Research Article

Manufacturer’s Business Strategy: Interaction of Sharing Economy and Product Rollover

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Received 22 April 2020; Accepted 22 July 2020; Published 19 August 2020

Academic Editor: Quanmin Zhu

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This paper develops an analytical framework to study a manufacturer’s optimal business strategy involving the interaction of product sharing and product rollover in the sharing economy. Our analysis reveals that, regardless of whether the production cost is high or low, the manufacturer will participate in product sharing as long as the new product’s rental cost is not very high. Furthermore, under the manufacturer’s participation in product sharing, the manufacturer will sell two generations of products when the production cost is low. When the production cost is moderate, the manufacturer will adopt single rollover. Finally, the manufacturer’s participation in product sharing will increase the old/new product’s price and the manufacturer’s total profits but will decrease the old/new product’s demand in the sales market. Meanwhile, the manufacturer’s product sharing will decrease the price and demand of idle products from owners in the sharing market.

1. Introduction

There is increasing interest in how a traditional manufacturer responds to the sharing economy. The sharing economy, as an emerging phenomenon, is disrupting various industries. For example, in the hotel industry, the valuation of Airbnb is more than $30 billion, which exceeds the market value of Hilton Worldwide, the most valuable hospitality company [1]. The same situation occurs in the transportation industry with companies like Uber and Didi. Some authors have focused on the impacts of product sharing on a traditional manufacturer and how the manufacturer adjusts its price, capacity, and quality to respond to the sharing economy [2–6]. Others have focused on whether and when the manufacturer enters the sharing market [7, 8]. Furthermore, in response to sales’ cannibalisation and consumers’ nonownership under the sharing economy, several manufacturers, such as Mercedes-Benz [9], General Motors [10], BMW [11], and Tesla [12], have introduced a sharing program to increase market share. Finally, other authors have focused on how product sharing affects the manufacturer’s product line design—higher-end manufacturers provide lower-end product sharing [13]. Taken as a whole, the literature—whether focused on impact, entry, or product line—has generated important insights into how manufacturers respond to the sharing economy.

However, many of these insights have not been combined with the traditional manufacturer’s product rollover strategy. As an important means of increasing market share, many manufacturers introduce upgraded products frequently [14]. More frequent product introductions result in more continual product rollovers—the process of phasing out the old generation while introducing a new one to the market [15–17]. In reality, there exists a single product rollover where unsold old products are phased out of the market along with a new product’s introduction; this is also a dual product rollover where the old product remains for some time in the market together with the new product [18]. Based on the effects of the sharing economy on a manufacturer who engages in the dual product rollover strategy, Liu et al. [6] found that the sharing market has an insignificant effect on the upgraded products, but there is a bumping-down effect on the old product’s sales. This was the first contribution to examine the impact of the sharing
economy on the manufacturer’s product rollover strategy. Nevertheless, they did not discuss the manufacturer’s product rollover strategy in the sharing market and whether the manufacturer participates in product sharing.

The purpose of the present research is to study the manufacturer’s optimal business strategy where there is an interaction of product sharing and product rollover in the sharing economy. In particular, we explore the following three questions. (i) When should a manufacturer participate in product sharing? (ii) Which product rollover strategy does the manufacturer adopt in the sharing market? (iii) How will the manufacturer’s product sharing affect the sales market and the sharing market? To describe the interaction of product sharing and product rollover, this research distinguishes four business strategy segments based on a manufacturer’s product rollover and whether the manufacturer participates in product sharing.

We developed an analytical framework to address these research questions. We studied a sales market with consumers purchasing products from a manufacturer and a sharing market with consumers renting products from owners or a manufacturer. The manufacturer introduces two generations of products. The new product is an improvement of the old one. In addition to deciding which rollover strategy to adopt, the manufacturer also decides whether to provide a rental service for the new product (i.e., in order to participate in product sharing). Owners decide whether to rent out an idle product that they previously purchased from the manufacturer. Consumers make a purchase or rent decision according to the net utility of different products/services. The consumers’ decisions affect the manufacturer’s price, demand, and profit. We analysed and compared the manufacturer’s optimal profit, which further affects the manufacturer’s business strategy. To the best of our knowledge, this is the first study to account for the manufacturer’s business strategy involving the interaction of product sharing and product rollover.

Our analysis generates new and interesting insights not identified by previous research. Firstly, regardless of whether the production cost is high or low, the manufacturer will participate in product sharing as long as the new product’s rental cost is not very high. When the production cost is low and the rental cost is lower than the production cost, the manufacturer will participate in product sharing under dual product rollover, i.e., an S-DR strategy. Meanwhile, the manufacturer’s participation in product sharing under single product rollover (S-SR strategy) depends on the relatively lower rental cost rather than the production cost.

Secondly, under the manufacturer’s participation in product sharing, the manufacturer will sell two generations of products when the production cost is low. When the production cost is moderate, the manufacturer will adopt single rollover. When the production cost is very low, there is little distinction in product performance between old and new products. Furthermore, the new product’s higher price pushes some consumers to delay purchasing it. The manufacturer will choose the S-DR strategy. Then, as the manufacturer’s production cost increases, the high price leads to more intense competition for these products/services in the market. The old product will be squeezed out by the new product’s sales and rental services. Thus, the manufacturer will switch to the S-SR strategy.

Thirdly, the manufacturer’s participation in product sharing will increase the old/new product’s price and the manufacturer’s total profits but will decrease the old/new product’s demand in the sales market. The manufacturer’s product sharing will decrease the price and demand of the idle product from owners in the sharing market. The manufacturer offering a rental service of the new product will result in some consumers transferring from the sales market to the sharing market and others transferring their renting demand from the idle product to the new product. However, participating in product sharing may help the manufacturer increase total profits in the sales market. Meanwhile, in the sharing market, owners will lower the idle product’s price to attract consumers and compete with the manufacturer.

The remainder of the paper is organised as follows. We review the related literature in Section 2. Section 3 introduces the proposed model and highlights our assumptions. Section 4 analyses the manufacturer’s sharing strategy, product rollover strategy, and market performance under different business strategies. Finally, Section 5 summarises the paper, illustrates its contribution, and provides directions for future research. All proofs of results are included in Appendix.

2. Literature Review

Extant research on how the sharing economy impacts a manufacturer has been conducted by scholars in recent years. Weber [2] showed that a sharing market tends to increase the price of new products and that a firm benefits from high-cost products rather than low-cost products in the sharing market. Jiang and Tian [3] examined the impact of product sharing where a monopolist manufacturer sells the product directly to consumers and showed that whether the manufacturer benefits from the sharing market depends on the product’s unit cost and sharing transaction cost. Then, Tian and Jiang [4] found that product sharing affects the distribution channel and showed that there exists a threshold for the capacity cost coefficient above which product sharing will increase the manufacturer’s optimal capacity and below which product sharing will reduce the manufacturer’s optimal capacity. Besides, Liu et al. [19] examined the effects of the sharing economy on a manufacturer who engages in the dual product rollover strategy and found that the sharing market has an insignificant effect on the new products; however, there is a bumping-down effect on the old product’s sales. Furthermore, the risk of moral hazard in the sharing market affects the manufacturer’s product rollover strategy. The authors found a lower limit for the moral hazard cost of sharing and discovered that the manufacturer will change its dual rollover to single rollover below this lower limit. These aforementioned studies discuss how the manufacturer should adjust its price, capacity, or quality to respond to the sharing economy.
Other scholars have examined whether and when manufacturers enter the sharing market. Abhishek et al. [7] highlighted the important role of consumer heterogeneity and showed that if heterogeneity in usage rates is too low, the manufacturer prefers to offer sales only; conversely, if heterogeneity is too high, the manufacturer prefers to offer sales and shares directly to consumers. Ke et al. [20] discussed whether introducing car sharing is a better business for automobile manufacturers and found that car sharing only (CO) is a dominated strategy. Tian et al. [8] showed that when a sharing transaction cost is low and a manufacturer’s marginal production cost is not very high, the manufacturer only sells the produce; meanwhile, when the transaction cost sharing is high or the manufacturer’s marginal production cost is high, the manufacturer should offer both sales and product sharing. Razeghian and Weber [5] used an overlapping-generations model to show that the sharing culture shapes the firm’s portfolio design. As the sharing propensity grows, the firm switches from unbundling (exclusively renting), via mixed bundling (renting and selling), to pure bundling (exclusively selling).

However, there is a lack of study on the manufacturer’s business strategy in the context of the sharing economy. The sharing economy constitutes an alternative or disruption to existing practices of production, sometimes challenging the dominant corporate-driven business strategy [21, 22]. The extant research has mainly focused on diversification or categorisation of sharing business strategies [23, 24] and has rarely considered how the manufacturer changes/adjusts their business strategy in the sharing economy. In the context of the sharing economy, the manufacturer’s business strategy can be extrinsically and intrinsically driven. The extrinsic factors include sales market cannibalisation and consumer’s nonownership, whereas the intrinsic factors are product innovation strategies such as product line or product rollover. Nevertheless, only product innovation is not enough; innovative business strategies are also important. Many firms continuously innovate their products but do not survive in the long term. Moreover, business strategy innovators have been found to be more profitable by an average of 6% compared to pure product innovators [25]. Consequently, business strategy selections are the fundamental decisions made by the manufacturer that shape the purpose and the value propositions of the manufacturer [26]. Bellos et al. [13] accounted for the automotive manufacturer’s sharing business strategy and product line design based on the fact that major manufacturers such as Daimler and BMW have included car sharing in their business strategies.

