligned} \tag{10}$$

By expanding $y_j(t) = A \exp(ikj + zt)$ into (10), we can get

$$\begin{aligned}
 & [1 + a\lambda\tau(2p-1)]z^2 - \gamma|\rho_0^2V'(\rho_0)|[1 + a\lambda\tau(2p-1)]ze^{ik} \\
 & + [2\gamma|\rho_0^2V'(\rho_0)|(1 + a\lambda\tau(2p-1)) + a(\lambda+1)]z \\
 & - \gamma|\rho_0^2V'(\rho_0)|[1 + a\lambda\tau(2p-1)]ze^{-ik} \\
 & + a\rho_0^2V'(\rho_0)\left[(ik) + \frac{1}{2}(ik)^2\right] \\
 & - a(\lambda+1)\gamma|\rho_0^2V'(\rho_0)|(e^{ik} - 2 + e^{-ik}) = 0.
 \end{aligned} \tag{11}$$

Taking $z = z_1(ik) + z_2(ik)^2 + \dots$, the first- and second-order terms of the coefficients (ik) and $(ik)^2$ can be written as

$$\begin{aligned}
 z_1 &= \frac{\rho_0^2V'(\rho_0)}{\lambda+1}, \\
 z_2 &= -\frac{[1 - a\lambda\tau(2p-1)]\rho_0^4V'^2(\rho_0)}{a(\lambda+1)^3} - \frac{\rho_0^2V'(\rho_0)}{2(\lambda+1)} + \gamma|\rho_0^2V'(\rho_0)|.
 \end{aligned} \tag{12}$$

According to the above results, it can be found that the unstable region increases and the uniform steady state becomes unstable when $z_2 < 0$. On the contrary, when $z_2 > 0$, the stability region is constantly increasing and the traffic flow maintains a stable state. Therefore, the neutral stability condition for traffic flow is given by

$$a = -\frac{2\rho_0^4 V'^2(\rho_0)}{2\lambda\tau(2p-1)\rho_0^4 V'^2(\rho_0) - (\lambda+1)^2 [2\gamma|\rho_0^2 V'(\rho_0)|(\lambda+1) - \rho_0^2 V'(\rho_0)]}. \quad (13)$$

The stability condition of the model can be obtained by adding disturbance:

$$a > -\frac{2\rho_0^4 V'^2(\rho_0)}{2\lambda\tau(2p-1)\rho_0^4 V'^2(\rho_0) - (\lambda+1)^2 [2\gamma|\rho_0^2 V'(\rho_0)|(\lambda+1) - \rho_0^2 V'(\rho_0)]}. \quad (14)$$

The neutral stability curves are shown for different values of p , λ , and γ , respectively. When $\lambda = 0$, the novel model degenerates into the classical two-lane lattice hydrodynamic model proposed by Nagatani [4]. From Figure 2(a), with the increasing of p from 0.1 to 0.3, the apex of curves decrease when $\lambda = 0.1$ and $\gamma = 0.1$, this means that the stability area of the traffic flow is increasing. That is to say, it is meaningful to account for the effect of the traffic interruption probability on traffic flow. In Figure 2(b), the change of density curves is also obvious with the change value of λ when $p = 0.2$, $\gamma = 0.1$. When $\lambda = 0.1$, the vertex value of the density curve is the maximum, and it reaches the minimum when the value of λ increases to 0.3. This phenomenon shows the stability of traffic flow can be efficiently enhanced by considering driver's characteristics in the two-lane lattice hydrodynamic model. From Figure 2(c), the stability condition is discussed for different value of γ when $p = 0.2$ and $\lambda = 0.1$. When $\gamma = 0.1$, the stability region is the smallest. As the value of γ increases to 0.3, the stability region tends to reach the maximum. In the future research, we can also use the control method to discuss [36]. Above all, we can conclude that the extended two-lane lattice hydrodynamic model is valuable, and the consideration of the traffic interruption probability under honk environment can enhance the stability of traffic flow.

4. Nonlinear Analysis

To describe the traffic evolution pattern of the novel model, nonlinear analysis is presented near the critical point (ρ_c, a_c) with reductive perturbation method in this section. For $0 < \varepsilon \ll 1$, the slow variables are X and T as follows:

$$X = \varepsilon(j + bt), \quad T = \varepsilon^3 t, \quad (15)$$

where b is an indeterminate constant.