Our research is different from three related studies [7, 8, 13] that focused on the scenario where a manufacturer provides a single type or single rollover product. These studies did not discuss the situation where a manufacturer offers a single or dual rollover strategy or what business strategy the manufacturer should adopt in the context of the sharing economy. Specific to this research, business strategies include decisions pertaining to product rollover and whether to participate in product sharing.

3. Model

3.1. Assumptions. To obtain growth, renewal, and competitive advantage, a manufacturer may introduce sequential new products. We suppose that the manufacturer launches a product at a constant production cost \(c\) and sells it at price \(p_{new}\). If the manufacturer adopts dual rollover, there simultaneously exists a new product and an old product in the sales market. The unit production cost \(c\) is the same for both generations of products [14]. However, the manufacturer sells the old product at price \(p_{old}\) (\(p_{old} < p_{new}\)) because of product performance losses due to technological advances [27, 28].

Furthermore, with the manufacturer participating in product sharing, we assume that the manufacturer only rents out new products. This assumption is attributable to the following reasons. To begin with, if the manufacturer chooses single rollover, it has to rent out new products in order to enter the sharing market. Next, if the manufacturer adopts dual rollover, it should rent out new products rather than old products to reflect the scarcity of its shared products. Finally, new product sharing can serve as a form of marketing, promoting the product by allowing the customer to experience it before making the large purchase necessary to acquire something so high end [13]. We suppose that the manufacturer considers selling the product priority and then rentals. As a consequence, there are new products and idle products in the sharing market. For the new product, the manufacturer rents it out to consumers at a rental cost \(r\) and price \(p_{renting}\) (\(p_{renting} < p_{new}\)\). \(r\) represents maintenance costs and performance losses during the sharing process. For the idle product, the owner rents it out at a rental cost \(t\) and price \(p_{idle}\) (\(p_{idle} < p_{old}\)). \(t\) represents the owner’s transaction cost including moral hazard cost, platform fees, and the hassle cost of delivering and picking up the product [8]. The sharing economy, in essence, is reusing access to underutilised products, which prioritises utilisation and accessibility over ownership [29]. Therefore, in a sharing transaction, a moral hazard problem requires some extra cost [30, 31]. For example, when other conditions are the same, an owner will assess a higher transaction cost when sharing an expensive high-quality product than sharing a cheap low-quality product.

In order to distinguish business strategies involving the interaction of product sharing and product rollover, we analyse the equilibria between a profit-maximising monopoly manufacturer and its consumers in a single-period model under four different scenarios, as summarised in Table 1. Based on the manufacturer’s product rollover (the first row) and whether it participates in sharing (the first column), a four-field matrix is derived. Firstly, the manufacturer adopts single rollover but does not participate in sharing; we denote this scenario by N-SR. Secondly, the manufacturer adopts dual rollover with no product sharing; this scenario is denoted by N-DR. Thirdly, the manufacturer adopts single rollover and participates in sharing (denoted S-SR). Fourthly, the manufacturer adopts dual rollover and participates in sharing (denoted S-DR).
consumers’ net utility is as follows:

$$U_{new} = v - p_{new} + \lambda (p_{idle} - t) + \varepsilon,$$  

where \(\varepsilon\) represents the salvage value of the product and \(\lambda\) represents a probability that the product is being leased by the owner.

1. Purchase the old product:

$$U_{old} = \beta v - p_{old} + \varepsilon.$$  

2. Rent the new product from the manufacturer:

$$U_{renting} = (1 - \varphi \beta)v - p_{renting}.$$  

3. Share the idle product from the owner:

$$U_{idle} = \varphi \beta v - p_{idle}.$$  

4. Do not purchase or rent any product, and the consumer’s net utility is zero.

We summarise the notations in this paper, as shown in Table 2.

4. Manufacturer’s Business Strategy

We obtain feedback equilibrium solutions using backwards induction for the above four business strategies. The equilibrium solutions and the method for obtaining them are summarised in Appendix A.

4.1. Manufacturer’s Sharing Strategy under Product Rollover

In this section, we analyse a manufacturer’s optimal participation strategy in the sharing market. As shown in Table 1, we focus on under what conditions the manufacturer should change its business strategy from N-DR to S-DR or from N-SR to S-SR. When the sharing market exists, the manufacturer wants to rent out the new product to consumers as well. This means that the sharing product’s transaction cost is low enough to ensure the efficiency of the sharing market [8]. Therefore, the manufacturer’s business strategy mainly depends on its production cost or rental cost. Proposition 1 and Proposition 2 illustrate how two factors \(c\) and \(r\) affect the manufacturer’s business strategy. These are the conditions for the manufacturer to participate in product sharing.

**Proposition 1.** If the manufacturer adopts dual product rollover, there exists \(c_2\) such that \(\pi_{SM}^{DR} > \pi_{SN}^{DR}\) when \(c < c_2\) and \(r < c\). The manufacturer will participate in product sharing and choose the S-DR strategy, where

\[
        c_2 = (-2 (2 - 8 \varepsilon + t \lambda) + \beta (8 - 8 \varepsilon + 4t \lambda + \varphi (8 + 2 \varepsilon (-8 + \lambda) + 2 \beta (-4 + 4 \varepsilon + \lambda - 2t \lambda) + \lambda (-2 + t (4 + \lambda)))
        - \beta \varphi^2 (4 + 2 \lambda (-1 + t + \beta + \varepsilon) + \lambda^2 (t + \beta + \varepsilon))) \cdot (8 - 4 \beta + 2 \beta \varphi [-4 + \beta (2 + \lambda)])^{-1}. 
\]
The manufacturer can create a sustainable competitive advantage and achieve a large market share by taking into account a dual rollover strategy [34]. Idle products from owners in the sharing market and new products from the manufacturer will cannibalise previous-generation product’s demand [6]. Moreover, the manufacturer’s offered rental of new products will further aggravate this demand cannibalisation through changing consumers’ decisions from purchasing to renting. The phenomenon is attributable to the following two reasons: first, the manufacturer’s rental service can benefit consumers who previously could not afford the new product. For instance, a new car’s sharing scheme may allow a consumer to enjoy the superior driving performance without spending more money to purchase it. Second, the new product’s rental service enables a strategic consumer to derive a larger utility through renting the new product instead of delaying buying the new/old product.

However, the manufacturer’s dual rollover strategy ensures that both new and old products are sold. Sales of the old product phases out a lot of inventory and helps the manufacturer meet the lower-end consumers’ needs. Therefore, the manufacturer expects to maintain competitive advantage for previous-generation products. For the manufacturer, the production cost of the new product is the same as the old product. Yet the introduction of the new product will lead to the old product losing product performance, and thus, the manufacturer marks down the old product’s price to attract some strategic consumers. Furthermore, there is no difference in product functional attributes between the old and idle product sharing when the production cost is high. Therefore, only with high-cost performance (i.e., low cost), old products can be not withdrawn from the market. There exists $c_1$ such that $d_{old}^{S-DR} > 0$ when $c < c_1$, where

\[
c_1(r) = (2(2 - 8\varepsilon + t\lambda) + \beta^3\lambda(2 + \lambda)\phi^2 - \beta(8 + 8\phi - 2\lambda\phi + 2\varepsilon(-4 + (-8 + \lambda)\phi + t\lambda(4 + (4 + \lambda)\phi)))
+ r(-8 + \beta(4 - 2(-4 + \lambda)\phi) + \beta^2\phi(-4 + \lambda^2\phi + 2\lambda(1 + \phi)))\beta^2\phi(4 + 2 + \varepsilon(-8 + 2\lambda\phi + \lambda^2\phi))
+ 2\lambda(-1 - \phi + t(2 + \phi))) - (16 + \beta(8 - 2(-8 + \lambda)\phi) + \beta^2\phi(-8 + 2\lambda\phi + \lambda^2\phi))^{-1}.
\]

Table 2: Notations and definitions used in the model.

<table>
<thead>
<tr>
<th>$j$</th>
<th>Products, $j = \text{new}$ (the new product), $\text{old}$ (the old product), $\text{renting}$ (rent the new product from the manufacturer), $\text{idle}$ (share the new/old product)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_j$</td>
<td>Price of product $j$</td>
</tr>
<tr>
<td>$c$</td>
<td>New/old products production cost</td>
</tr>
<tr>
<td>$r$</td>
<td>New product’s rental cost</td>
</tr>
<tr>
<td>$t$</td>
<td>Idle product’s transaction cost</td>
</tr>
<tr>
<td>$v$</td>
<td>Consumer’s valuation for purchasing the new product</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Renter’s acceptance of the used product, $\phi \in (0, 1)$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Product’s durability, $\beta \in (0, 1)$</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Salvage value of the new/old product</td>
</tr>
<tr>
<td>$U_j$</td>
<td>Consumer’s utility of choosing product $j$</td>
</tr>
<tr>
<td>$d_j$</td>
<td>Demand of product $j$</td>
</tr>
<tr>
<td>$\pi_M$</td>
<td>Manufacturer’s profit</td>
</tr>
<tr>
<td>$\pi_O$</td>
<td>Owner’s earnings</td>
</tr>
<tr>
<td>$N-SR$</td>
<td>No product sharing under single rollover, denoted as a superscript N-SR</td>
</tr>
<tr>
<td>$N-DR$</td>
<td>No product sharing under dual rollover, denoted as a superscript N-DR</td>
</tr>
<tr>
<td>$S-DR$</td>
<td>Product sharing under single rollover, denoted as a superscript S-DR</td>
</tr>
<tr>
<td>$S-SR$</td>
<td>Product sharing under dual rollover, denoted as a superscript S-SR</td>
</tr>
</tbody>
</table>

Note that the manufacturer should benefit from its offered rental service. The manufacturer’s product sharing not only ensures sales of new products but also attracts renters in the sharing market. High rental costs will lead to the new product’s high rental price, which prevents consumers from benefiting from renting the new product. Therefore, there exists $r_1$ such that there is a demand for the new product’s rental service when $r < r_1$. Meanwhile, the rental cost is a main factor influencing the manufacturer’s decision to participate in product sharing. Besides the transaction cost of idle products in the sharing market, the rental cost includes performance losses caused by the new product’s sharing. Previous studies showed that the owner should rent out products with high production cost. Otherwise, the sharing market is not efficient because the transaction cost exceeds the production cost. However, in reality, the transaction cost or the rental cost is not always lower than the production cost. As a result, product sharing with low production cost can lead to a loss such as that realised by ofo, a bike sharing company in Beijing. Therefore, the manufacturer will participate in product sharing only if the rental cost is low and lower than the production cost ($r < c$). Because of $c < r_1$, the manufacturer will choose the S-DR strategy when $c < c_2$ and $r < c$, where $c_2$ is the maximum production cost for the manufacturer’s S-DR strategy.
Furthermore, the equilibrium outcomes illustrate that the manufacturer can benefit more when it participates in product sharing under dual rollover. According to Proposition 1, the manufacturer may choose the S-DR strategy when $c < c_2$ and $r < c$. The proof of Proposition 1 is in Appendix B.

Similarly, the analysis of the equilibrium solution indicates that when manufacturers will choose the S-SR strategy, as shown in Proposition 2.

**Proposition 2.** If the manufacturer adopts single product rollover, there exist $r_2$ and $r_3$ such that $\pi_S^{S-SR} > \pi_S^{N-SR}$ when $r_2 < r < r_3$ and $r < c$. The manufacturer will participate in product sharing and choose the S-SR strategy, where

$$
\begin{align*}
  r_2 &= \frac{(-1 + \beta \phi)(2 - 4c + 4\epsilon - 2\lambda \phi + \beta(2 + \lambda)\phi)}{2 + \beta(-2 + \lambda)\phi}, \\
  r_3 &= \frac{4 + 4c - 4\epsilon + 2\lambda \phi - \beta(-2 + \lambda)(2c - 2\epsilon + \lambda)\phi - \beta^2\lambda(2 + \lambda)\phi^2}{8 + 4\beta \lambda \phi}.
\end{align*}
$$

The manufacturer introducing the new product results in product unsustainability and maintains the market share. When the production cost is high, the new product's high price means that only high-end consumers can afford it, while low-end consumers will rent the new product if they want to derive the usage value. However, if the new product's rental cost is low enough, all consumers may obtain the usage value via renting instead of buying the new product. This will reduce the new product's demands until it withdraws from the traditional sales market. The result is that the manufacturer only benefits from the new product's rental. Therefore, when the new product's rental cost is not very low ($r > r_2$), the manufacturer will benefit from the sales market and the sharing market simultaneously.

When the production cost is low and the new product's rental cost is high, there is no significant difference between the rental price and the selling price of the new product. At this point, giving up ownership and renting the new product cannot bring consumers high utility, and they will tend to either purchase the new product or rent the idle product. This means that the manufacturer cannot benefit from renting out the new product. Therefore, when the new product's rental cost is not very high ($r < r_3$), the consumer will obtain high utility from renting the new product. Mathematically, there exists $r_3$ such that $d_{new}^{SR} > 0$ when $r < r_3$. Similarly, the manufacturer participates in product sharing only if the rental cost is lower than the production cost. Therefore, the equilibrium outcomes show that the manufacturer can benefit more when it participates in product sharing under single rollover. According to Proposition 2, the manufacturer may choose the S-SR strategy when $r_2 < r < r_3$ and $r < c$. The proof of Proposition 2 is presented in Appendix B.

In contrast to Proposition 1, Proposition 2 shows that the manufacturer's participation in product sharing under single rollover is not subject to the production cost. Since the market share under single rollover is smaller than that under dual rollover, the manufacturer's participation in product sharing under single rollover cannibalises the existing sales market less obviously. When the production cost is high, the manufacturer's rental service also makes the market segmentation efficiency under single rollover. Meanwhile, the old product's demand will be cannibalised such that dual rollover cannot be adopted by the manufacturer.

Proposition 1 and Proposition 2 are different from those in previous studies that the manufacturer/owner should rent out products with high production cost [3, 6, 8] and with low production cost [13] in the sharing market. The present research reveals that regardless of whether the production cost is high or low, the manufacturer will participate in product sharing as long as the new product's rental cost is not very high. In the sales market, when the production cost is low, the manufacturer will sell two generations of products. Moreover, when the production cost is moderate, the manufacturer will only sell the new product. Furthermore, when the production cost is high, the manufacturer will not sell any product and consumers only rent the new product from the manufacturer in the sharing market. As a consequence, the manufacturer's participation in product sharing only depends on the new product's rental cost.

**4.2. Manufacturer’s Product Rollover Strategy.** Based on the fact that the sharing market is efficient, we analyse joint effects of the rental cost ($r$) and the production cost ($c$) on the manufacturer's decision to participate in product sharing (Section 4.1). In this section, our main focus is on analysing how the transaction cost ($t$) and the production cost ($c$) jointly impacts the sharing market’s efficiency and the manufacturer's product rollover strategy.

**Proposition 3.** (i) There exists $t_1$ such that the sharing market is efficient, and the manufacturer participates in product sharing when $0 < t \leq t_1$. (ii) There exists $t_2$ such that the sharing market is efficient, and the manufacturer does not participate in product sharing when $t_1 < t \leq t_2$. (iii) The sharing market does not exist when $t > t_2$.

$$
\begin{align*}
  t_1 &= \frac{\beta \phi(2(1 + c + r - \epsilon) - \beta \phi(2(1 - r - \epsilon) - \lambda(1 + r - \epsilon) + c(2 + \lambda)))}{(6 + 6 \beta \phi(2 + \lambda))(1 - \beta \phi)}, \\
  t_2 &= \frac{\beta \phi(2(1 + c - \epsilon) + \beta \phi(2 + c - \epsilon))}{4 + 3\lambda \beta \phi}.
\end{align*}
$$

The low transaction cost means that the sharing market is relatively efficient. Since the transaction cost for sharing is low, owners benefiting more from sharing will find it worthwhile to rent the idle product out, and renters deriving a higher utility will rent the product. There are two cases in the sharing market. In the first case, the transaction cost is very low ($t \leq t_1$). The total demand for sharing is large. The manufacturer expects to offer the new product's rental service to meet this demand and obtain more profits. Note that, in this case, the manufacturer's market performance changes with respect to the production cost. This is especially true for the manufacturer who adopts different rollover strategies. When the production cost is very low, there is little distinction in product performance between old and new products. However, the new product's higher price pushes some consumers to delay purchasing it. As a consequence, the new/old product and the new/idle product's rental.
service will coexist in the market. Therefore, the manufacturer will choose S-DR when the production cost is very low. One notable exception exists where the manufacturer chooses the S-SR strategy. When the transaction cost is close to \( t_1 \), a higher transaction cost compared to a low product cost will make the manufacturer reluctant to rent out new products under dual rollover. In this case, the manufacturer will benefit less from renting out new products than from selling old products. However, under single rollover, the manufacturer will obtain nonnegative profit from renting out new products despite the higher transaction cost. Then, as the manufacturer’s production cost increases, the high price leads to more intense competition for these products/services in the market. Consumers prefer the new product. The old product will be squeezed out by the new product’s sales and rental services. Therefore, the manufacturer will choose S-SR when the production cost is moderate. Furthermore, when the production cost continues to increase to a very high level, the manufacturer does not sell any product in the sales market and only rents out the new product in the sharing market. Since the high production cost means a high price, no consumers can afford to buy the product but instead rent it.

In the second case, the transaction cost increases from \( t_1 \) to \( t_2 \). This results in the manufacturer being unable to benefit from renting out the new product, and it does not participate in product sharing. Hence, in the presence of the sharing market, the manufacturer decides on the rollover strategy mainly based on the production cost. Note that the old product’s sales only occur when the transaction cost and the production cost are synchronous low. The reason for this is that the rental service of both the idle product and the new product in the sharing market mitigate the competition from the old product’s sales. In this case, the manufacturer will choose the N-DR strategy. Otherwise, the manufacturer will only sell the new product and choose the N-SR strategy.

As the transaction cost continues to rise, neither owners nor renters can obtain more earnings or utility, respectively, from the sharing market. Therefore, the sharing market will not exist due to the high transaction cost. We summarise the findings of our analysis in Figure 1. To simplify the analysis, we assume \( \beta = 0.5 \), \( \phi = 0.5 \), \( \lambda = 0.5 \), and \( r = 0.5c \).

In summary, as illustrated in Figure 1, a high transaction cost prevents consumers from obtaining a high utility via renting a product. If the transaction cost is low, the manufacturer and owners will rent out their unutilised products, and the sharing market is efficient. In the presence of the sharing market, to maximise profit, the manufacturer may consider to provide rentals of the new product. In the sharing region of the figure, including S-SR and S-DR, the manufacturer not only sells products in the sales market but also offers rental service of the new product in the sharing market. In the no sharing region, including N-SR and N-DR, the manufacturer only sells products in the sales market and does not offer rental services.

### 4.3. Market Performance under Different Business Strategies

#### 4.3.1. Market Performance in the Sales Market

Having discussed the manufacturer’s business strategies including sharing strategy and rollover strategy, we will now examine the different products’ market performance under each business strategy. This is shown in Proposition 4.