Let $\rho_j(t)$ satisfy the equation:

$$\rho_j(t) = \rho_c + \varepsilon R(X, T). \quad (16)$$

Substituting (15) and (16) into (7) and expending each team to the fifth order of ε , we can obtain

$$\begin{aligned} & \varepsilon^2 k_1 \partial_X R + \varepsilon^3 k_2 \partial_X^2 R + \varepsilon^4 (\partial_T R + k_3 \partial_X^3 R + k_4 \partial_X R^3) \\ & + \varepsilon^5 (k_5 \partial_X \partial_T R + k_6 \partial_X^2 R^3 + k_7 \partial_X^4 R) = 0. \end{aligned} \quad (17)$$

The coefficients k_i ($i = 1, 2, 3, \dots, 7$) are given in Table 1 where $V' = (\partial V(\rho)/\partial \rho)|_{\rho=\rho_c}$ and $V''' = (\partial^3 V_0(\rho)/\partial \rho^3)|_{\rho=\rho_c}$.

Near the critical point (ρ_c, a_c) , we assume the value of a_c as

$$a_c = (1 + \varepsilon^2)a. \quad (18)$$

By taking $b = -(\rho_0^2 V'(\rho_0)/(1 + \lambda))$ and eliminating the second- and third-order terms of ε , we obtain

$$\begin{aligned} & \varepsilon^4 (-g_1 \partial_X^3 R + g_2 \partial_X R^3 + \partial_T R) \\ & + \varepsilon^5 (g_4 \partial_X^4 R + g_5 \partial_X^2 R^3 + g_3 \partial_X^2 R) = 0, \end{aligned} \quad (19)$$

where the coefficients g_i ($i = 1, 2, \dots, 5$) are given in Table 2.

Equation (19) can be transformed into the standard mKdV equation by using the following transformations:

$$\begin{aligned} T &= \frac{1}{g_1} T', \\ R &= \sqrt{\frac{g_1}{g_2}} R'. \end{aligned} \quad (20)$$

The mKdV equation can be obtained with the correction term of $O(\varepsilon)$:

$$\begin{aligned} \partial_{T'} R' &= \partial_X^3 R' - \partial_X R'^3 \\ &+ \varepsilon \left[\frac{g_3}{g_1} \partial_X^2 R' + \frac{g_4}{g_1} \partial_X^4 R' + \frac{g_5}{g_2} \partial_X^2 R'^3 \right]. \end{aligned} \quad (21)$$

And, the mKdV equation can be rewritten simply as

$$\partial_{T'} R' = \partial_X^3 R' - \partial_X R'^3 + \varepsilon M[R'], \quad (22)$$

where $M[R'] = (g_3/g_1) \partial_X^2 R' + (g_4/g_1) \partial_X^4 R' + (g_5/g_2) \partial_X^2 R'^3$.

After ignoring the $O(\varepsilon)$, a kink solution of the mKdV equation can be obtained:

$$R'_0(X, T') = \sqrt{c} \tanh\left(\sqrt{\frac{c}{2}}(X - cT')\right), \quad (23)$$

where c is the propagation velocity.

Subsequently, the value of c should be solved. The following condition needs to be satisfied:

$$(R'_0, M[R'_0]) \equiv \int_{-\infty}^{+\infty} dX R'_0 M[R'_0] = 0. \quad (24)$$

When we consider the correction of $O(\varepsilon)$, and assuming the condition as $R_l(X, T_l) = R'_0(X, T_l) + \varepsilon R'_1(X, T_l)$. The value of c is obtained after solving the integral equation of (24) while $M[R'_0] = M[R]$:

$$c = \frac{5g_2g_3}{2g_2g_4 - 3g_1g_5}. \quad (25)$$

Substituting all the relevant values that have been solved into (16), the general density kink-antikink solution can be obtained:

$$\begin{aligned} \rho_j(t) = \rho_c + \sqrt{\frac{g_1c}{g_2}\left(\frac{a_c}{a} - 1\right)} \times \tanh\sqrt{\frac{c}{2}\left(\frac{a_c}{a} - 1\right)} \\ \times \left[j + (1 - cg_1)\left(\frac{a_c}{a} - 1\right)t\right]. \end{aligned} \quad (26)$$

In addition, the amplitude A of the density is

$$A = \sqrt{\frac{g_1c}{g_2}\left(\frac{a_c}{a} - 1\right)}. \quad (27)$$

From nonlinear analysis above, the mKdV equation and its kink-antikink solution of the model are obtained, which is closely related to different values of p , λ , and γ . It shows the practicality of the new model. Therefore, in the process of studying traffic congestion, we can boldly consider traffic interruption probability on two-lane highway under honk environment, and we have reason to believe that it can effectively alleviate traffic jams.