**Proposition 4.** The manufacturer’s participation in product sharing will increase the new product’s price \( p_{new}^{S-DR} > p_{new}^{N-DR} \) and \( p_{new}^{S-DR} > p_{new}^{N-SR} \), the old product’s price \( p_{old}^{N-DR} > p_{old}^{N-SR} \), and the manufacturer’s profit \( (\Pi_{SR}^{N} > \Pi_{SR}^{M} \) and \( \Pi_{DR}^{N} > \Pi_{DR}^{M} \) but will decrease the new product’s demand \( (d_{new}^{S-DR} < d_{new}^{N-SR} \) and \( d_{new}^{S-DR} < d_{new}^{N-DR} \) \) and the old product’s demand \( (d_{old}^{S-DR} < d_{old}^{N-DR} \) in the sales market.

The manufacturer’s offered rental service for the new product will provide an alternative for consumers who want to purchase the new product. Some consumers may find that renting the new product brings a higher utility than purchasing it, and therefore, they will transfer from the sales market to the sharing market. This will lead to a drop in the new product’s demand, i.e., \( d_{new}^{S-DR} < d_{new}^{N-SR} \) and \( d_{new}^{S-DR} < d_{new}^{N-DR} \). Nevertheless, in order to ensure the new product’s scarcity and rental service’s profitability, the manufacturer will increase the new product’s price, instead of lowering it. This maintains the product’s scarcity and attracts high-end consumers, i.e., \( p_{new}^{S-DR} > p_{new}^{N-SR} \) and \( p_{new}^{S-DR} > p_{new}^{N-DR} \). This is contrary to our general intuition that the manufacturer will lower the new product’s price to attract consumers to purchase it if its demand decreases.

Meanwhile, as previously discussed, the manufacturer’s rental service for the new product enables strategic consumers to derive a higher utility via renting the new product instead of delaying buying the previous-generation product. Therefore, there is no doubt that the old product’s demand will decrease when the manufacturer adopts product sharing, i.e., \( d_{old}^{S-DR} < d_{old}^{N-DR} \). Similarly, in order to maximise the total profit, the manufacturer will not mark down the old product’s price, i.e., \( p_{old}^{S-DR} > p_{old}^{N-DR} \).

Thus, we find that participating in product sharing may help the manufacturer increase its per-unit profit from selling products regardless of whether they are new or old. Furthermore, although the sales demand is harmed, the rental service for the new product may offset the demand loss. As a result, for the manufacturer, the more products/services they offer, the higher the total profits, i.e., \( \Pi_{M}^{S-DR} > \Pi_{M}^{N-SR} \) and \( \Pi_{M}^{S-DR} > \Pi_{M}^{N-DR} \).

#### 4.3.2. Market Performance in the Sharing Market

Having discussed how the manufacturer’s business strategies affect the product’s market performance in the sales market, we will now examine how these business strategies affect the sharing market. The equilibrium outcomes of the sharing market show that the maximum transaction cost for ensuring the existence of the sharing market may vary with the manufacturer’s different business strategies. As shown in Figure 2, four curves represent the boundaries of the sharing market under each business strategy. When the manufacturer’s rollover strategy remains unchanged, offering rentals of the new product requires the sharing market to be more efficient, which is accomplished by lowering the transaction cost, i.e., the boundary goes from \( t^{N-DR} \) to \( t^{S-DR} \). When only the idle product exists in the sharing market, consumers have no choice but rent the idle
product despite its high price due to the relatively high transaction cost, i.e., $t \in (t_{S-DR}, t_{N-DR})$ or $t \in (t_{S-SR}, t_{N-SR})$. Once consumers may rent the new product, the idle product’s price must be marked down to compete with the manufacturer’s new product. Then, the relatively high transaction cost leads to no earning for owners; consequently, these owners do not rent out the idle product, and the sharing market does not exist. The idle product’s market performance is shown in Proposition 5.

**Proposition 5.** The manufacturer’s participation in product sharing will decrease the idle product’s price ($p_{S-idle}^{SR} < p_{N-idle}^{SR}$ and $p_{S-idle}^{DR} < p_{N-idle}^{DR}$) and the idle product’s demand ($d_{S-idle}^{SR} < d_{N-idle}^{SR}$ and $d_{S-idle}^{DR} < d_{N-idle}^{DR}$) in the sharing market.

When the manufacturer offers the rental service of the new product, the lower-end consumer will turn to rent the new product because of its excellent product performance. Thus, consumers transfer their renting demand from the idle product to the new product, i.e., $d_{S-idle}^{DR} < d_{N-idle}^{DR}$ and $d_{S-idle}^{SR} < d_{N-idle}^{SR}$. Compared to the manufacturer with a dominated position in the market, owners have to mark down the idle product’s price, i.e., $p_{S-idle}^{SR} < p_{N-idle}^{SR}$ and $p_{S-idle}^{DR} < p_{N-idle}^{DR}$. The decline in both demand and price of the idle product will reduce owners’ earnings in the sharing market. The reasons are as follows. First, for consumers in the sharing market, the superior performance of the new product is more attractive than that of the idle product. Second, in the face of competition with the manufacturer’s rental service, owners have no choice but to reduce the idle product’s price despite its low cost. This will attract more lower-end consumers to the sharing market.

5. Discussion and Conclusions

This research is motivated by the fact that traditional manufacturers may introduce upgraded products frequently to obtain competitive advantage and more continual product rollovers. In response to the sharing economy, these
manufacturers also enter the sharing market by offering rental services. For example, Mercedes-Benz, General Motors, BMW, and Tesla have introduced their own car sharing programs. This study developed an analytical framework to study the manufacturer’s optimal business strategy involving the interaction of product sharing and product rollover in the sharing economy. We examined consumers’ purchasing and sharing decisions and analysed how these three key factors—the production cost, the new product’s rental cost, and the idle product’s transaction cost—affect the manufacturer’s business strategy.

Our analysis revealed that, regardless of whether the production cost is high or low, the manufacturer will participate in product sharing only if the new product’s rental cost is not very high. When the production cost is low and the rental cost is lower than the production cost, the manufacturer will participate in product sharing under dual product rollover, i.e., the S-DR strategy. Moreover, the manufacturer’s participation in product sharing under single product rollover (the S-SR strategy) depends on the relatively lower rental cost rather than the production cost. This finding is different from that in previous studies that determined the manufacturer/owner should rent out products with high production cost [3, 6, 8] and with low production cost [13] in the sharing market. The manufacturer’s participation in product sharing under a product rollover strategy is only subject to the new product’s rental cost.

Furthermore, under the manufacturer’s participation in product sharing, the manufacturer will sell two generations of products when the production cost is low. When the production cost is moderate, the manufacturer will adopt single rollover. When the production cost is very low, there is little distinction in product performance between old and new products, but the new product’s higher price pushes some consumers to delay purchasing it. As a consequence, the new product, the old product, the rental service for the new product, and rental service for the idle product will all coexist in the market. Therefore, the manufacturer will choose the S-DR strategy when the production cost is very low. Then, as the manufacturer’s production cost increases, the high price leads to more intense competition for these products/services in the market. The old product will be squeezed out by the new product’s sales and rental services. Therefore, the manufacturer will choose the S-SR strategy when the production cost is moderate. When the production cost continues to increase and becomes very high, the manufacturer does not sell any products in the sales market and only rents out the new product in the sharing market. Note that when the manufacturer does not participate in product sharing, the manufacturer will choose the N-DR strategy when both the transaction cost and the production cost are synchronous low. Otherwise, the manufacturer will choose the N-SR strategy.

Finally, the manufacturer’s participation in product sharing will increase the old/new product’s price and the manufacturer’s total profits but will decrease the old/new product’s demand in the sales market. Meanwhile, the manufacturer’s product sharing will decrease the idle product’s price and demand in the sharing market. In essence, the sharing market is a secondary market in which the idle product is rented out from owners via temporary access to nonownership models [35]. Participating in product sharing means that the manufacturer will expand into the sharing market and hopes to meet consumers’ needs in the sharing market. This will impact the sales market and the sharing market in two ways. First, some consumers may find that renting the new product brings a higher utility than purchasing it, and therefore, they will transfer from the sales market to the sharing market. Second, others will turn to rent the new product because of its excellent product performance and thus transfer their renting demand from the idle product to the new product. Therefore, we find that participating in product sharing may help the manufacturer increase its per-unit profit in the sales market. Meanwhile, stakeholders in the sharing market will reduce the transaction cost to ensure sharing efficiency, and owners will lower the idle product’s price to attract consumers and compete with the manufacturer.

6. Limitations
We have simplified our model specification to derive meaningful analytical results. Some of our assumptions can be relaxed to deal with more complex situations. First, the manufacturer is a monopolistic firm who provides differentiated products and services. The competition among manufacturers should be taken into account in the future. Second, the impact of a sharing platform on the manufacturer’s participation in product sharing is not considered. In particular, the sharing platform’s service fee directly affects the transaction cost of the idle product or the rental cost of the new product. This, in turn, will affect which business strategy is chosen by the manufacturer.

Appendix
A. The Equilibrium Solutions under Four Different Scenarios
In this appendix, we solve the manufacturer’s profit-maximisation problem. We obtain feedback equilibrium solutions using backwards induction for four games.

A.1. Product’s Optimal Equilibrium Solutions with N-SR Strategy
We will first solve the manufacturer’s maximum profit under the N-SR strategy. First, the manufacturer sets the new product’s price \( p_{\text{new}} \). Second, the owner who previously purchased the product sets the idle product’s price \( p_{\text{idle}} \). We use a superscript N-SR to denote the equilibrium results under this scenario. The solution process is as follows.

The owner decides the idle product’s price to maximise her earnings. If \( t \geq p_{\text{idle}} \), the owner’s expected earnings are negative and the sharing market do not exist. If \( t < p_{\text{idle}} \), the owner will rent out the idle product. As a result, all owners’ earnings in the sharing market can be written as
\[ \pi_O = \begin{cases} (P_{idle} - t) \left[ p_{new} - \epsilon - \lambda (P_{idle} - t) \frac{P_{idle}}{\phi \beta} \right], & \text{if } t < P_{idle}, \\ 0, & \text{if } t \geq P_{idle}, \end{cases} \]

where \( \pi_O \) is a strictly concave function of the owner’s decision variable \( P_{idle} \). From the first-order optimality conditions for the owner’s earnings-maximisation, the following expression can be derived:

\[ P_{idle} = \frac{t + \phi \beta (P_{new} - \epsilon + 2 \lambda t)}{2 (1 + \lambda \phi \beta)}. \]  

(A.2)

Replacing the expression into the manufacturer’s profit function

\[ \pi_M = \begin{cases} (P_{new} - c) \left[ 1 - P_{new} - \epsilon - \lambda (P_{idle} - t) \right], & \text{if } t < P_{idle}, \\ (P_{new} - c) \left[ 1 - P_{new} - \epsilon \right], & \text{if } t \geq P_{idle}, \end{cases} \]

we get

\[ \pi_M = \frac{(P_{new} - c) \left[ t \lambda - 2 - 2 \lambda \phi \beta + (P_{new} - \epsilon) (2 + \lambda \phi \beta) \right]}{2 (1 + \lambda \phi \beta)}, \]  

(A.3)

where \( \pi_M \) is strictly concave in the manufacturer’s decision variable \( P_{new} \). From the first-order optimality conditions, the following expression can be derived:

\[ P_{N-\text{SR}}^{new} = \frac{1}{2} \left( 2 + c + \epsilon - \frac{2 + \lambda t}{2 + \lambda \phi \beta} \right). \]  

(A.5)

Replacing the expression into equation (A.2), we get

\[ P_{N-\text{SR}}^{idle} = \frac{4 t + \phi \beta (2 + 2 c - 2 \epsilon + 9 \lambda t) + \lambda \phi \beta^2 (2 + c - \epsilon + 4 \lambda t)}{4 (1 + \lambda \phi \beta) (2 + \lambda \phi \beta)}. \]  

(A.6)

Furthermore, we obtain

\[ \pi_M = \begin{cases} (P_{new} - c) \left[ 1 - P_{new} + \epsilon + \lambda (P_{idle} - t) \right] + (P_{renting} - r) \left[ p_{new} - \epsilon - \lambda (P_{idle} - t) - \frac{P_{renting}}{1 - \phi \beta} \right], & \text{if } t < P_{idle}, \\ (P_{new} - c) \left[ 1 - P_{new} - \epsilon \right], & \text{if } t \geq P_{idle}. \end{cases} \]

Letting \( \frac{\partial \pi_M}{\partial P_{new}} = 0 \) and \( \frac{\partial \pi_M}{\partial P_{renting}} = 0 \), we get

\[ d_{N-\text{SR}}^{new} = \frac{2 - \lambda t + 2 \lambda \phi \beta - c (2 + \lambda \phi \beta) + \epsilon (2 + \lambda \phi \beta)}{4 (1 + \lambda \phi \beta)}, \]  

\[ d_{N-\text{SR}}^{idle} = \frac{\phi \beta [2 + 2 \lambda \phi \beta + (2 + \lambda \phi \beta) (c - \epsilon)] - t (4 + 3 \lambda \phi \beta)}{4 \phi \beta (2 + \lambda \phi \beta)}, \]  

\[ \pi_{N-\text{SR}}^{M} = \frac{[2 - \lambda t + 2 \lambda \phi \beta - (c - \epsilon) (2 + \lambda \phi \beta)]^2}{8 (1 + \lambda \phi \beta) (2 + \lambda \phi \beta)}. \]  

(A.7)

A.2. Product’s Optimal Equilibrium Solutions with S-SR Strategy. We will second solve the manufacturer’s maximum profit under the S-SR strategy. First, the manufacturer sets the products’ price, \( P_{new} \) and \( P_{renting} \). Second, the owner who previously purchased the product sets the idle product’s price \( P_{idle} \). We use a superscript S-SR to denote the equilibrium results under this scenario. The solution process is as follows.

The owner decides the idle product’s price to maximise her earnings. If \( t \geq P_{idle} \), the owner’s expected earnings are negative, and the sharing market do not exist. If \( t < P_{idle} \), the owner will rent out the idle product. As a result, all owners’ earnings in the sharing market can be written as

\[ \pi_O = \begin{cases} \left( P_{idle} - t \right) \left[ \frac{P_{renting}}{(1 - \phi \beta)} - \frac{P_{idle}}{\phi \beta} \right], & \text{if } t < P_{idle}, \\ 0, & \text{if } t \geq P_{idle}, \end{cases} \]  

(A.8)

where \( \pi_O \) is a strictly concave function of the owner’s decision variable \( P_{idle} \). From the first-order optimality conditions, the following expression can be derived:

\[ P_{idle} = \frac{t + \phi \beta (P_{renting} - t)}{2 (1 - \phi \beta)}. \]  

(A.9)

Substituting (A.9) in the manufacturer’s profit function, we get

\[ \pi_M = \begin{cases} (P_{new} - c) \left[ 1 - P_{new} - \epsilon - \lambda (P_{idle} - t) \right] + (P_{renting} - r) \left[ p_{new} - \epsilon - \lambda (P_{idle} - t) - \frac{P_{renting}}{1 - \phi \beta} \right], & \text{if } t < P_{idle}, \\ (P_{new} - c) \left[ 1 - P_{new} - \epsilon \right], & \text{if } t \geq P_{idle}. \end{cases} \]  

(A.10)
Complexity

\[ p_{\text{new}}^{\text{S-SR}} = \frac{r(2 + \lambda \phi \beta)(\phi \beta(2 + \lambda) - 2) + (1 - \phi \beta)[8 + 4\lambda \phi \beta + (2\varepsilon - \lambda t)](2 + 2\phi \beta + \lambda \phi \beta) + c[8 - \phi \beta(8 - 2\lambda + 2\lambda \phi \beta + \lambda^2 \phi \beta)]}{12 - \phi \beta[8 - 4\lambda + \phi \beta(2 + \lambda)^2]}, \]

(A.11)

\[ p_{\text{renting}}^{\text{S-SR}} = \frac{(1 - \phi \beta)(4(1 + c + r - \varepsilon) + 2\lambda t + \phi \beta[4(r + \varepsilon - 1) - 2\lambda(1 - r - t + \varepsilon) + t\lambda^2 + 4c + 2\lambda])}{12 - \phi \beta[8 - 4\lambda + \phi \beta(2 + \lambda)^2]}, \]

(A.12)

Substituting (A.12) in equation (A.9), we get

\[ p_{\text{idle}}^{\text{S-SR}} = \frac{\phi \beta[2(1 + c + r - \varepsilon) - \phi \beta(2 + \lambda) + \lambda(1 + r + \varepsilon) - c(2 + \lambda)]}{12 - \phi \beta[8 - 4\lambda + \phi \beta(2 + \lambda)^2]}, \]

(A.13)

Furthermore, we obtain

\[ d_{\text{new}}^{\text{S-SR}} = \frac{2(1 - \phi \beta)[2 + 4\varepsilon - 2\lambda t + \phi \beta(2 + \lambda)] - 8c(1 - \phi \beta) + 2r[2 - \phi \beta(2 - \lambda)]}{12 - \phi \beta[8 - 4\lambda + \phi \beta(2 + \lambda)^2]}, \]

\[ d_{\text{renting}}^{\text{S-SR}} = \frac{4 + 4c - 8\varepsilon - 4\lambda t - \lambda(2 + \lambda)\phi^2 \beta^2 + \phi \beta[4\varepsilon - 2c(2 + \lambda) - 4 - 2\lambda(2r + t + \varepsilon) - t\lambda^2]}{12 - \phi \beta[8 - 4\lambda + \phi \beta(2 + \lambda)^2]}, \]

\[ n_m^{\text{S-SR}} = 4c^2(1 - \phi \beta) + 2r^2(2 + \lambda \phi \beta) - r[4 - 4\varepsilon + 2\lambda t - 4\phi \beta + \beta \phi(2 - \lambda)(2\varepsilon - \lambda t) - \lambda \beta^2 \phi^2(2 + \lambda)] - 2c[r[2 - \phi \beta(2 - \lambda)] - (1 - \phi \beta)[2 + 4\varepsilon - 2\lambda t + \phi \beta(2 + \lambda)]]
+ (1 - \beta \phi)[4 + 4\varepsilon^2 - \lambda t(2 - \lambda t) + \lambda \phi \beta(2 - 2t - \lambda t) + \varepsilon(4 - 4\lambda t + 4\phi \beta + 2\lambda \phi \beta)]
\cdot (12 - \phi \beta[8 - 4\lambda + \phi \beta(2 + \lambda)^2])^{-1}. \]

(A.14)

A.3. Product’s Optimal Equilibrium Solutions with N-DR Strategy. We will third solve the manufacturer’s maximum profit under the N-DR strategy. First, the manufacturer sets the products’ price, \( p_{\text{new}} \) and \( p_{\text{old}} \). Second, the owner who previously purchased the product sets the idle product’s price \( p_{\text{idle}} \). We use a superscript N-DR to denote the equilibrium results under this scenario. The solution process is as follows.