5. Numerical Simulation

To further verify the above results of the theoretical analysis, numerical simulation is discussed to investigate the impact of the traffic interruption probability on a two-lane highway under honk environment. The novel lattice hydrodynamic model of equation (7) is discretized through the difference scheme:

$$\begin{aligned} \rho_j(t + 2\Delta t) = \gamma|\rho_0^2 V'| \Delta t [\rho_{j+1}(t + \Delta t) - \rho_{j+1}(t) + \rho_{j-1}(t + \Delta t) - \rho_{j-1}(t)] \\ - \Delta t \left[2\gamma|\rho_0^2 V'| + \frac{a(1+\lambda)}{1+a\lambda\tau(2p-1)} \right] [\rho_j(t + \Delta t) - \rho_j(t)] \\ + \frac{a(1+\lambda)\Delta t^2}{1+a\lambda\tau(2p-1)} \gamma|\rho_0^2 V'| [\rho_{j+1}(t) - 2\rho_j(t) + \rho_{j-1}(t)] - \rho_j(t) \\ - \frac{a\rho_0^2\Delta t^2}{1+a\lambda\tau(2p-1)} [V(\rho_{j+1}(t)) - V(\rho_j(t))] + 2\rho_j(t + \Delta t). \end{aligned} \quad (28)$$

We can get initial conditions:

$$\rho_j(1) = \rho_j(0) = \begin{cases} \rho_0, & j \neq \frac{N}{2}, \frac{N}{2} + 1, \\ \rho_0 - 0.05, & j = \frac{N}{2}, \\ \rho_0 + 0.05, & j = \frac{N}{2}, \end{cases} \quad (29)$$

where the total number of lattice $N = 100$, $t = 10^4$ s, $\rho_0 = \rho_c = 0.25$, $\tau = 0.1$, $a = 2.1$.

Figure 3 shows the influence on the traffic system caused by changed value of p , which is reflected by the amplitude change of the density curve. When $p = 0$, it is clear that the amplitude of the density curve is the highest compared to others in Figure 3(a). Subsequently, in Figures 3(b) to 3(d), the amplitude variation trend of density curves is obvious.

The larger value of p is, the more stable and regular the change of density curves become. The phenomenon can be explained by that the traffic interruption probability is helpful to strengthen the stability of traffic flow. That is to say, the traffic interruption probability can be considered to ease traffic congestion on traffic system.

Figure 4 is the two-dimensional density diagram corresponding to Figure 3 at $t = 10300$ s. The different values of p for Figures 4(a) to 4(d) are taken 0, 0.1, 0.2, 0.3, respectively. With the increasing value of p , the density waves gradually become smooth and uniform. It verifies that the traffic interruption probability is effective to be accounted for two-lane highway under honk environment.

Figure 5 shows the evolution of density waves on traffic system when $p = 0.2$, $\gamma = 0.1$, which considers the difference of traffic flow for aggressive and negative drivers under honk environment. From Figures 5(a) to 5(c), the changes of density waves are obvious, which include the decrease of the

FIGURE 2: Phase diagram in (ρ, a) space for (a) $p = 0.1, 0.2, 0.3$ with $\lambda = 0.1, \gamma = 0.1$. (b) $\lambda = 0.1, 0.2, 0.3$ with $p = 0.2$ and $\gamma = 0.1$. (c) $\gamma = 0.1, 0.2, 0.3$ with $p = 0.2$ and $\lambda = 0.1$.

TABLE 1: The coefficients k_i of the model.

Symbol	Corresponding expression
k_1	$b + (\rho_c^2 V' / (1 + \lambda))$
k_2	$((2b^2 [1 + a\lambda\tau(2p - 1)] + a\rho_c^2 V''') / 2a(1 + \lambda)) - \gamma \rho_c^2 V' $
k_3	$\rho_c^2 V' / 6(1 + \lambda)$
k_4	$\rho_c^2 V''' / 6(1 + \lambda)$
k_5	$2b[1 + a\lambda\tau(2p - 1)] / a(1 + \lambda)$
k_6	$(\rho_c^2 V' - 2(1 + \lambda) \rho_c^2 V') / 24(1 + \lambda)$
k_7	$\rho_c^2 V''' / 12(1 + \lambda)$

amplitude and traffic congestion is improved. But it also indicates that the traffic flow has not reached a stable state until the density wave curve becomes a straight line in

TABLE 2: The coefficients g_i of the model.