Then, all owners’ earnings in the sharing market can be written as

\[ n_O = \begin{cases} (p_{\text{idle}} - t)(\frac{p_{\text{old}} - \varepsilon}{\beta} - \frac{p_{\text{idle}}}{\beta^2}), & \text{if } t < p_{\text{idle}}, \\ 0, & \text{if } t \geq p_{\text{idle}}, \end{cases} \]

(A.15)

where \( n_O \) is strictly concave in the owner’s decision variable \( p_{\text{idle}} \). From the first-order optimality conditions, the following expression can be derived:

\[ p_{\text{idle}} = \frac{1}{2}(t + \phi p_{\text{old}} - \phi \varepsilon). \]

(A.16)

The manufacturer’s profit function is
\[
\pi_M = \begin{cases} 
(p_{\text{new}} - \epsilon)[1 - p_{\text{new}} + \epsilon + \lambda(p_{\text{idle}} - t)] + (p_{\text{old}} - \epsilon) \left[ p_{\text{new}} - \epsilon - \lambda(p_{\text{idle}} - t) - \frac{P_{\text{old}} - \epsilon}{\beta} \right], & \text{if } t < P_{\text{idle}}, \\
(p_{\text{new}} - \epsilon)[1 - p_{\text{new}} - \epsilon] + (p_{\text{old}} - \epsilon) \left( p_{\text{new}} - \epsilon - \frac{p_{\text{old}} - \epsilon}{\beta} \right), & \text{if } t \geq P_{\text{idle}}.
\end{cases}
\]

(A.17)

Replacing (A.16) into equation (A.17) and letting \( \partial \pi_M / \partial p_{\text{new}} = 0 \) and \( \partial \pi_M / \partial p_{\text{old}} = 0 \), we get

\[
p_{\text{new}}^{N-\text{DR}} = \frac{8 - 2\lambda t (2 - \beta) + \lambda \varphi \beta (4 - \lambda t) + 2d(2 + \lambda \varphi) + \varphi \left[ 2(6 - \lambda \varphi) + \beta (2 - \lambda \varphi)^2 \right]}{16 - \beta (2 - \lambda \varphi)^2},
\]

(A.18)

\[
p_{\text{old}}^{N-\text{DR}} = \frac{8c + 8\epsilon - \beta \left[ \epsilon t^2 + \varphi (2 - \lambda \varphi)^2 - 2(2 + \lambda t + \lambda \varphi) \right]}{16 - \beta (2 - \lambda \varphi)^2},
\]

(A.19)

Substituting (A.19) in equation (A.16), we get

\[
p_{\text{old}}^{N-\text{DR}} = \frac{\varphi (4c + 2\beta - 4 \epsilon + \lambda \varphi \beta) + t \left[ 8 - \beta (2 - \lambda \varphi + \lambda^2 \varphi^2) \right]}{16 - \beta (2 - \lambda \varphi)^2}.
\]

(A.20)

Furthermore, we obtain

\[
d_{\text{new}}^{N-\text{DR}} = \frac{4(2 - c - \beta + \epsilon - \lambda t) + 2 \lambda \varphi (c + \beta - \epsilon)}{16 - \beta (2 - \lambda \varphi)^2},
\]

\[
d_{\text{old}}^{N-\text{DR}} = -\left( \left( 2c - \lambda (4 - \varphi) \right) - 2 \left[ 4 - 6 \lambda t + \epsilon (2 + 4 \lambda + \lambda \varphi) \right] \beta \left[ 4 + \lambda \left[ -4 - \lambda \varphi (2 - \varphi) + t (\lambda \varphi - \lambda - 1) (2 - \lambda \varphi) + \epsilon (2 - \lambda \varphi)^2 \right] \right] \right) 
\cdot \left( 16 - \beta (2 - \lambda \varphi)^2 \right)^{1/2},
\]

\[
d_{\text{idle}}^{N-\text{DR}} = \frac{\varphi (4c + 2\beta - 4 \epsilon + \lambda \varphi \beta) - t (8 - 2\beta + \lambda \varphi \beta)}{\varphi \beta \left[ 16 - \beta (2 - \lambda \varphi)^2 \right]},
\]

\[
p_{\text{m}}^{N-\text{DR}} = \left( 4c^2 + 4 \epsilon^2 - \beta^2 \left[ \lambda^2 \varphi t^2 - 2 \lambda (t + \varphi) + \epsilon (2 - \lambda \varphi)^2 \right] + \beta \left[ 2 - \lambda t \right] + \epsilon (12 - 2 \lambda \varphi - \lambda t (2 - \lambda \varphi)) \right) 
- c \cdot \left( 8 \epsilon + \varphi \left[ 12 - 2 \lambda \varphi - \lambda t (2 - \lambda \varphi) - \beta (2 - \lambda \varphi)^2 \right] \right) \cdot \left( \beta \left[ 16 - \beta (2 - \lambda \varphi)^2 \right] \right)^{-1}.
\]

(A.21)

A.4. Product’s Optimal Equilibrium Solutions with S-DR Strategy. We will fourth solve the manufacturer’s maximum profit under the S-DR strategy. First, the manufacturer sets the products’ price, \( p_{\text{new}}, p_{\text{old}}, \) and \( p_{\text{renting}} \). Second, the owner who previously purchased the product sets the idle product’s price \( p_{\text{idle}} \). We use a superscript S-DR to denote the equilibrium results under this scenario. The solution process is as follows.
Then, all owners’ earnings in the sharing market can be written as

\[
\pi_O = \begin{cases} 
(p_{idle} - t) \left( \frac{P_{renting}}{1 - \phi\beta} - \frac{p_{idle}}{\phi\beta} \right), & \text{if } t < p_{idle}, \\
0, & \text{if } t \geq p_{idle}.
\end{cases}
\] (A.22)

From the first-order optimality conditions, we get

\[
P_{idle} = \frac{t(1 - \phi\beta) + \phi\beta P_{renting}}{2(1 - \phi\beta)}.
\] (A.23)

Substituting (A.23) in the manufacturer’s profit function, we get

\[
\pi_M = \begin{cases} 
(p_{new} - c)[1 - P_{new} + \epsilon + \lambda(p_{idle} - t)] + (p_{old} - c)[P_{new} - \epsilon - \lambda(p_{idle} - t) - \frac{P_{old} - \epsilon}{\beta}] + (P_{renting} - r) \left[ \frac{P_{old} - \epsilon}{\beta} - \frac{P_{renting}}{1 - \phi\beta} \right], & \text{if } t < p_{idle}, \\
(p_{new} - c)(1 - P_{new} - \epsilon) + (p_{old} - c)(P_{new} - \epsilon - \frac{P_{old} - \epsilon}{\beta}) + (P_{renting} - r) \left[ \frac{P_{old} - \epsilon}{\beta} - \frac{P_{renting}}{1 - \phi\beta} \right], & \text{if } t \geq p_{idle}.
\end{cases}
\] (A.24)

Replacing (A.23) into equation (A.24) and letting \((\partial\pi_M/\partial P_{new}) = 0, (\partial\pi_M/\partial P_{old}) = 0, \) and \((\partial\pi_M/\partial P_{renting}) = 0, \) we get

\[
P_{new}^{S-DR} = (4 + 8\epsilon - 2\lambda t + \beta[4(-4 - 2c + r + 2\epsilon(-3 + \beta) - \lambda t(-2 + \beta)] + [-4\phi[2 + 4\epsilon + \beta(-4 - 2c + r + 2\epsilon(-3 + \beta))]
\]

\[+ 2\lambda[-c + (r + 2t)(1 - \beta)^2 + \epsilon - 2\beta(1 + \epsilon) + \lambda^2t\beta)] + \beta \phi^2 [(2 + \lambda\beta)2 + \lambda(c - r - t + \beta)
\]

\[+ \epsilon[8 + \lambda(-2 + 4\beta + \lambda\beta)]]]) \cdot (8 + 2\beta[-8(2 + \phi) + \phi\beta[4 - 2(-8 + 2\beta + \lambda) + \phi^2(4 + 2\lambda\beta + 2\lambda^2\beta)]])^{-1},
\] (A.25)

\[
P_{old}^{S-DR} = (8\epsilon + \beta[-4[4c - 2r + 4\epsilon + \beta(2 - 2\epsilon + \lambda t)] + \phi[8\beta(2c - r + \beta) - 8\epsilon(2 - 2\beta + \lambda^2\beta)\lambda t + 2\lambda\beta(-1 + r\beta + 2t\beta - 2\epsilon)
\]

\[+ \lambda^2t\beta] + \phi^2\beta[\lambda\beta[2 + \lambda(c - r - t + \beta)] + \epsilon[8 + \lambda\beta(4 + \lambda)]])
\]

\[\cdot (8 + 2\beta[-8(2 + \phi) + \phi\beta[4 - 2(-8 + 2\beta + \lambda) + \phi^2(4 + 2\lambda\beta + 2\lambda^2\beta)]])^{-1},
\] (A.26)

\[
P_{rent}^{S-DR} = \frac{(1 - \phi\beta)[-4c + 2r(2 - 4\beta + \beta^2) + 4\epsilon - 2\beta + \lambda t \beta + \phi\beta(-4\epsilon + 4c\lambda \beta - r(4 + \lambda\beta)) + \beta[2 + \lambda(-2 + t + \beta - et\lambda)]]}{4 + \beta[-8(2 + \phi) + \phi\beta[4 - 2\phi(-8 + 2\beta + \lambda) + \phi^2(4 + 2\lambda\beta + 2\lambda^2\beta)]]},
\] (A.27)

Substituting (A.27) in equation (A.23), we get

\[
P_{idle}^{S-DR} = \left( \phi\beta[4c + 2\beta + 2r(2 - 4\beta + \beta^2) + 4\epsilon + \phi\beta[2(2c - 2r + \beta - 2\eta) + \lambda \beta(-2 + c - r + \beta - \epsilon)]
\]

\[+ t \left[ 4 + \beta[-8(2 + \phi) + \phi(16 - 3 + 4\phi + \phi\beta(-4 + 3\lambda + 6\lambda^2))] \right] \right),
\] (A.28)

Furthermore, we obtain
Proof of Proposition 1.

This appendix contains all proofs of the propositions in the paper.

**Proof of Proposition 1.** If the manufacturer switches from the N-DR strategy to the S-DR strategy, the equilibrium solutions for all products’ price and demand are nonnegative. As shown in Appendices A.3 and A.4, we just analyse under what conditions $d^S_{\text{old}}>0$ and $d^S_{\text{renting}}>0$. We analyse the case that there exists $c_1$ such that $d^S_{\text{new}}<0$ when $c<c_1$, where

\[
c_1 (r) = (2 - 8 + 4\lambda + \beta\lambda (2 + \lambda)^2 - \beta(8 + 4\lambda - 2\lambda\phi + e(4 - 4\lambda + (4 + 4\lambda)\phi) + t\lambda (4 + (4 + 4\lambda)\phi)
\]
\[+ r(-8 + \beta(-4 - 4\lambda\phi + \beta^2\lambda(4 - 4\lambda + (4 + 4\lambda)\phi))\beta^2\phi(t\lambda^3\phi + 4 + 4\lambda + e(-8 + 2\lambda\phi + \lambda^2\phi))\]
\[+ 2\lambda(-1 - \phi + t(2 + \phi))) \cdot (-16 + \beta(-8 - 2\lambda\phi + 4 + 2\lambda\phi + \lambda^2\phi))^{-1}.
\]

There exists $r_1$ such that $d^S_{\text{renting}}>0$ when $r<r_1$, where

\[
 r_1 = (8\epsilon + 4 + 4\lambda + \beta^2\lambda(2 + \lambda)^2 - \beta(8 + 8\lambda - 2\lambda\phi + 8\epsilon(4 - 4\lambda + (4 + 4\lambda)\phi) + t\lambda (4 + (4 + 4\lambda)\phi)
\]
\[+ \beta^2\phi(4 + 2\lambda(2 + t + \epsilon + \phi) - \lambda^2(4\phi + t(2 + \phi))) \cdot (-16 + \beta(-2 - 2\lambda\phi + 4 + 2\lambda\phi + \lambda^2\phi))^{-1}.
\]
Yet \( r < c \) and \( r_1 > c \). Letting \( c_1 = r \), we obtain there exists \( c_2 \) such that \( d_{S-\text{old}}^{SR} > 0 \) when \( c < c_2 \), where

\[
c_2 = \frac{(-2(2 - 8e + t\lambda) + \beta(8 - 8e + 4\pi + \phi(8 + 2e(8 + 2e - 8 + \lambda) + 2\beta(-4 + 4e + \lambda - 2t\lambda) + \lambda(-2 + t(4 + \lambda)))
- \beta\varphi^2(4 + 2\lambda(-1 + t + \beta + e) + \lambda^2(t + \beta + e)))}{0.8} \cdot (8 - 4\beta + 2\beta\varphi[-4 + \beta(2 + \lambda)])^{-1}.
\] (B.3)

Meanwhile, we easily get \( \pi_{M - S}^{S-\text{DR}} - \pi_{M}^{N-\text{DR}} > 0 \) in the case where \( \pi_{M}^{S-\text{DR}} > \pi_{M}^{N-\text{DR}} > 0 \) when \( c < c_2 \) and \( r < c \).

Proof of Proposition 2. If the manufacturer switches from the N-SR strategy to the S-SR strategy, the equilibrium solutions for all products’ price and demand are nonnegative. As shown in Appendices A.1 and A.2, we just analyse under what conditions \( d_{S-\text{new}}^{SR} > 0 \) and \( d_{S-\text{renew}}^{SR} > 0 \). We analyse the case \( d_{S-\text{new}}^{SR} > 0 \). There exists \( r_2 \) such that \( d_{S-\text{new}}^{SR} > 0 \) when \( r > r_2 \), where

\[
r_2 = \frac{(-1 + \beta\varphi)(2 - 4c + 4e - 2t\lambda + \beta(2 + \lambda)\varphi)}{2 + \beta(-2 + \lambda)\varphi}.
\] (B.4)

There exists \( r_3 \) such that \( d_{S-\text{renew}}^{SR} > 0 \) when \( r < r_3 \), where

\[
r_3 = \frac{4 + 4c - 4e + 2t\lambda - 4\beta\varphi + \beta(-2 + \lambda)(2c - 2e + t\lambda)\varphi - \beta^2\lambda(2 + \lambda)\varphi^2}{8 + 4\beta\lambda\varphi}.
\] (B.5)