Symbol	Corresponding expression
g_1	$-\rho_c^2 V' / 6(1 + \lambda)$
g_2	$\rho_c^2 V''' / 6(1 + \lambda)$
g_3	0
g_4	$(\rho_c^2 V' - 2(1 + \lambda)\gamma \rho_c^2 V') / 24(1 + \lambda)$
g_5	$\rho_c^2 V''' / 12(1 + \lambda)$

Figure 5(d). At this moment, the value of λ satisfies the condition for the extended model to reach a steady state. Therefore, we can conclude that considering aggressive and

FIGURE 3: The phase diagram of the model according to different values of parameter p . (a) $p=0$, (b) $p=0.1$, (c) $p=0.2$, (d) $p=0.3$.

FIGURE 4: Continued.

FIGURE 4: Density profile of the density wave at $t = 10300$ s corresponds to Figure 3. (a) $p = 0$, (b) $p = 0.1$, (c) $p = 0.2$, (d) $p = 0.3$.

FIGURE 5: The phase diagram of the model according to different values of parameter λ . (a) $\lambda = 0$, (b) $\lambda = 0.1$, (c) $\lambda = 0.2$, (d) $\lambda = 0.3$.

negative drivers in two-lane highway under honk environment is necessary.

Figure 6 shows the density profile for fixed time $t = 10300$ s, which corresponds to Figure 5. The amplitudes of the density waves decreased until it reached a steady state. It means traffic jams can be improved when accounting for

aggressive and negative drivers in two-lane highway under honk environment.

From Figure 7, the traffic flow can be improved gradually from Figures 7(a) to 7(d), it shows different values of lane-changing rate γ under $p = 0.2$, $\lambda = 0.1$. According to Figure 7(a), the kink-antikink density wave is strong, and the

FIGURE 6: Density profile of the density wave at $t = 10300s$ corresponds to Figure 5. (a) $\lambda = 0$, (b) $\lambda = 0.1$, (c) $\lambda = 0.2$, (d) $\lambda = 0.3$.

FIGURE 7: Continued.

FIGURE 7: The phase diagram of the model according to different values of parameter γ . (a) $\gamma = 0$, (b) $\gamma = 0.1$, (c) $\gamma = 0.2$, (d) $\gamma = 0.3$.

FIGURE 8: Density profile of the density wave at $t = 10300$ s corresponds to Figure 7. (a) $\gamma = 0$, (b) $\gamma = 0.1$, (c) $\gamma = 0.2$, (d) $\gamma = 0.3$.

traffic flow is unstable. When increasing the value of γ from 0 to 0.3, it is found that the amplitudes of density waves reduce gradually. This phenomenon indicates that the steady state of traffic flow is approaching the steady state, and the traffic flow will reach the final stable state at future. Regarding the lane-changing rate, the different study proposed by Wang

et al. [37] also reached the same conclusion. That is to say, the consideration of lane-changing in two-lane system is valuable to reduce traffic jams.

Figure 8 shows the density profile at $t = 10300$ s, which corresponds to Figure 7. It is apparent that the amplitudes of density waves reduce when we increase the number of γ in

two-lane system. It indicates that the effect of two-lane lattice hydrodynamic model can suppress the traffic jams more effectively.

6. Conclusion

The paper proposed an extended lattice hydrodynamic model, which considers the traffic interruption probability on two-lane highway under honk environment. To demonstrate the reachability of the model, a series of theoretical analysis and numerical simulations of the extended model are carried out. By linear analysis, the stability condition and stability curves are shown. And the kink-antikink soliton solution for the mKdV equation can be obtained. Then, the evolution is studied in numerical simulation with different values of the traffic interruption probability, the control constant of the honking effect, and lane-changing rate. The variation of density curves show that the new model is researchable, which is consistent with the results of linear analysis and nonlinear analysis. We can conclude that the lattice hydrodynamic model considering the traffic interruption probability on two-lane highway under honk environment is effective, it is beneficial to enhance traffic jams on traffic system.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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