\[
A_1 = 2\left[12 + 4\beta\varphi(-2 + \lambda) - \beta^2\varphi^2(2 + \lambda)^2\right],
\]
\[
A_2 = 4 - 8\beta(2 + \varphi) + \beta^3\varphi(2\lambda\varphi + \lambda^2\varphi - 4) + 2\beta^2\left[2 - \varphi(\lambda - 8) + 2\varphi^2\right],
\]
\[
A_3 = \beta\varphi\left[4(2c + \beta - 2\varphi + \beta\lambda\varphi) + t\lambda\left(2 - 2\beta + t\lambda\right)\right] + \lambda^2\varphi^2\left[2 + c\lambda - (r + t - \beta + e)\right] - 2\left[4(2 - \beta + 2\beta\varphi + t\lambda)\right],
\]
\[
A_4 = 8 - 17r - 2e + 4t\lambda - 4\beta\varphi[4 + e(-4 + \lambda) + 2r(-2 + \lambda) + t\lambda(2 + \lambda)] + \beta^2\varphi^2\left[8 + \lambda\left(-8 + 4t + 2\lambda - 3t\lambda^2 + 16r + 4r\lambda\right)
+ 2e(-4 + 4\lambda + \lambda^2)\right] + \beta\lambda\varphi^3\left[8 - 4e + 4\lambda\left(r + t\right) + \lambda^2\left(2\left(1 + r + t\right) + e\right)\right] - c\left[2 + \beta\varphi(-2 + \lambda)\right][2 + \beta\lambda\varphi](-2 + \beta\varphi(2 + \lambda)].
\]
\[
A_5 = \left\{\lambda\beta\varphi^2(\lambda\varphi - 2) - 2(\lambda\varphi + 2) + 2\beta\varphi[2 + \lambda(2 + \lambda)]\right\} + \left\{8e + \beta^3\lambda\varphi(\lambda\varphi - 2) + c\left[8\beta\varphi - 8 + 3\lambda\varphi(\lambda\varphi - 2)\right]
+ \beta\left[16r + t\lambda(\lambda\varphi - 2) - 2(2 + 4e\varphi + \lambda\varphi)\right] - \beta^2\left[r(\lambda\varphi - 2)^2 + \varphi(-4 - 2\lambda(2 + t + e + \varphi) + \lambda^2(e\varphi + 2t + t\varphi))\right]\right\},
\]
\[
A_6 = 8e + \beta\left[-4(4c - 2r + 4e + \beta(2 - 2e + t\lambda))\right] + \left[8\beta\varphi(2c - r - \beta) - 8e(2 + t\beta\varphi(\beta - 2)) + 2\beta\varphi(2t\beta - 2e - 1) + t\beta\lambda^2\right]
+ \beta\varphi^2\left[\beta\lambda(2 + \lambda(c - r - t + \beta)) + e(8 + \beta\lambda(4 + \lambda))\right],
\]
\[
A_7 = \left\{\lambda\beta\varphi^2(\lambda\varphi - 2) - 2(\lambda\varphi + 2) + 2\beta\varphi[2 + \lambda(2 + \lambda)]\right\} + \left\{8e + \beta^3\lambda\varphi(\lambda\varphi - 2) + c\left[8\beta\varphi - 8 + 3\lambda\varphi(\lambda\varphi - 2)\right]
+ \beta\left[16r + t\lambda(\lambda\varphi - 2) - 2(2 + 4e\varphi + \lambda\varphi)\right] - \beta^2\left[r(\lambda\varphi - 2)^2 + \varphi(-4 - 2\lambda(2 + t + e + \varphi) + \lambda^2(e\varphi + 2t + t\varphi))\right]\right\},
\]
\[
A_8 = 8e + \beta\left[-4(4c - 2r + 4e + \beta(2 - 2e + t\lambda))\right] + \left[8\beta\varphi(2c - r - \beta) - 8e(2 + t\beta\varphi(\beta - 2)) + 2\beta\varphi(2t\beta - 2e - 1) + t\beta\lambda^2\right]
+ \beta\varphi^2\left[\beta\lambda(2 + \lambda(c - r - t + \beta)) + e(8 + \beta\lambda(4 + \lambda))\right],
\]
\[
A_9 = \left\{\lambda\beta\varphi^2(\lambda\varphi - 2) - 2(\lambda\varphi + 2) + 2\beta\varphi[2 + \lambda(2 + \lambda)]\right\} + \left\{8e + \beta^3\lambda\varphi(\lambda\varphi - 2) + c\left[8\beta\varphi - 8 + 3\lambda\varphi(\lambda\varphi - 2)\right]
+ \beta\left[16r + t\lambda(\lambda\varphi - 2) - 2(2 + 4e\varphi + \lambda\varphi)\right] - \beta^2\left[r(\lambda\varphi - 2)^2 + \varphi(-4 - 2\lambda(2 + t + e + \varphi) + \lambda^2(e\varphi + 2t + t\varphi))\right]\right\},
\]
\[
A_{10} = 8e + \beta\left[-4(4c - 2r + 4e + \beta(2 - 2e + t\lambda))\right] + \left[8\beta\varphi(2c - r - \beta) - 8e(2 + t\beta\varphi(\beta - 2)) + 2\beta\varphi(2t\beta - 2e - 1) + t\beta\lambda^2\right]
+ \beta\varphi^2\left[\beta\lambda(2 + \lambda(c - r - t + \beta)) + e(8 + \beta\lambda(4 + \lambda))\right],
\]
\[
A_{11} = \left\{\lambda\beta\varphi^2(\lambda\varphi - 2) - 2(\lambda\varphi + 2) + 2\beta\varphi[2 + \lambda(2 + \lambda)]\right\} + \left\{8e + \beta^3\lambda\varphi(\lambda\varphi - 2) + c\left[8\beta\varphi - 8 + 3\lambda\varphi(\lambda\varphi - 2)\right]
+ \beta\left[16r + t\lambda(\lambda\varphi - 2) - 2(2 + 4e\varphi + \lambda\varphi)\right] - \beta^2\left[r(\lambda\varphi - 2)^2 + \varphi(-4 - 2\lambda(2 + t + e + \varphi) + \lambda^2(e\varphi + 2t + t\varphi))\right]\right\},
\]
\[
A_{12} = 8e + \beta\left[-4(4c - 2r + 4e + \beta(2 - 2e + t\lambda))\right] + \left[8\beta\varphi(2c - r - \beta) - 8e(2 + t\beta\varphi(\beta - 2)) + 2\beta\varphi(2t\beta - 2e - 1) + t\beta\lambda^2\right]
+ \beta\varphi^2\left[\beta\lambda(2 + \lambda(c - r - t + \beta)) + e(8 + \beta\lambda(4 + \lambda))\right].
\]
\[ A_7 = \beta[4 + \varphi(8 - 2\lambda + \beta(-4 + \lambda(2 + 2\varphi + \lambda\varphi)))] - 8, \]
\[ A_8 = [4(2 + 2c - 4r - 2e + t\lambda) + 4\beta\left[4(r + e - c - 1) + \lambda(2 + 3c - 6r - 2t - 3\epsilon) + 3t\lambda^2\right] + \beta^2\varphi^2\left[8(1 - c + e) + 4\lambda(4r - 4c + t + 4e - 4) - 2\lambda^2(1 + c + 4r + 6t - e) + t\lambda^3\right] + \lambda\beta^2\varphi^2(2 + \lambda)(4 - 2\lambda + 2c + \lambdac - 2e - 2t)], \]
\[ A_9 = \left\{4(2 + 2c - 4r - 2e + t\lambda)^2 + 4\beta^2\varphi(-8(1 + c - e)(1 + c - 2r - e) + 4\lambda(2c^2 + 8r^2 - (1 - 2r - 2c) + 2r(t + 4e - 3) - c(8r + 2t + 4e - 3)) - 2\lambda^2(8r + t + 7e - 7c) + 5r^2\lambda^3\right\} + \beta^2\varphi^2\left[16(1 + c - e)^2 - 16\lambda[3c^2 - (1 - e)(t + 3e - 3) - r(8 - 6e - c(6r + t + 6e - 6))] + 4\lambda\left[1 + 2c^2 + 20r^2 - t(8 - t) - 2e + 16t - 2\lambda^2 - 2c(10r + 8t + 2e - 1) + 4r(-1 + 3t + 5e)\right] - 4t\lambda^3(7 - 7c + 10r + 5t + 7e) + 9t^2\lambda^4\right\} + 2\lambda\beta^3\varphi(8(1 + c - e)(2 + c - e) - 4\lambda^2 - 4t - 2c + 4t - 6e + \epsilon(t + e - 2r(5 - 2e) + c(4 - 4r - t - 2e))\lambda^2(2(1 + 2r)(1 + r + t) - c(3 + 4r + 6t) + \epsilon(3 + 4r + 6t)) - (2 - c + 4r + e - 2e)^2\lambda^3(6 - c + 4r + 4t + e)^2\right\} + \beta^4\lambda^3\varphi^3\left[4(2 + c - e)^2 + 4\lambda\left[4(r + t) + (c - e)^2\right] + \lambda^2\left[c^2 + 8r + 8t - 2(c + e) + (2 + e)^2\right]\right\}, \]

where \(A_1 > 0, A_2 > 0, A_3 < 0, A_4 > 0, A_5 > 0, A_6 < 0, A_7 < 0, \) and \(A_8 > 0.\)

Let \(A_9 > 0,\) and \(A_9 > 0.\)

\[ B_1 = [2\beta(2 - \lambda\varphi)^2 - 16\left\{4 + \beta\left[-8(2 + \varphi) + \beta\left(4 - 2\varphi(2\beta + \lambda - 8) + (4 + 2\beta\lambda\varphi + \beta\lambda^2\varphi^2)\right)\right]\right\}, \]
\[ B_2 = \beta(8 - 2\beta + 4\varphi + \beta\lambda\varphi - 4\beta\varphi\left[4(2c - \beta - 2c) + 2\lambda\left(2\beta - 1 - \beta c + 2r\beta + \lambda t - \beta^2 + \beta c\right)\right]\right\} + \beta^2\lambda\varphi^2\left[2c + \lambda - \lambda(r + t - \beta - e) - 2\lambda\epsilon - 4r + 2\lambda - 2\lambda t\right], \]
\[ B_3 = 4(2 - \lambda\varphi)(-2\lambda + \beta\varphi^2(2 + \lambda)^2), \]
\[ B_4 = 4(2 + 2c - 4r - 2e + 5t\lambda) - 4\beta^2\varphi\left[8(1 + \epsilon - 2\lambda) - 2c(-2 + \lambda)(2 + 3\lambda) + \lambda(8e + 2\lambda - 6e\lambda - 4r(2 + \lambda) + t(2 + \lambda)(2 + 3\lambda))\right] - \beta^3\lambda^2\varphi^3(2 + c - e)(2 + \lambda)^2, \]
\[ B_5 = 4(1 + \beta\varphi)(2 + \beta\varphi[8 + 4\lambda + \beta\varphi(2 + \lambda)^2]), \]
\[ B_6 = 8 - 8\epsilon + 28t\lambda - 4\beta\lambda\varphi e + 4r - 7\lambda) - \beta^3\varphi^4\left[8 + 14\lambda^2 + 2c(2 + \lambda)(-2 + 3\lambda) - 4r(1 + \lambda\beta\varphi)(2 + 2\beta\varphi + \lambda\beta\varphi) + c(2 + \beta\lambda\varphi)\right] + \beta^4\varphi^3(2 + \lambda)^2 + 2\lambda(2 + \lambda\varphi)(-2 + 3\lambda), \]

where \(B_1 > 0, B_2 < 0, B_3 < 0, B_4 > 0, B_5 < 0,\) and \(B_6 > 0.\)

We obtain
\[
\frac{\pi_{SR} - \pi_{DR}^N}{\Delta_{SR} - \Delta_{DR}^N} = \left\{ \frac{A_4}{(2 + \beta\lambda\varphi)A_1} > 0, \right\}
\]
\[
\frac{\pi_{SR} - \pi_{DR}^N}{\Delta_{SR}^S - \Delta_{DR}^N} = \frac{A_5}{B_1} > 0, \]
\[
\frac{A_8}{2\beta B_1} > 0, \]
\[
\frac{\Delta_{SR}^S - \Delta_{DR}^N}{\Delta_{SR}^S - \Delta_{DR}^N} = \frac{2A_3[2 - \lambda\varphi + \beta\varphi(2\lambda + \lambda\varphi - 2)]}{B_1} < 0, \]
\[
\frac{\pi_{SR} - \pi_{DR}^N}{\Delta_{SR}^S - \Delta_{DR}^N} = \frac{A_8}{4(1 + \beta\lambda\varphi)(2 + \beta\lambda\varphi)A_1} > 0. \]

We obtain
\[ P_{\text{idle}}^{S-DR} - P_{\text{idle}}^{N-DR} = \frac{\varphi B_2}{B_1} < 0, \]
\[ P_{\text{idle}}^{N-DR} - P_{\text{idle}}^{N-SR} = \frac{\varphi B_2}{B_3} < 0, \]  \hspace{1cm} (B.10)
\[ d_{\text{idle}}^{S-DR} - d_{\text{idle}}^{N-DR} = \frac{B_2}{B_1} < 0, \]
\[ d_{S}^{S-SR} - d_{S}^{N-SR} = \frac{B_2}{B_3} < 0. \]

Proof finished.

Data Availability

All data and models generated or analysed during this study are included in this submitted article (and its appendices).

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors’ Contributions

Jian Feng conceptualized the study, performed formal analysis, provided methodology, collected resources, and wrote the original draft. Bin Liu administrated and supervised the project. Jian Feng and Zhenfeng Liu reviewed and edited the manuscript.

Acknowledgments

The authors gratefully acknowledge the support from the Innovation Method Fund of China (Project no. 2018IM020300) and National Natural Science Foundation of China (Project no. 71971134).

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